High quality (classical) solution to the QCD relaxion problem

Gilad Perez, Weizmann Inst.

Based on work \w Abhishek Banerjee & Joshua Eby, 2210.05690
Also, have to mention work with Hyungjin Kim, 2205.12988
See also: Banerjee, Kim, Matsedonski, GP, Safranova (20)

Outline

- Strong CP problem?
- QCD axion and its quality problem, the QCD line …
- Hook's Z_n high quality ultra light QCD axion model, going outside the QCD line
- The classical relaxion and its (clockwork) quality problem (only classical evolution)
- Why the QCD relaxion doesn't work
- Relaxed-relaxion
- A high quality Z_n QCD relaxion model
- Outlook cool pheno

The strong CP “problem”
and
The axion solution

“QCD problem” & the SM

- In the SM the CKM phase is order 1 but $\bar{\theta} = \theta - \arg\left[\det\left(Y_u Y_d\right)\right] \lesssim 10^{-10}$
- Is this a problem? Not necessarily, different spurions at tree level they are orthogonal, as exploited in Nelson-Barr type of models
- At 7 loops the EDM receives log-div contributions but it is tiny, and the finite contribution predicts $\bar{\theta} \sim 10^{-16}$ so it doesn’t look like a serious problem at the moment, similar to the flavor problem …
- In fact in Nelson-Barr the two CP phases are related but not in axion models …

SM vs. QCD axion model, quality

- Within the SM everything is immune against UV (Planck) suppressed contribution

- Neutrino masses are the closest but they require lower scale …

- The QCD axion in fact is not, to see let's look at the axion-QCD para':

$$\mathscr{L}_a = \frac{(a/f_a + \bar{\theta})}{32\pi^2} G\tilde{G} \;\Leftrightarrow\; V(\bar{\theta} \leftrightarrow a/f_a + \bar{\theta}) = -\frac{m_\pi^2 f_\pi^2}{(1+z)^2}\sqrt{1 + z^2 + 2\,z\cos(\bar{\theta})} + \mathcal{O}\left(\frac{10^{-3}\,m_u}{\Lambda_{\rm QCD}}\right)$$

with $z \equiv m_u/m_d$ and $V(a/f_a) \sim -\,z m_\pi^2 f_\pi^2 \cos(a/f_a + \bar{\theta}) \;\;\Rightarrow\;\; \text{QCD line}: \;\; m_a \sim \Lambda_{\rm QCD}^2/f$

Let's also mention: $m_\pi^2(\bar{\theta}) = m_\pi^2|_{\bar{\theta}=0}\sqrt{\frac{1 + z^2 + 2z\,\cos\bar{\theta}}{(1+z)^2}} \sim m_\pi^2|_{\bar{\theta}=0}\left(1 - z\,\bar{\theta}^2/2\right)$

QCD axion's quality problem

$$V = \Lambda_{\rm QCD}^4 \cos(a/f + \bar{\theta}) + \frac{\Phi^n}{M_{\rm Pl}^n}(\Phi^\dagger \Phi)^2 \quad \Rightarrow \quad \Lambda_{\rm QCD}^4 \sin \delta\theta \sim \epsilon^N f^4 \quad \Rightarrow_{f \to 10^{10}\,{\rm GeV}} \quad \left(\frac{\Lambda_{\rm QCD}}{10^{10}\,{\rm GeV}}\right)^4 10^{-10} \sim \left(\frac{10^{10}\,{\rm GeV}}{M_{\rm Pl}}\right)^n$$

where with n<7 operators, $\delta\theta > 10^{-10}$ and the strong CP problem is not solve!

This may be solved if one impose a (gauged) discrete symmetry, respected by gravity

Z_n QCD axion model

Hook (2018)

See also: Di Luzio, Gavela, Quilez & Ringwald (21)

The magic of the model

- Consider the following model based on N copies of the SM

$$\mathscr{L} = \sum_{k=1}^{N} \mathscr{L}_{\rm SM}^{k} + \Phi \sum_{k} Q_k Q_k^c \exp(2\pi i k/N)\,,\ \ \langle\Phi\rangle = (f_a + \rho)\exp(ia/f_a)/\sqrt{2}$$

- Under the Z_N & U(1) sym':

$$\mathscr{L}_{\rm SM}^{i} \to \mathscr{L}_{\rm SM}^{i+1}\,, \quad Q_i^{(c)} \to Q_{i+1}^{(c)}\,, \quad \Phi \to e^{2\pi i/N}\Phi \quad \& \quad Q_i \to e^{i\theta} Q_i\,, \quad \Phi \to e^{-i\theta}\Phi$$

- Resulting with the following QCD axion potential

$$\mathscr{L}_k = \left(\bar\theta + a/f_a + 2\pi k/N\right)\frac{1}{32\pi^2} G\tilde{G} \quad \Rightarrow \quad V(\bar\theta) = -\sum_k \frac{m_\pi^2 f_\pi^2}{(1+z)^2}\sqrt{1 + z^2 + 2\,z\cos(\bar\theta + 2\pi k/N)}$$

Naturally light high quality QCD axion

- Expanding in z makes it obvious that the leading contribution arise at order z^N
- Thus the axion mass is suppressed by $z^{N/2}$ allow to go above the QCD line: $m^2 f_a^2 \sim \Lambda_{\text{QCD}}^4 \, z^N \ll \Lambda_{\text{QCD}}^4$
- For sufficient large N it also avoid the quality problem

The relaxion mechanism in a nutshell

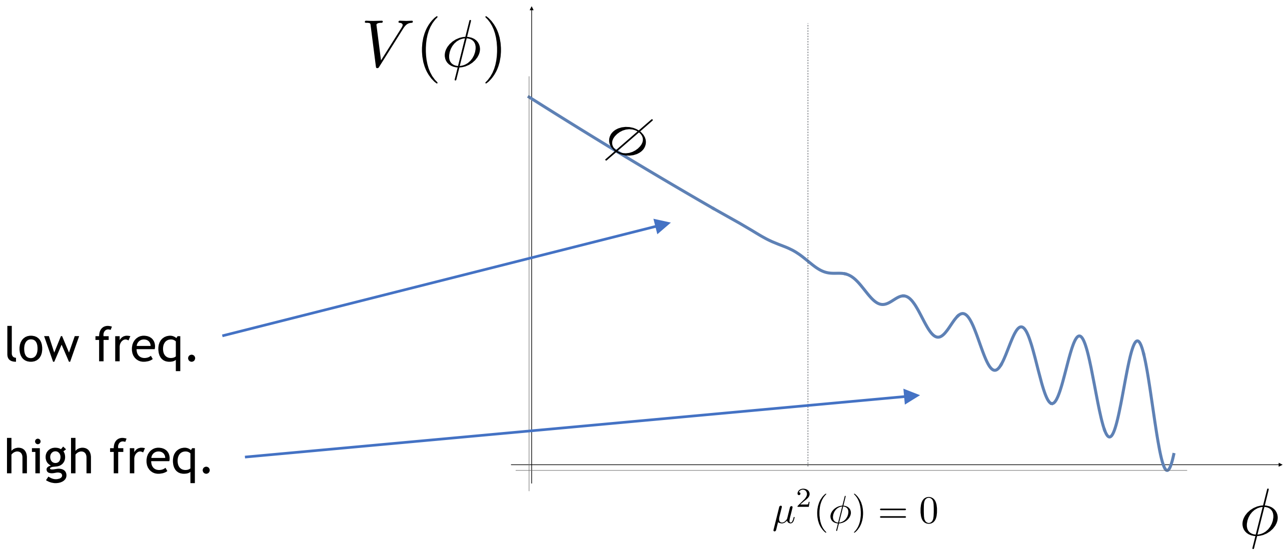
Graham, Kaplan & Rajendran (15)

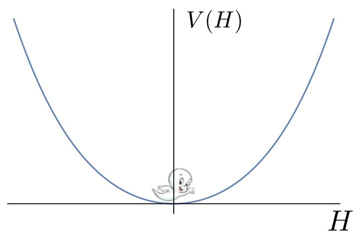
Relaxion mechanism (inflation based, slow rolling)

Graham, Kaplan & Rajendran (15)

(*i*) Add an ALP (relaxion) Higgs dependent mass: $\overbrace{\left(\Lambda^2 - g^2\phi^2\right)}^{\mu^2(\phi)} H^\dagger H$.

(ii) ϕ roles till μ^2 changes sign $\Rightarrow \langle H \rangle \neq 0 \Rightarrow$ stops rolling.





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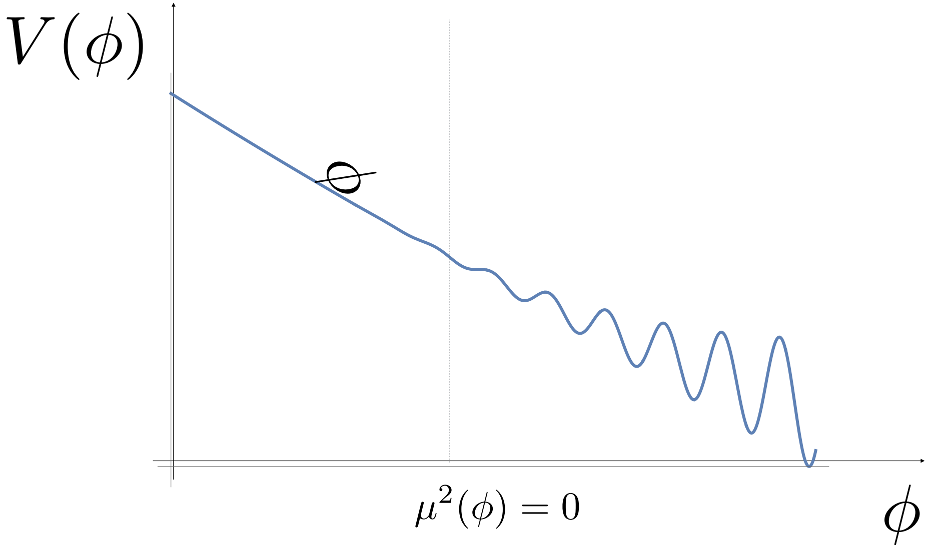

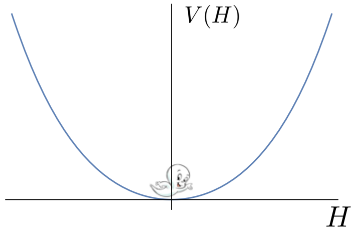

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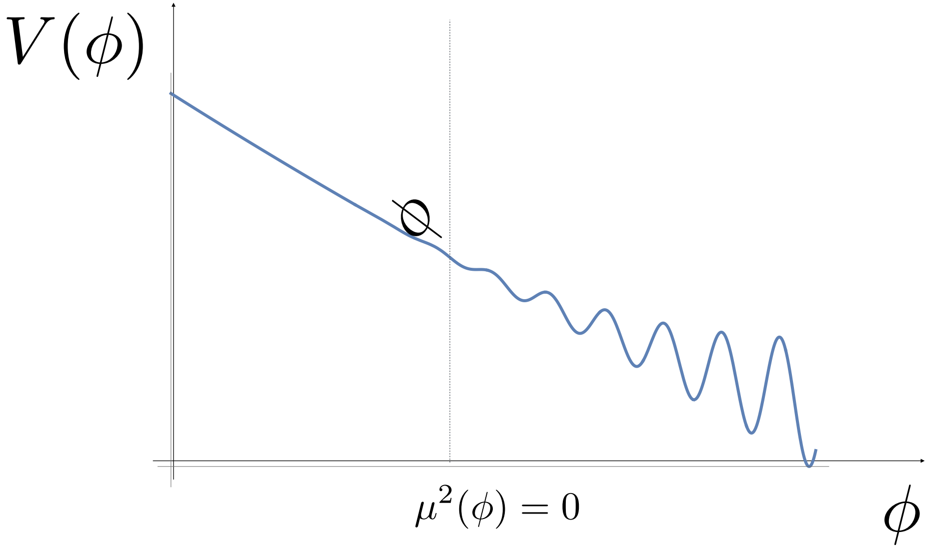

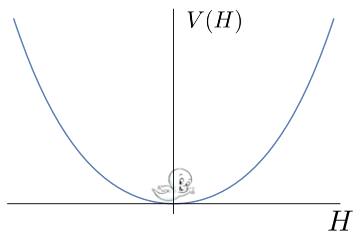

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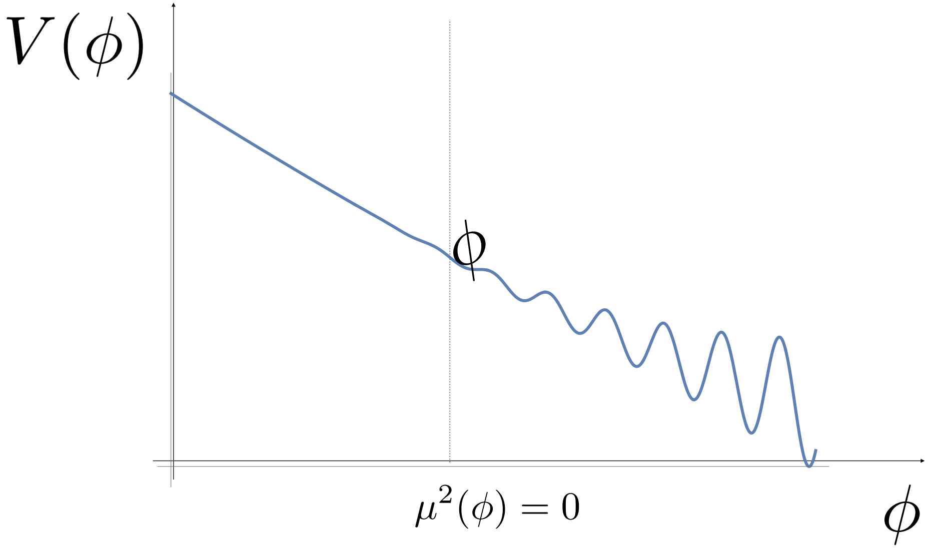

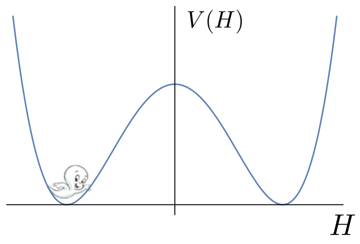

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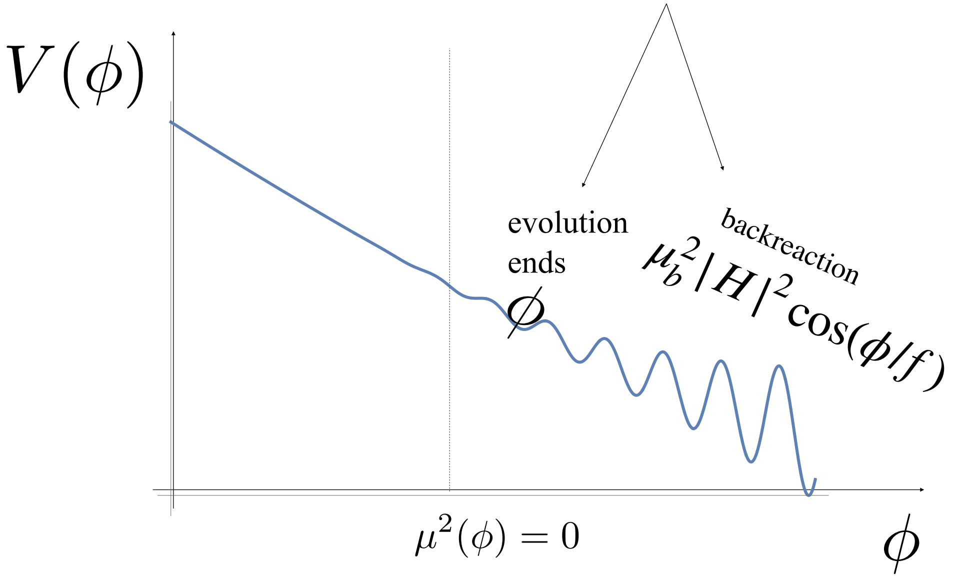

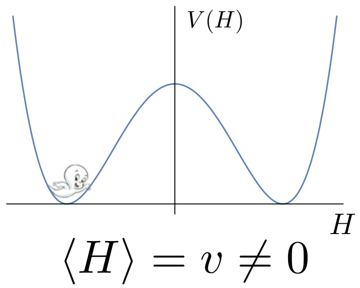

The basic relations & parametric dependence

- As the relaxion is an axion, the potential must be a periodic function of it:

$$V(\phi)^{\rm rol} \sim M^4 \cos(\phi/F) \leftrightarrow g\Lambda^3\phi \qquad \mu^2(\phi) = \Lambda^2 + M^2\cos(\phi/F) + m_{\rm back}^2 \cos(\phi/f + \alpha)$$

$$F \gg f \gtrsim M \sim \Lambda \gg v \gtrsim m_{\rm back}$$

Espinosa et al. (15)

- We start assuming $\phi \sim F$ and the stopping condition reads:

$$V'(\phi) = 0 \quad \Leftrightarrow \quad M^4/F = v^2 m_{\rm back}^2/f \quad \Rightarrow \quad v/\Lambda \lesssim (f/F)^{\frac{1}{4}}$$

Gupta, Komargodski, GP & Ubaldi (15)

- Require very big hierarchy between f and F

Clockwork

To have a cut-off of 10^4 v we need $f/F = 10^{-16}$

Choi, Kim & Yun (14); Choi & Im; Kaplan & Rattazzi (15)

Clockwork model

- The following linear sigma model:

$$V(\phi) = \sum_{j=0}^{N} \left(-m^2 \phi_j^\dagger \phi_j + \frac{\lambda}{4} |\phi_j^\dagger \phi_j|^2 \right) + \sum_{j=0}^{N-1} \left(\epsilon \phi_j^\dagger \phi_{j+1}^3 + h.c. \right)$$

In the $\epsilon \to 0$ limit have $U(1)^N \Rightarrow N$ Goldstones.

- However there is only one true Goldstone, upon the charge assignment:

$$Q = 1, 1/3, 1/9, \ldots 1/3^N$$

- Move to the non-linear sigma model:

$$\phi_j \to U_j \equiv f e^{i\pi_j/(\sqrt{2}f)}$$

Clockwork model at low energies

Choi, Kim & Yun (14); Choi & Im; Kaplan & Rattazzi (15)

- The following effective low energy non-linear sigma potential:

$$\mathcal{L}_{pNGB} = f^2 \sum_{j=0}^{N} \partial_\mu U_j^\dagger \partial^\mu U_j + \left(\epsilon f^4 \sum_{j=0}^{N-1} U_j^\dagger U_{j+1}^3 + h.c. \right) + \cdots$$
$$= \frac{1}{2} \sum_{j=0}^{N} \partial_\mu \pi_j \partial^\mu \pi_j + \epsilon f^4 \sum_{j=0}^{N-1} e^{i(3\pi_{j+1} - \pi_j)/(\sqrt{2} f)} + h.c. + \cdots$$

- There is only one true Goldstone with the following profile:

$$\vec{a}_{(0)}^T = \mathcal{N} \left(1 \quad \tfrac{1}{3} \quad \tfrac{1}{9} \quad \ldots \quad \tfrac{1}{3^N} \right)$$

The 0-mode/exact Goldstone profile & breaking

Choi, Kim & Yun (14); Choi & Im; Kaplan & Rattazzi (15)

- Add small breaking on first and last sites:

$$\frac{\pi_0}{32\pi^2 f} G_0 \tilde{G}_0 + \frac{\pi_N}{32\pi^2 f} G_N \tilde{G}_N$$

$$\downarrow$$

$$\Lambda_N \cos\frac{\pi_N}{f} + \Lambda_0 \cos\frac{\pi_0}{f}$$

$$\downarrow$$

$$\Lambda_N \cos\frac{\pi}{3^N f_a} + \Lambda_0 \cos\frac{\pi}{f_a}$$

The 0-mode/exact Goldstone profile & breaking

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- Add small breaking on first and last sites:

$$\Lambda_N \cos\frac{\pi}{3^N f_a} + \Lambda_0 \cos\frac{\pi}{f_a}$$

$$f \equiv f_a \quad \& \quad F = 3^N f$$

To have a cut-off of $10^4\, v$ we need $f/F = 10^{-16}$ => 35 cites …

Relaxion and cosmology

- Must not disturb inflation $H^2 > \Lambda^4/M_{\rm Mpl}^2$

- Dominated by classical evolution $H < \dot{\phi}/H \sim V'/H^2 \lesssim v^4/fH^2 \;\;\Rightarrow\;\; \Lambda < f < v^4/H^3$

- Combining the two $\Lambda \lesssim M^{\frac{3}{7}} v^{\frac{4}{7}} \sim 10^8\,\text{GeV}$

- There is also an interesting relation between the cutoff and the number of e-folds

$$\Delta\phi \sim F \quad \Rightarrow \quad N_{\rm ef} \sim F/\dot{\phi} \times H \sim FH^2/V' \sim F^2H^2/\Lambda^4 \gtrsim F^2/M_{\rm Pl}^2$$

$$\sim (\Lambda/v)^8 f^2/M_{\rm Pl}^2 \gtrsim \Lambda^{10}/v^8 M_{\rm Pl}^2 \sim \left(\frac{\Lambda}{100\,\text{TeV}}\right)^{10}$$

Challenges of the relaxion

i. QCD relaxion CP problem
ii. Quality Problem

Relaxion and CP violation

- The relaxion is based on two breaking of the shift symmetry
- The Rolling potential and the backreaction potential
- As seen the stopping condition is when the derivative of the Rolling potential is equal to the one of the backreaction potential, where QCD axion require the the axion settles at the minimum of its potential => $a/f_a \sim 1$ (in fact very close to $\pi/2$)
- This is incompatible unless one is giving up on classical evolution, which my force us to think about the measure problem & eternal inflation

Nelson & Prescod-Weinstein (17) Gupta (18)
Chatrchyan & Servant (22)

Relaxed relaxion & some pheno

Relaxion's naive parameters (similar to ALP, backreaction domination)

$$m_\phi^2 \sim \partial_\phi^2 V_{br}(\phi,h) \sim \frac{\mu_b^2 v_{\rm EW}^2}{f^2} \cos\frac{\phi_0}{f}$$

$$\sin\theta_{h\phi} \sim \partial_\phi \partial_h V_{br}(\phi,h)/v_{\rm EW}^2 \sim \frac{\mu_b^2}{f v_{\rm EW}} \sin\frac{\phi_0}{f}$$

~ 1 ⇒ The relaxion is light and mixes with the Higgs

Flacke, Frugiuele, Fuchs, Gupta & GP; Choi & Im (16); Banerjee, Kim & GP (18)

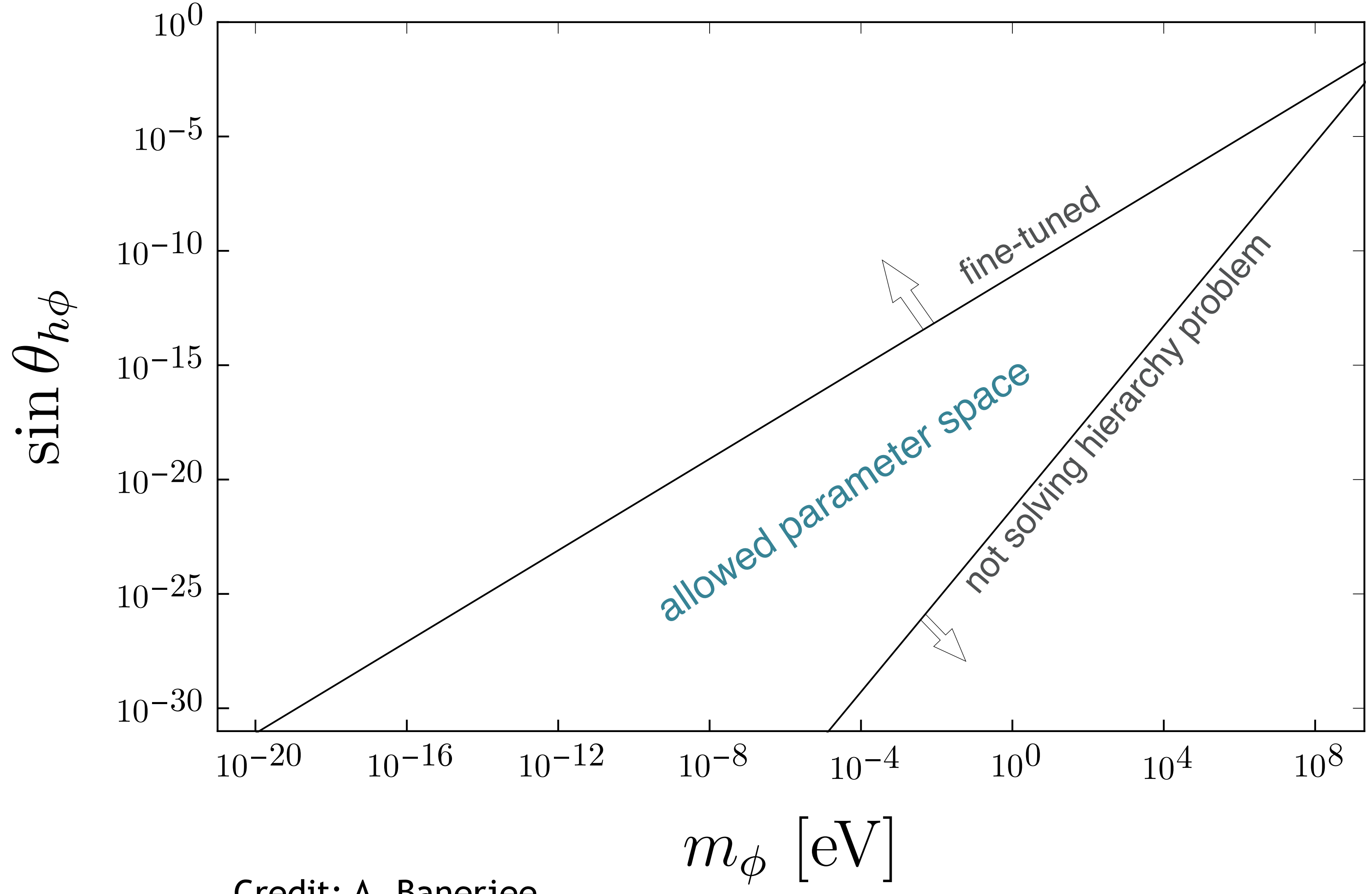
Naively: mixing angle in terms of mass $$\sin\theta_{h\phi} \sim \frac{m_\phi}{v_{\rm EW}} \frac{\mu_b}{v_{\rm EW}}$$

Maximum mixing angle $$(\sin\theta_{h\phi})_{\rm max} \sim \frac{m_\phi}{v_{\rm EW}}$$

Naturalness bound

Minimum mixing angle $$(\sin\theta_{h\phi})_{\rm min} \sim \frac{m_\phi^2 \Lambda_{\rm min}}{v_{\rm EW}^3}$$

The relaxion's naive parameter space

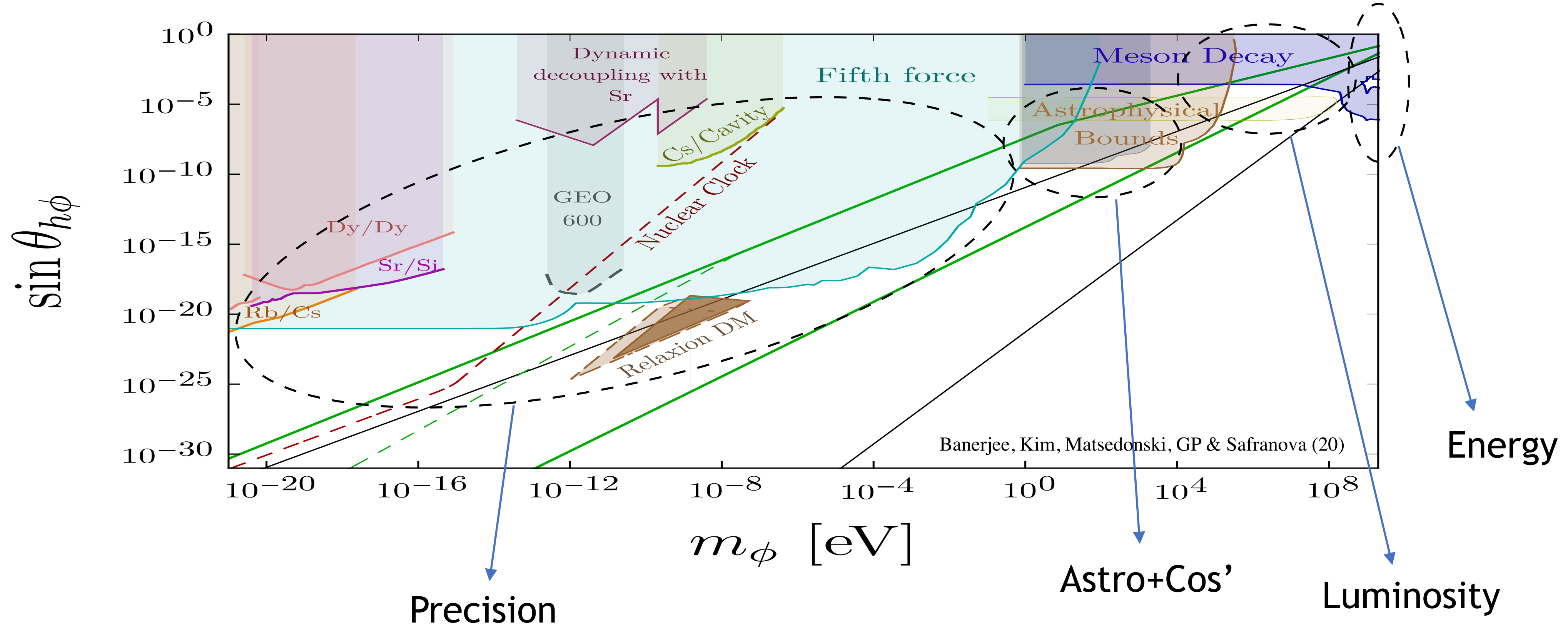


Credit: A. Banerjee

The log crisis

- Lesson 1 - finding NP requires diverse approach, searches across frontier

- Lesson 2 - experimentally, worth checking where many decades are covered:



Less naive treatment, the relaxed relaxion

$$V(\phi, h) = \left(\Lambda^2 - \Lambda^2 \frac{\phi}{F}\right) |H|^2 - \frac{\Lambda^4}{F}\phi - \mu_b^2 |H|^2 \cos\frac{\phi}{f} \qquad v^2(\phi) = \begin{cases} 0 \text{ when } \phi < f_{\rm eff} \\ > 0 \text{ when } \phi > f_{\rm eff} \end{cases}$$

Relaxion stopping point determines the EW scale $\frac{\Lambda^4}{F} \sim \frac{\mu_b^2 v_{EW}^2}{f}$

Resolution parameter

Higgs mass change for $\Delta\phi = 2\pi f$ $\quad \frac{\Delta v^2}{v^2} \sim \frac{\Lambda^2}{F}\frac{f}{v^2} \sim \boxed{\frac{\mu_b^2}{\Lambda^2}} \equiv \delta^2 \ll 1$

$$V_{br} = -\mu_b^2 |H|^2 \cos\frac{\phi}{f}$$

⇨ Potential height grows incrementally

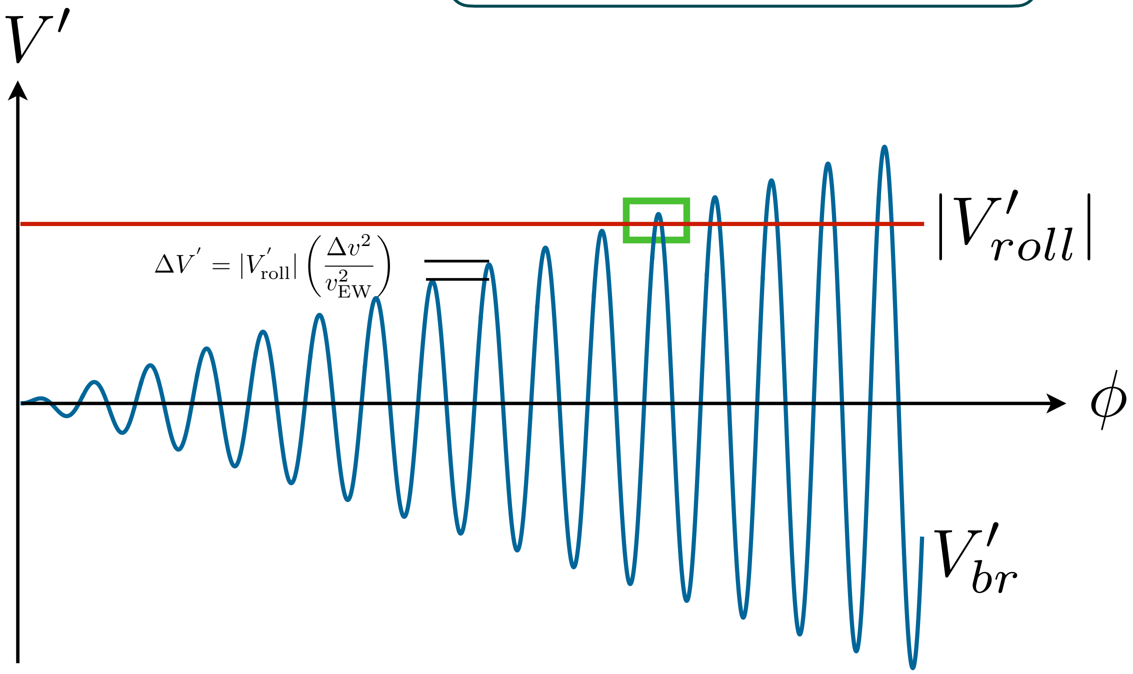
Stopping condition, fine resolution

Banerjee, Kim, Matsedonski, GP, Safranova (20)

$$V(\phi) = (v^2(\phi))^2 + g\Lambda^3\phi \;\Rightarrow\; V' \simeq v^2(\phi)\mu_{\rm br}^2/f \times \sin(\phi/f) + \Lambda_{\rm br}^4/f = 0$$

$$V_\phi' = 0 \Rightarrow \sin\theta = \frac{v_{\rm EW}^2}{v^2(\phi)} + \frac{v_{\rm EW}^2}{\Lambda^2}$$

$$\frac{\phi_0}{f} \sim \frac{\pi}{2} \text{ upto resolution factors}$$



Credit: A. Banerjee

The relaxion stops at ~ max' of the derivative of backreaction potential!

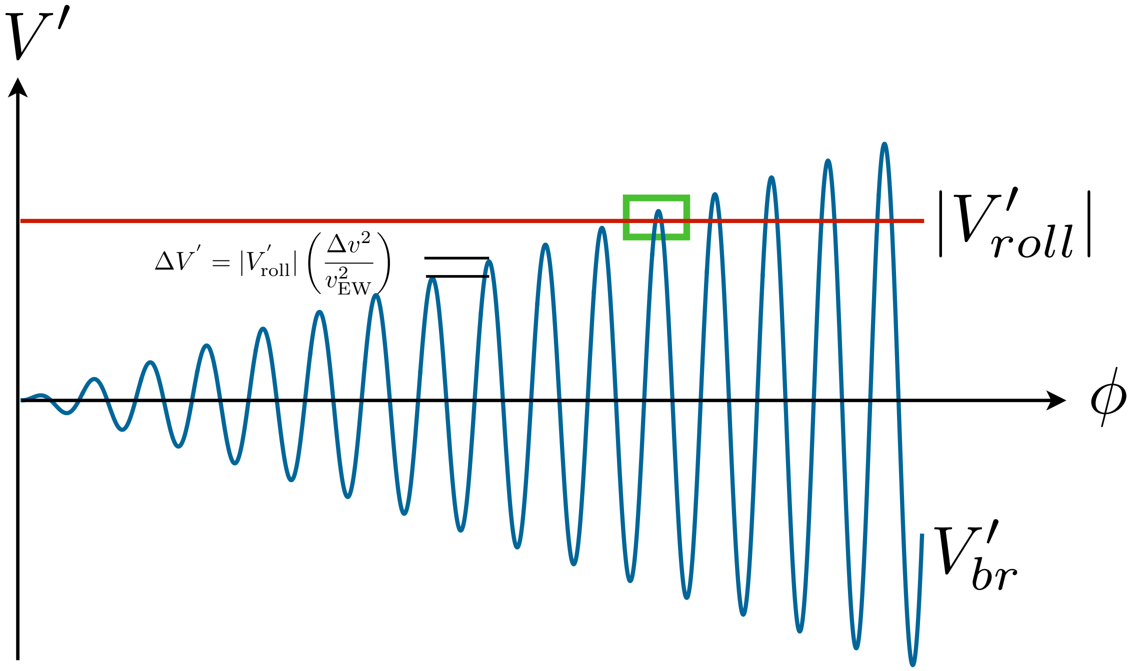
$$\bar{\theta} = \pi/2$$

Stopping condition, fine resolution

Banerjee, Kim, Matsedonski, GP, Safranova (20)

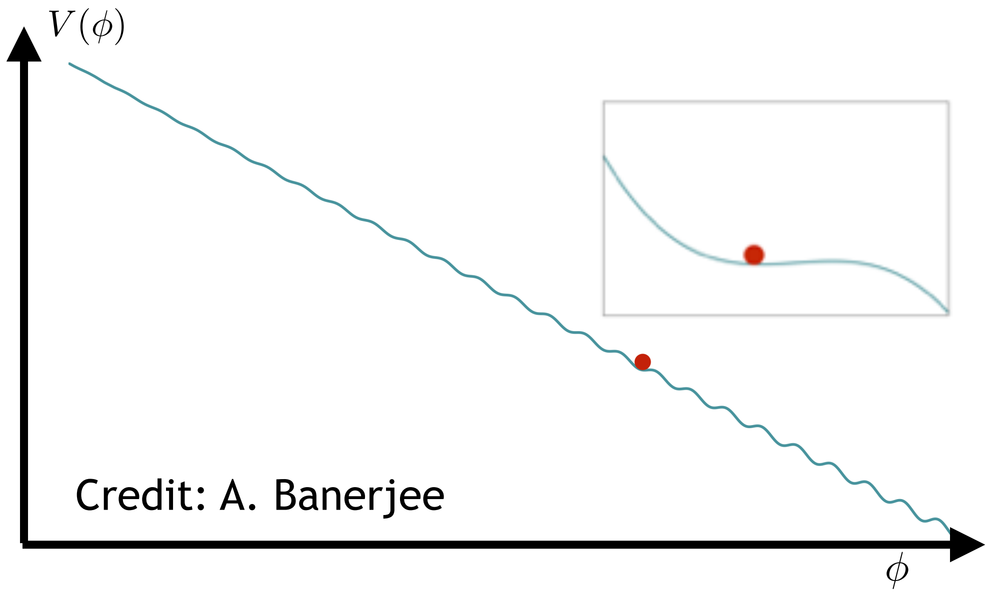
$$V'_\phi = 0 \Rightarrow \sin\theta = \frac{v^2_{\rm EW}}{v^2(\phi)} + \frac{v^2_{\rm EW}}{\Lambda^2} \quad \Longrightarrow \quad \frac{\phi_0}{f} \sim \frac{\pi}{2} \text{ upto resolution factors}$$

$$m^2_\phi \approx \delta \times \left(m^2_\phi\right)_{\rm naive} \ll \left(m^2_\phi\right)_{\rm naive}$$



Relaxion: barriers increase incrementally:

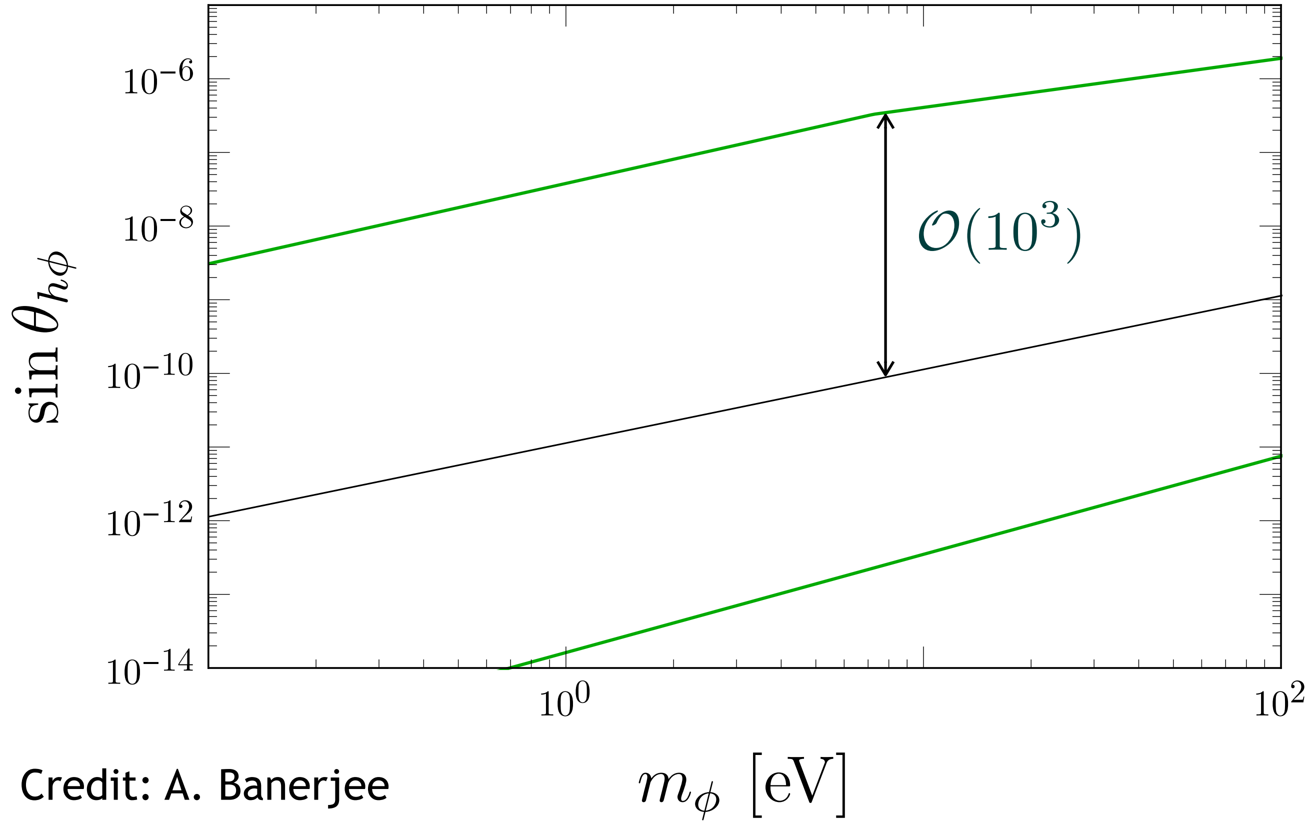
relaxion stops at shallow region => small mass



Relaxed mass => natural violation of naturalness bound

Banerjee, Kim, Matsedonski, GP, Safranova (20)

Max. Mixing angle: $\sin\theta_{h\phi}^{\max} = \left(\frac{m_\phi}{v_{EW}}\right)^{\frac{2}{3}} \gg \left(\frac{m_\phi}{v_{EW}}\right)_{\text{naturalness}}$



Credit: A. Banerjee

High quality (classical) solution to the QCD relaxion problem

Banerjee, Eby & GP, last month

Combine ingredients to avoid the QCD relaxion CP problem

- Basic idea: asume Z_N sym' QCD relaxion model, say Z_{2n}, and make the backreaction dominated by a single sector, k, which is not the SM.
- As we showed (ignoring higher z) the relaxion will stop the evolution at $\bar{\theta}_k \sim \pi/2$
- Consider for instance having the SM at the Nth site and the site with the dominant backreaction on the site after:

$$V(\bar{\theta}) \sim -\Lambda'^4_{\mathrm{QCD}'} \cos(\bar{\theta} + a/f + \pi/2) \;\; \Leftrightarrow \;\; \mathscr{L}_k = \left(\bar{\theta} + a/f_a + \pi/2\right) \frac{1}{32\pi^2} G'\tilde{G}' \;\; \Longrightarrow \;\; (a/f_a)_{\mathrm{relax}} \approx -\bar{\theta}$$

- If this is just the sector after the SM then the SM will have

$$\mathscr{L}_{\mathrm{SM}} = \left(\bar{\theta} + a/f_a\right) \frac{1}{32\pi^2} G\tilde{G}\,,$$ solving the strong CP problem.

Higher harmonics (thanks to Serra-Stelzl-Weiler)

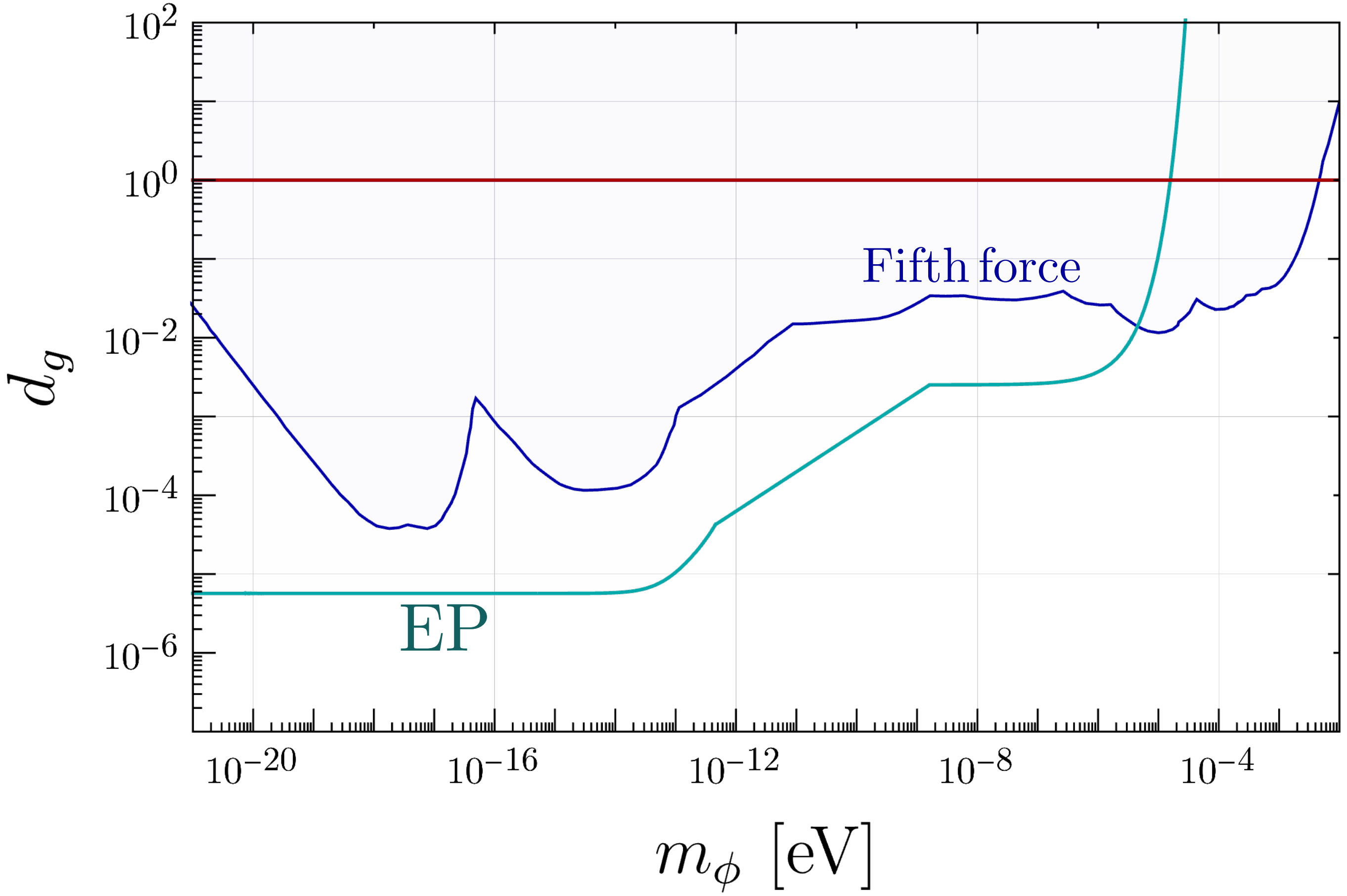
- To suppress the higher harmonics we need
  $m'_u/\Lambda'_{QCD'} \lesssim 10^{-6}$ & keeping $m'_u\Lambda'^3_{QCD'} \sim$ const $\Leftrightarrow$ $\Lambda'_{QCD'} \to (10-100)\Lambda'_{QCD'}$
- Viable model requires ($\delta\theta \lesssim 10^{-9}$): $\dfrac{m_u\Lambda^3_{QCD}}{\Lambda'^4_{br} \equiv m'_u\Lambda'^3_{QCD'}} \lesssim \dfrac{10^{-9}}{z'-z}$, $\dfrac{\Lambda'^4_{br}}{v'^4} \lesssim 10^{-9}$, $\dfrac{\Lambda'^2_{br}}{v'\Lambda} \lesssim 10^{-9}$
- Suppressing tunneling (+ quantum): $\Delta V \sim \Lambda^4_{back}\delta^3 \equiv \left(m'_u\Lambda'^3_{QCD'}\right)^4 \delta^3 \gg H_I^4$
- Ensuring inflation domination $H_I^2 \gg \Lambda^4/M^2_{Pl}$ $\Rightarrow$ $\Lambda^4_{back}\left(\mu^2_{back}/\Lambda^2\right)^3 \ll \Lambda^8/M^4_{Pl}$
- The max of the derivative is at $\cos\theta_{relax} = -z$
- Consider 2 models with $z = 1,1/2$ leading to $\theta_{relax} = 2\pi/3,\ \pi$

$$\theta_{CP} = 10^{-9}, \frac{v}{v'} = 4\times 10^{-3}, \frac{\Lambda_{QCD}}{\Lambda'_{QCD}} = 7\times 10^{-4} \quad \Rightarrow \quad \begin{array}{l} \theta_{CP} = 10^{-9},\ \ z = 0.455,\ \ v' = 13\,\text{TeV} \\ \theta_{CP} = 10^{-9},\ \ z = 0.493,\ \ v' = 7\,\text{TeV} \end{array}$$

Pheno

- Let me highlight two interesting effect the first is just related to the fact that the QCD axion coupled quadratically to masses:

Quality problem, 5th force vs EP violation, gluon



EP: Planck suppressed operators are excluded for $m_\phi \lesssim 10^{-5}\,\mathrm{eV}$

5th force: ***operators are excluded for*** $m_\phi \lesssim 10^{-3}\,\mathrm{eV}$

Oscillations of energy levels induced by QCD-axion-like DM

Kim & GP (22)

- Consider axion model \w $\left(\alpha_s/8\right)\left(a/f\right)\, G\tilde{G}$ coupling, usually searched by magnetometers
- However, spectrum depends on $\theta^2 = (a(t)/f)^2$: $m_\pi^2(\theta) = B\sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos\theta}$

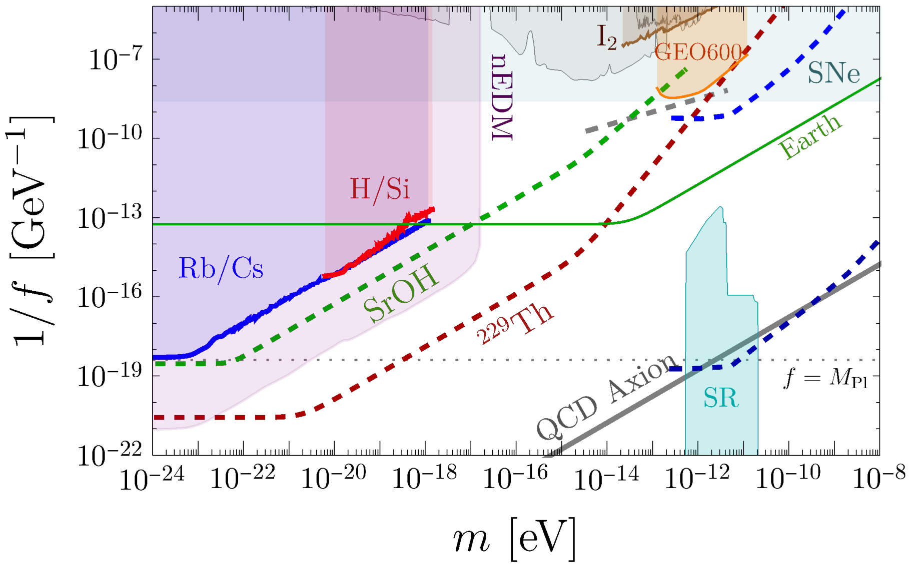
Brower, Chandrasekharanc, Negele & Wiese (03)

$$\mathrm{MeV} \times \theta^2 \bar{n}n \Rightarrow \frac{\delta f}{f} \sim \frac{\delta m_N}{m_N} \sim 10^{-16} \times \cos(2m_a) \times \left(\frac{10^{-15}\,\mathrm{eV}}{m_\phi}\frac{10^9\,\mathrm{GeV}}{f}\right)^2 \quad \text{vs} \quad m_N \frac{a}{f}\bar{n}\gamma^5 n \Rightarrow \left(f \gtrsim 10^9\,\mathrm{GeV}\right)_{\mathrm{SN}}$$

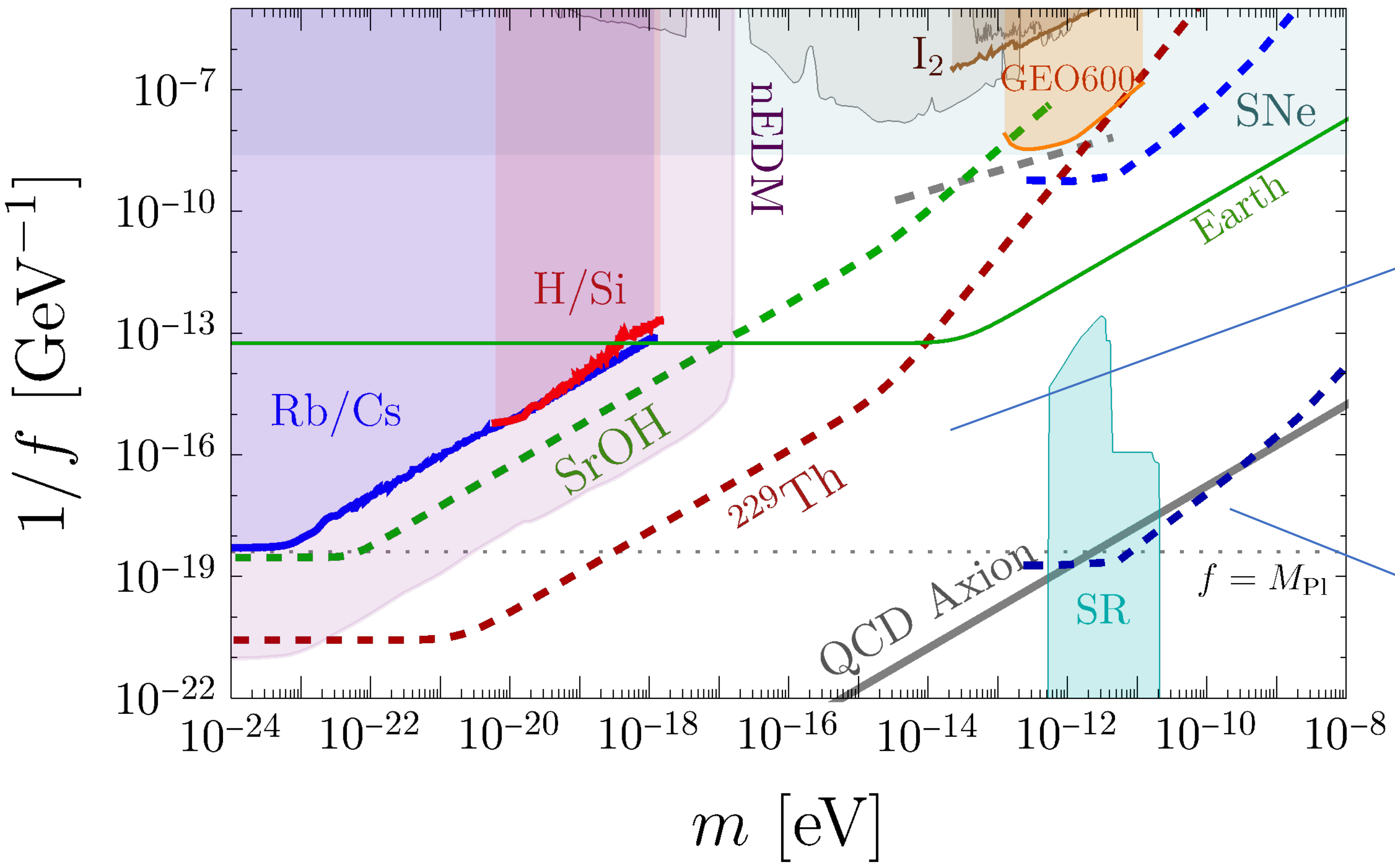
$$\frac{\delta m_\pi^2}{m_\pi^2} \approx \frac{1}{4}\theta^2$$

$$\frac{\delta m_N}{m_N} \simeq 0.13\frac{\delta m_\pi}{m_\pi}$$

$$\frac{\delta f_{\mathrm{Th}}}{f_{\mathrm{Th}}} \simeq 2\times 10^5 \frac{\delta m_\pi^2}{m_\pi^2}$$



Non-magnetometer-based probes of QCD-like axion



Unnatural (Hook's) models

$$1/f \gtrsim m_a/\Lambda^2_{\rm QCD}$$

Natural models

$$1/f \lesssim m_a/\Lambda^2_{\rm QCD}$$

Pheno

- Let me highlight two interesting effect the first is just related to the fact that the QCD axion coupled quadratically to masses: see next page
- The 2nd is due to the fact that the min' of relaxion potential deviates from pi/2.

The QCD axion also induces a scalar interaction with the nucleon in the presence of a CP-violating phase of the form of

$$g_{\phi NN} \simeq 1.3 \times 10^{-2} \frac{m_N}{f} \delta_\theta \, .$$

$$\frac{9 \times 10^{-24}}{f_{11}} \lesssim g_{\phi NN} \lesssim \frac{4 \times 10^{-23}}{f_{11}} \, ,$$

where, $f_{11} = f/(10^{11}$ GeV) and we have used $m_N \sim 1$ GeV. The strongest bound on $g_{\phi N N}$ comes from the experiments looking for the existence of fifth force and/or violation of equivalence principle (EP). The bound from EP violation searches, for the axion mass around 10^{-6} eV, is $g_{\phi NN}$? 10^{-21}

There's also a bound for axion-proton pseudoscalar and axion-nucleon scalar coupling which will be probed by Ariadne

$$\frac{4 \times 10^{-35}}{f_{11}^2} \lesssim |g_p^a g_{\phi NN}| \lesssim \frac{2 \times 10^{-34}}{f_{11}^2} \, .$$

Pheno

- Finally it could be probed by combination of scalar oscillation as well as pseudo scalar …

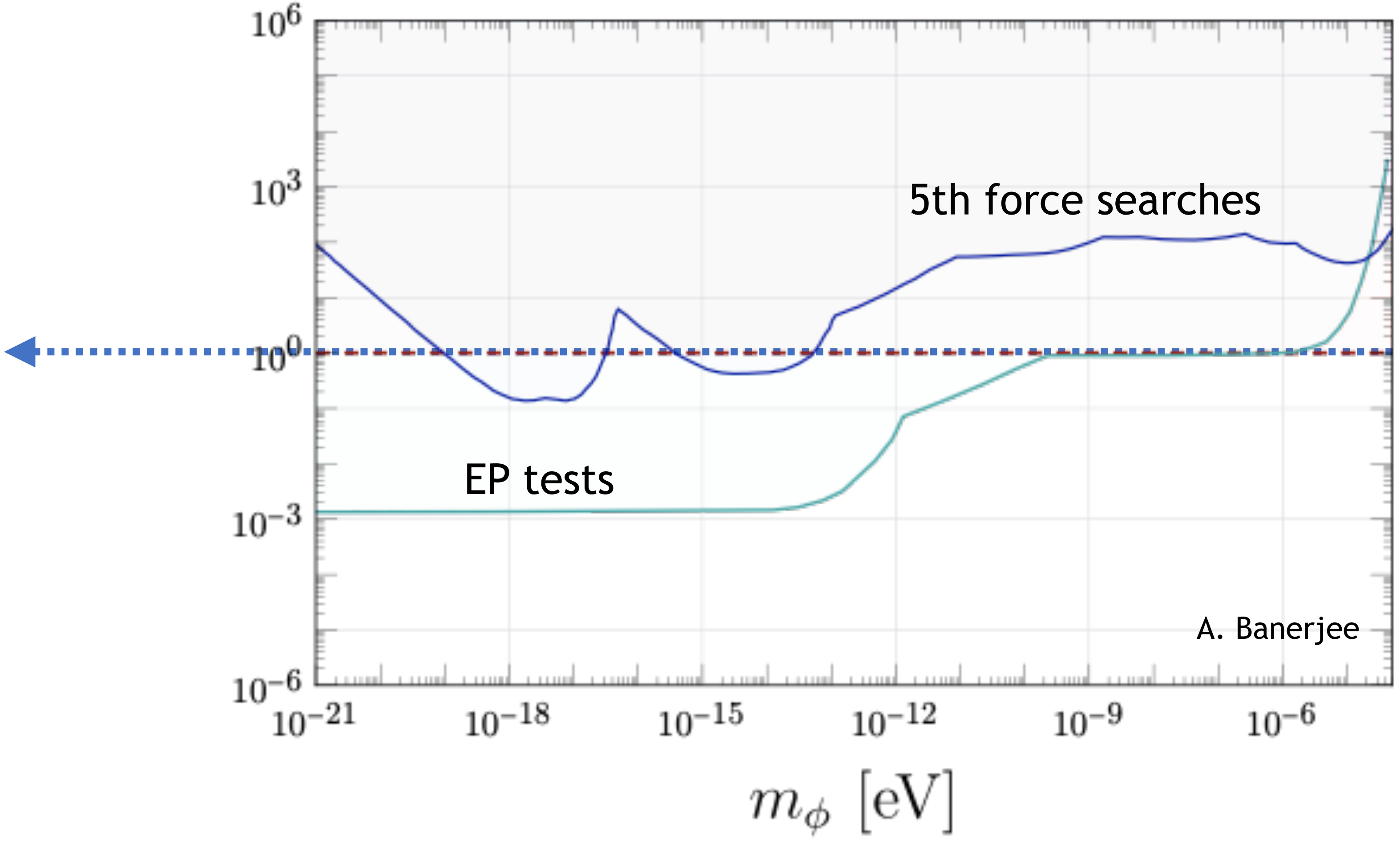
Conclusions

- Strong CP + hierarchy problem, why the QCD relaxion doesn't work
- Relaxed-relaxion
- Hook's Z_n high quality ultra light QCD axion model
- A high quality Z_{4n} QCD relaxion model
- Outlook cool pheno

Backups

Quality problem, 5th force vs EP violation, electron coupling

$$d_{m_e} \sim 1 \text{ or } g_e \sim \frac{m_e}{M_{\rm Pl}}$$



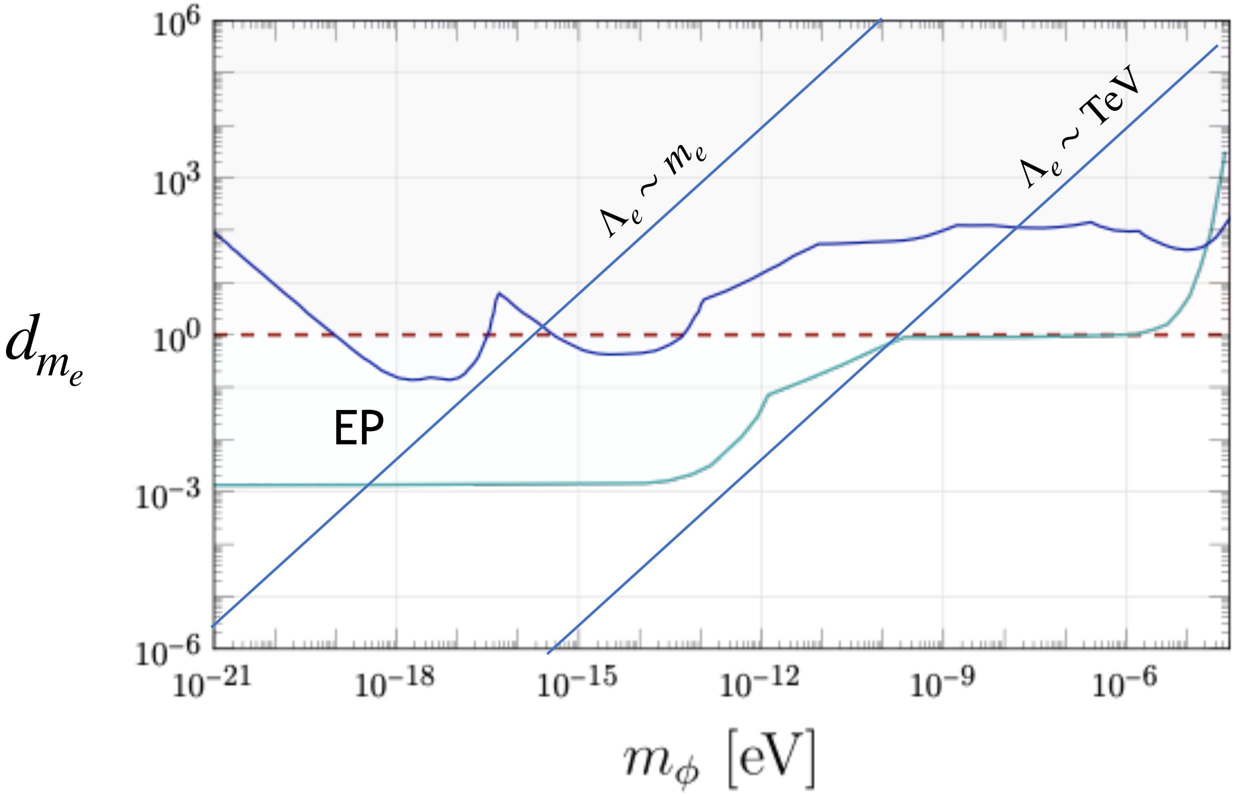
EP: Planck suppressed operators are excluded for $m_\phi \lesssim 10^{-6}\,\text{eV}$

5th force: ***operators are excluded for*** $10^{-19} \lesssim m_\phi \lesssim 10^{-13}\,\text{eV}$

ultralight spin 0 field & naturalness

- For this action there's also an issue of naturalness: $d_{m_e} < 4\pi m_\phi/\Lambda_e \times M_{\rm Pl}/m_e$

With $\Lambda_e \gtrsim m_e$ (for mirror model) => $d_{m_e} \lesssim 10^{6,0} \times \dfrac{m_\phi}{10^{-10}\,\text{eV}} \times \dfrac{m_e, \text{TeV}}{\Lambda_e}$



QCD low energy (2 gen ignoring eta')

At low energies: $$\mathcal{L} \supset \frac{\theta g_s^2}{32\pi^2} G\tilde{G} + qMq^c, \qquad M = \begin{pmatrix} , m_u & 0 \\ 0 & m_d \end{pmatrix}$$

	$SU(3)$	$SU(2)_L$	$SU(2)_R$	$U(1)_B$	$U(1)_A$
A_μ	adj				
q	$\square$	$\square$		1	1
q^c	$\bar{\square}$		$\bar{\square}$	-1	1
M		$\bar{\square}$	$\square$		-2

$$\langle qq^c \rangle \neq 0,$$ Breaks SU(2) L x R to diagonal

Chiral Goldstone action

$$U = e^{i \frac{\Pi^a}{\sqrt{2} f_\pi} \sigma^a},$$

	$SU(2)_L$	$SU(2)_R$	$U(1)_B$	$U(1)_A$
U	$\square$	$\overline{\square}$		2

$$U \propto q q^c$$

$$\mathcal{L} = f_\pi^2 \ \mathrm{Tr}\ \partial_\mu U \partial^\mu U^\dagger + a f_\pi^3 \ \mathrm{Tr}\ M U + h.c.,$$

Axial sym transformation

$$u \to e^{i\alpha} u, \qquad u^c \to e^{i\alpha} u^c,$$

$$\mathcal{L} \to \mathcal{L} + \alpha \frac{g^2}{16\pi^2} G\tilde{G}.$$

$$u \to e^{i\alpha} u, \qquad d \to e^{i\alpha} d, \qquad \theta \to \theta - 2\alpha.$$

$$U \to e^{i\alpha} U, \qquad M \to e^{-i\alpha} M.$$

Removing the GGdual coupling, phase freedom

$$U = e^{i\pi^a\tau^a/f_\pi} = \cos\frac{|\vec{\pi}|}{f_\pi} + i\frac{\pi^a}{|\vec{\pi}|}\tau^a \sin\frac{|\vec{\pi}|}{f_\pi}.$$

$$\begin{aligned} u &\rightarrow e^{i\phi_u}u \\ d &\rightarrow e^{i\phi_d}d, \end{aligned}$$

$$\phi_u + \phi_d = \theta.$$

$$U_0 = \begin{pmatrix} e^{i\phi_u} & 0 \\ 0 & e^{i\phi_d} \end{pmatrix}.$$

$$V = -B_0\mathrm{Tr}[(MU_0)U + (MU_0)^\dagger U^\dagger] = -B_0\left[4A\cos\frac{|\vec{\pi}|}{f_\pi} - 4D\frac{\pi^3}{|\vec{\pi}|}\sin\frac{|\vec{\pi}|}{f_\pi}\right].$$

$$D = \frac{1}{2}\mathrm{Tr}\left[\tau^3\begin{pmatrix} m_u\sin\phi_u & 0 \\ 0 & m_d\sin\phi_d \end{pmatrix}\right] = \frac{1}{2}(m_u\sin\phi_u - m_d\sin\phi_d) = 0$$

QCD parameter space

$$\begin{aligned}
\sin\phi_u &= \frac{m_d \sin\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}} \\
\sin\phi_d &= \frac{m_u \sin\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}} \\
\cos\phi_u &= \frac{m_u + m_d \cos\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}} \\
\cos\phi_d &= \frac{m_d + m_u \cos\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}}.
\end{aligned}$$

$$A = \frac{1}{2}\mathrm{Tr}\begin{pmatrix} m_u \cos\phi_u & 0 \\ 0 & m_d \cos\phi_d \end{pmatrix} = \frac{1}{2}(m_u \cos\phi_u + m_d \cos\phi_d).$$

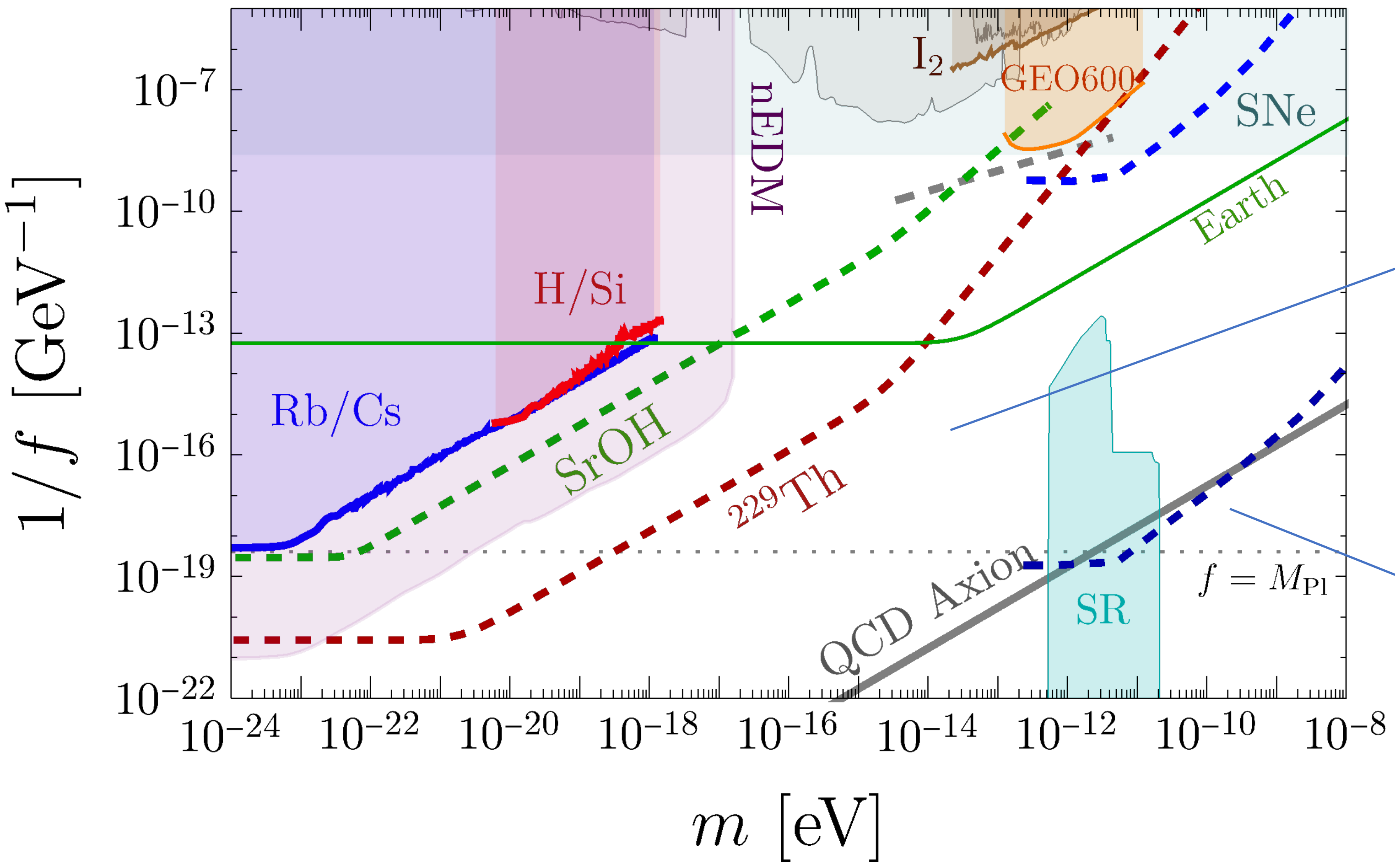
The QCD line

$$m_a \sim \frac{1}{f} \times \Lambda^2_{\mathrm{QCD}} \quad \text{or} \quad m_a \sim g_{\mathrm{gluon}} \times \Lambda^2_{\mathrm{cutoff,\,shiftsym}}$$

$$m_a \gtrsim g_{\mathrm{gluon}} \times \Lambda^2_{\mathrm{cutoff,\,shiftsym}} \quad \text{or} \quad 1/f \lesssim m_a/\Lambda^2_{\mathrm{QCD}}$$

It is not hard to go naturally below the QCD line but it is very hard to go above it.

The QCD line



Unnatural (Hook's) models

$$1/f \gtrsim m_a/\Lambda_{\rm QCD}^2$$

Natural models

$$1/f \lesssim m_a/\Lambda_{\rm QCD}^2$$

Simplest possible model, free massive scalar

- Most minimal model would be just a free massive scalar :

$$\mathscr{L} \in m_\phi^2 \phi^2 \,, \; \rho_{\rm Eq}^{\rm DM} \sim {\rm eV}^4 \sim m_\phi^2 \phi_{\rm Eq}^2 = m_\phi^2 \phi_{\rm init}^2 ({\rm eV}/T_{\rm osc})^3$$

$$T_{\rm os} \sim \sqrt{M_{\rm Pl} m_\phi} \implies \phi_{\rm init} \sim M_{\rm Pl} \left(\frac{10^{-27}\,{\rm eV}}{m_\phi} \right)^{\frac{1}{4}}$$

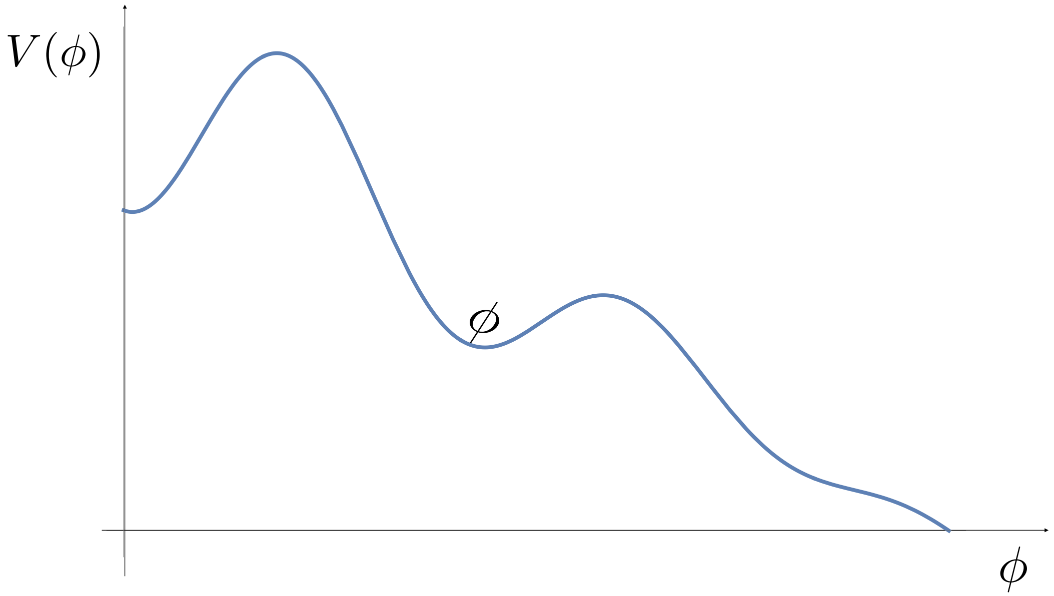
(can add a few more bounds, SR, isogurvature but still large parameter space, reasonable field excursion)

- Just remind you that if we add Planck suppressed operators then we did find bounds …
- Also, in the presence of these coupling if it's too light there will be naturalness issues …

The relaxion DM dynamical missalignment

Banerjee, Kim & GP (18)

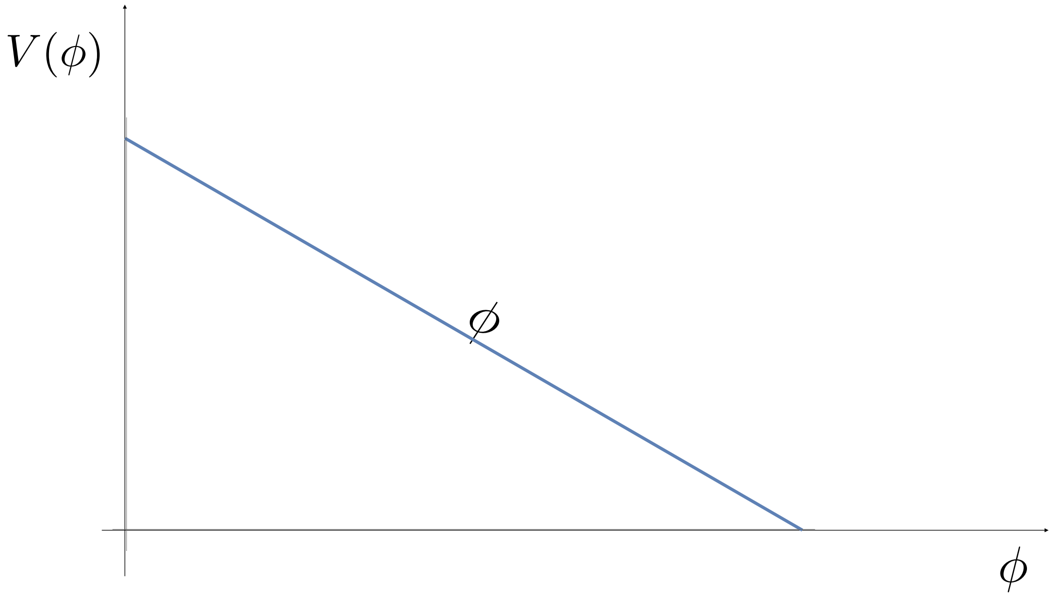
- Basic idea is similar to axion DM:



Concrete ex.: relaxion dark matter (DM)

♦ Basic idea is similar to axion DM (but avoiding missalignment problem):

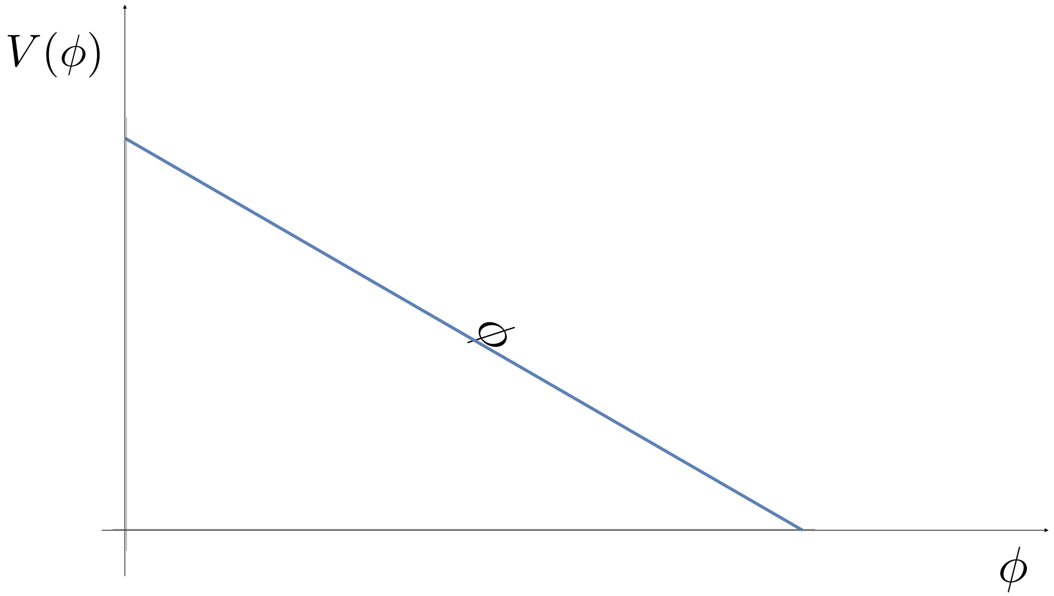
After reheating the wiggles disappear (sym' restoration):



Concrete ex.: relaxion dark matter (DM)

- Basic idea is similar to axion DM (but avoiding missalignment problem):

  After reheating the wiggles disappear: and the relaxion roles a bit.



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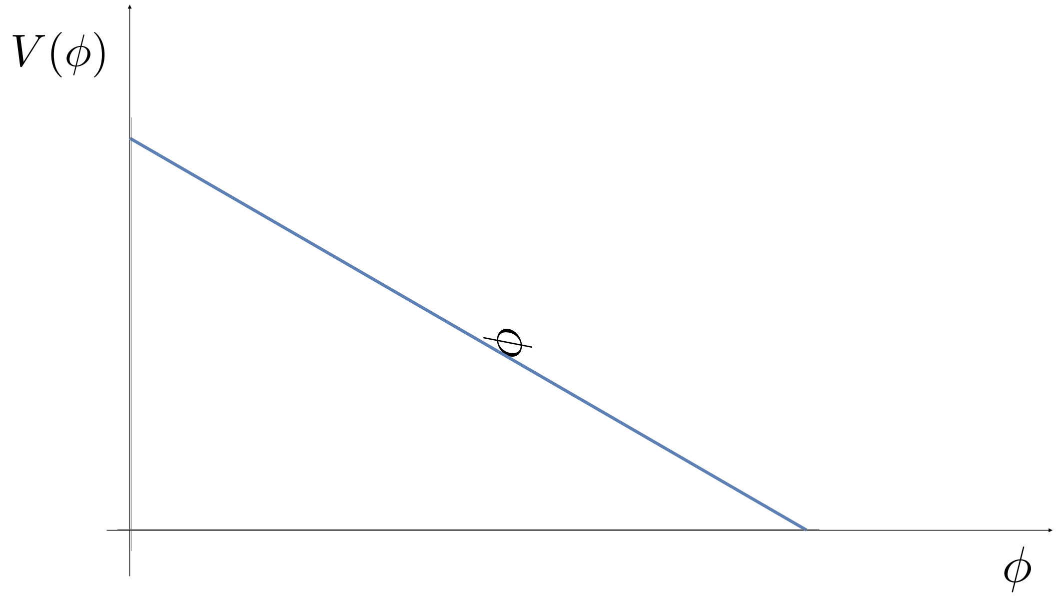
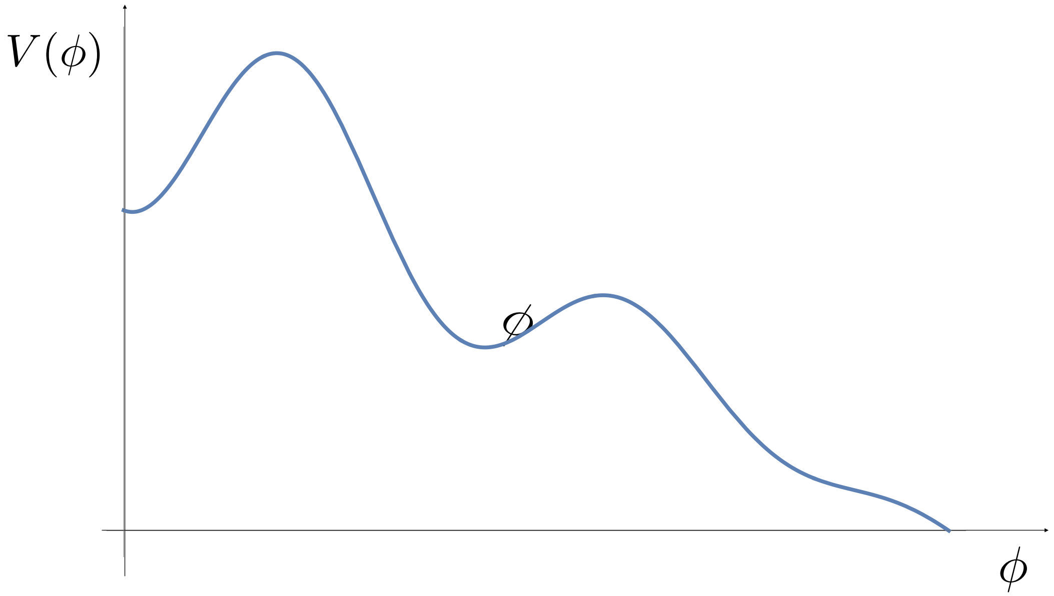


Concrete ex.: relaxion dark matter (DM)

When the universe cools the electroweak symmetry is broken, brings back the wiggles.
Now the relaxion not at the min', if trapped it starts to oscillates = DM



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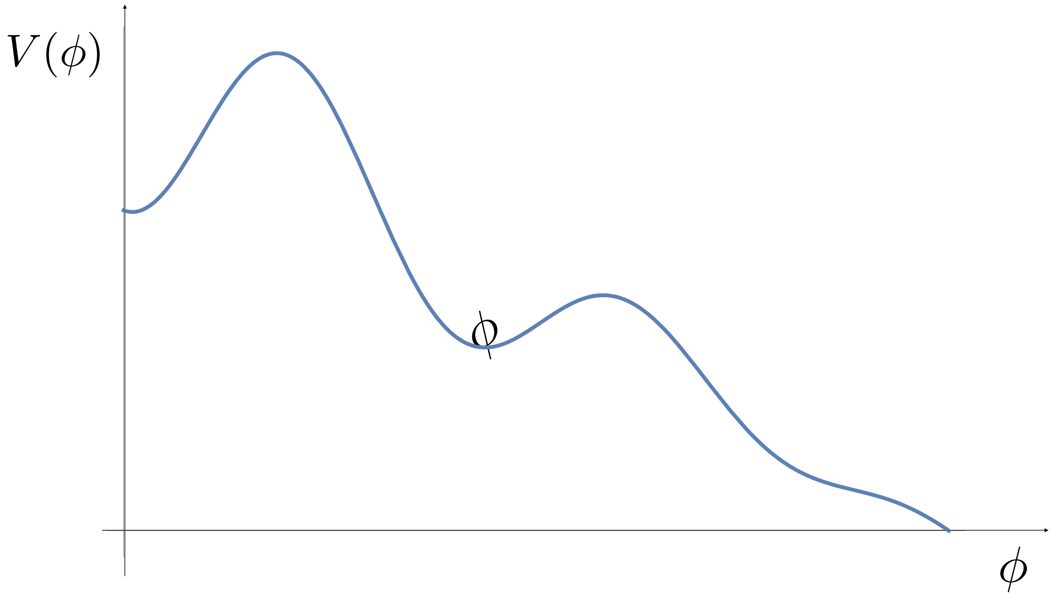

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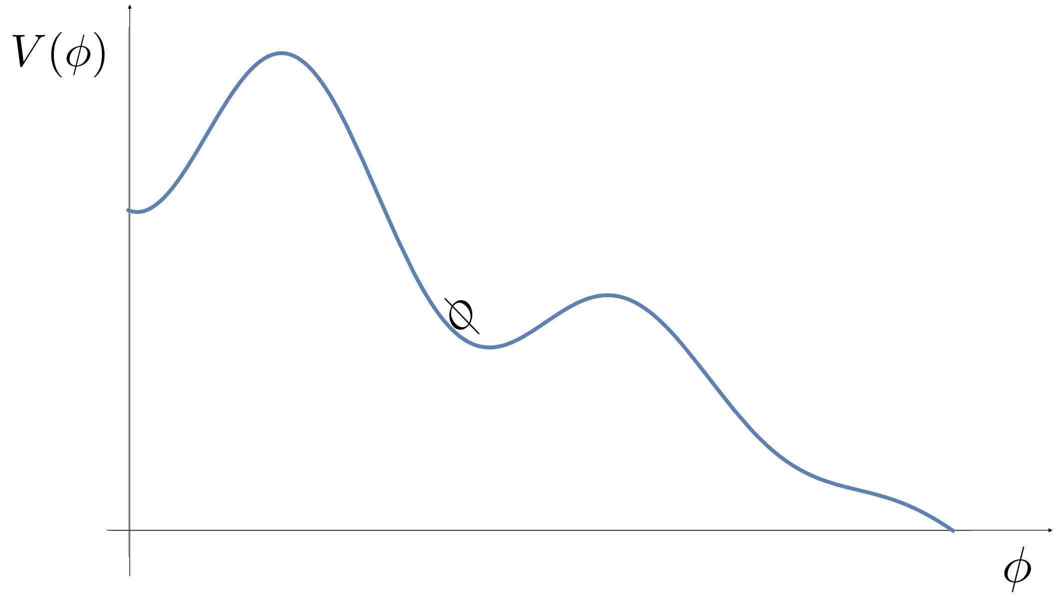

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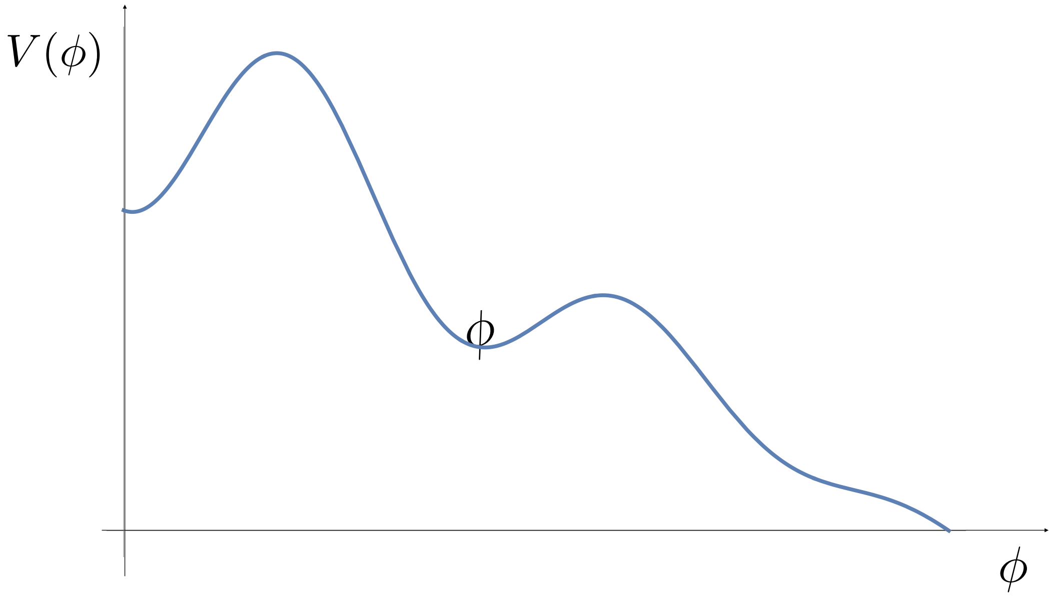
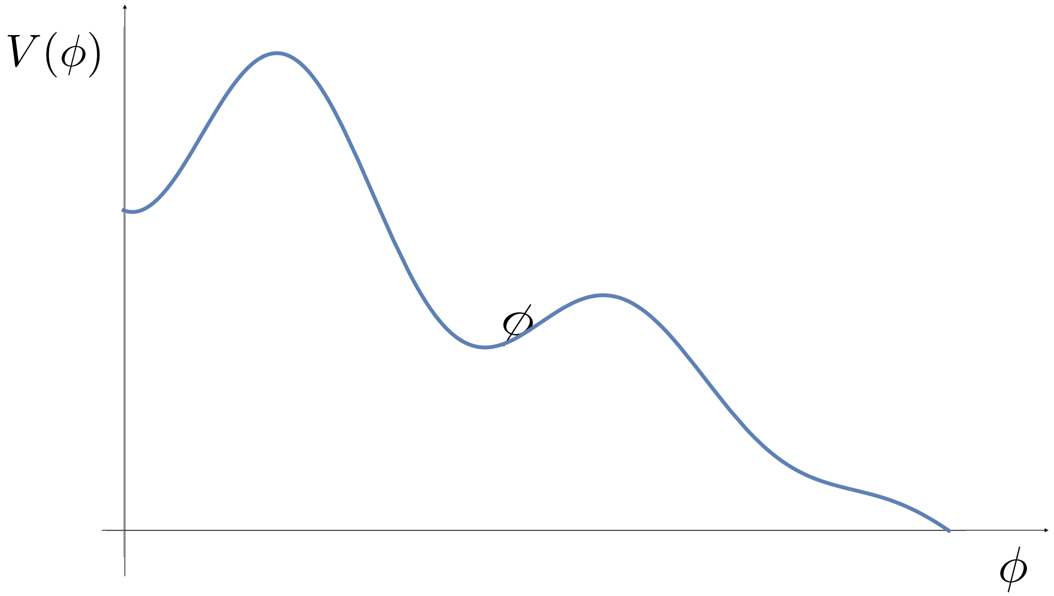


Concrete ex.: relaxion dark matter (DM)

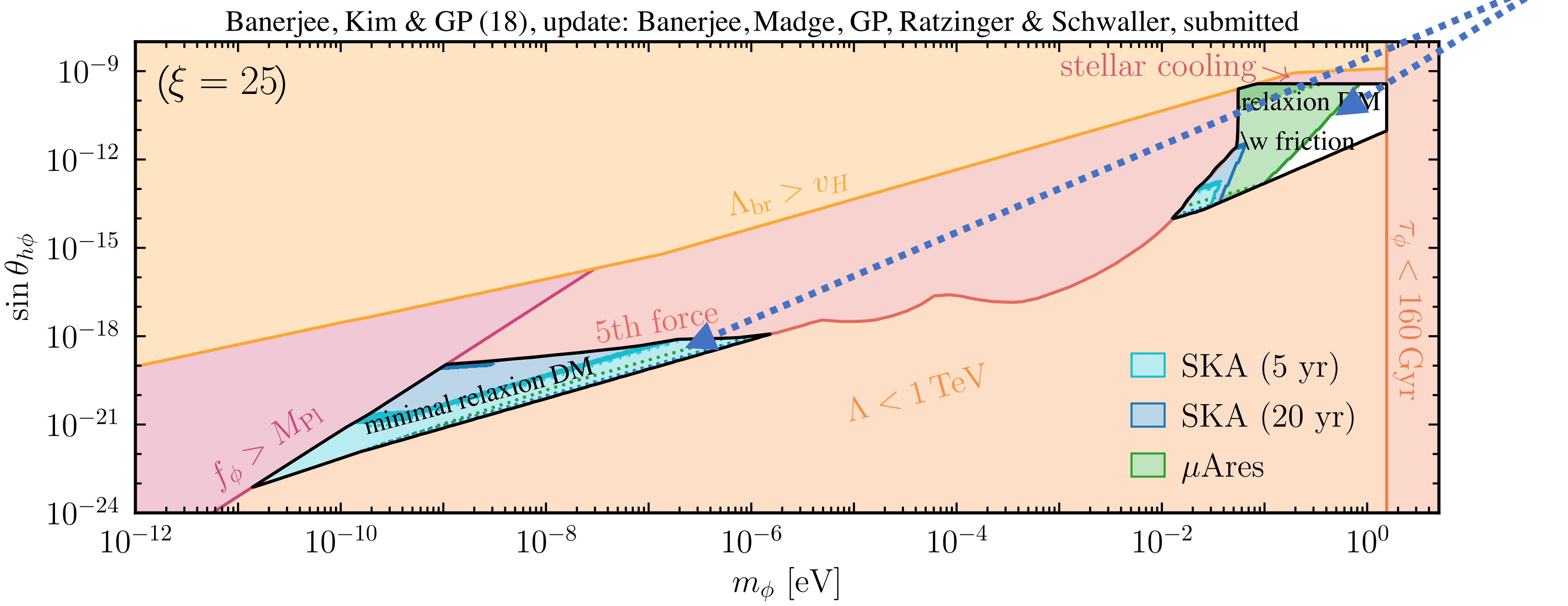
- Basic idea is similar to axion DM (but avoiding missalignment problem):

  Now the relaxion not at the min' & if it is trapped it starts to oscillates = DM.



relaxion DM+GW

DM window



The black solid line encompass the DM relaxion parameter space. The colored regions inside the viable DM space can be probed via GWs in μAres (green) or SKA (blue/turquoise). The light shading and solid lines indicate points that can be probed for a subrange of reappearance temperatures, whereas the darker shaded parts enclosed by dotted lines are accessible for all valid T_{ra}.

Equivalence principle (EP) tests, prelim

- Consider the following effective action for scalar DM: $\mathscr{L}_\phi \in d_{m_e}\dfrac{\phi}{M_{\rm Pl}} m_e \bar{e}e + d_g \dfrac{\phi}{2gM_{\rm Pl}}\beta_g GG$
- The leading action in the non-relativistic limit, say, of the electron is

$$\mathscr{L}_e^{\rm NR} = m_e(\phi) + \frac{1}{2}m_e v^2 = m_e^0 + d_{m_e}\frac{\phi}{M_{\rm Pl}} \quad \Rightarrow \quad a = d_{m_e}\frac{\phi'}{M_{\rm Pl}}$$

- Inside an atom we can rewrite it as:

$$\mathscr{L}_{\rm atm}^{\rm NR} = M_{\rm Nuc}(\phi) + Nm_e(\phi) + B \;\Rightarrow\; M_{\rm atm}a = \phi'\left(\partial_\phi M_{\rm Nuc}(\phi) + N\partial_\phi m_e(\phi)\right) \;\Rightarrow\; a = \phi'\partial_\phi \ln M_{\rm atm} \equiv \sqrt{G_N}\phi'\alpha_{\rm atm}$$

which can be readily generalised to any system.

- For a test particle at distances such that $m_\phi R \ll 1$ and say $R \gtrsim R_{\rm Earth}$ have $\phi' \propto 1/R^2$ and the acceleration is given by $a = G_N M_{\rm test}\alpha_{\rm test}M_{\rm Earth}\alpha_{\rm Earth}/R^2$

Damour & Donoghue (10)

Equivalence principle (EP) tests

- We would compare two bodies, A and B, to search for a differential acceleration effect via the EotWash parameter $\frac{\delta a_{AB}}{a} = \alpha_{\mathrm{Earth}}(\alpha_A - \alpha_B)$

- Or if we switch on one coupling d_i it is useful to define the corresponding individual "diatonic charge" $d_i Q_i \equiv \alpha_i$

- The experiment test is very simple, let's search for masses smaller than the inverse size of the Earth then we can use two test bodies on a satellite that are free falling with the satellite and just track them. That's exactly what the Microscope mission is doing some 700km above earth

- After >5 yrs of running they've achieve precision of better than $\eta_{\mathrm{EP}} < 10^{-14}$, which can be translated to the following bounds on generic scalar models

Equivalence principle (EP) tests

- For variety of coupling it can be expressed as:

$$\text{EP bounds}: \left(\frac{\delta a^{\text{test}}}{a}\right) < \eta_{\text{EP}} \sim 10^{-14} \quad \Leftrightarrow \quad \left(d_i^{(1)} d_j^{(1)}\right) \Delta Q_i^{\text{test}} Q_j^{\text{Earth}}$$

$$\vec{Q}^{\mathbf{a}} \approx F^{\mathbf{a}} \left(3\,10^{-4} - 4\,r_I + 8\,r_Z\,, 3\,10^{-4} - 3\,r_I\,, 0.9\,, 0.09 - \frac{0.04}{A^{1/3}} - 2 \times 10^6 r_I^2 - r_Z, 0.002\,r_I\right)$$

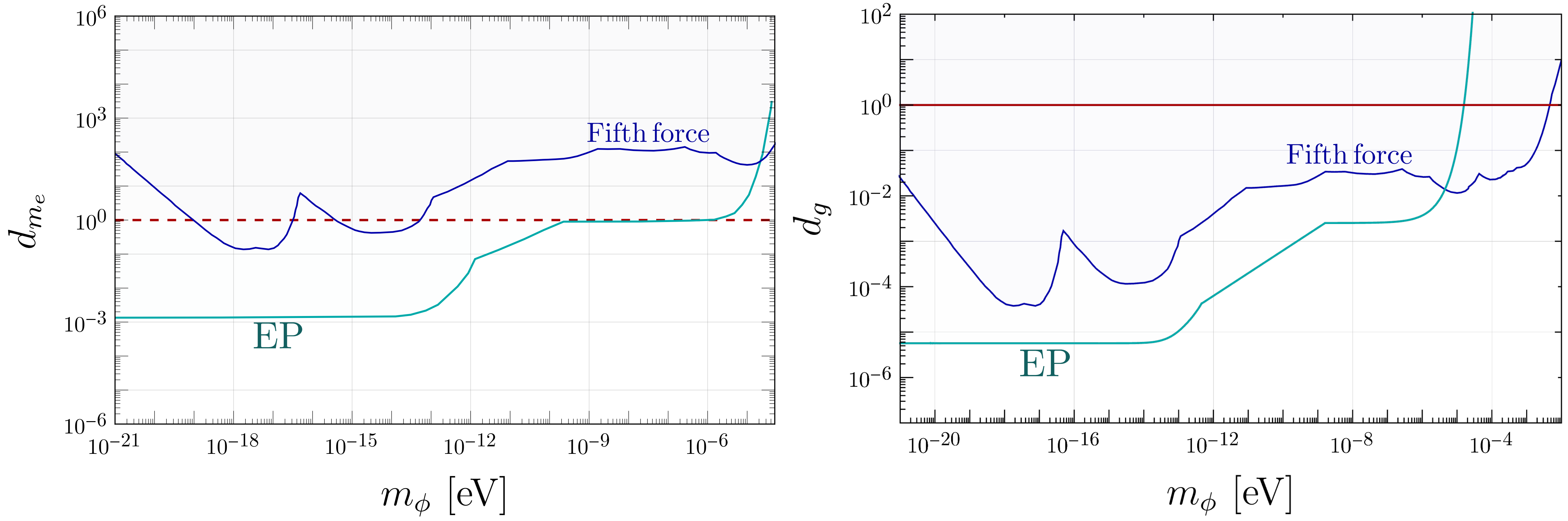
Where $\vec{X} \equiv X_{e,m_e,g,\hat{m},\delta m}$, with $\hat{m} \equiv (m_d + m_u)/2\,, \delta m \equiv (m_d - m_u)\,,\ 10^4\,r_{I;Z} \equiv 1 - 2Z/A; Z(Z-1)/A^{4/3}$, & $F^{\mathbf{a}} = 931\,A^{\mathbf{a}}/(m^{\mathbf{a}}/\text{MeV})$ with $A^{\mathbf{a}}$ being the atomic number of the atom $\mathbf{a}$

$$\Delta \vec{Q}^{\text{Mic}} \simeq 10^{-3}(-1.94\,, 0.03\,, 0.8\,, -2.61\,, -0.19)$$

Tretiak, et al.; Oswald, et al (22)

Equivalence principle (EP) tests

Banerjee, GP, Safronova, Savoray & Shalit (to appear)



Where one can find models that avoid the strongest EP bounds and for a pure dilaton the EP bound can be avoided

Tretiak, et al.; Oswald, et al (22)

Direct dark matter searches, sensitivity

- How do we search for ULDM directly?

Take for example the Lagrangian $\mathscr{L}_\phi \in d_{m_e}\frac{\phi}{M_{\rm Pl}}m_e\bar{e}e + d_g\frac{\phi}{2gM_{\rm Pl}}\beta_g GG$ and focus first about the electron coupling?

- The most sensitive way is with clocks, because $\phi \sim \frac{\sqrt{2\rho_{\rm DM}}}{m_\phi}\cos(m_\phi t)$ then the electron mass oscillates with time => energy levels oscillates with time: $E_n \sim m_e\alpha^2 1/2n^2$

- For instance: $\Delta E_{21} \sim m_e\alpha^2 1/2 \times 3/4 \times \left[1 + d_{m_e}\frac{\sqrt{2\rho_{\rm DM}}}{m_\phi M_{\rm Pl}}\left(\sim 10^{-15} \times \frac{d_{m_e}}{10^{-3}}\frac{10^{-15}\,{\rm eV}}{m_\phi}\right) \times \cos(m_\phi t)\right]$

Direct dark matter searches via clocks

- Which implies that clocks can win over EP for precision of roughly 1:10^{15} for about 1 Hz DM mass
- How the clock works: for this school it's just creating a state which is a superposition of the two states and thus oscillates with time and picking up the above phase: $\exp^{i\Delta E(m_e(t))t}$
- However, to see the effect you need to compare it to another system that would not have the above precise dependence …

Enhanced sensitivity

- The most robust coupling is to the gluons:

Mixing with the Higgs, dilaton and even QCD axion have coupling to the gluons

- How to be sensitive to the coupling to QCD?
- Could be via reduced mass, or via g-factor, magnetic moment-spin interactions-hyperfine or vibrational model in molecules, or the queen of all nuclear clock , ^{229}Th
- It is super sensitive because $E_{\rm nu-clock} \sim E_{\rm nu} - E_{\rm QED} \sim 8\,{\rm eV} \ll E_{\rm nu} \sim {\rm MeV}$

$$\frac{\Delta E}{E} = \frac{E_{\rm nu}(t) - E_{\rm QED}}{E_{\rm nu-clock}} \Rightarrow \frac{\Delta E_{\rm nu}(t)}{E_{\rm nu-clock}} \sim \frac{E_{\rm nu}}{E_{\rm nu-clock}} \times d_g \frac{m_N}{M_{\rm Pl}} \cos(m_\phi t) \sim 10^5 d_g \frac{m_N}{M_{\rm Pl}} \cos(m_\phi t)$$