A new approach to color-coherent QCD evolution

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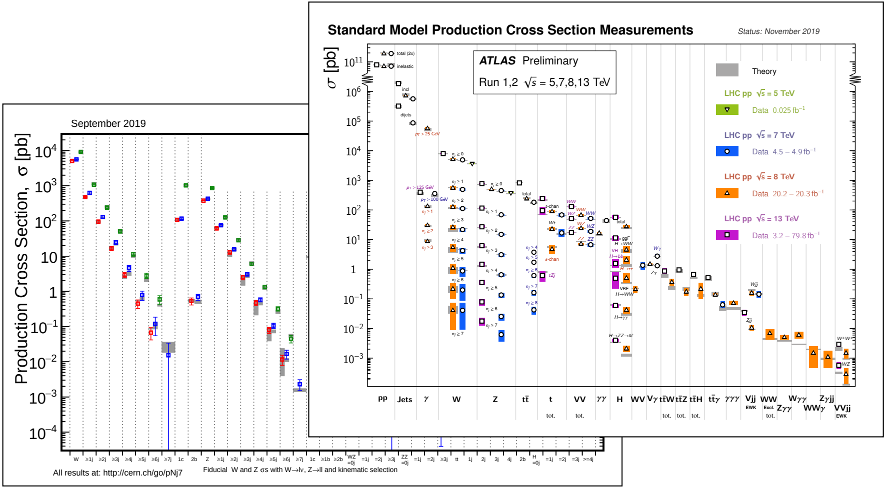
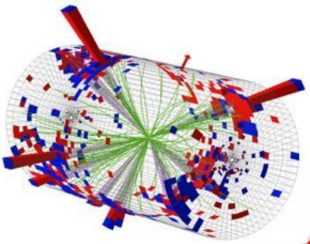
The Standard Model as we know it

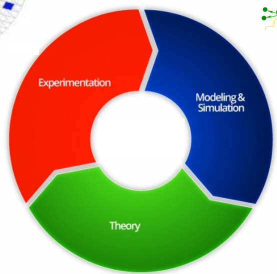


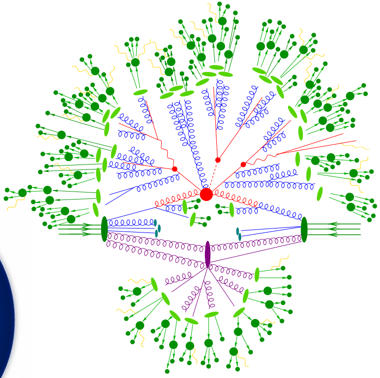
[ATLAS] https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicResults

[CMS] https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsCombined

The toolkit



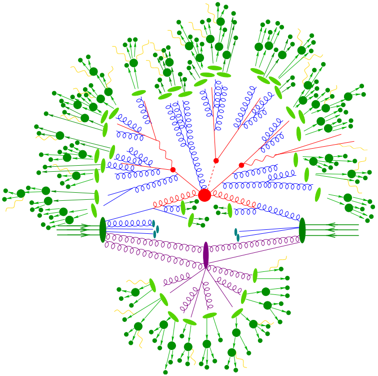




$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{D}\psi + h.c.$$

LHC event generators

[Buckley et al.] arXiv:1101.2599
[Campbell et al.] arXiv:2203.11110



- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays

Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \to X} = \sum_{i,j \in \{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1,\mu_F^2) f_{p_2,j}(x_2,\mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij \to X}(x_1 x_2, \mu_F^2)}_{\text{short distance}}$$

Connection to QCD theory

- $\hat{\sigma}_{ij\to n}(\mu_F^2) \to$ Collinearly factorized fixed-order result at N^{x}LO
  Implemented in fully differential form to be maximally useful
- $f_i(x,\mu_F^2) \to$ Collinearly factorized PDF at N^{y}LO
  Evaluated at $O(1\mathrm{GeV}^2)$ and expanded into a series above $1\mathrm{GeV}^2$
  DGLAP: $\frac{\mathrm{d}x\, x f_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\, \frac{\alpha_s}{2\pi}\big[zP_{ab}(z)\big]_+ \,\tau f_b(\tau,t)\,\delta(x-\tau z)$
  Implemented by parton showers, dipole showers, antenna showers, ...
  - In the spotlight recently due to missing systematic error budget
    - Logarithmic precision [PanScales,Deductor,Herwig,Sherpa,...]
    - Higher-order corrections [Vincia,Sherpa,Herwig,...]
  - Uncontrollable effects from on-shell kinematics mapping
  - Various possibilities of matching to collinear limit

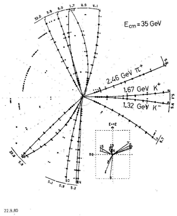
$$P_{aa}(z) = C_a\frac{2z}{1-z} + \ldots \qquad \leftrightarrow \qquad J^\mu = \sum_i \mathbf{T}_i \frac{p_i^\mu}{p_i p_j}$$

Co-design of simulations over the years

Accuracy → Precision

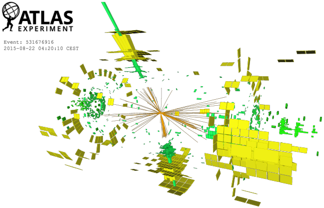
Color coherent evolution
Backward evolution
ME corrections
Color dipole formalism
Spin correlations
Color dipole showers
LO merging
NLO matching
Dipole-like showers
Antenna showers
NLO merging
Color ME corrections
NNLO matching
Threshold effects
NLO splitting kernels
Higher-order PS framework
Accuracy quantification
Amplitude evolution
NLL & NGLs in one framework
Efficient $1/N_c$ corrections

1985 — 1990 — 1995 — 2000 — 2005 — 2010 — 2015 — 2018 — 2022 →



$\sqrt{s} \times 500$ →

e^+e^- vs. pp



Simulation of QCD dipole radiation

Standard approaches and their problems

Semi-classical radiation pattern

- $U(1)$ point charge on trajectory $y^\mu(s) \to$ conserved current j^μ(x)

$$j^\mu(x) = g \int \mathrm{d}t \, \frac{dy^\mu(t)}{dt} \, \delta^{(4)}(x - y(t)) \; , \qquad\qquad g = \sqrt{4\pi\alpha}$$

- Particle receives 'kick' at $t = 0 \to$ momentum change $p_k \to p_i$

$$j^\mu(k) = \int \mathrm{d}^4x \, e^{ikx} \, j^\mu(x) = ig \left(\frac{p_k^\mu}{p_k k + i\varepsilon} - \frac{p_i^\mu}{p_i k - i\varepsilon} \right)$$

- Probability of single-gluon final state sourced by $j^\mu(k)$ at $\mathcal{O}(\alpha_s)$

$$\int \mathrm{d}W^{2\,(1)}_{a\to bc}(p_j) = \int \frac{\mathrm{d}^3\vec{p}_j}{(2\pi)^3\,2E_j} \left| \, i \int \mathrm{d}^4x \, j^\mu(x) \langle \vec{p}_j | A_\mu(x) | 0 \rangle \right|^2$$

$$= -\int \frac{\mathrm{d}^3\vec{p}_j}{(2\pi)^3\,2E_j} \sum_{\lambda=\pm} \big(j(p_j)\varepsilon_\lambda(p_j) \big) \big(j(p_j)\varepsilon_\lambda(p_j) \big)^*$$

$$\to |g|^2 \left(\frac{\mu^2 e^{\gamma_E}}{4\pi} \right)^\varepsilon \int \frac{\mathrm{d}^D\vec{p}_j}{(2\pi)^D} \, \frac{2p_i p_k}{(p_i p_j)(p_k p_j)} \, \delta(p_j^2)$$

  - Universal, semi-classical integrand
  - Originates in gauge boson radiation off conserved charge
  - Leads to double logarithm $1/2\log^2(2p_ip_k/\mu^2)$

Semi-classical radiation pattern

[Marchesini,Webber] NPB310(1988)461

- Soft gluon radiator can be written in terms of energies and angles

$$J_\mu J^\mu \to \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{W_{ik,j}}{E_j^2}$$

  Angular “radiator” function

$$W_{ik,j} = \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})}$$

- Divergent as $\theta_{ij} \to 0$ and as $\theta_{jk} \to 0$

  $\to$ Expose individual collinear singularities using $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

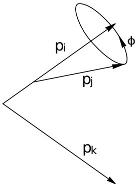
$$\tilde{W}^i_{ik,j} = \frac{1}{2}\left[\frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{kj})} + \frac{1}{1 - \cos\theta_{ij}} - \frac{1}{1 - \cos\theta_{kj}}\right]$$

  - Divergent as $\theta_{ij} \to 0$, but regular as $\theta_{kj} \to 0$
  - Convenient properties upon integration over azimuthal angle

Semi-classical radiation pattern

- Work in a frame where direction of $\vec{p}_i$ aligned with z-axis

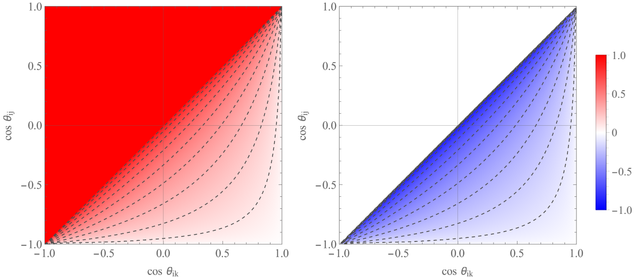
$$\cos\theta_{kj} = \cos\theta_k^i \cos\theta_j^i + \sin\theta_k^i \sin\theta_j^i \cos\phi_{kj}^i$$



- Integration over ϕ_j yields

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_{kj}^i \tilde{W}_{ik,j}^i = \frac{1}{1-\cos\theta_j^i} \times \begin{cases} 1 & \text{if} \quad \theta_j^i < \theta_k^i \\ 0 & \text{else} \end{cases}$$

  - On average, no radiation outside cone defined by parent dipole
  - Differential radiation pattern more intricate:
    Positive & negative contributions outside cone sum to zero



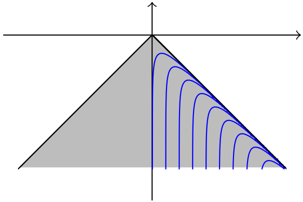
Angular ordered parton showers

- Differential radiation probability

$$\mathrm{d}\mathcal{P} = \mathrm{d}\Phi_{+1}|M|^2 \approx \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\,\mathrm{d}z\,\frac{\alpha_s}{2\pi}\,P_{\widetilde{ij}i}(z)$$

  - Ordering parameter $\tilde{q}^2 = \frac{2p_ip_j}{z(1-z)} \approx 4E_{\widetilde{ij}}^2 \sin^2\frac{\theta_{ij}}{2}$
  - Splitting variable $z = \frac{1+\cos\theta_{ik}}{2} = \frac{p_ip_k}{(p_i+p_j)p_k}$

- Lund plane filled from center to edges
  - Random walk in p_T^2
  - Color factors correct for observables insensitive to azimuthal correlations
  - Small dead zone at $\ln(p_T^2/\tilde{s}) \approx 0$



- Usually disfavored due to dead zones
  Not suitable to resum non-global logartihms

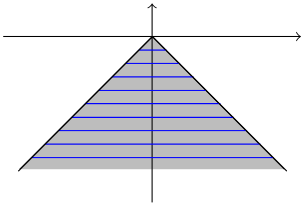
Dipole showers

- Differential radiation probability for the dipole

$$\mathrm{d}\mathcal{P} = \mathrm{d}\Phi_{+1}|M|^2 \approx \frac{\mathrm{d}p_T^2}{p_T^2}\,\mathrm{d}\eta\,\frac{\alpha_s}{2\pi}\,\tilde{P}_{\widetilde{ij}}(z)$$

  - Ordering parameter p_T^2
  - Splitting variable $z = 1 - \frac{s_{ij}}{s - s_{ij}}\,\mathrm{e}^{-2\eta}$

- Lund plane filled from top to bottom
  - Random walk in η
  - Color factors in CFFE approximation
  - Both ends of dipole evolve simultaneously
  - No dead zones

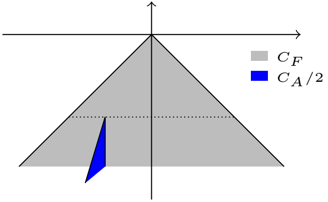


- Solves problem of dead zones
  Known issues with color coherence

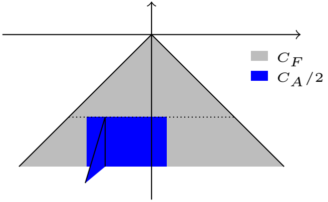
Problems with average color charges

[Gustafsson] NPB392(1993)251

- In angular ordered showers angles are measured in the event center-of-mass frame
$\rightarrow$ coherence effects modeled by angular ordering variable agree on average with matrix element



- In dipole-like showers angles effectively measured in center-of-mass frame of emitting color dipole
$\rightarrow$ angular coherence not reflected by setting average QCD charge

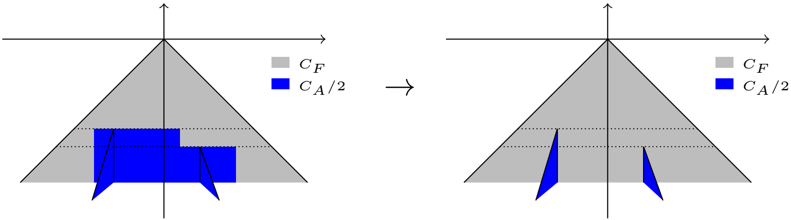


- Emission off "back plane" in Lund diagram should be associated with C_F, but is partly associated with $C_A/2$ in dipole showers
- All-orders problem that appears first in 2-gluon emission case

Solutions for average color charges

[Gustafsson] NPB392(1993)251
[Hamilton,Medves,Salam,Scyboz,Soyez] arXiv:2011.10054

- Analyze rapidity of gluon emission in event center-of-mass frame
- Sectorize phase space and assign gluon to closest parton
  $\rightarrow$ choose corresponding color charge for evolution
- Same technology for higher number of emissions

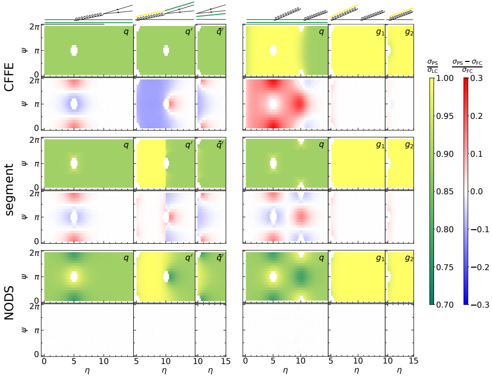


- Starting with 4 emissions, there be “color monsters”
  - Quartic Casimir operators (easy)
  - Non-factorizable contributions (hard)

Solutions for average color charges

[Hamilton,Medves,Salam,Scyboz,Soyez] arXiv:2011.10054

- Can include double-soft corrections via reweighting [Giele,Kosower,Skands] arXiv:1102.2126
- Algorithm scales as N^2 but can be simplified while retaining formal accuracy
- Implementation as nested corrections in rapidity segments of parent dipole
- Excellent agreement with full matrix element



- Good agreement with full-color evolution [Hatta,Ueda] arXiv:1304.6930

Problems with momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam] arXiv:1805.09327

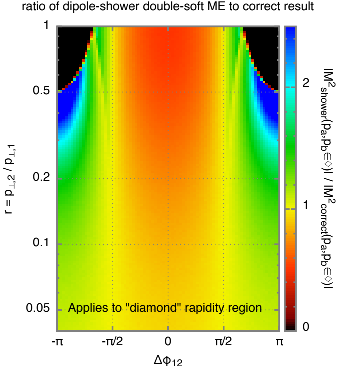
- Subtle problems in standard dipole-like momentum mapping

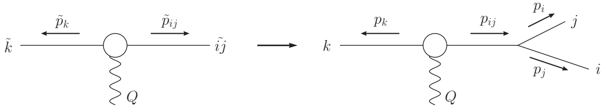
$$p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\right)\tilde{p}_k^\mu$$

$$p_i^\mu = \tilde{z}\,\tilde{p}_{ij}^\mu + (1-\tilde{z})\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu + k_\perp^\mu$$

$$p_j^\mu = (1-\tilde{z})\,\tilde{p}_{ij}^\mu + \tilde{z}\,\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu - k_\perp^\mu$$

- Induces angular correlations across multiple emissions
- Spoils agreement w/ analytic resummation





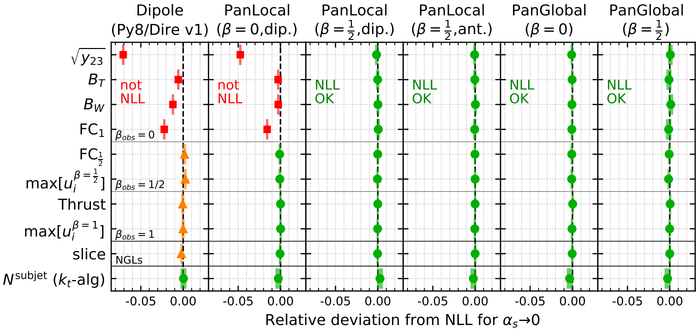
Solutions for momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] arXiv:2002.11114

- Problem can be solved by partitioning of antenna radiation pattern and choosing a suitable evolution variable ($\beta \sim 1/2$)

$$k_T = \rho v e^{\beta|\bar{\eta}|} \qquad \rho = \left(\frac{s_i s_j}{Q^2 s_{ij}}\right)^{\beta/2}$$

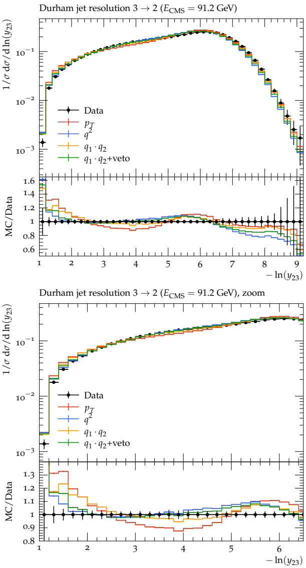
- Different recoil schemes can lead to NLL result if β chosen appropriately: Local dipole, local antenna, and global antenna
- NLL correct for global and non-global observables in $e^+e^- \to$hadrons



Solutions for momentum mapping

[Bewick,Ferrario-Ravasio,Richardson,Seymour] arXiv:1904.11866

- Recoil schemes affect logarithmic accuracy but impact also phase-space coverage
- In context of angular ordered Herwig 7 (NLL accurate for global observables)
  - q_T preserving scheme:
    Maintains logarithmic accuracy
    Overpopulates hard region
  - q^2 preserving scheme:
    Breaks logarithmic accuracy
    Good description of hard region
  - Dot product preserving scheme (new):
    Maintains logarithmic accuracy
    Good description of hard radiation



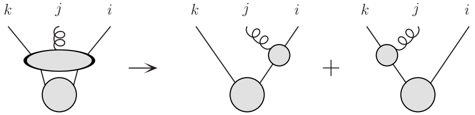
A new perspective on old ideas

Identified partons & azimuthal angle dependence

The semi-classical matrix element revisited

- Alternative to additive matching: partial fraction matrix element & match to collinear sectors [Ellis,Ross,Terrano] NPB178(1981)421, [Catani,Seymour] hep-ph/9605323

$$\frac{W_{ik,j}}{E_j^2} \quad \rightarrow \quad \frac{1}{p_i p_j}\,\frac{p_i p_k}{(p_i+p_k)p_j} + \frac{1}{p_k p_j}\,\frac{p_i p_k}{(p_i+p_k)p_j}$$



  - Captures matrix element both in angular ordered and unordered region
  - Caveat: Oversampling difficult for certain kinematics maps

- Separate into energy & angle first [Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057
  Partial fraction angular radiator only: $W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}$

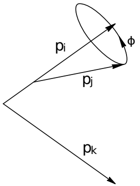
$$\bar{W}^i_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{kj})}$$

  - Bounded by $(1-\cos\theta_{ij})\bar{W}^i_{ik,j} < 2$
  - Strictly positive

The semi-classical matrix element revisited

- Integration over ϕ_j yields

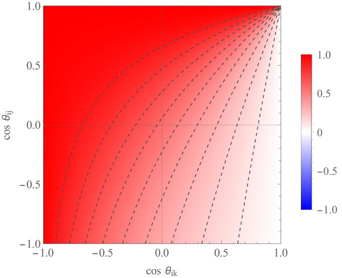
$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_{kj}^i \bar{W}_{ik,j}^i = \frac{1}{1-\cos\theta_j^i}\frac{1}{\sqrt{(\bar{A}_{ij,k}^i)^2 - (\bar{B}_{ij,k}^i)^2}}$$



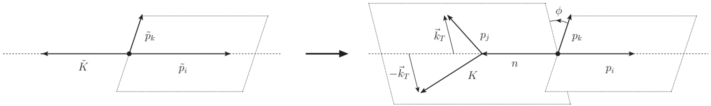
  - Radiation across all of phase space
  - Probabilistic radiation pattern

$$\bar{A}_{ij,k}^i = \frac{2-\cos\theta_j^i(1+\cos\theta_k^i)}{1-\cos\theta_k^i}$$

$$\bar{B}_{ij,k}^i = \frac{\sqrt{(1-\cos^2\theta_j^i)(1-\cos^2\theta_k^i)}}{1-\cos\theta_k^i}$$



Kinematics mapping revisited



- In collinear limit, splitting kinematics defined by ($n \to$ auxiliary vector)

$$p_i \xrightarrow{i||j} z\,\tilde{p}_i\;, \qquad p_j \xrightarrow{i||j} (1-z)\,\tilde{p}_i \qquad \text{where} \qquad z = \frac{p_i n}{(p_i+p_j)n}$$

- Parametrization, using hard momentum $\tilde{K}$

$$p_i = z\,\tilde{p}_i\;, \qquad n = \tilde{K} + (1-z)\,\tilde{p}_i$$

- Using on-shell conditions & momentum conservation ($\kappa = \tilde{K}^2/(2\tilde{p}_i\tilde{K})$)

$$\begin{aligned} p_j &= (1-z)\,\tilde{p}_i + v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) + k_\perp \\ K &= \tilde{K} - v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) - k_\perp \end{aligned}$$

- Momenta in $\tilde{K}$ Lorentz-boosted to new frame K [Catani,Seymour] hep-ph/9605323

$$p_l^\mu \to \Lambda^\mu{}_\nu(K,\tilde{K})\,p_l^\nu\;, \qquad \Lambda^\mu{}_\nu(K,\tilde{K}) = g^\mu{}_\nu - \frac{2(K+\tilde{K})^\mu(K+\tilde{K})_\nu}{(K+\tilde{K})^2} + \frac{2\tilde{K}^\mu K_\nu}{K^2}\;.$$

Logarithmic accuracy – Analytic proof

- Logarithmic accuracy of parton shower can be quantified by comparing results to (semi-)analytic resummation e.g. [Banfi,Salam,Zanderighi] hep-ph/0407286
- Example: Thrust or FC_0 in $e^+e^- \to$hadrons

- Define a shower evolution variable $\xi = k_T^2/(1-z)$
- Parton-shower one-emission probability for $\xi > Q^2\tau$

$$R_{\text{PS}}(\tau) = 2\int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \int_{z_{\text{min}}}^{z_{\text{max}}} dz\, \frac{\alpha_s\left(k_T^2\right)}{2\pi}\, C_F \left[\frac{2}{1-z} - (1+z)\right] \Theta(\eta)$$

- Approximate to NLL accuracy

$$R_{\text{NLL}}(\tau) = 2\int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \left[\int_0^1 dz\, \frac{\alpha_s\left(k_T^2\right)}{2\pi}\, \frac{2\,C_F}{1-z}\Theta(\eta) - \frac{\alpha_s(\xi)}{\pi}\, C_F B_q\right]$$

Logarithmic accuracy – Analytic proof

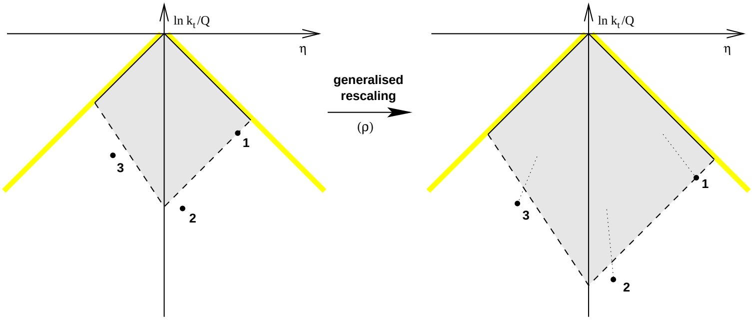
- Cumulative cross section $\Sigma(\tau) = e^{-R(\tau)}\mathcal{F}(\tau)$ obtained from all-orders resummed result by Taylor expansion of virtual corrections in cutoff ε

$$\mathcal{F}(\tau) = \int \mathrm{d}^3k_1|M(k_1)|^2\, e^{-R'\ln\frac{\tau}{\varepsilon v_1}} \sum_{m=0}^{\infty}\frac{1}{m!}\left(\prod_{i=2}^{m+1}\int_{\varepsilon v_1}^{v_1}\mathrm{d}^3k_i|M(k_i)|^2\right) \times \Theta\big(\tau - V(\{p\},k_1,\ldots,k_n)\big)$$

  - $\mathcal{F}(\tau)$ is pure NLL & accounts for (correlated) multiple-emission effects
- In order to make $\mathcal{F}(\tau)$ calculable, make the following assumptions
  - Observable is recursively infrared and collinear safe
  - Hold $\alpha_s(Q^2)\ln\tau$ fixed, while taking limit $\tau\to 0$
    $\rightarrow$ Can factorize integrals and neglect kinematic edge effects

    Can be interpreted as $\boldsymbol{\alpha_s \to 0}$ or $\boldsymbol{s\to\infty}$ limit

Logarithmic accuracy – Analytic proof



- $\alpha_s \to 0$ / $s \to \infty$ limit taken by similarity transformation of Lund plane
- Can be parametrized in terms of scaling parameter ρ
  a, b – observable-dependent resummation parameters

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)}$$

$$\eta_l \to \eta'_l = \eta - \xi_l \frac{\ln\rho}{a+b}\,, \qquad \text{where} \qquad \xi = \frac{\eta}{\eta_{\max}}$$

- NLL precision requires scaling to be maintained after additional emissions

Logarithmic accuracy – Analytic proof

- Lorentz transformation defined by shift $\tilde{K} \to K$

$$K^\mu = \tilde{K}^\mu - X^\mu \,, \qquad \text{where} \qquad X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu$$

- X is small, but is it small enough? Rewrite

$$\Lambda^\mu{}_\nu(K,\tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

- In NLL limit, coefficients scale as

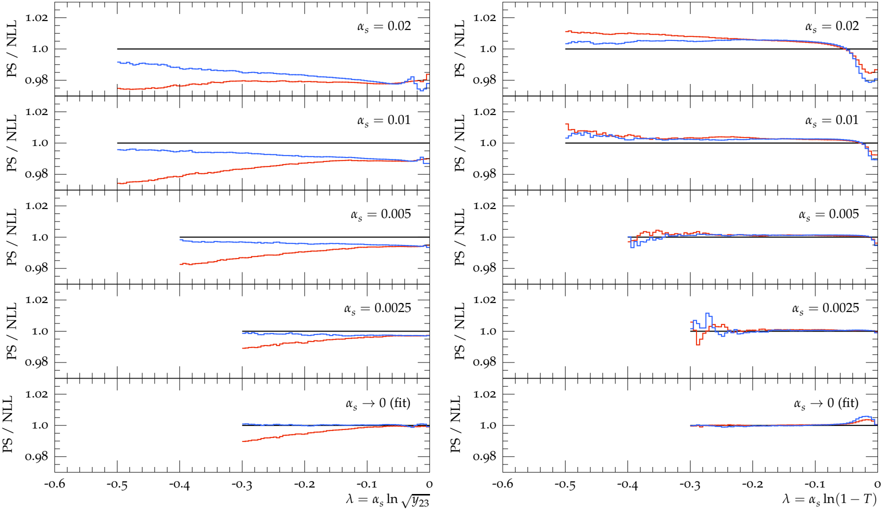
$$A^\nu \xrightarrow{\rho\to 0} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2}\,, \qquad \text{and} \qquad B^\nu \xrightarrow{\rho\to 0} \frac{\tilde{K}^\nu}{\tilde{K}^2}\,.$$

- Simplify situation by taking $a=1$, $b=0$ (worst offenders)
  Relative momentum shift of soft emission particle l becomes

$$\begin{aligned}
\Delta p_l^0/\tilde{p}_l^0 &\sim 2X^0 + \rho^{1-\max(\xi_i,\xi_j)}\tilde{K}^0 &&\sim \rho^{1-\max(\xi_i,\xi_j)}\\
\Delta p_l^3/\tilde{p}_l^3 &\sim X^3 &&\sim \rho^{1-\max(\xi_i,\xi_j)}\\
\Delta p_l^{1,2}/\tilde{p}_l^{1,2} &\sim \rho^{-\xi_l}X^{1,2} &&\sim \rho^{1-\xi_l}
\end{aligned}$$

- For hard momenta, leading terms in X^μ cancel exactly
  Remaining components scale as ρ or stronger

Logarithmic accuracy – Numerical checks

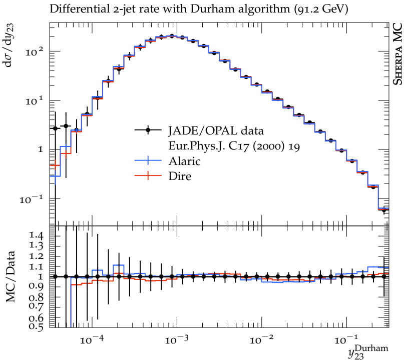


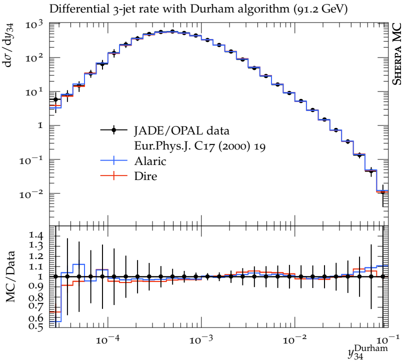
- At fixed $\lambda = \alpha_s \log v$, deviation from NLL should be proportional to α_s
- Dire algorithm (red) fails, Alaric (blue) passes

Comparison to experimental data: LEP I

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

- Comparison to experimental data from LEP

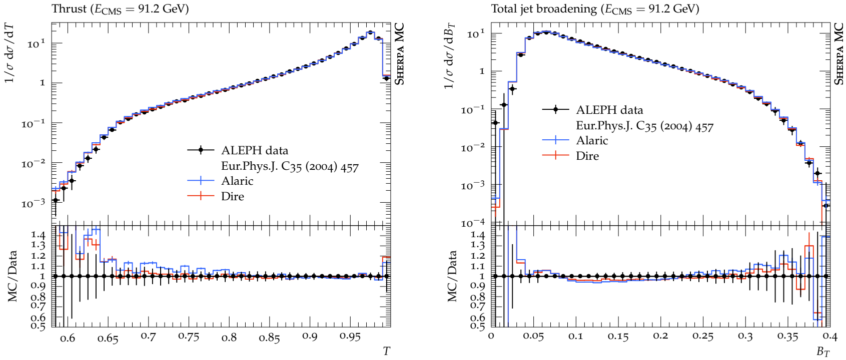




Comparison to experimental data: LEP I

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

- Comparison to experimental data from LEP



Matching to fixed-order calculations

Modified subtraction and MC@NLO

- Leading-order calculation for observable O

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\,O(\Phi_B)$$

- NLO calculation for same observable

$$\langle O\rangle = \int \mathrm{d}\Phi_B\left\{\mathrm{B}(\Phi_B) + \tilde{V}(\Phi_B)\right\}O(\Phi_B) + \int \mathrm{d}\Phi_R\,\mathrm{R}(\Phi_R)\,O(\Phi_R)$$

- Parton-shower result until first emission

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\left[\Delta^{(\mathrm{K})}(t_c)\,O(\Phi_B) + \int_{t_c}\mathrm{d}\Phi_1\,\mathrm{K}(\Phi_1)\,\Delta^{(\mathrm{K})}(t(\Phi_1))\,O(\Phi_R)\right]$$

$$\overset{\mathcal{O}(\alpha_s)}{\to}\int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\left\{1 - \int_{t_c}\mathrm{d}\Phi_1\mathrm{K}(\Phi_1)\right\}O(\Phi_B) + \int_{t_c}\mathrm{d}\Phi_B\mathrm{d}\Phi_1\,\mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)\,O(\Phi_R)$$

Phase space: $\mathrm{d}\Phi_1 = \mathrm{d}t\,\mathrm{d}z\,\mathrm{d}\phi$

Splitting functions: $\mathrm{K}(t,z) \to \alpha_s/(2\pi t)\sum \mathrm{P}(z)\,\Theta(\mu_Q^2 - t)$

Sudakov factors: $\Delta^{(\mathrm{K})}(t) = \exp\left\{-\int_t \mathrm{d}\Phi_1\mathrm{K}(\Phi_1)\right\}$

Modified subtraction and MC@NLO

[Frixione,Webber] hep-ph/0204244

- Matching achieved by subtracting PS at $\mathcal{O}(\alpha_s)$ from NLO ...

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \left[\mathrm{B}(\Phi_B)\, \mathrm{K}(\Phi_1) - \mathrm{S}(\Phi_R) \right]$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \left[\mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\, \mathrm{K}(\Phi_1) \right]$$

- ... and combining with PS, written as a generating functional $\mathcal{F}$

$$\langle O \rangle = \int \mathrm{d}\Phi_B\, \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\, \mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2, O) + \int \mathrm{d}\Phi_R\, \mathrm{H}^{(\mathrm{K})}(\Phi_R)\, \mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R), O)$$

- Key ingredient needed: Integrated parton-shower splitting functions
- Utilize similarity to CS identified particle scheme [Catani,Seymour] hep-ph/9605323
  Integrals known, but some counterterms must be evaluated numerically

Monte-Carlo counterterms

- From CS identified particle scheme [Catani,Seymour] hep-ph/9605323

$$\int_{m+1} \mathrm{d}\sigma^S + \int_m \mathrm{d}\sigma^C = \frac{1}{2} \sum_{i=g,q,\bar{q}} \sum_{\tilde{\imath}=1}^{m} \int_0^1 \frac{\mathrm{d}z}{z^{2-2\epsilon}} \int_m \mathrm{d}\sigma^B\left(\ldots, \frac{p_i}{z}, \ldots\right) \otimes \hat{\mathbf{I}}_{\tilde{\imath}i}^{(\mathrm{FS})}$$

- In $\overline{\mathrm{MS}}$ scheme, $\hat{\mathbf{I}}_{\tilde{\imath}i}^{(\mathrm{FS})} = \delta(1-z)\mathbf{I}_{\tilde{\imath}i} + \mathbf{P}_{\tilde{\imath}i} + \mathbf{H}_{\tilde{\imath}i}$
  Integration of $\mathbf{P}_{\tilde{\imath}i}$ yields zero, only non-trivial term is

$$\int_0^1 \mathrm{d}z\, \mathbf{H}_{\tilde{\imath}i} = -\frac{\alpha_s}{2\pi} \sum_{k=1, k\neq\tilde{\imath}}^{m} \frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2} \left\{ \mathcal{K}^{\tilde{\imath}i} + \delta_{\tilde{\imath}i}\,\mathrm{Li}_2\left(1 - \frac{2\tilde{p}_i\tilde{p}_k\,\tilde{K}^2}{(\tilde{p}_i\tilde{K})(\tilde{p}_k\tilde{K})}\right) - \int_0^1 \mathrm{d}z\, P_{\mathrm{reg}}^{qq}(z) \ln\frac{n^2\tilde{p}_i\tilde{p}_k}{2z(\tilde{p}_i n)^2} \right\}$$

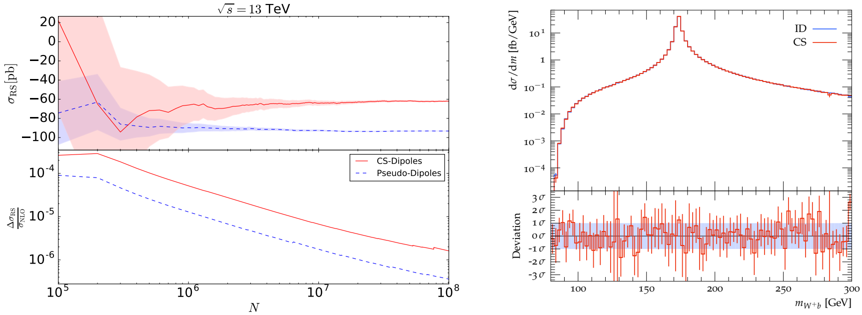
- $\mathcal{K}^{\tilde{\imath}i}$ for final-state evolution

$$\mathcal{K}^{\tilde{\imath}i} = \int_0^1 \mathrm{d}z\,\left(\bar{K}^{\tilde{\imath}i}(z) + \tilde{K}^{\tilde{\imath}i}(z) + 2P_{\tilde{\imath}i}(z)\ln z\right)\,.$$

  Initial-state result obtained from known breaking of
  Gribov-Lipatov relation [Curci,Furmanski,Petronzio] NPB175(1980)27

Application at fixed order

[Liebschner,Siegert,SH] arXiv:1807.04348



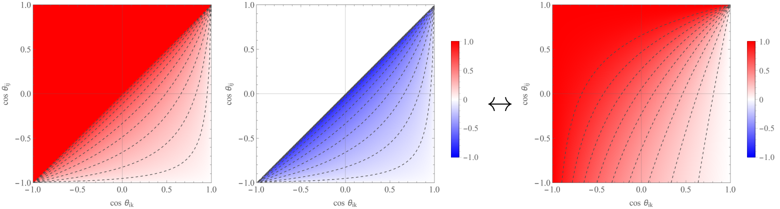
- Slightly more complicated than standard CS subtraction, but
- Improved convergence for example in $pp \to t\bar{t}$ due to easily constructible resonance-awareness

The Alaric scheme

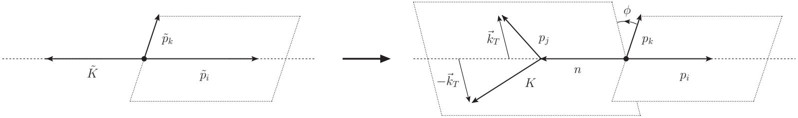
Differences to existing parton showers

Summary of Alaric scheme

- Partial fractioning of eikonal allows full phase-space coverage while keeping splitting function positive definite



- Disentangling matrix element and kinematics allows appropriate choice of recoiler and enables analytic proof of NLL correctness



- Induces azimuthal angle dependence of soft splitting function
$\rightarrow$ natural and most general form of the evolution kernel

$$\bar{W}^{i}_{ik,j} = \frac{1}{1-\cos\theta_j}\,\frac{1-\cos\theta_k}{2-\cos\theta_j-\cos\theta_k\cos\theta_j-\sin\theta_k\sin\theta_j\cos\phi_{jk}}$$

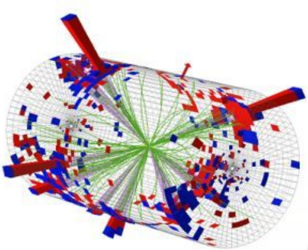
Summary and Outlook

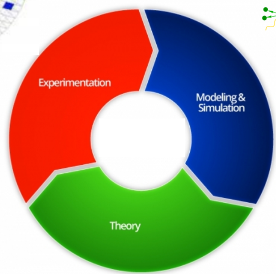
- Lots of activity in parton shower development right now
  - Logarithmic precision [PanScales,Deductor,Herwig,Sherpa,...]
  - Higher-order kernels [Vincia,Sherpa,Herwig,...]
  - Interplay w/ NNLL, CMW [PanScales,Sherpa,...]

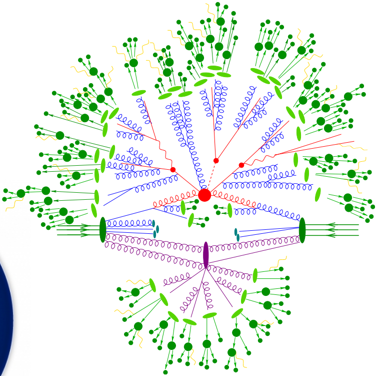
- New Alaric scheme contributes
  - Intuitive understanding & connection to angular ordering
  - Simple, analytic proof of NLL precision
  - Unified treatment of FSR & ISR
  - Easy matching to NLO

- Next steps
  - Spin correlations & $1/N_c$ terms
  - Higher-order splitting functions
  - Heavy flavor evolution

Exciting times ahead!



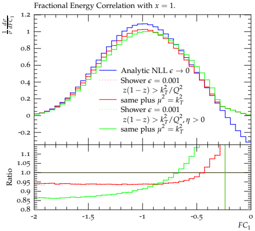




$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi + h.c.$$

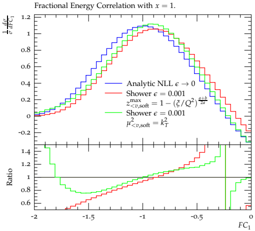
Numerical effects away from the $\alpha_s \to 0$ limit

[Reichelt,Siegert,SH] arXiv:1711.03497



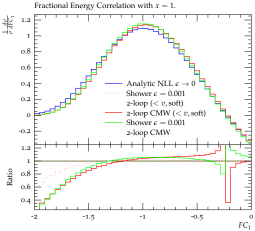
Single emission effects

- 4-mom conservation
- PS sectorization
- k_T scale in coll. terms



Multiple emission effects

- z bounds by unitarity
- k_T scale by unitarity



Effects of scale choice

- 2-loop CMW in all soft terms
- 2-loop CMW overall

- Simplest process and simplest type of observable, still sizable differences away from $\tau \to 0$ limit
- How do we proceed to quantify precision in the intermediate region (“between” NLL and NLO) ?