Quarkonium rapidity distributions via matching of NLO and High-Energy Factorisation

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Fixed-target experinemts at the LHC
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Outline

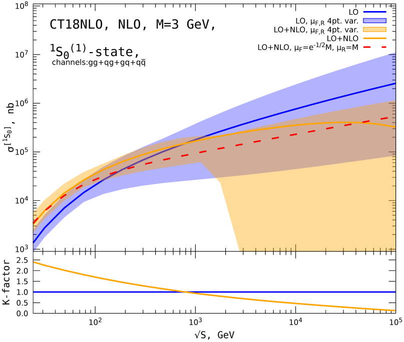
1. Reminder: energy-dependence of η_c total cross section in HEF
2. Rapidity-differential cross section of η_c-production

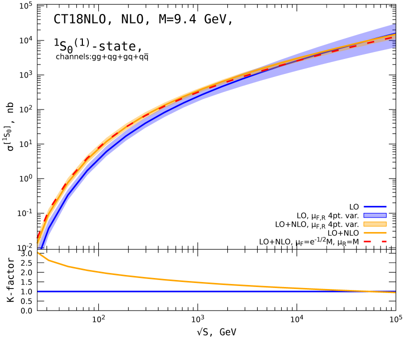
Perturbative instability of the η_c total cross section

For the p_T**-integrated** cross section of η_Q hadroproduction, the LO partonic subprocess is simply:

$$g + g \to Q\bar{Q}\left[{}^1S_0^{(1)}\right].$$

The NLO correction can be computed in closed form [Kuhn, Mirkes, 93'; Petrelli *et.al.*, 98'], and:





Why?

Collinear factorization for total CS for the state $m = ^{2S+1}L_J^{(0)}$:

$$\sigma^{[m]}(\sqrt{S}) = \int\limits_{z_{\min}}^{1} \frac{dz}{z} \mathcal{L}_{ij}(z, \mu_F) \hat{\sigma}_{ij}^{[m]}(z, \mu_F, \mu_R),$$

where $i, j = q, \bar{q}, g$, $\boxed{z = M^2/\hat{s}}$ and *partonic luminosity*:

$$\mathcal{L}_{ij}(z, \mu_F) = \int\limits_{-y_{\max}}^{+y_{\max}} dy\ \tilde{f}_i\left(\frac{M}{\sqrt{Sz}}e^{y}, \mu_F\right) \tilde{f}_j\left(\frac{M}{\sqrt{Sz}}e^{-y}, \mu_F\right),$$

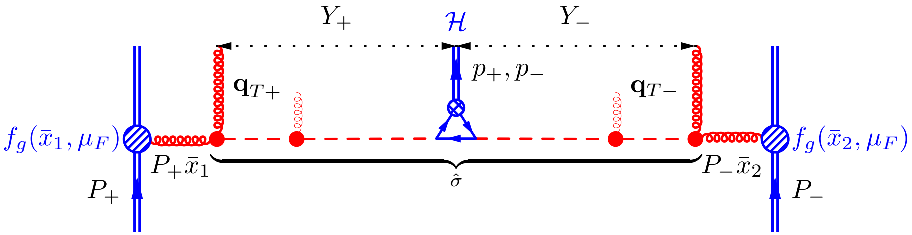
with $\tilde{f}_j(x, \mu_F)$ – momentum density PDFs.
NLO coefficient function [Kuhn, Mirkes, 93'; Petrelli *et.al.*, 98'] in the $z \to 0$ limit

$$\hat{\sigma}_{ij}^{[m]} = \sigma_{\rm LO}^{[m]} \left[A_0^{[m]} \delta(1-z) + C_{ij} \frac{\alpha_s(\mu_R)}{\pi} \left(A_0^{[m]} \ln \frac{M^2}{\mu_F^2} + A_1^{[m]} \right) + O(z\alpha_s, \alpha_s^2) \right],$$

where $C_{gg} = 2C_A = 2N_c$, $C_{qg} = C_{gq} = C_F = (N_c^2 - 1)/(2N_c)$, $C_{q\bar{q}} = 0$ and $A_1^{[m]} < 0$.

High-Energy factorization, the picture

High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']:



Small parameter $\boxed{z = \frac{M^2}{\hat{s}} = \frac{M^2}{M_T^2} z_+ z_-,}$ where $M_T^2 = M^2 + \mathbf{p}_T^2$ and

$$z_+ = \frac{p_+}{P_+\bar{x}_1},\ z_- = \frac{p_-}{P_-\bar{x}_2}$$

Using the BFKL formalism one resums corrections to $\hat{\sigma}$ enhanced by

$$Y_\pm \sim \ln \frac{1}{z_\pm},\ \text{in LP w.r.t. } z_\pm.$$

Resumed coefficient function

Small parameter: $\boxed{z = \frac{M^2}{\hat{s}}.}$

LLA ($\alpha_s^n \ln^{n-1} \frac{1}{z}$) in High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']:

$$\hat{\sigma}_{ij}^{[m],\ \mathrm{HEF}}(z, \mu_F, \mu_R) = \int\limits_{-\infty}^{\infty} d\eta \int\limits_{0}^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2\ \mathcal{C}_{gi}\left(\frac{M_T}{M}\sqrt{z}e^{\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right)$$

$$\times \mathcal{C}_{gj}\left(\frac{M_T}{M}\sqrt{z}e^{-\eta}, \mathbf{q}_{T2}^2, \mu_F, \mu_R\right) \int\limits_{0}^{2\pi} \frac{d\phi}{2} \frac{H^{[m]}(\mathbf{q}_{T1}^2, \mathbf{q}_{T2}^2, \phi)}{M_T^4} + O(z) + \mathrm{NLL},$$

The coefficient functions $H^{[m]}$ are known at LO in α_s [Hagler *et.al*, 2000; Kniehl, Vasin, Saleev 2006] for $m = {}^1S_0^{(1,8)}, {}^3P_J^{(1,8)}, {}^3S_1^{(8)}$.
The $H^{[m]}$ is a tree-level "squared matrix element" of the $2 \to 1$-type process:

$$R_+(\mathbf{q}_{T1}, q_1^+) + R_-(\mathbf{q}_{T2}, q_2^-) \to c\bar{c}[m].$$

LLA evolution w.r.t. $\ln 1/z$

In the LL($\ln 1/z$)-approximation, the $Y = \ln 1/z$-evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$-factor has the form:

$$\tilde{\mathcal{C}}(x, \mathbf{q}_T) = \delta(1-z)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int_x^1 \frac{dz}{z} \int d^{2-2\epsilon}\mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2) \tilde{\mathcal{C}}\left(\frac{x}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A/\pi$ and

$$K(\mathbf{k}_T^2, \mathbf{p}_T^2) = \delta^{(2-2\epsilon)}(\mathbf{k}_T) \, \frac{(\mathbf{p}_T^2)^{-\epsilon}}{\epsilon} \frac{(4\pi)^\epsilon \Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} + \frac{1}{\pi(2\pi)^{-2\epsilon}\mathbf{k}_T^2}.$$

It is convenient to go from $(z, \mathbf{q}_T)$-space to $(N, \mathbf{x}_T)$-space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon}\mathbf{q}_T \; e^{i\mathbf{x}_T \mathbf{q}_T} \int_0^1 dx \; x^{N-1} \; \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

- Mellin convolutions over z turn into products: $\int \frac{dz}{z} \to \frac{1}{N}$
- Large logs map to poles at $N = 0$: $\boxed{\alpha_s^{k+1} \ln^{k} \frac{1}{z} \to \frac{\alpha_s^{k+1}}{N^{k+1}}}$
- All *collinear divergences* are contained inside $\mathcal{C}$ in $\mathbf{x}_T$-space.

Exact LL solution

In $(N, \mathbf{q}_T)$-space, subtracted $\mathcal{C}$, which resums all terms $\propto (\hat{\alpha}_s/N)^n$ has the form:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N, \alpha_s)) \frac{\gamma_{gg}(N, \alpha_s)}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}(N, \alpha_s)},$$

where $\gamma_{gg}(N, \alpha_s)$ is the solution of [Jaroszewicz, 82']:

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}(N, \alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma),$$

where $\psi(\gamma) = d \ln \Gamma(\gamma)/d\gamma$ – Euler's ψ-function. The first few terms:

$$\gamma_{gg}(N, \alpha_s) = \underbrace{\underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA}} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \ldots}_{\text{LLA}}$$

The function $R(\gamma)$ is

$$R(\gamma_{gg}(N, \alpha_s)) = 1 + O(\alpha_s^3).$$

Does this work?

The resummation has to reproduce the $A_1^{[m]}$ NLO coefficient when expanded up to NLO in α_s. And it does. We have performed expansion up to NNLO:

State	$A_0^{[m]}$	$A_1^{[m]}$	$A_2^{[m]}$	$B_2^{[m]}$
${}^1S_0$	1	$\mathbf{-1}$	$\frac{\pi^2}{6}$	$\frac{\pi^2}{6}$
${}^3S_1$	0	1	0	$\frac{\pi^2}{6}$
${}^3P_0$	1	$\mathbf{-\frac{43}{27}}$	$\frac{\pi^2}{6}+\frac{2}{3}$	$\frac{\pi^2}{6}+\frac{40}{27}$
${}^3P_1$	0	$\frac{5}{54}$	$-\frac{1}{9}$	$-\frac{2}{9}$
${}^3P_2$	1	$\mathbf{-\frac{53}{36}}$	$\frac{\pi^2}{6}+\frac{1}{2}$	$\frac{\pi^2}{6}+\frac{11}{9}$

for e.g.

$$\hat{\sigma}_{gg}^{[m],\ \mathrm{HEF}}(z\to 0) = \sigma_{\mathrm{LO}}^{[m]}\left\{A_0^{[m]}\delta(1-z) + \frac{\alpha_s}{\pi}2C_A\left[A_1^{[m]} + A_0^{[m]}\ln\frac{M^2}{\mu_F^2}\right]\right.$$
$$\left. + \left(\frac{\alpha_s}{\pi}\right)^2 \ln\frac{1}{z}\cdot C_A^2\left[2A_2^{[m]} + B_2^{[m]} + 4A_1^{[m]}\ln\frac{M^2}{\mu_F^2} + 2A_0^{[m]}\ln^2\frac{M^2}{\mu_F^2}\right] + O(\alpha_s^3)\right\},$$

Matching with NLO of CF

The HEF works only at $z \ll 1$, misses power corrections $O(z)$, while NLO CF is exact in z, but only NLO in α_s. **We need to match them.**

- Simplest prescription: just subtract the overlap at $z \ll 1$:

$$\sigma^{[m]}_{\rm NLO+HEF} = \sigma^{[m]}_{\rm LO\ CF} + \int\limits_{z_{\rm min}}^{1} \frac{dz}{z} \left[\check{\sigma}^{[m],ij}_{\rm HEF}(z) + \hat{\sigma}^{[m],ij}_{\rm NLO\ CF}(z) - \hat{\sigma}^{[m],ij}_{\rm NLO\ CF}(0)\right] \mathcal{L}_{ij}(z),$$

- Or introduce **smooth weights**:

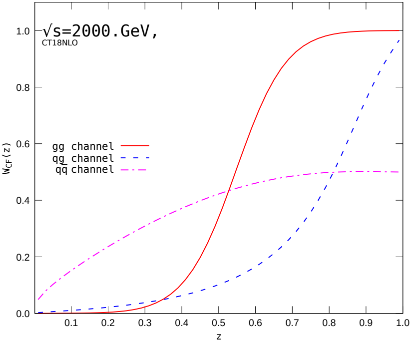
$$\sigma^{[m]}_{\rm NLO+HEF} = \sigma^{[m]}_{\rm LO\ CF} + \int\limits_{z_{\rm min}}^{1} dz \left\{ \left[\check{\sigma}^{[m],ij}_{\rm HEF}(z) \frac{\mathcal{L}_{ij}(z)}{z}\right] w^{ij}_{\rm HEF}(z) + \left[\hat{\sigma}^{[m],ij}_{\rm NLO\ CF}(z) \frac{\mathcal{L}_{ij}(z)}{z}\right] (1 - w^{ij}_{\rm HEF}(z)) \right\},$$

Inverse error weighting method

In the InEW method [Echevarria, *et.al.*, 2018] the weights are calculated from **estimates of the error** of each contribution:

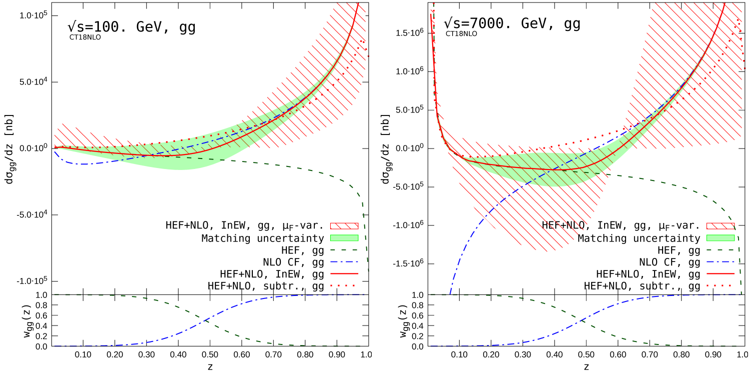
$$w^{ij}_{\rm HEF}(z) = \frac{[\Delta\sigma^{ij}_{\rm HEF}(z)]^{-2}}{[\Delta\sigma^{ij}_{\rm HEF}(z)]^{-2} + [\Delta\sigma^{ij}_{\rm CF}(z)]^{-2}},$$

- For $\Delta\sigma_{\rm CF}$ we take the NNLO $\alpha_s^2 \ln \frac{1}{z}$ term of $\hat{\sigma}(z)$ predicted by HEF,
- For $\Delta\sigma_{\rm HEF}$ we take the $\alpha_s O(z)$ part of the NLO CF result for $\hat{\sigma}(z)$.
- In both cases, stability against $O(\alpha_s^2)$ (constant in z, unknown) corrections is checked

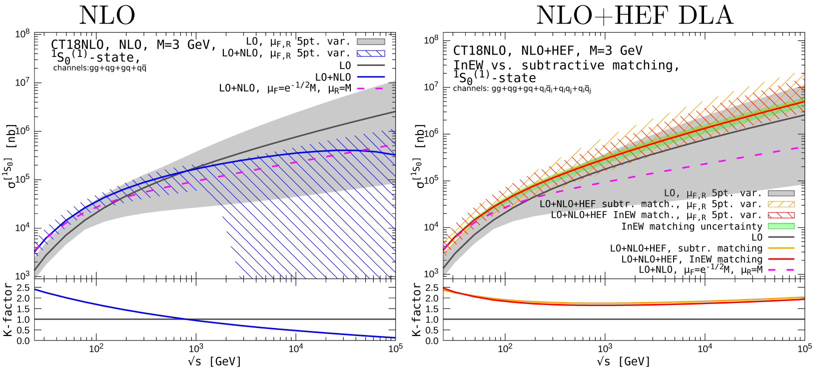


Matching plots

Plots of the integrand of the total cross section (gg channel) as function of $z = M^2/\hat{s}$:



Matched results for η_c



Uniformly-accurate predictions over the wide range of energies are achieved.

Rapidity-differential cross section at NLO

Convenient kinematic variables:

$$z_+ = \frac{p^+}{q_1^+}, \quad z_- = \frac{p^-}{q_2^-},$$

the NLO cross section is given by:

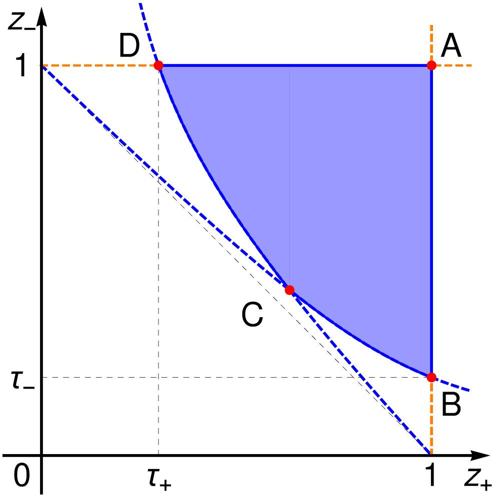
$$\frac{d\sigma_{\rm NLO}}{dy} = \frac{\alpha_s}{4\pi}\hat{\sigma}_{\rm LO}\int\limits_0^1 \frac{dz_+}{z_+}\tilde{f}(x_1)\int\limits_0^1 \frac{dz_-}{z_-}\tilde{f}(x_2)I_{\rm NLO}(z_+, z_-),$$

with $x_{1,2}$ expressed through $z_\pm$ and

$$\begin{aligned}
I_{\rm NLO}^{{}^1S_0^{[1]}}(z_+, z_-) \quad &= \quad \delta(z_+ - 1)\delta(z_- - 1)\left[\left(\pi^2 - 20\right)C_F + \left(\pi^2 + 4\right)C_A + 2\beta_0 \ln\frac{\mu_R^2}{\mu_F^2}\right] \\
&- \quad 4C_A \frac{(1 - z_-(1 - z_-))^2}{(1 - z_-)_+}\ln\frac{z_-\mu_F^2}{M^2}\delta(z_+ - 1) + (z_+ \leftrightarrow z_-) \\
&+ \quad 4C_A\left(\frac{\ln(1 - z_-)}{(1 - z_-)}\right)_+ (1 - z_-(1 - z_-))^2\delta(z_+ - 1) + (z_+ \leftrightarrow z_-) \\
&+ \quad \frac{4C_A}{(1 - z_+)_+(1 - z_-)_+}\frac{(z_+ + z_- - 1)(z_+z_- - 1)^2}{z_+z_-(z_+ + z_- - 2)^2}\left[(1 + a + a^2)^2 + b^2 - 2a^2 b\right].
\end{aligned}$$

with $a = z_+ + z_- - 2$, $b = (1 - z_+)(1 - z_-)$.

NLO phase-space



$$z_+ = \frac{p^+}{q_1^+}, \quad z_- = \frac{p^-}{q_2^-},$$

NLO:

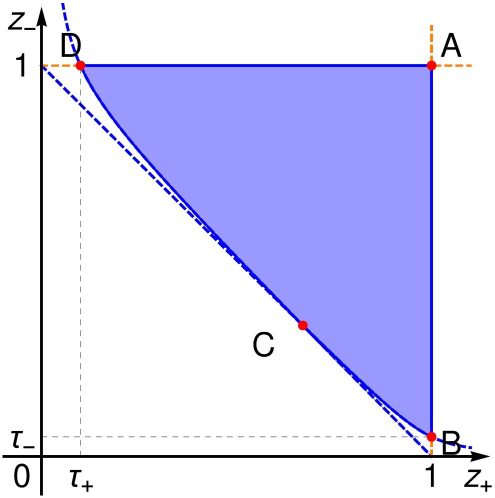
$$g+g \to c\bar{c}\left[{}^1S_0^{(1)}\right]+g,$$

$$x_1 = \tau_+\sqrt{\frac{z_-}{z_+(z_+ + z_- - 1)}},$$

$$x_2 = \tau_-\sqrt{\frac{z_+}{z_-(z_+ + z_- - 1)}},$$

with
$\tau_\pm = Me^{\pm y}/\sqrt{S}$.

NLO phase-space



$$z_+ = \frac{p^+}{q_1^+}, \quad z_- = \frac{p^-}{q_2^-},$$

NLO:

$$g+g \to c\bar{c}\left[{}^1S_0^{(1)}\right]+g,$$

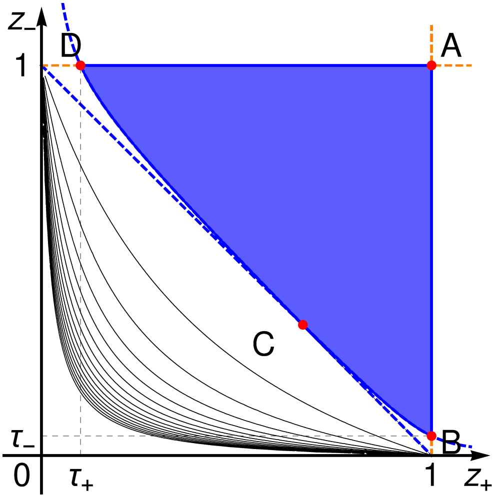
$$x_1 = \tau_+\sqrt{\frac{z_-}{z_+(z_+ + z_- - 1)}},$$

$$x_2 = \tau_-\sqrt{\frac{z_+}{z_-(z_+ + z_- - 1)}},$$

with

$\tau_\pm = Me^{\pm y}/\sqrt{S}$.

Phase-space beyond NLO



$$z_+ = \frac{p^+}{q_1^+}, \quad z_- = \frac{p^-}{q_2^-},$$

N..NLO:

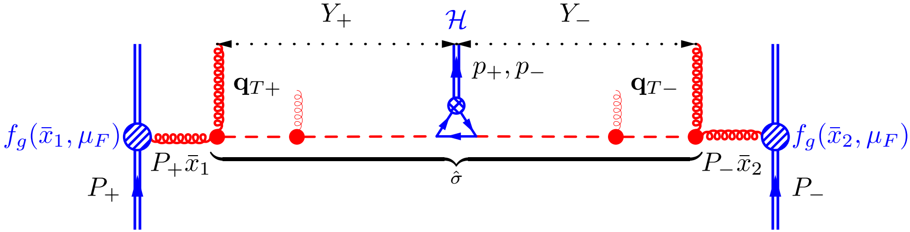
$$g+g \to c\bar{c}\left[{}^1S_0^{(1)}\right]+X,$$

lower bound for M_X:

$$M_X \geq M\sqrt{\frac{(1-z_+)(1-z_-)}{z_+z_-}} \times\theta(1-z_+-z_-)$$

High-Energy factorization, the picture

High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']:



Small parameters:

$$z_+ = \frac{p_+}{P_+\bar{x}_1},\ z_- = \frac{p_-}{P_-\bar{x}_2}$$

Using the BFKL formalism one resums corrections to $\hat{\sigma}$ enhanced by

$$Y_\pm \sim \ln\frac{1}{z_\pm},\ \text{in LP w.r.t. } z_\pm.$$

HEF resummation formula

$$\frac{d\sigma^{(\mathrm{HEF})}}{dy} = \sum_{i,j=g,q,\bar{q}} \int\limits_0^{2\pi} \frac{d\phi}{2} \int\limits_0^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2 \int\limits_{x_1}^{1} \frac{dz_+}{z_+} \tilde{f}_i\left(\frac{x_1}{z_+}, \mu_F\right) \mathcal{C}_{gi}(z_+, \mathbf{q}_{T1}^2, \mu_F, \mu_R)$$

$$\times \int\limits_{x_2}^{1} \frac{dz_-}{z_-} \tilde{f}_j\left(\frac{x_2}{z_-}, \mu_F\right) \mathcal{C}_{gj}(z_-, \mathbf{q}_{T2}^2, \mu_F, \mu_R) \frac{H^{[m]}(\mathbf{q}_{T1}^2, \mathbf{q}_{T2}^2, \phi)}{M_T^4},$$

with $x_{1,2} = M_T e^{\pm y}/\sqrt{S}$, $M_T^2 = M^2 + (\mathbf{q}_{T1} + \mathbf{q}_{T2})^2$.

At $O(\alpha_s)$ reproduces high-energy asymptotics of $I_{\mathrm{NLO}}(z_+, z_-)$:

$$I_{\mathrm{asy.}}(z_+, z_-) = 4C_A \left[-\frac{1}{z_+} - \frac{1}{z_-} + \frac{1}{(1-z_-)_+} + \frac{1}{(1-z_+)_+} - \ln\frac{z_+\mu_F^2}{M^2}\delta(1-z_-) - \ln\frac{z_-\mu_F^2}{M^2}\delta(1-z_+) \right]$$

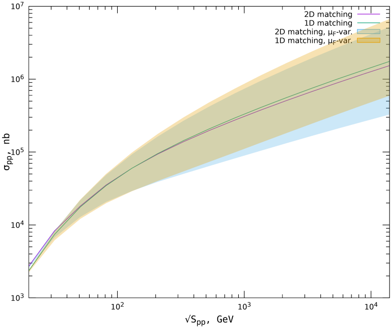
2D subtractive matching

$$\frac{d\sigma_{\mathrm{NLO,\ sub.}}}{dy} = \frac{\alpha_s}{4\pi}\hat{\sigma}_{\mathrm{LO}} \int\limits_0^1 \frac{dz_+}{z_+}\tilde{f}(x_1) \int\limits_0^1 \frac{dz_-}{z_-}\tilde{f}(x_2) \left[I_{\mathrm{NLO}}(z_+, z_-) - I_{\mathrm{asy.}}(z_+, z_-)\right],$$

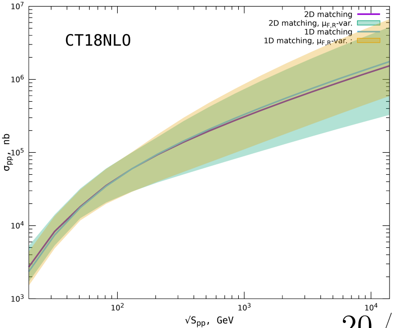
$$\frac{d\sigma}{dy} = \frac{d\sigma_{\mathrm{NLO,\ sub.}}}{dy} + \frac{d\sigma_{\mathrm{HEF}}}{dy}$$

Validation: y-integrated **matched** cross section plots with 1D and 2D subtractive matching:

μ_F-variation

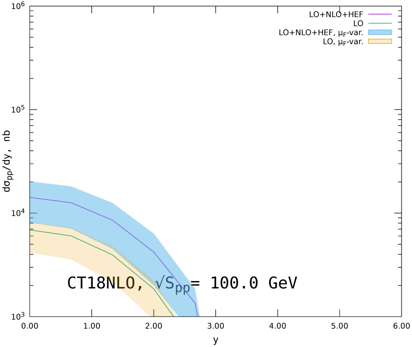


μ_F and μ_R 5pt. variation

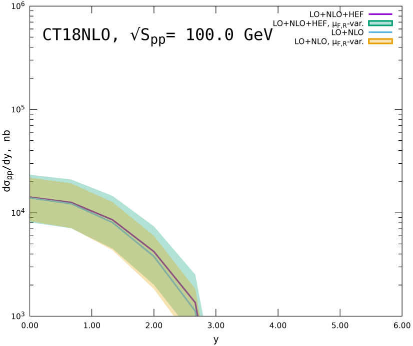


Rapidity distributions, $\sqrt{S} = 100$ GeV

μ_F-variation, LO vs. matching

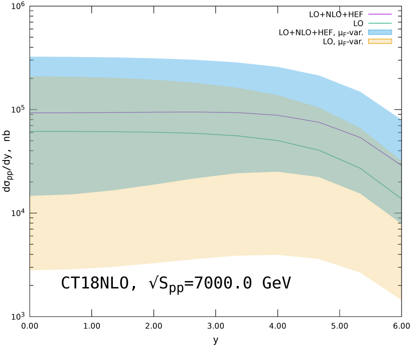


μ_F and μ_R 5pt. variation, NLO vs. matching

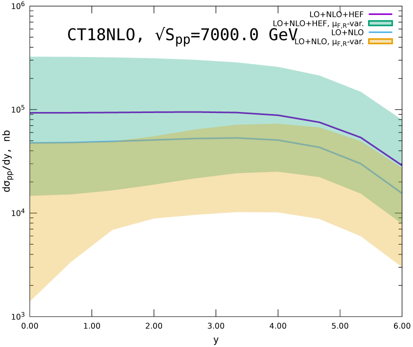


Rapidity distributions, $\sqrt{S} = 7$ TeV

μ_F-variation, LO vs. matching

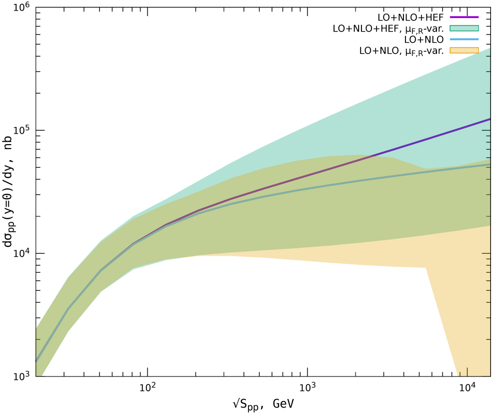


μ_F and μ_R 5pt. variation, NLO vs. matching



$d\sigma(y=0)/dy$ vs. $\sqrt{S}$

μ_F and μ_R 5pt. variation, NLO vs. matching



Conclusions and outlook

- The high-energy instability of the NLO cross section is related with lack of the $\alpha_s^n \ln^{n-1} \frac{\hat{s}}{M^2}$ corrections in $\hat{\sigma}$ at $\hat{s} \ll M^2$.
- The HEF at DLA is the formalism to solve this problem if the standard fixed-order PDFs are to be used.
- Matching between NLO CF and HEF has to be performed.
- Formalism and numerical framework for computation of **rapidity distributions** with *2D subtractive matching* is demonstrated on example of η_c-production
- Future plans:
  - *InEW-matching*
  - χ_{cJ} production in DLA+NLO CF
  - p_T-distribution of J/ψ in DLA+NLO CF
  - Scale-variation is way too large... NLO CF+NLL HEF calculation is in progress.

Thank you for your attention!