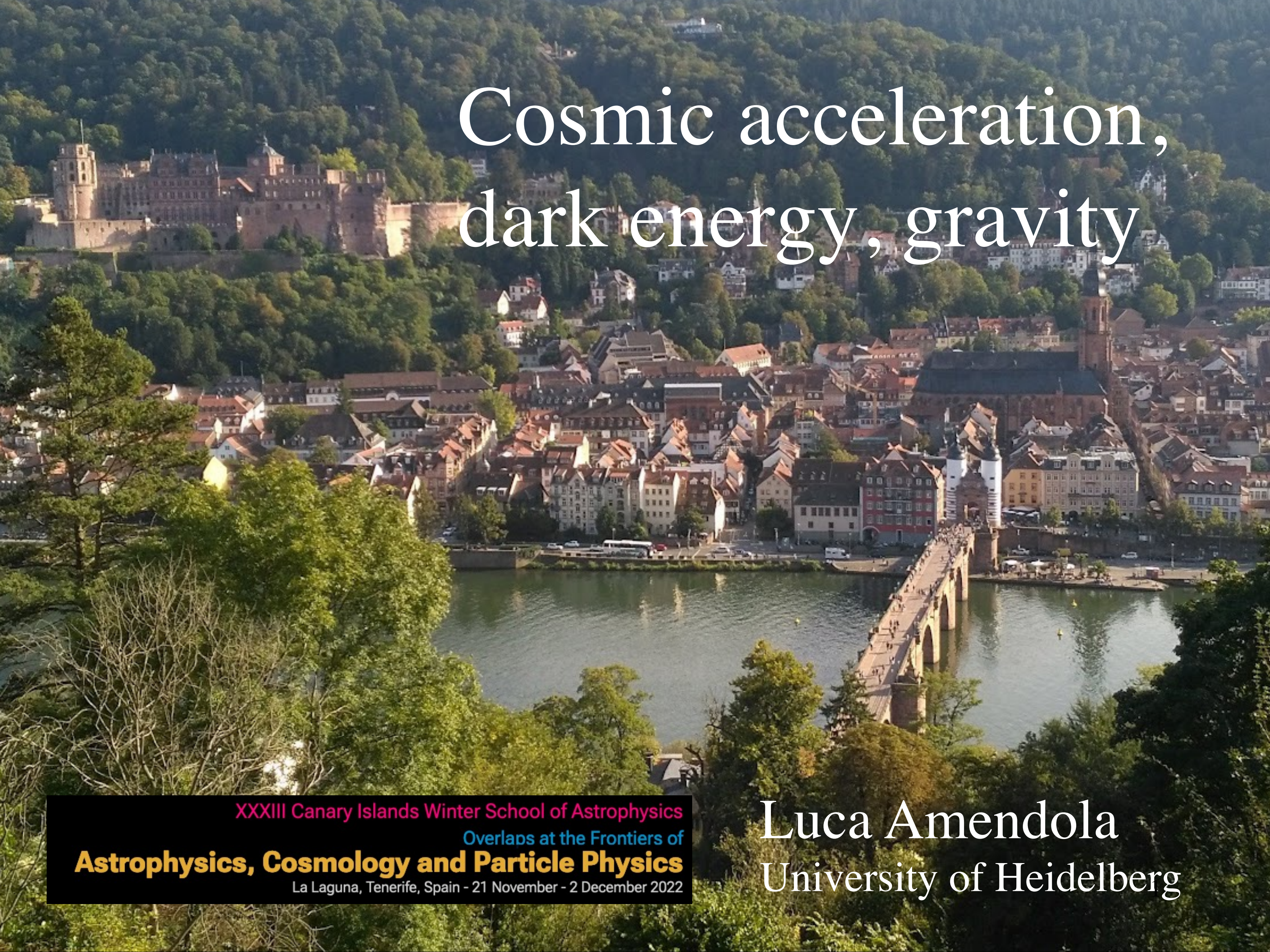

Cosmic acceleration, dark energy, gravity

XXXIII Canary Islands Winter School of Astrophysics
Overlaps at the Frontiers of
Astrophysics, Cosmology and Particle Physics
La Laguna, Tenerife, Spain - 21 November - 2 December 2022

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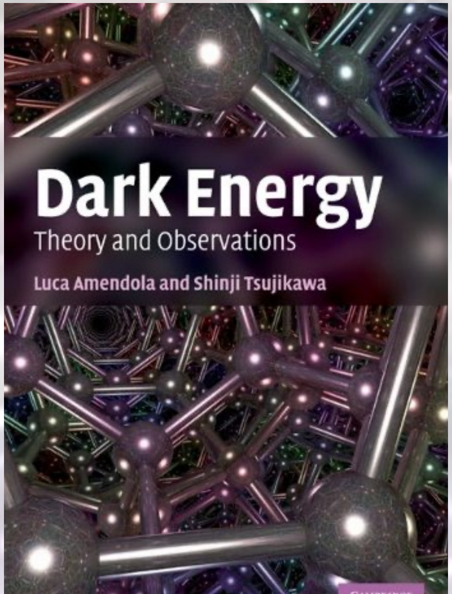
University of Heidelberg

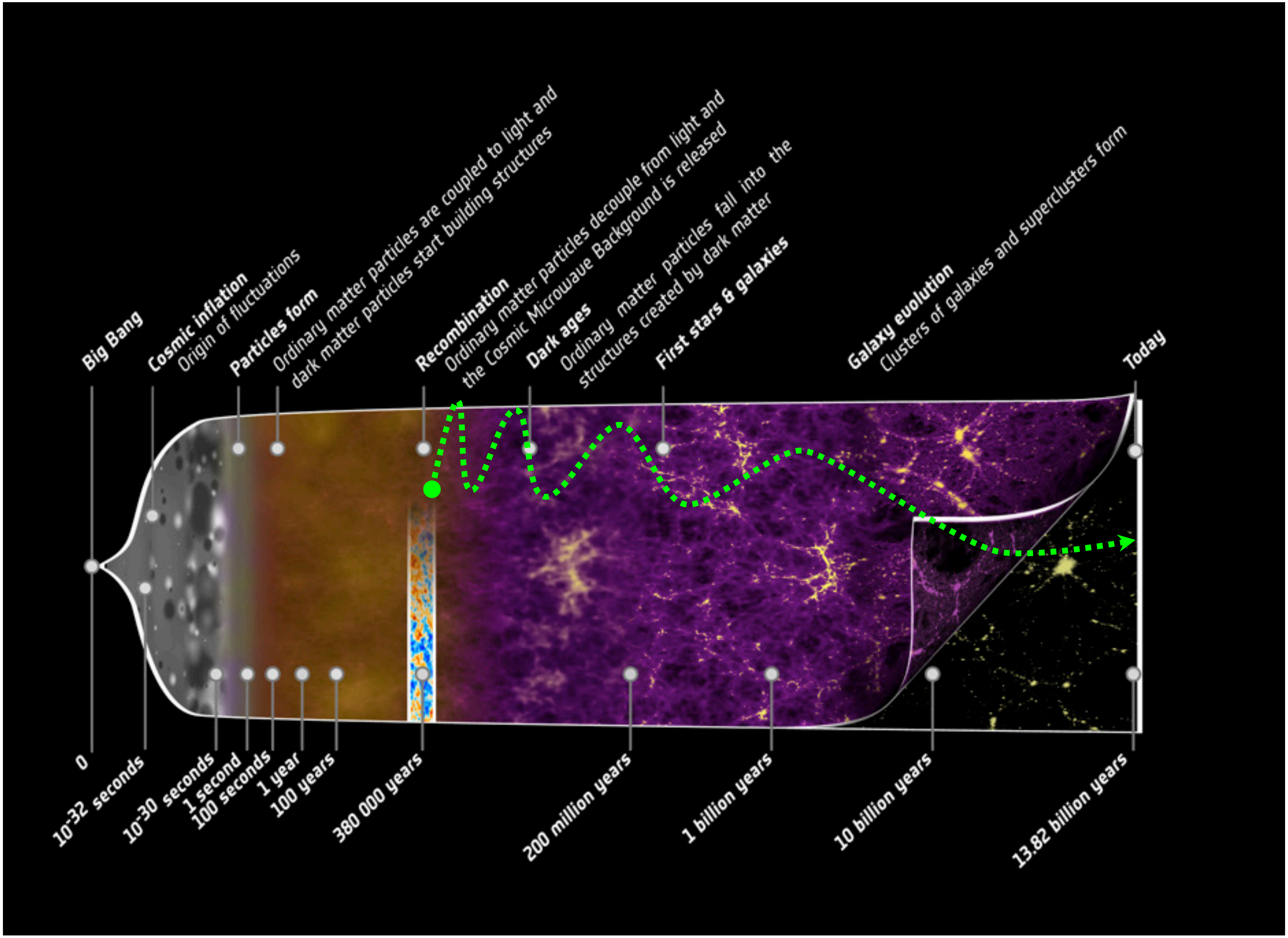
Outline

- Lecture 1 Introduction to Dark Energy
- Lecture 2 Beyond Einstein's Gravity, 1
- Lecture 3 Beyond Einstein's Gravity, 2
- Lecture 4 Testing dark energy

Texts

- **Dodelson**, Modern Cosmology
- **Amendola & Tsujikawa**, Dark Energy. Theory and Observations, Cambridge UP (rev. ed. 2015)
- **Euclid Theory WG**, Cosmology and Fundamental Physics with the Euclid Satellite, arXiv 1206.1225+1606.00180





Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

Friedmann equation

GR+Homogeneity+Isotropy:
Friedmann equation in flat space

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi}{3}\rho$$

$$\Omega_{m,0} = \frac{8\pi\rho_{m0}}{3H_0^{\ 2}}, \quad \Omega_{rad,0} = \frac{8\pi\rho_{rad,0}}{3H_0^{\ 2}}, \quad \Omega_{\Lambda,0} = \frac{8\pi\rho_{\Lambda,0}}{3H_0^{\ 2}}, \quad a = (1+z)^{-1}$$

$$H^2 = H_0^2\left[\Omega_{m,0}(1+z)^3 + \Omega_{rad,0}(1+z)^4 + \Omega_{\Lambda,0}\right]$$

$$1 = \Omega_{m,0} + \Omega_{rad,0} + \Omega_{\Lambda,0}$$

$$H^2 = H_0^2 \sum \Omega_{i,0}(1+z)^{3(1+w_i)}$$

Luminosity distance…

$$f = \frac{L}{4\pi r^2(z)(1+z)^2}$$

$$d_L = r(z)(1+z)$$

$$ds^2 = 0 \quad \Longrightarrow \quad r(z)$$

$$d_L(z) = \frac{1+z}{H_0\sqrt{\Omega_{k0}}}\sinh\left(H_0\sqrt{\Omega_{k0}}\int\frac{dz}{H(z)}\right)$$

$$H(z) = H_0(\Omega_{m0}a^{-3} + \Omega_\Lambda + \Omega_{k0}a^{-2})^{1/2}$$

…and magnitudes

$$f = \frac{L}{4\pi d_L^2}$$

Apparent magnitude

$$m = -2.5 \log f + c_1$$

Absolute magnitude

$$M = -2.5 \log L + c_2$$

$$m = M + 25 + 5 \log d_L (Mpc)$$

expansion rate

Background cosmology depends entirely
on the expansion rate E(z)

$$E\left(z; \Omega_{i,0}, w_i\right) \equiv \frac{H}{H_0}$$

Observations directly give E(z);
all the rest depend on the cosmological model

Cosmology Executive Summary

Dark matter 27%

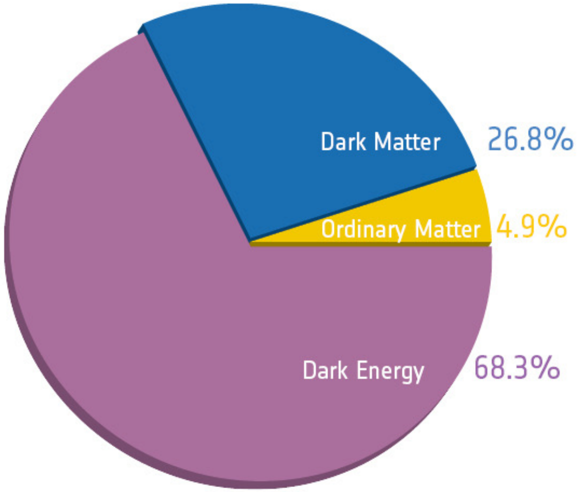
Baryons 5%

Massive neutrinos: 0.1%

Photons: 0.01%

Humans: 10^{-39} %

Spatial curvature: very close to 0



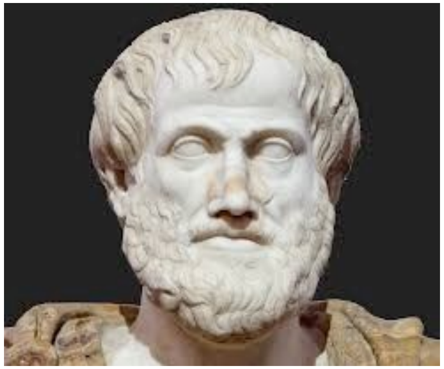
Something else: ≈ 70%

Cosmic inventory

name	density	EOS w
baryons	0.05	≈ 0
CDM	0.27	≈ 0
radiation	0.0001	1/3
Massive neutrinos	<0.05	≈ 0 today
Cosm. const.	0.68	-1
curvature	<0.01	-1/3
Other ?	?	?

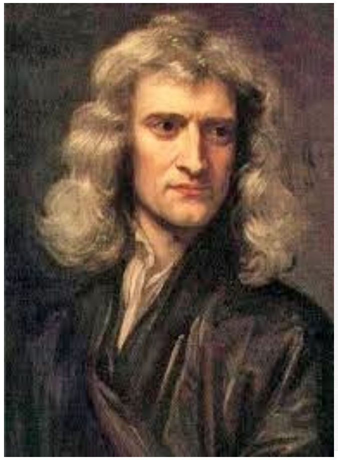
Back to the classics

Historical perspective, circa 350 BCE



- Gravity is always attractive: how to avoid that the **sky** falls on our head?
- **Aristotle**'s answer: quintessence

Historical perspective, circa 1700



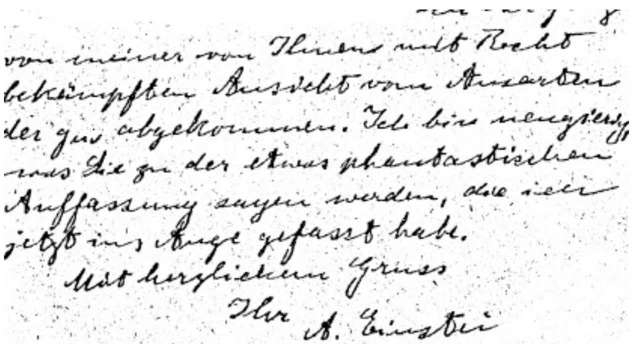
- Gravity is always attractive: how to avoid that the **stars** fall on our head?
- **Newton**'s answer: initial conditions

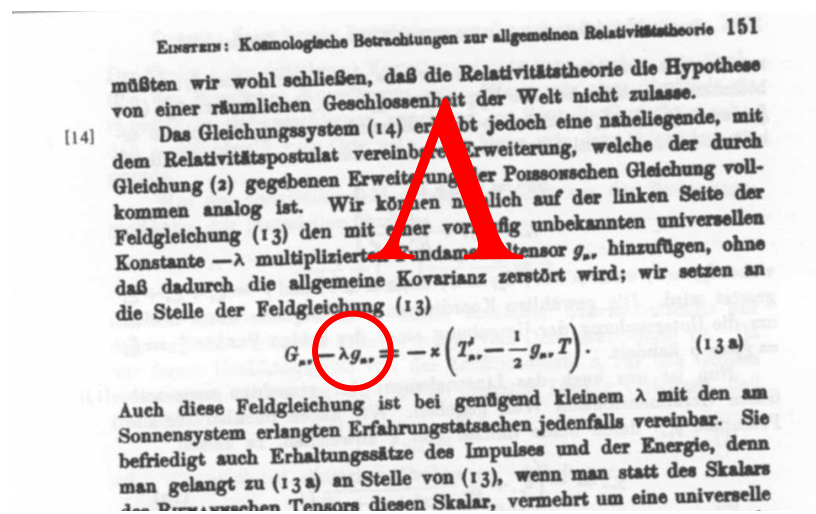
Historical perspective, circa 1900

- Gravity is always attractive: how to avoid that the **Universe** fall on our head?
- Einstein's answer: modify GR introducing a form of repulsive gravity

Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$


von meiner von Ihnen mit Recht bekämpften Ansicht vom Ausarten der $g_{\mu\nu}$ abgekommen. Ich bin neugierig, was Sie zu der etwas phantastischen Auffassung sagen werden, die ich jetzt ins Auge gefasst habe.
Mit herzlichem Gruss
Ihr A. Einstein


EINSTEIN: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie 151

müßten wir wohl schließen, daß die Relativitätstheorie die Hypothese von einer räumlichen Geschlossenheit der Welt nicht zulasse.

[14] Das Gleichungssystem (14) erlaubt jedoch eine naheliegende, mit dem Relativitätspostulat vereinbare Erweiterung, welche der durch Gleichung (2) gegebenen Erweiterung der Poissonschen Gleichung vollkommen analog ist. Wir können nämlich auf der linken Seite der Feldgleichung (13) den mit einer vorläufig unbekannten universellen Konstante $-\lambda$ multiplizierten Fundamentaltensor $g_{\mu\nu}$ hinzufügen, ohne daß dadurch die allgemeine Kovarianz zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

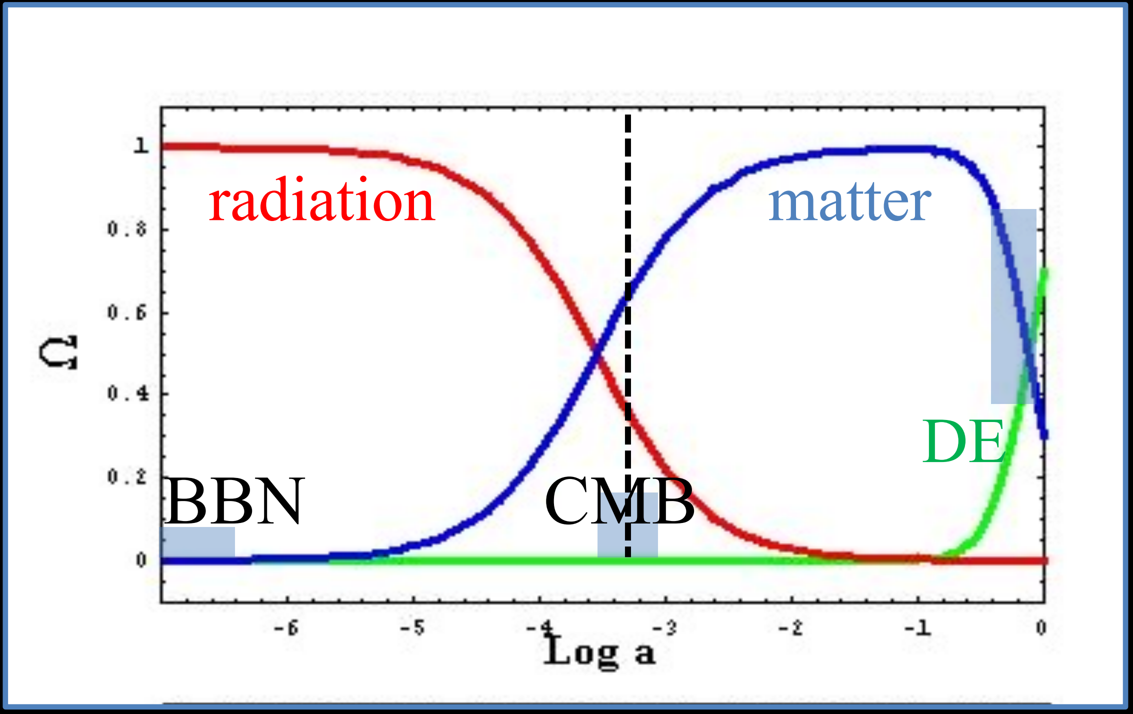
$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \qquad (13a)$$

Auch diese Feldgleichung ist bei genügend kleinem λ mit den am Sonnensystem erlangten Erfahrungstatsachen jedenfalls vereinbar. Sie befriedigt auch Erhaltungssätze des Impulses und der Energie, denn man gelangt zu (13a) an Stelle von (13), wenn man statt des Skalars des Riemannschen Tensors diesen Skalar, vermehrt um eine universelle

Einstein 1917

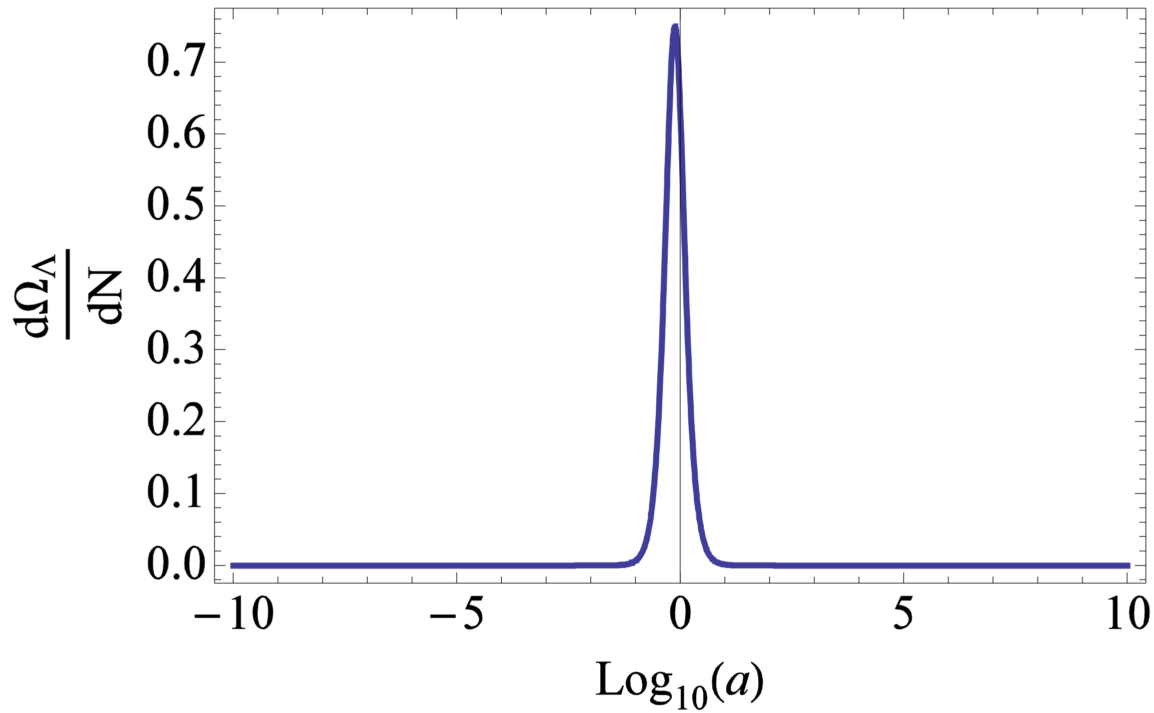
Time view

We know so little about the evolution of the universe!

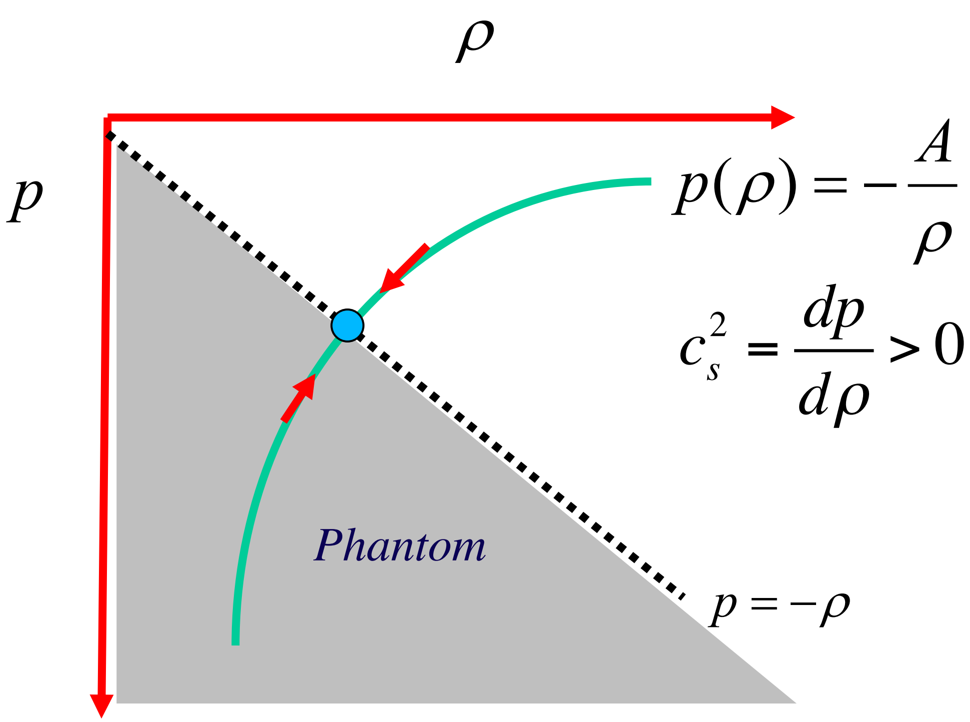


Why now?

The coincidence problem



Beyond the cosmological constant



$$\dot{\rho} = -3H(p+\rho)$$

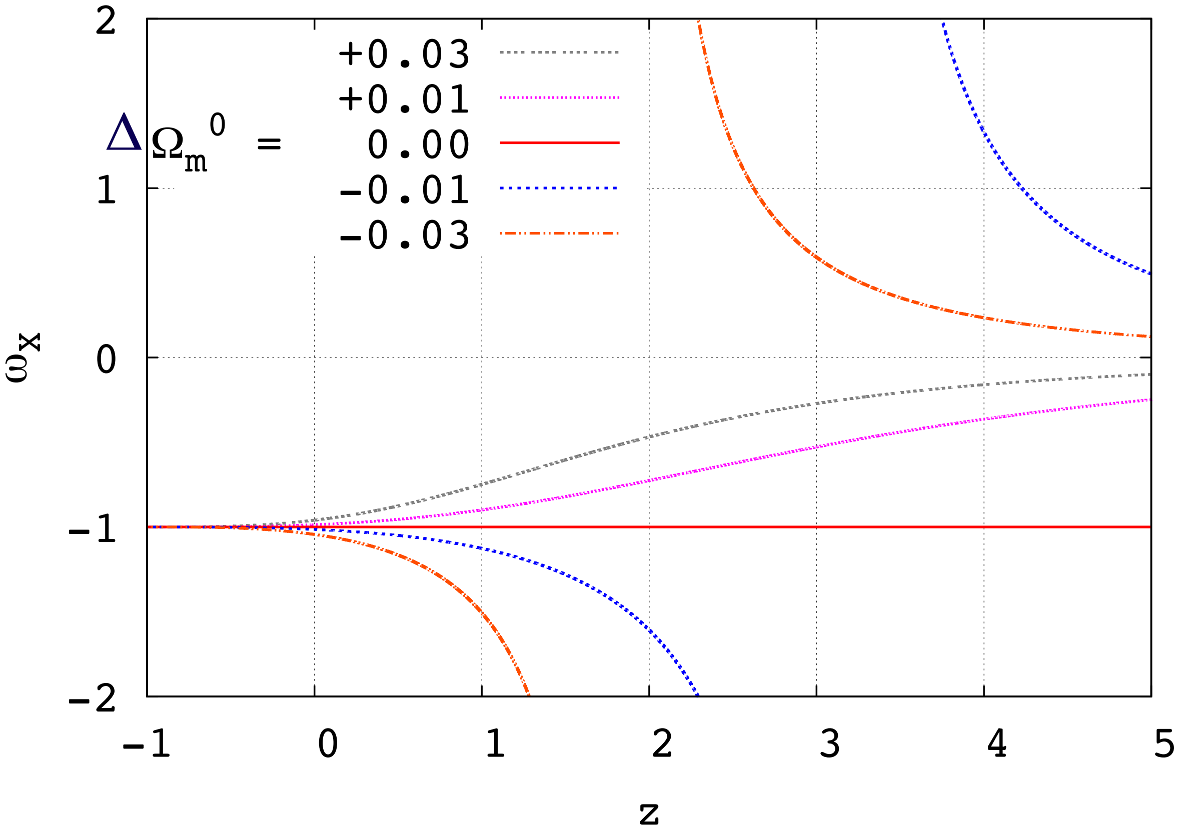
Warning 1

- Not everything we insert in our equations is necessarily an observable!
- Consider only two components, pressureless uncoupled matter and "something else".
- Then the EOS of this "something else" is related to the expansion $E(z)=H(z)/H_0$ as

$$w_{DE} = \frac{(1+z)(E^2)' - 3E^2}{3[E^2 - \Omega_{m0}(1+z)^3]}$$

- Even a perfect knowledge of E(z) is not sufficient to obtain w(z): one needs also Ω_{m0}
- But all you get from distance indicators is E(z), not Ω_{m0}
- Conclusion: either you "know" Ω_{m0} and obtain w(z) or you "know" w(z) and obtain Ω_{m0}

Example

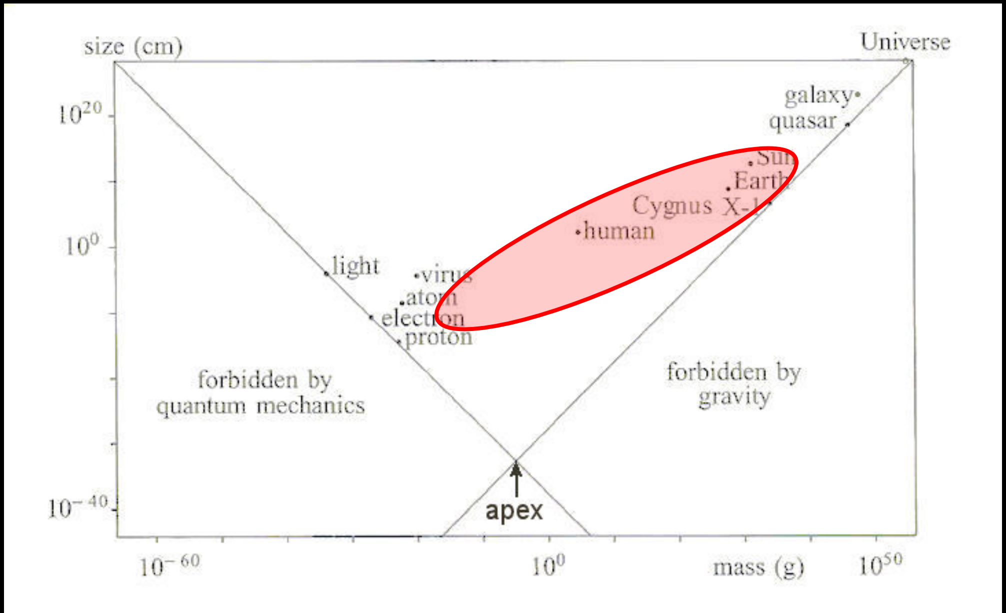


Ω=0.3

ΛCDM with different Ω

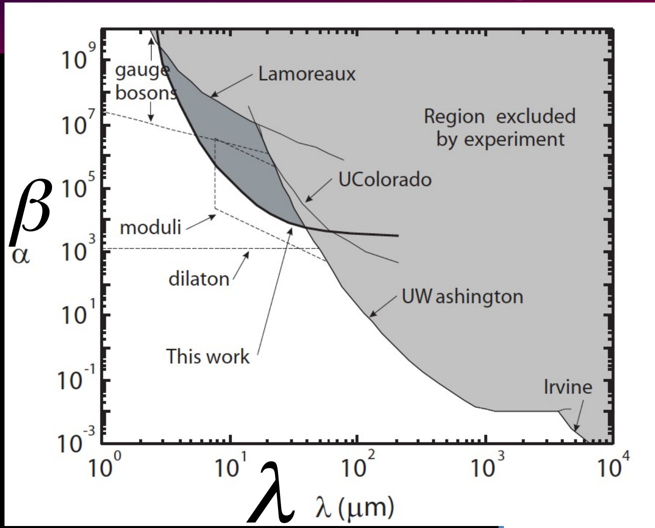
Warning 2

- We only directly test gravity within the solar system, at the present time, and with “baryons”



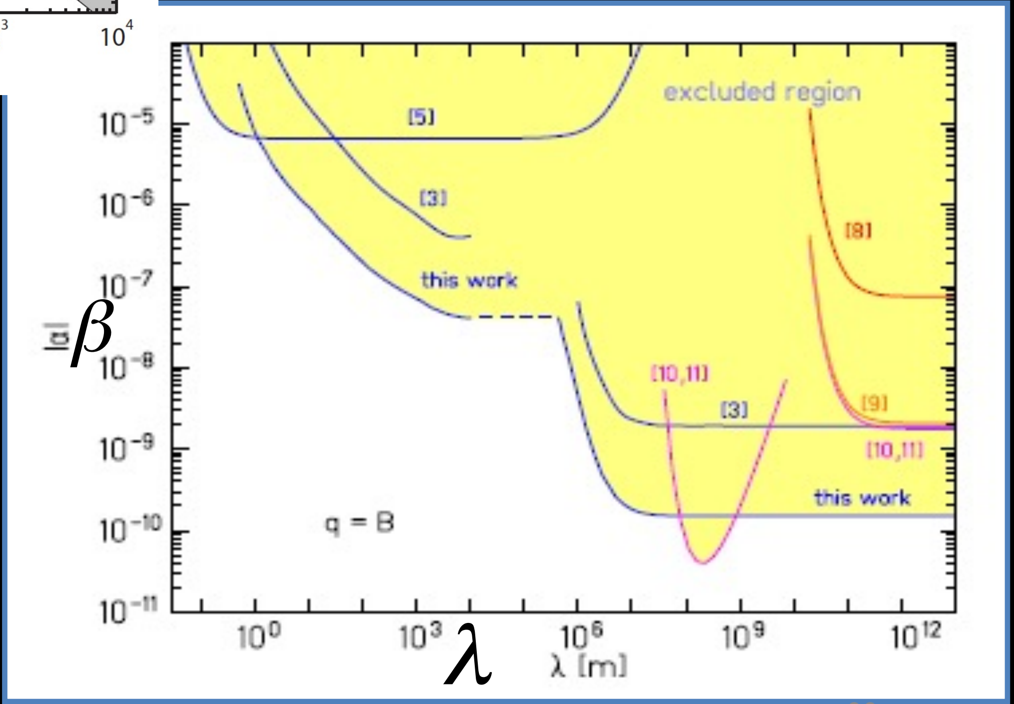
On Space and Time, Edited by Shahn Majid

Testing Gravity



Smullin et al. 2004

$$\frac{GM}{r} \rightarrow \frac{GM}{r}(1+\beta e^{-r/\lambda})$$



Schlamminger et al 2008

The fourfold way out of local gravity

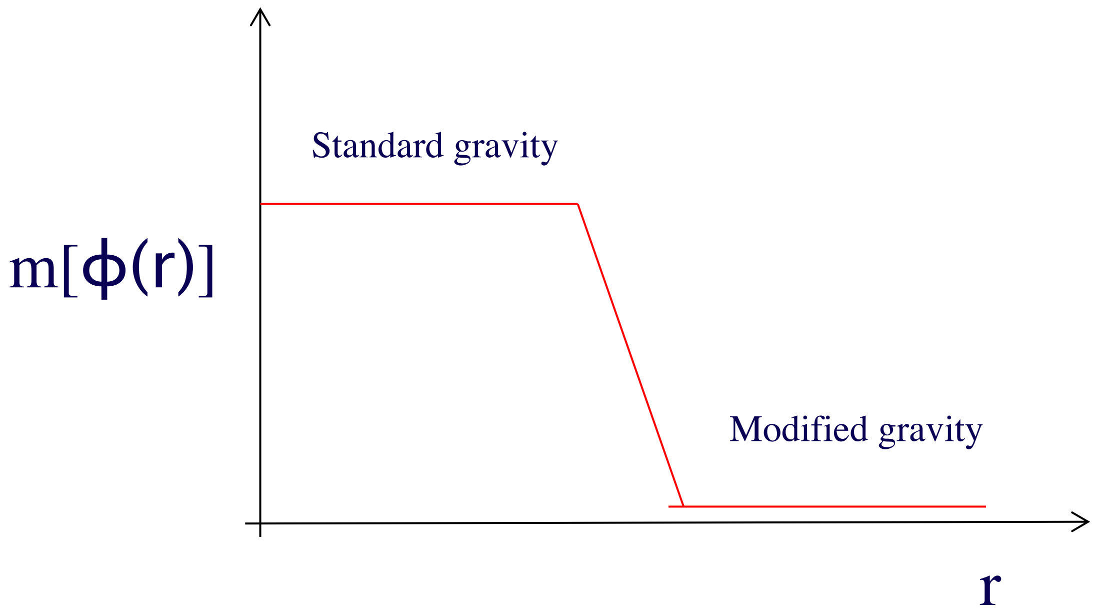
$$\Psi = -\frac{MG^*}{r}(1+\beta\ e^{-m_\varphi r})$$

$$m_\phi, \beta \left\{ \begin{array}{l} \text{depends on time} \\ \text{depends on space} \\ \text{depends on local density} \\ \text{depends on species} \end{array} \right.$$

Chameleon mechanism for a scalar field

$$\Psi = -\frac{MG^*}{r}(1+\beta(\phi)\ e^{-m_\varphi(\phi)r}) \qquad m(\phi)^2 = \frac{d^2V}{d\phi^2}$$

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\phi}{\mathrm{d}r} = \frac{\mathrm{d}V_{\rm eff}(\phi)}{\mathrm{d}\phi}\,, \qquad V_{\rm eff}(\phi) \equiv V(\phi) + \sum_i \rho_i e^{Q_i\phi}\,.$$



Mapping the expansion rate

$$f = \frac{L}{4\pi d_L^2}$$

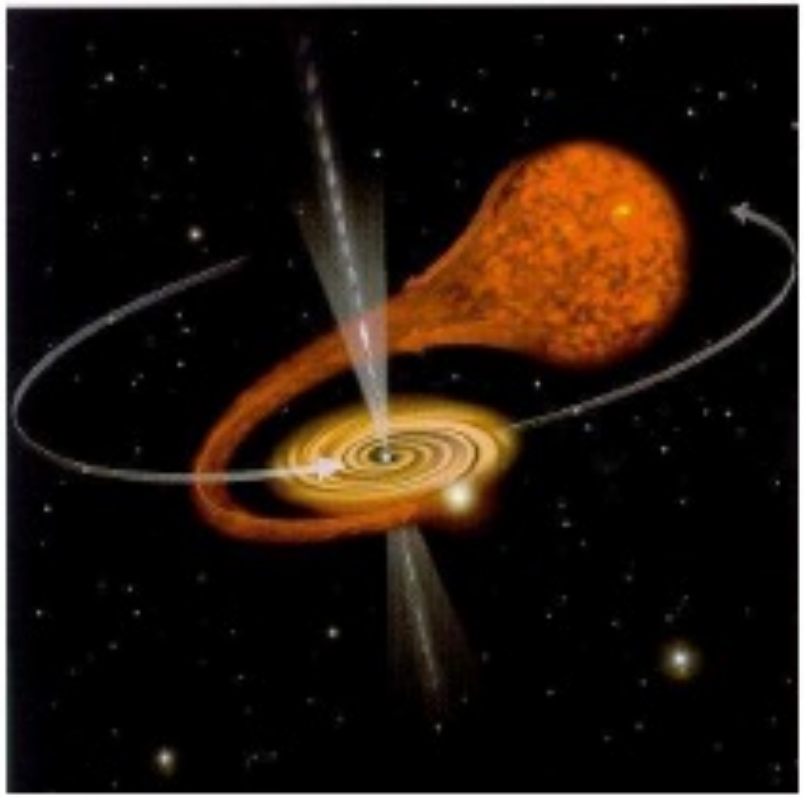
$$d_L \sim \int \frac{dz}{E(z; \Omega_{i,0}, w_i)}$$

Flux ---> distance ---> cosmology

Lighthouses in the dark

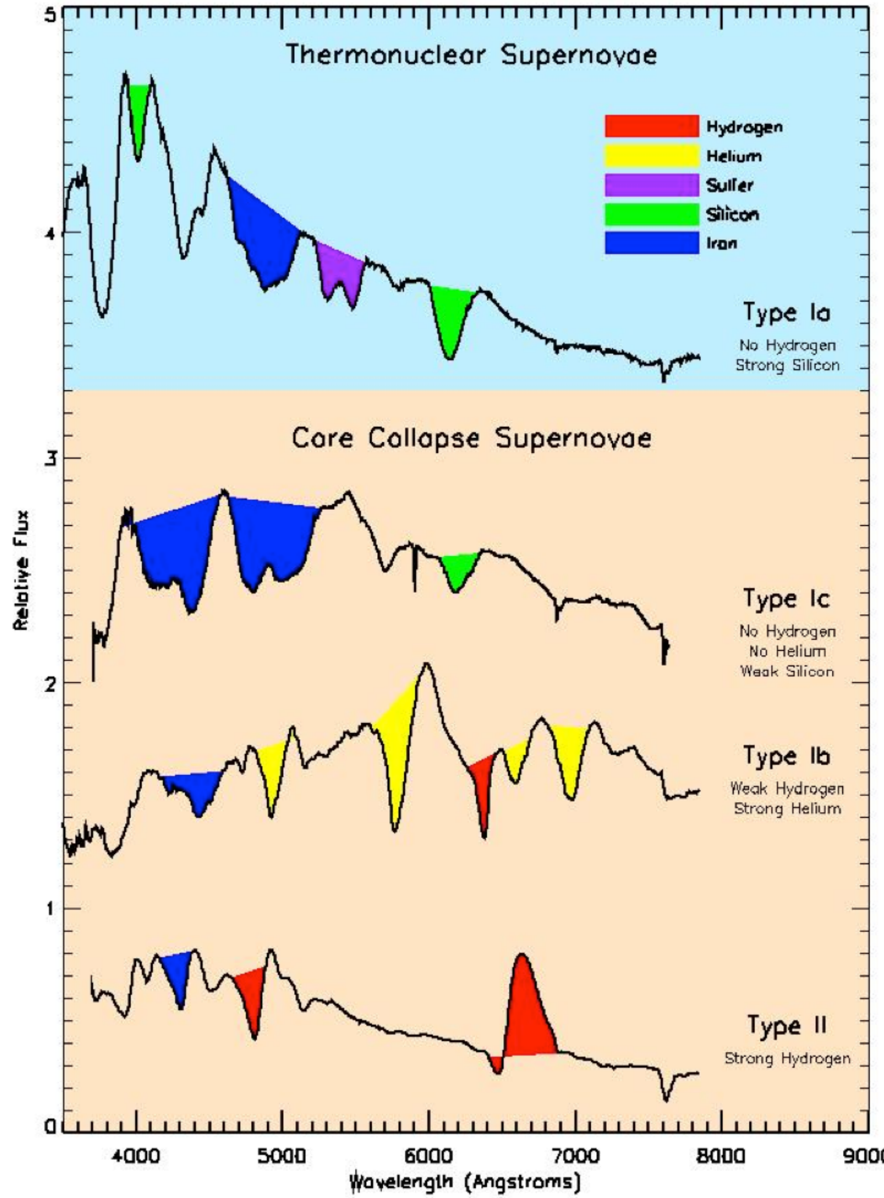
$$d^2 = \frac{L}{4\pi f}$$

Supernovae Ia $m - M = 5\log d_L + 25$

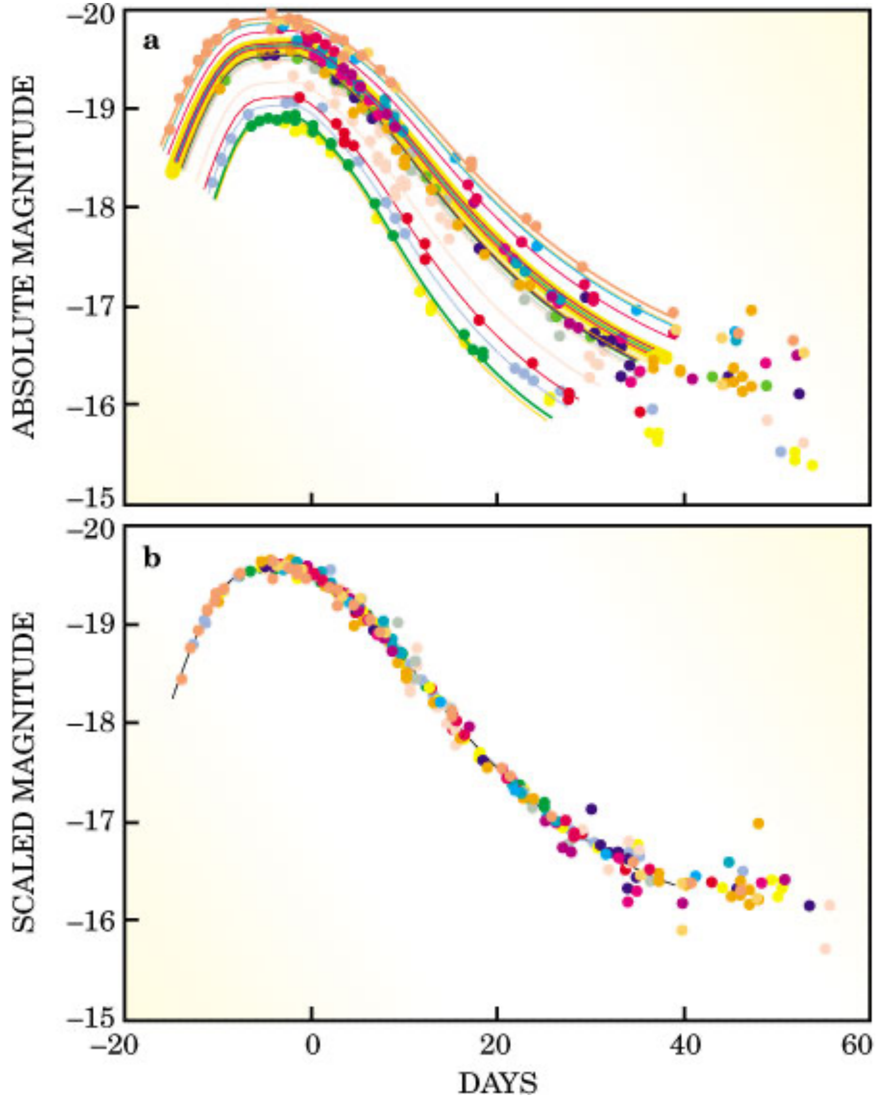


- This hypothesis can be tested and calibrated through a local sample whose distance we know by other means.

Types of supernovae



http://supernova.lbl.gov/~dnkasen/tutorial/



Phillips, Hamuy, et al. 1993, 1995

- Then, we compare $m_{obs}(z)$ with

$$m_{theor}(z) = M + 25 + 5 \log d(z; \Omega_M, \Omega_\Lambda, ..)$$

magnitudes

$$f = \frac{L}{4\pi d_L^2}$$

Apparent magnitude

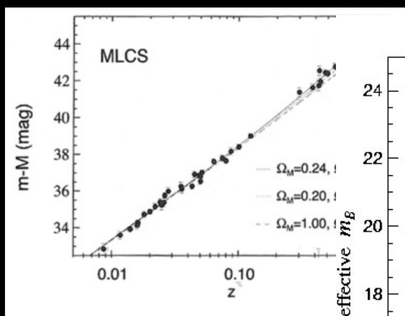
$$m = -2.5 \log f + c_1$$

Absolute magnitude

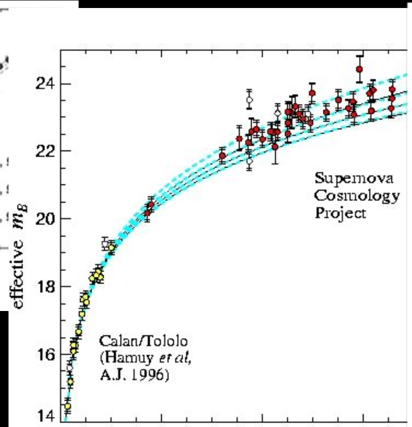
$$M = -2.5 \log L + c_2$$

$$m = M + 25 + 5 \log d_L(Mpc)$$

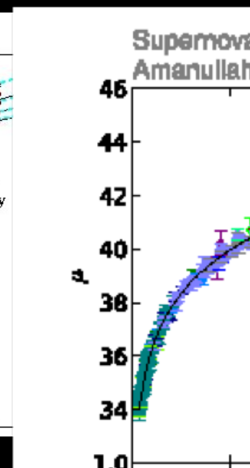
Hubble diagram



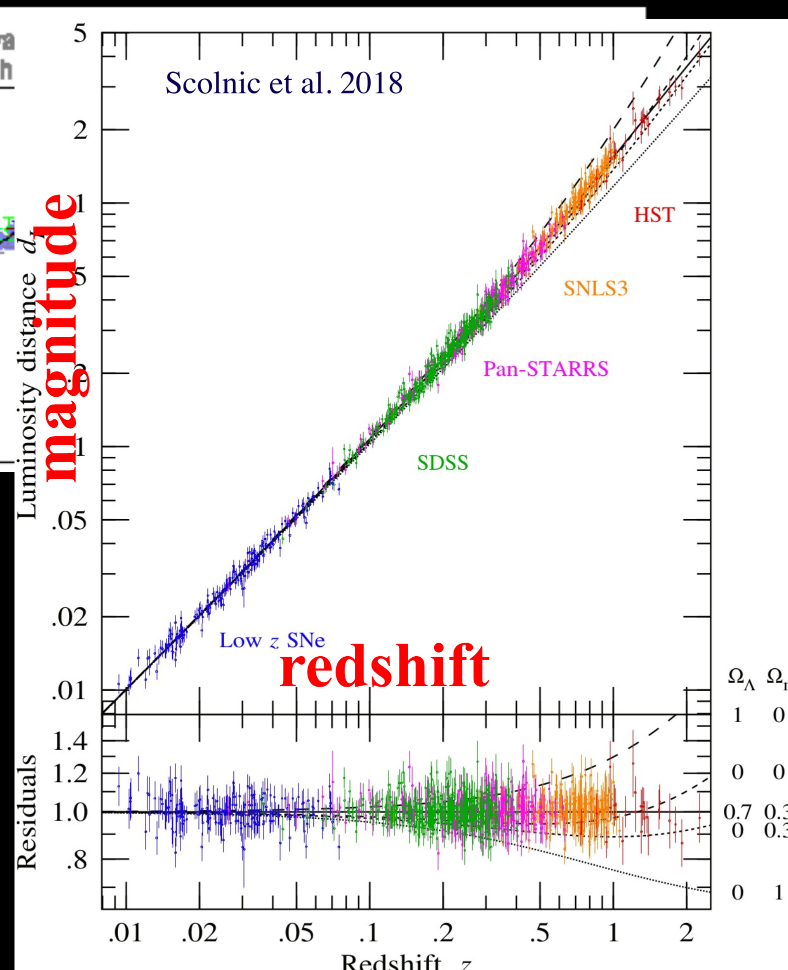
1997



1998

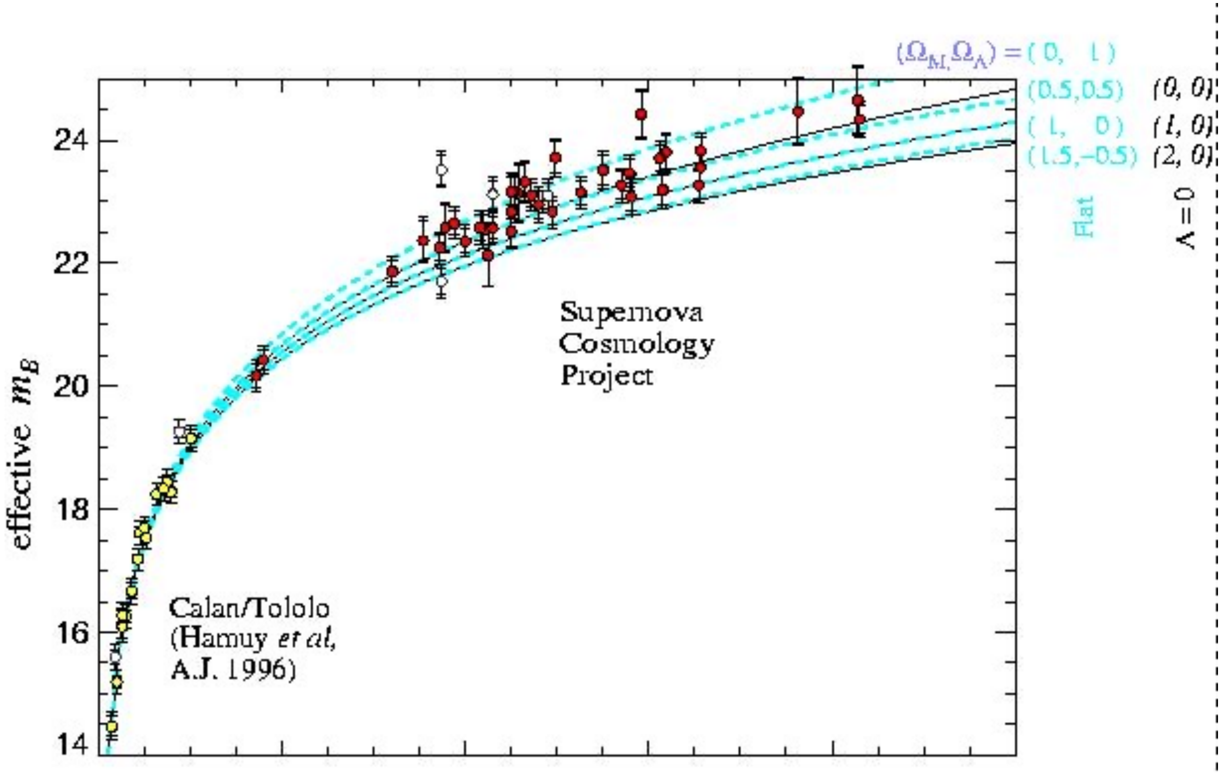


2010



2018

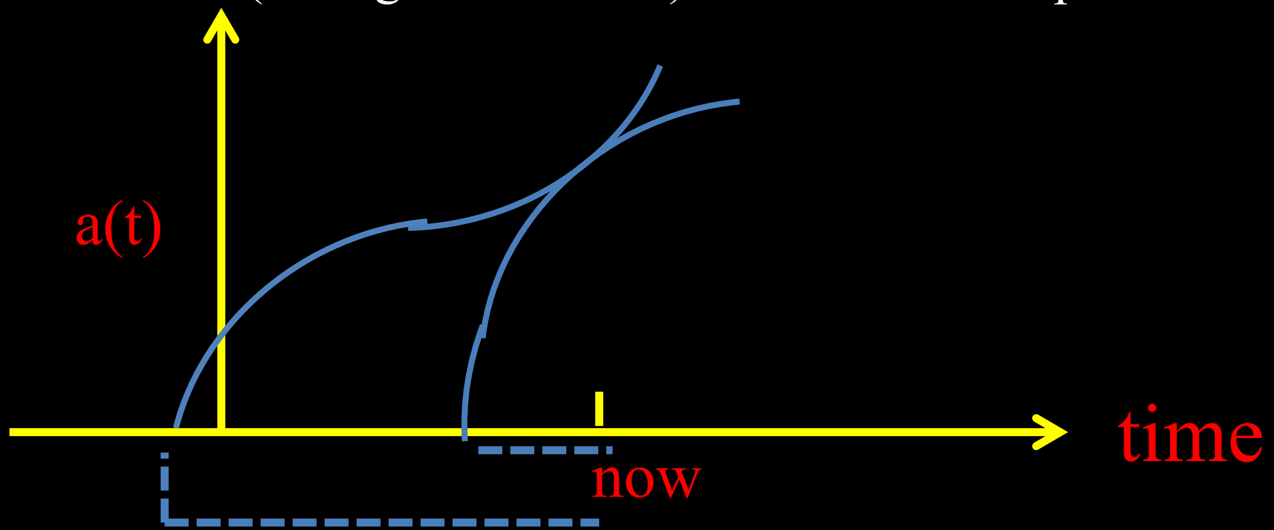
SNIa are dimmer than expected!



Basic property 1

Local Hubble law $r(z) = \frac{z}{H_0}$ ⟶ $r(z) = \int \frac{dz}{H(z)}$ Global Hubble law

If H(z) in the past is smaller (i.e. acceleration), then r(z) is larger: larger distances (for a given redshift) make dimmer supernovae



LambdaCDM

$$H^2 = H_0^2[\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + \left(1 - \Omega_{m,0} - \Omega_{\Lambda,0}\right)(1+z)^2]$$

Only two parameters!

$$d_L\left(z; \Omega_{m,0}, \Omega_{\Lambda,0}\right) \sim \int \frac{dz}{[\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + \left(1 - \Omega_{m,0} - \Omega_{\Lambda,0}\right)(1+z)^2]^{1/2}}$$

Statistics in three steps

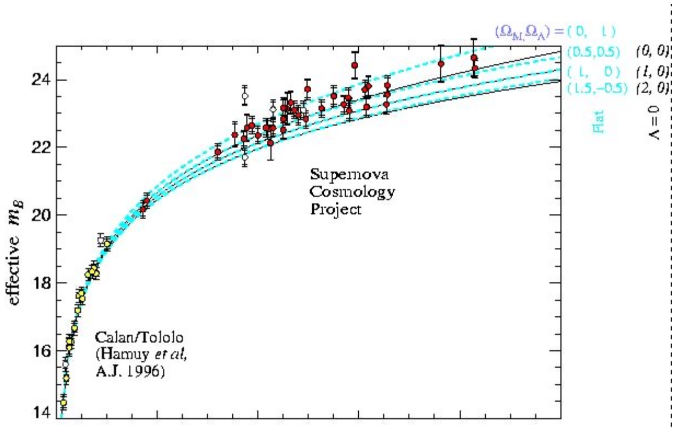
1: take a model

$$H^2 = H_0^2 E^2(z; params)$$

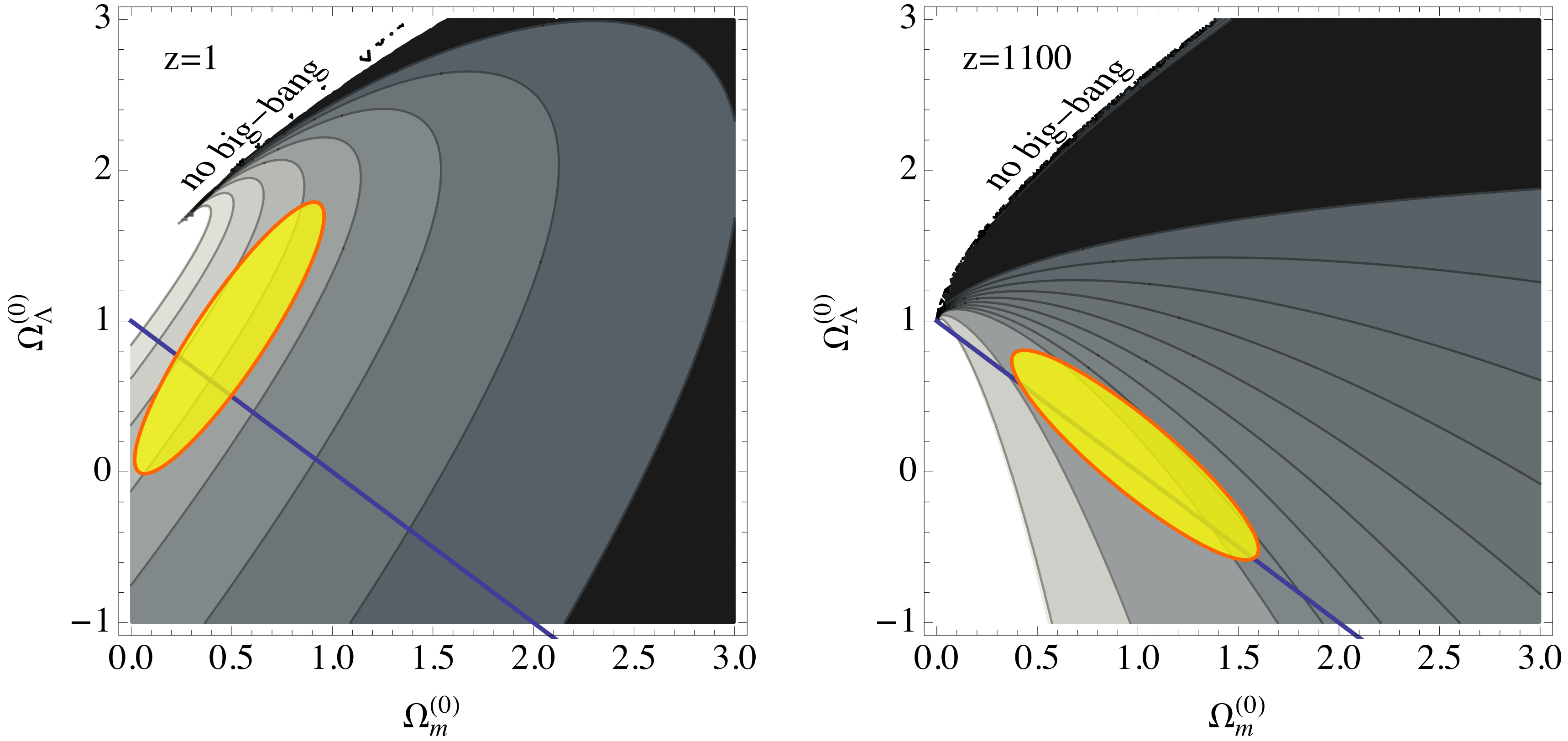
2: find the distance

$$d_L(z; params) \sim \int \frac{dz}{E(z)}$$

3: vary the parameters and
minimize the chi-squared with data

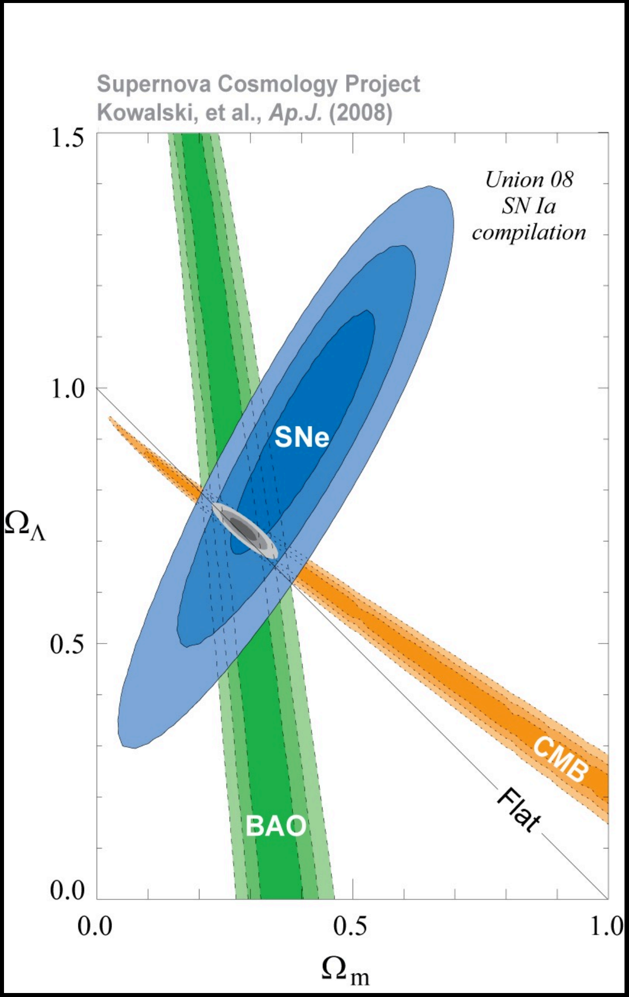


Basic property 2

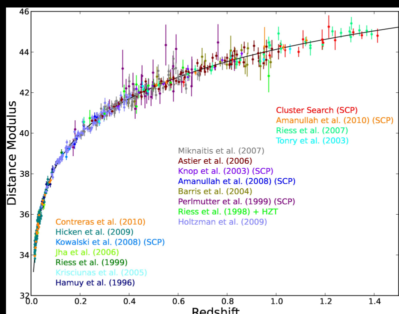


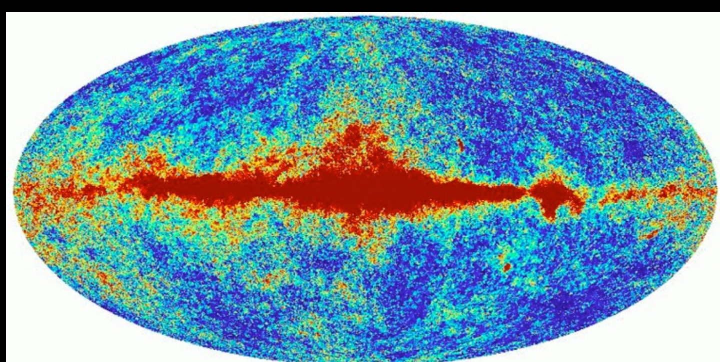
Curves of constant luminosity distance

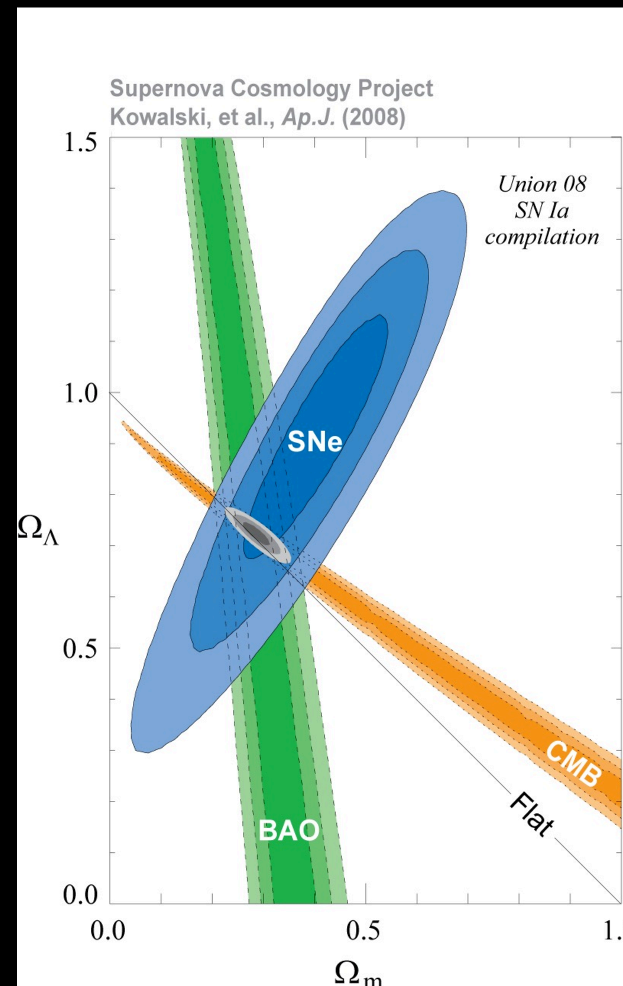
Properties of Dark Energy



Properties of Dark Energy





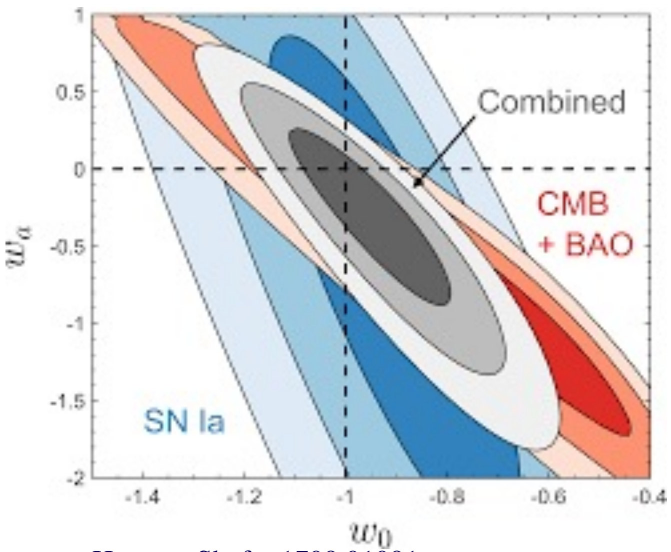


XCDM

Three parameters?

$$H^2 = H_0^2[\Omega_{m,0}(1+z)^3 + \Omega_{X,0}(1+z)^{3(1+w_X)} + \left(1-\Omega_{m,0}-\Omega_{\Lambda,0}\right)(1+z)^2]$$

Four parameters?



Huterer, Shafer 1709.01091

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

Chevalier-Polarski-Linder param.

Etc.

Problems

Parametrizations of w(z) are simple but arbitrary

They only specify the background, not the perturbations

They do not correspond to fundamental degrees of freedom (fields)

Properties of Dark Energy

Isotropy

Abundance

Observational requirements

Slow evolution

Weak clustering

Properties of Dark Energy

Observations:

- Isotropy
- Large abundance
- Slow evolution
- Weak clustering

Theory:

- Scalar field?
- $\Omega_{DE} \approx \Omega_m$
- $w_{eff} \approx -1$
- $c_s \approx 1$

The magic of Lagrangians

$$\int dx^4 \sqrt{-g}\left[R + L_{matter}\right] \quad \xrightarrow{\text{variation}} \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

The two laws of Lagrangian-cooking:
a) form a scalar: Eqs are covariant
b) no explicit functions of coords: Energy-momentum tensor is conserved

Random example:

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + R_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + V(\phi) + R_{\mu\nu\sigma\tau}\phi^{,\nu}\phi^{,\mu}\phi^{,\sigma}\phi^{,\tau} + R_{\mu\nu\sigma\tau}R^{,\sigma\tau}\phi^{,\nu\mu} + ... + L_{matter}\right]$$

The magic of Fourier space

Fourier space

$$\phi(x,y,z,t) \rightarrow \phi(t)\exp(i\vec{k}\,\vec{x})$$

therefore

$$\frac{\partial}{\partial x}\phi(x,y,z,t) \rightarrow \phi(t)ik_x \exp(i\vec{k}\,\vec{x})$$

$$\nabla\phi(x,y,z,t) \rightarrow \phi(t)i\vec{k} \exp(i\vec{k}\,\vec{x})$$

$$\nabla\nabla\phi(x,y,z,t) \rightarrow -\phi(t)k^2 \exp(i\vec{k}\,\vec{x})$$

Very useful for linear equations because the space part drops out!

The past ten years of DE research

$$\int dx^4 \sqrt{-g}\left[R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi,\frac{1}{2}\phi_{,\mu}\phi^{,\mu})R + G_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

Cosmological constant, Dark energy w=const, Dark energy w=w(z),quintessence, scalar-tensor model, coupled quintessence, k-essence, f(R), Gauss-Bonnet, Galileons, KGB,

The Horndeski Lagrangian

The most general 4D scalar field theory with second order equations of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

$$\mathcal{L}_2 = K(\phi, X)\,,$$
$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi\,,$$
$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,,$$
$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].$$

$$X = \frac{1}{2}\phi_{,\mu}\phi^{,\mu}$$

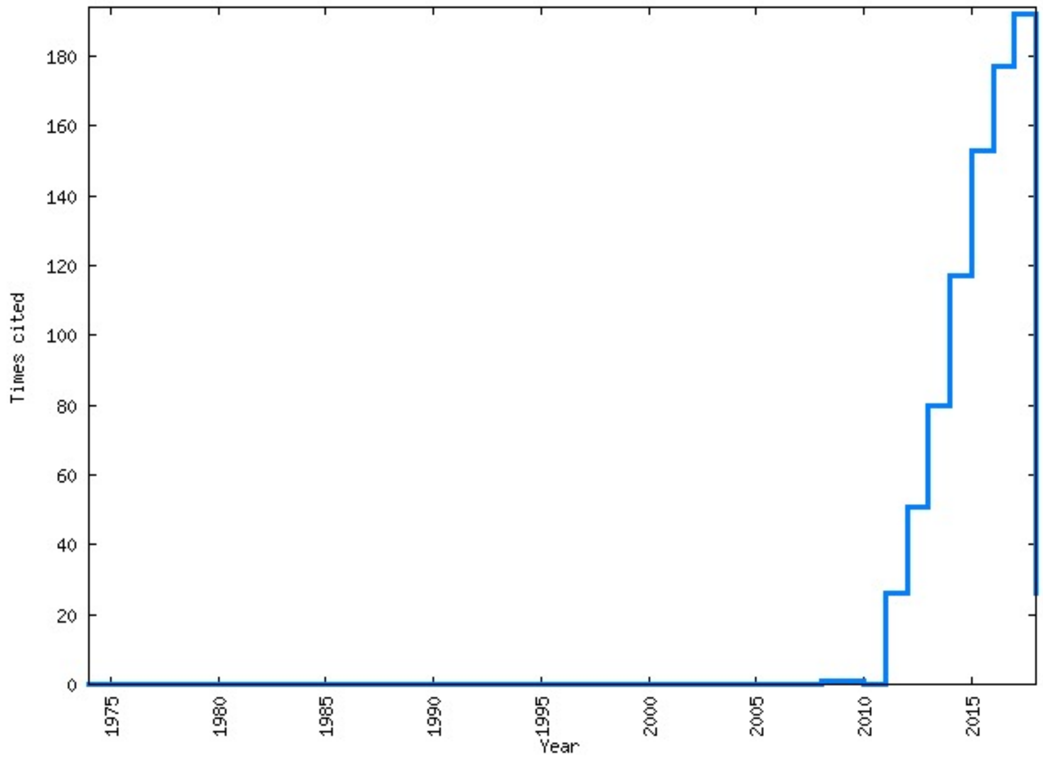
- ✓ First found by Horndeski in 1975
- ✓ rediscovered by Deffayet et al. in 2011
- ✓ no ghosts, no classical instabilities
- ✓ it modifies gravity!
- ✓ it includes f(R), Brans-Dicke, k-essence, Galileons, etc etc etc
- ✓ Invariant under conformal and disformal transformations

Horndeski…

Second-order scalar-tensor field equations in a four-dimensional space

Gregory Walter Horndeski (Waterloo U.)

Int.J.Theor.Phys. 10 (1974) 363-384



Limits of the Horndeski Lagrangian

- If $G_4 = 1/2$ and $G_5 = 0$ (it is actually sufficient $G_5 = const$) the HL reduces to ordinary gravity with a scalar field having a non-canonical kinetic sector given by $\mathcal{L}_2, \mathcal{L}_3$. The canonical form is obtained for $K = X - V(\phi)$ and $G_3 = 0$ ($G_3 = const$ is sufficient). ΛCDM is recovered for $K = -2\Lambda$.
- The "minimal" form of modified gravity within the HL is provided by $G_4 = G_4(\phi)$ and $G_5 = const$: this is then equivalent to a Brans-Dicke scalar-tensor model, again with a non-canonical kinetic sector.
- The original Brans-Dicke model is recovered assuming a kinetic sector, $K = (\omega_{BD}/\phi)X, G_3 = 0$, and $G_4(\phi) = \phi/2$.
- If the kinetic sector vanishes, $K_{,X} = G_3 = 0$, then we reduce ourselves to a $f(R)$ model [13], whose Lagrangian is $\mathcal{L}_R = (R + f(R))/2$. In fact, this model is equivalent to a scalar-tensor theory with $G_4(\phi) = e^{2\phi/\sqrt{6}}/2$ and a potential $K(\phi) = -(Rf_{,R} - f)/2$ where $\phi = \sqrt{6}/2 \log(1 + f_{,R})$. This relation should then be inverted to get $R = R(\phi)$ and used to replace R with ϕ in $K(\phi)$.
- If one sets $G_i(\phi, X) = G_i(X)$ then the Lagrangian is invariant under the shift $\phi \to \phi + c$ with $c = const$. This shift-symmetric version of the HL is connected to the Covariant Galileon when the functional dependence of the G_i is fixed [5] and is able to produce the accelerated expansion without a potential that makes the field slow roll.

The Ostrogradski theorem

Higher-than-second order equations of motion are unstable

$$L = L(q, \dot{q}, \ddot{q})$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}} = 0$$

The Hamiltonian contains a term linear in a canonical momentum

$$H = P_1 Q_2 + P_2 a(Q_1, Q_2, P_2) - L(Q_1, Q_2, a)$$

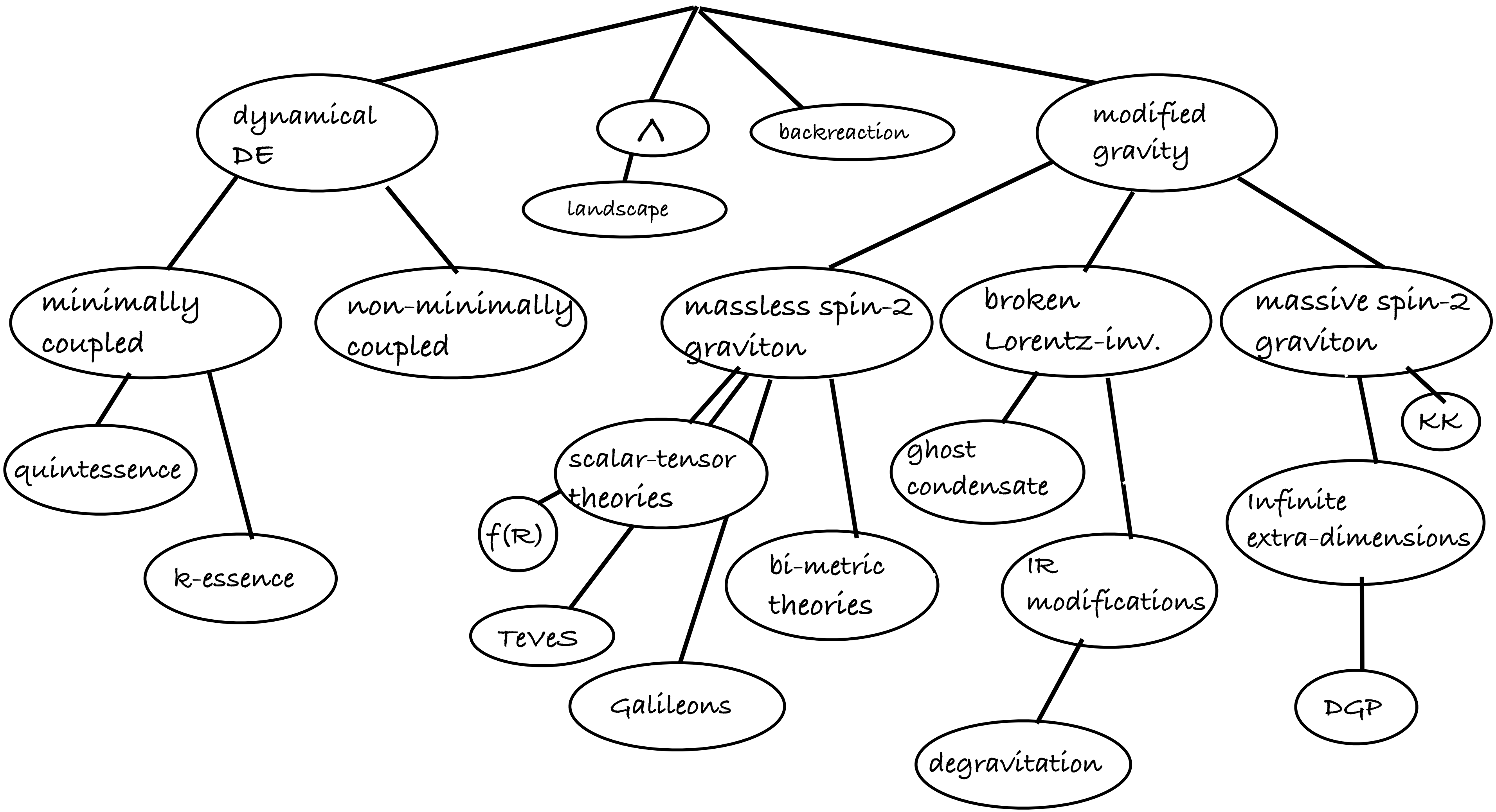
Energy states are unbounded from below!

Woodard astro-ph/0601672

Caveats…

- The dangerous term in the Hamiltonian
  can actually be absent in degenerate cases
- The instability can manifest itself in very long time scales
- The proof assumes locality

The theoretical landscape



Courtesy Alessandra Silvestri

The next ten years of DE research

Combine observations of background, linear and non-linear perturbations to reconstruct as much as possible the Horndeski model

The Great Horndeski Hunt

Let us assume we have only

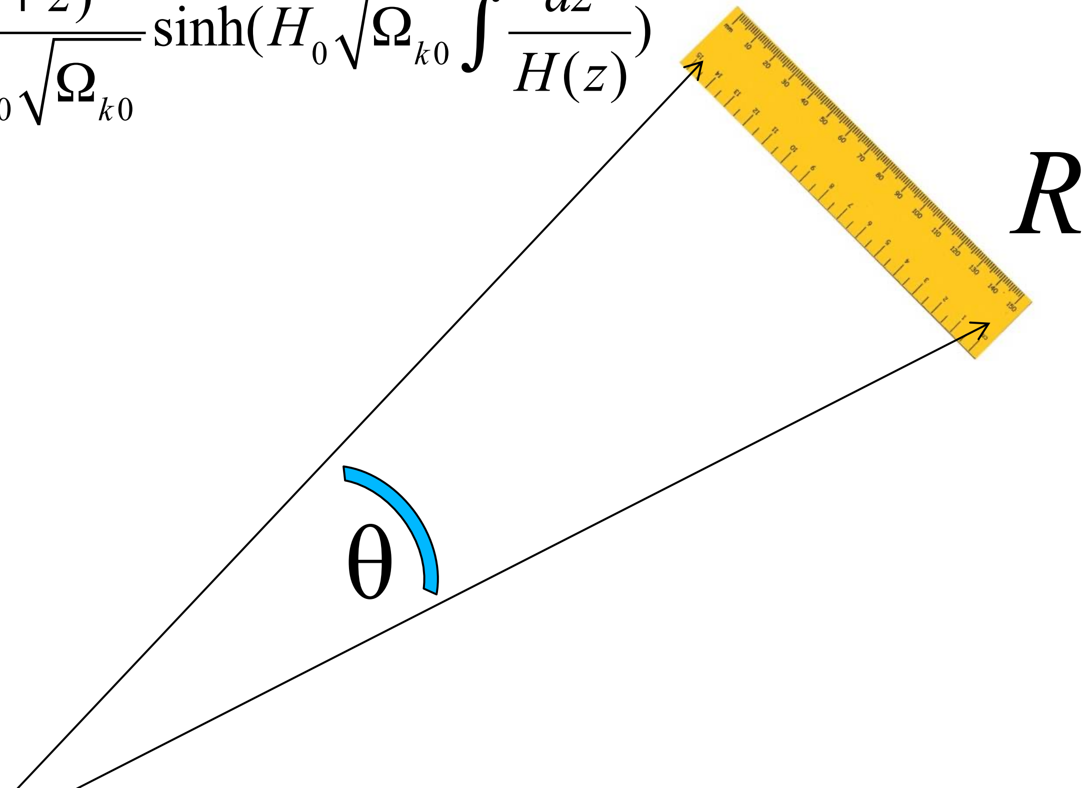
1) a perturbed FRW metric

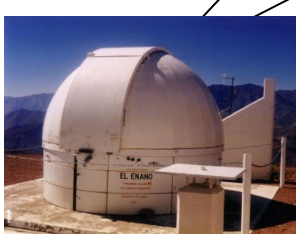
2) pressureless matter

3) the Horndeski field

Standard rulers

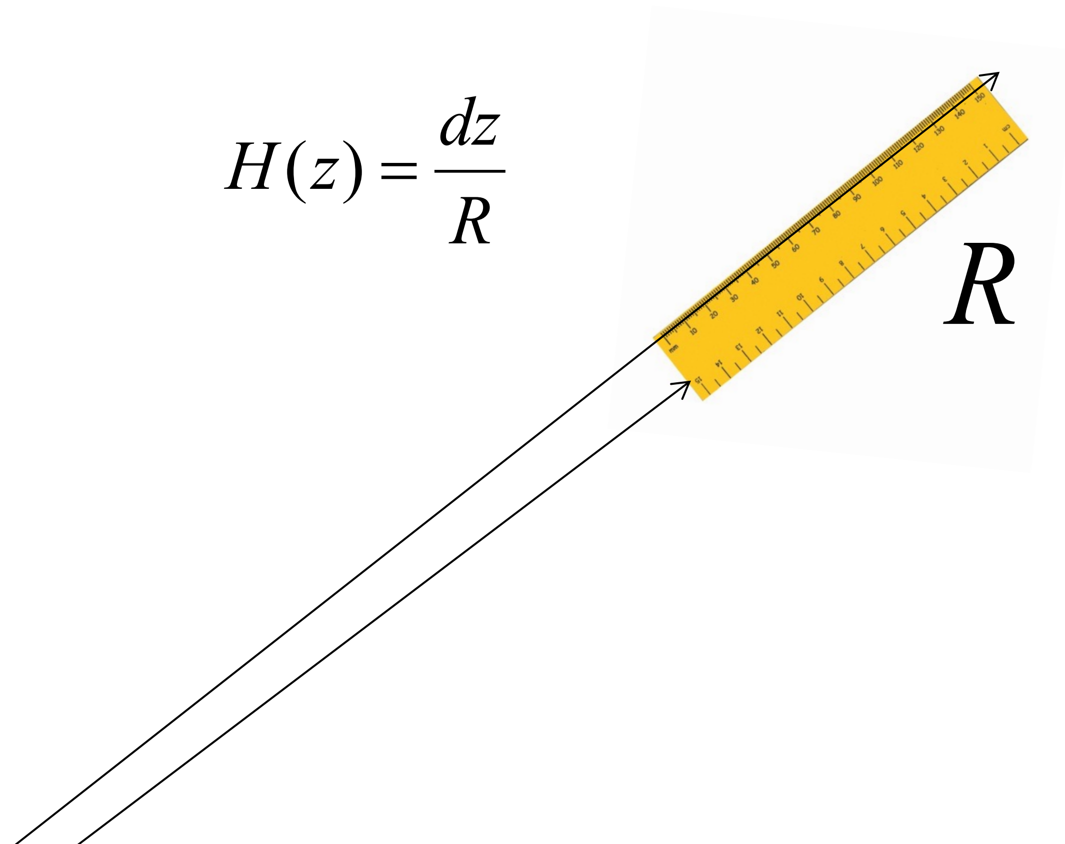
$$D(z) = \frac{R}{\theta} = \frac{(1+z)^{-1}}{H_0\sqrt{\Omega_{k0}}}\sinh(H_0\sqrt{\Omega_{k0}}\int\frac{dz}{H(z)})$$

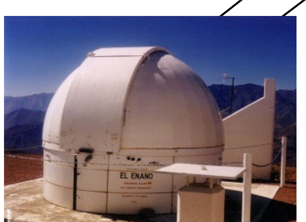




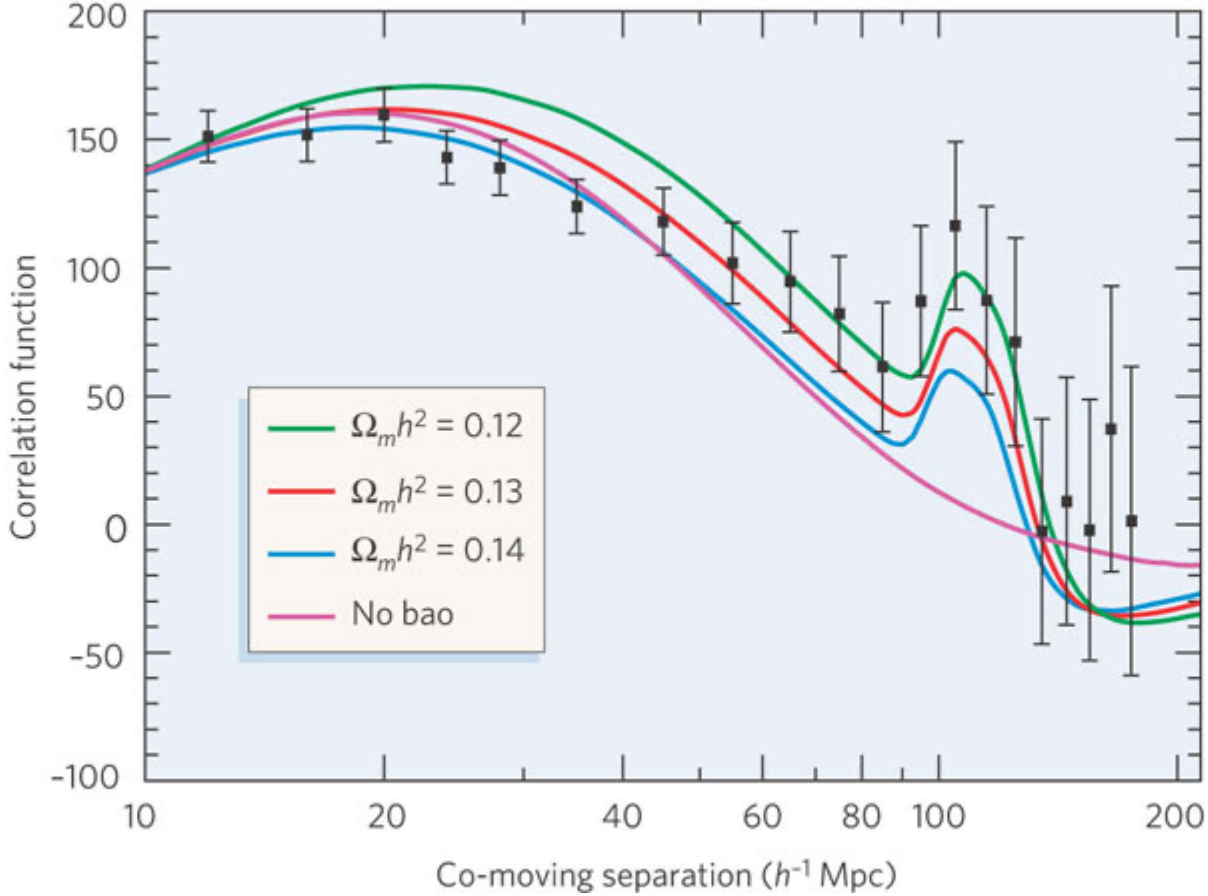
Standard rulers

$$H(z)=\frac{dz}{R}$$





BAO ruler



Charles L. Bennett
Nature 440, 1126-1131(2006)

Background: SNIa, BAO, ...

Then we can measure H(z) and

$$D(z)=\frac{1}{H_0\sqrt{\Omega_{k0}}}\sinh(H_0\sqrt{\Omega_{k0}}\int\frac{dz}{H(z)})$$

and therefore we can reconstruct the full FRW metric

$$ds^2=dt^2-\frac{a(t)^2}{\left(1-\frac{\Omega_{k0}}{4}r^2\right)^2}(dx^2+dy^2+dz^2)$$

Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2+dy^2+dz^2)]$$

- Poisson's equation

$$\nabla^2\Psi = 4\pi G\rho_m\delta_m$$

- anisotropic stress

$$1 = -\frac{\Psi}{\Phi}$$

Warning: all the perturbation variables in this talk are root mean squares!

The story so far

The most general way of introducing a new field in cosmology

$$
\begin{aligned}
\mathcal{L}_2 &= K(\phi, X)\,, \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,, \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{aligned}
$$

The most general way of introducing a new field in cosmology

$$
\begin{aligned}
\mathcal{L}_2 =& K(\phi, X)\,, \\
\mathcal{L}_3 =& -G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 =& G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,, \\
\mathcal{L}_5 =& G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{aligned}
$$

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

The most general parametrization of gravity at linear level

- Poisson's equation $$\nabla^2\Psi = 4\pi G\, Y(k,a)\rho_m\delta_m$$
- anisotropic stress $$\eta(k,a) = -\frac{\Phi}{\Psi}$$

Modified Gravity at the linear level

- standard gravity

$$Y(k,a)=1$$
$$\eta(k,a)=1$$

- scalar-tensor models

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{2(F+F'^2)}{2F+3F'^2}$$
$$\eta(a)=1+\frac{F'^2}{F+F'^2}$$

Boisseau et al. 2000
Acquaviva et al. 2004
Schimd et al. 2004
L.A., Kunz &Sapone 2007

- f(R)

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{1+4m\frac{k^2}{a^2R}}{1+3m\frac{k^2}{a^2R}},\quad \eta(a)=1+\frac{m\frac{k^2}{a^2R}}{1+2m\frac{k^2}{a^2R}}$$

Bean et al. 2006
Hu et al. 2006
Tsujikawa 2007

- DGP

$$Y(a)=1-\frac{1}{3\beta};\quad \beta=1+2Hr_c w_{DE}$$
$$\eta(a)=1+\frac{2}{3\beta-1}$$

Lue et al. 2004;
Koyama et al. 2006

- massive bi-gravity

$$Y(a)=....$$
$$\eta(a)=....$$

F. Koennig and L. A. 2014,
Y. Akrami et al. 2014

Modified Gravity at the linear level

In the quasi-static limit, every Horndeski model is characterized at linear scales by the two functions

$$\eta(k,a) = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

k = wavenumber

h_i = time-dependent functions

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

Modified Gravity at the linear level

$$h_1 \equiv \frac{w_4}{w_1^2} = \frac{c_T^2}{w_1}, \qquad h_2 \equiv \frac{w_1}{w_4} = c_T^{-2},$$

$$h_3 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2 w_2 H - w_2^2 w_4 + 4w_1 w_2 \dot{w}_1 - 2w_1^2(\dot{w}_2 + \rho_m)}{2w_1^2},$$

$$h_4 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2 H^2 - w_2 w_4 H + 2w_1 \dot{w}_1 H + w_2 \dot{w}_1 - w_1(\dot{w}_2 + \rho_m)}{w_1},$$

$$h_5 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2 H^2 - w_2 w_4 H + 4w_1 \dot{w}_1 H + 2\dot{w}_1{}^2 - w_4(\dot{w}_2 + \rho_m)}{w_4},$$

$$\begin{aligned}
w_1 &\equiv 1 + 2(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}XHG_{5,X}),\\
w_2 &\equiv -2\dot{\phi}(XG_{3,X} - G_{4,\phi} - 2XG_{4,\phi X}) + 2H(w_1 - 4X(G_{4,X} + 2XG_{4,XX} - G_{5,\phi} - XG_{5,\phi X})) - 2\dot{\phi}XH^2(3G_{5,X} + 2XG_{5,XX}),\\
w_3 &\equiv 3X(K_{,X} + 2XK_{,XX} - 2G_{3,\phi} - 2XG_{3,\phi X}) + 18\dot{\phi}XH(2G_{3,X} + XG_{3,XX}) - 18\dot{\phi}H(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2G_{4,\phi XX})\\
&\quad - 18H^2(1/2 + G_4 - 7XG_{4,X} - 16X^2G_{4,XX} - 4X^3G_{4,XXX}) - 18XH^2(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2G_{5,\phi XX})\\
&\quad + 6\dot{\phi}XH^3(15G_{5,X} + 13XG_{5,XX} + 2X^2G_{5,XXX}),\\
w_4 &\equiv 1 + 2(G_4 - XG_{5,\phi} - XG_{5,X}\ddot{\phi}). \qquad \text{(A2)}
\end{aligned}$$

De Felice et al. 2011; L.A. et al.,PRD, arXiv:1210.0439, 2012

Yukawa Potential

$$\eta(k,a) = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

Momentum space

$$k^2\Psi = 4\pi G Y(a,k)\rho_m(k)$$

$$k^2\Phi = 4\pi G Y(a,k)\eta(a,k)\rho_m(k)$$

$$\Psi = -\frac{GM}{r}h_2\left(1+\frac{h_4-h_5}{h_5}e^{-r/\sqrt{h_5}}\right) = -\frac{\bar{G}M}{r}\left(1+Qe^{-mr}\right)$$

$$\Phi = -\frac{GM}{r}h_1\left(1+\frac{h_3-h_5}{h_5}e^{-r/\sqrt{h_5}}\right) = -\frac{\hat{G}M}{r}\left(1+\hat{Q}e^{-mr}\right)$$

Real space

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

2nd order perturbed Lagrangians

An elegant way to derive the perturbation equations is to
write down the perturbed Lagrangian at second order:
when you differentiate it, you get first order perturbation equations!

For instance standard EH Lagrangian gives in Minkowski

$$ds^2 = -(1+2\Psi)dt^2 + 2B_{,i}dx^i dt + ((1+2\Phi)\delta_{ij} + 2E_{,ij})dx^i dx^j$$

$$S_g = \frac{1}{2}\int d^3x dt[-8B_i\Phi_i' + 4\Phi_i\Psi_i - 4\Phi'\Box E' + 2\Phi_i^2 - 6\left(\Phi'\right)^2]$$

Definition

Propagating degree of freedom: a field with two time derivatives in the 2nd order Lagrangian, **once all the constraints have been taken into account**

Identifying the degrees of freedom

Classifying theories by their dof

Standard gravity: no scalar degree of freedom, 2 tensor dofs (beside matter)
Horndeski: one scalar dof, 2 tensor dofs (beside matter)
Massive grav: one scalar dof, 2+2 tensor dofs (beside matter)
Bimetric models: one scalar dof, 2 vector dofs, 2+2 tensor dofs (beside matter)

2nd order pert. Lagrangian shows explicitly the dofs

For Horndeski:

$$S = \int d^3x dt \{ Q_S [\dot{\varphi} - \frac{c_s^2}{a^2} (\partial_i \varphi)^2] + \sum_{\alpha=1}^{2} Q_T [\dot{h_\alpha}^2 - \frac{c_T^2}{a^2} (\partial_i h_\alpha)^2] \}$$

De Felice & Tsujikawa 2011

Identifying the degrees of freedom

2nd order pert. Lagrangian shows explicitly the dofs

$$S=\int d^3x dt\{Q_S[\dot{\varphi}-\frac{c_s^2}{a^2}(\partial_i\varphi)^2]+\sum_{\alpha=1}^{2}Q_T[\dot{h_\alpha}^2-\frac{c_T^2}{a^2}(\partial_i h_\alpha)^2]\}$$

scalar tensor

…and the minimal number of free functions: two for each dof (plus a function for the background, i.e. 5 functions for Horndeski)

The magic of 2nd order perturbed Lagrangians

2nd order pert. Lagrangian also clarifies the stability conditions

$$S = \int d^3x dt \{ Q_S [\dot{\varphi} - \frac{c_s^2}{a^2} (\partial_i \varphi)^2] + \sum_{\alpha=1}^{2} Q_T [\dot{h_\alpha}^2 - \frac{c_T^2}{a^2} (\partial_i h_\alpha)^2] \}$$

Equation of motion in Fourier space $\ddot{\phi} - c_s^2 k^2 \phi = 0$

Positive squared sound speeds $c_{s,T}^2 \geq 0$

Positive kinetic terms $Q_{s,T} \geq 0$

Horndeski Lagrangian

$$\mathcal{L}_2 = K(\phi, X)\,,$$
$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi\,,$$
$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,,$$
$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right].$$

Linearized Horndeski: Bellini-Sawicki parametrization

$$
\begin{aligned}
M_*^2 \equiv & 2\left(G_4 - 2XG_{4X} + XG_{5\phi} - \dot{\phi}HXG_{5X}\right) \\
HM_*^2\alpha_{\rm M} \equiv & (\dot{M_*^2}) \\
H^2M_*^2\alpha_{\rm K} \equiv & 2X\left(K_X + 2XK_{XX} - 2G_{3\phi} - 2XG_{3\phi X}\right) + \\
& + 12\dot{\phi}XH\left(G_{3X} + XG_{3XX} - 3G_{4\phi X} - 2XG_{4\phi XX}\right) + \\
& + 12XH^2\left(G_{4X} + 8XG_{4XX} + 4X^2G_{4XXX}\right) - \\
& - 12XH^2\left(G_{5\phi} + 5XG_{5\phi X} + 2X^2G_{5\phi XX}\right) + \\
& + 4\dot{\phi}XH^3\left(3G_{5X} + 7XG_{5XX} + 2X^2G_{5XXX}\right) \\
HM_*^2\alpha_{\rm B} \equiv & 2\dot{\phi}\left(XG_{3X} - G_{4\phi} - 2XG_{4\phi X}\right) + \\
& + 8XH\left(G_{4X} + 2XG_{4XX} - G_{5\phi} - XG_{5\phi X}\right) + \\
& + 2\dot{\phi}XH^2\left(3G_{5X} + 2XG_{5XX}\right) \\
M_*^2\alpha_{\rm T} \equiv & 2X\left(2G_{4X} - 2G_{5\phi} - \left(\ddot{\phi} - \dot{\phi}H\right)G_{5X}\right)
\end{aligned}
$$

sound speeds

$$S=\int d^3xdt\{Q_S[\dot\varphi-\frac{c_s^2}{a^2}(\partial_i\varphi)^2]+\sum_{\alpha=1}^{2}Q_T[\dot{h_\alpha}^2-\frac{c_T^2}{a^2}(\partial_i h_\alpha)^2]\}$$

$$Q_\mathrm{S}=\frac{M_*^2(2\alpha_K+3\alpha_B^2)}{(2-\alpha_\mathrm{B})^2},$$

$$c_\mathrm{S}^2=\frac{(2-\alpha_\mathrm{B})\,\alpha_1+2\alpha_2}{2\alpha_K+3\alpha_B^2},$$

$$Q_\mathrm{T}=\frac{M_*^2}{8},$$

$$c_\mathrm{T}^2=1+\alpha_\mathrm{T}$$

$$\alpha_1\equiv\alpha_B+(\alpha_B-2)\,\alpha_T+2\alpha_M$$

$$\alpha_2\equiv\alpha_B\xi+\alpha_B'-2\xi-3\tilde\Omega_m$$

Bellini-Sawicki parametrization

Model Class		α_K	α_B	α_M	α_T
ΛCDM		***0***	***0***	***0***	***0***
cuscuton ($w_X \neq -1$)	[71]	0	0	0	0
quintessence	[1, 2]	$(1-\Omega_m)(1+w_X)$	0	0	0
k-*essence*/perfect fluid	[45, 46]	$\frac{(1-\Omega_m)(1+w_X)}{c_s^2}$	0	0	0
kinetic gravity braiding	[47–49]	$\frac{m^2(n_m+\kappa_\phi)}{H^2 M_{Pl}^2}$	$\frac{m\kappa}{H M_{Pl}^2}$	0	0
galileon cosmology	[57]	$-\frac{3}{2}\alpha_M^3 H^2 r_c^2 e^{2\phi/M}$	$\frac{\alpha_K}{6} - \alpha_M$	$\frac{-2\dot\phi}{HM}$	0
BDK	[26]	$\frac{\dot\phi^2 K_{,\dot\phi\dot\phi} e^{-\kappa}}{H^2 M^2}$	$-\alpha_M$	$\frac{\dot\kappa}{H}$	0
metric $f(R)$	[3, 72]	0	$-\alpha_M$	$\frac{B\dot H}{H^2}$	0
MSG/Palatini $f(R)$	[73, 74]	$-\frac{3}{2}\alpha_M^2$	$-\alpha_M$	$\frac{2\dot\phi}{H}$	0
f(Gauss-Bonnet)	[52, 75, 76]	0	$\frac{-2H\dot\xi}{M^2+H\dot\xi}$	$\frac{\dot H\dot\xi+H\ddot\xi}{H(M^2+H\dot\xi)}$	$\frac{\ddot\xi-H\dot\xi}{M^2+H\dot\xi}$

Beyond Horndeski, I

Look now at the matter equations of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

Standard sub-horizon matter equations

$$\theta = ik_j v^j$$

$$\dot{\delta} = -\theta$$ continuity

$$\dot{\theta} = -\mathcal{H}\theta + k^2\Psi$$ Euler

$$k^2\Phi = 4\pi G a^2 \rho\delta$$ Poisson

Modifying gravity

$$k^2\Phi = 4\pi G a^2 \rho\delta Y\eta$$

Modifying continuity equation

$$\dot{\delta} = -\theta + \Delta$$

$$\Delta = 4\alpha'_H \frac{\mathcal{H}M_P^2}{\rho_m a^2} k^2\Phi$$

Gleyzes et al 2014;
Lombriser et al 2015

Beyond Horndeski, II

Or, we can add several Horndeski fields!

$$\int dx^4 \sqrt{-g}\left[L_{H1} + L_{H2} + \ldots + L_{matter}\right]$$

$$Y \equiv A_1 \frac{1 + A_2 k^2 + A_3 k^4}{1 + A_4 k^2 + A_5 k^4},$$

$$\eta \equiv -\frac{\Phi}{\Psi} = B_1 \frac{1 + B_2 k^2 + B_3 k^4}{1 + B_4 k^2 + B_5 k^4},$$

A. Silvestri et al. 2013
T. Baker et al. 2013
V. Vardanyan & L.A., 2015

Beyond Horndeski, III

- Torsion
- Non-metricity
- Non-universal coupling
- Palatini
- Vectors
- Tensors

Beyond Horndeski, IV

Pauli-Fierz (1939) action: the only ghost-free quadratic action for a massive spin two field

$$\int d^4x\sqrt{g}R_g + m^2\int d^4x h_{\mu\nu}h_{\alpha\beta}\left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

The three deadly sins of Pauli-Fierz theory:

- It does not reduce to massless gravity for $m \to 0$ (vDVZ disc.)
- It violates diffeomorphism invariance
- It contains a ghost when extended to non-linear level (Boulware-Deser ghost)

Ghost-free Bigravity

- The first problem was solved by Vainshtein (1972): there exists a radius below which the linear theory cannot be applied;
  - For the Sun, this radius is larger than the solar system!

- The second and third problems can be solved introducing a second metric:

$$
\begin{aligned}
S =& -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f) \\
&+ m^2 M_g^2 \int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n(\sqrt{g^{\alpha\beta}f_{\beta\gamma}}) + \int d^4x\sqrt{-\det g}L_m(g,\Phi)
\end{aligned}
$$

The only ghost-free local non-linear massive gravity theory!

deRham, Gabadadze, Tolley 2010
Hassan & Rosen, 2011

Modified Gravity in bimetric gravity

In the quasi-static limit, every Horndeski model and bimetric gravity model is characterized at linear scales by the two functions

$$\eta(k,a) = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

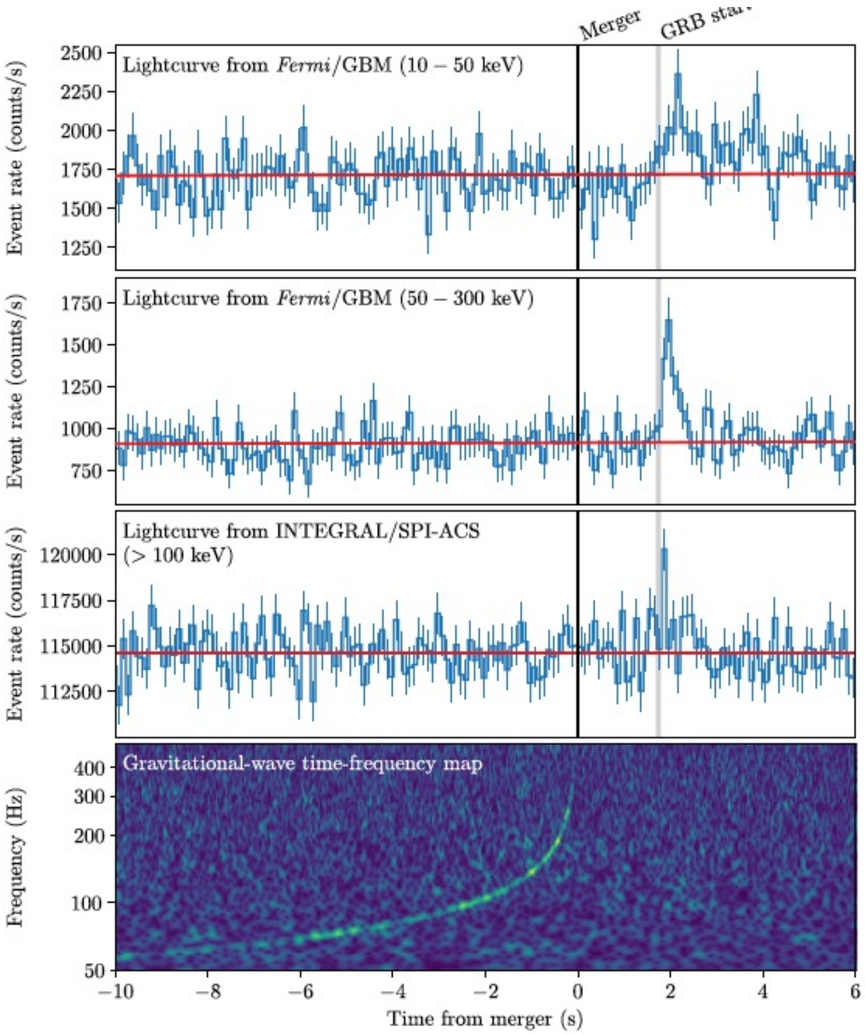
k = wavenumber

$\boldsymbol{h_i}$ **= time-dependent functions**

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

The speed of gravitational waves

2017:
GW and gamma-ray simultaneous detection



The speed of gravitational waves

$$S=\int d^3x dt\{Q_S[\dot{\varphi}-\frac{c_s^2}{a^2}(\partial_i\varphi)^2]+\sum_{\alpha=1}^{2}Q_T[\dot{h_\alpha}^2-\frac{c_T^2}{a^2}(\partial_i h_\alpha)^2]\}$$

$$Q_{\rm S}=\frac{M_*^2(2\alpha_K+3\alpha_B^2)}{(2-\alpha_{\rm B})^2},$$

$$c_{\rm S}^2=\frac{(2-\alpha_{\rm B})\,\alpha_1+2\alpha_2}{2\alpha_K+3\alpha_B^2},$$

$$Q_{\rm T}=\frac{M_*^2}{8},$$

$$c_{\rm T}^2=1+\alpha_{\rm T}$$

$$\alpha_1\equiv\alpha_B+(\alpha_B-2)\,\alpha_T+2\alpha_M$$

$$\alpha_2\equiv\alpha_B\xi+\alpha_B'-2\xi-3\tilde{\Omega}_m$$

The speed of gravitational waves

$$c^2_T = 1 + \alpha_T$$

$$M_*^2 \alpha_T \equiv 2X \left(2G_{4X} - 2G_{5\phi} - \left(\ddot{\phi} - \dot{\phi} H\right) G_{5X}\right)$$

Only way to have c_T=1 is

$$G_{4,X} = 0$$

$$G_5 = 0$$

Conformal coupling!

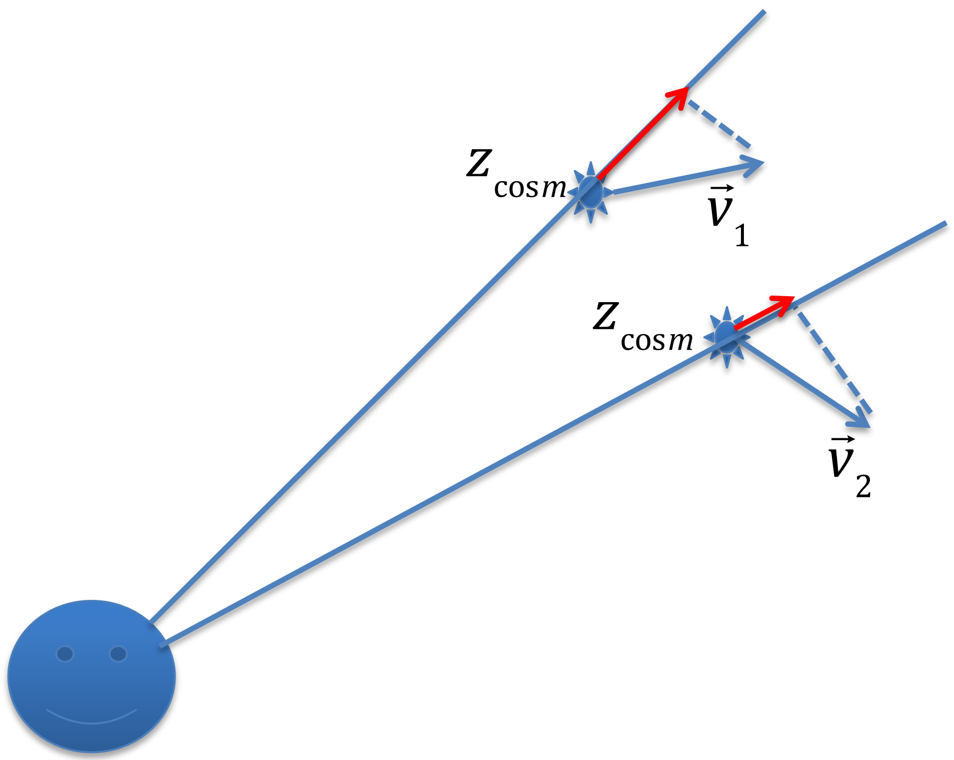
Caveats...

The speed of GW has been measured only at $z = 0$!

Measuring H(z)

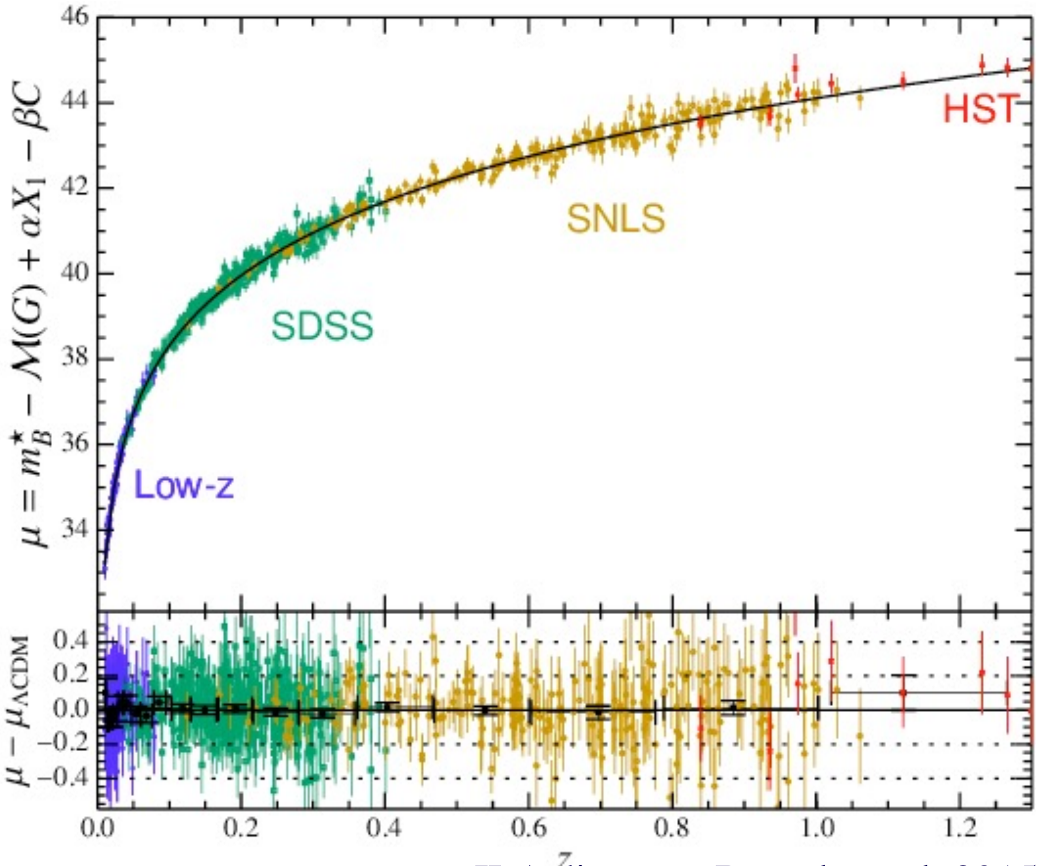
Standard candles give us d_L

But their scatter around z also give us their peculiar velocity:



SNIa scatter

$$d_L(z) = d_L(z_{cosm} + z_{pec})$$
$$\approx d_L(z_{cosm}) + \Delta d_L(z_{pec})$$



JLA diagram, Betoule et al. 2015

From luminosity to velocity

Hui & Greene 2006
astro-ph/0512159
Davis et al. 2011
1012.2912

$$\frac{\delta D_L}{D_L} = v_s \left[2 - \frac{d\log D_L}{d\log(1+z)}\right]$$

$$\frac{\delta D_L}{D_L} = \frac{\log 10}{5}\delta m$$

$$v_s = \frac{\log 10}{5}\delta m \left[2 - \frac{d\log D_L}{d\log(1+z)}\right]^{-1}$$

Measuring H(z)

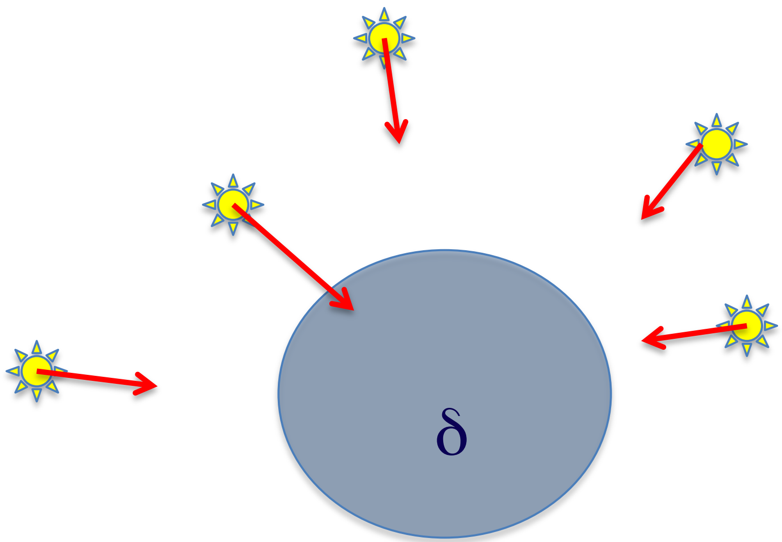
What is the variance of *v*?

$$\sigma^2_{v,\rm eff} \equiv \left[\frac{\log 10}{5}\sigma_{\rm int}\right]^2 \left[2 - \frac{d\log D_L}{d\log(1+z)}\right]^{-2} + \frac{\sigma^2_{v,\rm nonlin}}{c^2}.$$

Hui & Greene 2006
astro-ph/0512159
Davis et al. 2011
1012.2912

Intrinsic scatter — conversion — non-linear scatter

Peculiar velocity field



correlation between linear velocity field and density contrast

Measuring H(z)

Standard candles give us d_L

But we can also measure their space correlation and their peculiar velocity!

cosine of angle between k and r

In the linear regime:

$$v_{\|} = i\frac{H}{k(1+z)}\beta\frac{\boldsymbol{k}\cdot\boldsymbol{r}}{kr}\delta_g = i\frac{H\mu}{k(1+z)}\beta\delta_g\,,$$

$$\beta = \frac{f}{b} = \frac{growth}{bias}$$

Measuring H(z)

So given a large number of SNIa we can measure the correlations of their positions and of their peculiar velocity field for every *k*,*μ*

$$\bar{n}V\langle\delta\delta^*\rangle = \bar{n}\left(1+\beta\mu^2\right)^2 S_\delta^2 b^2 P_m + 1 = B^2\, PN_\delta\,,$$

$$i\bar{n}V\langle\delta v^*\rangle = \frac{\bar{n}H\mu}{k(1+z)}\left(1+\beta\mu^2\right)S_\delta S_v\,\beta b^2 P_m = ABP,$$

$$\bar{n}V\langle vv^*\rangle = \bar{n}\left[\frac{H\mu}{k(1+z)}\right]^2 S_v^2\beta^2 b^2 P_m + \sigma_{v,\mathrm{eff}}^2 = A^2 PN_v,$$

D. Burkey and A. N. Taylor, Mon. Not. Roy. Astron. Soc. **347**, 255 (2004), arXiv:astro-ph/0310912 [astro-ph].

C. Howlett, A. S. G. Robotham, C. D. P. Lagos, and A. G. Kim, Astrophys. J. **847**, 128 (2017), arXiv:1708.08236 [astro-ph.CO].

3x2 Covariance matrix

two random fields (density contrast and peculiar velocity), three correlation functions

$$L=[(2\pi)^N C]^{-1/2}\exp(-\frac{1}{2}x_i C_{ij}^{-1} x_j)$$

$$x=\{\delta,v\}$$

$$C_{ij}=\begin{pmatrix} B^2 PN_\delta & ABP \\ ABP & A^2 PN_v \end{pmatrix}$$

Measuring H(z)

However, we do not have access to k,μ
but only to k_r,μ_r given a reference cosmology

For a generic cosmology one has

$$k = \alpha k_r$$

$$\mu = \mu_r H / (H_r \alpha)$$

where

$$\alpha = \frac{\sqrt{H^2 D^2 \mu_r^2 - H_r^2 D_r^2 (\mu_r^2 - 1)}}{H_r D}$$

All together: Clustering of Standard Candles (CSC)

observables

$$\bar{n}V\langle\delta\delta^*\rangle = \bar{n}\left(1+\beta\mu^2\right)^2 S_\delta^2 b^2 P_m + 1 = B^2 PN_\delta\,,$$

$$i\bar{n}V\langle\delta v^*\rangle = \frac{\bar{n}H\mu}{k(1+z)}(1+\beta\mu^2)S_\delta S_v\,\beta b^2 P_m = ABP,$$

$$\bar{n}V\langle vv^*\rangle = \bar{n}\left[\frac{H\mu}{k(1+z)}\right]^2 S_v^2\beta^2 b^2 P_m + \sigma_{v,\mathrm{eff}}^2 = A^2 PN_v,$$

depend entirely on the following parameters

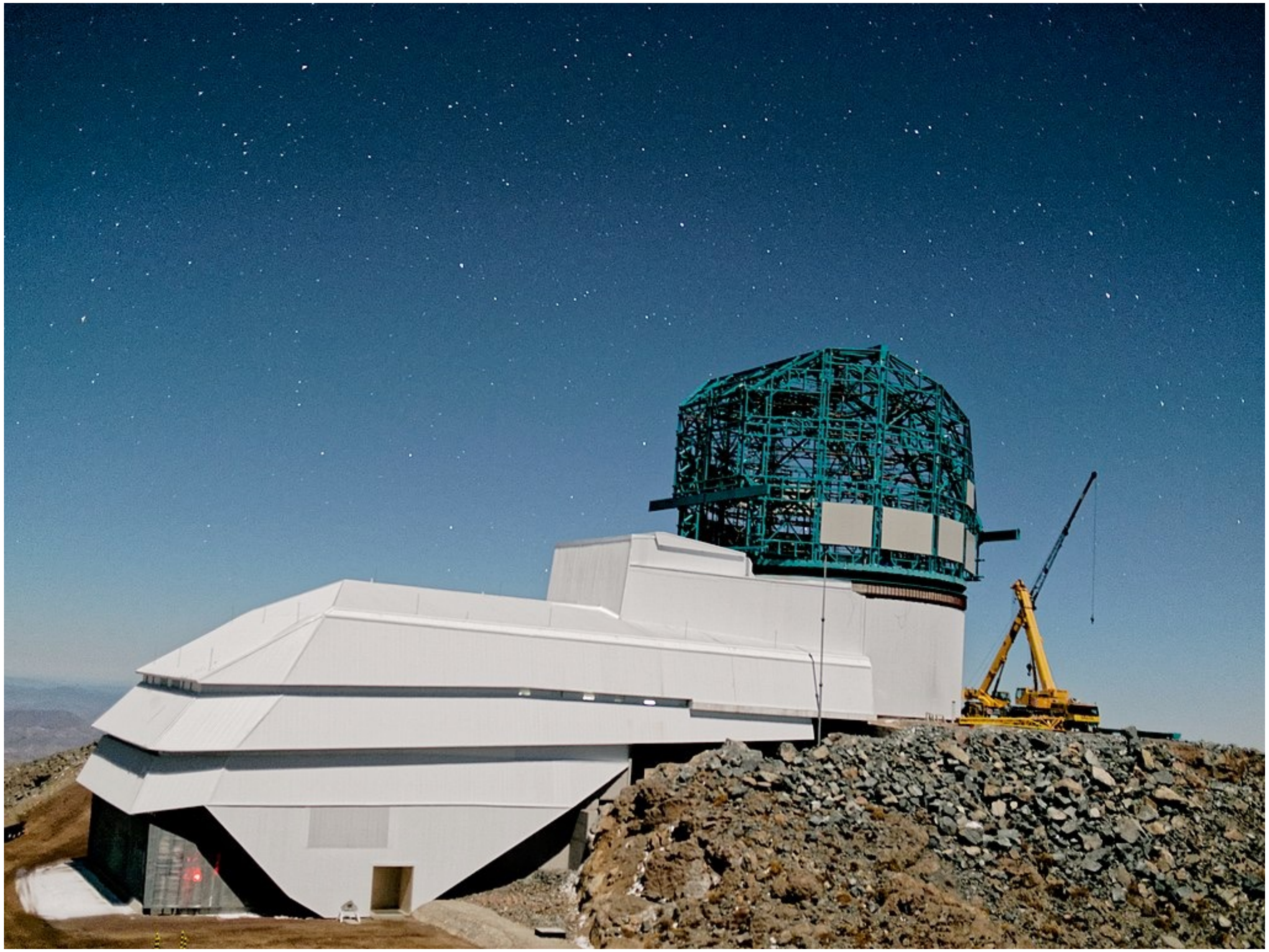
$$H_0 D(z),\quad E(z),\quad \beta(k,z),\quad \sigma_\delta(z),\sigma_v(z),\quad P(k,z)$$

non-linear velocity smoothing (fixed)

galaxy power spectrum

They completely specify the correlations at any (linear) k,μ,z without assuming any cosmology !

LSST at
Vera Rubin Observatory



By Wil O'Mullane 2019-09-11, CC BY-SA 4.0,
https://commons.wikimedia.org/w/index.php?curid=97813768

Observing η

$$\eta(k,a) = -\frac{\Phi}{\Psi}.$$

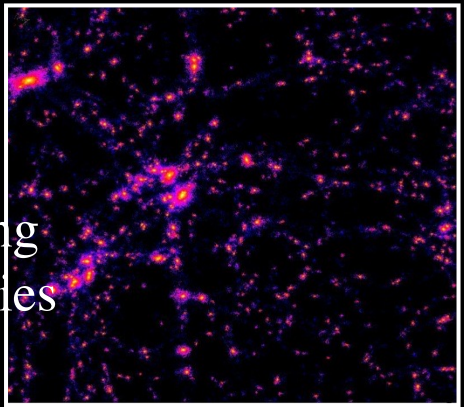
Reconstruction of the metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

Non-relativistic particles respond to Ψ

$$\delta'' + (1 + \frac{H'}{H})\delta' = k^2\Psi$$

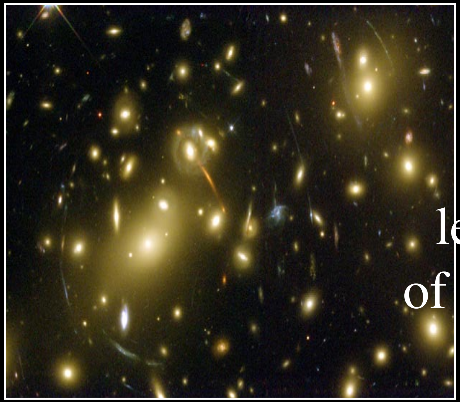
$$\delta = \frac{\rho - \hat{\rho}}{\hat{\rho}}$$



clustering of galaxies

Relativistic particles respond to Φ-Ψ

$$\alpha = \int \nabla_{perp}(\Psi - \Phi)dz$$



lensing of galaxies

Observing Y

Modified-gravity
sub-horizon matter equations

$$\theta = ik_j v^j$$

$$\delta = \frac{\rho(x) - \rho_0}{\rho_0}$$

$$\dot{\delta} = -\theta$$ continuity

$$\dot{\theta} = -\mathcal{H}\theta + k^2\Psi$$ Euler

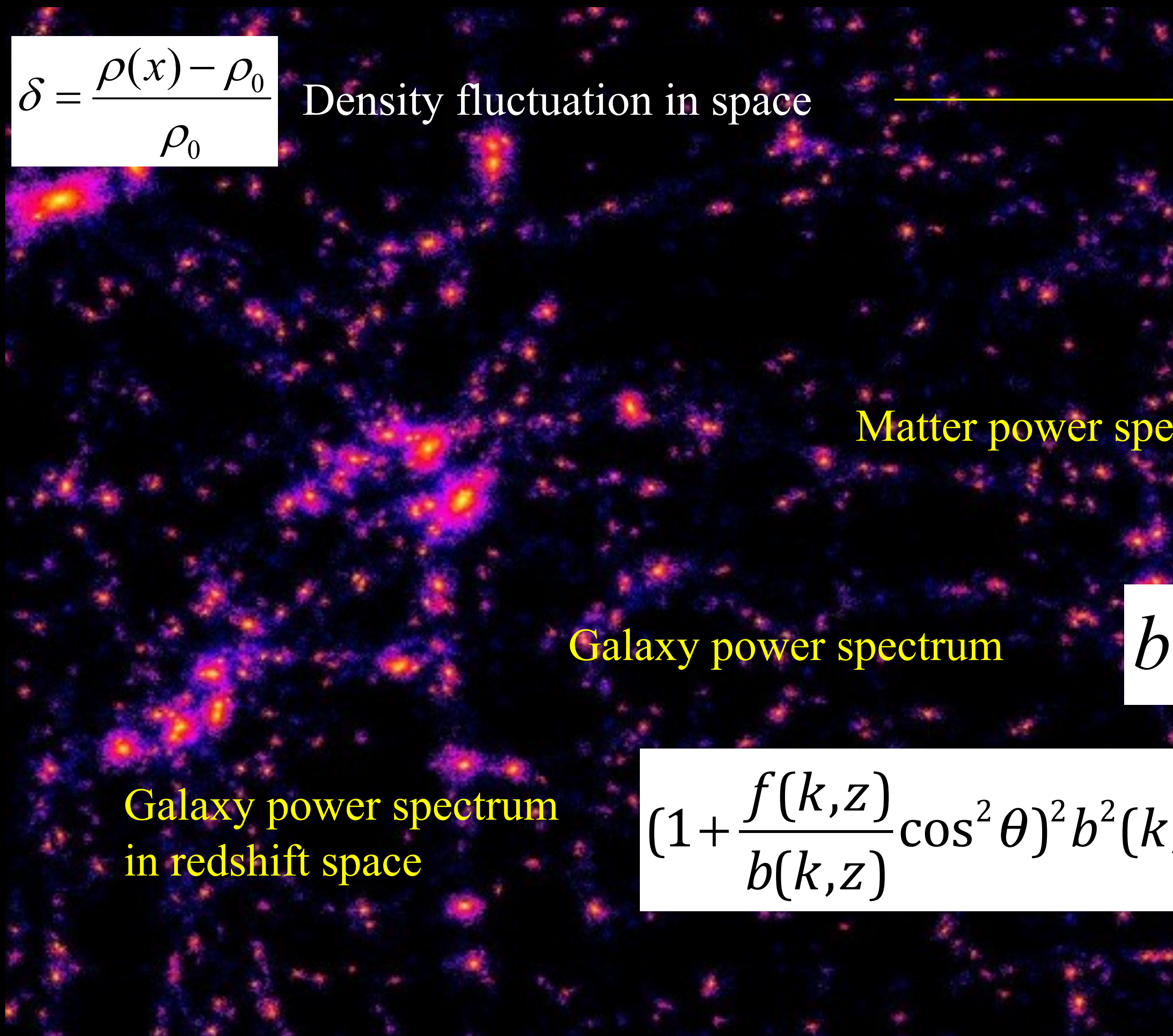
$$k^2\Phi = 4\pi G a^2 \rho\delta Y\eta$$ Poisson

This can be written as a single equation:

$$\delta'' + (1 + \frac{H'}{H})\delta' - \frac{3}{2}\Omega_m Y\delta = 0$$

prime is d/dlog(a)

Reality check



$$\delta = \frac{\rho(x) - \rho_0}{\rho_0}$$ Density fluctuation in space → $$\left\langle {\delta_k}^2 \right\rangle = P(k,z)$$

Matter power spectrum $$P_{matter}(k,z)$$

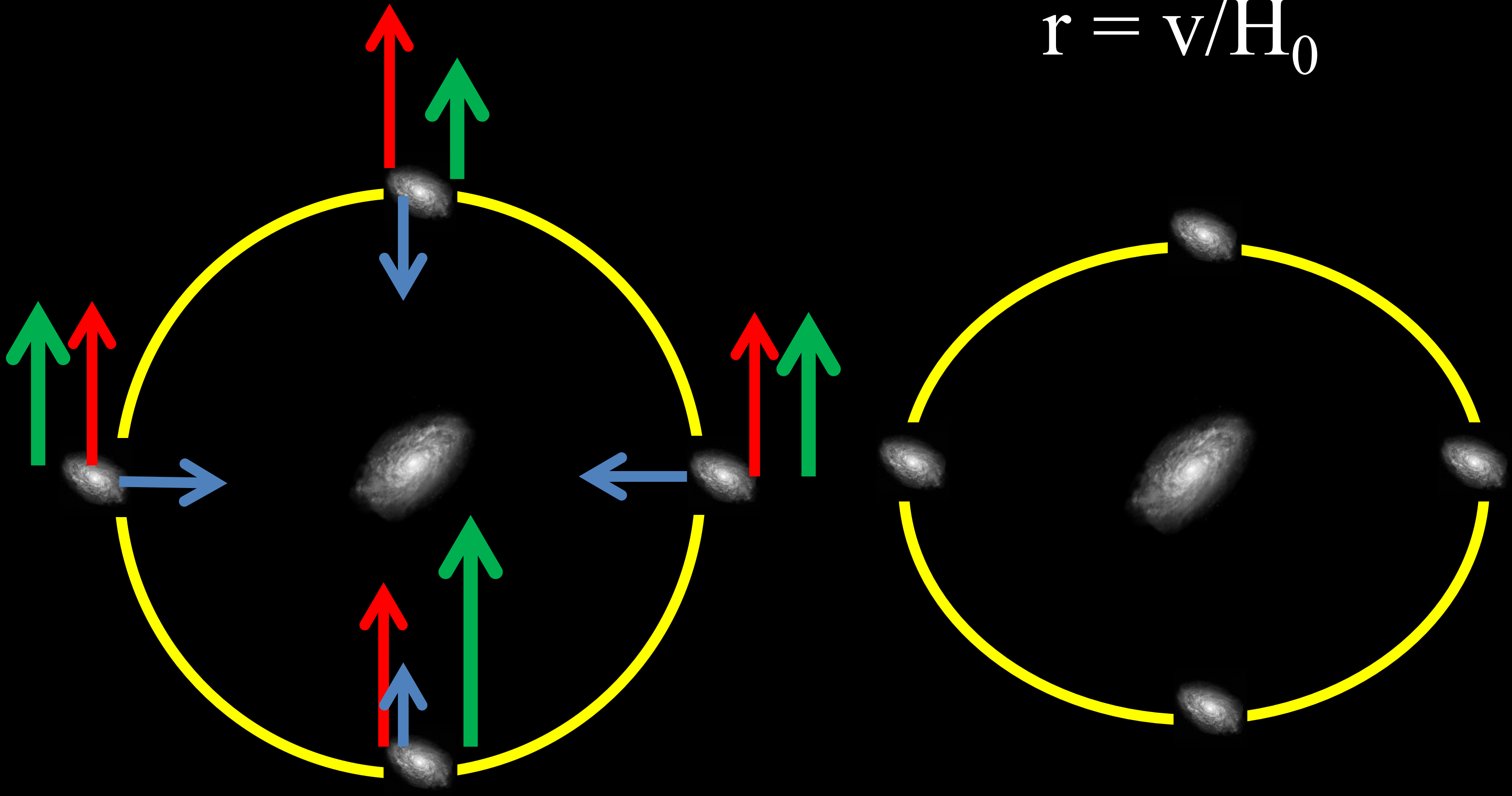
Galaxy power spectrum $$b^2(k,z) P_{matter}(k,z)$$

Galaxy power spectrum in redshift space

$$\left(1 + \frac{f(k,z)}{b(k,z)} \cos^2\theta\right)^2 b^2(k,z) G^2(k,z) P^{matter}_{initial}(k, z_{in})$$

Peculiar velocities

$r = v/H_0$



Real space

Redshift space

Deconstructing the galaxy power spectrum

$$P_{galaxy}(k,z,\mu=\cos\theta)=(1+\frac{f(k,z)}{b(k,z)}\cos^2\theta)^2 b^2(k,z)G^2(k,z)P_{initial}^{matter}(k,z_{in})$$

$b(k,z)$

$P_{initial}^{matter}(k,z_{in})$

do not depend on gravity

$G(k,z)$

$f(k,z)$

depend on gravity

Deconstructing the galaxy power spectrum

Line-of-sight decomposition

$$\mu \equiv \cos\theta$$

$$\delta_{galaxy}(k,z,\mu) = G(k,z)(1+\frac{f(k,z)}{b(k,z)}\mu^2)\ b(k,z)\delta_{initial}^{matter}(k,z_{in})$$

$$\equiv A + R\mu^2$$

$$\delta_{\text{lensing}}(k,z) = -\frac{3}{2}Y(1+\eta)G(k,z)\Omega_m\delta_{initial}^{matter}(k,z_{in}) \equiv L$$

Three linear observables: A, R, L

galaxy clustering

$$A = Gb\delta_{m,0}(k)$$

$$R = Gf\delta_{m,0}(k)$$

weak gravitational lensing

$$L = -\frac{3}{2}Y(1+\eta)G\Omega_m\delta_{m,0}(k)$$

The only model-independent ratios

Redshift distortion/Amplitude $$P_1 = \frac{R}{A} = \frac{f}{b}$$

Lensing/Redshift distortion $$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f}$$

Redshift distortion rate $$P_3 = \frac{R'}{R} = \frac{f'}{f} + f$$

Expansion rate $$E = \frac{H}{H_0}$$

How to combine them to test the theory?

Summarizing....

Matter conservation equation independent of gravity theory

$$\delta''+(1+\frac{H'}{H})\delta'-\frac{3}{2}\Omega_m Y\delta=0$$

Observables

$$P_2=\frac{L}{R}=\frac{\Omega_{m0}Y(1+\eta)}{f} \qquad P_3=\frac{R'}{R}=\frac{f'}{f}+f \qquad E=\frac{H}{H_0}$$

Testing the entire Horndeski Lagrangian

A unique combination of observables

Independent of the bias, of the initial conditions, of the specific model

$$\frac{3P_2(1+z)^3}{2E^2(P_3+2+\frac{E'}{E})}-1=\eta=h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

Observables

Theory

L.A., M. Motta, I. Sawicki, M. Kunz, I. Saltas, 1210.0439 1305.0008

Testing the entire Horndeski Lagrangian

$$\eta_{obs} \equiv \frac{3P_2(1+z)^3}{2E^2(P_3+2+\frac{E'}{E})} - 1$$

$\eta_{obs} \neq 1$ → gravity is modified

$\eta_{obs}(k) \neq \eta(Horndeski)$ → The entire Horndeski model is falsified

L.A., M. Motta, I. Sawicki, M. Kunz, I. Saltas, 1210.0439 1305.0008

Can we measure it?

The first measurement ever

A compilation of all available datasets of lensing and growth

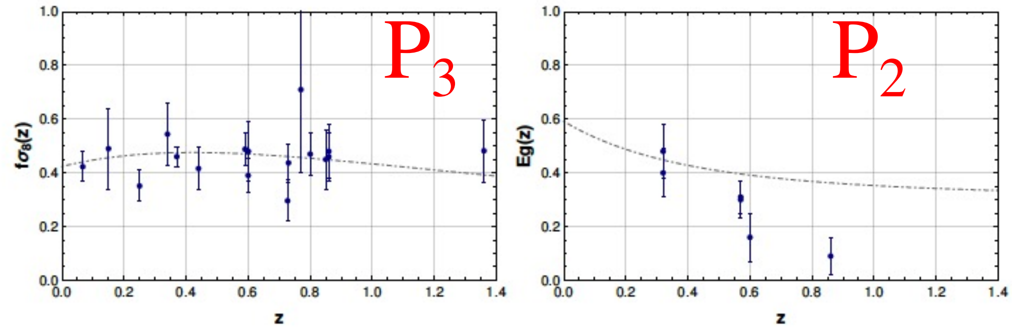


Figure 1. Left: Plot of the full $f\sigma_8$ data (table II) as a function of z with the theoretical curve in dashed dot. Right: Plot of the full E_g data (table III) as a function of z with the theoretical curve in dashed dot.

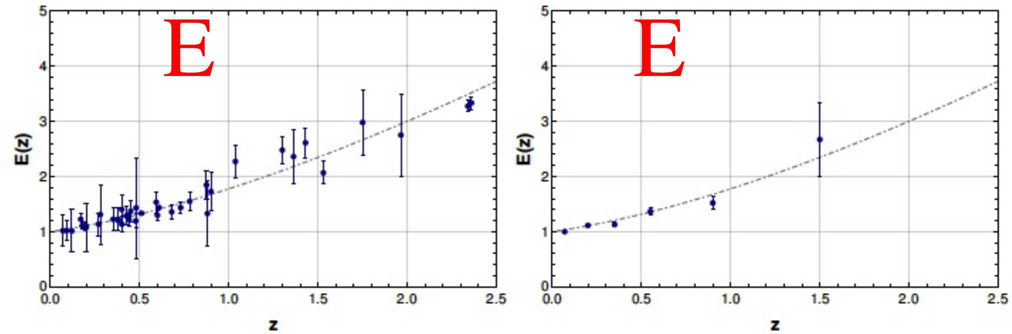
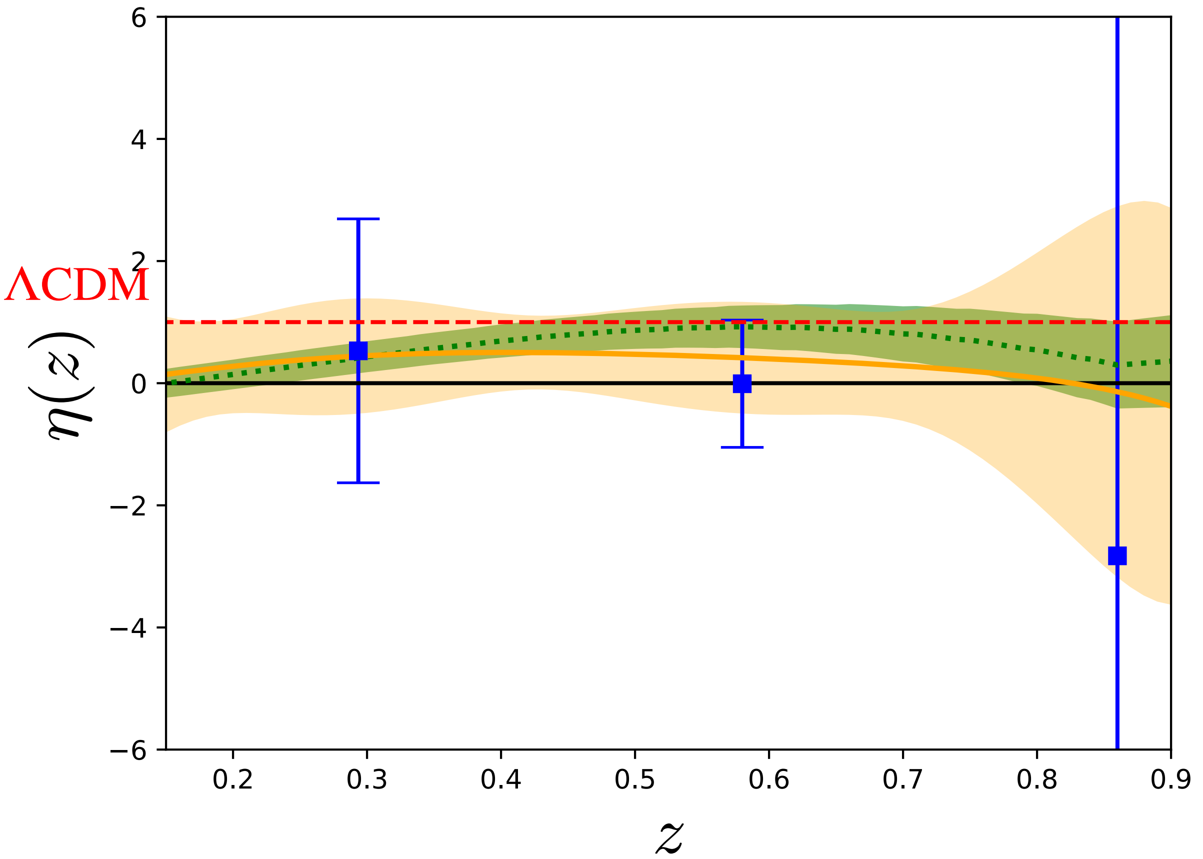


Figure 2. Right: Plot of the full $H(z)$ data (table IV) as a function of z with the theoretical curve in dashed dot. Right: Plot of the full $E(z)$ data (table II) as a function of z with the theoretical curve in dashed dot.

Collab. with Santiago Casas and Ana M. Pinho
arXiv:1805.00025

The first model-independent measurement ever



Result compressed
in a single central bin

$$\eta = 0.40 \pm 0.60$$

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy

15,000 square degrees

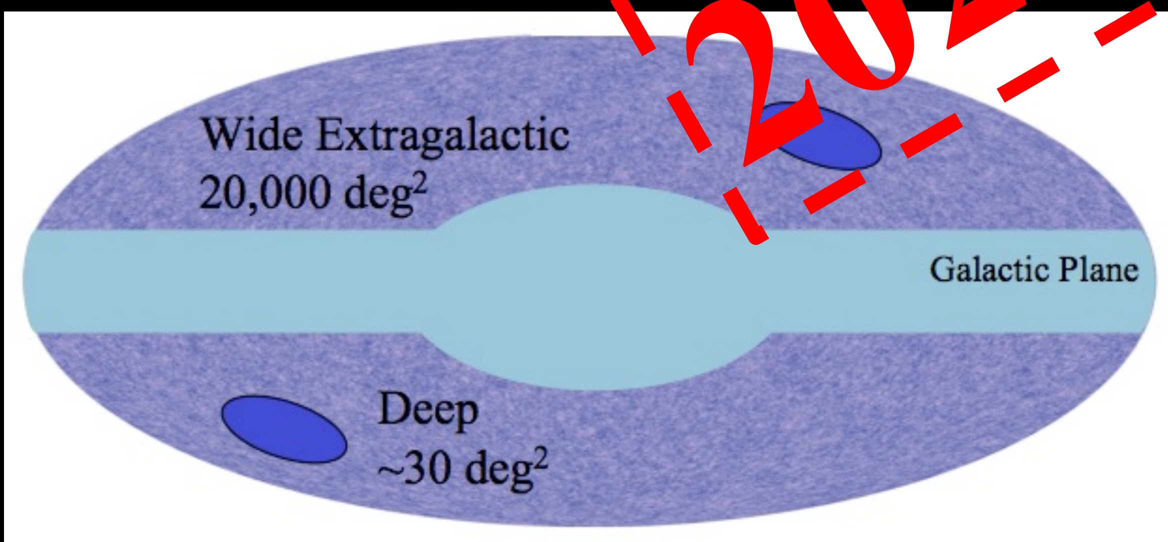
70 million redshifts, 2 billion images

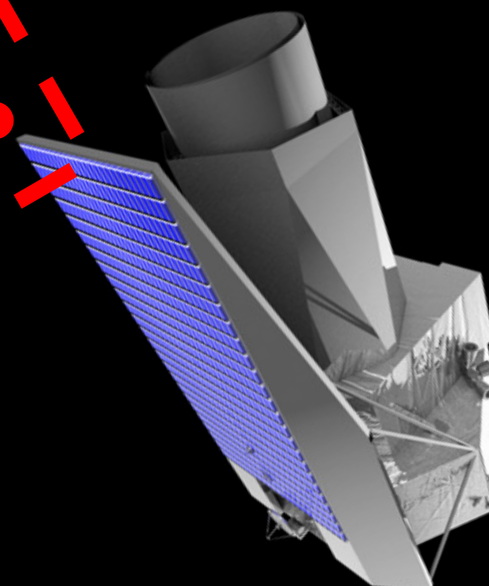
Median redshift z = 1

PSF FWHM ~0.18’’

>1000 peoples, >10 countries

2023!





Euclid satellite

arXiv Red Book 1110.3193

Results...

$$\eta(k,a) = H_2\left(\frac{1+k^2H_4}{1+k^2H_5}\right)$$

Model 1: η constant for all z, k
Error on η around 2%

Model 2: η varies in z
Error on η

η=1/2 for f(R)!
(at small scales)

TABLE X. Fiducial values and errors for the parameters $P_1, P_2, P_3, E'/E$ and $\bar{\eta}$ for every bin. The last bin has been omitted since R' is not defined there.

$\bar{z}$	P_1	ΔP_1	$\Delta P_1(\%)$	P_2	ΔP_2	$\Delta P_2(\%)$	P_3	ΔP_3	$\Delta P_3(\%)$	(E'/E)	$\Delta E'/E$	$\Delta E'/E(\%)$	$\bar{\eta}$	$\Delta\bar{\eta}$	$\Delta\bar{\eta}(\%)$
0.6	0.766	0.012	1.6	0.729	0.013	1.8	0.134	0.13	99	-0.920	0.022	2.4	1	0.11	11
0.8	0.819	0.010	1.2	0.682	0.011	1.6	0.317	0.12	38	-1.04	0.046	4.4	1	0.091	9.1
1.0	0.859	0.0093	1.1	0.650	0.011	1.7	0.460	0.12	26	-1.13	0.099	8.7	1	0.090	9.0
1.2	0.888	0.0092	1.0	0.628	0.014	2.3	0.569	0.13	23	-1.21	0.12	10	1	0.097	9.7
1.4	0.911	0.010	1.1	0.613	0.020	3.3	0.654	0.11	16	-1.26	0.09	7.1	1	0.073	7.3

L.A, M. Kunz, A. Vollmer,
A. Guarnizo, S. Fogli, 1311.4765, 2013

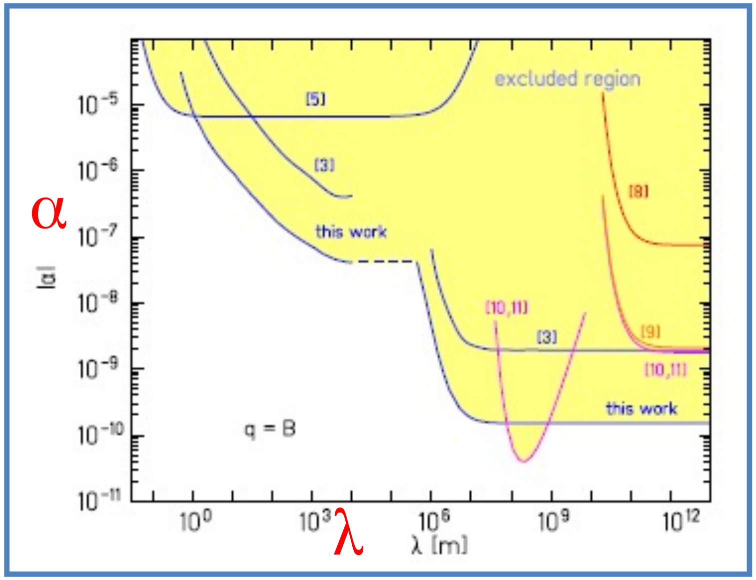
Observing Y

$$\nabla^2\Psi = 4\pi G\, Y(k,a)\rho_m\delta_m$$

In collab. with Laura Taddei and Matteo Martinelli

Observing Y

Excluded
in the Lab



$$\Psi(r) = -\frac{GM}{r}\left(1+\alpha e^{-r/\lambda}\right)$$

Observing Y

Modified-gravity
sub-horizon matter equations

$\theta = ik_j v^j$

$$\dot{\delta} = -\theta$$ continuity

$$\dot{\theta} = -\mathcal{H}\theta + k^2\Psi$$ Euler

$$k^2\Phi = 4\pi G a^2 \rho\delta Y\eta$$ Poisson

This can be written as a single equation:

$$\delta'' + \left(1 + \frac{H'}{H}\right)\delta' - \frac{3}{2}\Omega_m Y\delta = 0$$

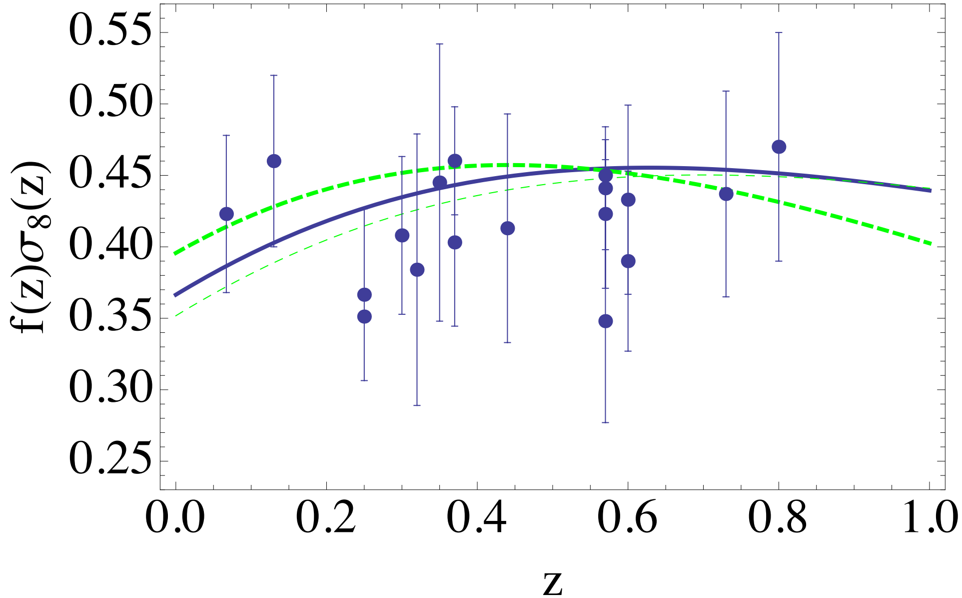
prime is d/dlog(a)

$f\sigma_8(z)$

$$R = Gf\delta_{m,0}(k) = const \times \delta'(z)$$

$$G = \frac{\delta(z)}{\delta(0)} \qquad f = \frac{d\log\delta}{d\log a}$$

$$\delta'' + \left(1 + \frac{H'}{H}\right)\delta' - \frac{3}{2}\Omega_m Y\delta = 0$$



By measuring the growth of fluctuations
we can find Y!

Yukawa Potential

$$Y(k,a) = h_1 \left(\frac{1+k^2 h_5}{1+k^2 h_3} \right)$$

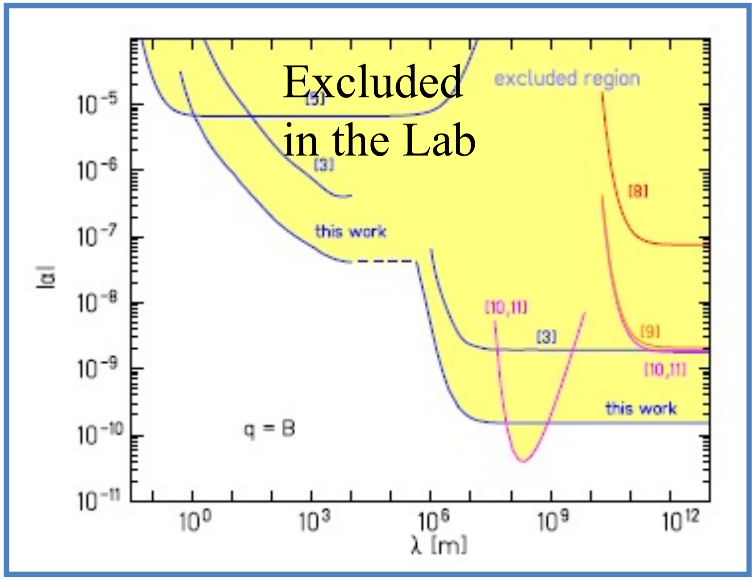
Momentum space

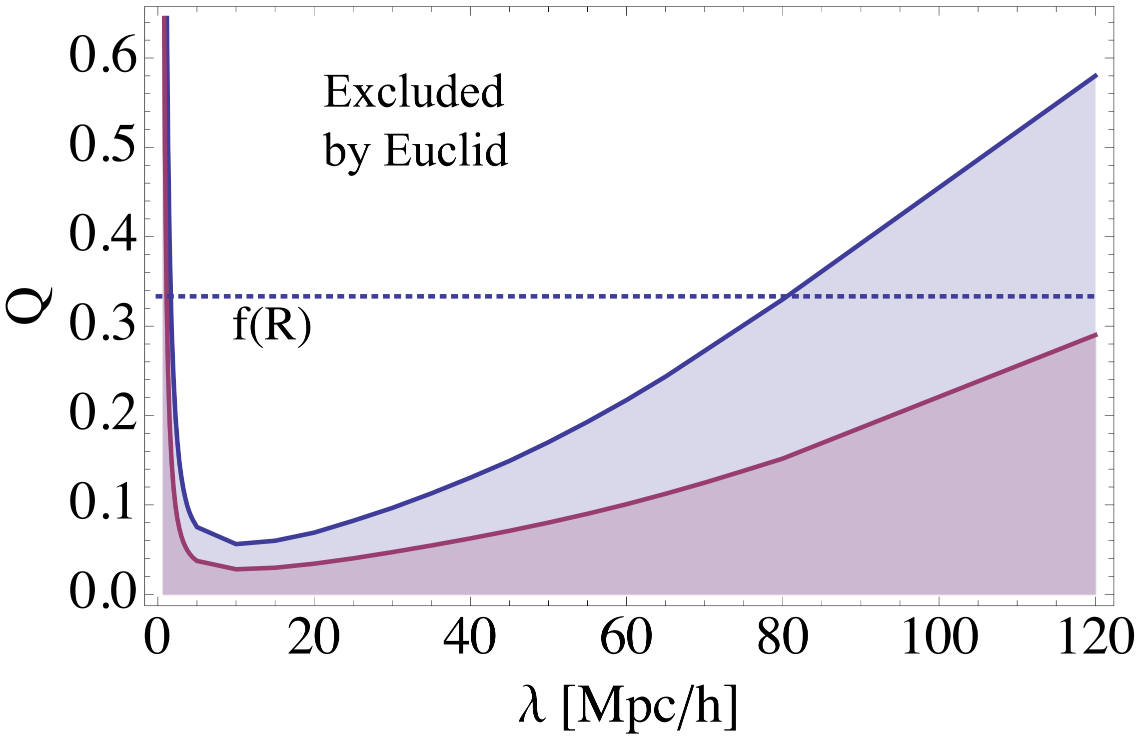
$$\nabla^2 \Psi = 4\pi G\, Y(k,a) \rho_m \delta_m$$

$$\Psi = -\frac{GM}{r} h_2 \left(1 + \frac{h_4 - h_5}{h_5} e^{-r/\sqrt{h_5}}\right) = -\frac{\bar{G}M}{r}\left(1 + Q e^{-mr}\right)$$

Real space

Cosmological exclusion plot





L. Taddei, L.A., 2014

$$\Psi(r) = -\frac{GM}{r}(1 + Qe^{-r/\lambda})$$

f(R)

$\lambda^{-2} = m^2 = \frac{R}{3}(\frac{f_{,R}}{Rf_{,RR}} - 1).$

Q=1/3

Conclusion

Next generation experiments like Euclid can test gravity at cosmological scales in a model-independent way