

XXXIII Canary Islands Winter School of Astrophysics

Fundamental Physics with Galaxies



The Standard Model of Particle Physics

No gravity

Does not include neutrino mass

Does not include dark matter

Cosmic inflation (origin of Universe)

Baryon asymmetry (origin of matter)

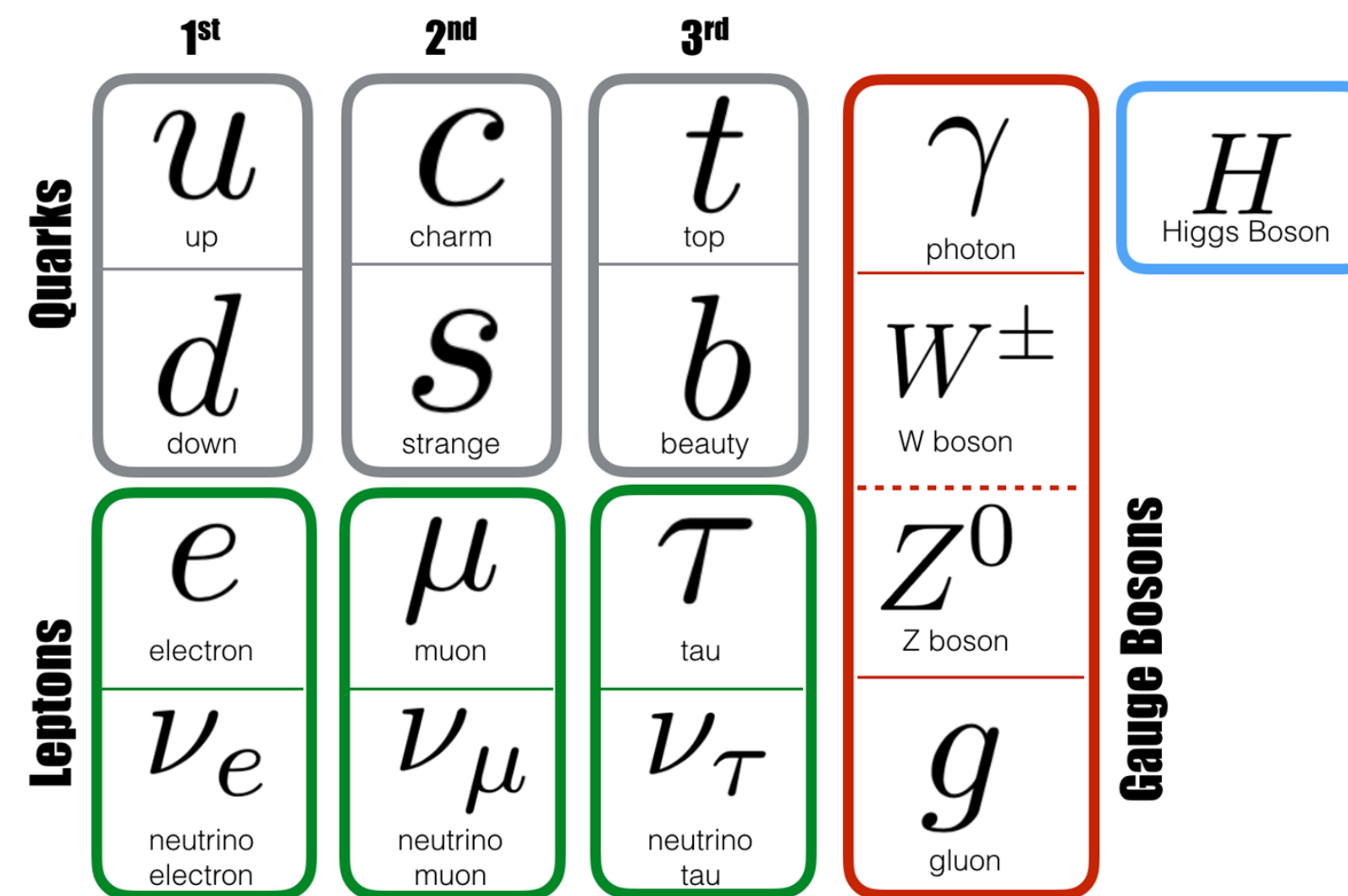
Why vacuum energy *just (not) zero*?

Why weak scale \ll Planck scale?

Why no strong CP violation?

Almost grand unification?

Why Higgs vacuum *just* metastable?



The Standard Model of Particle Physics and Astrophysics

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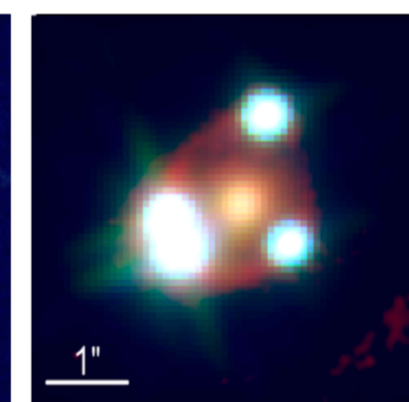
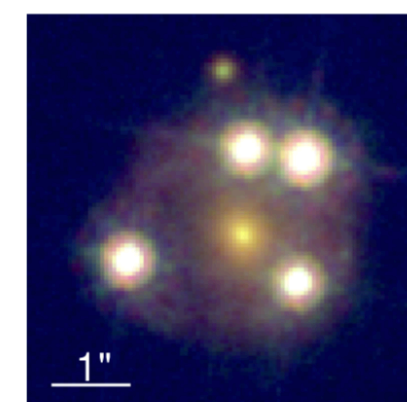
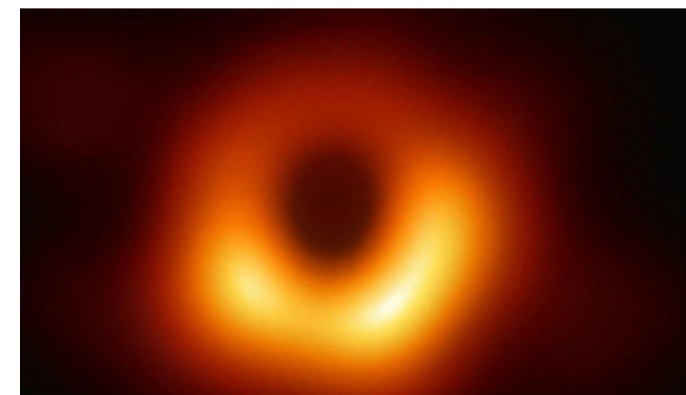
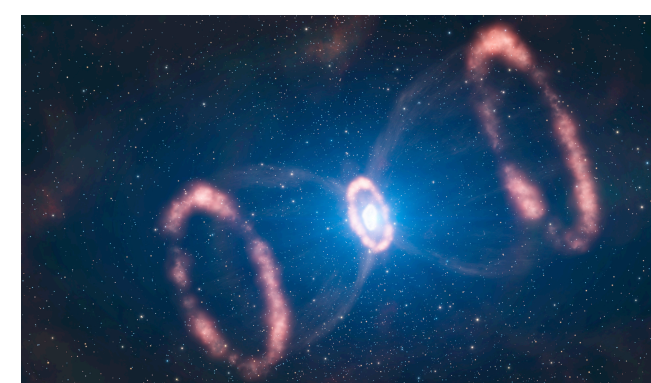
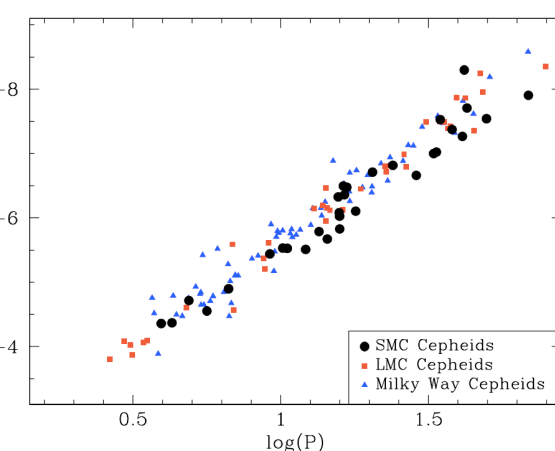
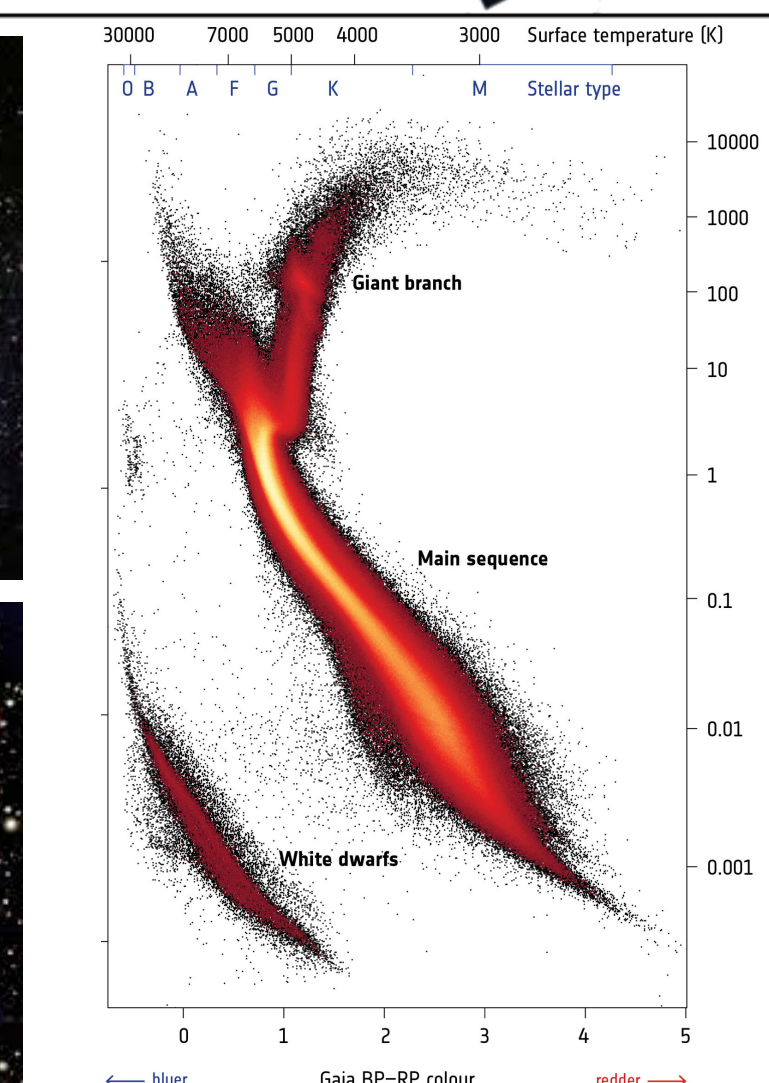
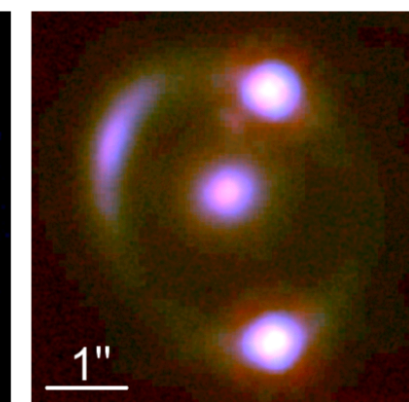
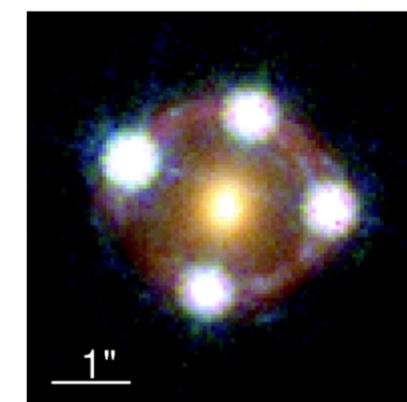
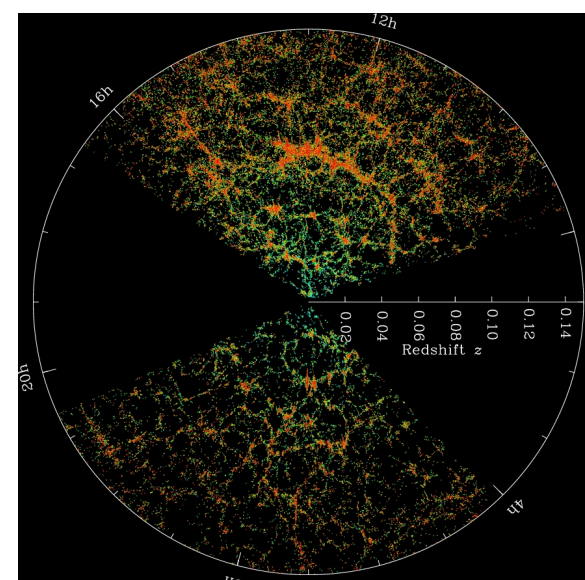
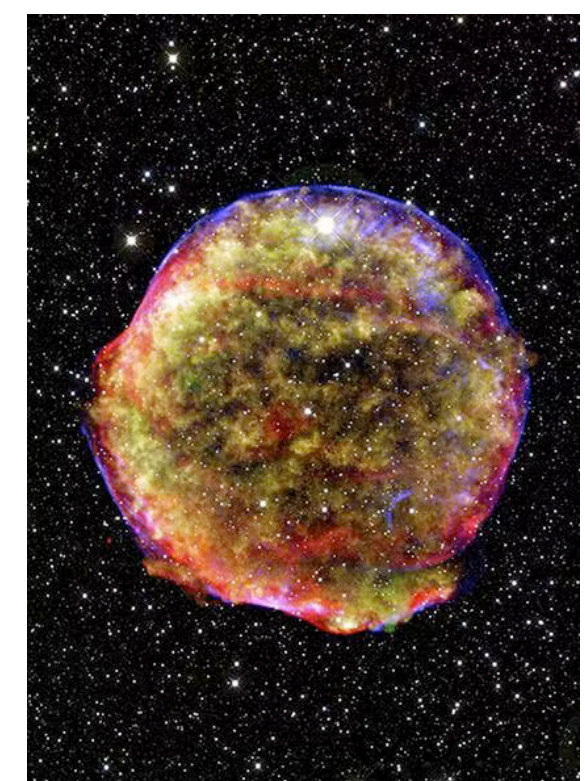
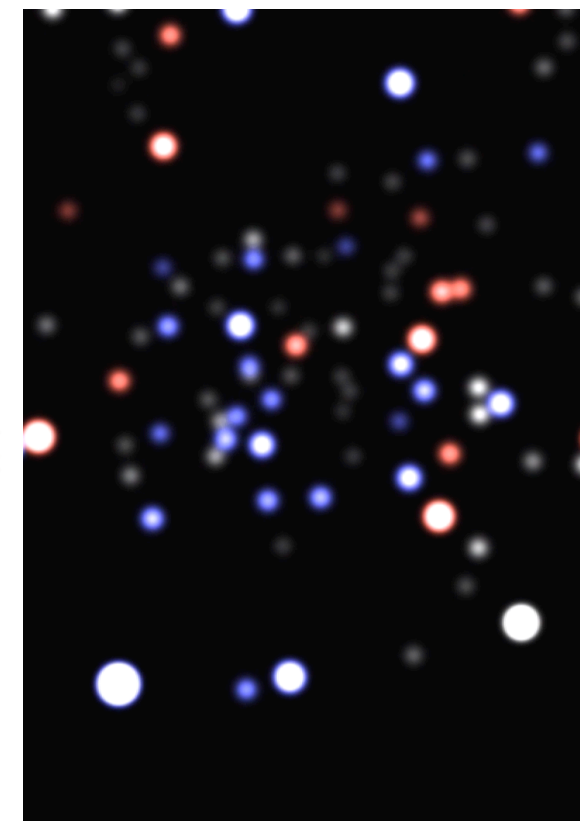
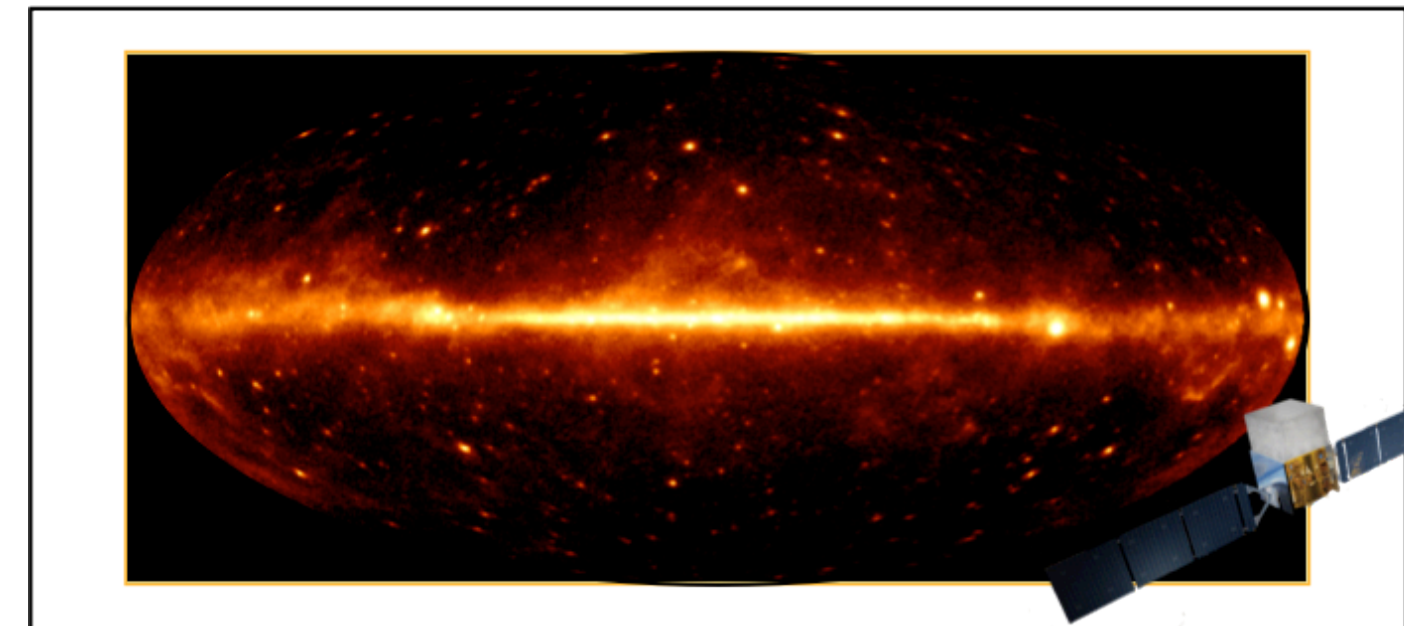
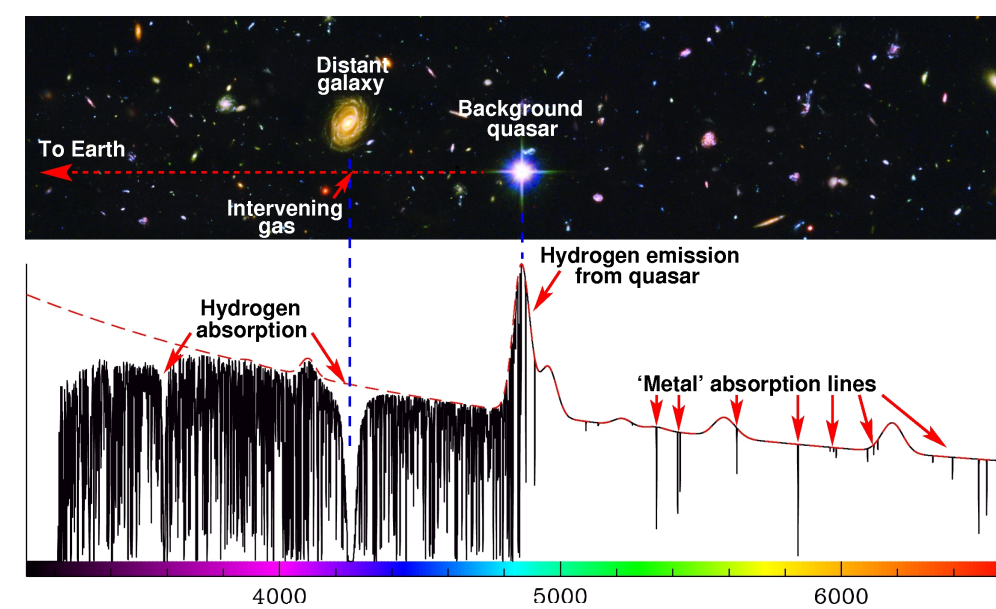
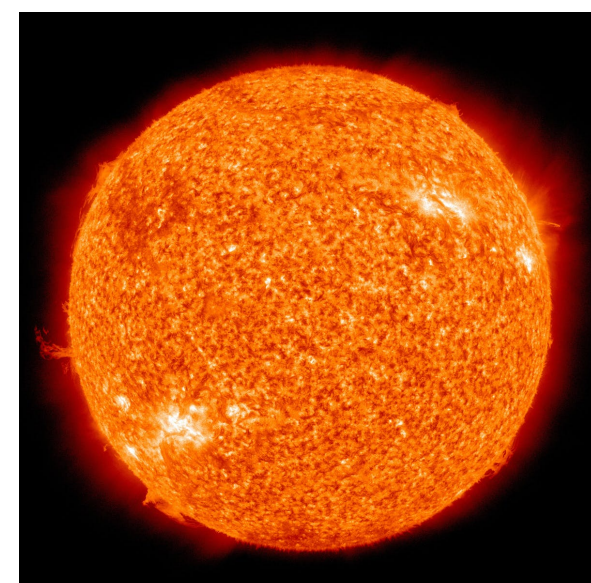
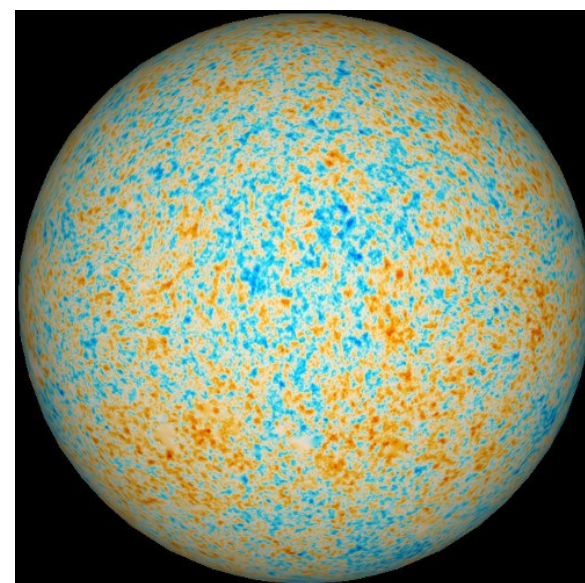
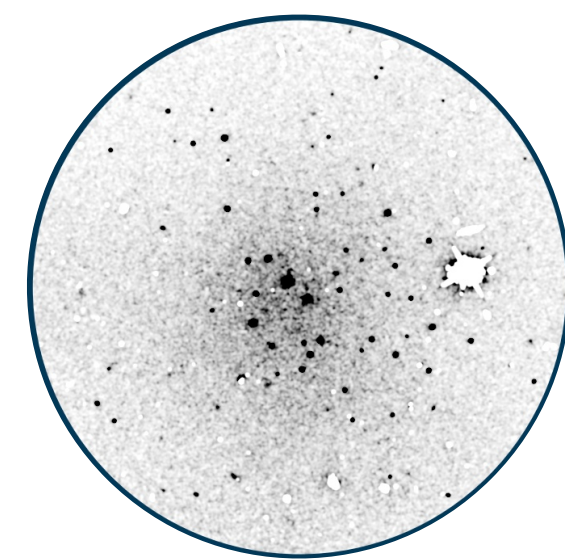
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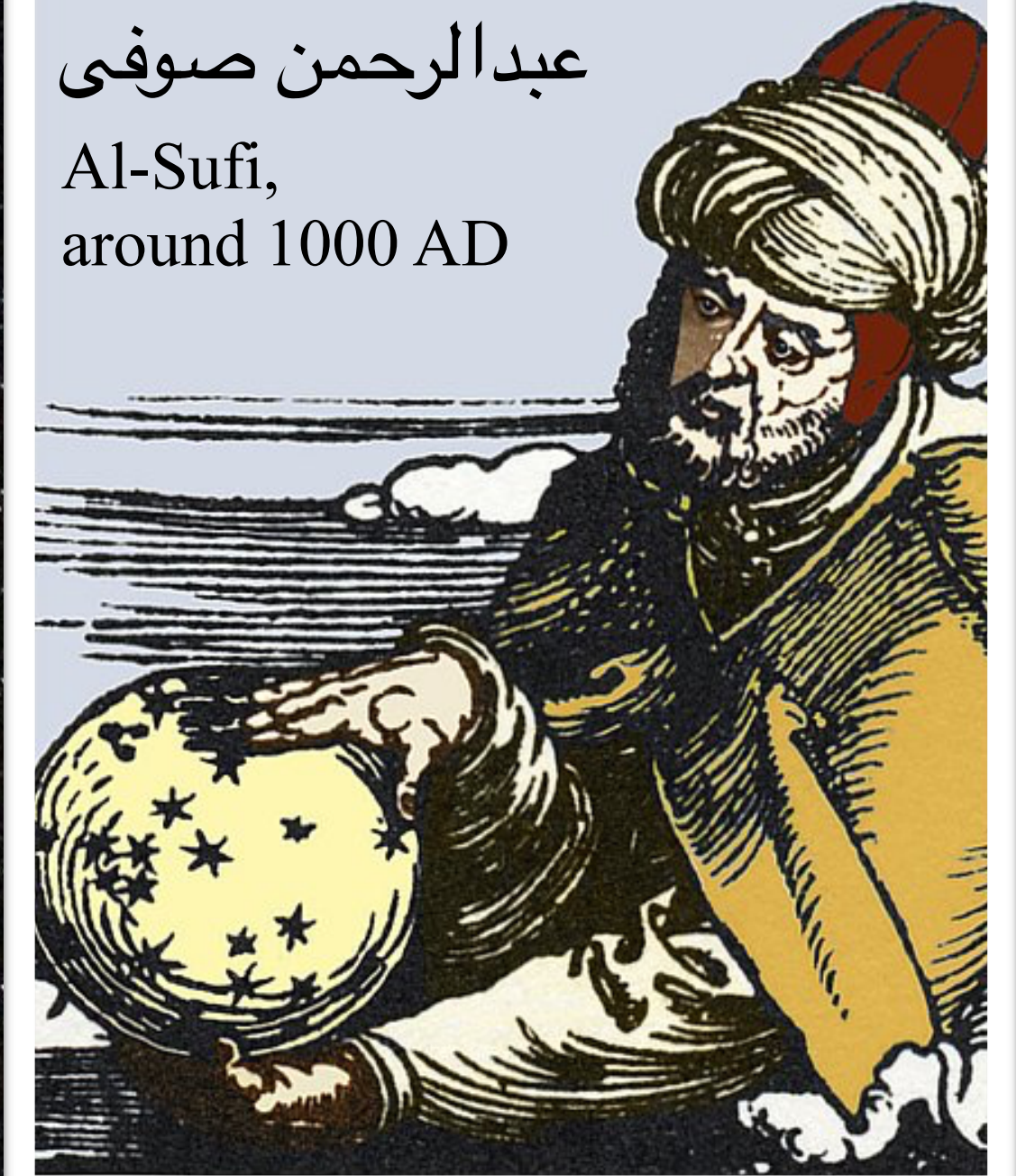
Almost grand unification?

Why Higgs vacuum *just* metastable?

- Observed first in astrophysics
- Observed only in astrophysics
- Constrained by astrophysics



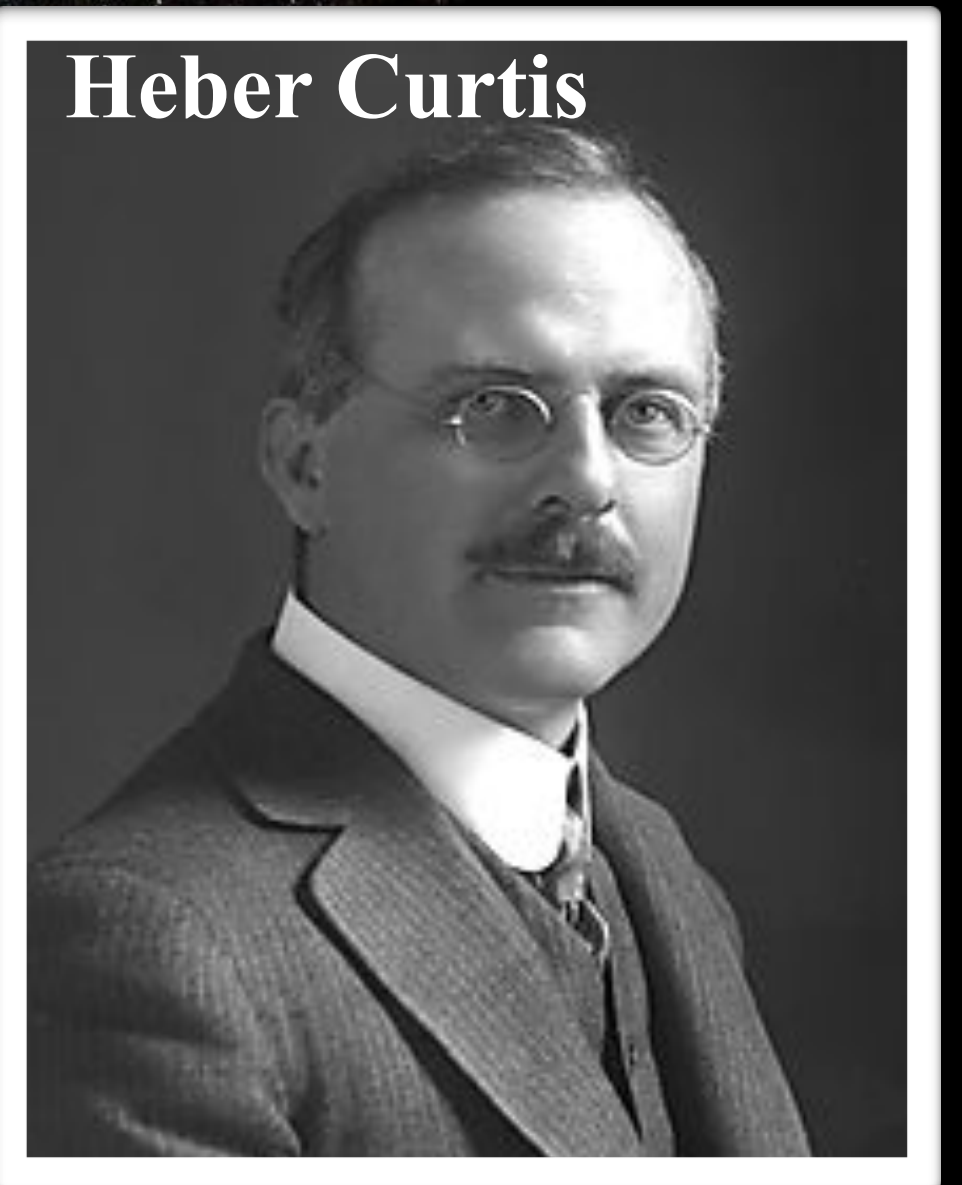
عبدالرحمن صوفى
Al-Sufi,
around 1000 AD



Harlow Shapley



Heber Curtis

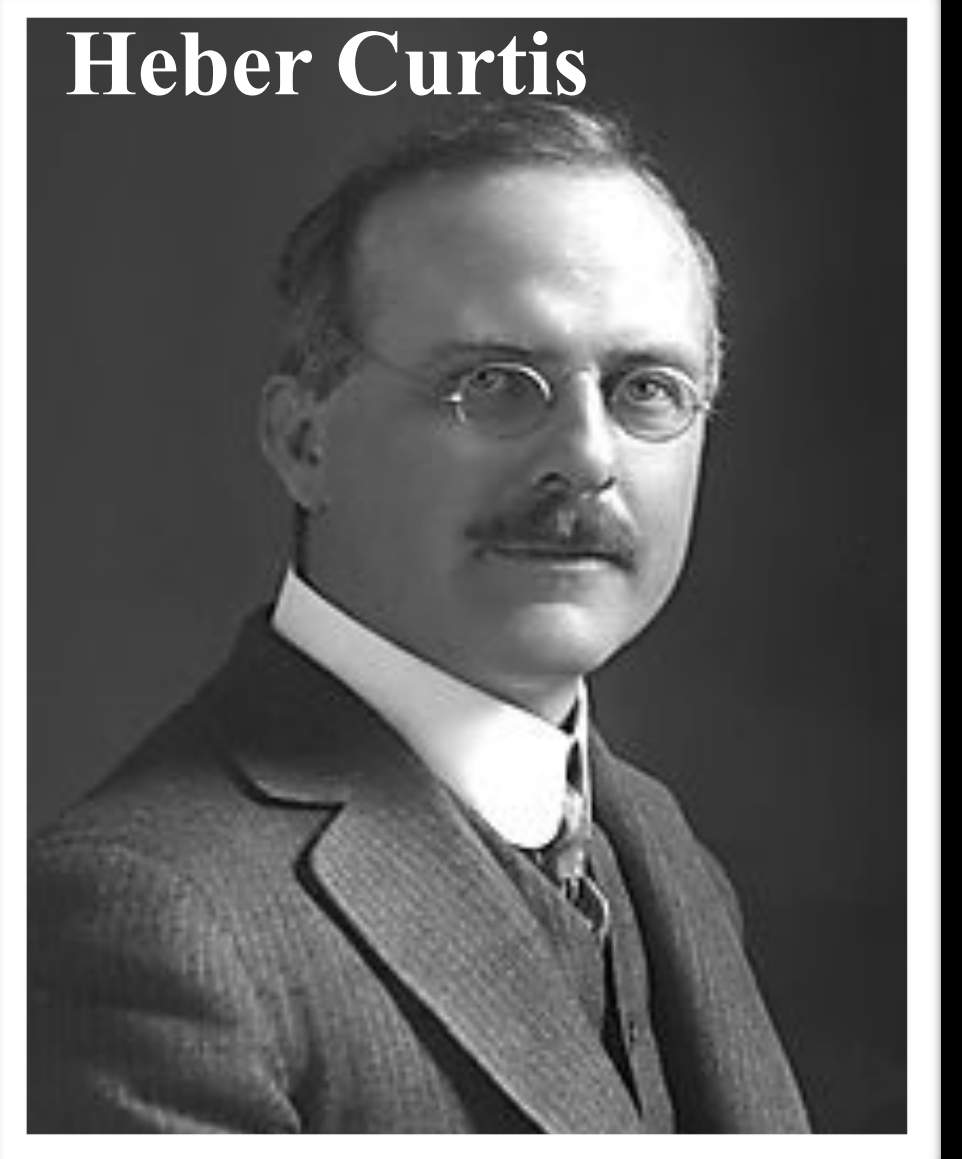


The Great Debate 1920

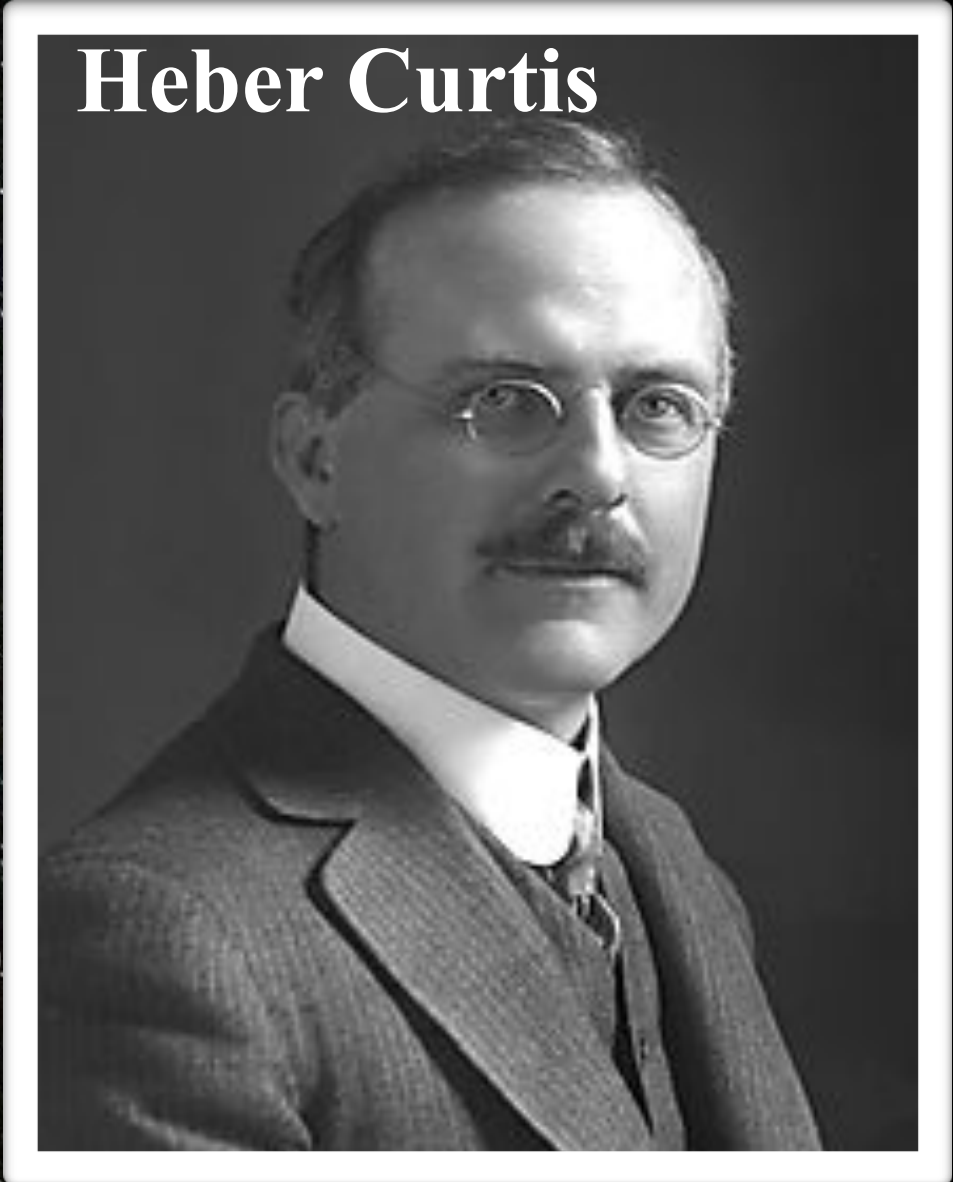
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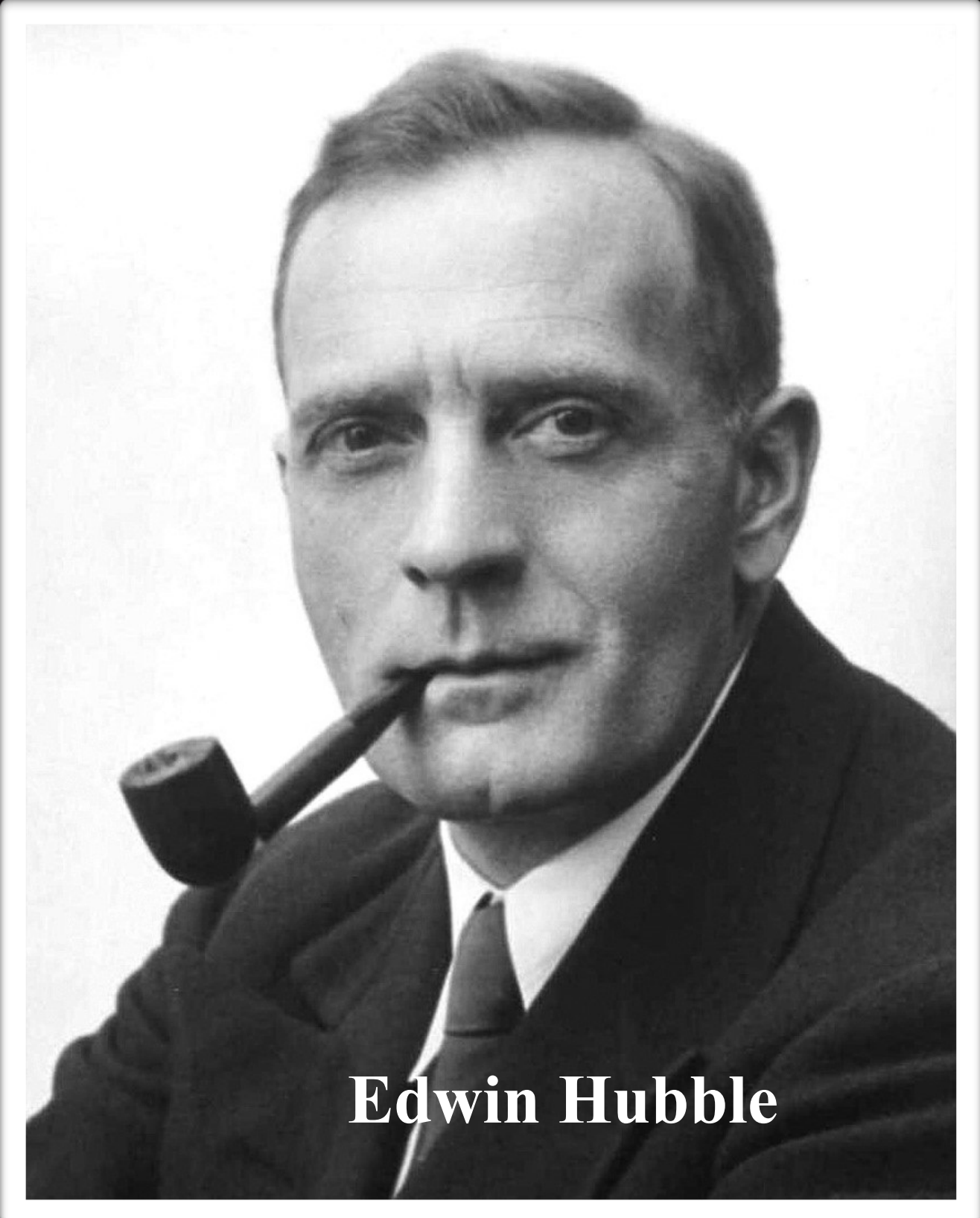
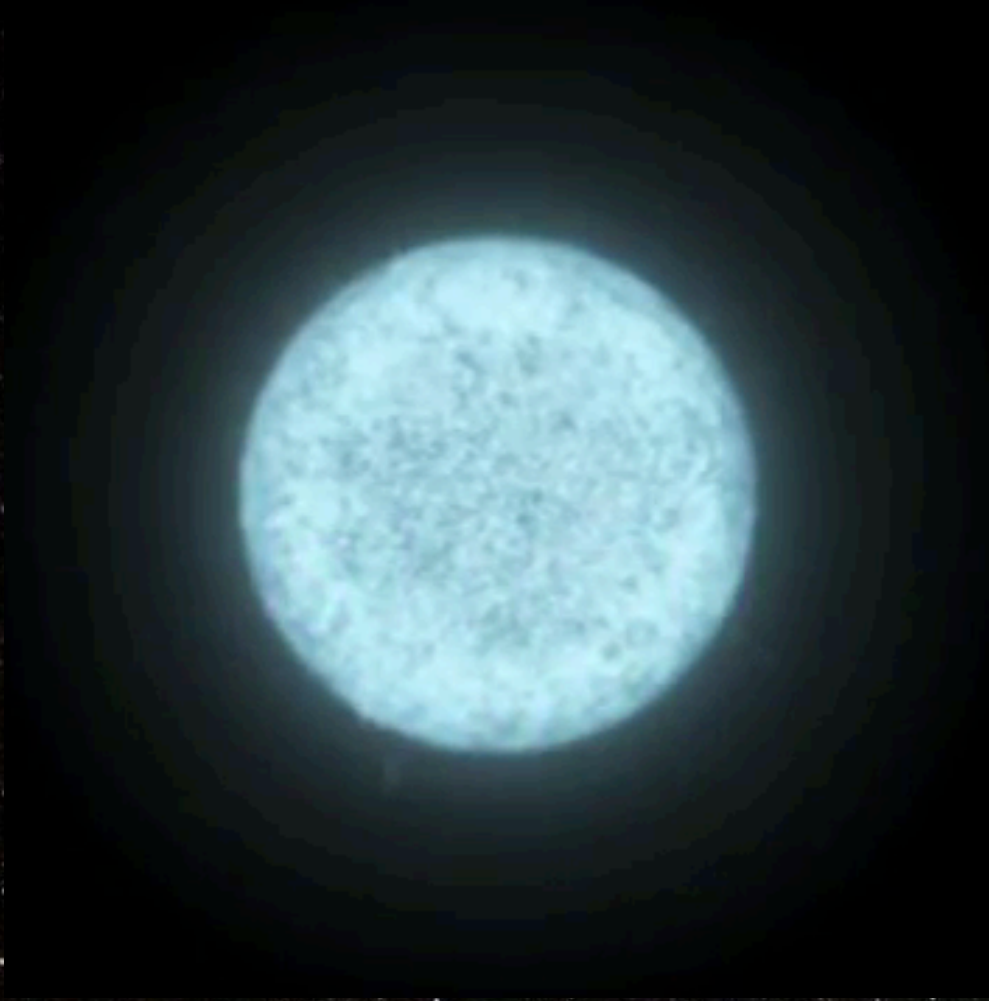
Heber Curtis



**...a good friendly “scrap” is an excellent thing...
sort of clears out the atmosphere.
To shake hands at the beginning and conclusion,
but use our shillelaghs in the interim to the best of
our ability.**

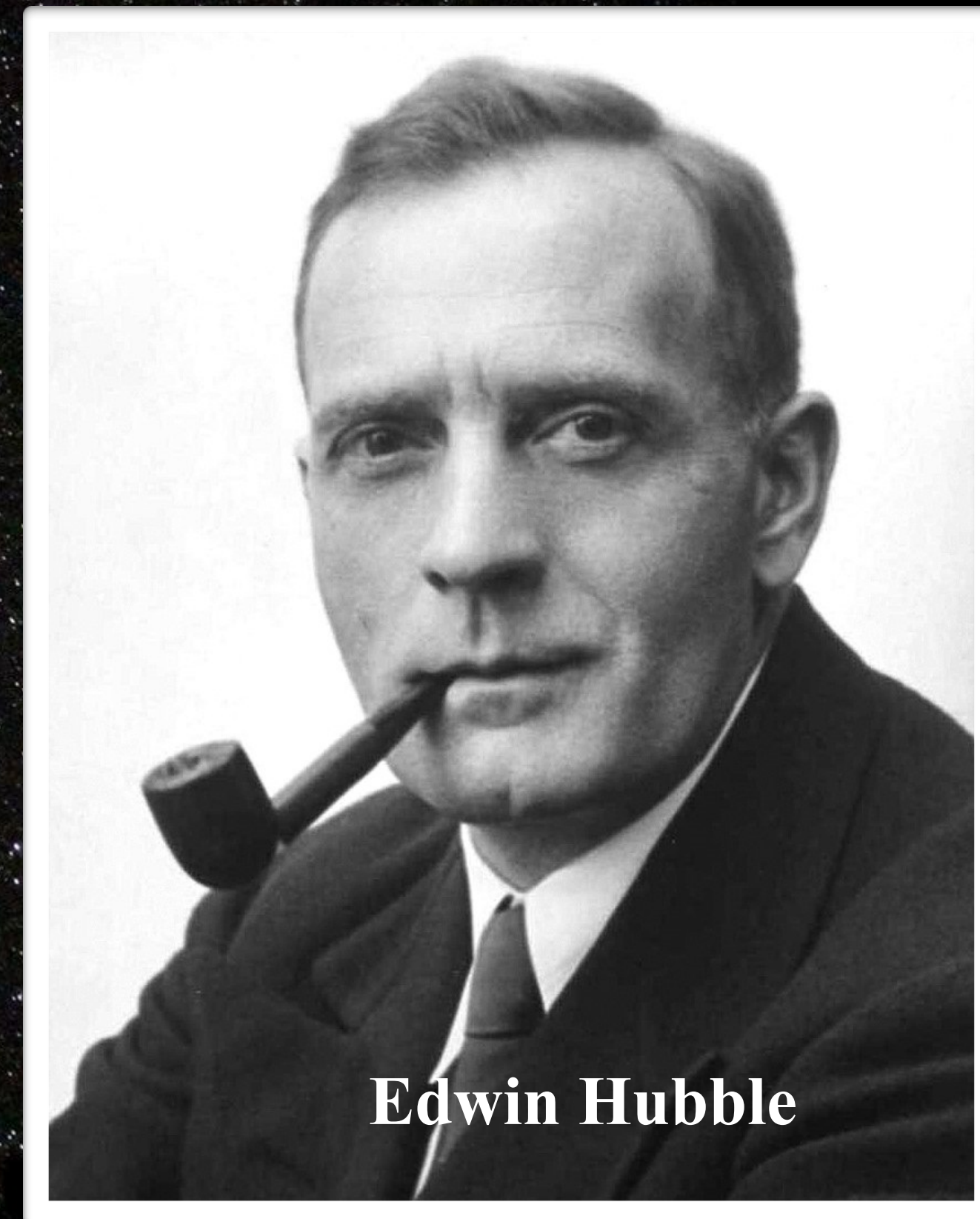
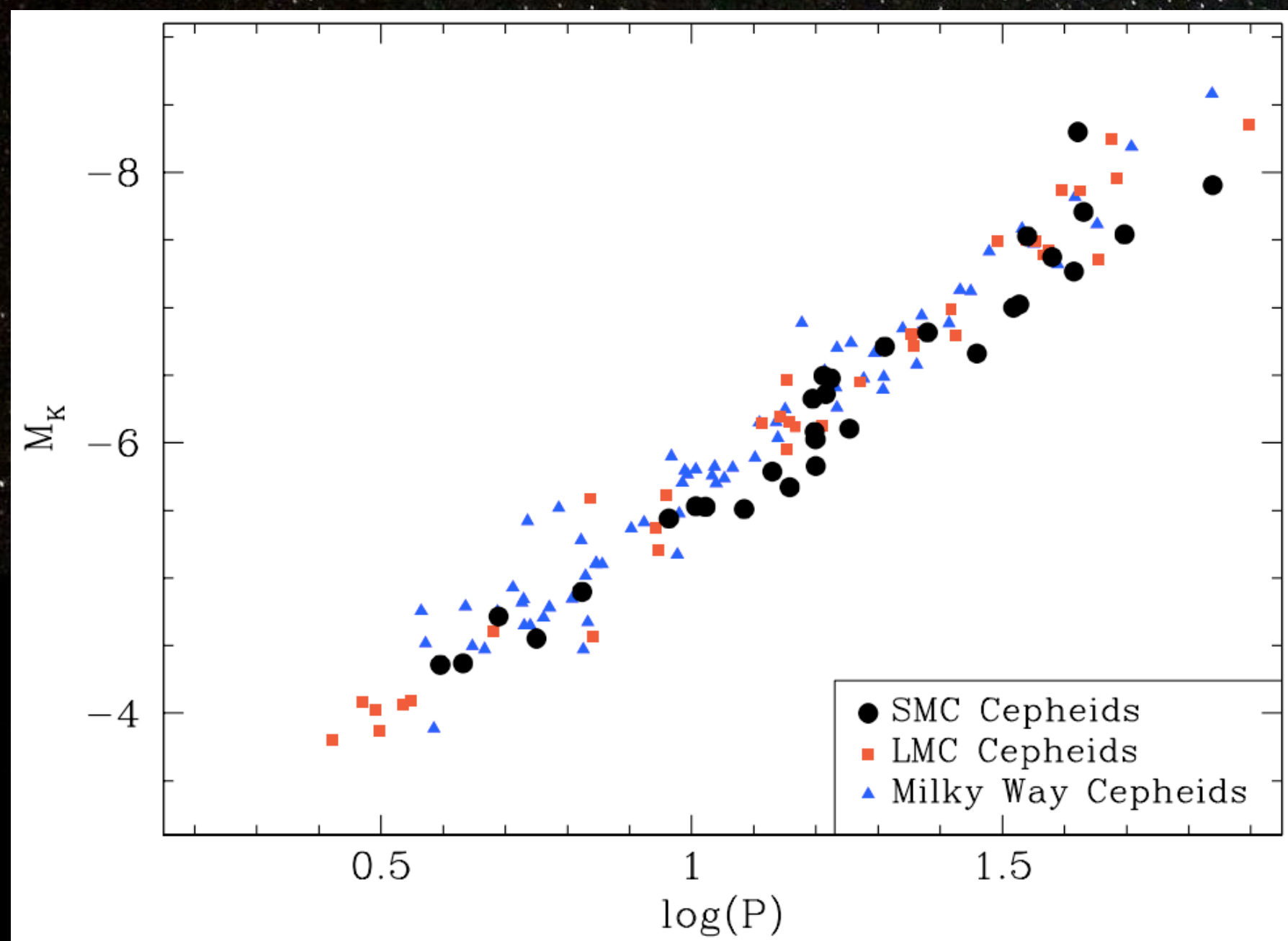
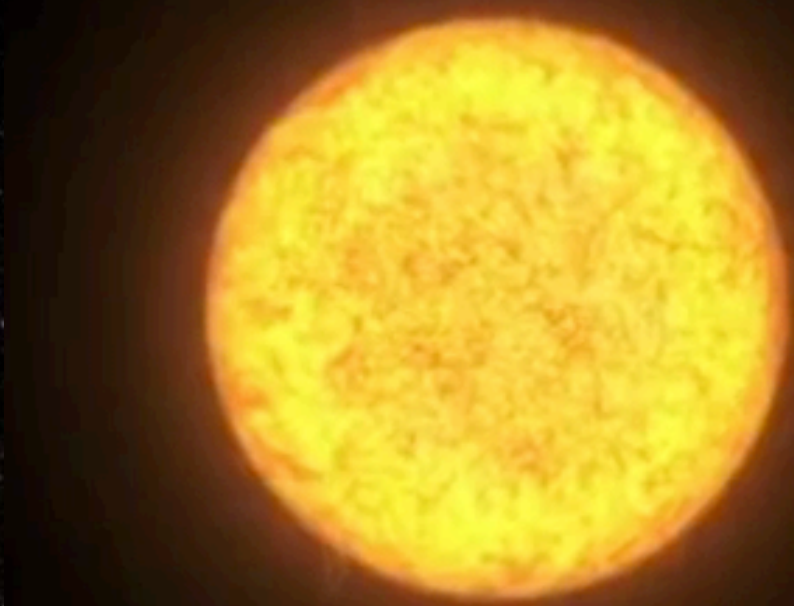


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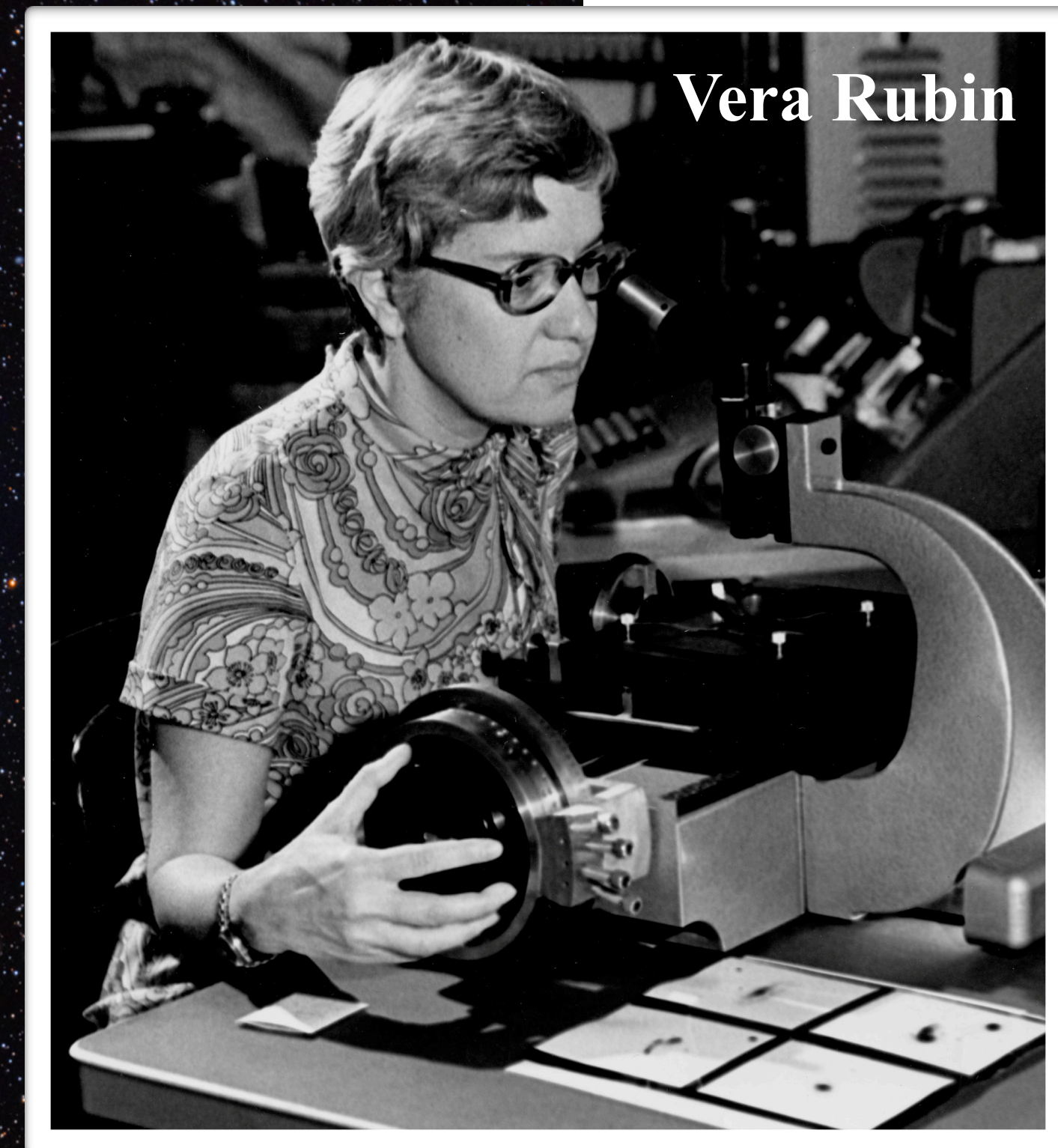
Edwin Hubble

1923



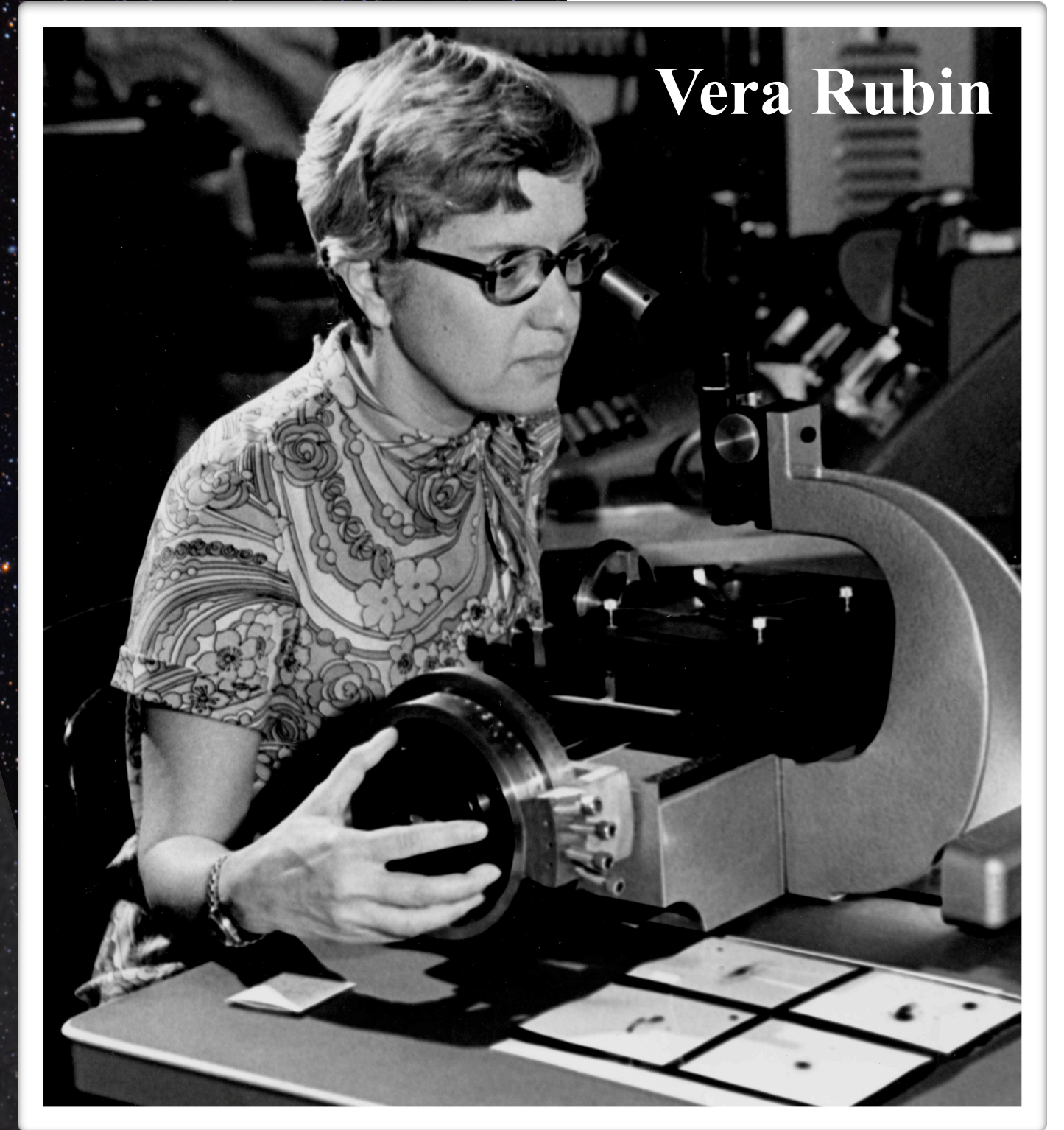
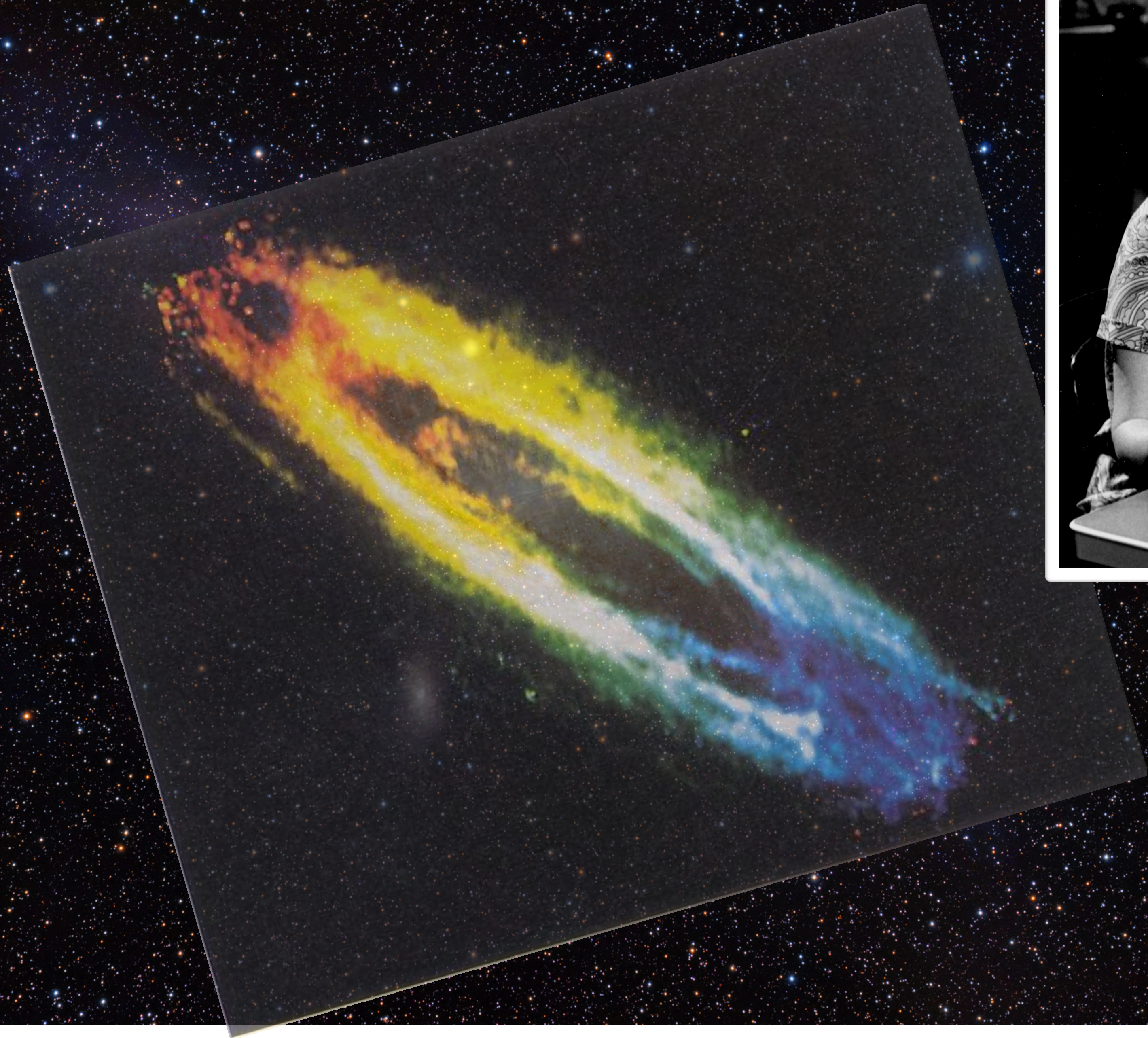
Edwin Hubble

1923



Vera Rubin

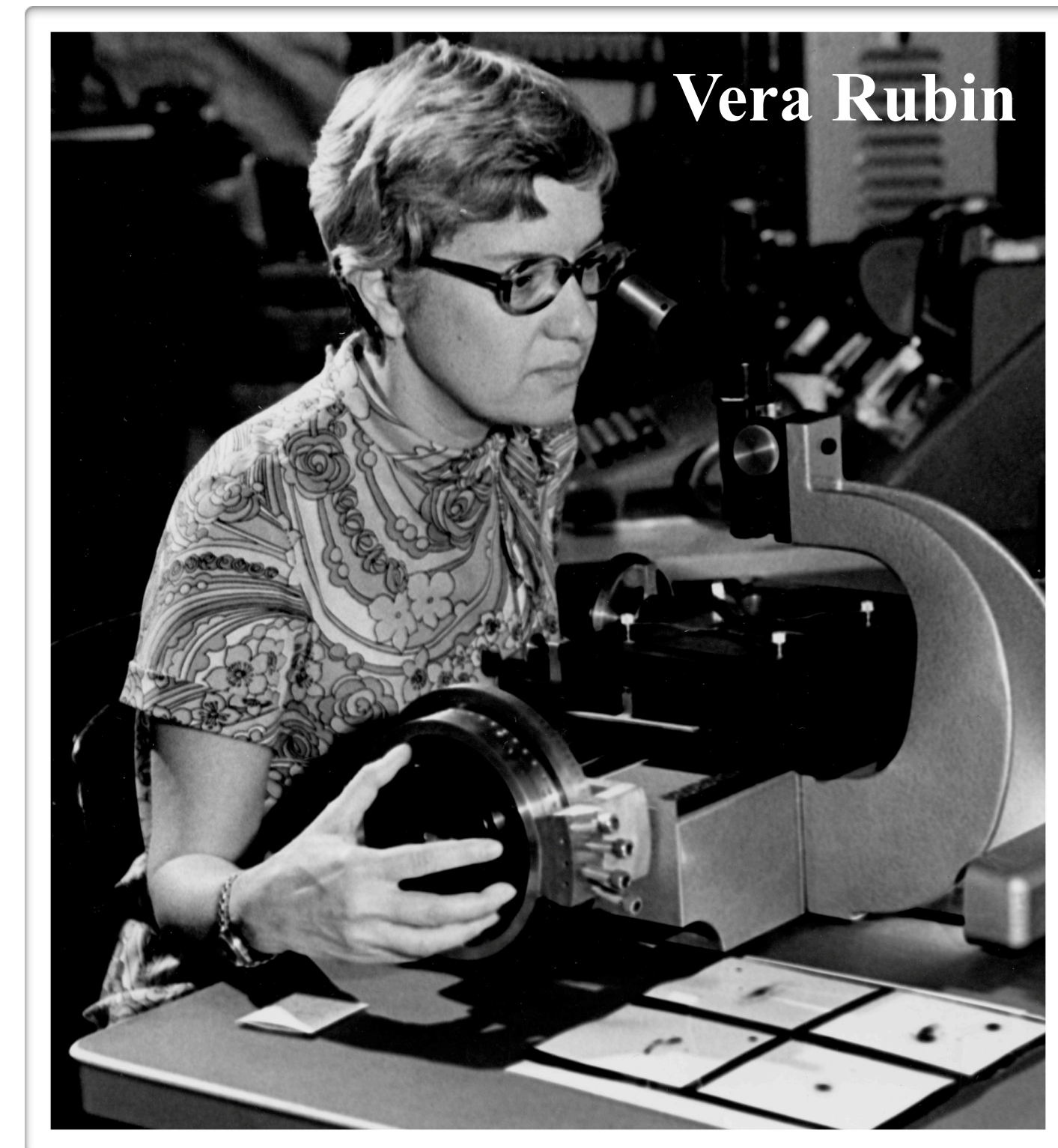
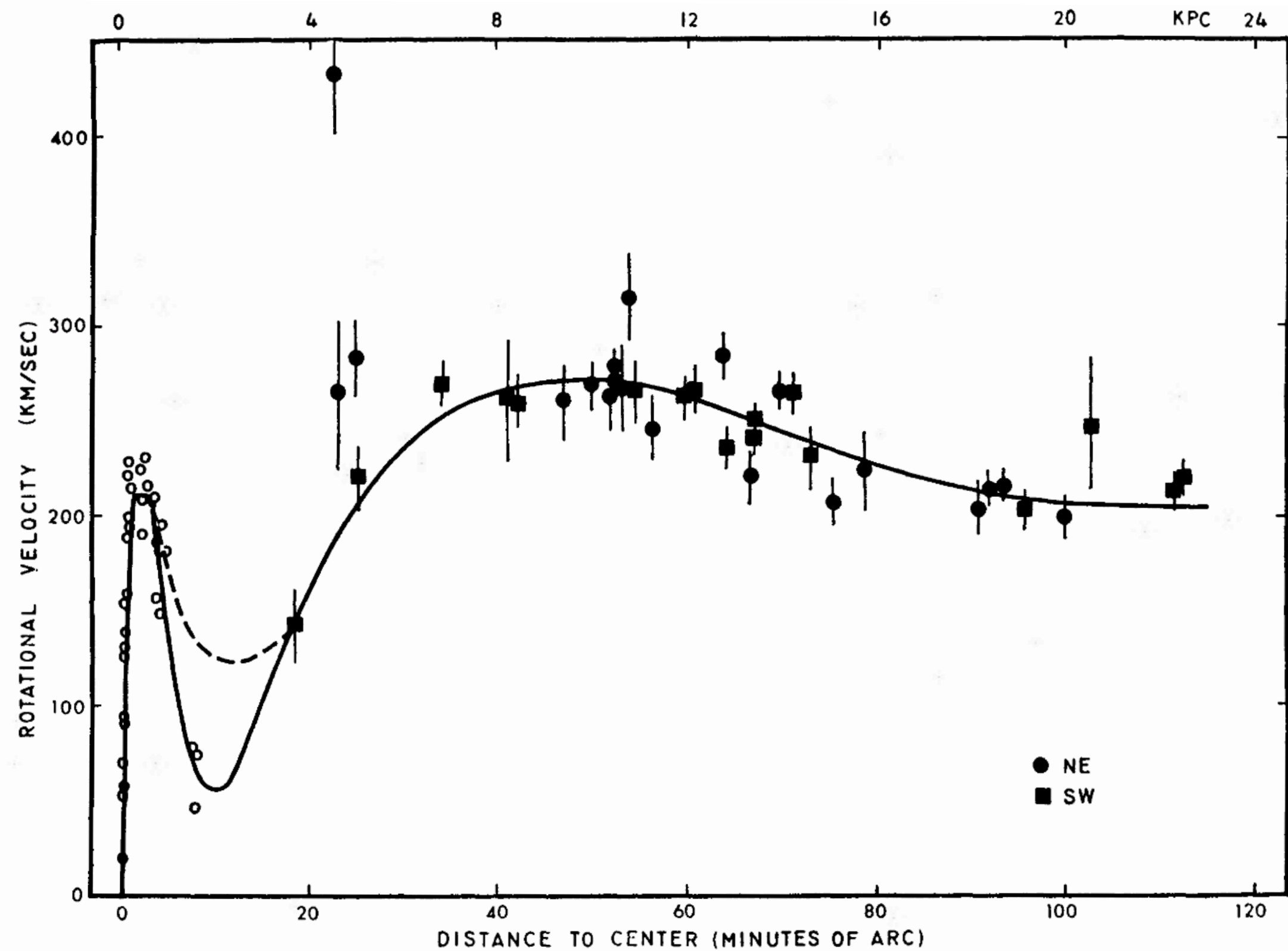
Dark matter



Dark matter

ROTATION OF THE ANDROMEDA NEBULA FROM A SPECTROSCOPIC
SURVEY OF EMISSION REGIONS*

VERA C. RUBIN† AND W. KENT FORD, JR.†
Department of Terrestrial Magnetism, Carnegie Institution of Washington and
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Received 1969 July 7; revised 1969 August 21



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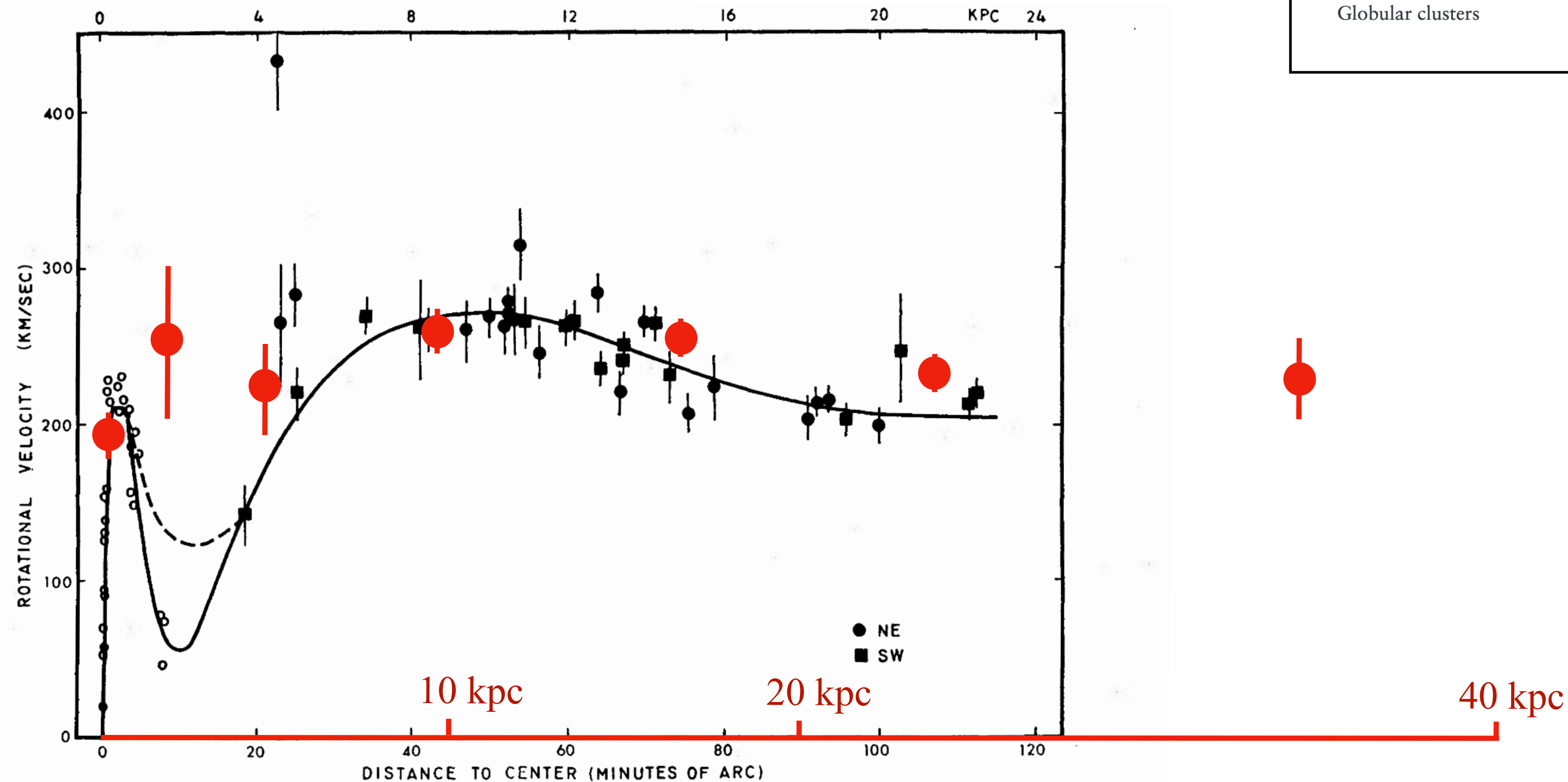
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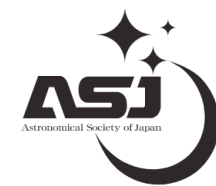
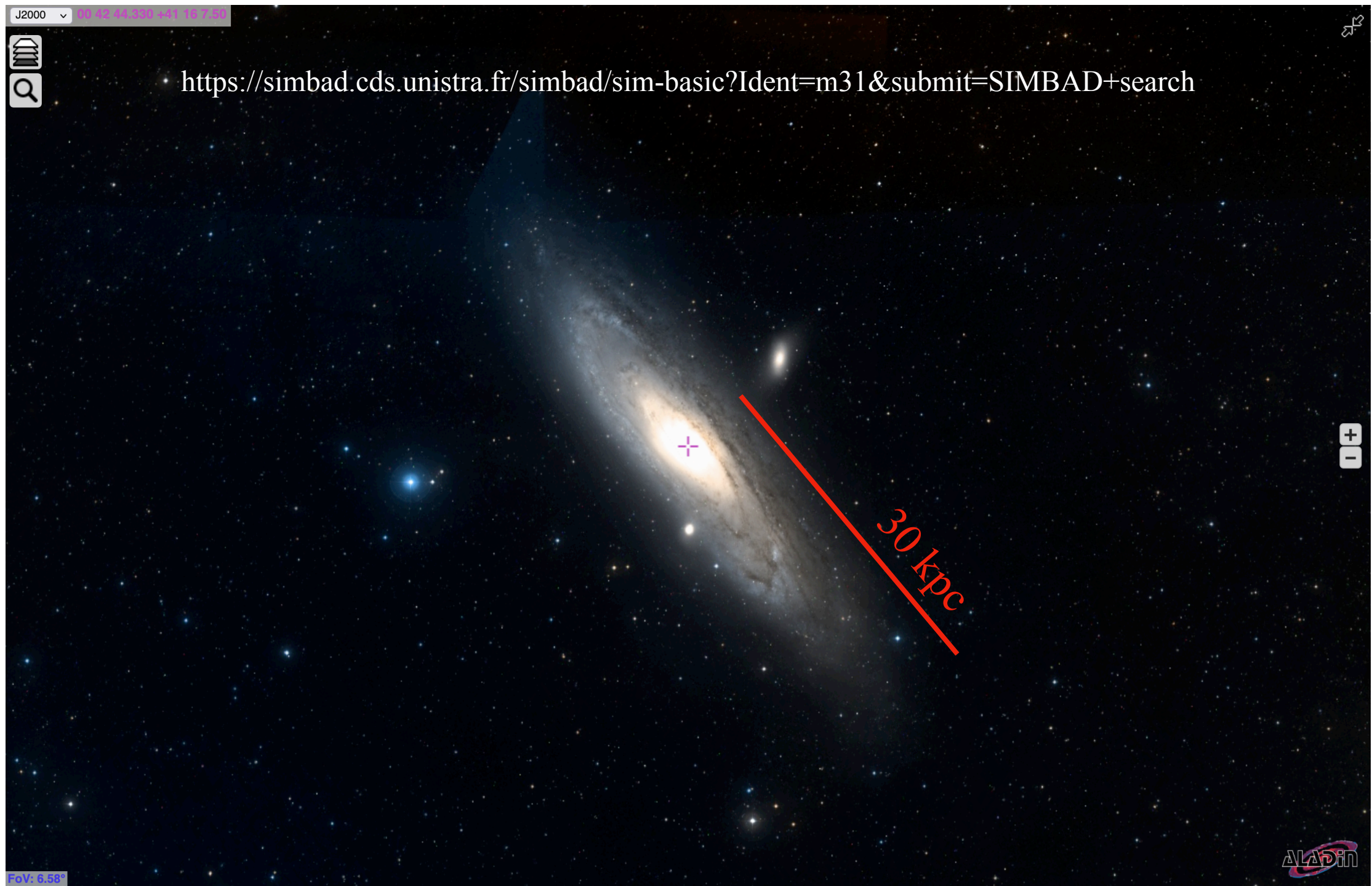


Dark halos of M31 and the Milky Way

Yoshiaki Sofue*

Disk RC (CO)	Loinard, Allen, and Lequeux (1995)
Ibid (combi: HI, CO, opt)	Sofue et al. (1981, 1999)
Ibid (HI)	Carignan et al. (2006)
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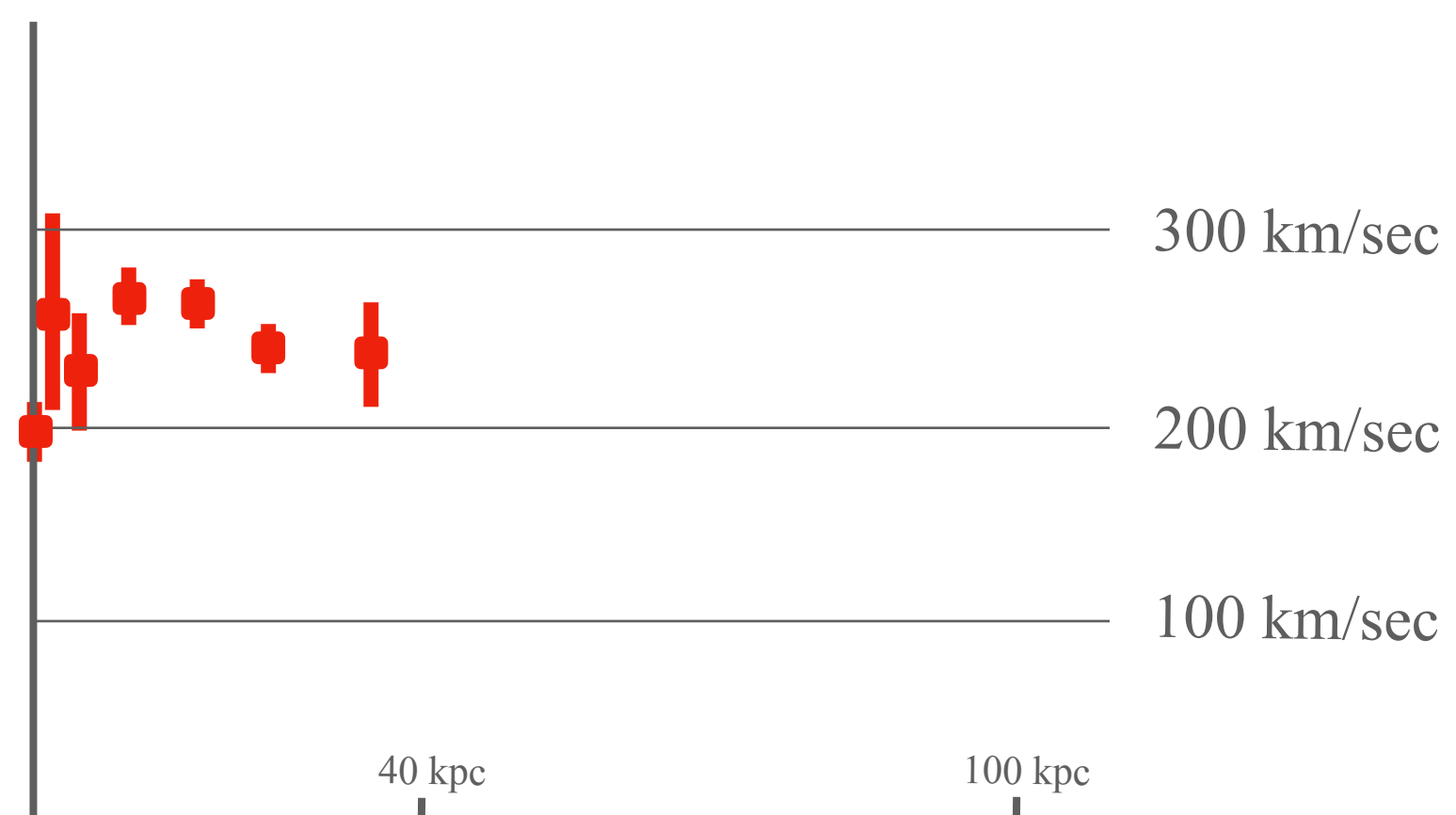


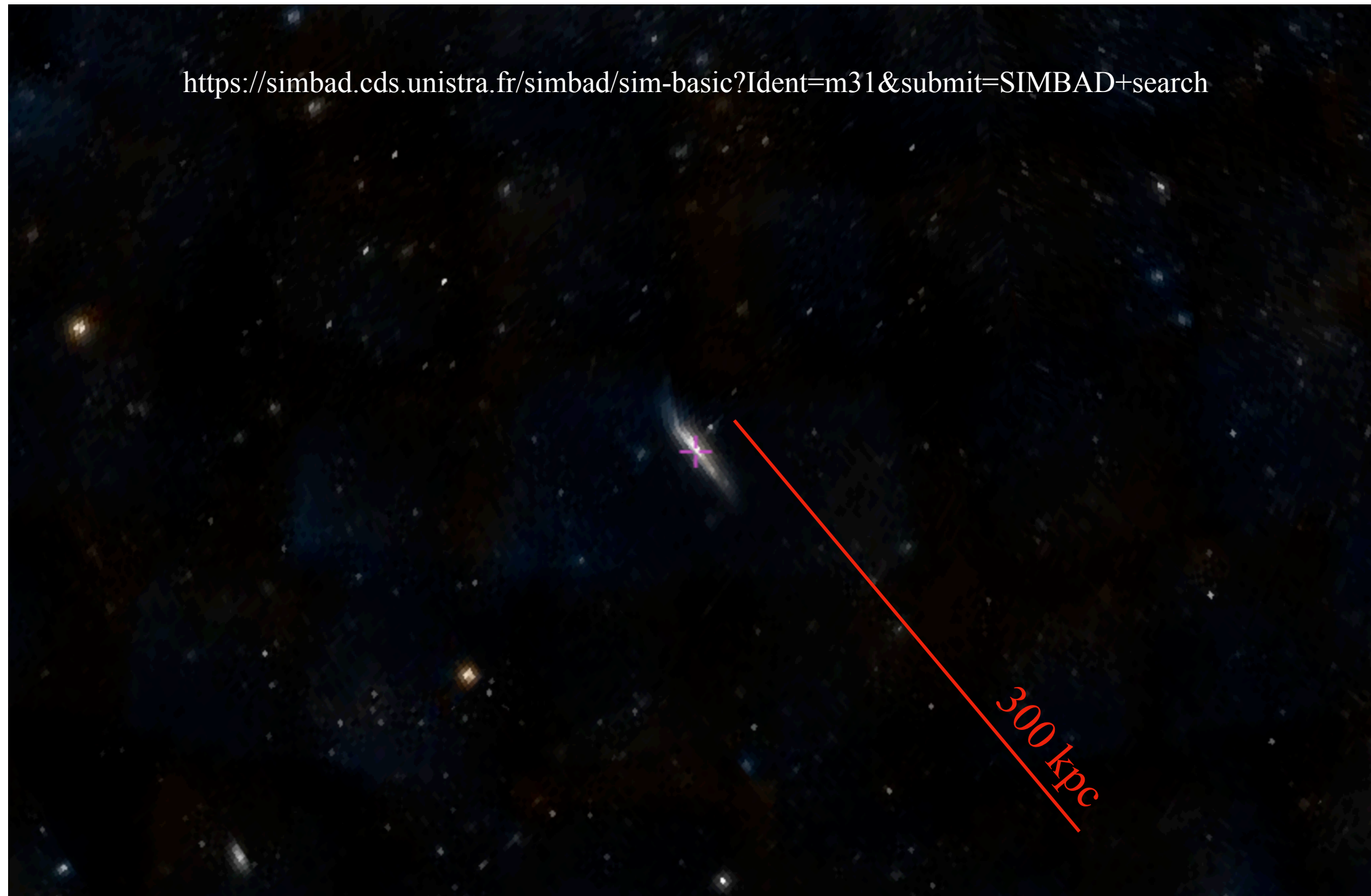


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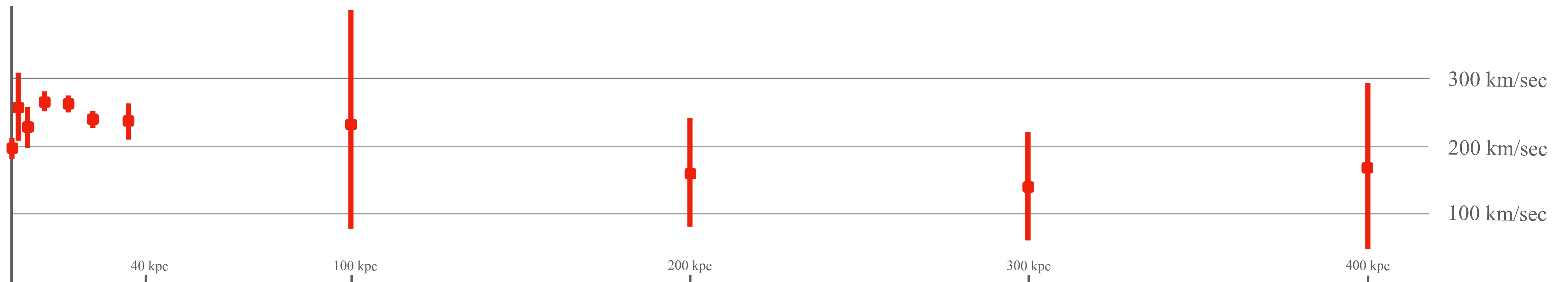


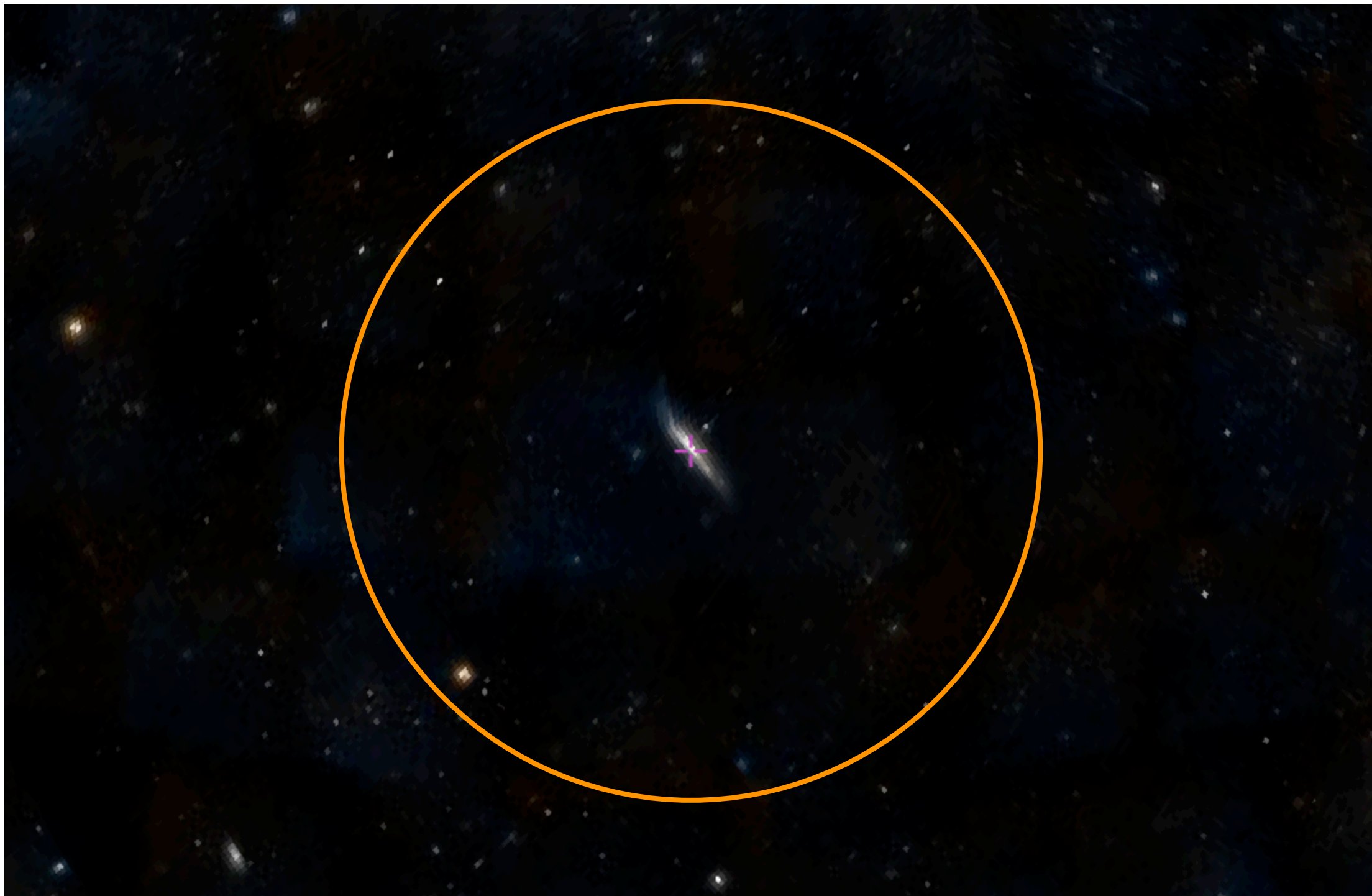


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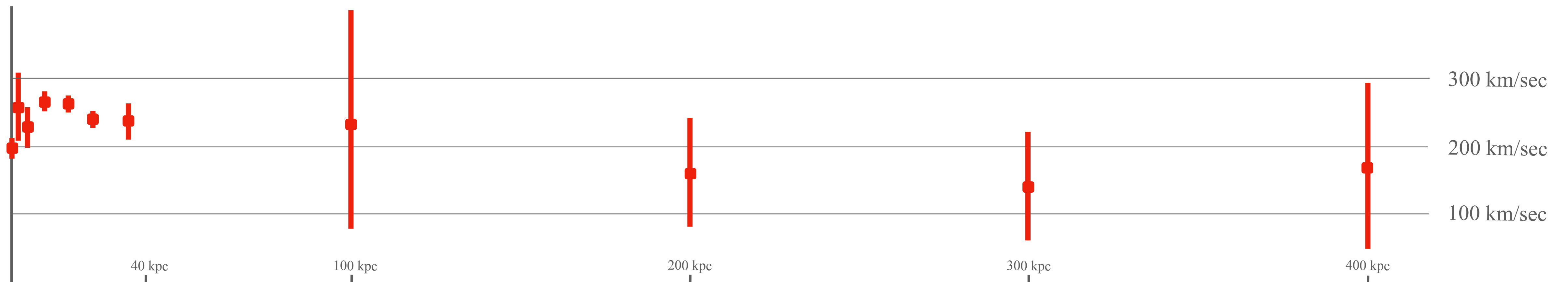
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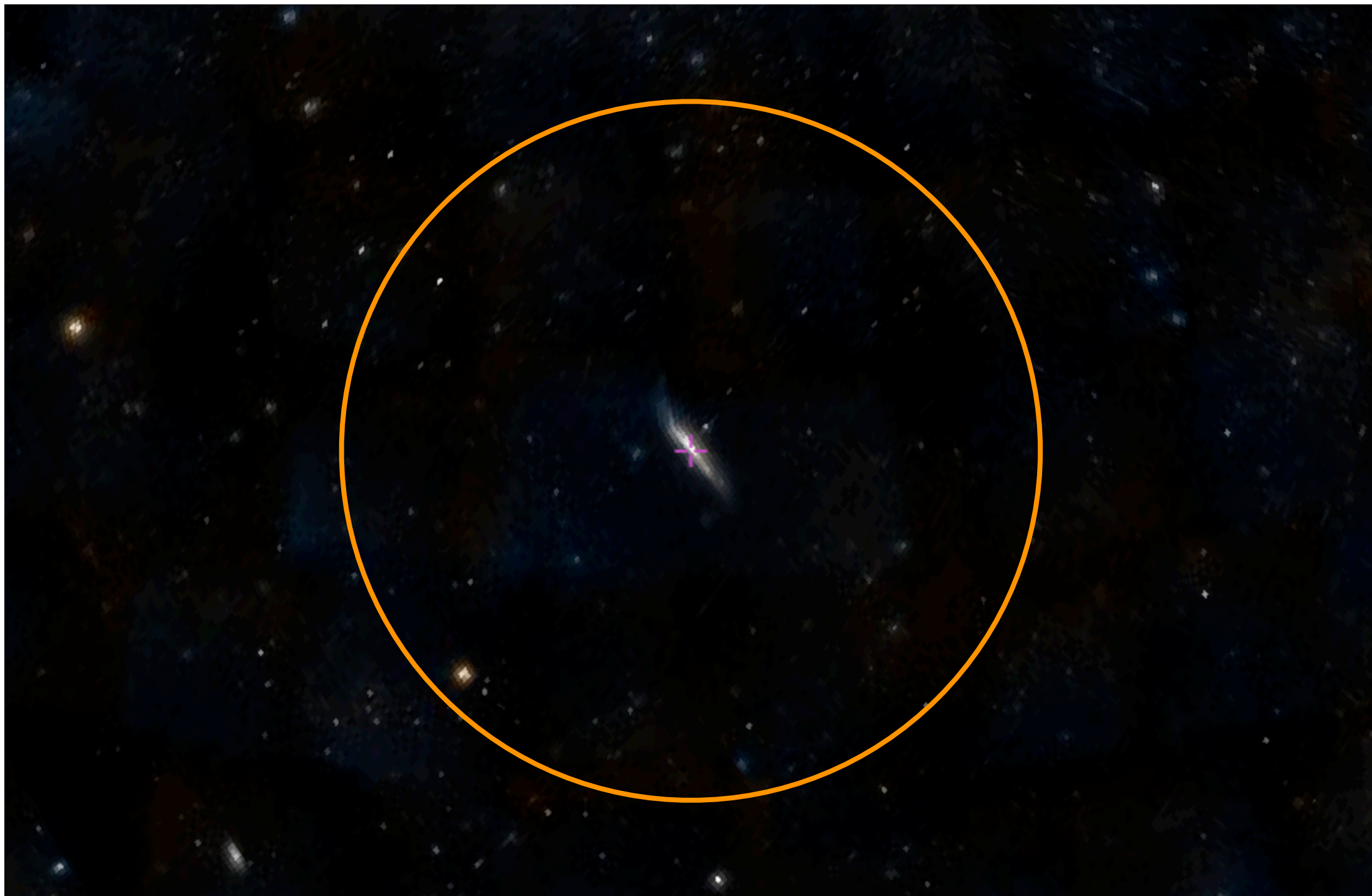
**Astronomy
&
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Stellar mass map and dark matter distribution in M31

A. Tamm¹, E. Tempel^{1,2}, P. Tenjes^{1,3}, O. Tihhonova^{1,3}, and T. Tuvikene¹

$$R_{200} = (189 - 213) \text{ kpc} \quad M_{200} = (0.8 - 1.1) \times 10^{12} M_{\odot}$$





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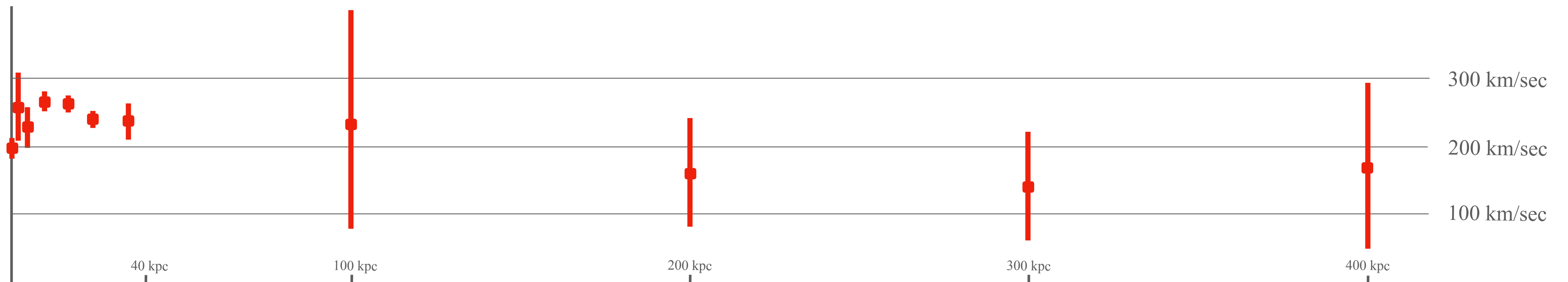
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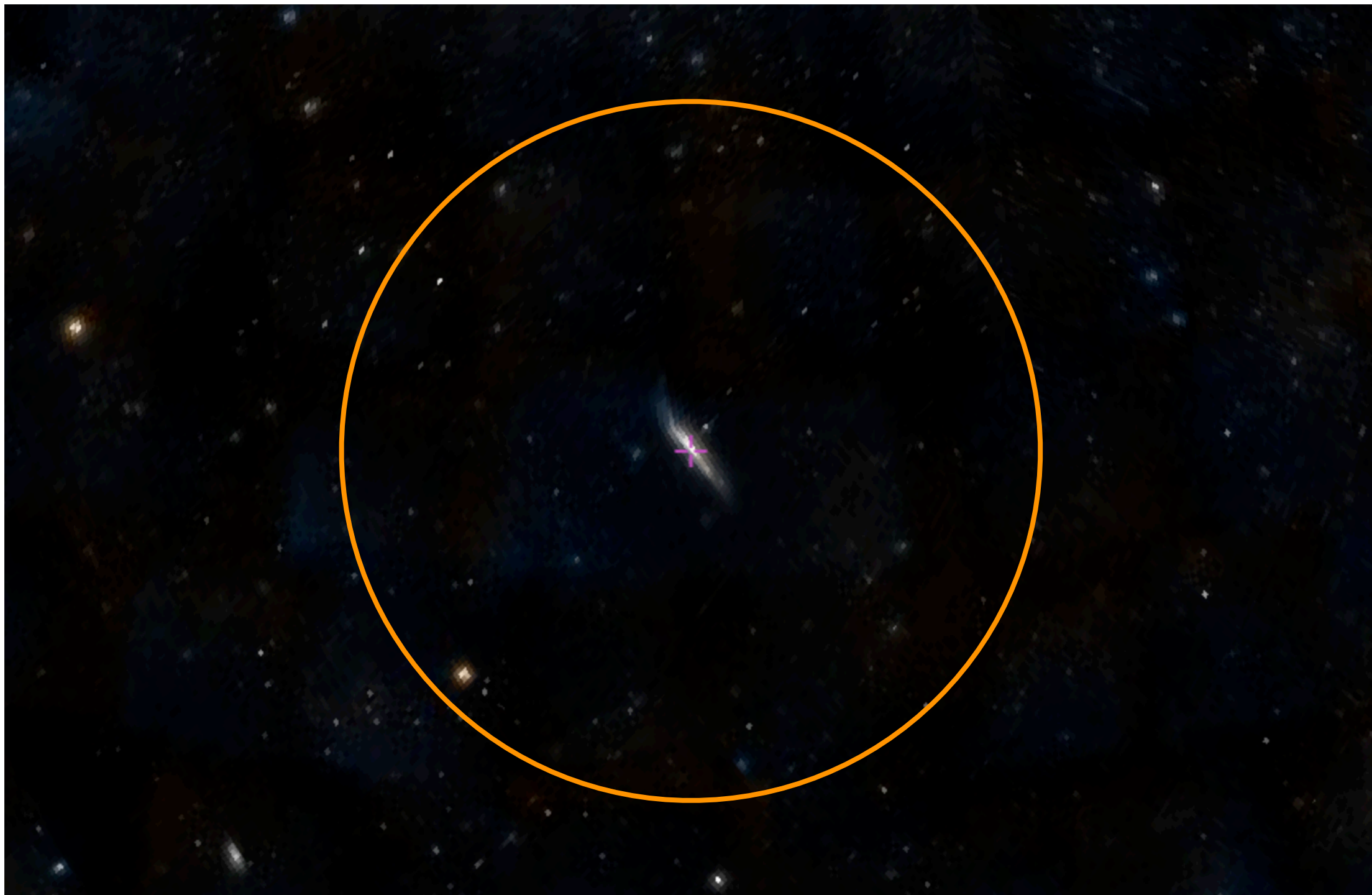
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$$\rho(R_{200}) = 200 \rho_c \approx 200 \times 8.5 \times 10^{-30} \frac{\text{g}}{\text{cm}^3} \approx 2.5 \times 10^4 \frac{M_{\odot}}{\text{kpc}^3}$$





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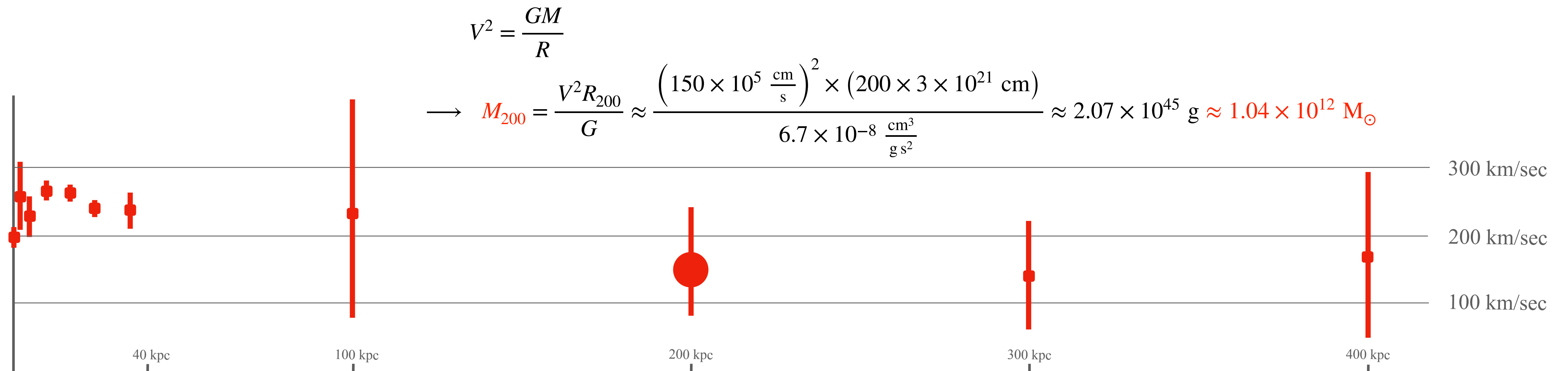
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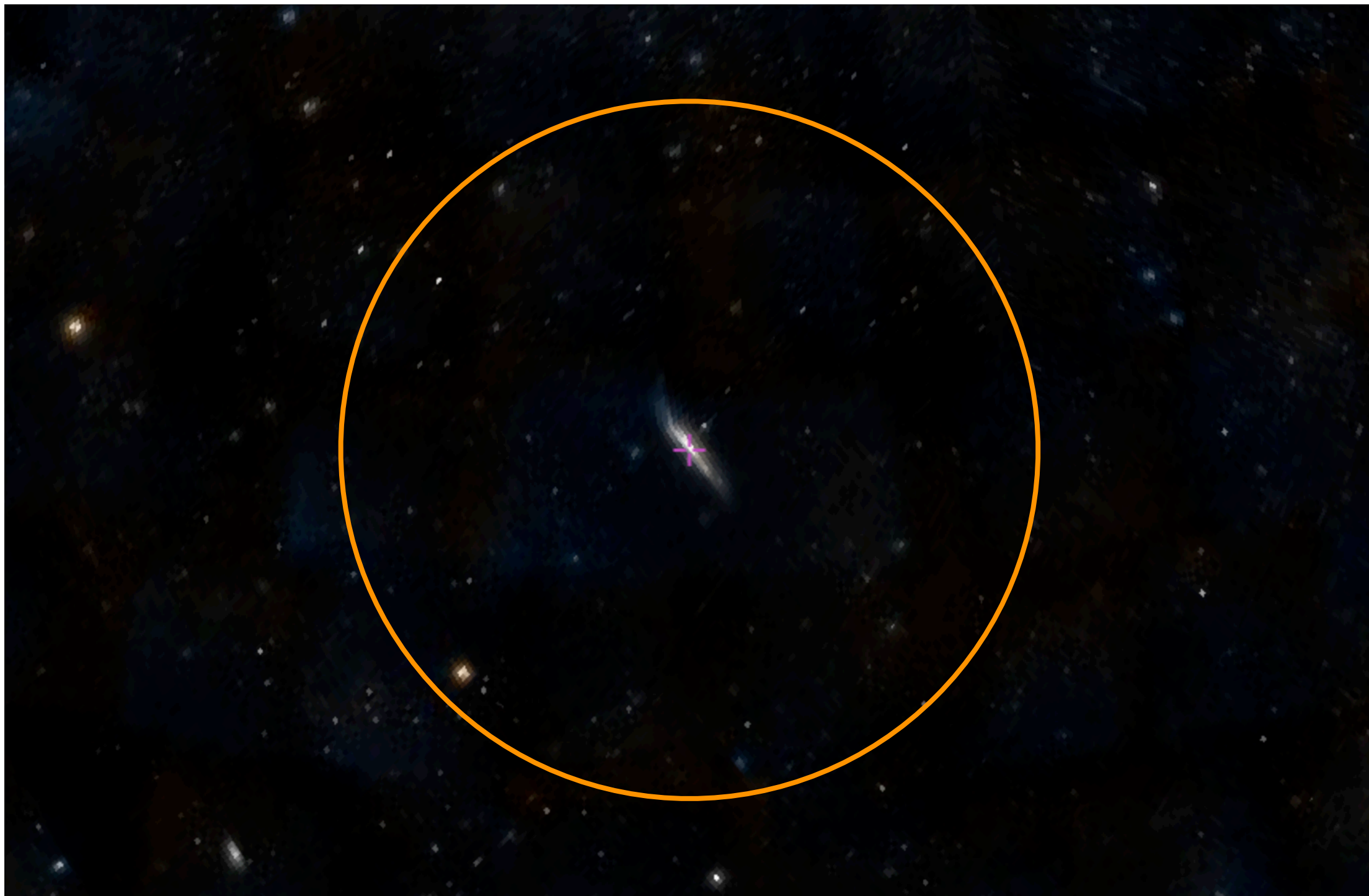
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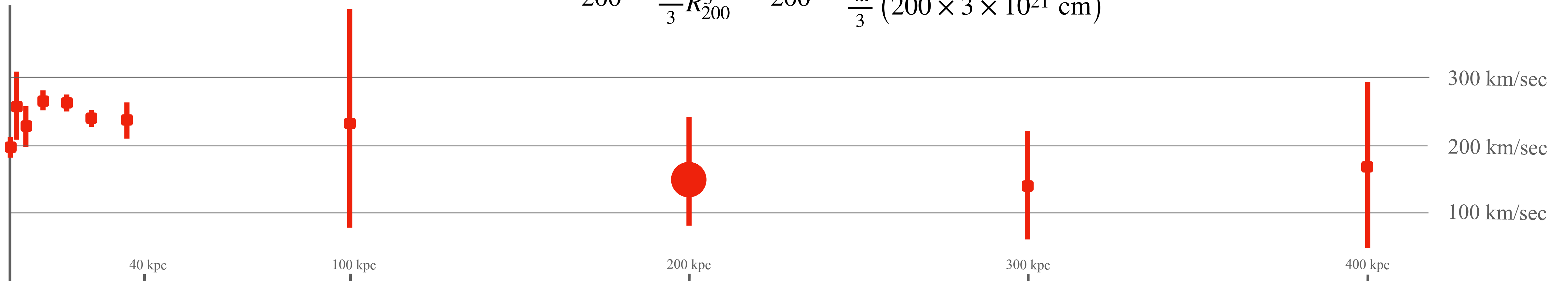
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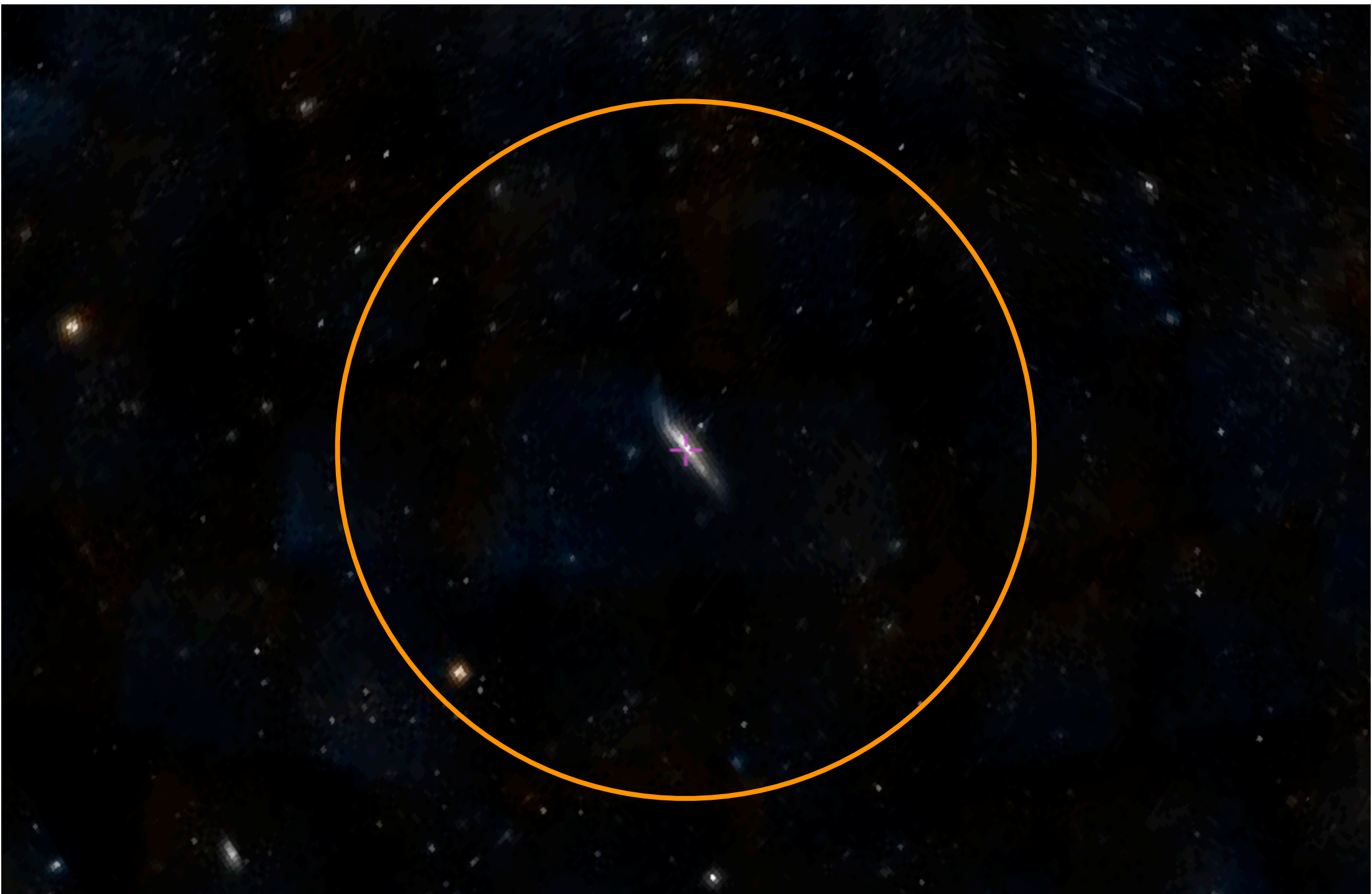
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$$\frac{1}{200} \times \frac{M_{200}}{\frac{4\pi}{3} R_{200}^3} \approx \frac{1}{200} \times \frac{10^{12} \times 2 \times 10^{33} \text{ g}}{\frac{4\pi}{3} (200 \times 3 \times 10^{21} \text{ cm})^3} \approx 1.1 \times 10^{-29} \text{ g/cm}^3 \sim \rho_c$$



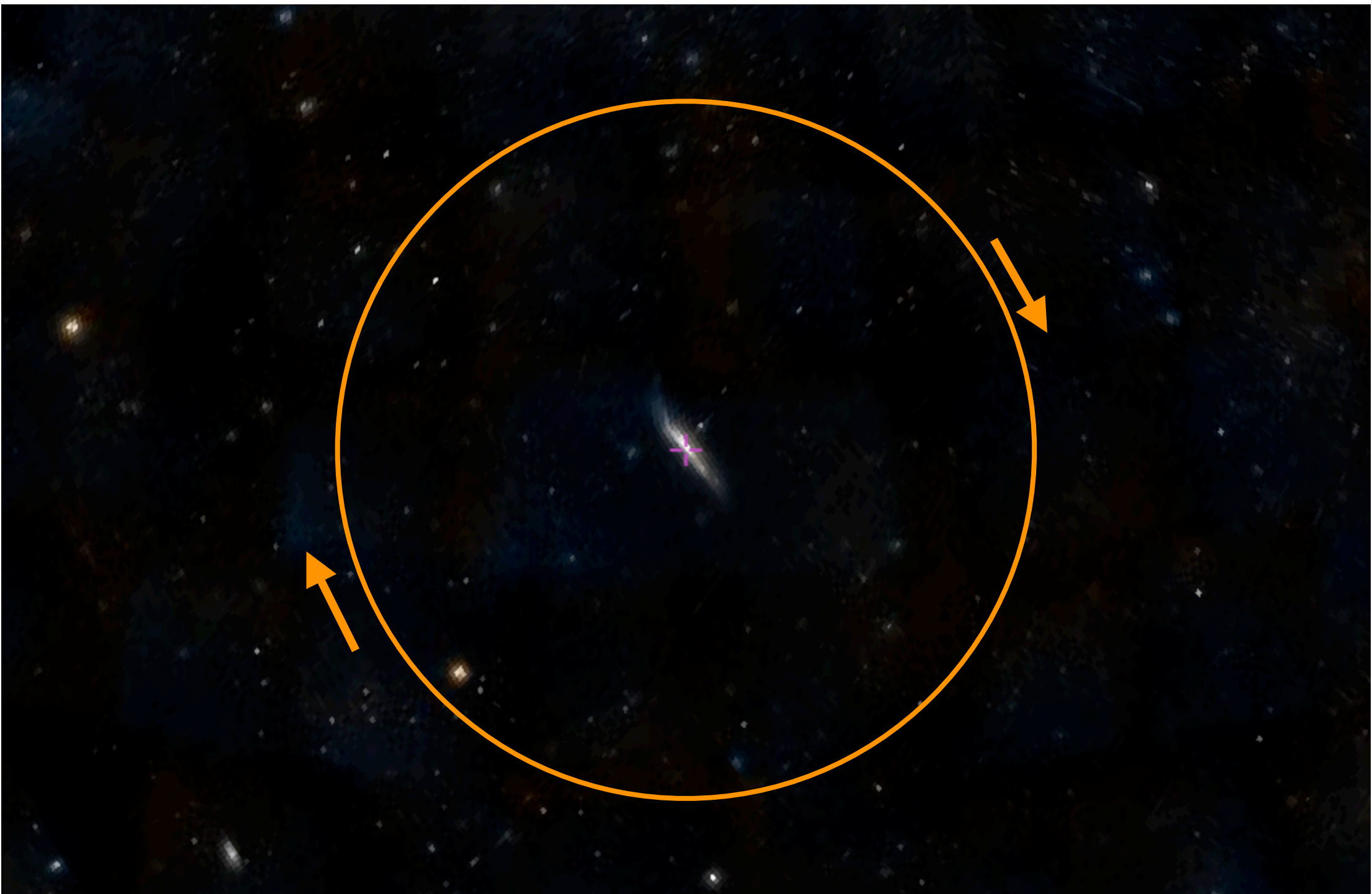
Where does it end?



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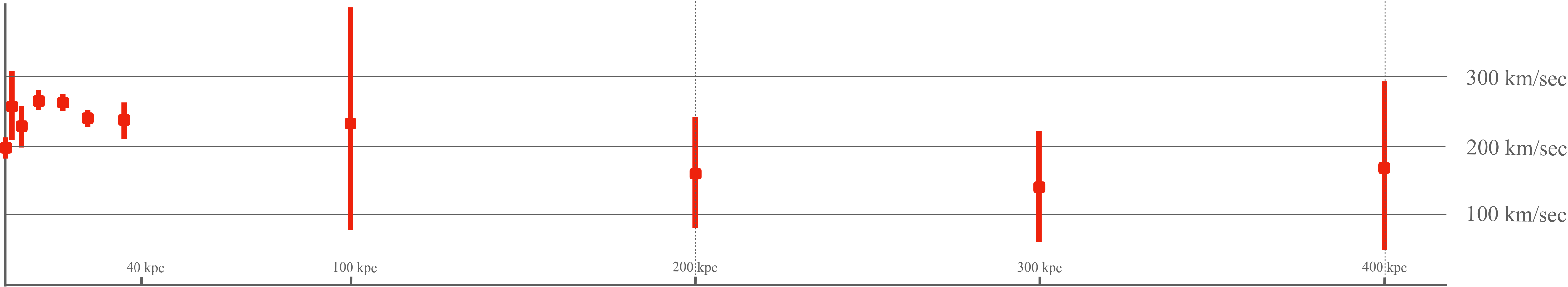
Time to make 1 revolution:

$$T = \frac{2\pi R}{V}$$



$$T = \frac{2\pi R}{V} \approx \frac{2\pi \times (200 \text{ kpc})}{(150 \text{ km/s})} \approx 8.2 \text{ Gyr}$$

$$\frac{2\pi \times (400 \text{ kpc})}{(150 \text{ km/s})} \approx 16.4 \text{ Gyr}$$



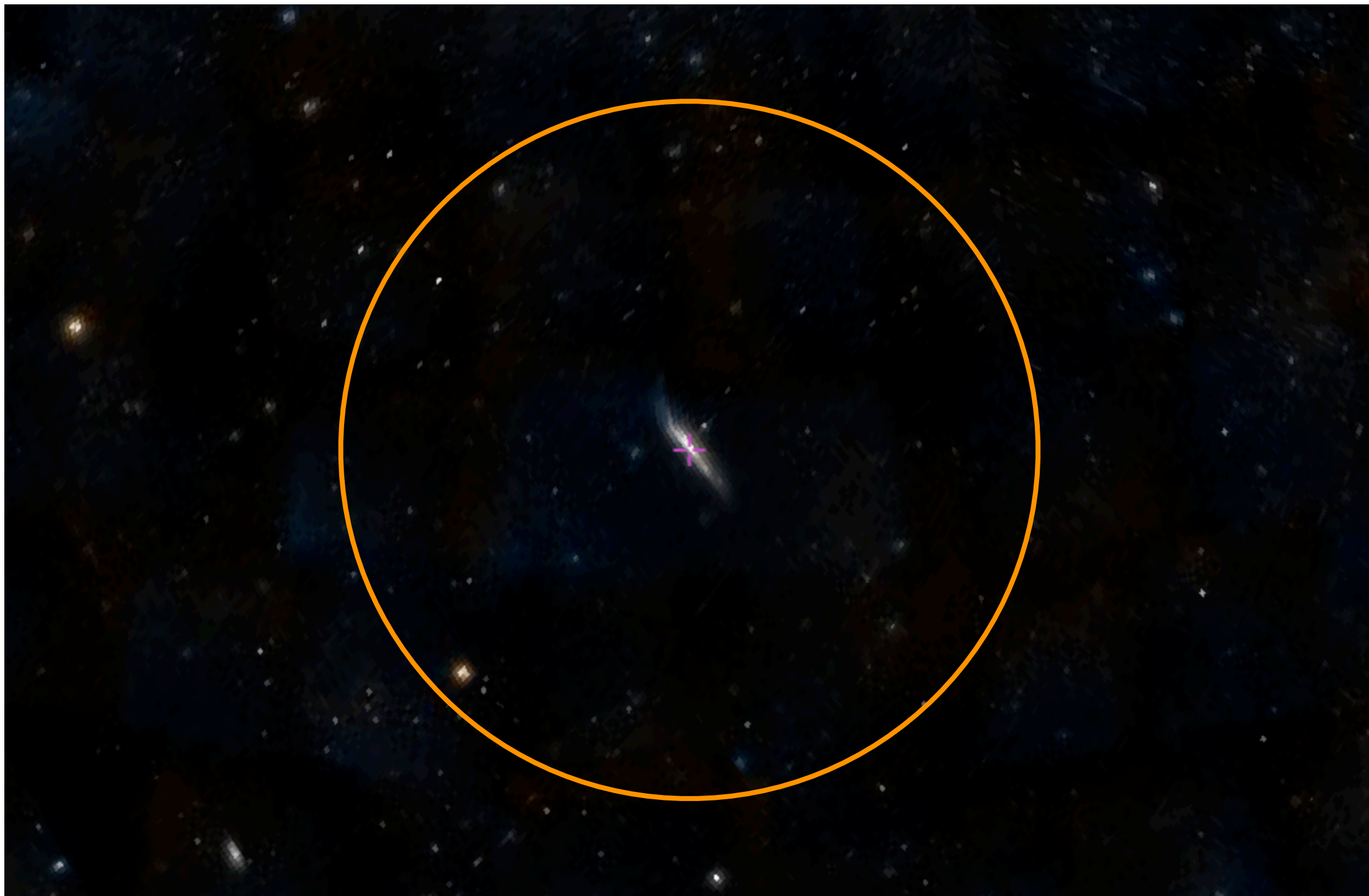
Where does it end?

Cosmic neighbors:

$$\rho_m \approx 0.3 \rho_c \approx \frac{10^{12} M_\odot}{(3 \text{ Mpc})^3}$$

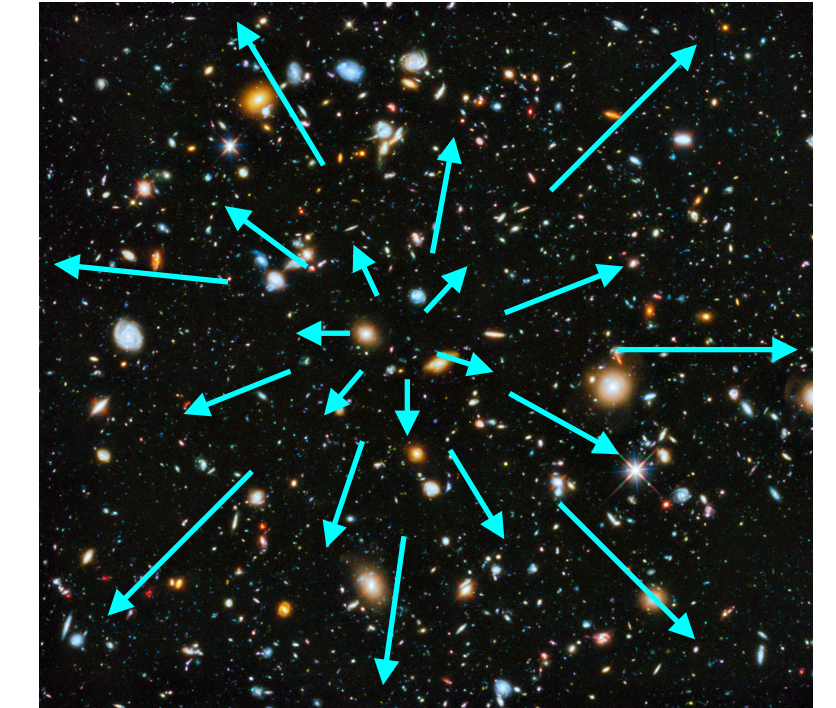
MW-M31 distance ~ 770 kpc



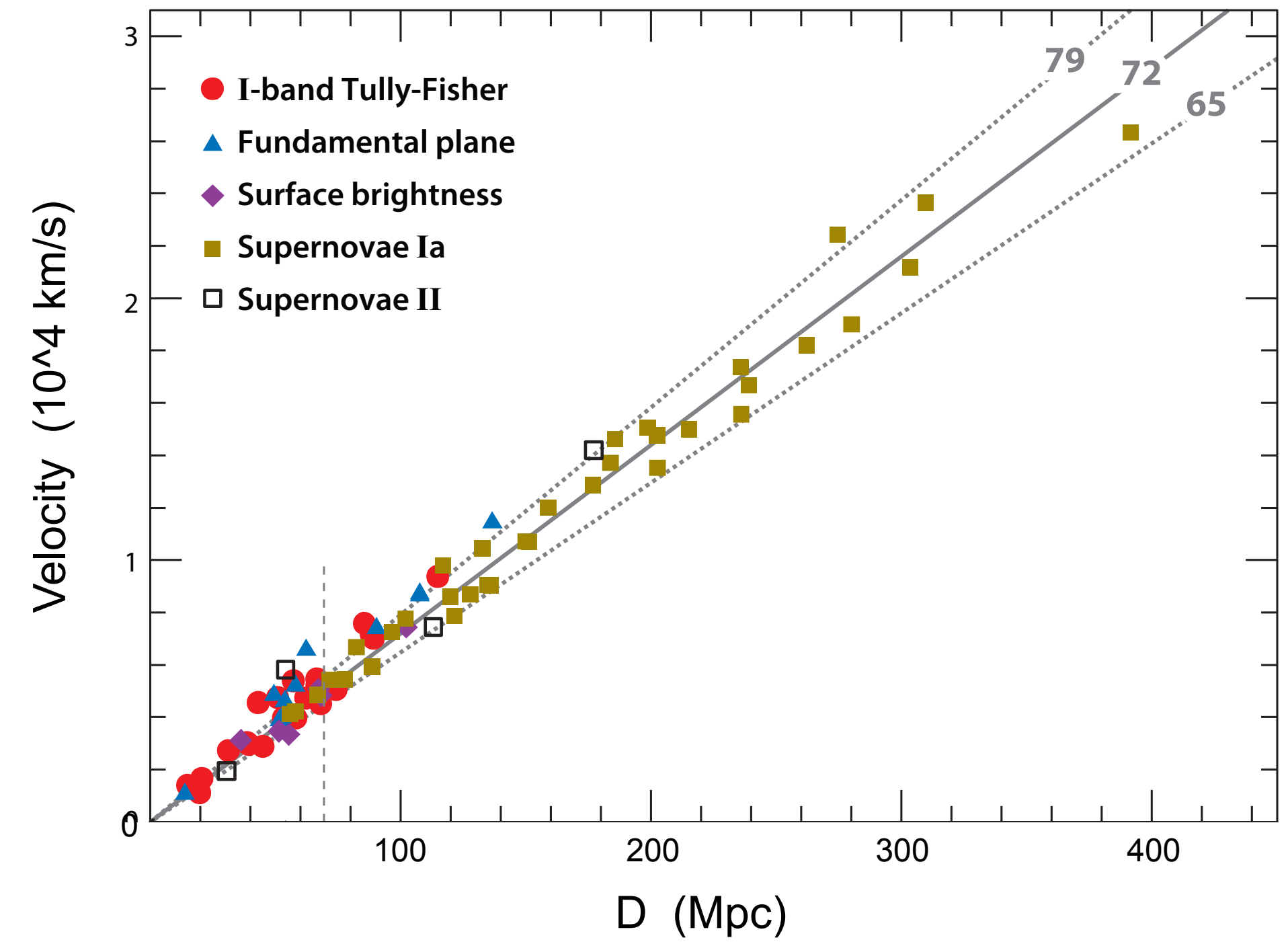


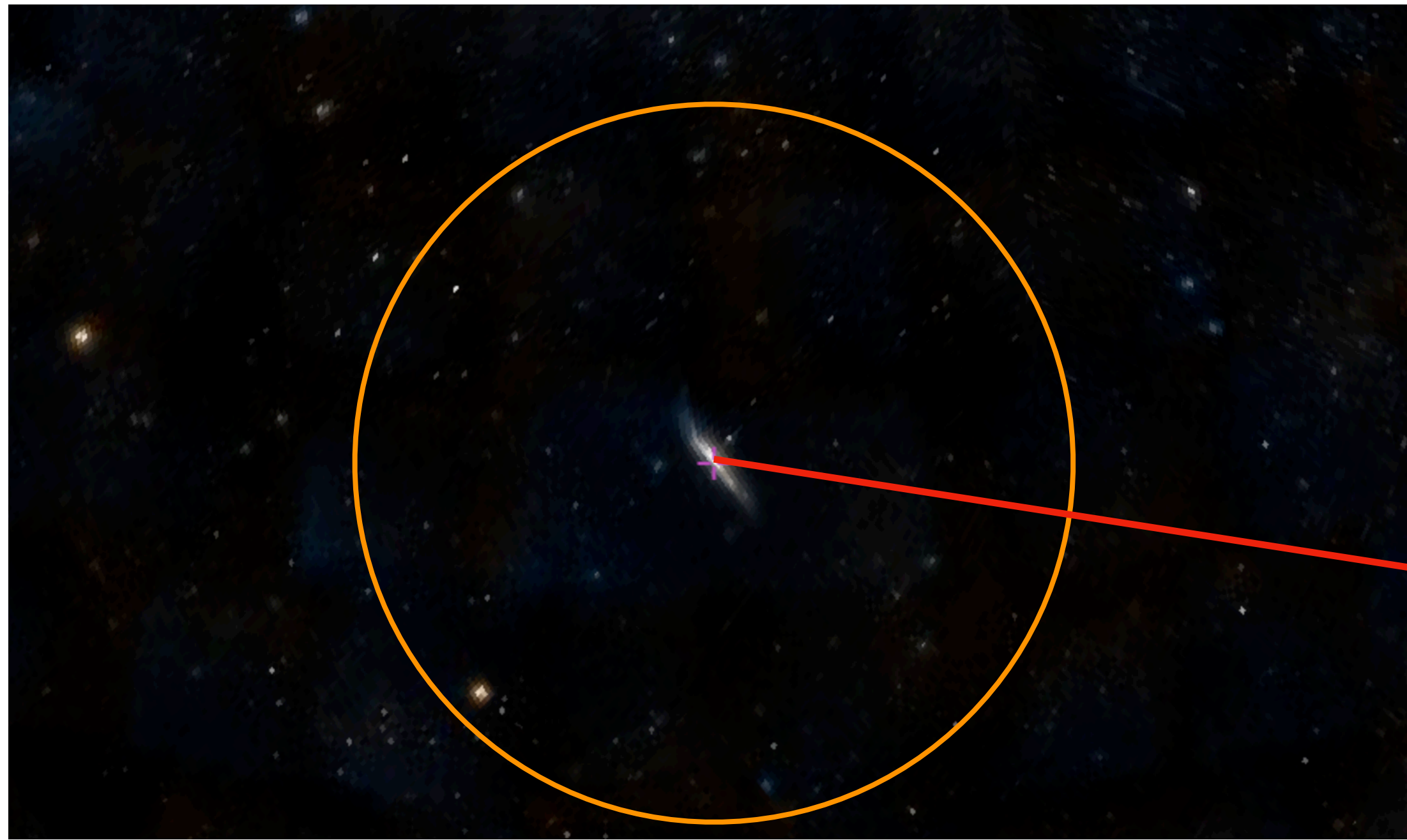
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Cosmic expansion:



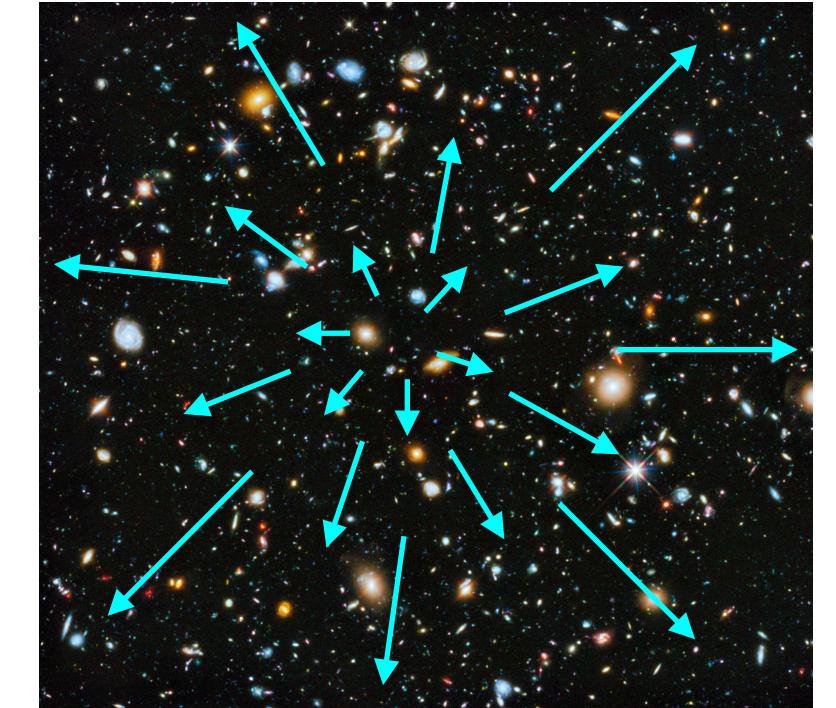
Freedman & Madore, Ann. Rev. A&A, vol. 48, p.673-710





Where does it end?

Cosmic expansion:



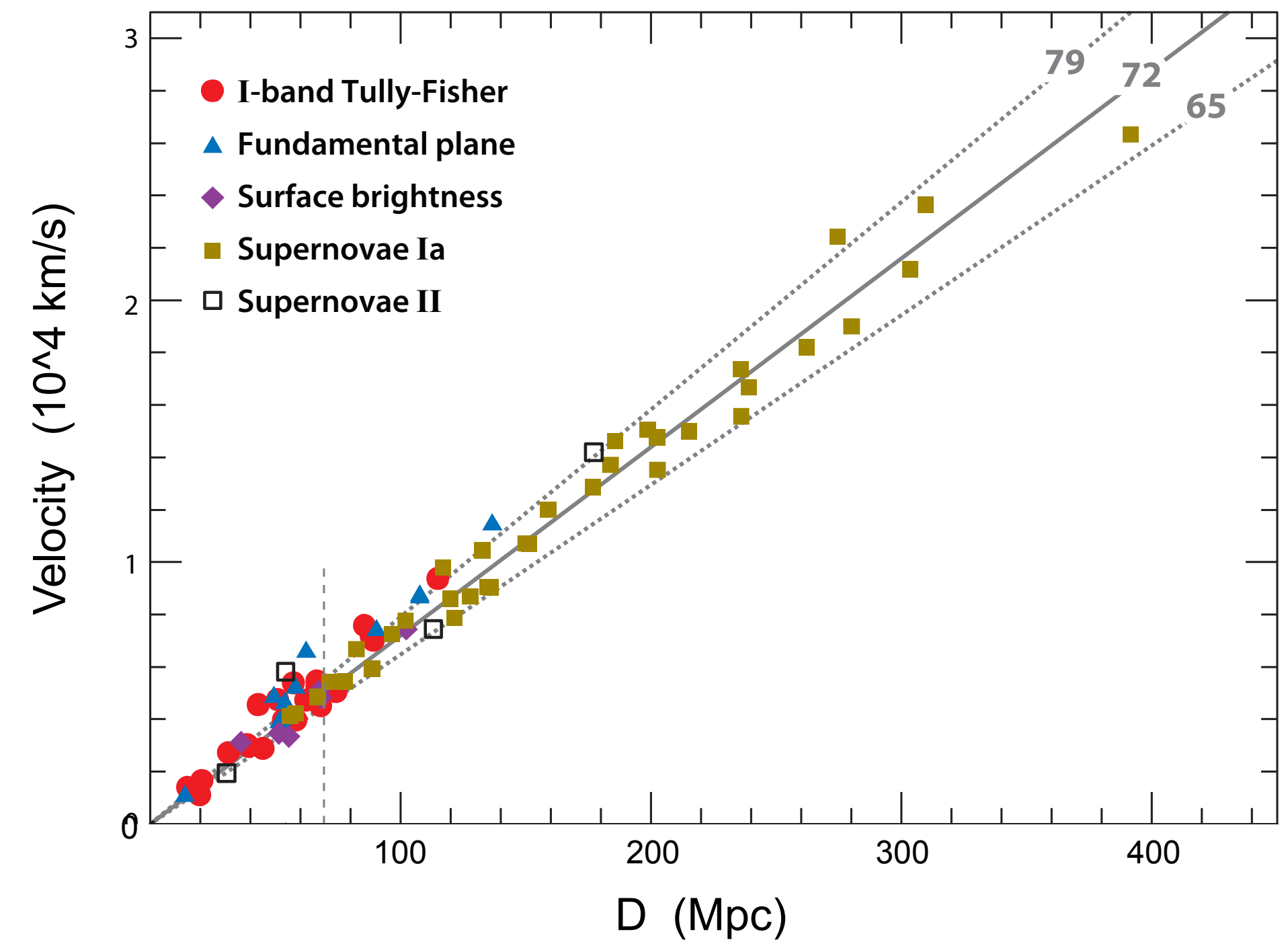
Expansion modifies law of rotation around a point mass M:

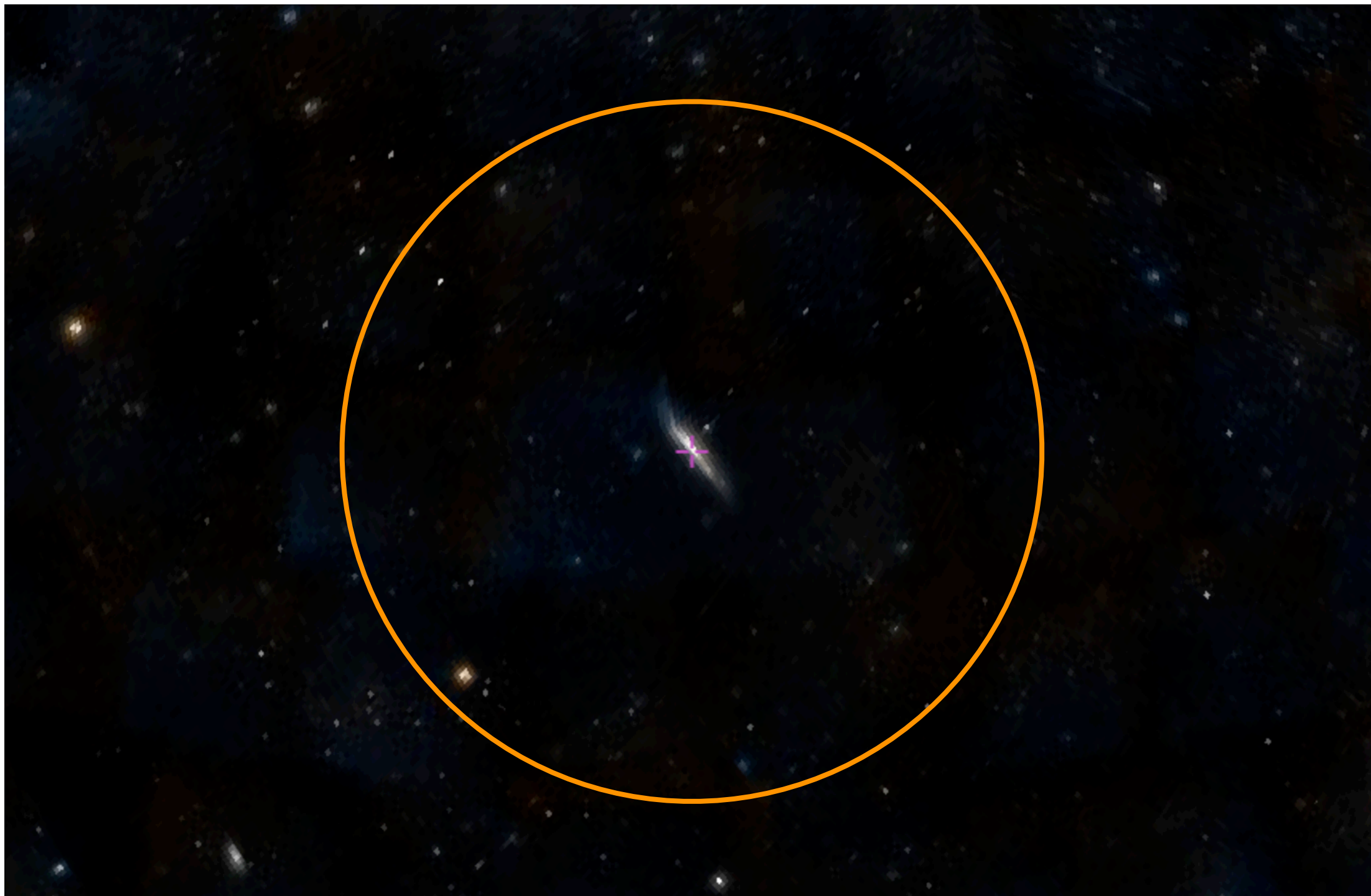
$$\Omega^2 = \frac{GM}{R^3} \left(1 - \frac{H_0^2 R^3}{GM} \right)$$

$$V = \Omega R \quad \longrightarrow \quad V^2 = \frac{GM}{R} \left(1 - \frac{R^3}{R_c^3} \right)$$

$$R_c = \left(\frac{GM}{H_0^2} \right)^{\frac{1}{3}} \approx 0.9 \left(\frac{M}{10^{12} M_\odot} \right)^{\frac{1}{3}} \left(\frac{10^{-29} \text{ g/cm}^3}{\rho_c} \right)^{\frac{1}{3}} \text{ Mpc}$$

Freedman & Madore, Ann. Rev. A&A, vol. 48, p.673-710





Dark matter

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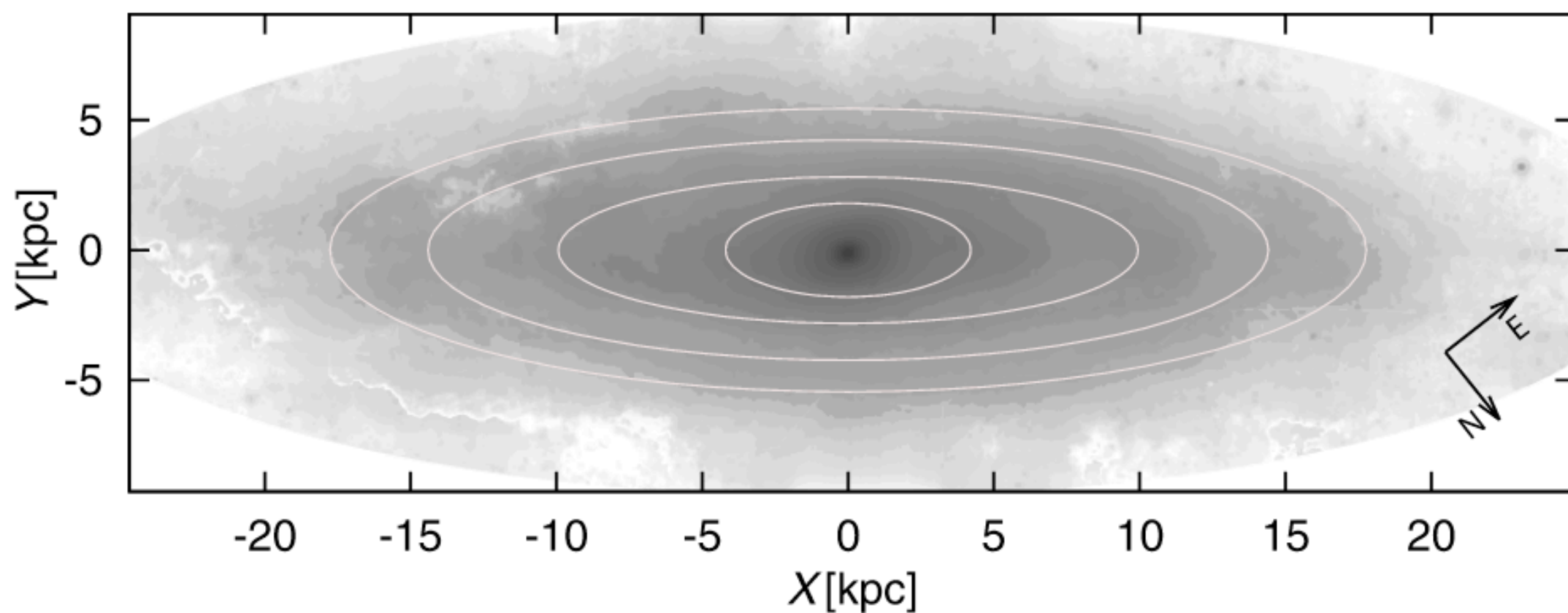
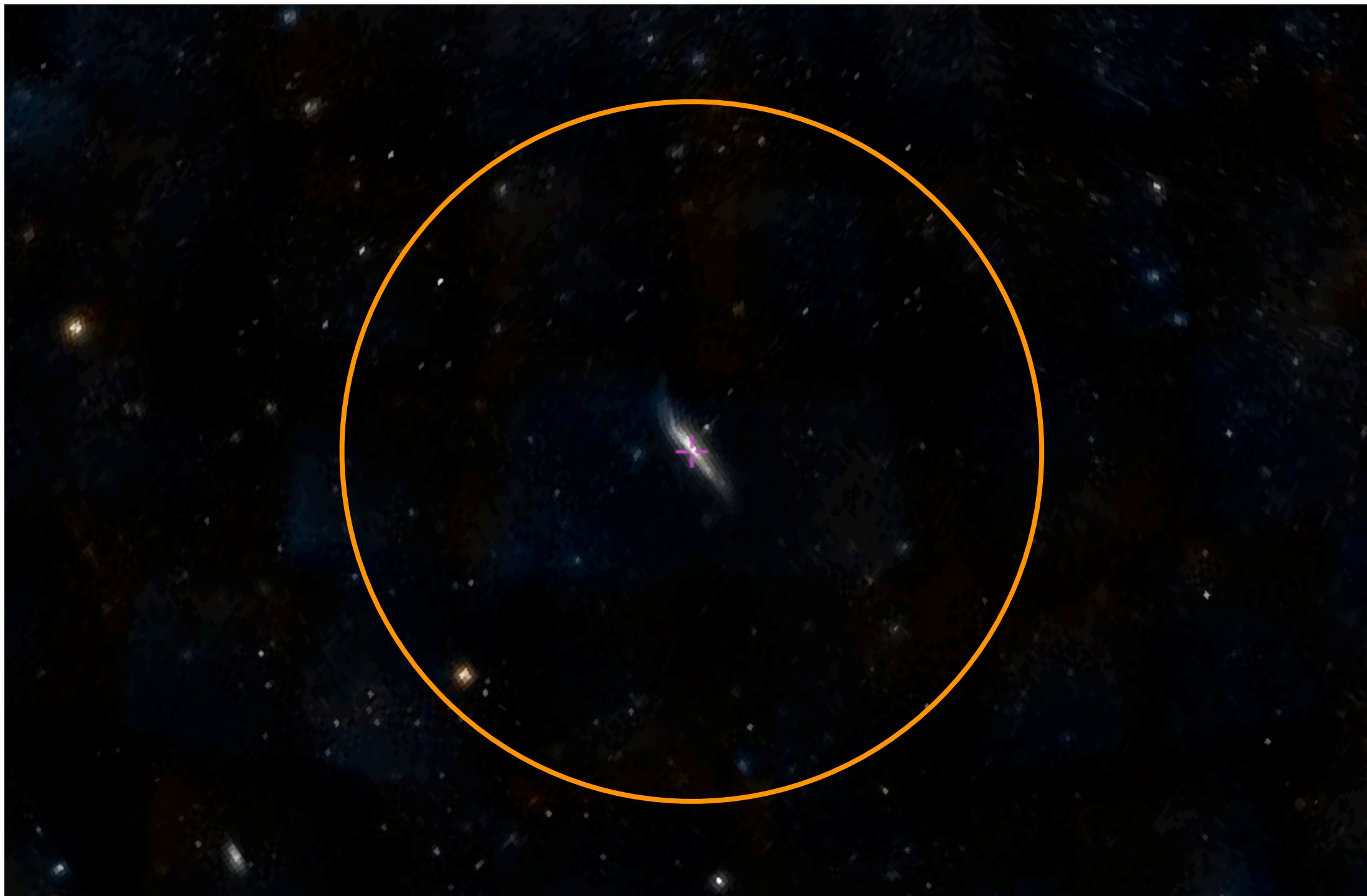


Fig. 2. Stellar mass-density map of M31. The ellipses enclose 50%, 75%, 90%, and 95% of the total mass, respectively.

Dark matter

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Table 1. Synthetic stellar populations used for SED fitting.

Name	Age	[Fe/H]	$\frac{M_{\text{tot}}}{L_g}$	$\frac{M_{\text{tot}}}{L_r}$	$\frac{M_{\text{tot}}}{L_i}$	Fract.
	[Gyr]		$\left[\frac{M_{\odot}}{L_{\odot}}\right]$	$\left[\frac{M_{\odot}}{L_{\odot}}\right]$	$\left[\frac{M_{\odot}}{L_{\odot}}\right]$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
B07-1	0.7	0.40	0.76	0.78	0.72	0.014
B07-3	0.4–1	0.05	0.47	0.50	0.56	0.003
B07-4	7–12	0.03	5.05	3.87	3.12	0.983
M05-1	1	0.00	1.11	1.00	0.85	0.008
M05-2	2	0.00	2.18	1.70	1.43	0.002
M05-3	4	0.00	3.99	3.03	2.56	0.214
M05-4	12	0.00	11.6	8.08	6.47	0.767
M05-5	12	−0.33	9.00	6.60	5.37	0.009
GALEV-1	1, 10	0.04	2.88	3.14	2.92	0.004
GALEV-2	2, 11	0.07	4.35	4.13	3.65	0.011
GALEV-3	4, 13	0.09	7.58	6.20	5.23	0.089
GALEV-4	12	0.12	4.63	4.55	4.05	0.015
GALEV-5	12	0.18	10.9	8.33	6.86	0.881

Notes. The columns contain the following: (1) stellar population model; for B07 models the number is as in the original paper; (2) approximate age of the dominant star-formation epoch(s); (3) average metallicity of the stars; (4)–(6) mass-to-light ratio in the *gri* filters; (7) total stellar mass fraction in M31 of the corresponding stellar population.

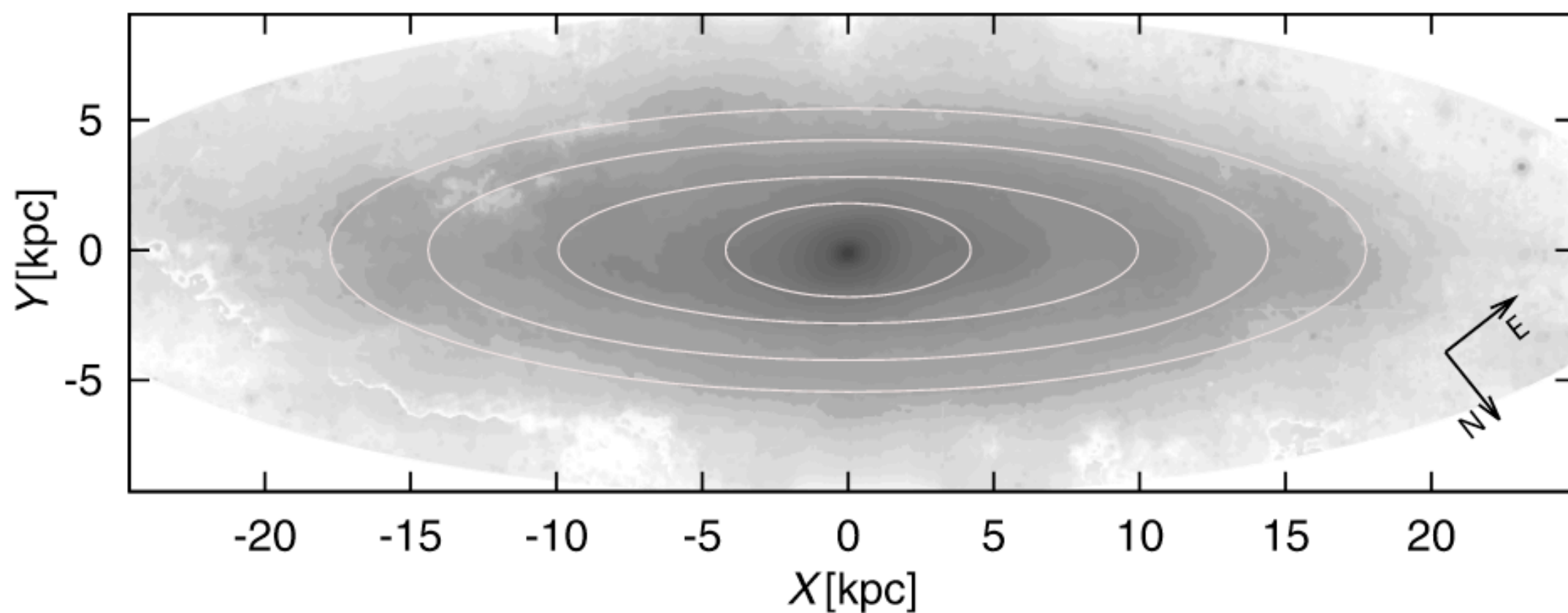
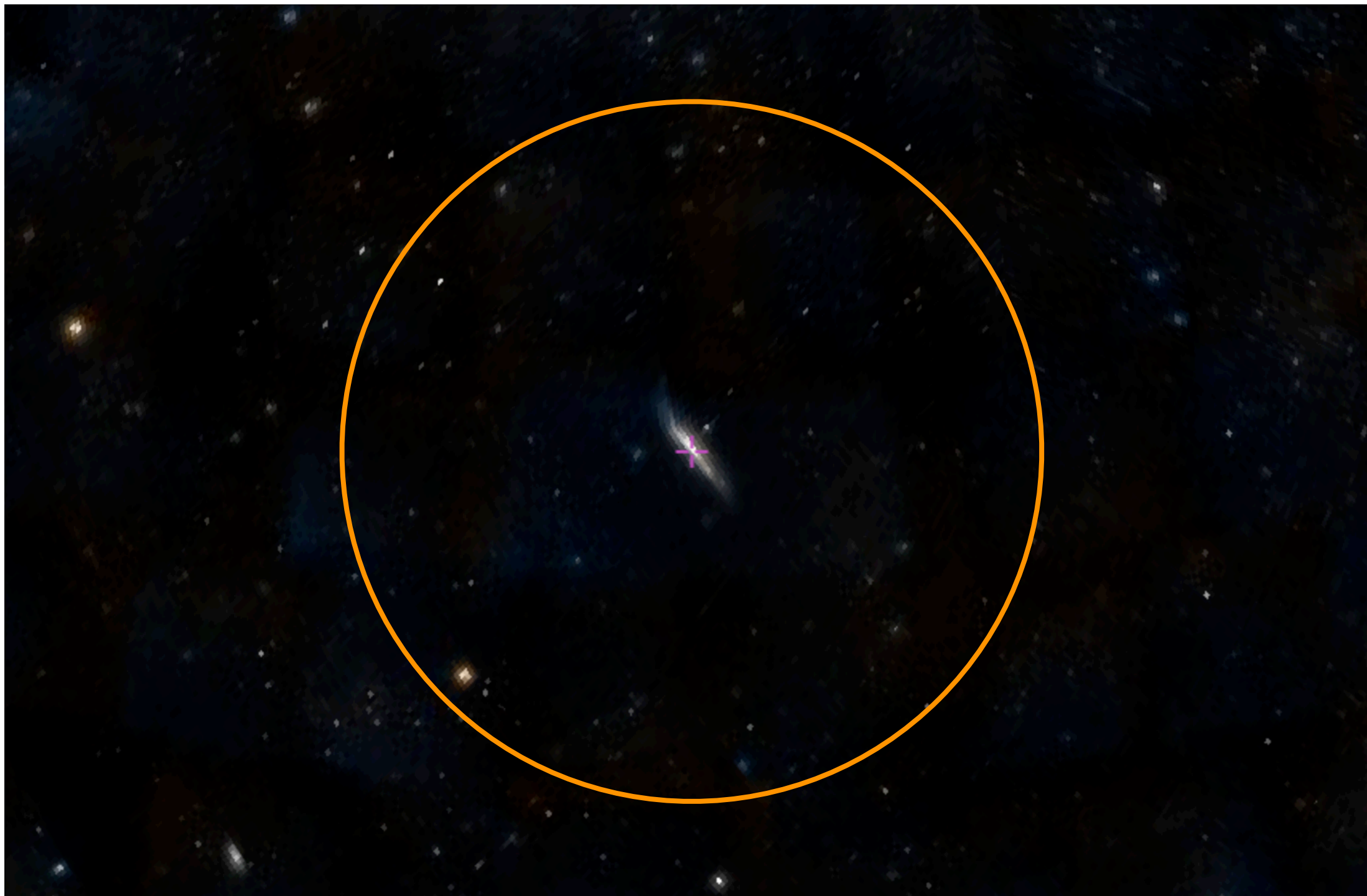


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Dark matter

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Astrophysics**

Stellar mass map and dark matter distribution in M31

A. Tamm¹, E. Tempel^{1,2}, P. Tenjes^{1,3}, O. Tihhonova^{1,3}, and T. Tuvikene¹

Table 1. Synthetic stellar populations used for SED fitting.

Name	Age	[Fe/H]	$\frac{M_{\text{tot}}}{L_g}$	$\frac{M_{\text{tot}}}{L_r}$	$\frac{M_{\text{tot}}}{L_i}$	Fract.
	[Gyr]		$\left[\frac{M_{\odot}}{L_{\odot}}\right]$	$\left[\frac{M_{\odot}}{L_{\odot}}\right]$	$\left[\frac{M_{\odot}}{L_{\odot}}\right]$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
B07-1	0.7	0.40	0.76	0.78	0.72	0.014
B07-3	0.4–1	0.05	0.47	0.50	0.56	0.003
B07-4	7–12	0.03	5.05	3.87	3.12	0.983
M05-1	1	0.00	1.11	1.00	0.85	0.008
M05-2	2	0.00	2.18	1.70	1.43	0.002
M05-3	4	0.00	3.99	3.03	2.56	0.214
M05-4	12	0.00	11.6	8.08	6.47	0.767
M05-5	12	−0.33	9.00	6.60	5.37	0.009
GALEV-1	1, 10	0.04	2.88	3.14	2.92	0.004
GALEV-2	2, 11	0.07	4.35	4.13	3.65	0.011
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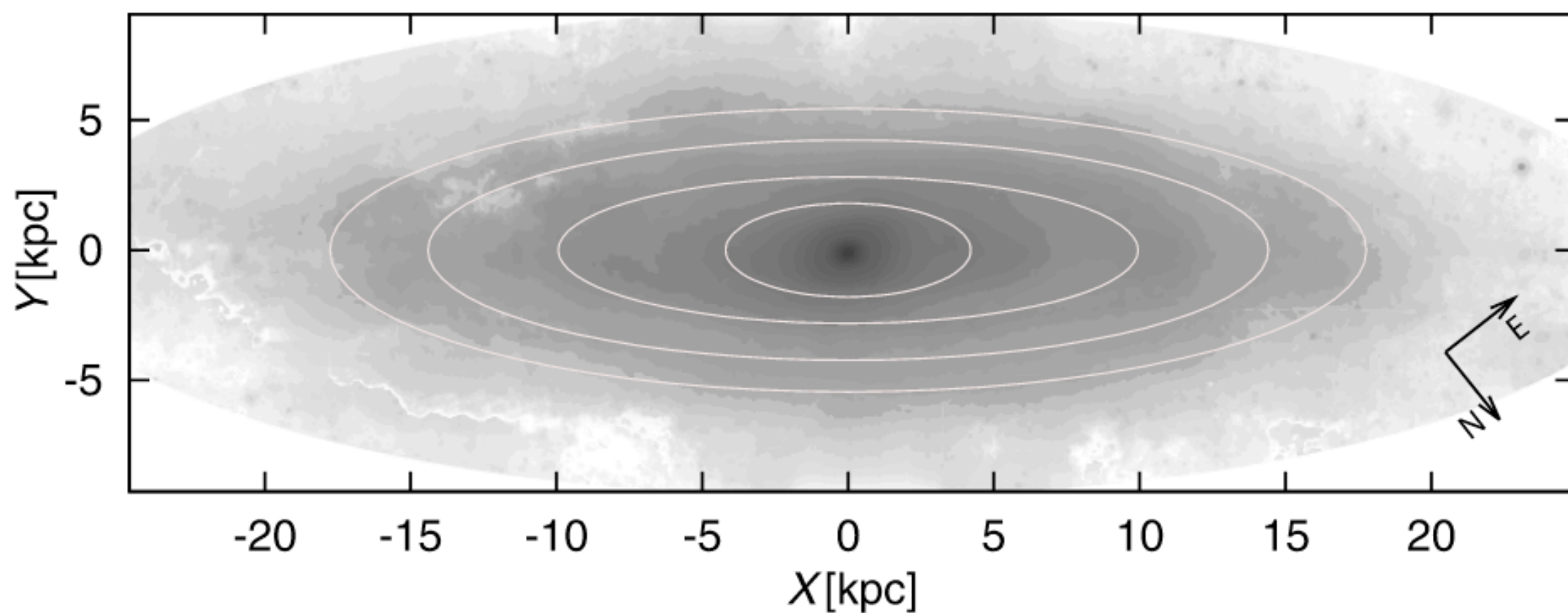
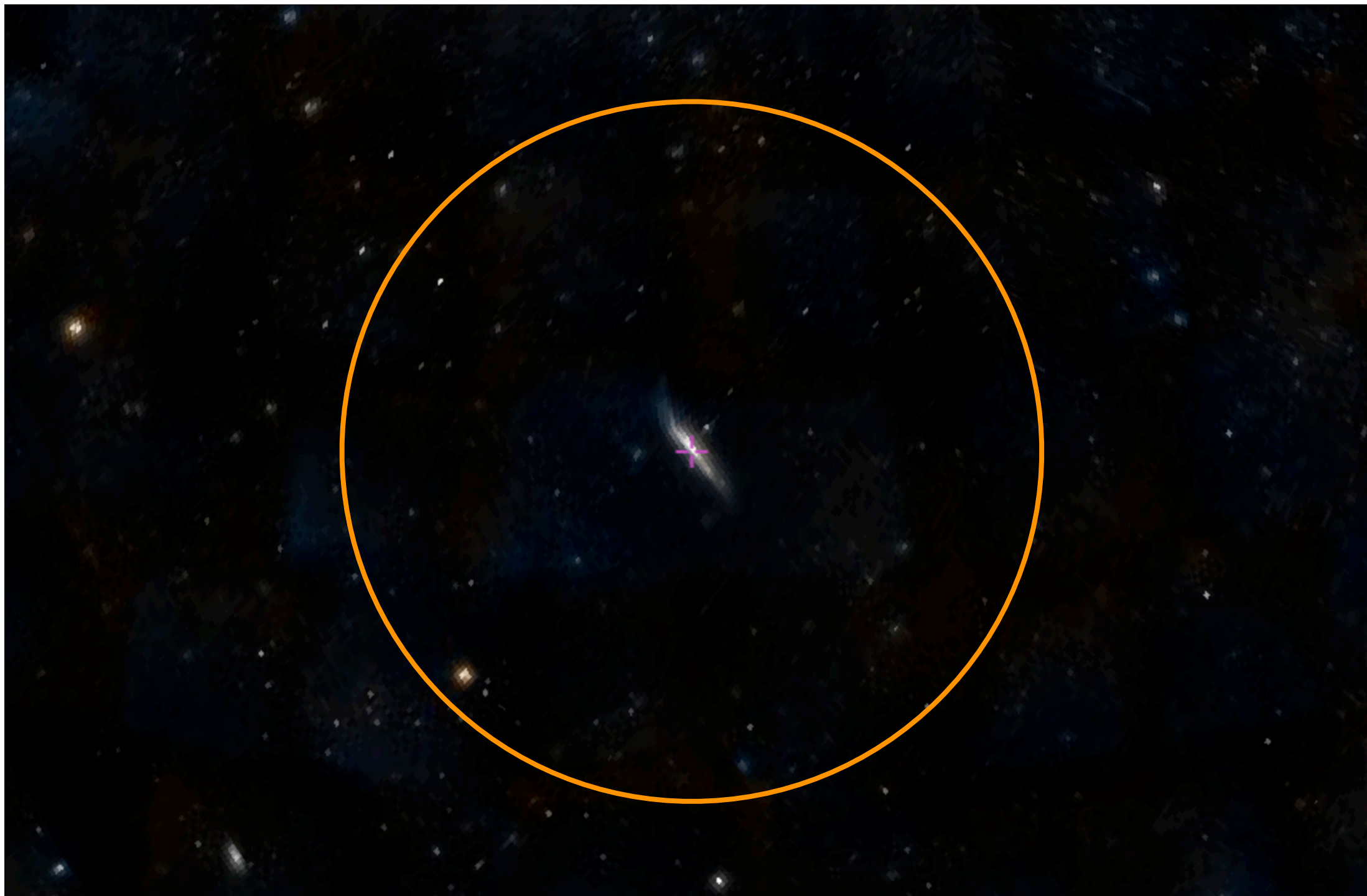


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B07-1	0.7	0.40	0.76	0.78	0.72	0.014
B07-3	0.4–1	0.05	0.47	0.50	0.56	0.003
B07-4	7–12	0.03	5.05	3.87	3.12	0.983
M05-1	1	0.00	1.11	1.00	0.85	0.008
M05-2	2	0.00	2.18	1.70	1.43	0.002
M05-3	4	0.00	3.99	3.03	2.56	0.214
M05-4	12	0.00	11.6	8.08	6.47	0.767
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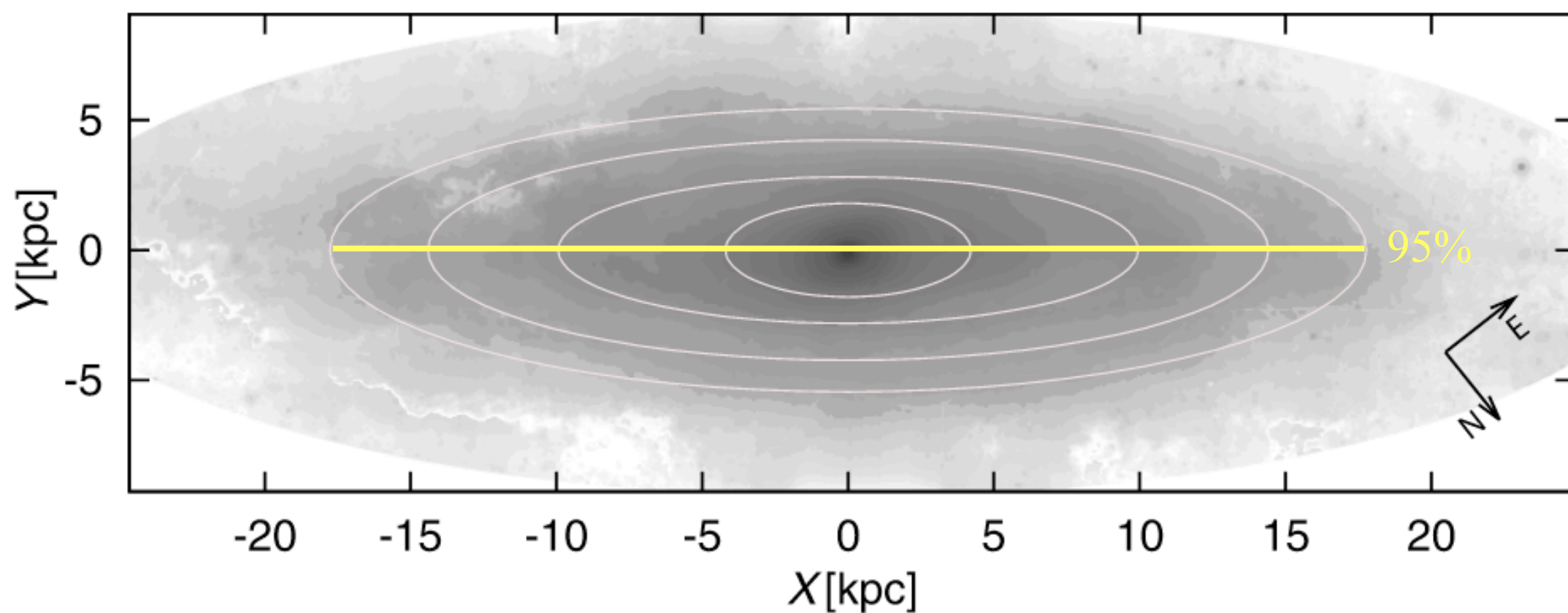
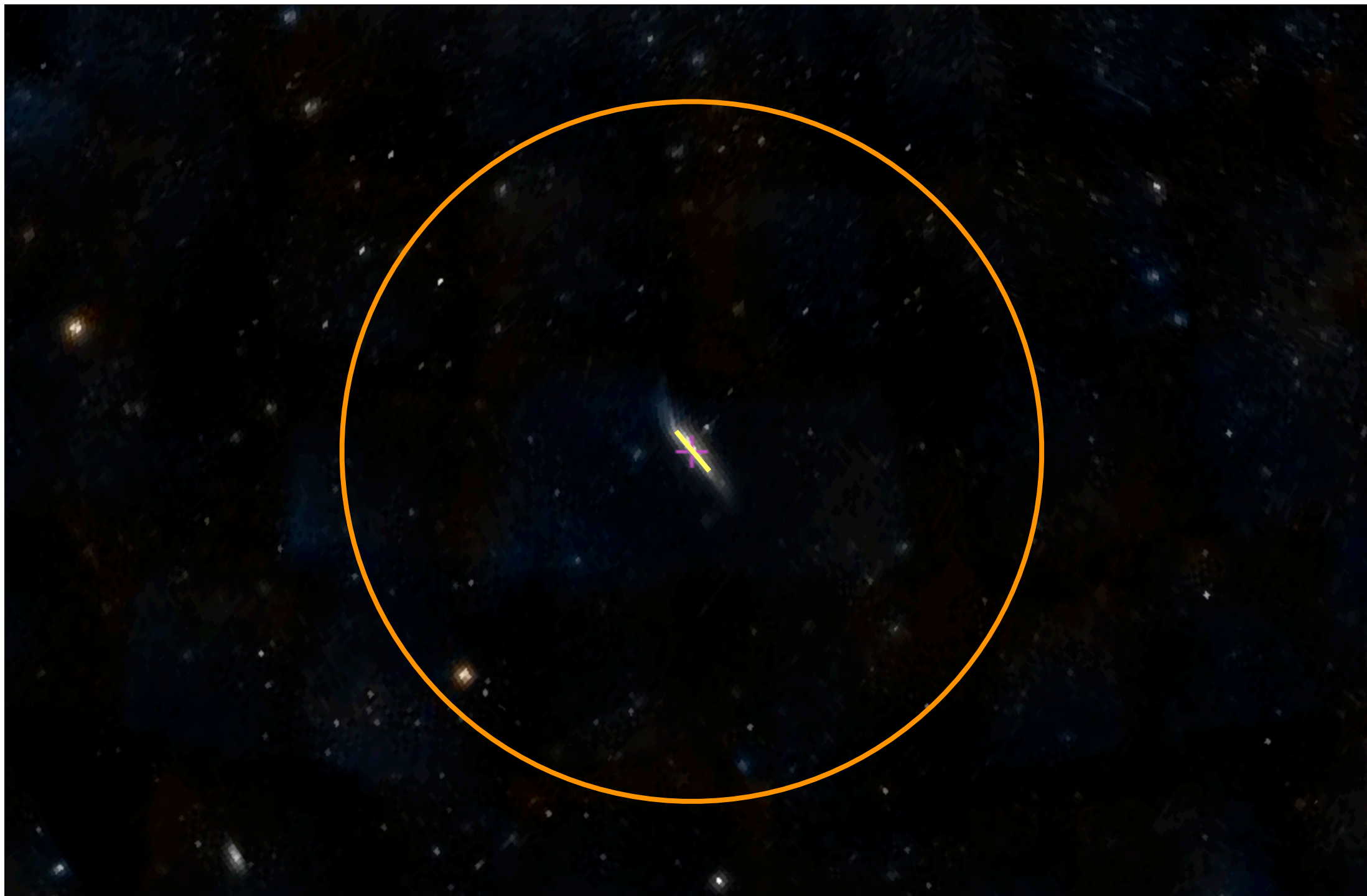


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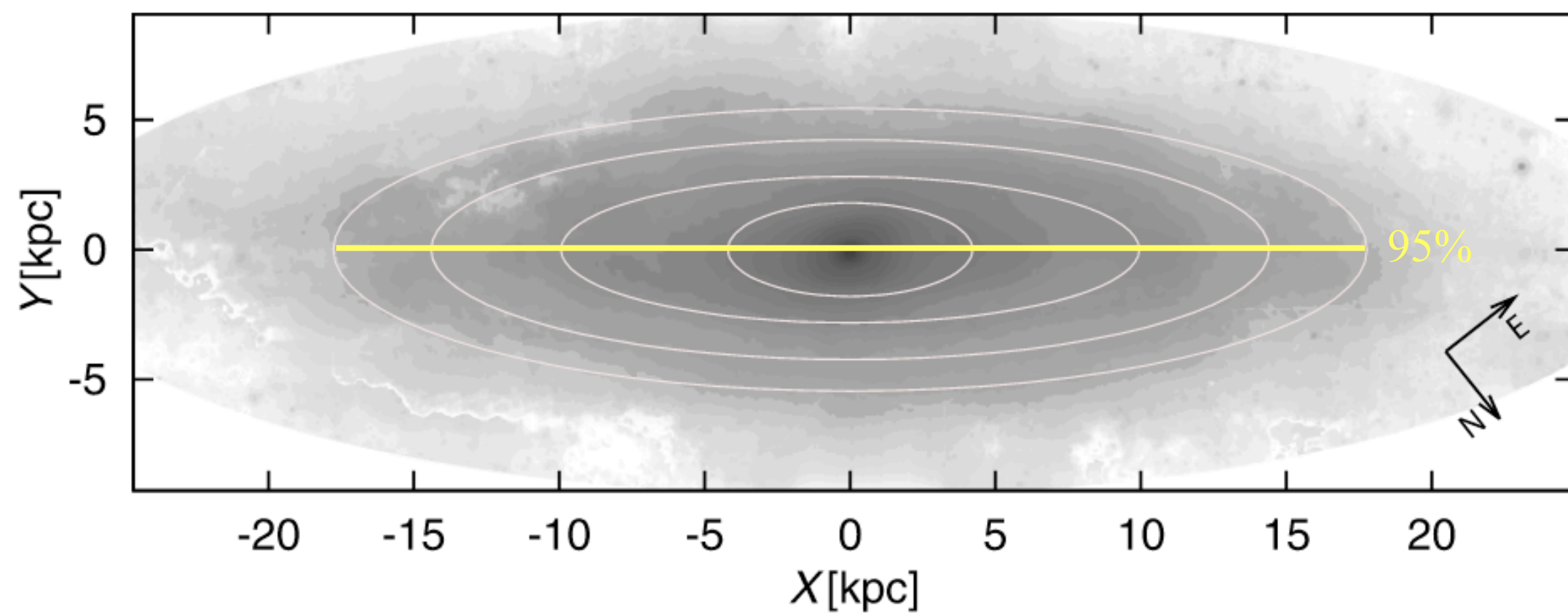
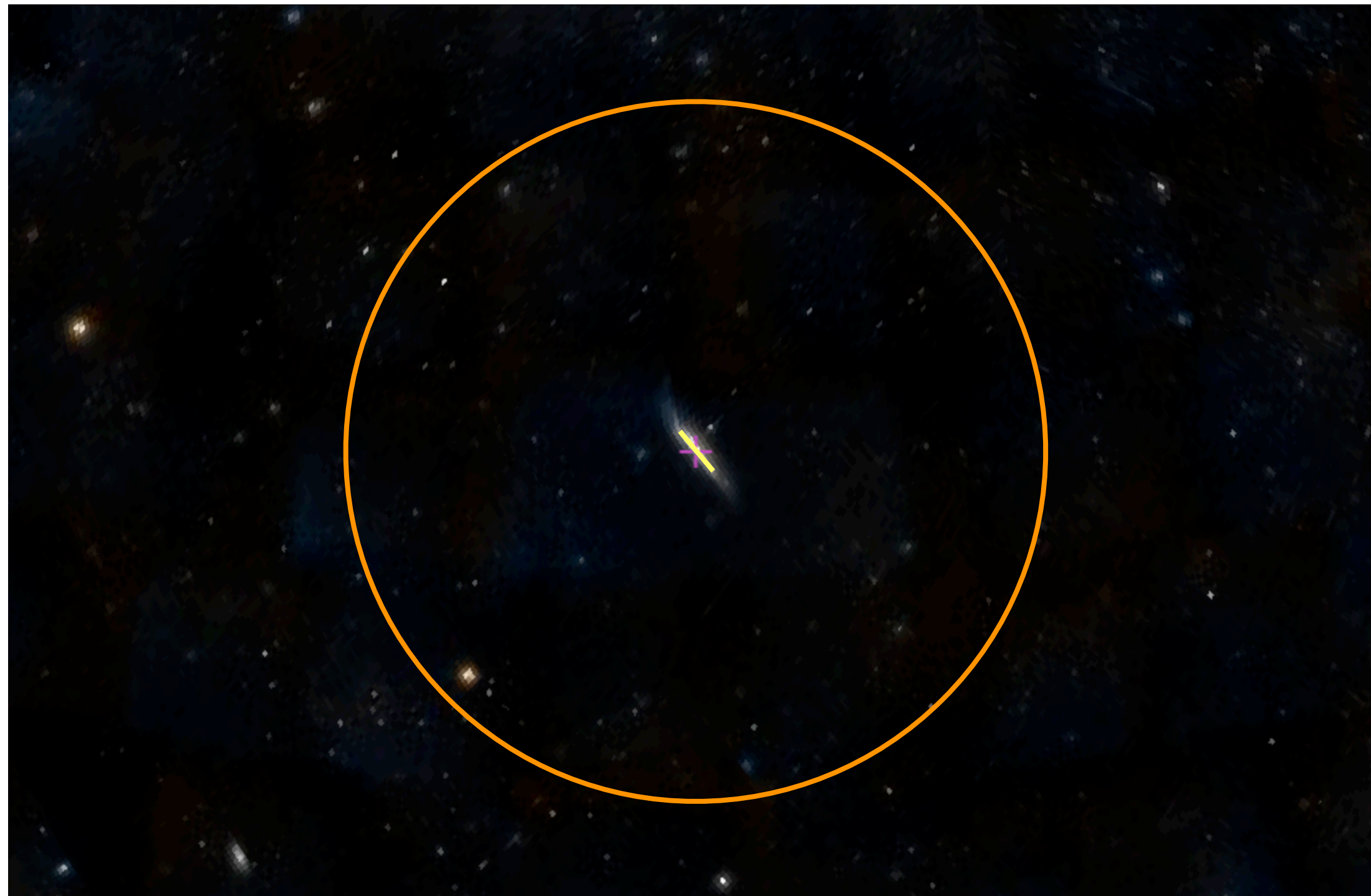


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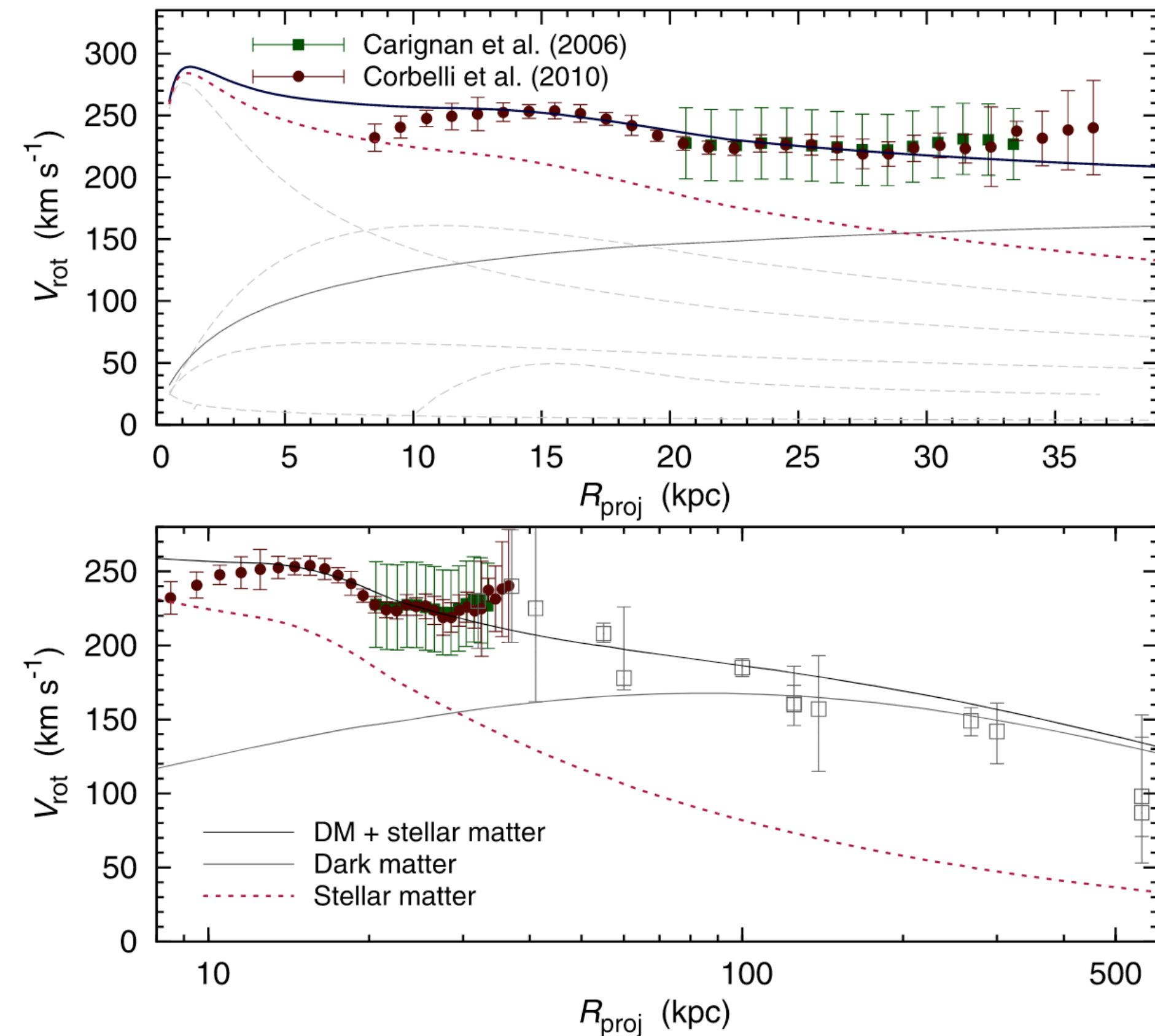
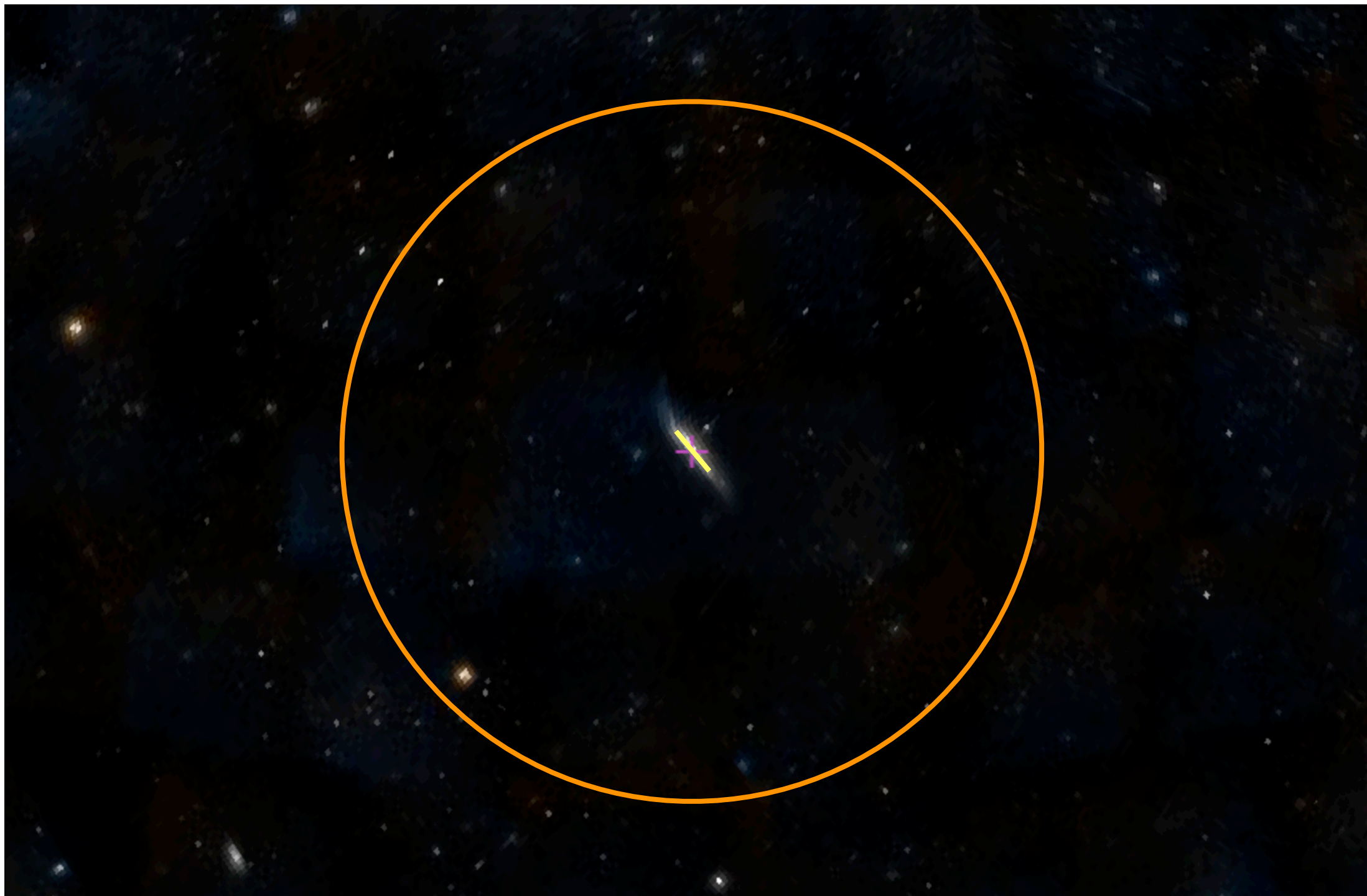


Fig. 7. Observed rotation curve together with the maximum-stellar model, in which the stellar masses are 1.5 times higher than in the B07 model.



At ~30 kpc:

M(DM)/M(stars) ~ 1

At ~300 kpc:

M(DM)/M(stars) ~ 10

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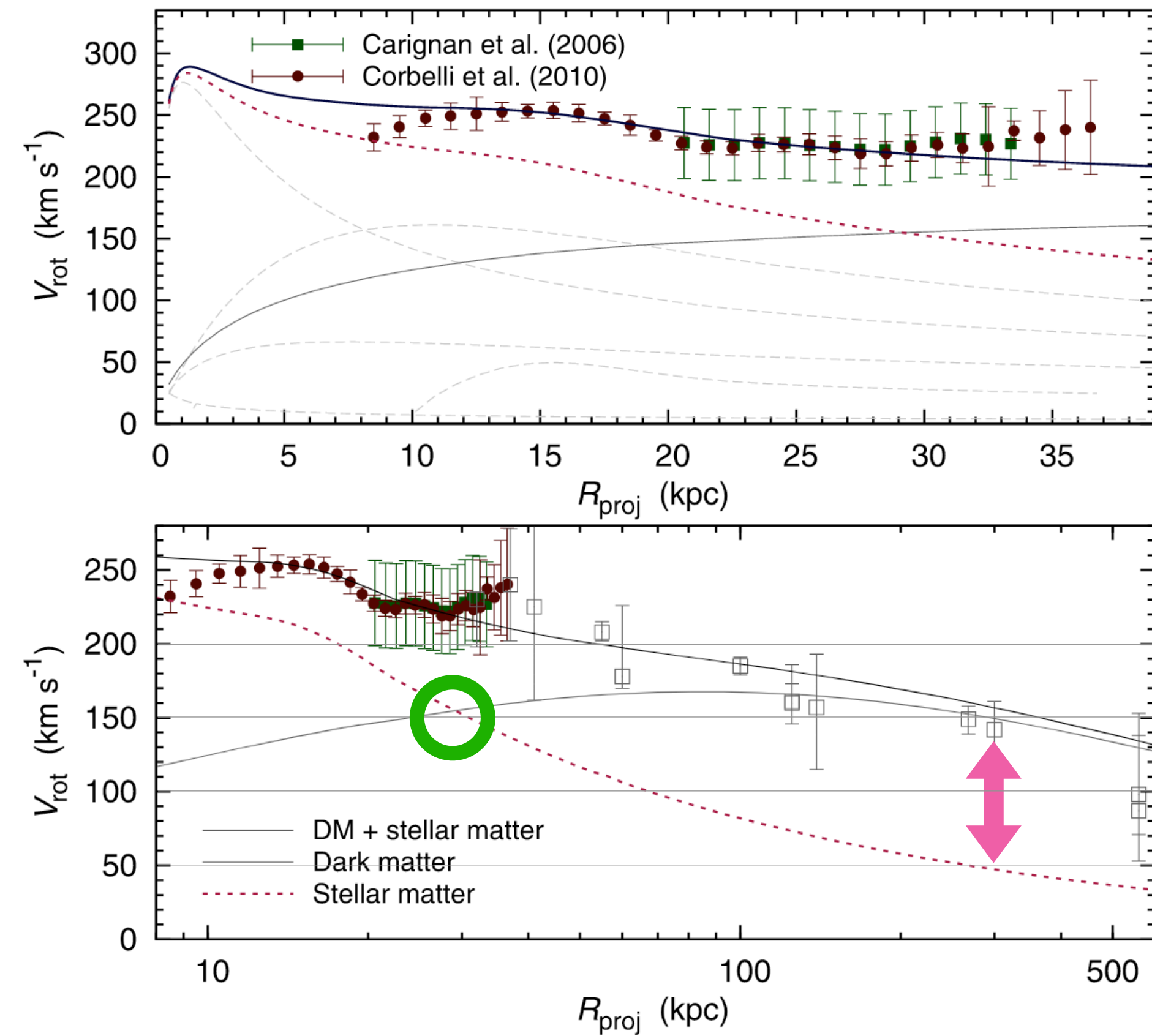
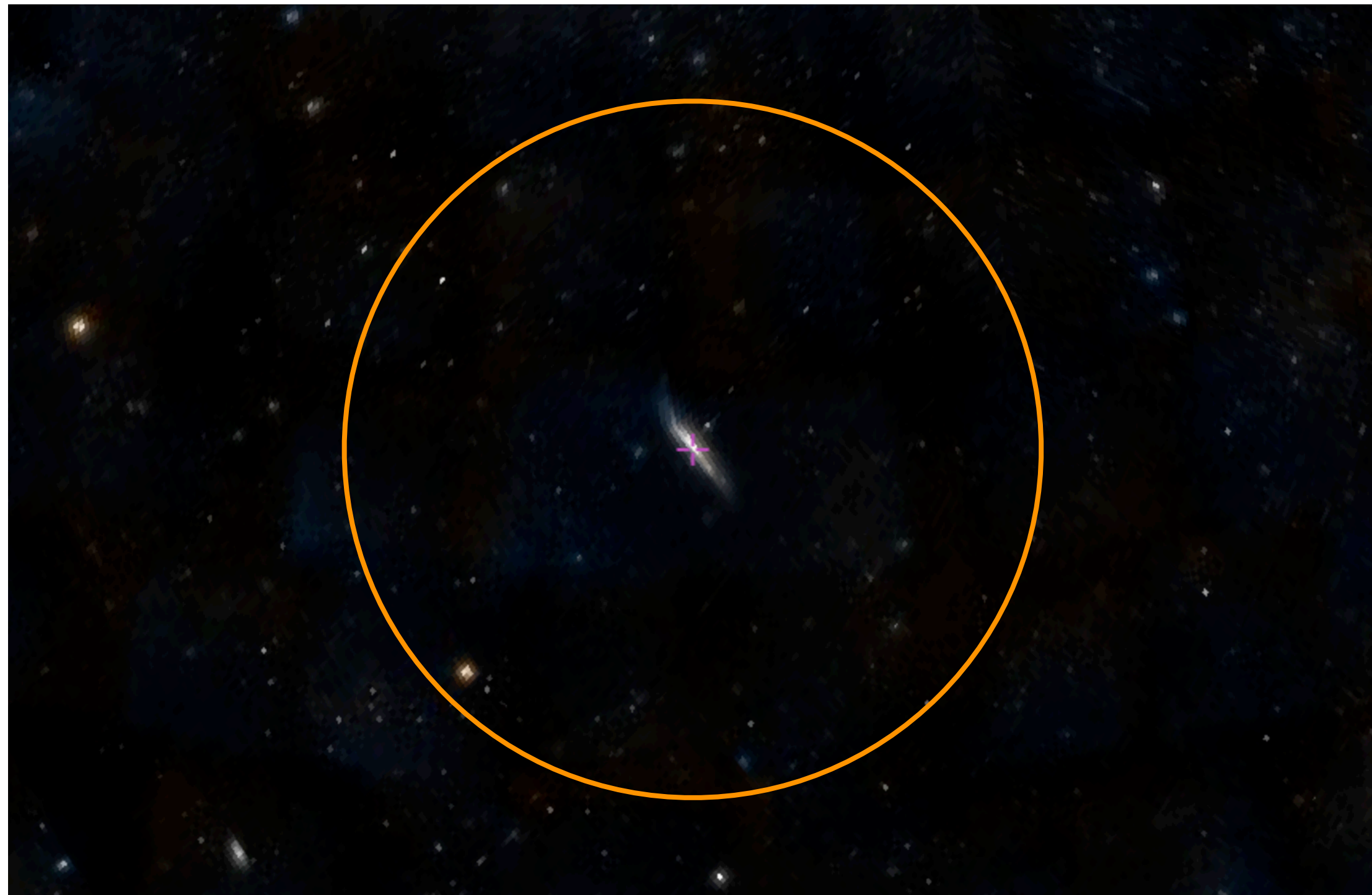
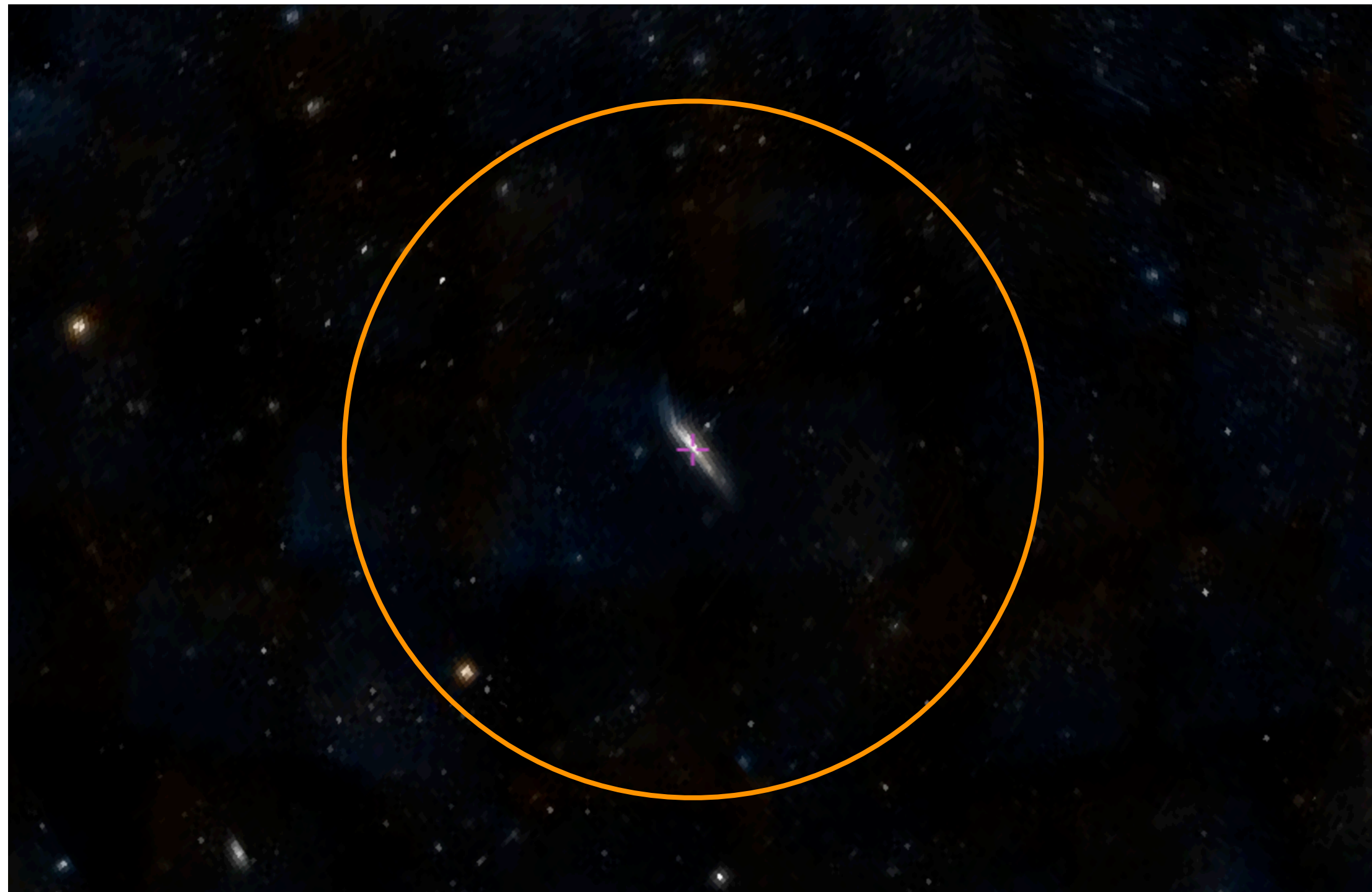


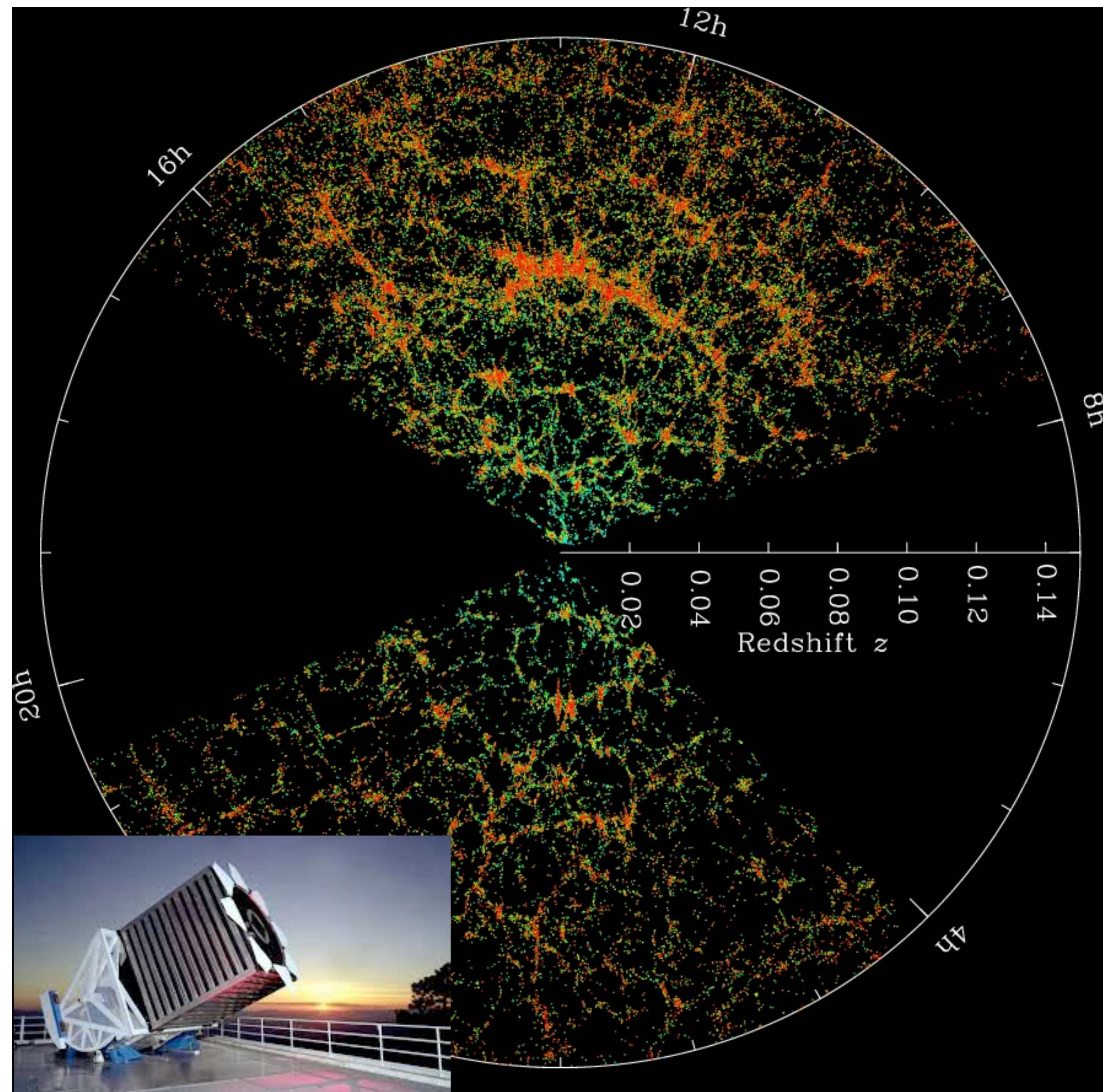
Fig. 7. Observed rotation curve together with the maximum-stellar model, in which the stellar masses are 1.5 times higher than in the B07 model.

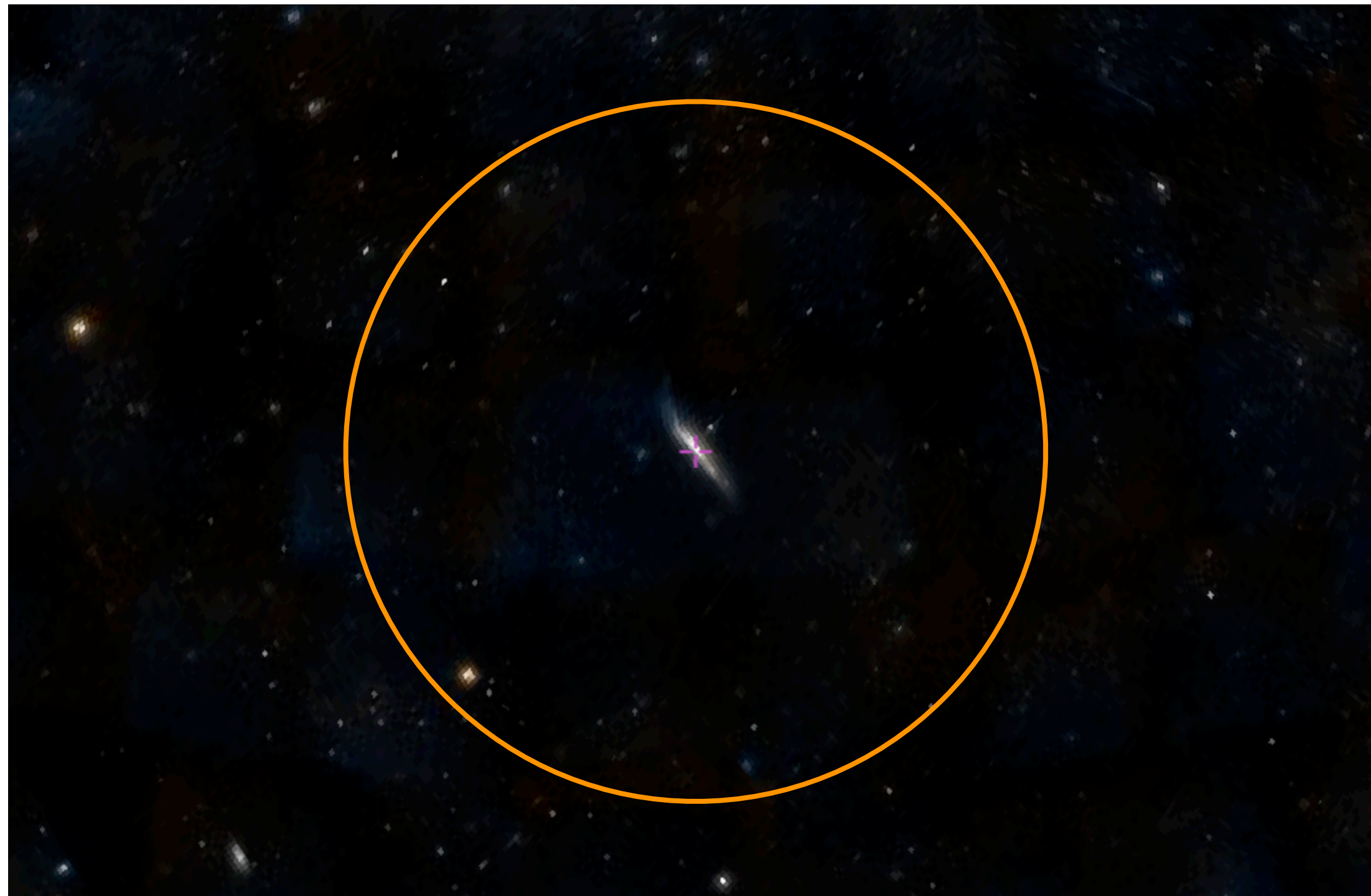


How does this fit into the big picture?



How does this fit into the big picture?

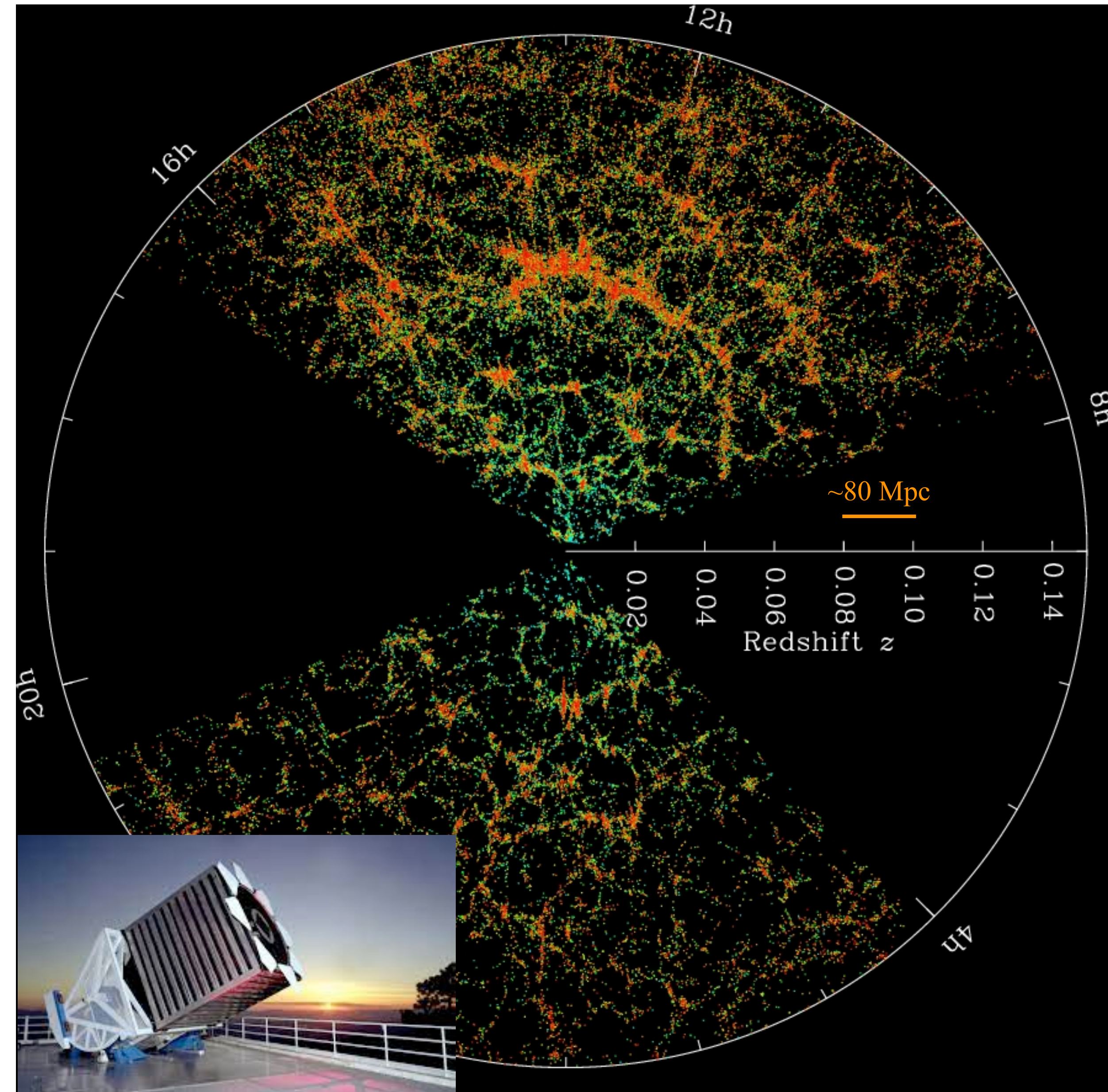


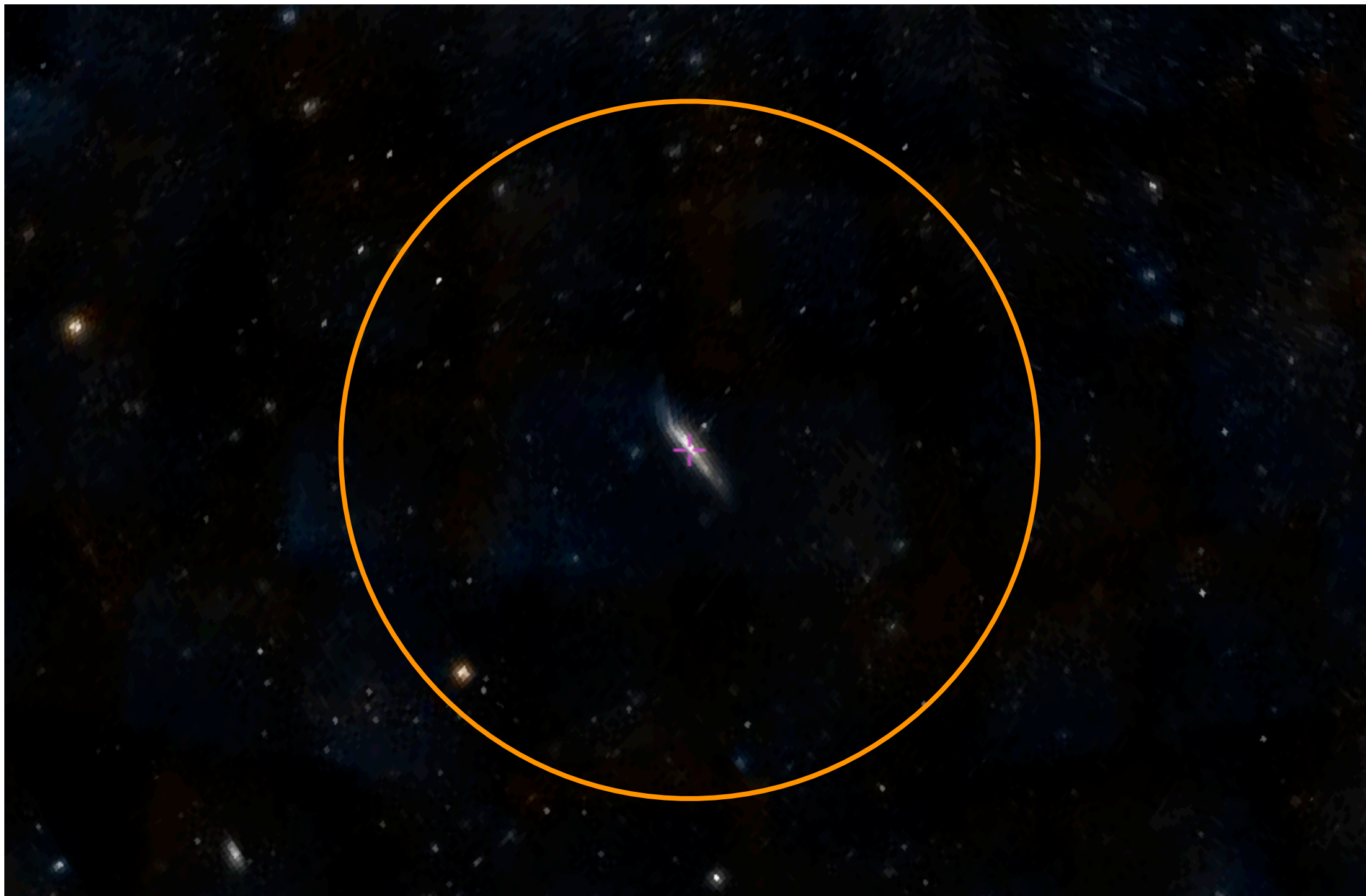


Map $d = 400$ kpc (today) into Δz near $z \sim 0$:

$$\begin{aligned} \Delta\eta &= c \int_{t_1}^{t_2} \frac{dt'}{a(t')} = c \int_z^{z+\Delta z} \frac{dz'}{H(z')} \approx \frac{c}{H_0} \int_z^{z+\Delta z} \frac{dz'}{\sqrt{\Omega_\Lambda + (1+z')^3 \Omega_m}} \\ &\approx \frac{c\Delta z}{H_0 \sqrt{\Omega_\Lambda + (1+z)^3 \Omega_m}} \\ &\approx 400 \text{ kpc} \frac{\Delta z}{10^{-4}} \end{aligned}$$

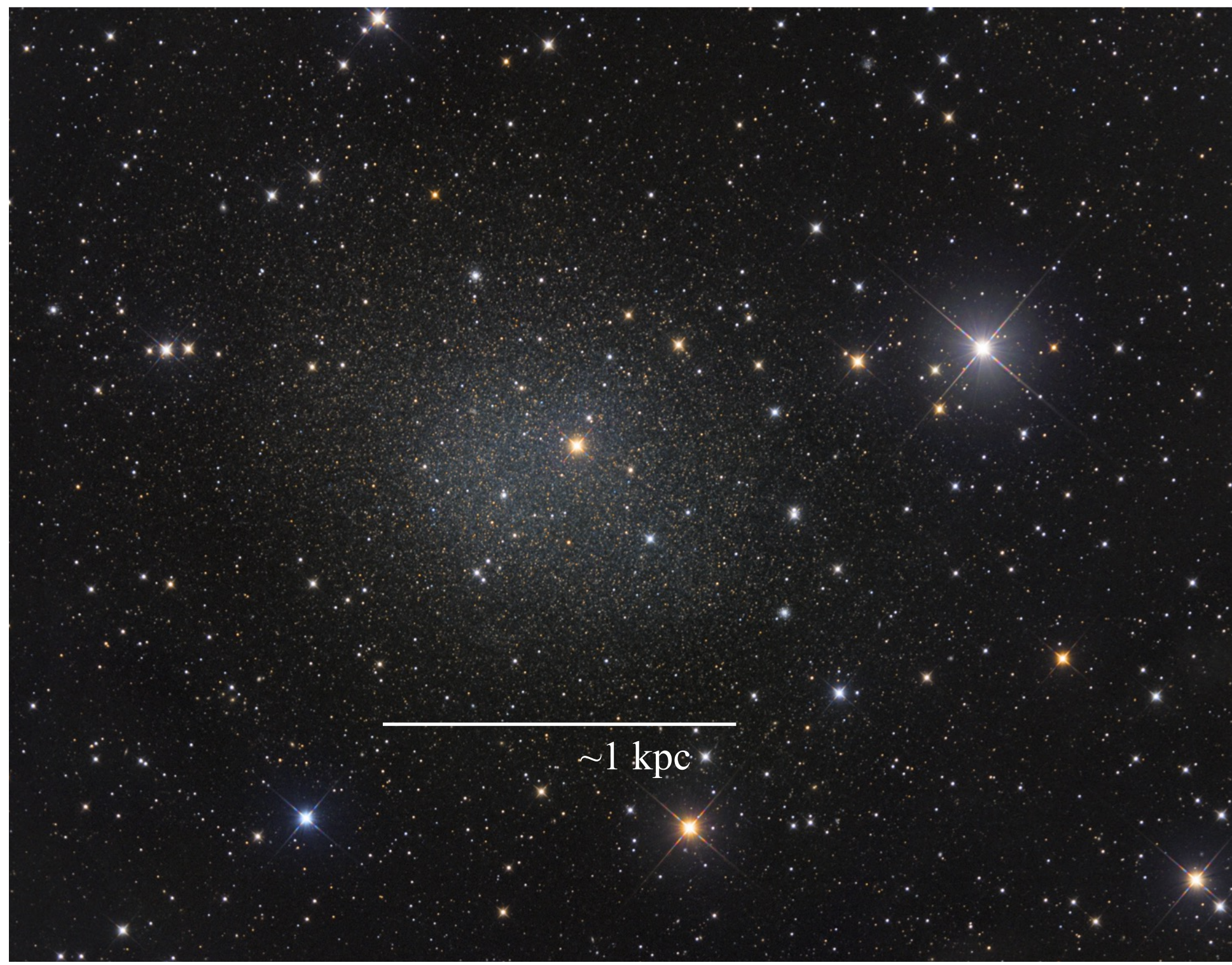
How does this fit into the big picture?



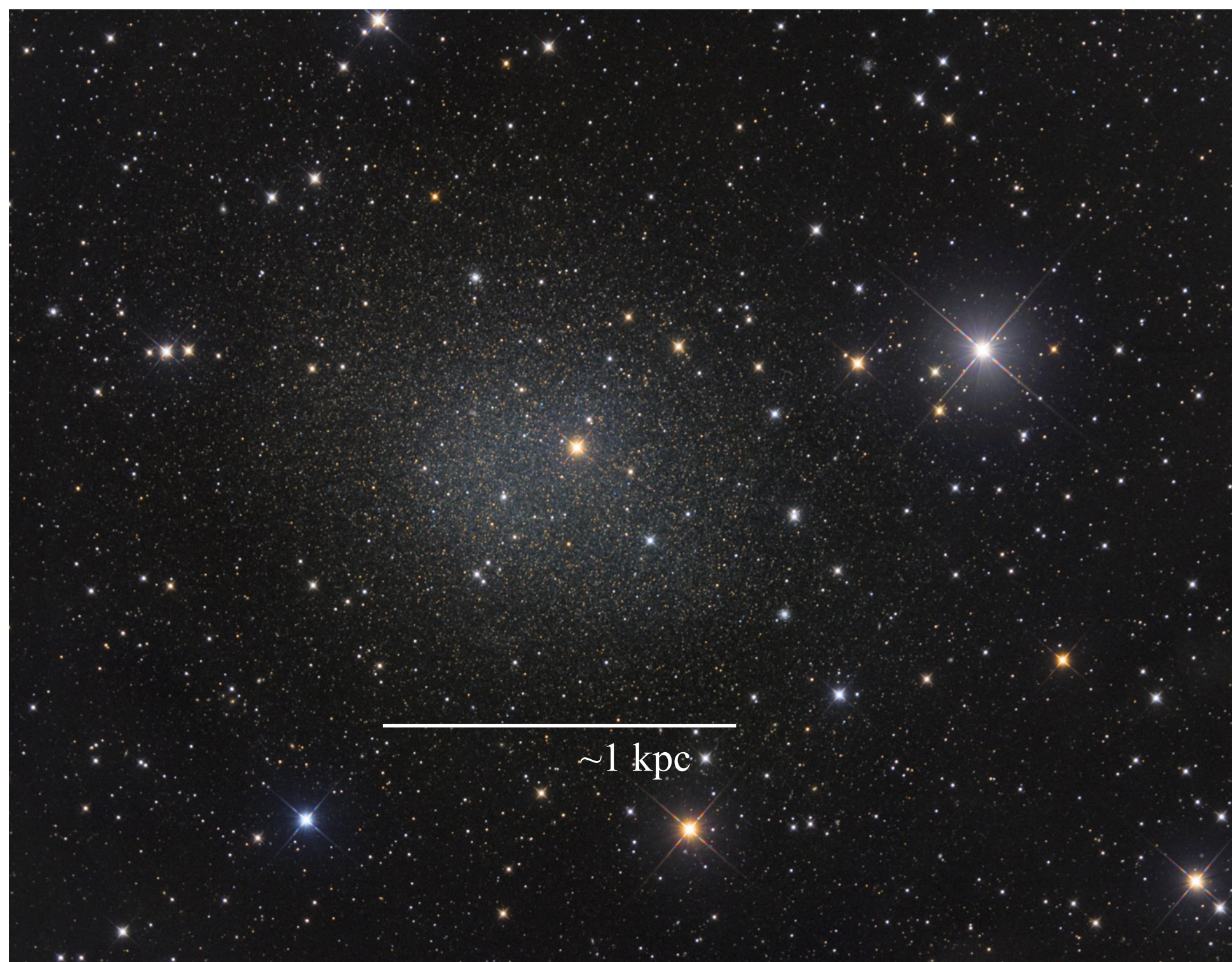


Map $d = 1$ Mpc (today) into Δz near $z \sim 0$:

$$\begin{aligned} \Delta\eta &= c \int_{t_1}^{t_2} \frac{dt'}{a(t')} = c \int_z^{z+\Delta z} \frac{dz'}{H(z')} \approx \frac{c}{H_0} \int_z^{z+\Delta z} \frac{dz'}{\sqrt{\Omega_\Lambda + (1+z')^3 \Omega_m}} \\ &\approx \frac{c\Delta z}{H_0 \sqrt{\Omega_\Lambda + (1+z)^3 \Omega_m}} \\ &\approx 1 \text{ Mpc} \frac{\Delta z}{2.5 \times 10^{-4}} \end{aligned}$$

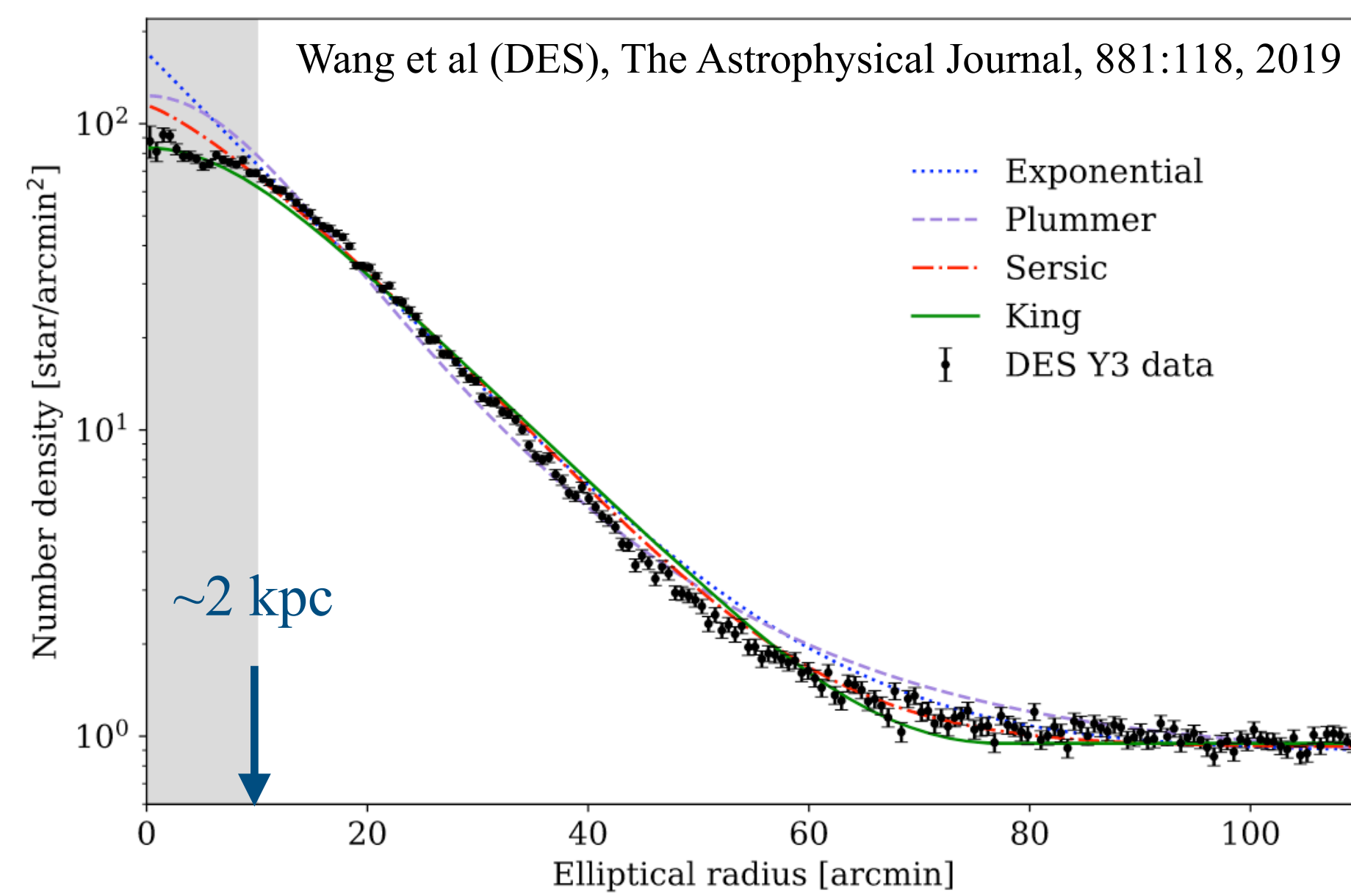
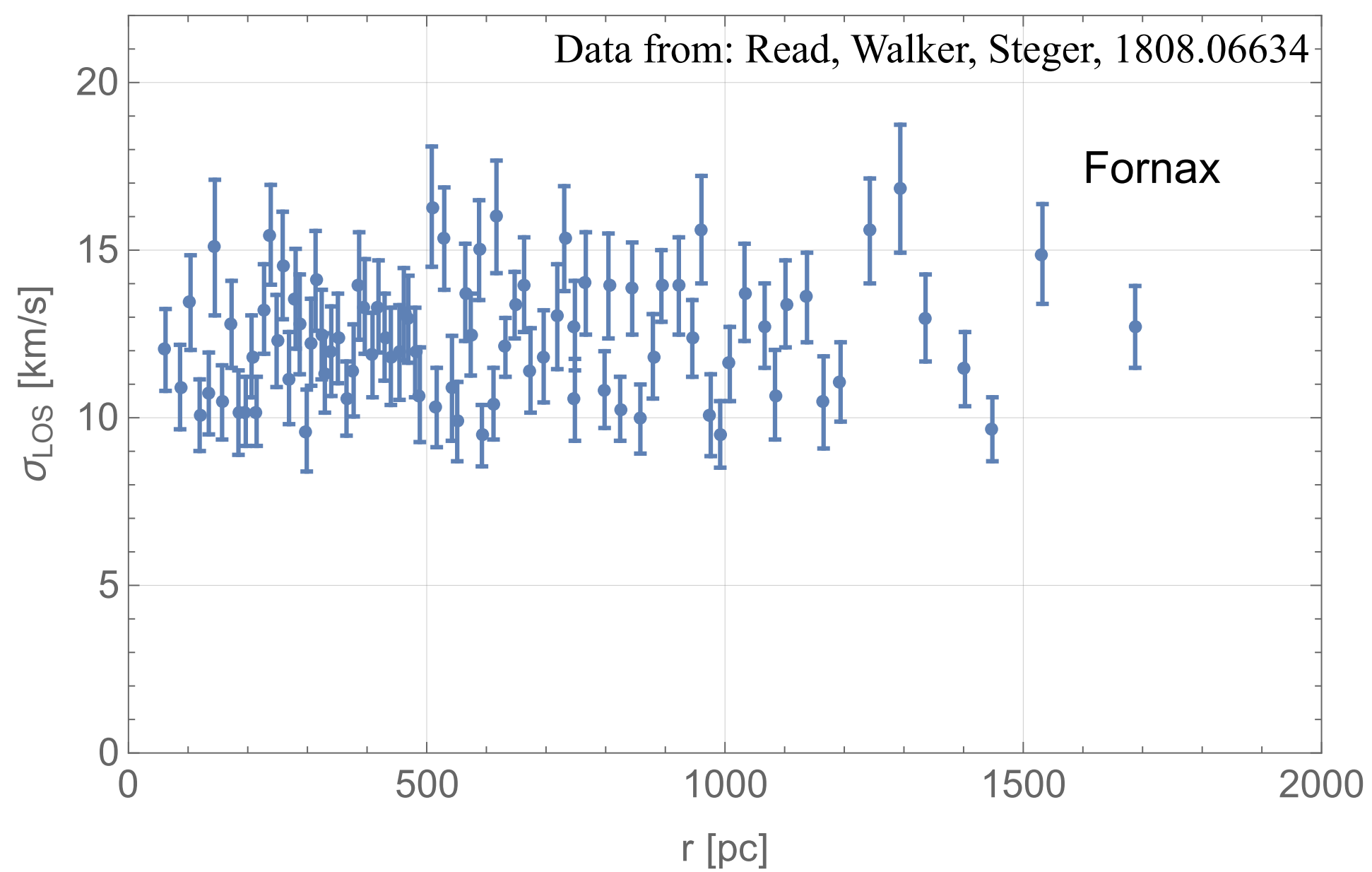


Fornax dwarf spheroidal, satellite galaxy of the Milky Way

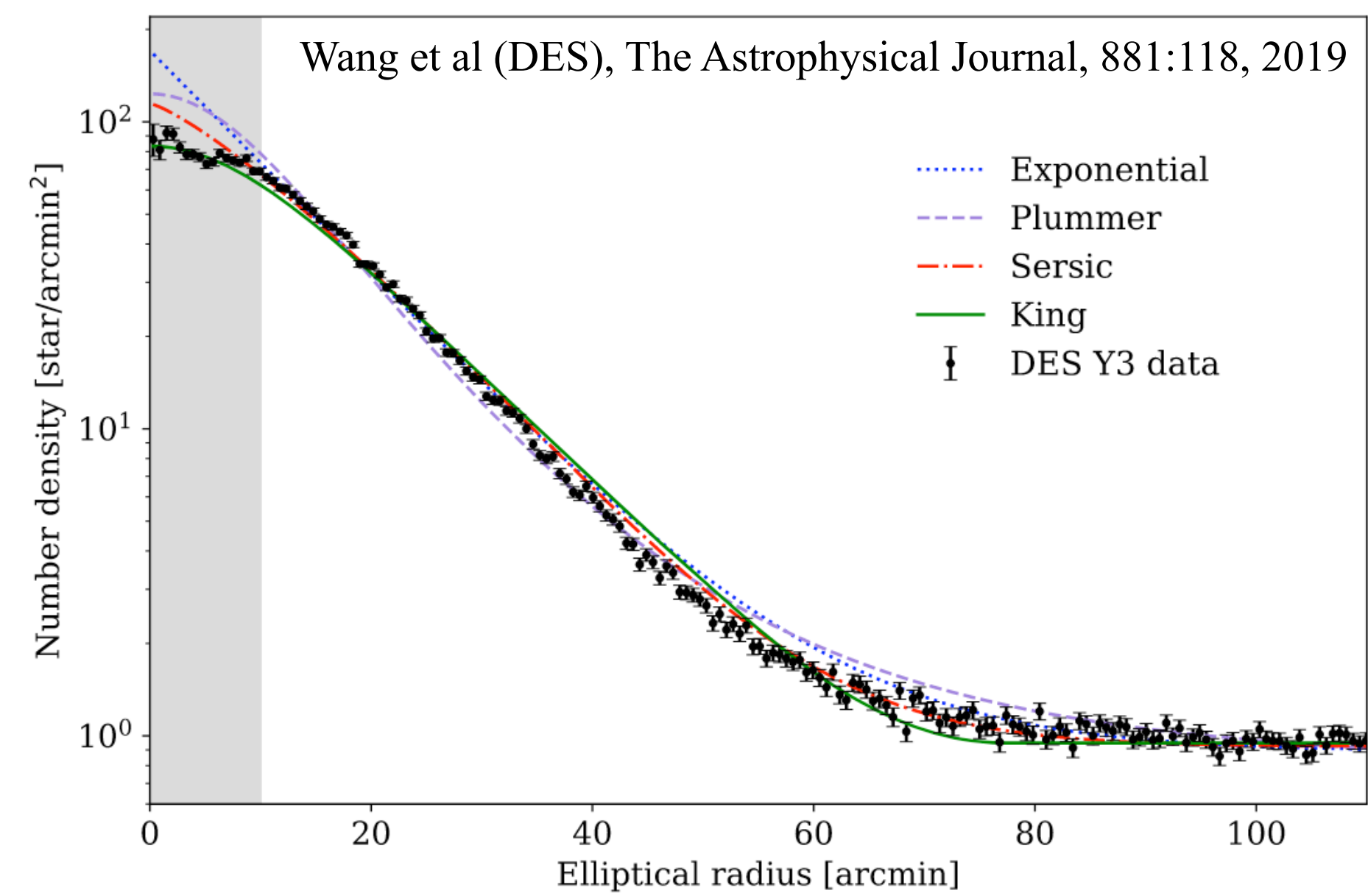
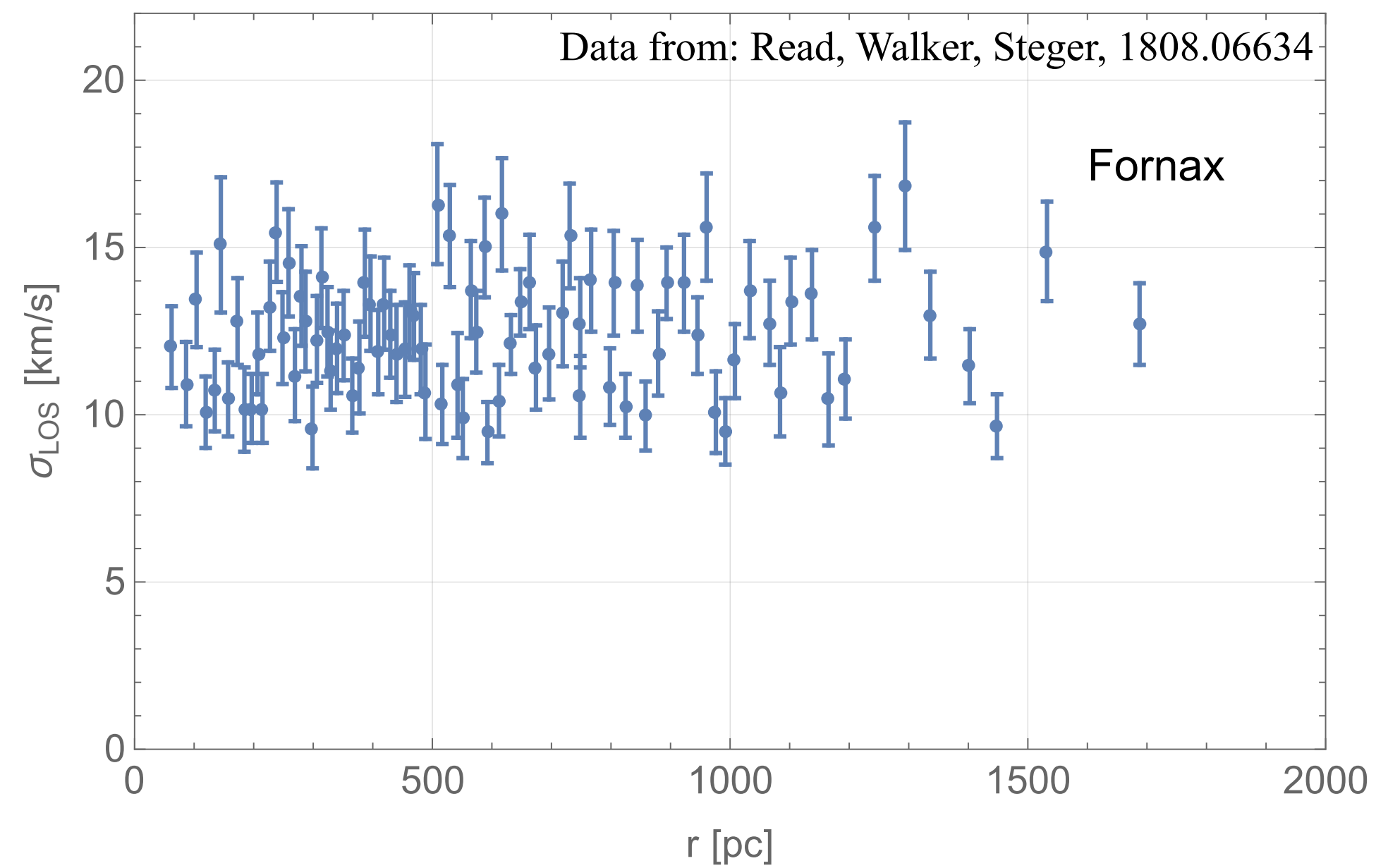


What is observed:

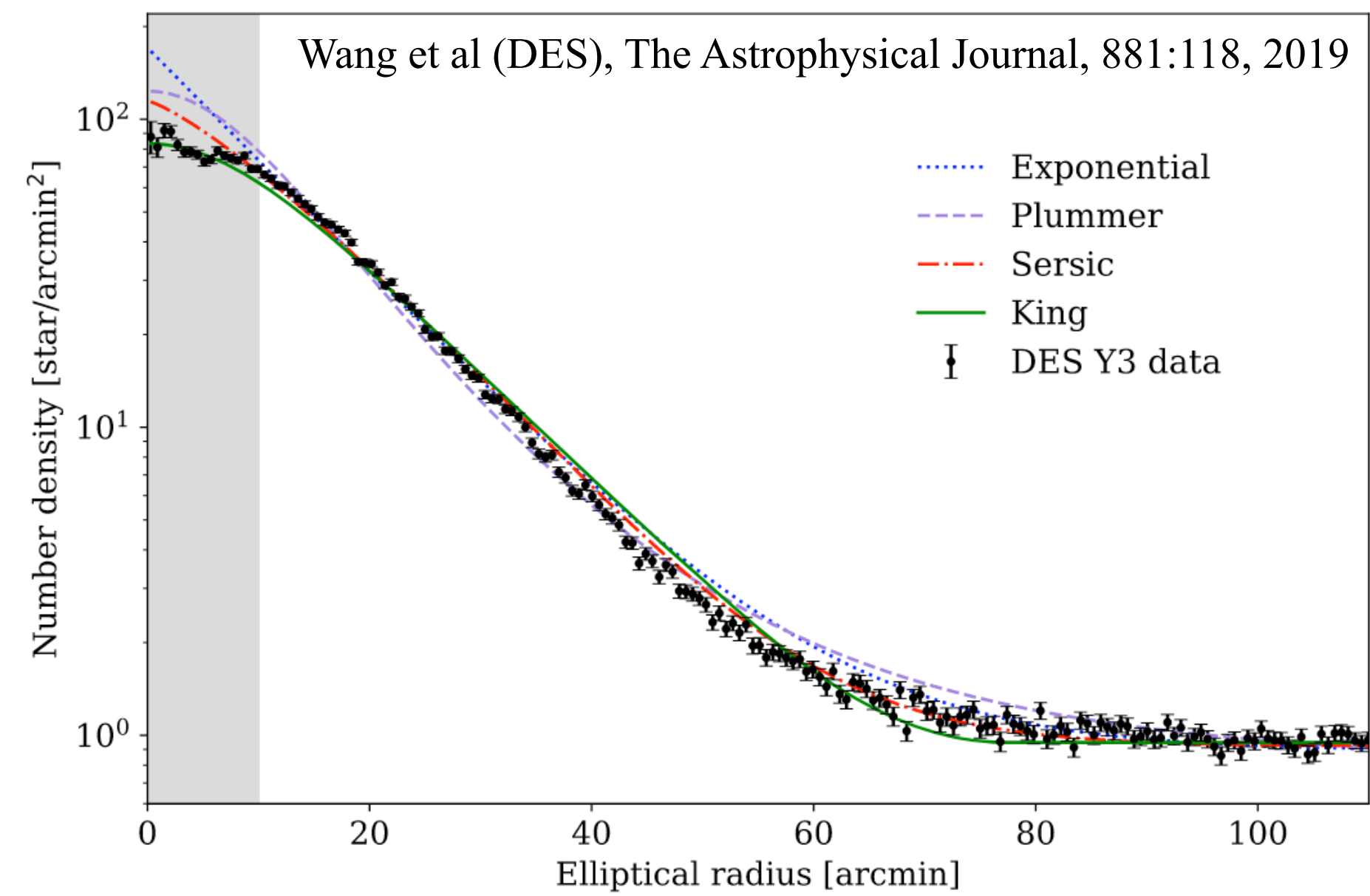
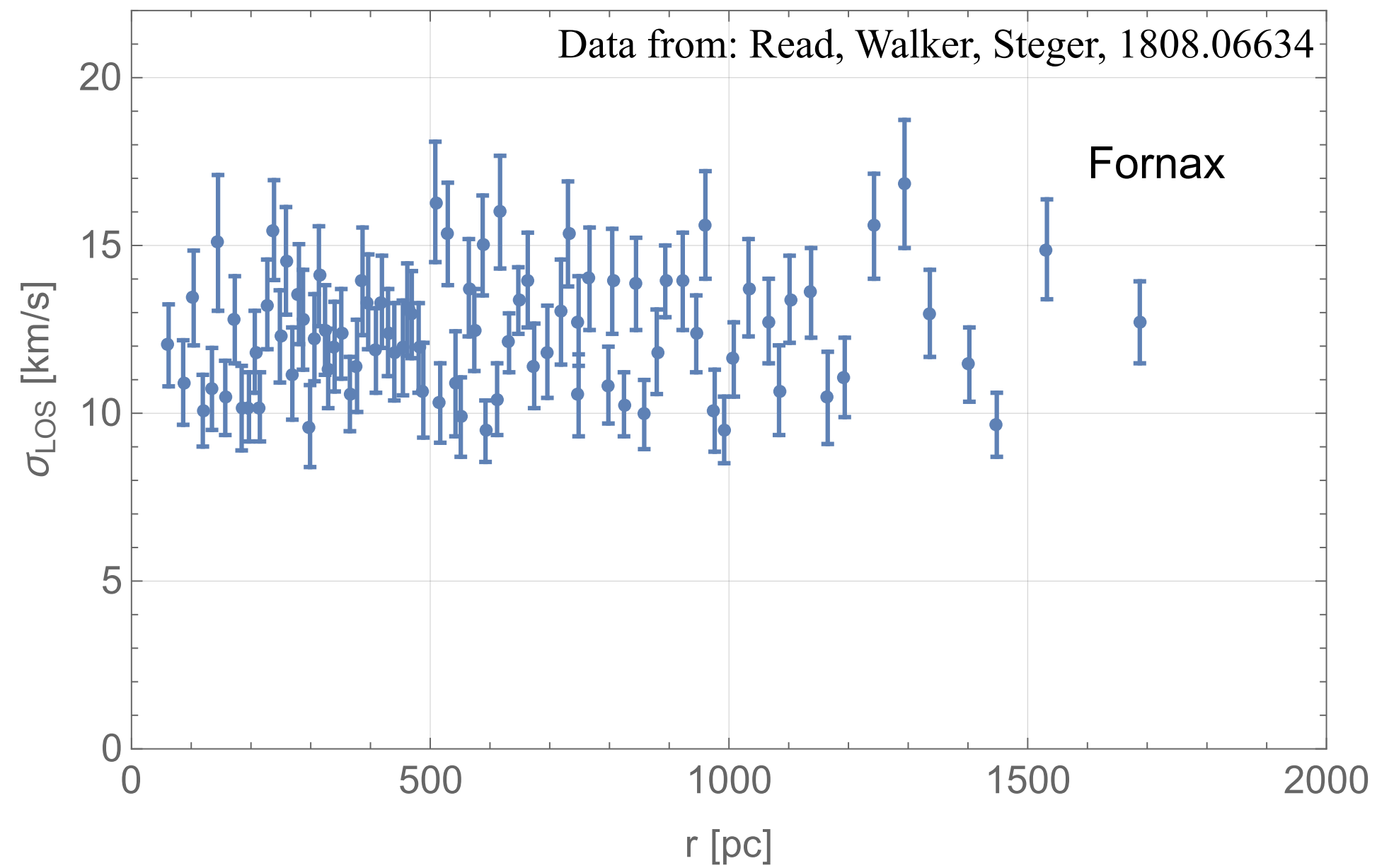
1. Line-of-sight velocities
2. Column density of stars



$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}_i \frac{\partial f}{\partial x_i} + \dot{p}_i \frac{\partial f}{\partial p_i} = 0$$



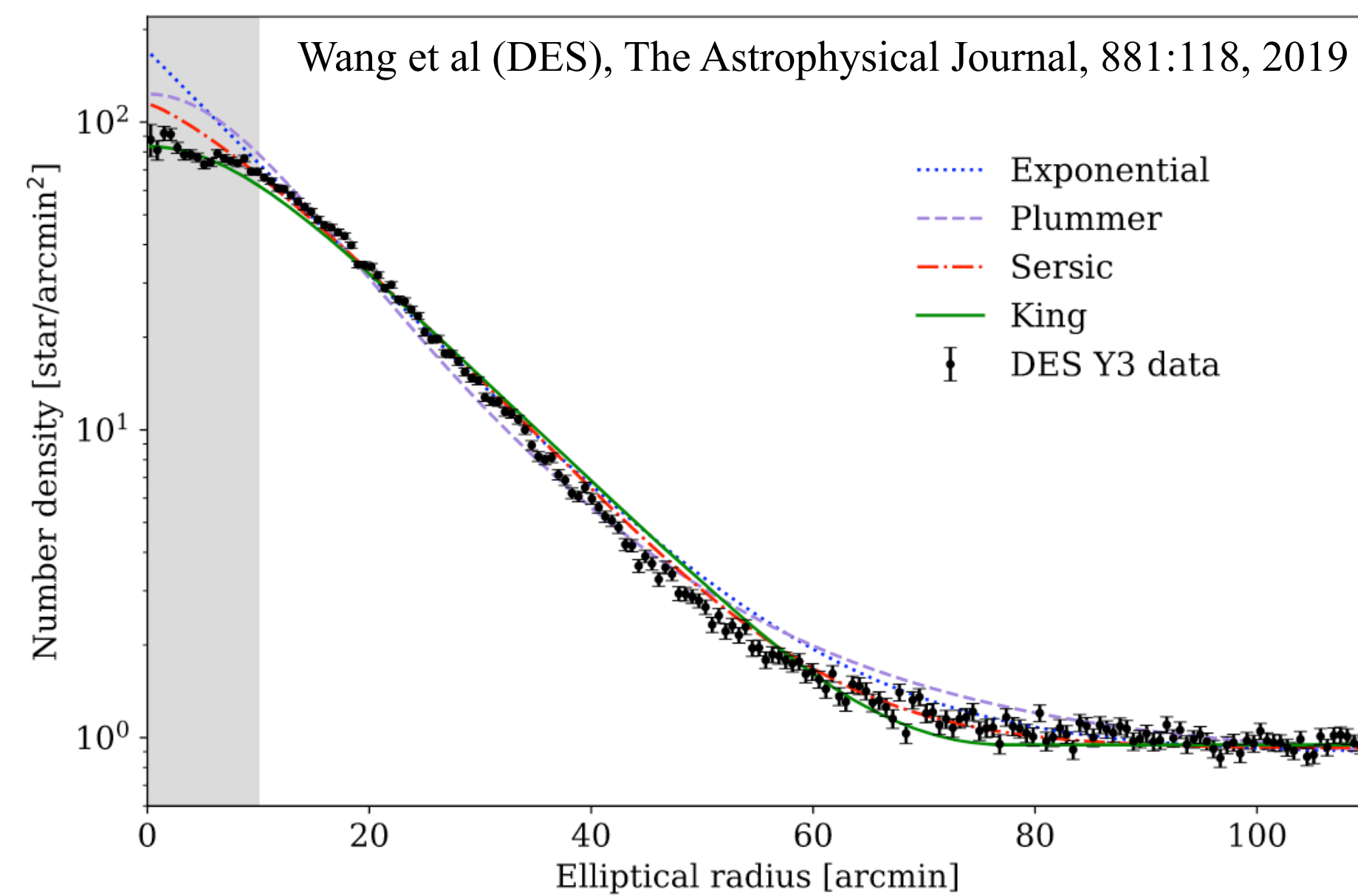
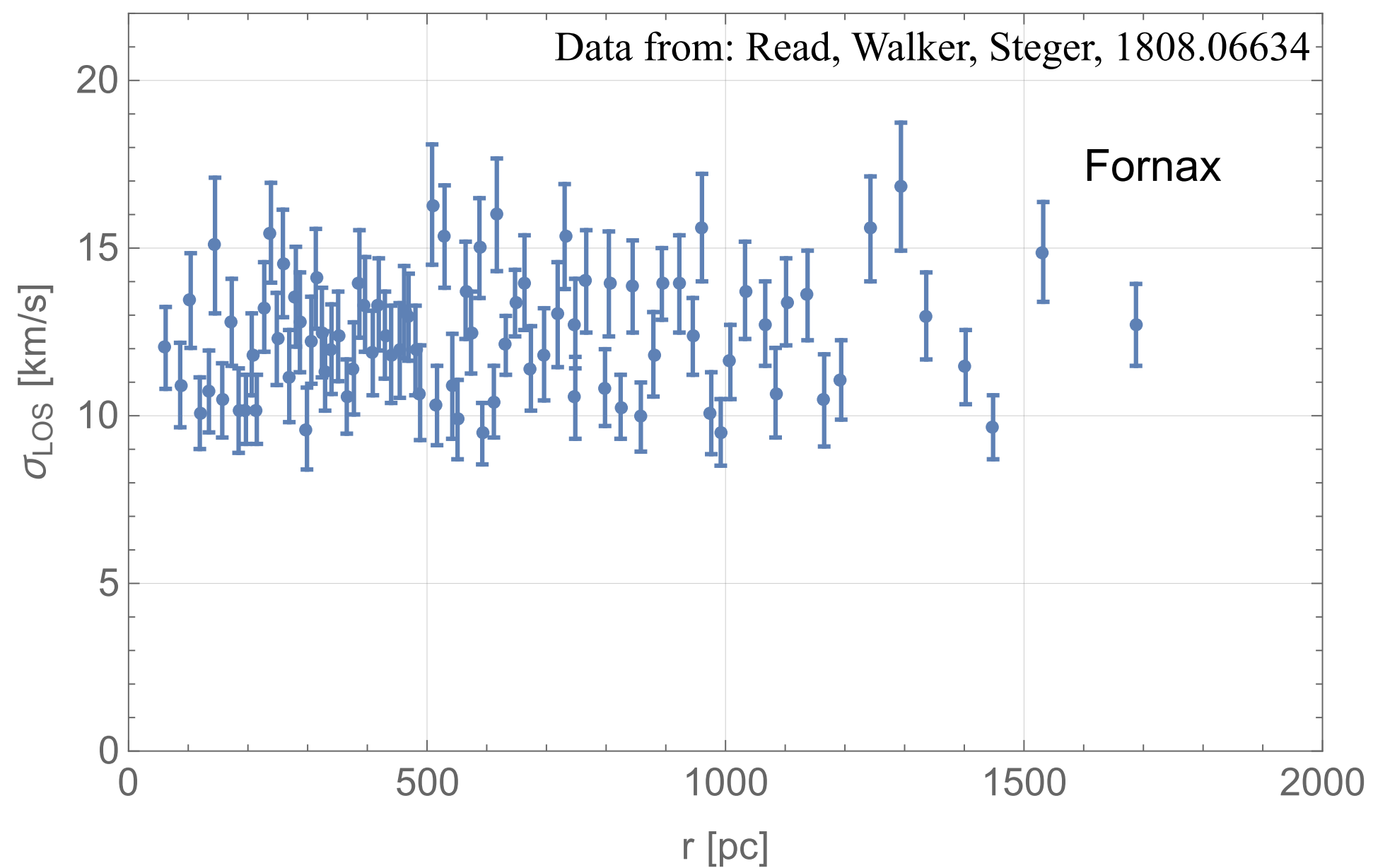
$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}_i \frac{\partial f}{\partial x_i} + \dot{p}_i \frac{\partial f}{\partial p_i} = \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} = 0$$



$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}_i \frac{\partial f}{\partial x_i} + \dot{p}_i \frac{\partial f}{\partial p_i} = \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} = 0$$

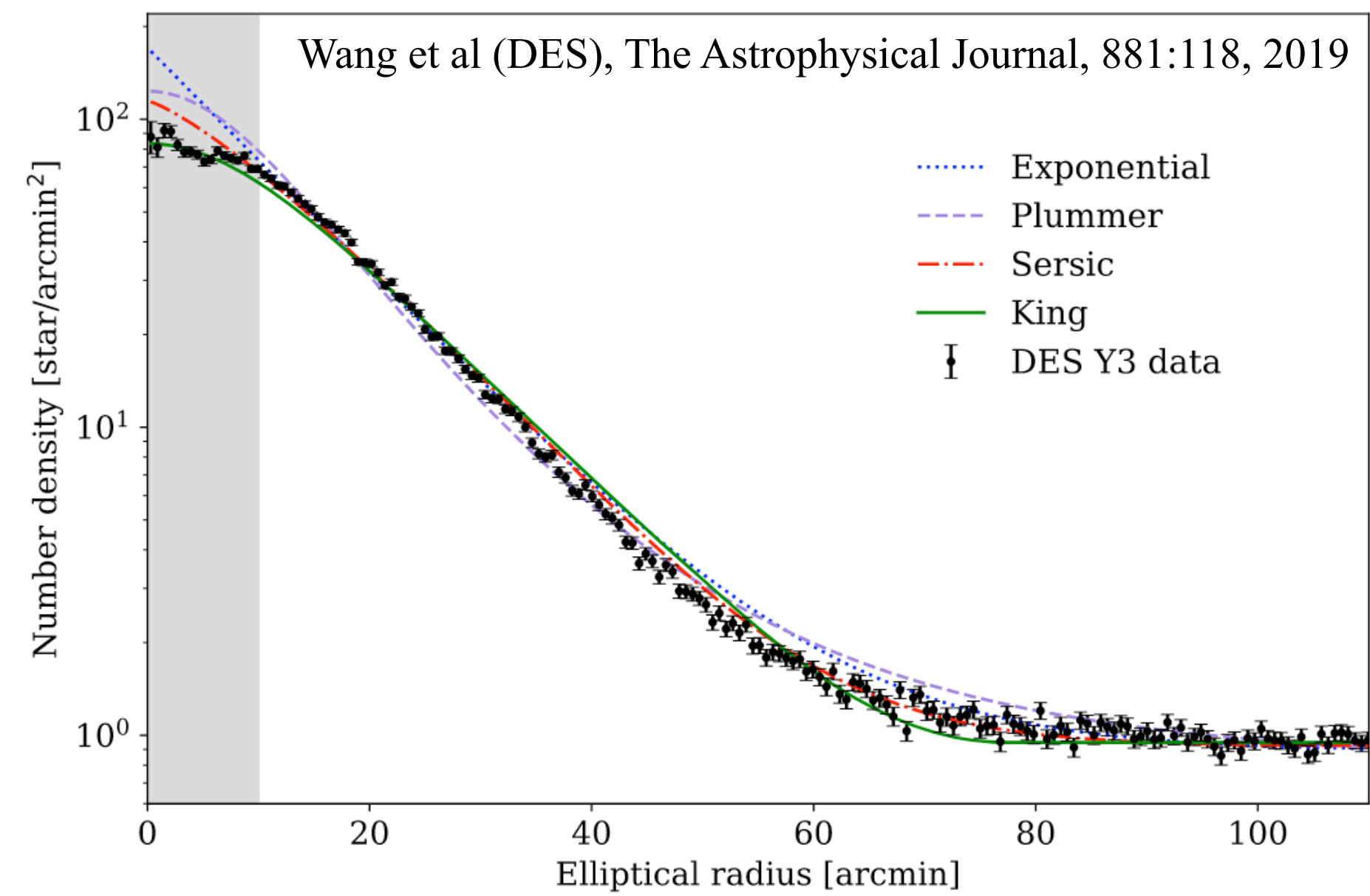
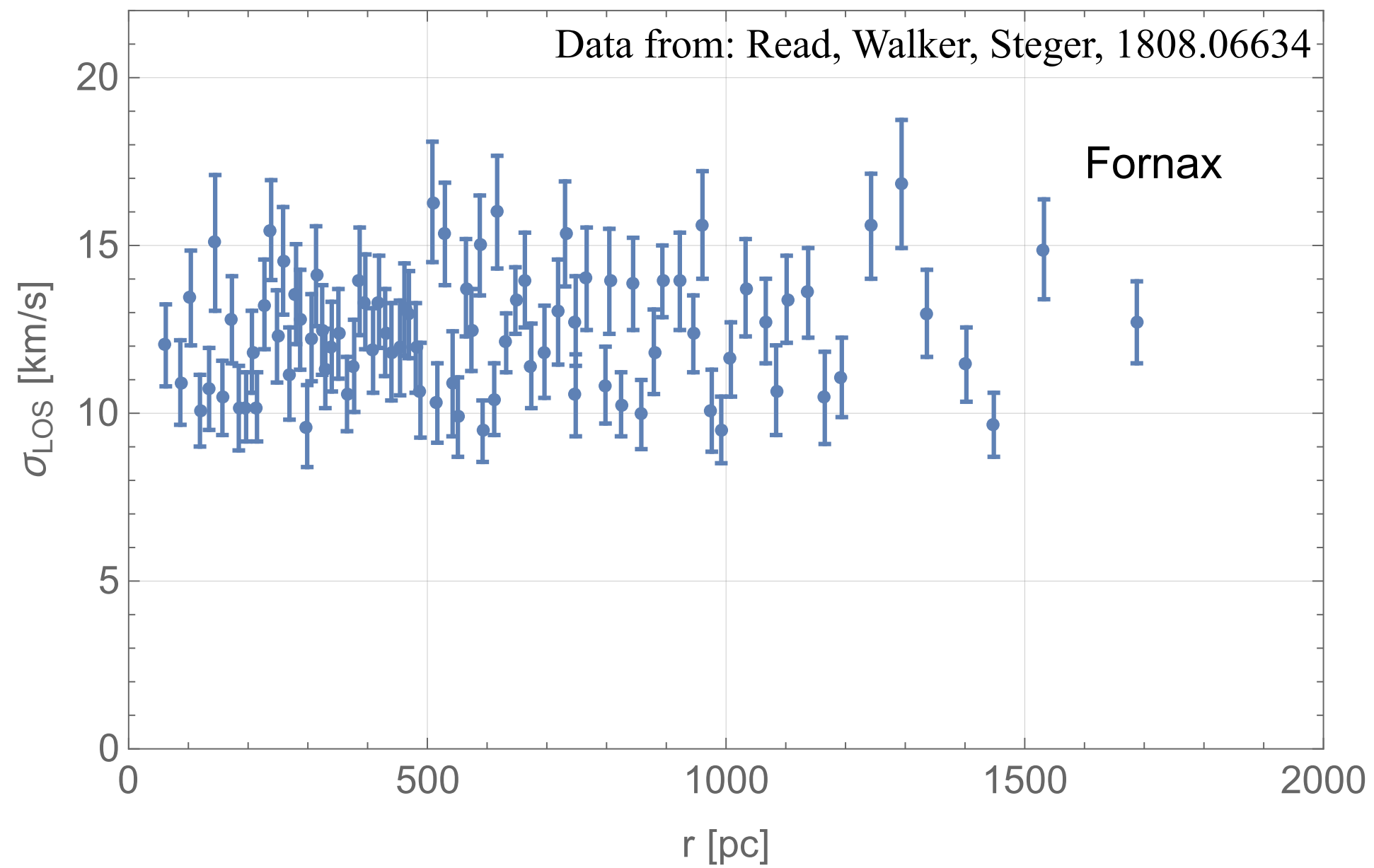
$$\int d^3p v_j \left(\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} \right) = \frac{\partial}{\partial t} (n \bar{v}_j) + \frac{\partial}{\partial x_i} (n \bar{v}_i \bar{v}_j) + n \frac{\partial \Phi}{\partial x_j} = 0$$

$$n \bar{v}_i \bar{v}_j(\mathbf{x}) = \int d^3p v_i v_j f(\mathbf{x}, \mathbf{v})$$



Assume steady state

$$\cancel{\frac{\partial}{\partial t} (n\bar{v}_j)} + \frac{\partial}{\partial x_i} (n\bar{v}_i\bar{v}_j) + n\frac{\partial\Phi}{\partial x_j} = 0$$



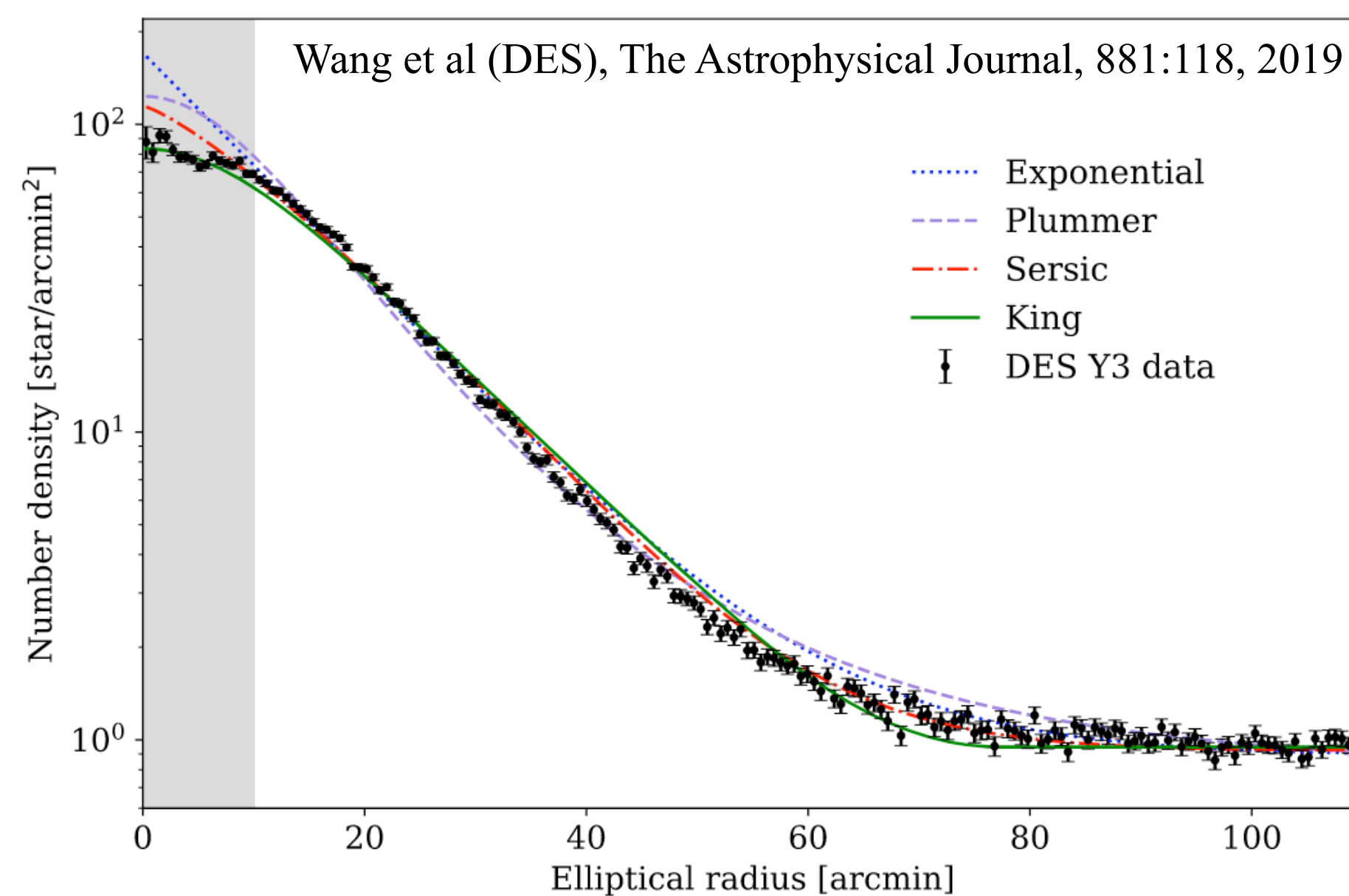
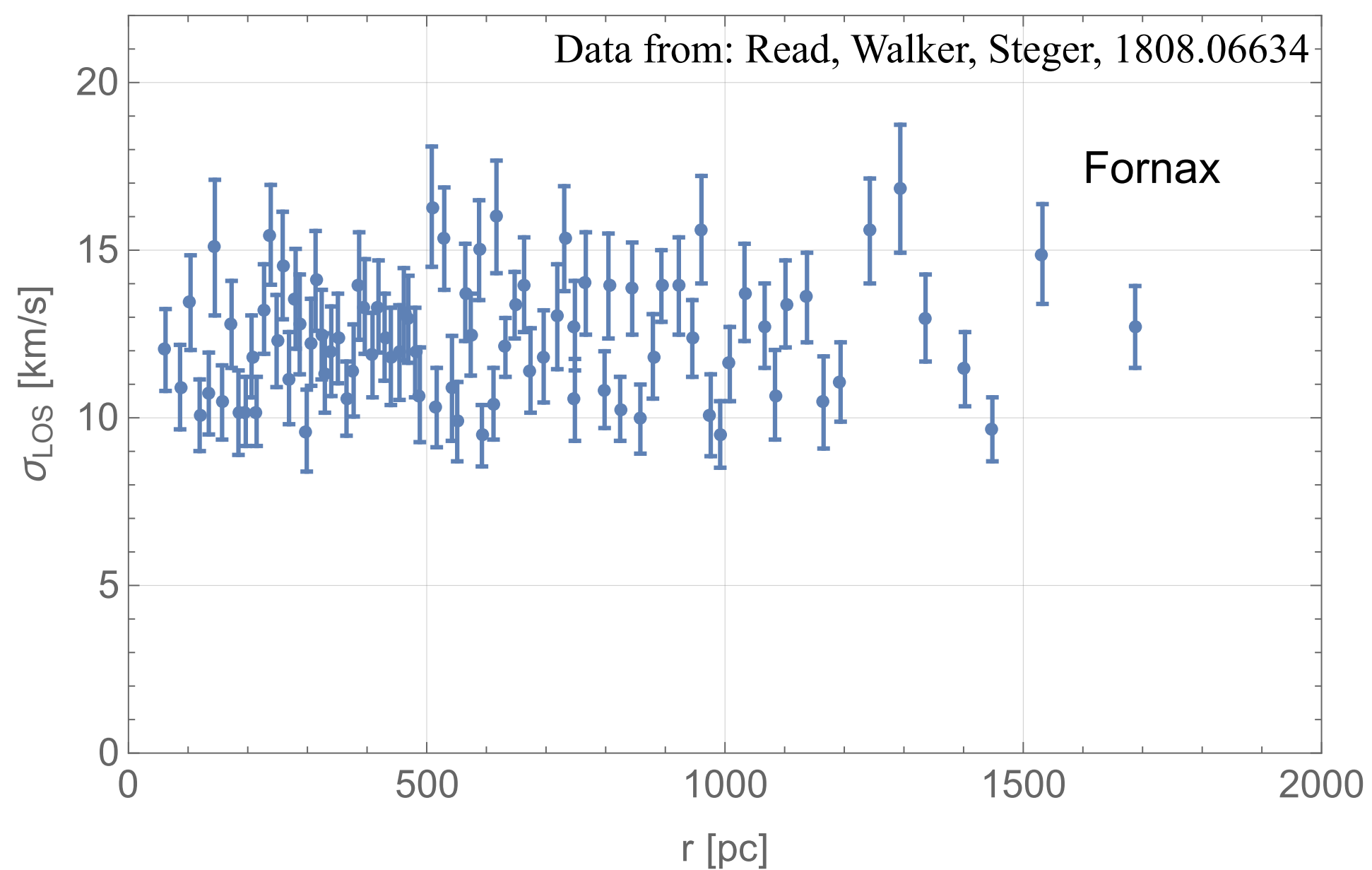
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$$\cancel{\frac{\partial}{\partial t} (n\bar{v}_j)} + \frac{\partial}{\partial x_i} (n\bar{v}_i\bar{v}_j) + n\frac{\partial\Phi}{\partial x_j} = 0$$

Spherical symmetry

$$\frac{\partial}{\partial x_i} (n\bar{v}_i\bar{v}_j) \rightarrow \frac{\partial}{\partial r} (n\bar{v}_r^2) + n\frac{2\bar{v}_r^2 - \bar{v}_t^2}{r} = \frac{\partial}{\partial r} (n\bar{v}_r^2) + \frac{2n\beta\bar{v}_r^2}{r}$$

$$\frac{\partial\Phi}{\partial x_j} \rightarrow \frac{GM(r)}{r^2} \quad \beta = 1 - \frac{\bar{v}_t^2}{2\bar{v}_r^2}$$



Assume steady state

$$\cancel{\frac{\partial}{\partial t} (n\bar{v}_j)} + \frac{\partial}{\partial x_i} (n\bar{v}_i\bar{v}_j) + n\frac{\partial\Phi}{\partial x_j} = 0$$

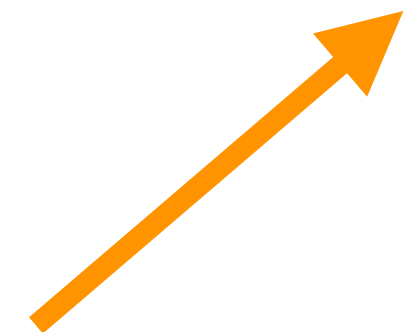


Jeans equation

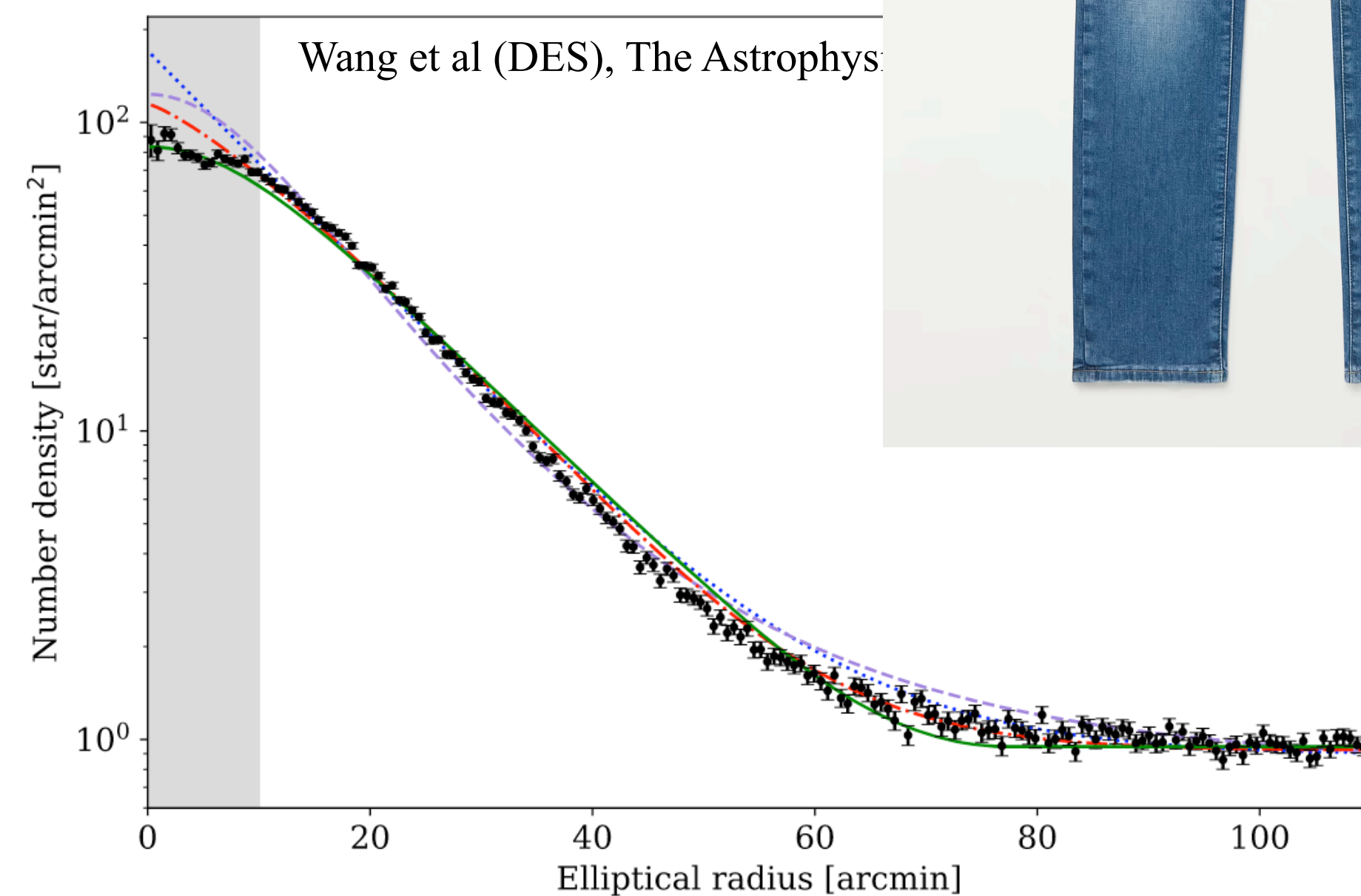
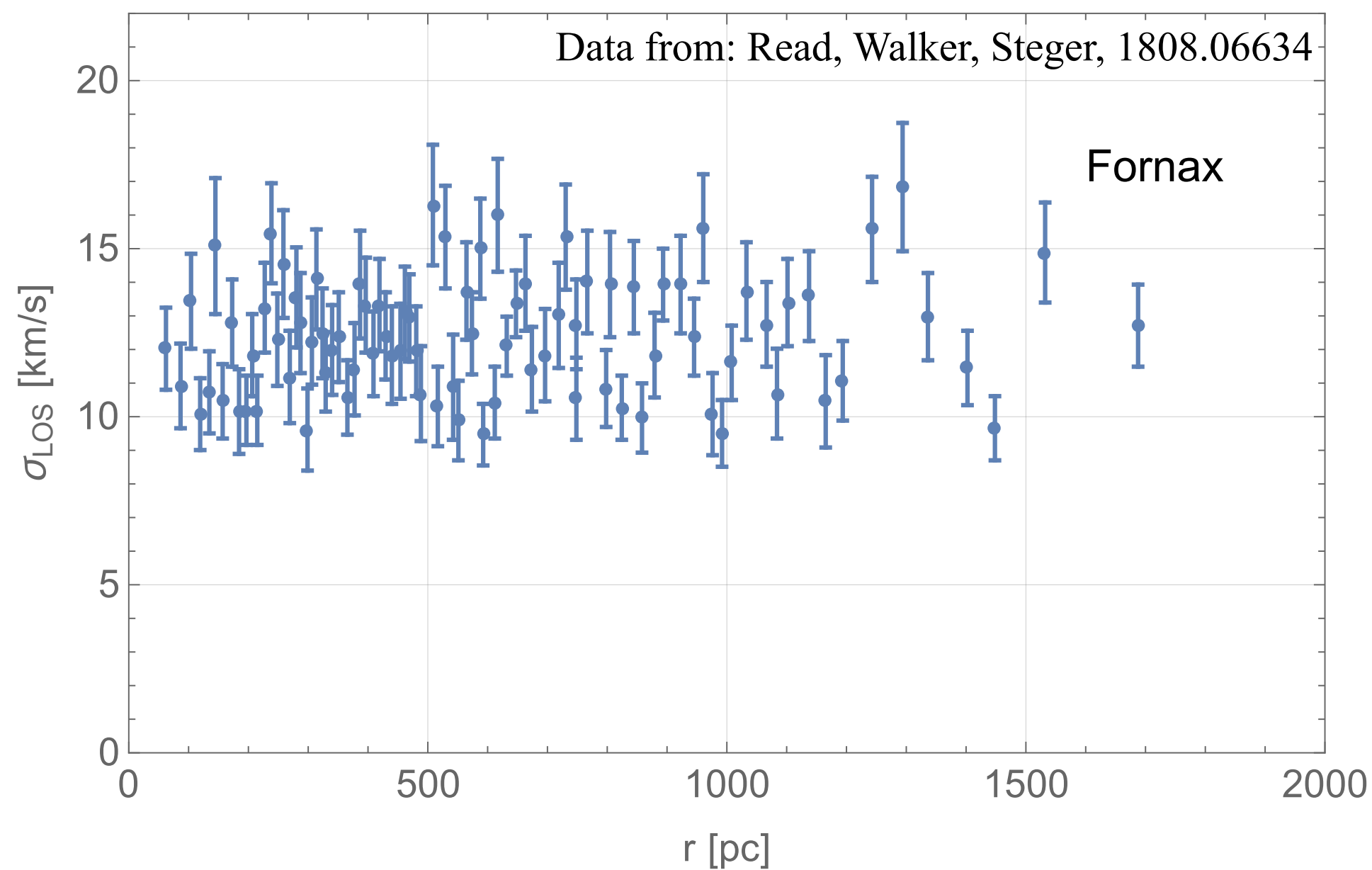
$$\frac{1}{n} \frac{d}{dr} (n\bar{v}_r^2) + \frac{2\beta}{r} \bar{v}_r^2 = -\frac{GM(r)}{r^2}$$

Spherical symmetry

$$\frac{\partial}{\partial x_i} (n\bar{v}_i\bar{v}_j) \rightarrow \frac{\partial}{\partial r} (n\bar{v}_r^2) + n\frac{2\bar{v}_r^2 - \bar{v}_t^2}{r} = \frac{\partial}{\partial r} (n\bar{v}_r^2) + \frac{2n\beta\bar{v}_r^2}{r}$$



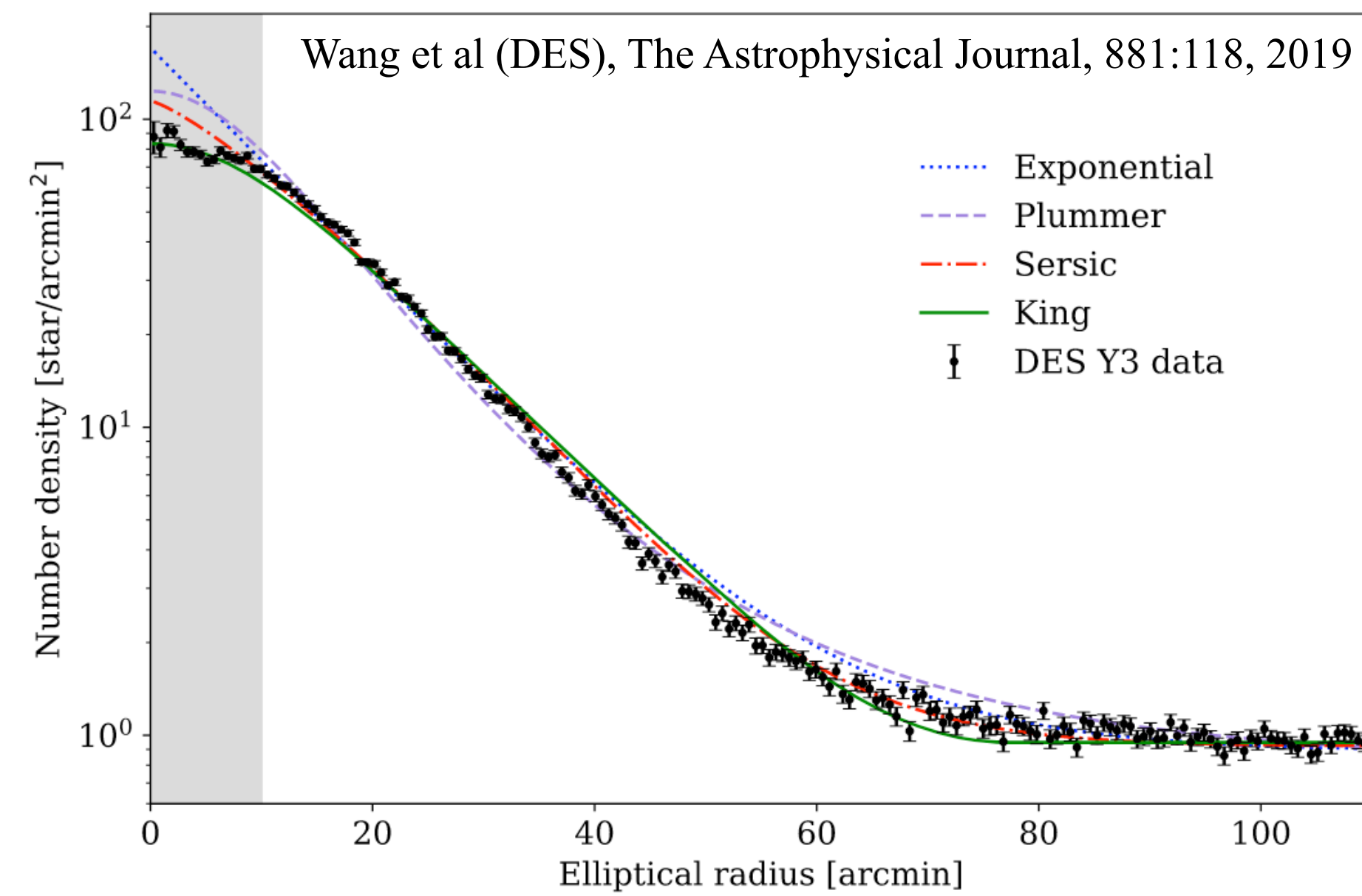
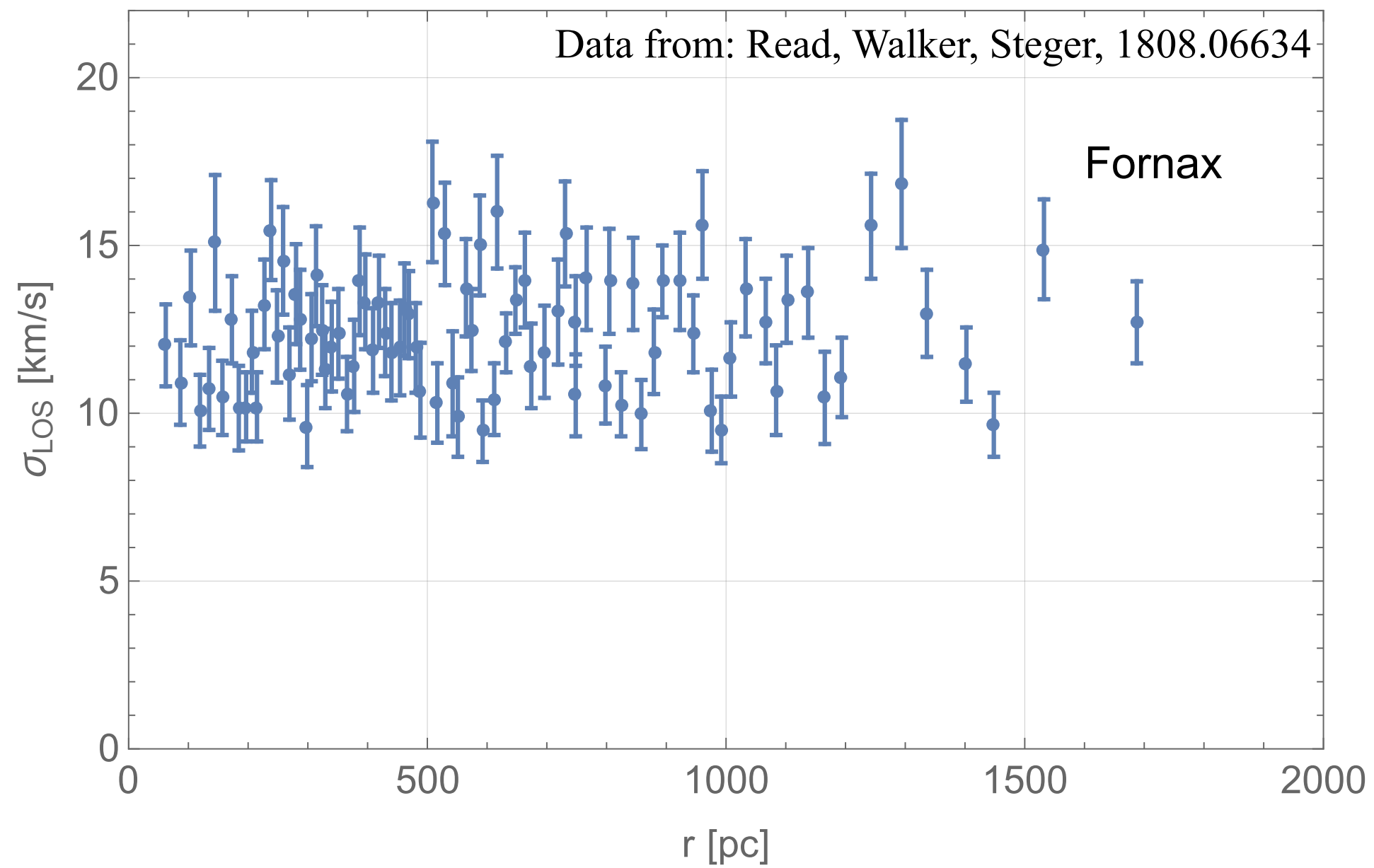
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$$\frac{1}{n} \frac{d}{dr} \left(n \overline{v_r^2} \right) + \frac{2\beta}{r} \overline{v_r^2} = - \frac{GM(r)}{r^2}$$

If β is constant in r :

$$n \overline{v_r^2} = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y)$$



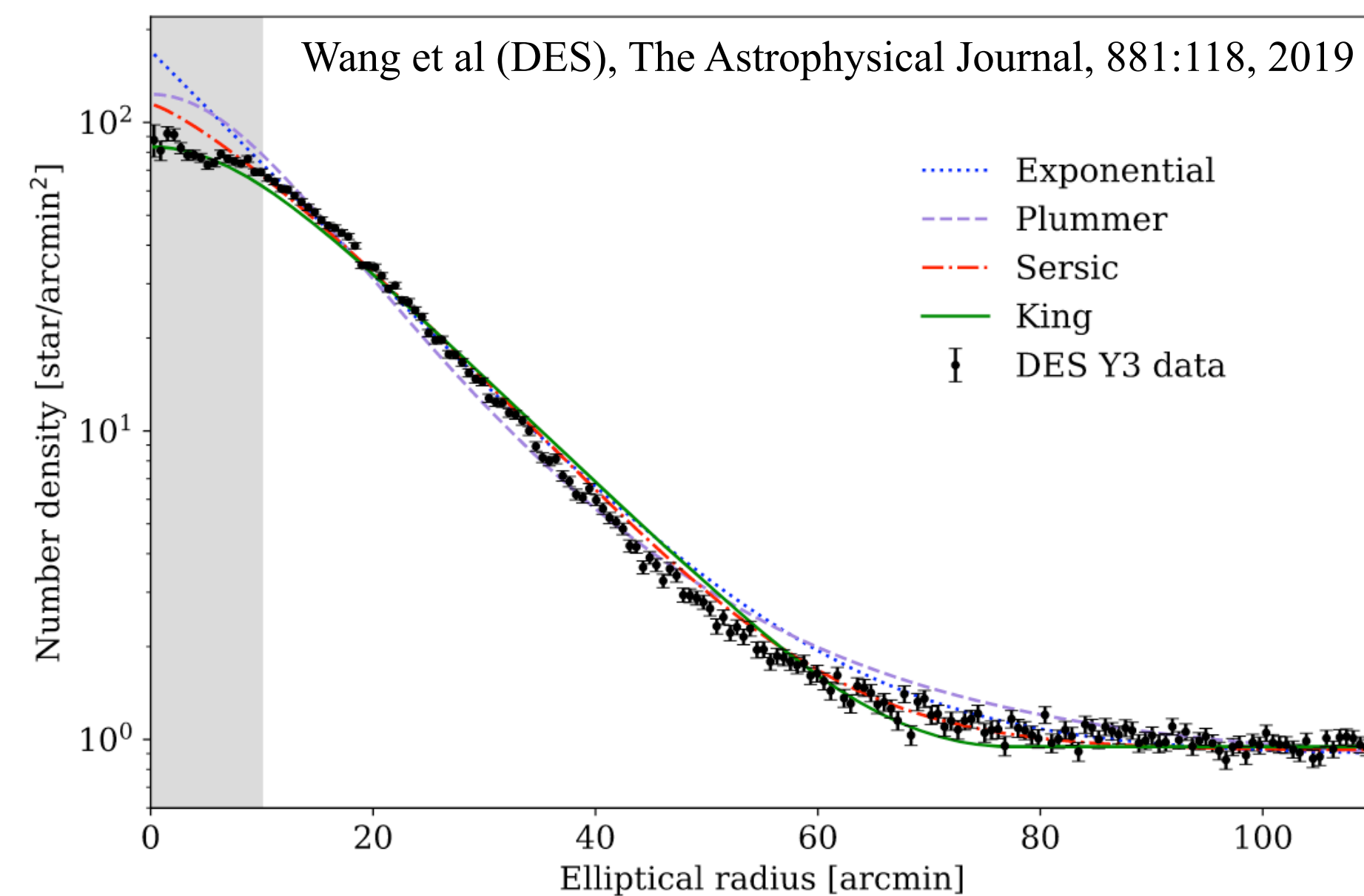
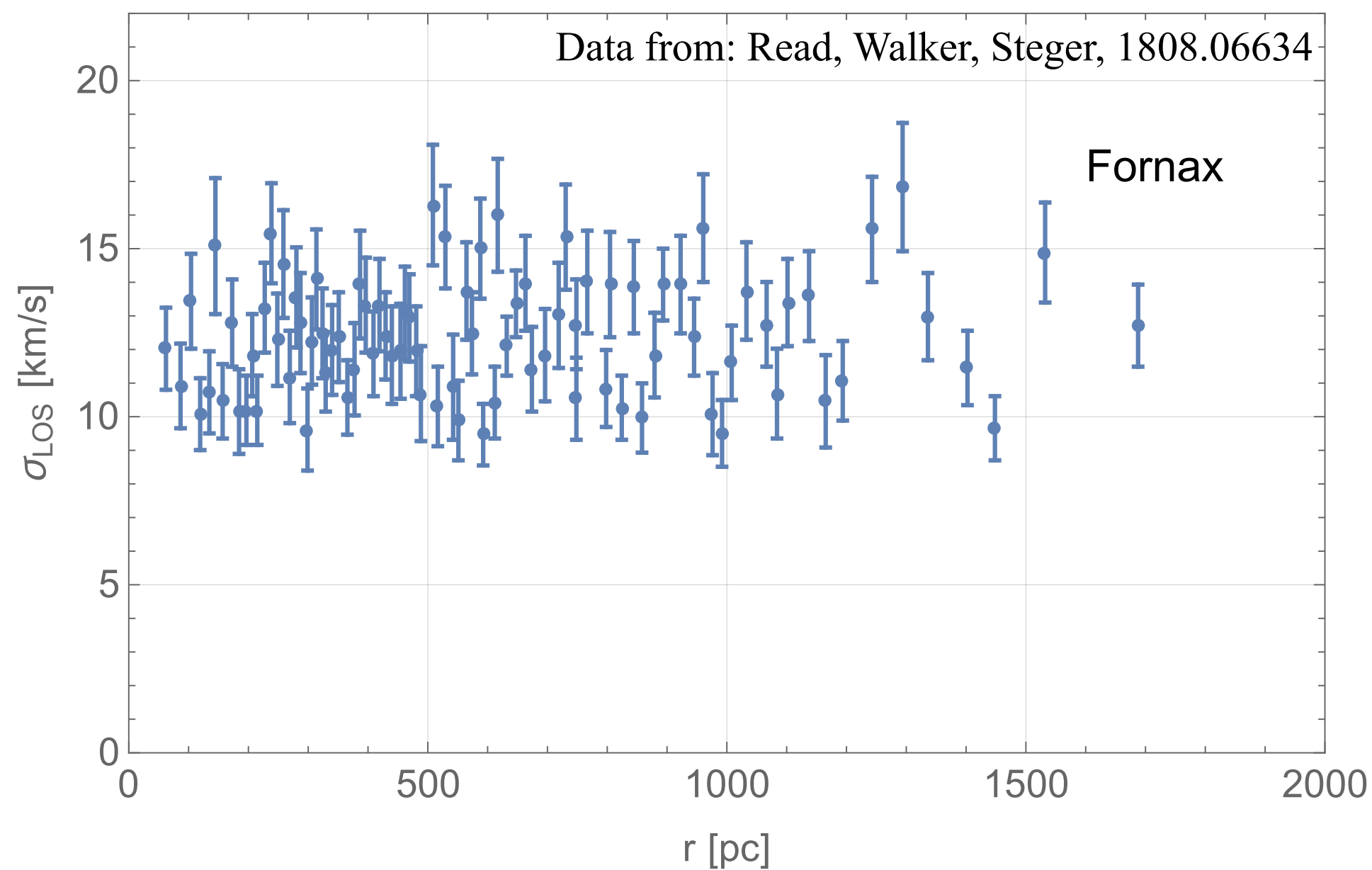
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If β is constant in r :

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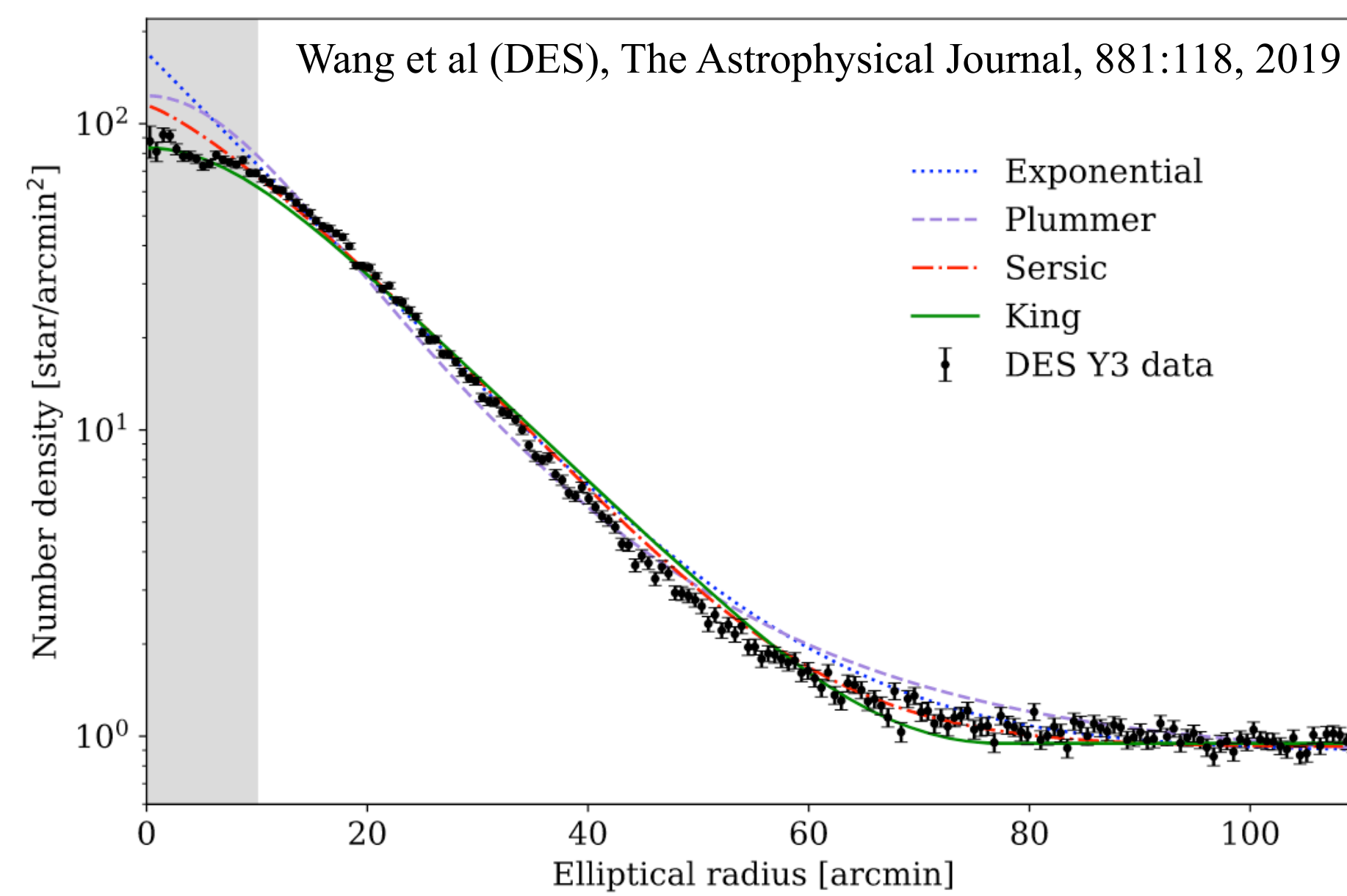
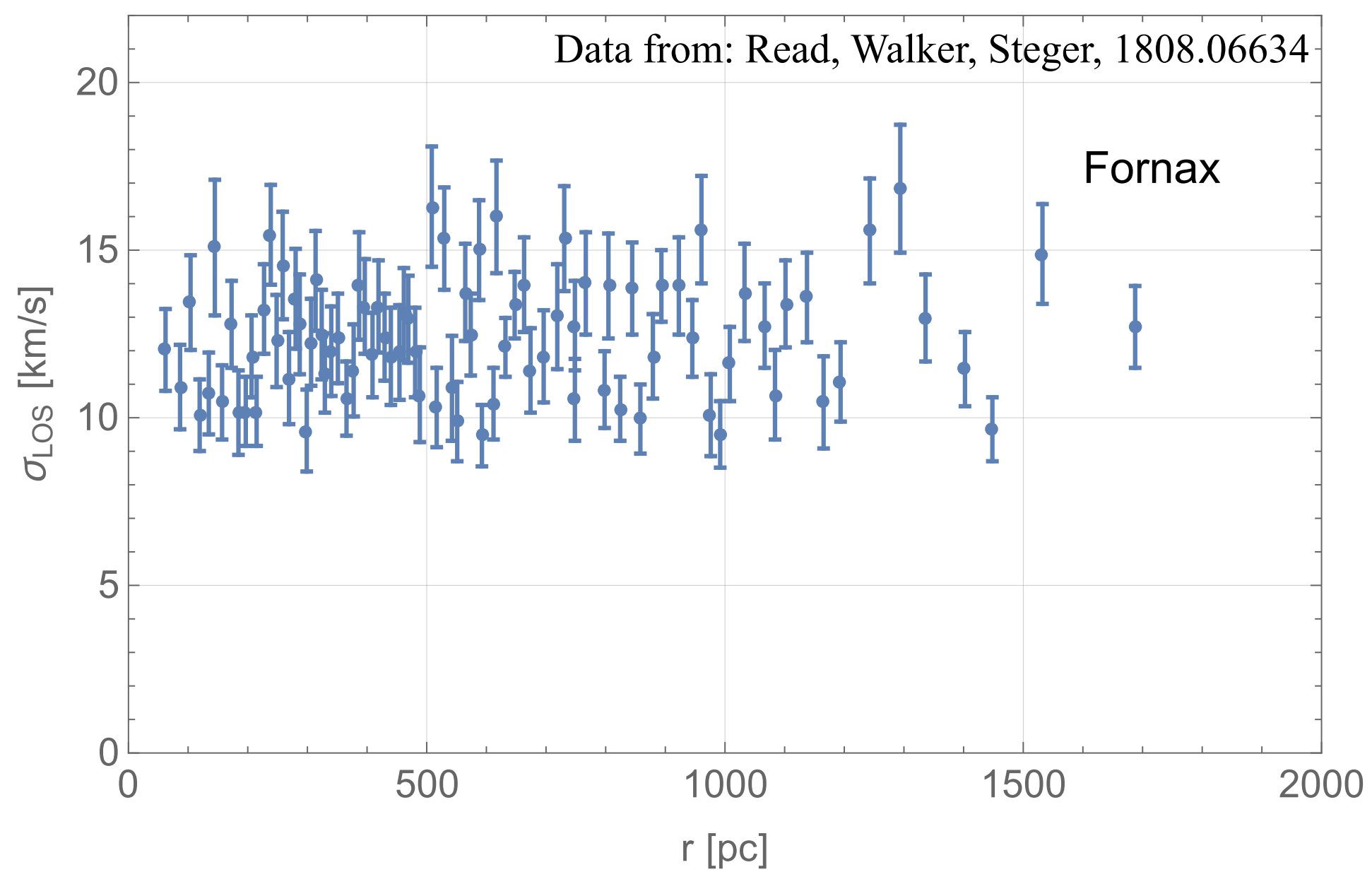
What is actually observed — line-of-sight velocity:

$$\sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left(1 - \frac{\beta r^2}{y^2} \right) \frac{y \overline{nv_r^2}(y)}{\sqrt{y^2 - r^2}}$$



$$n\overline{v_r^2} = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y)$$

$$\sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left(1 - \frac{\beta r^2}{y^2} \right) \frac{y n \overline{v_r^2}(y)}{\sqrt{y^2 - r^2}}$$



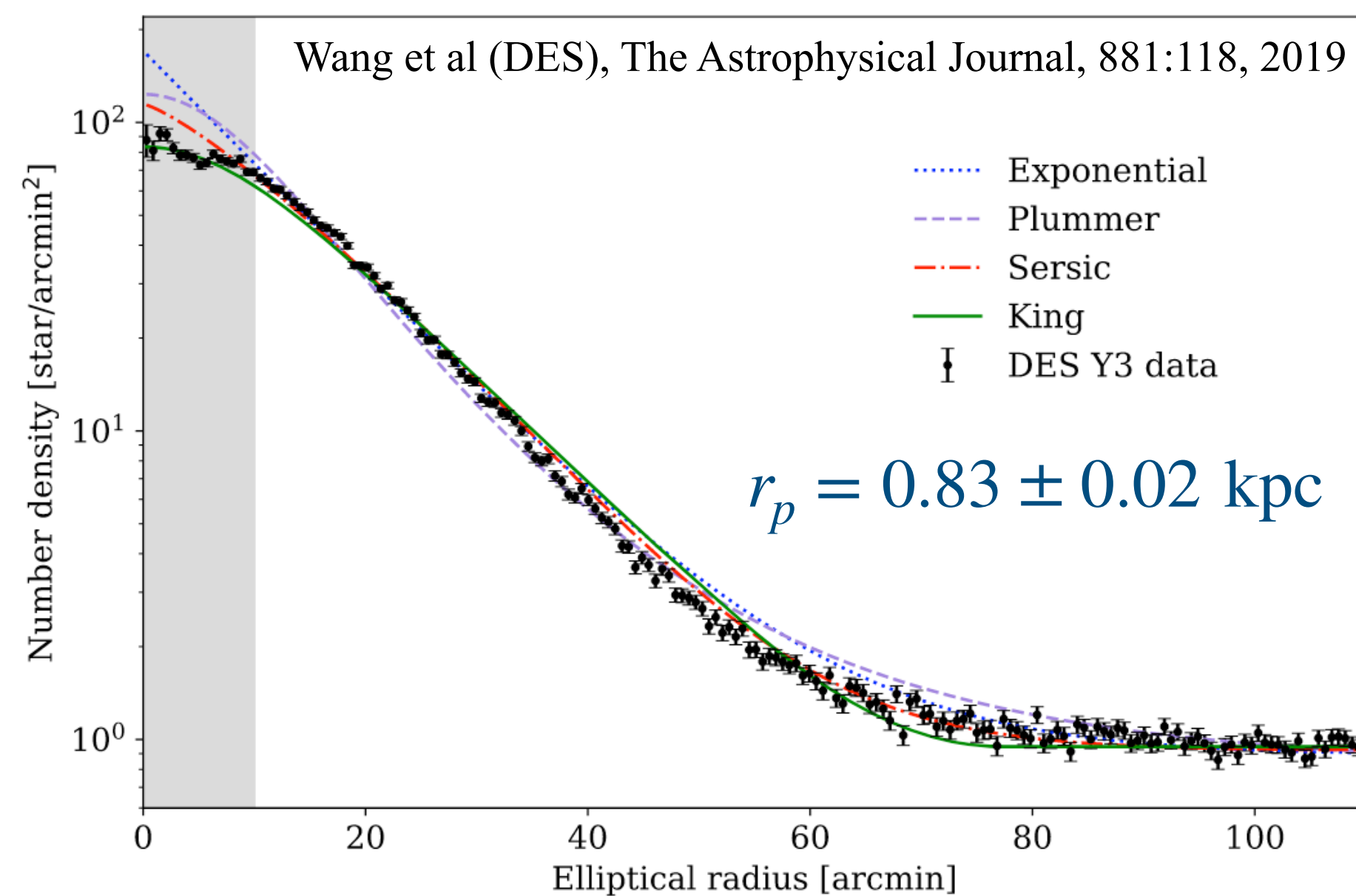
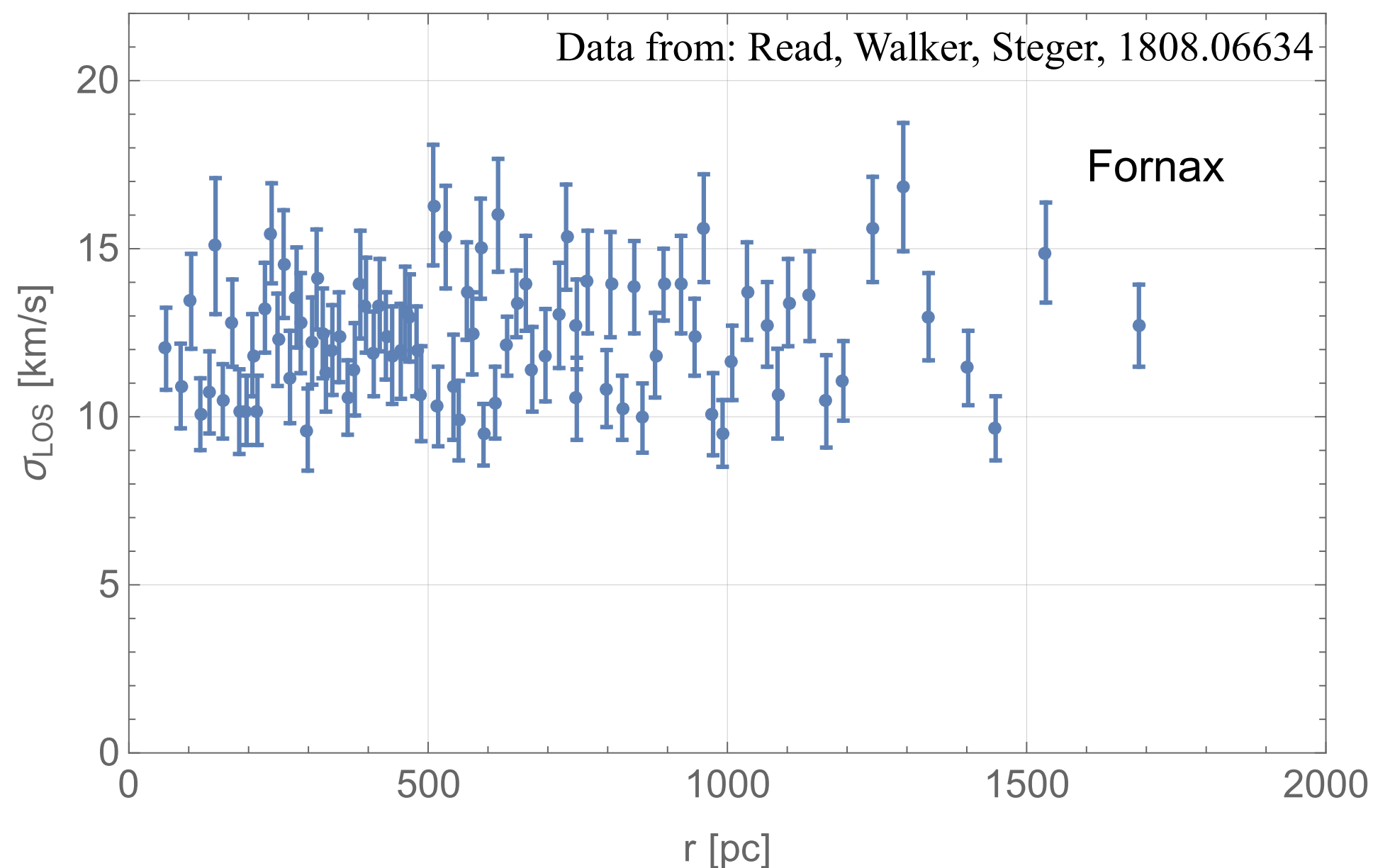
$$\overline{nv_r^2} = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y)$$

$$\sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left(1 - \frac{\beta r^2}{y^2} \right) \frac{y \overline{nv_r^2}(y)}{\sqrt{y^2 - r^2}}$$

E.g., Plummer profile:

$$I(r) = \frac{L}{\pi r_p^2 \left(1 + \frac{r^2}{r_p^2} \right)^2}$$

$$n(r) = \frac{3(L/l)}{4\pi r_p^3 \left(1 + \frac{r^2}{r_p^2} \right)^{\frac{5}{2}}}$$



$$\overline{nv_r^2} = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y)$$

$$\sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left(1 - \frac{\beta r^2}{y^2} \right) \frac{y \overline{nv_r^2}(y)}{\sqrt{y^2 - r^2}}$$

E.g., Navarro-Frenk-White (NFW):

$$\rho(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s} \right)^2}$$

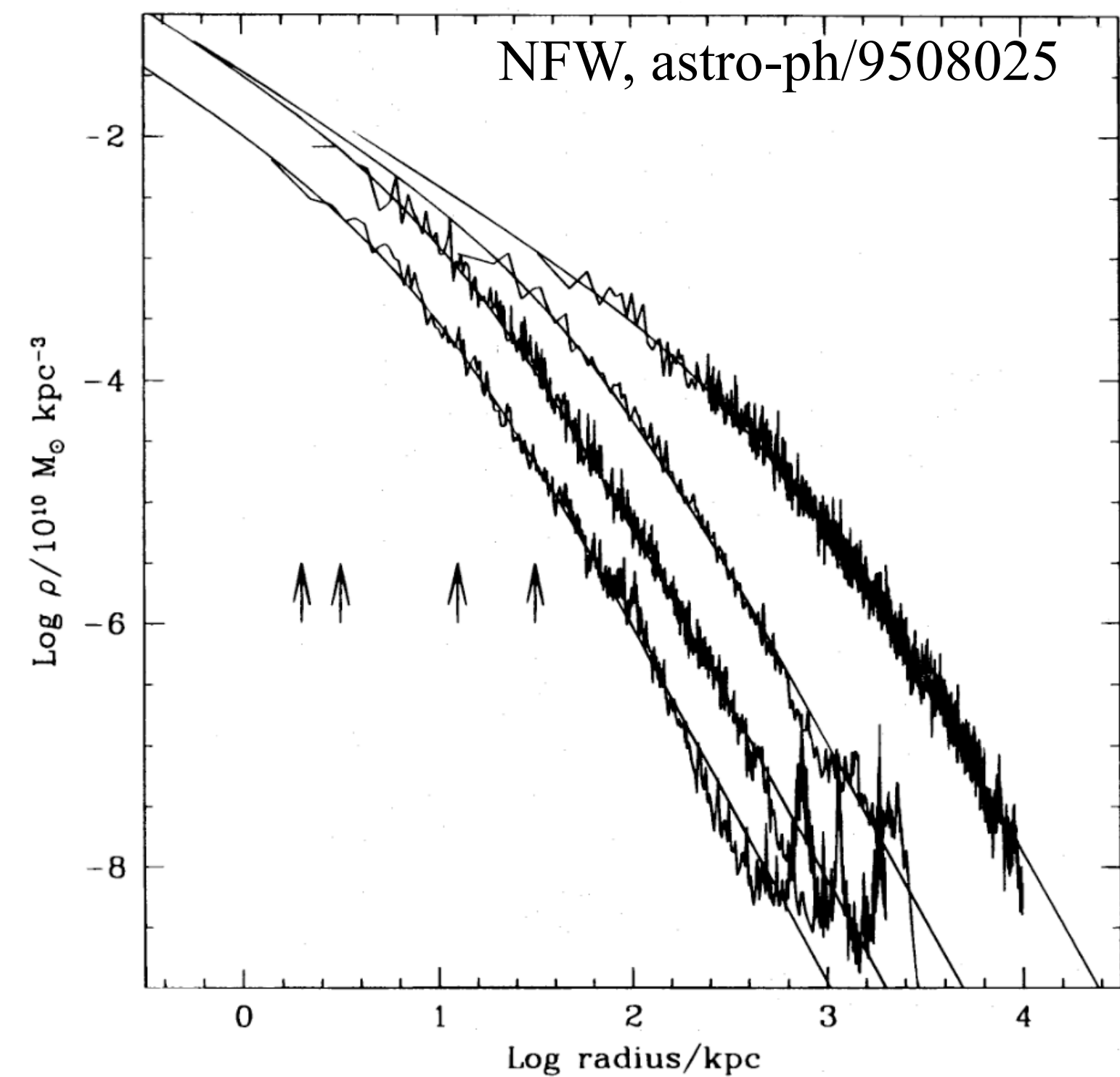
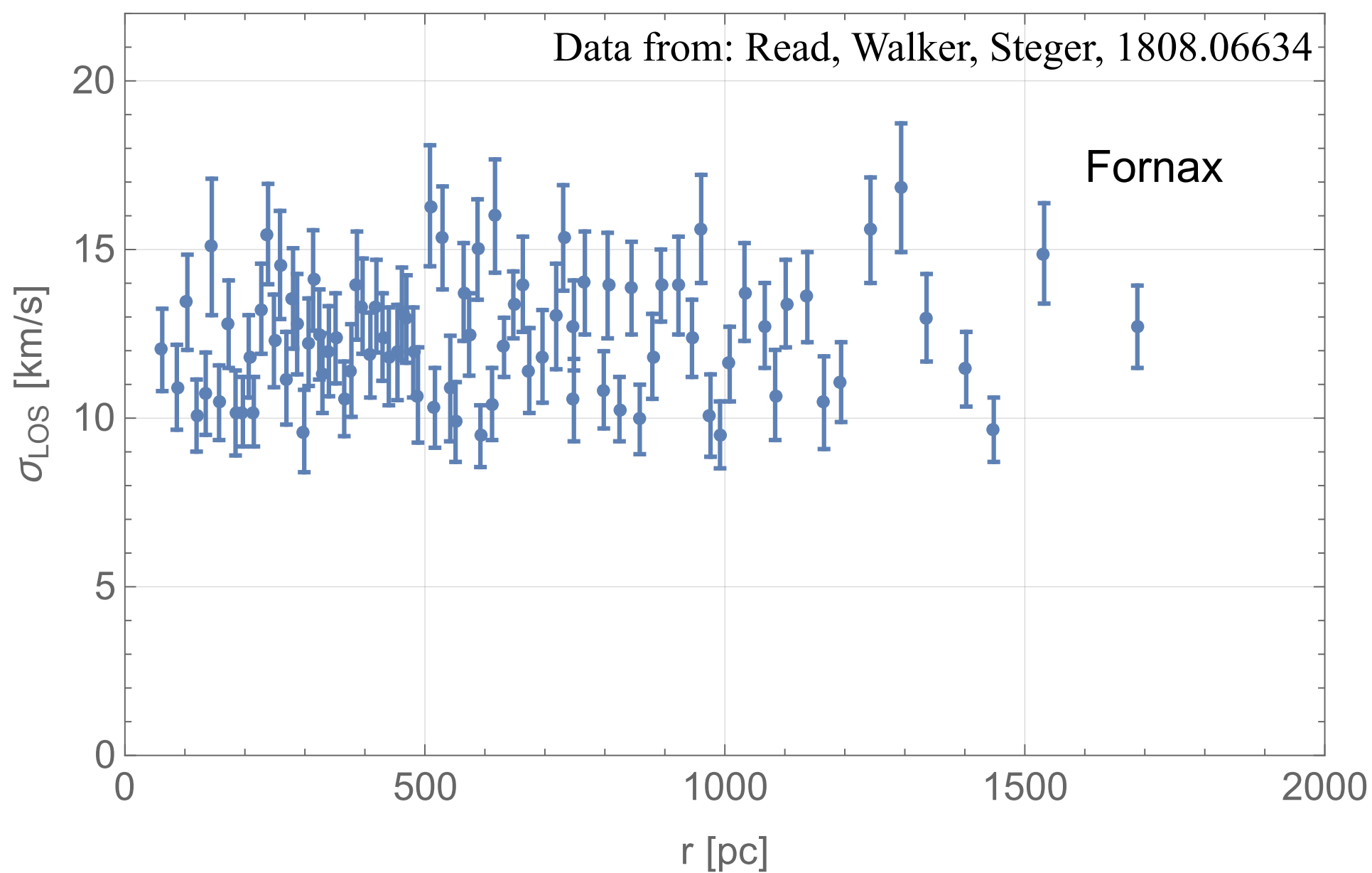


FIG. 3.—Density profiles of four halos spanning 4 orders of magnitude in mass. The arrows indicate the gravitational softening, h_g , of each simulation. Also shown are fits from eq. (3). The fits are good over two decades in radius, approximately from h_g out to the virial radius of each system.

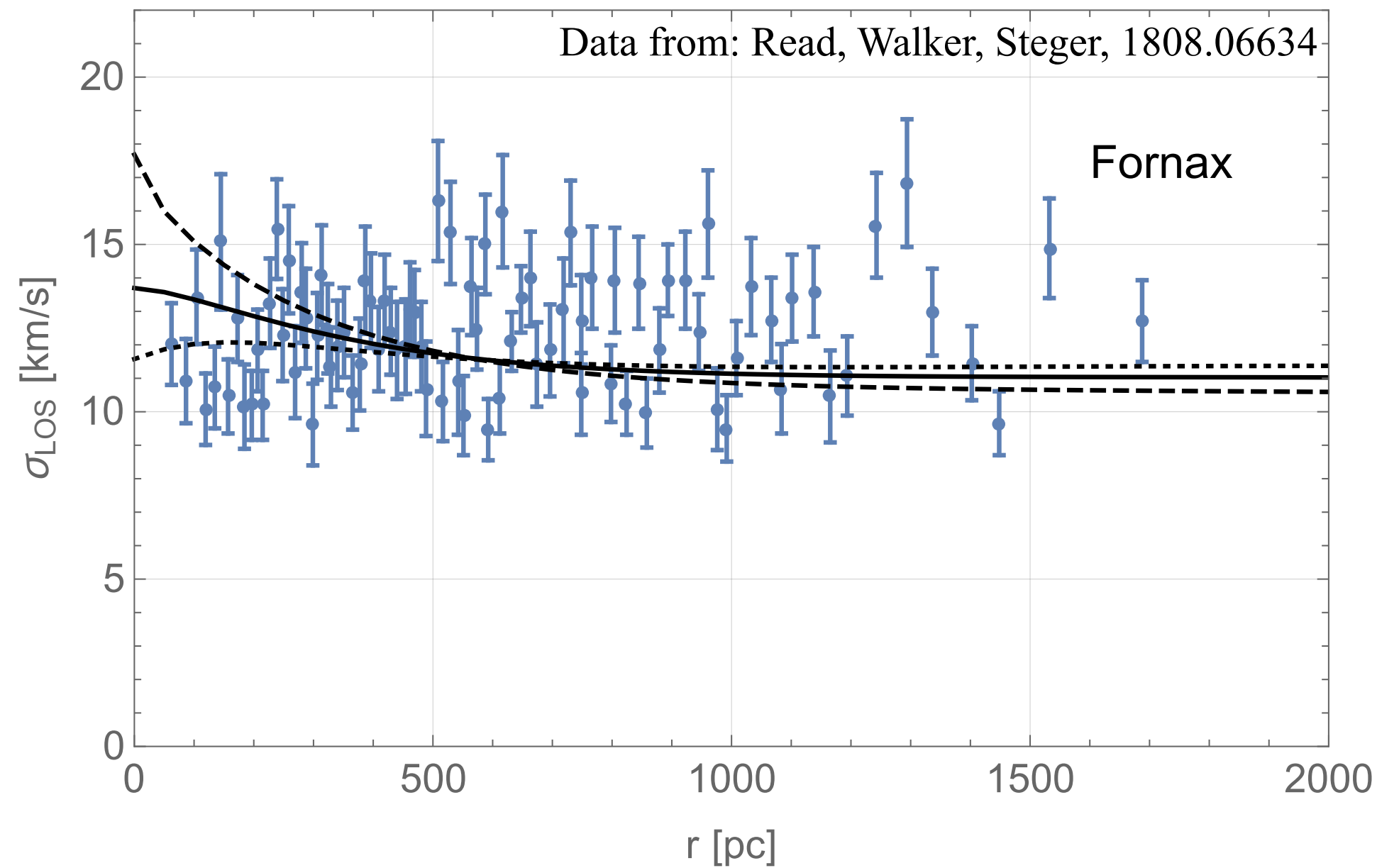
$$n\bar{v}_r^2 = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y)$$

$$\sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left(1 - \frac{\beta r^2}{y^2} \right) \frac{y n\bar{v}_r^2(y)}{\sqrt{y^2 - r^2}}$$

E.g., Navarro-Frenk-White (NFW):

$$\rho(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s} \right)^2} \quad r_s = 2 \text{ kpc}, \quad \rho_s = 1.2 \times 10^7 \frac{M_\odot}{\text{kpc}^3}$$

$$M(r) = 4\pi\rho_s r_s^3 \left(\ln \left(1 + \frac{r}{r_s} \right) - \frac{r}{r+r_s} \right)$$



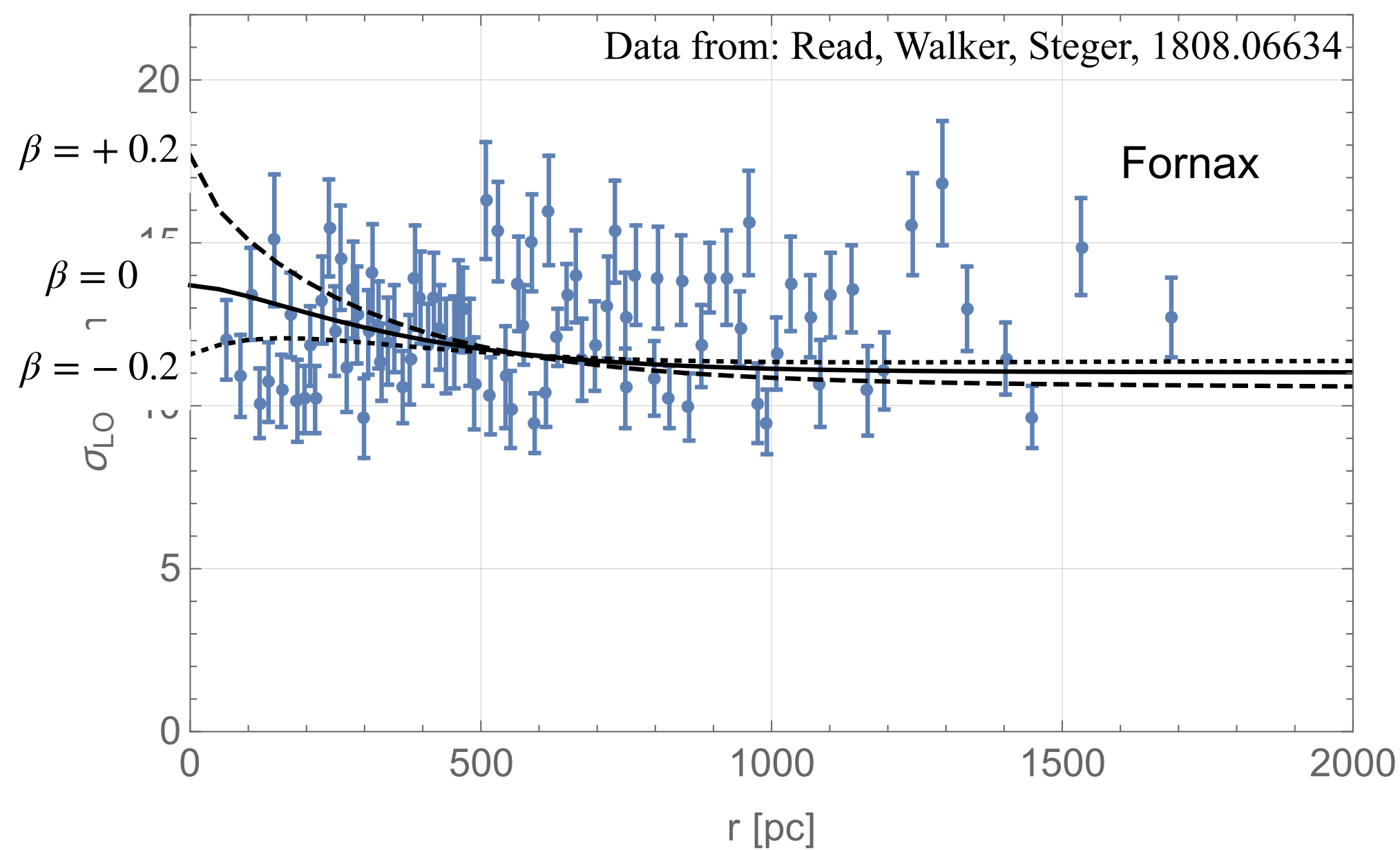
$$n\bar{v}_r^2 = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y)$$

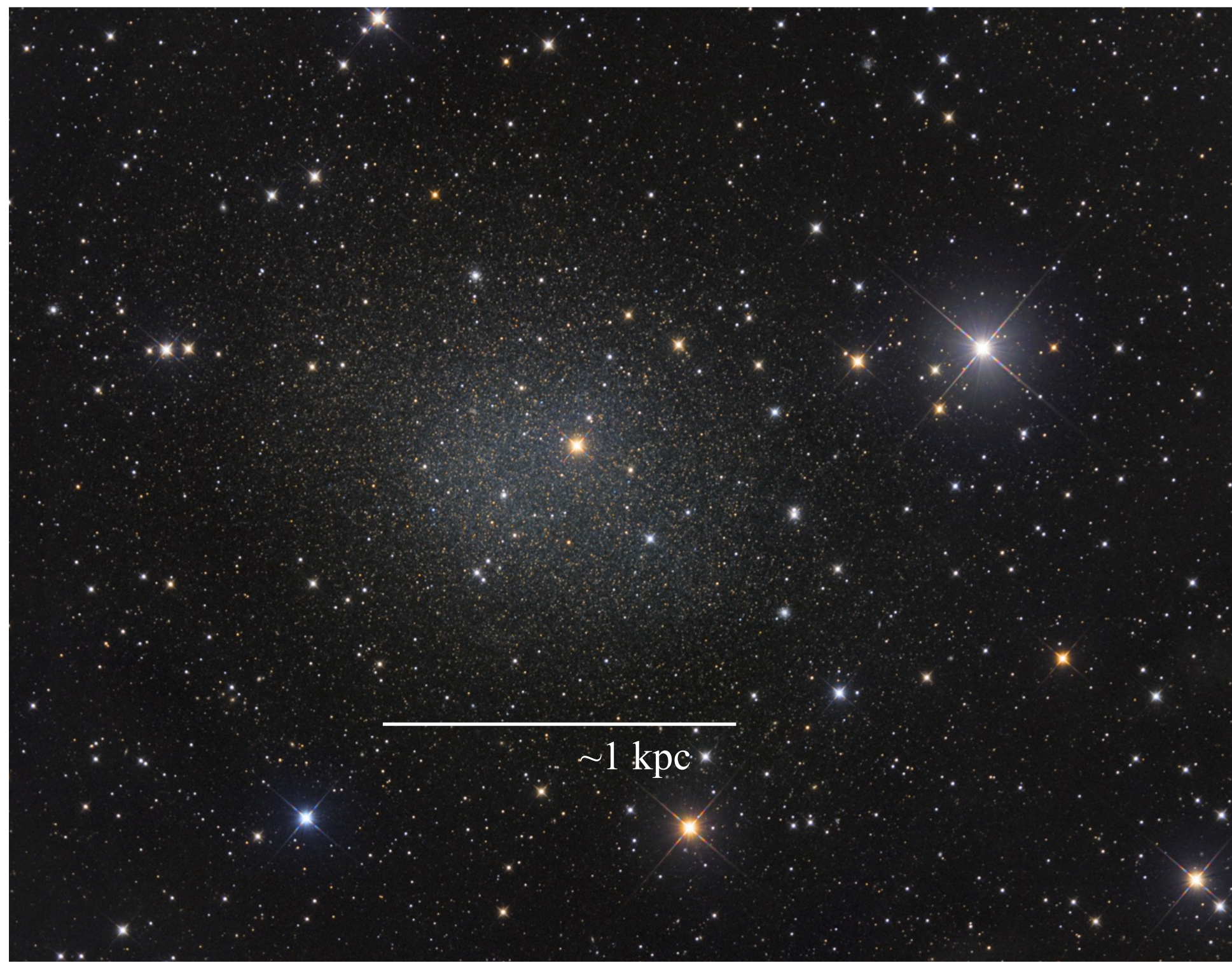
$$\sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left(1 - \frac{\beta r^2}{y^2} \right) \frac{y n\bar{v}_r^2(y)}{\sqrt{y^2 - r^2}}$$

E.g., Navarro-Frenk-White (NFW):

$$\rho(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s} \right)^2} \quad r_s = 2 \text{ kpc}, \quad \rho_s = 1.2 \times 10^7 \frac{M_\odot}{\text{kpc}^3}$$

$$M(r) = 4\pi\rho_s r_s^3 \left(\ln \left(1 + \frac{r}{r_s} \right) - \frac{r}{r+r_s} \right)$$





$$\overline{nv_r^2} = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y)$$

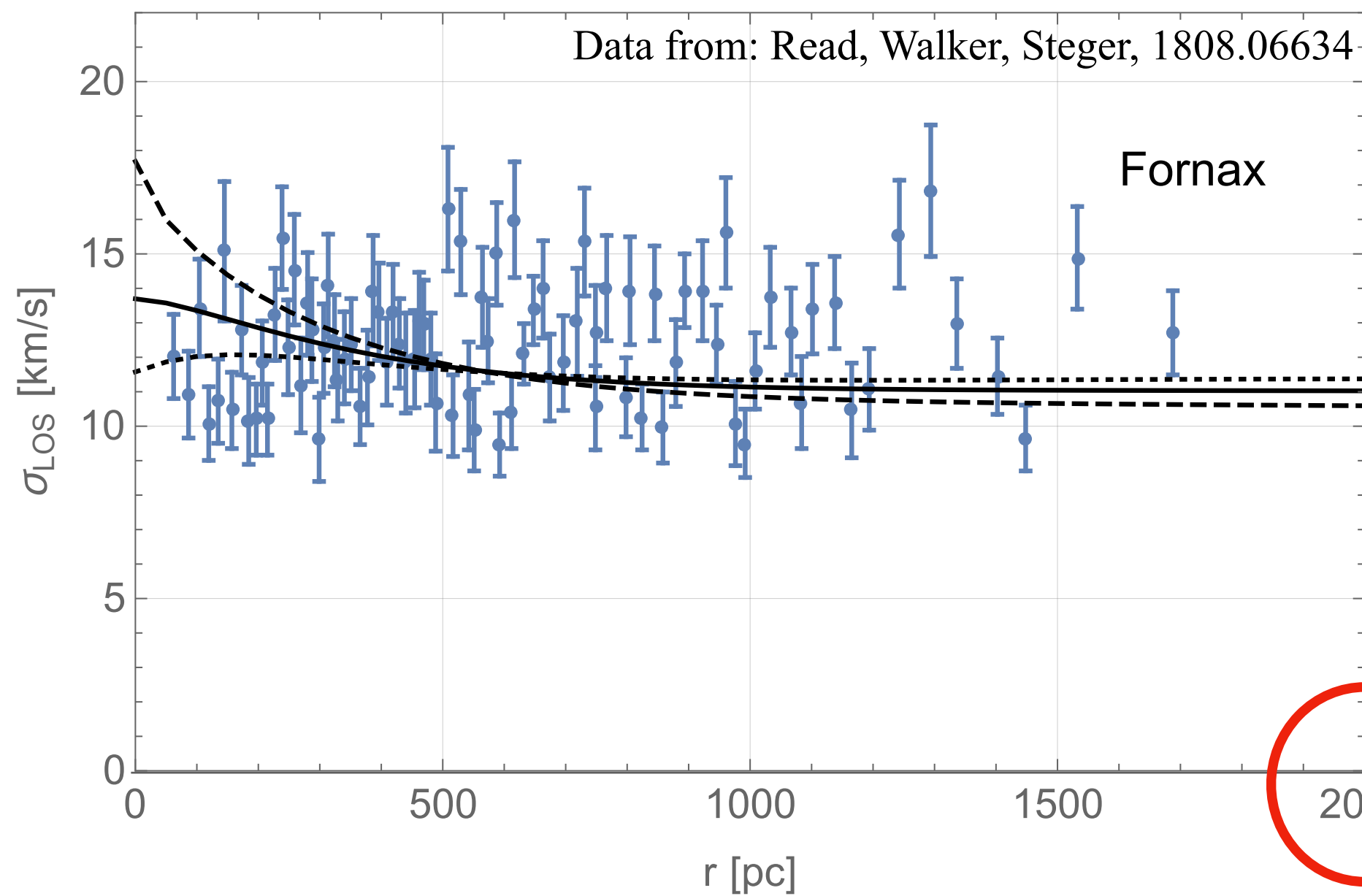
$$\sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left(1 - \frac{\beta r^2}{y^2} \right) \frac{y \overline{nv_r^2}(y)}{\sqrt{y^2 - r^2}}$$

E.g., Navarro-Frenk-White (NFW):

$$\rho(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s} \right)^2} \quad r_s = 2 \text{ kpc}, \quad \rho_s = 1.2 \times 10^7 \frac{M_\odot}{\text{kpc}^3}$$

$$M(r) = 4\pi\rho_s r_s^3 \left(\ln \left(1 + \frac{r}{r_s} \right) - \frac{r}{r+r_s} \right)$$

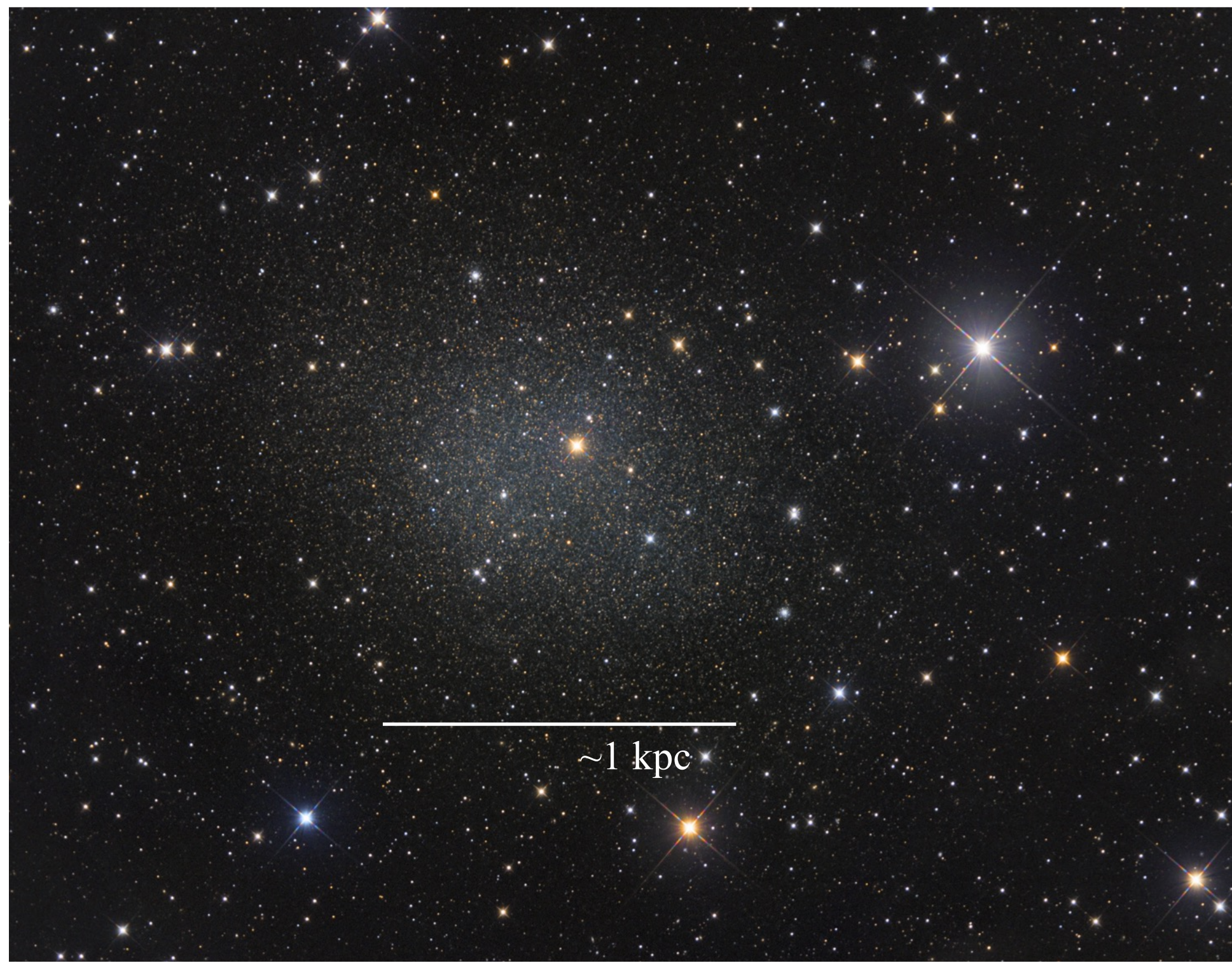
$$R_{200} \approx r_s \left(\frac{\rho_s}{200\rho_c} \right)^{\frac{1}{3}} \approx 16 \text{ kpc}, \quad M_{200} \approx 1.6 \times 10^9 M_\odot$$



$$M(2 \text{ kpc}) \approx 2.3 \times 10^8 M_\odot$$

//

16000



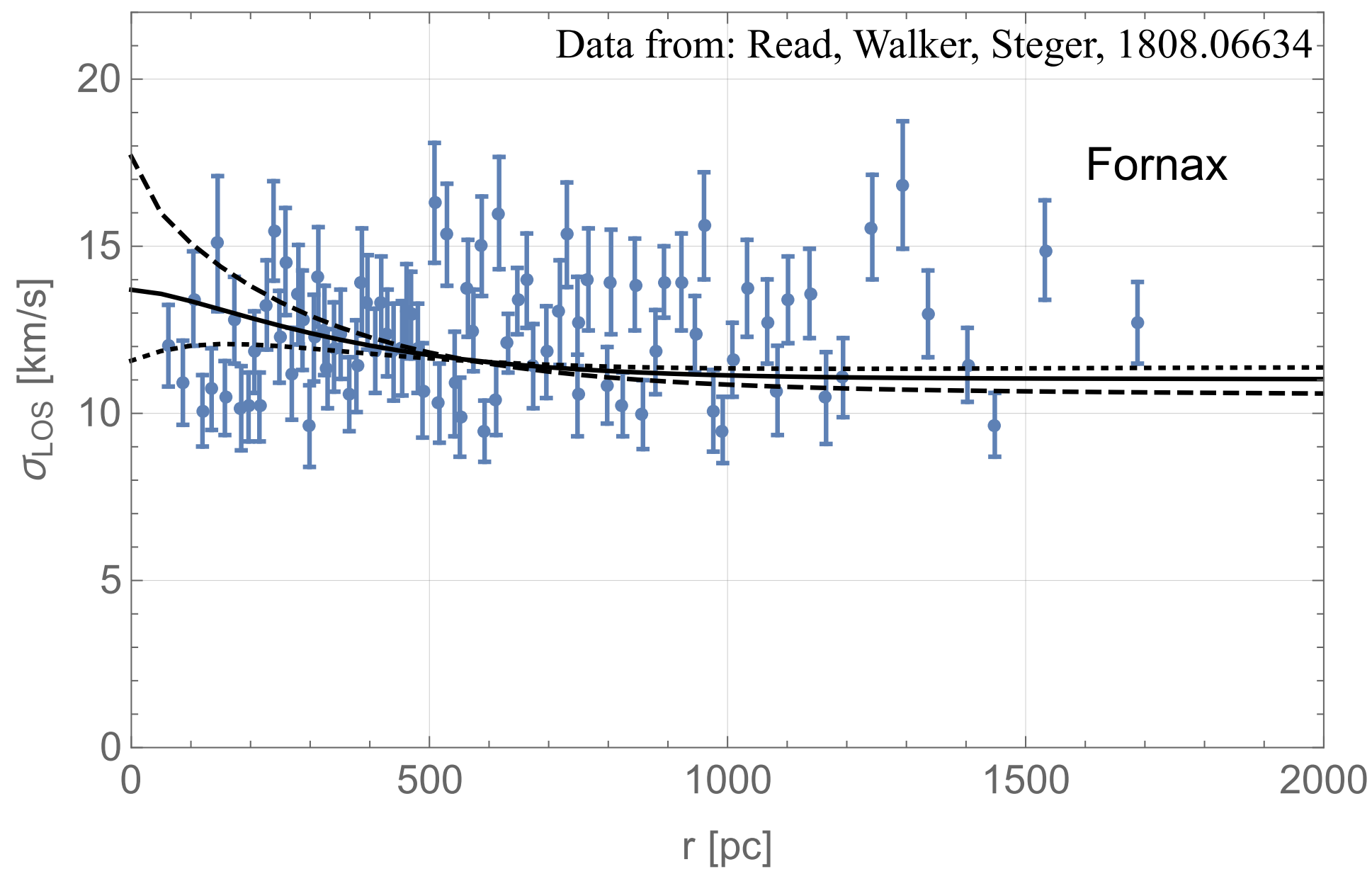
de Boer et al (A&A 544, A73 (2012)) — stellar population synthesis:

Total stellar mass $M_* \approx 4.3 \times 10^7 M_\odot$

This means:

$$\frac{M(r_s \approx 2 \text{ kpc})}{M_*} \approx 5.3$$

$$\frac{M(R_{200} \approx 16 \text{ kpc})}{M_*} \approx 37$$

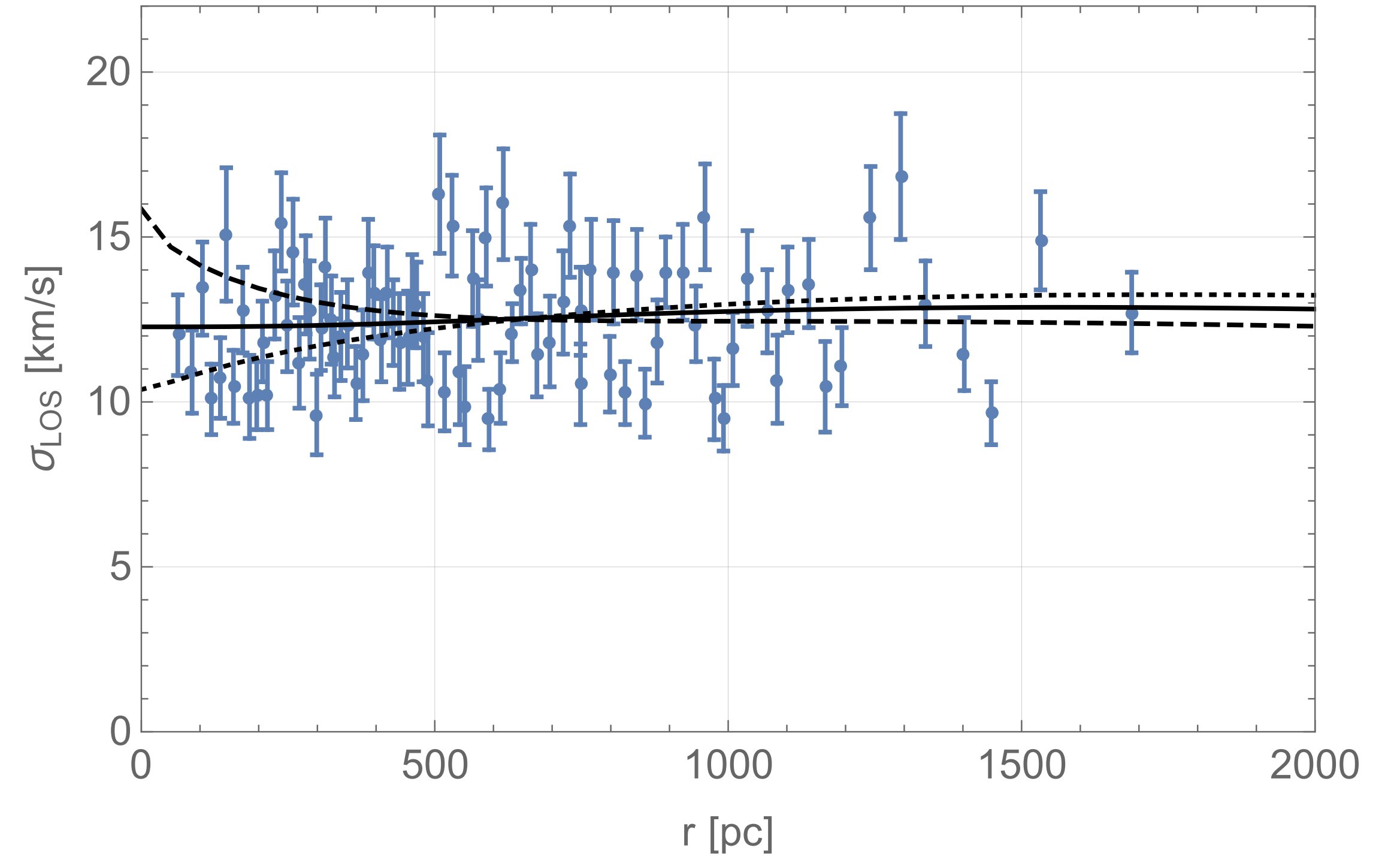
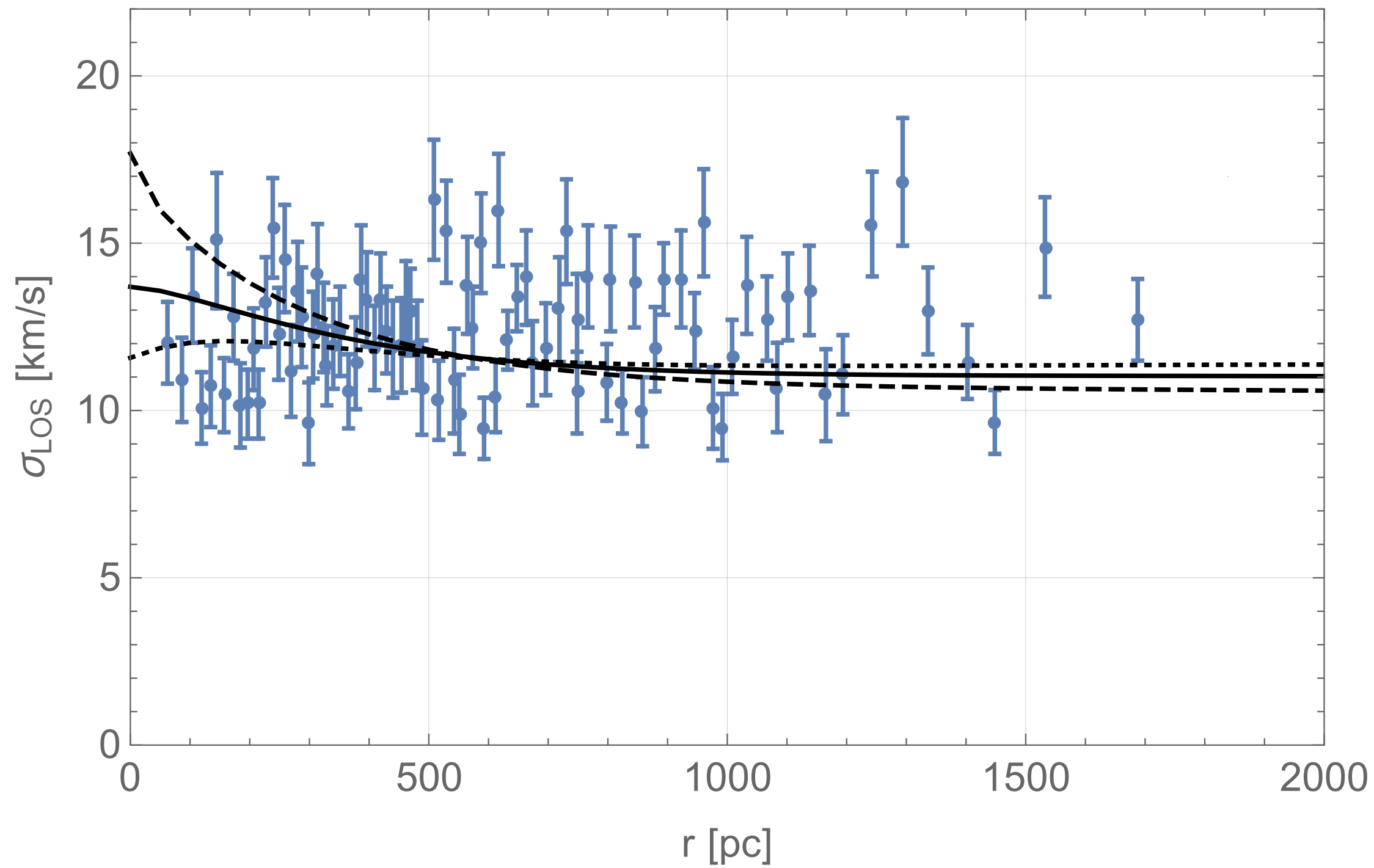


core / cusp

McGaugh et al 2001,
Gored et al 2006,
de Blok 2010,
Teyssier et al 2013,
Del Popolo & Face 2015,
Bullock & Boylan-Kolchin, 2017,
Meadows et al, 2019,
Santos-Santos et al 2020,...

$$\rho_{\text{NFW}} = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

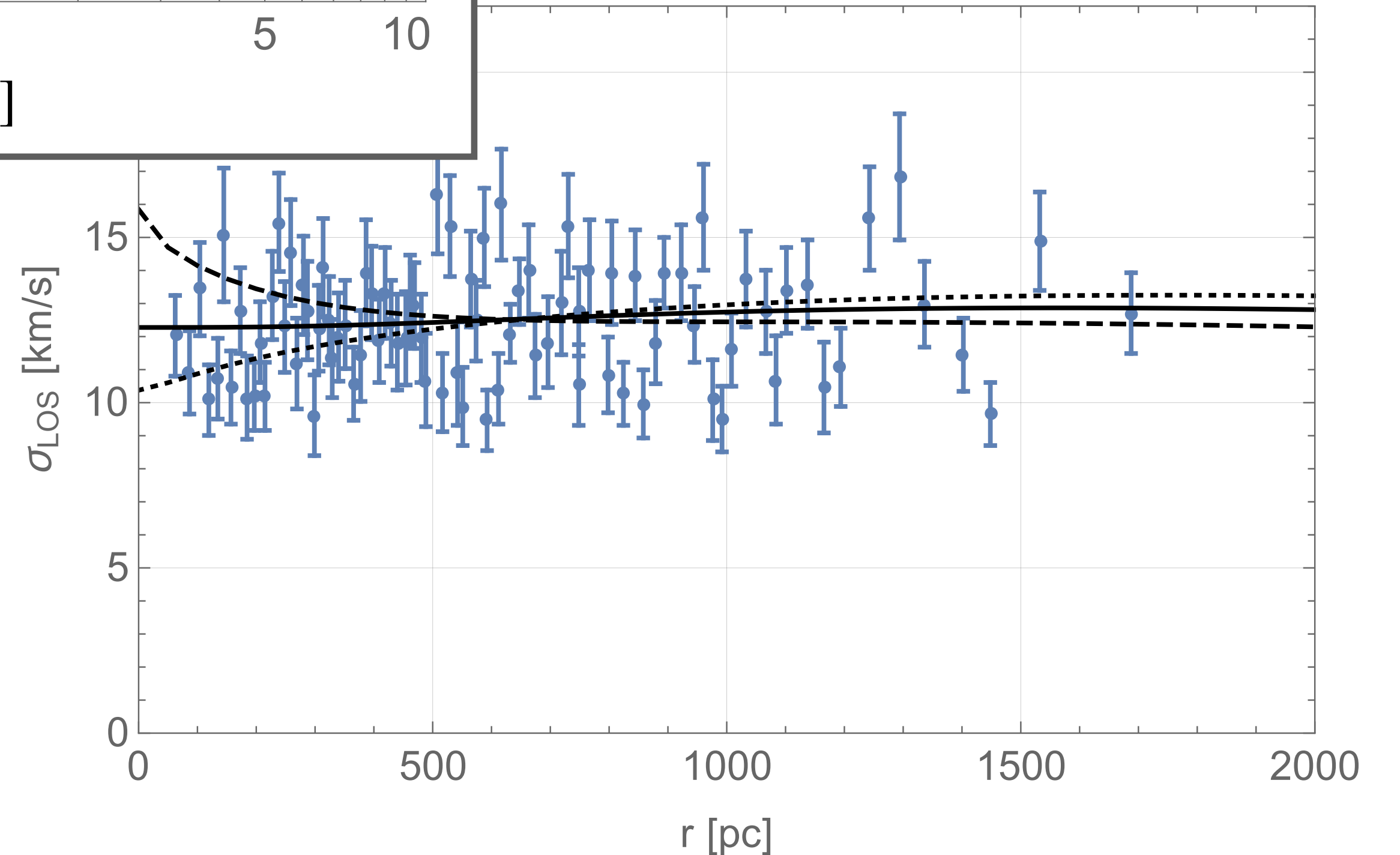
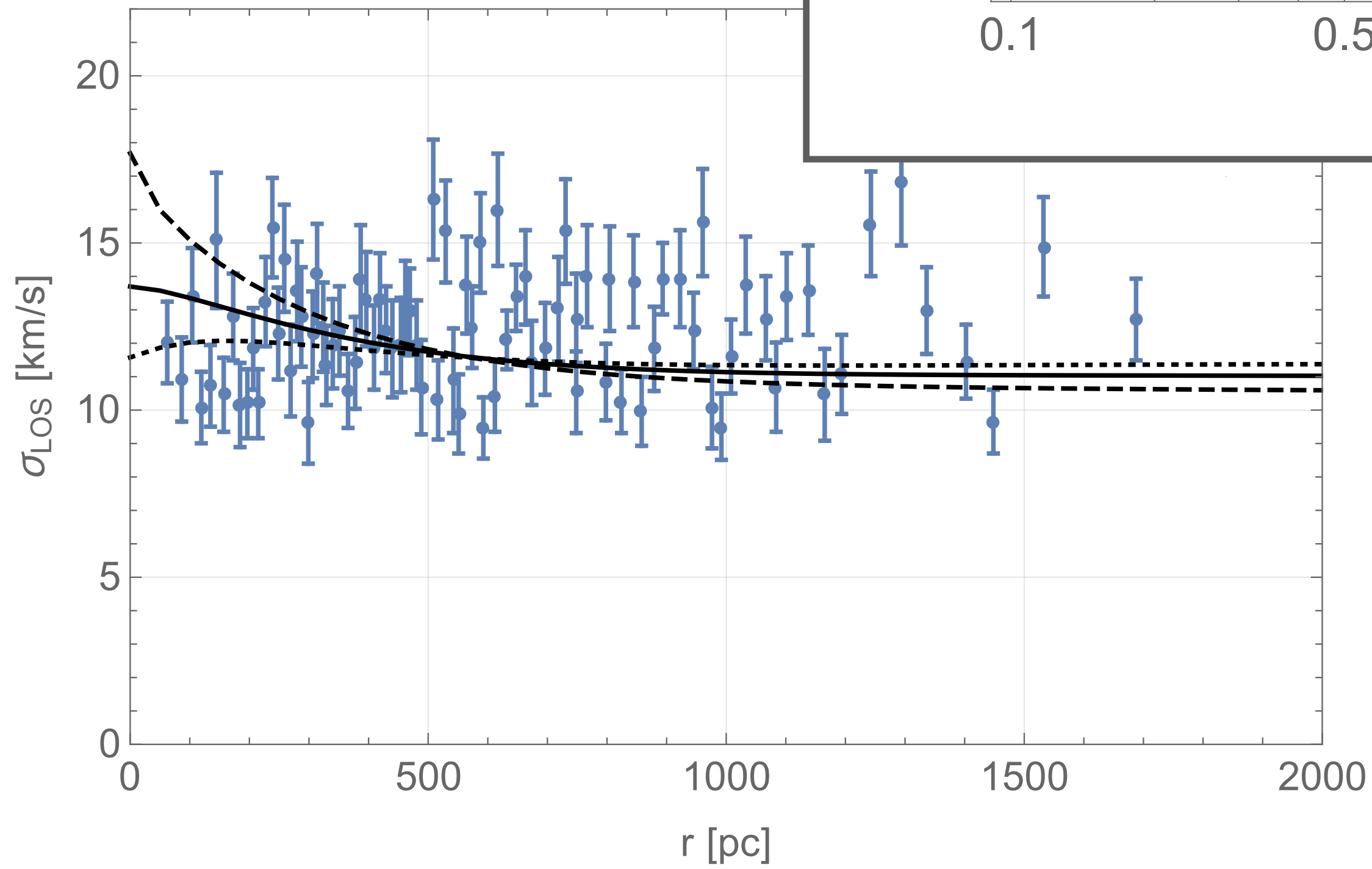
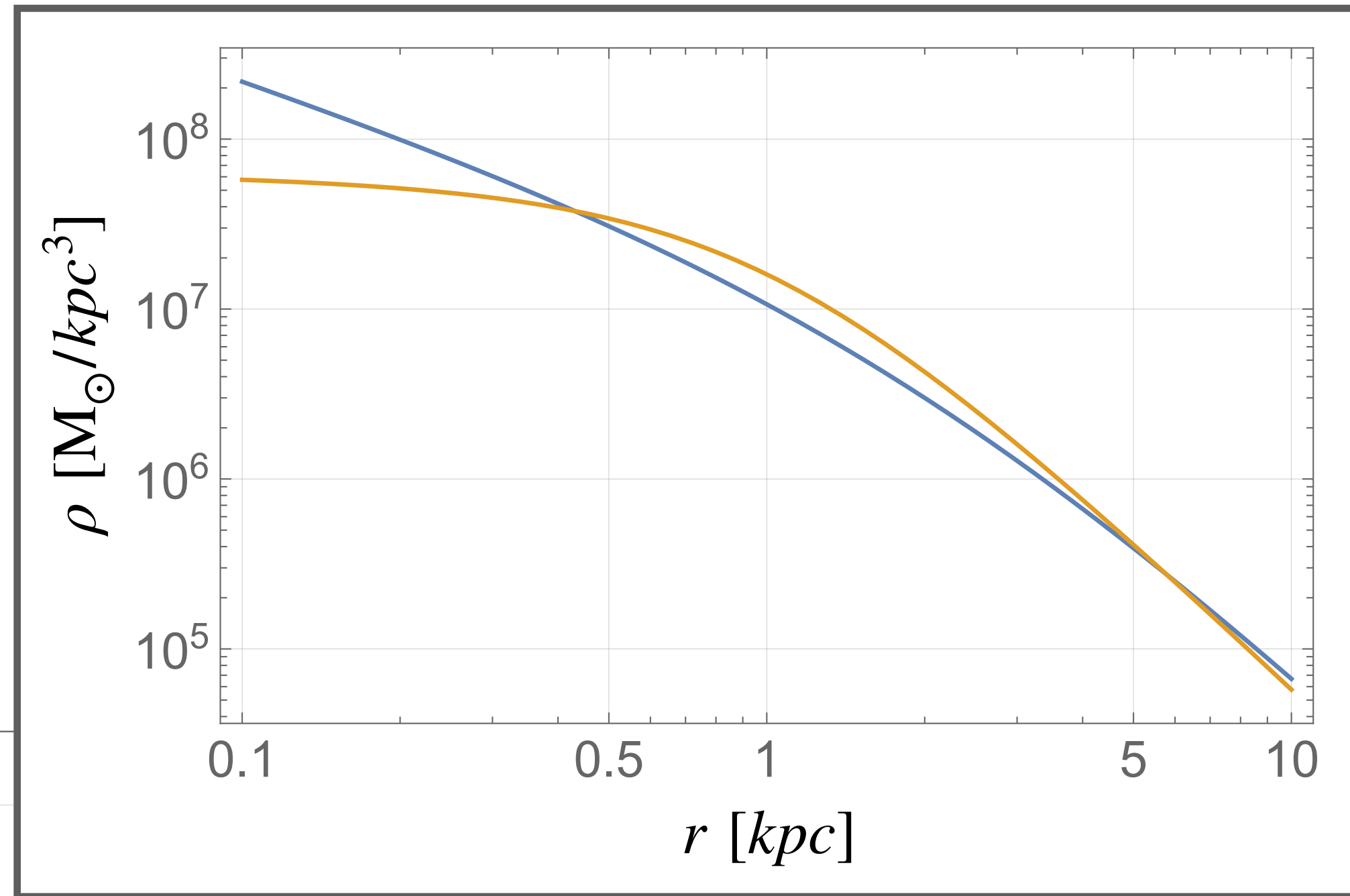
$$\rho_{\text{Burkert}} = \frac{\rho_0}{\left(1 + \frac{r}{r_0}\right) \left(1 + \frac{r^2}{r_0^2}\right)}$$



core / cusp

$$\rho_{\text{NFW}} = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

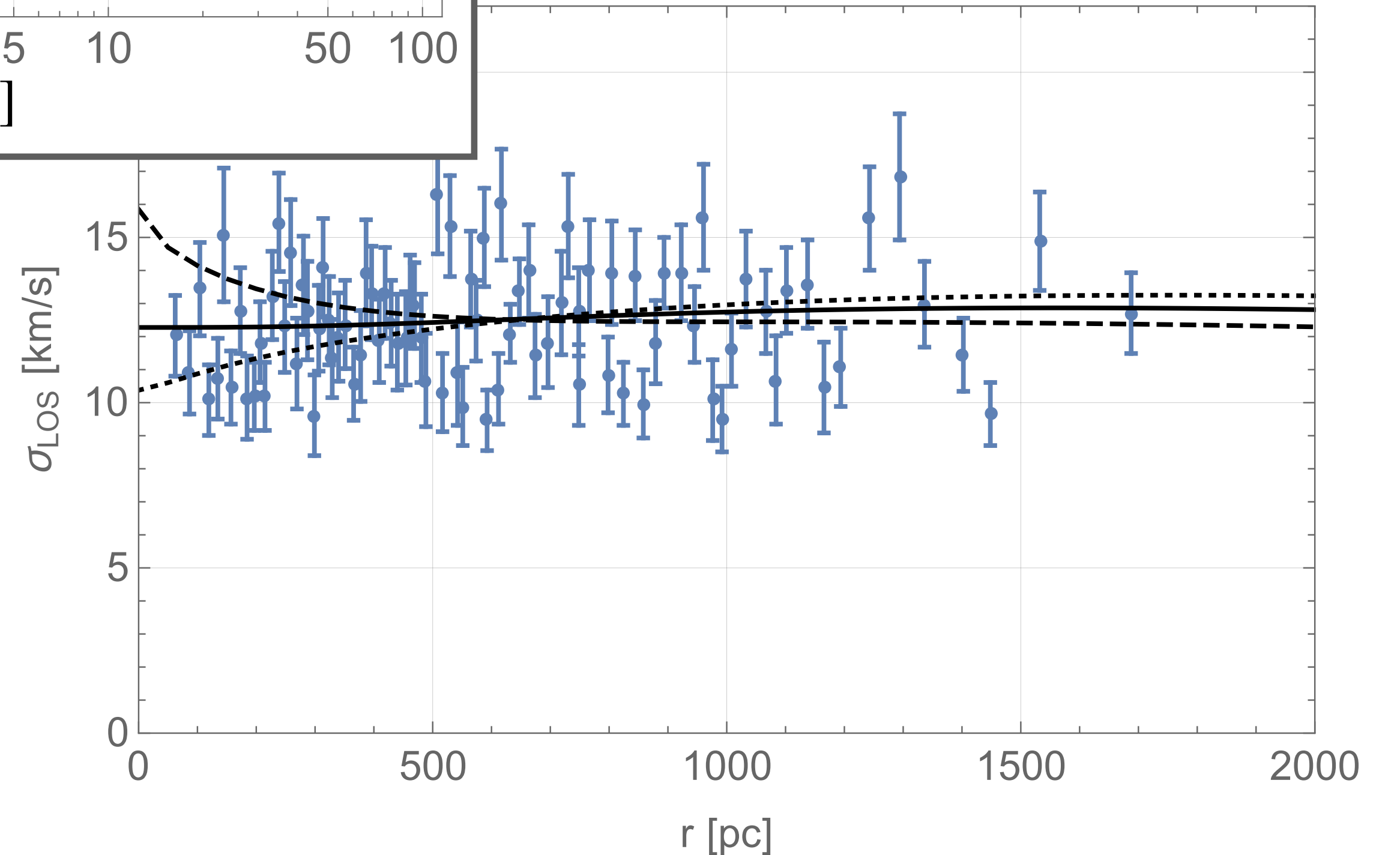
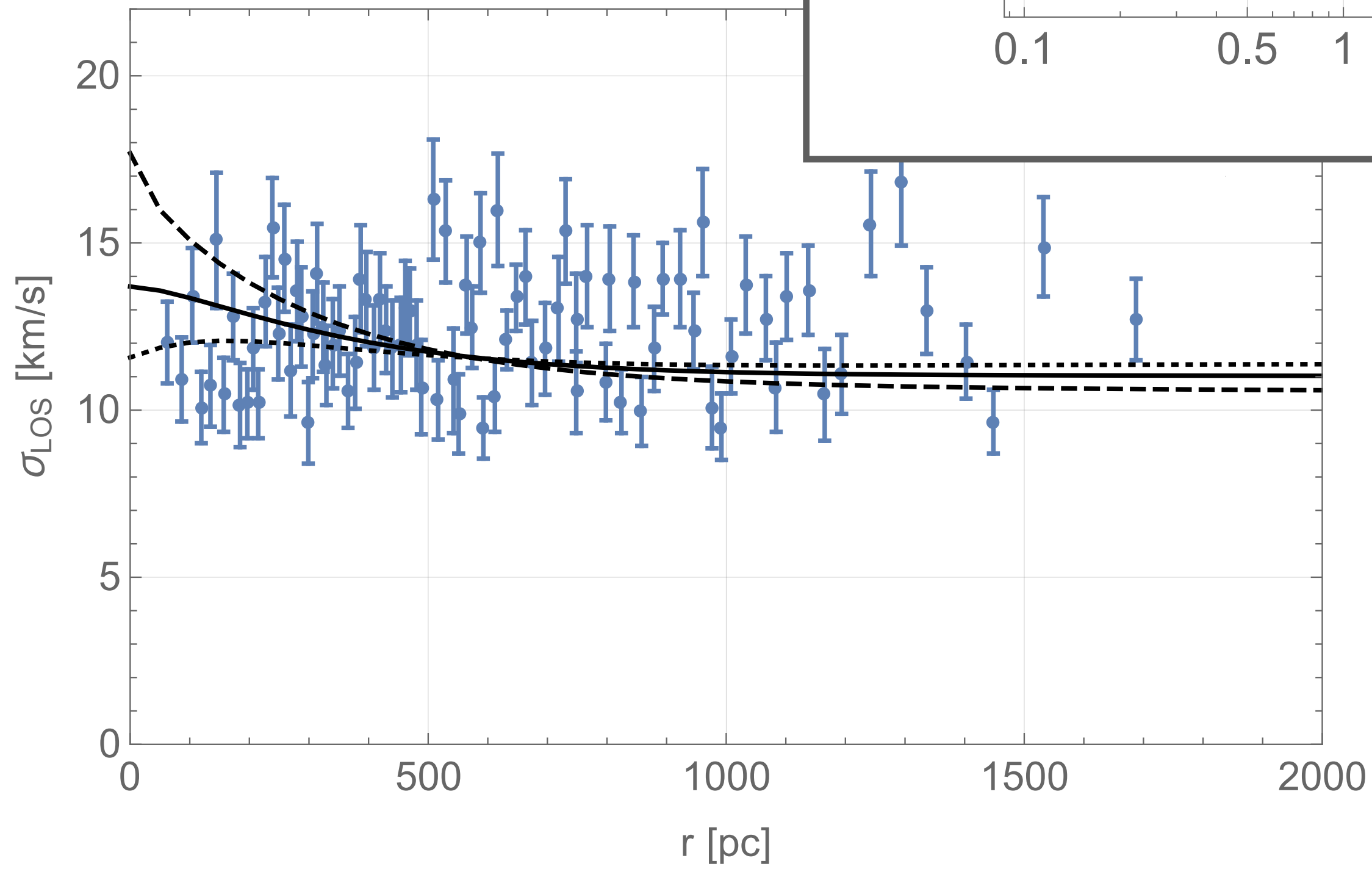
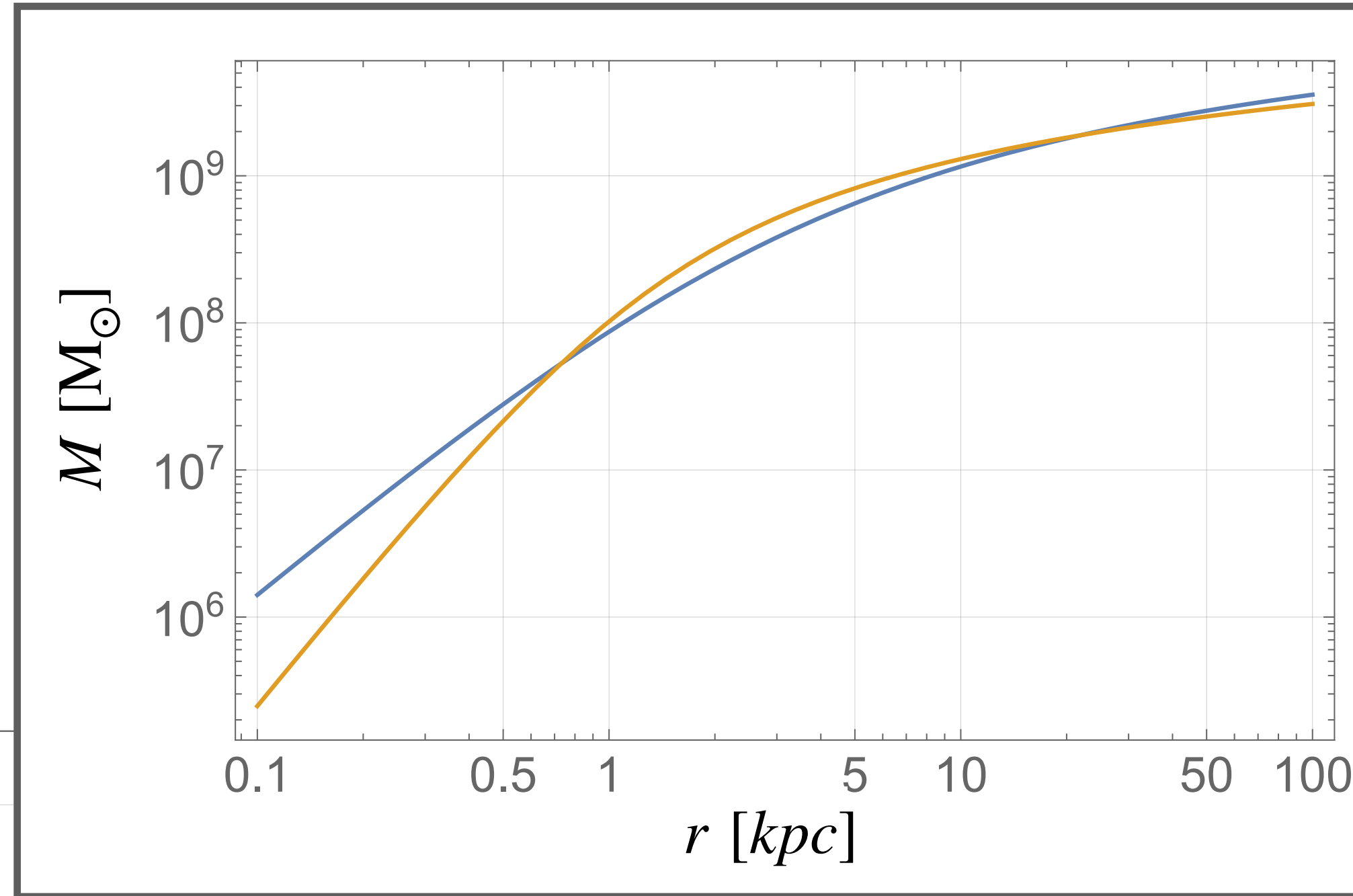
$$\rho_{\text{Burkert}} = \frac{\rho_0}{\left(1 + \frac{r}{r_0}\right) \left(1 + \frac{r^2}{r_0^2}\right)}$$

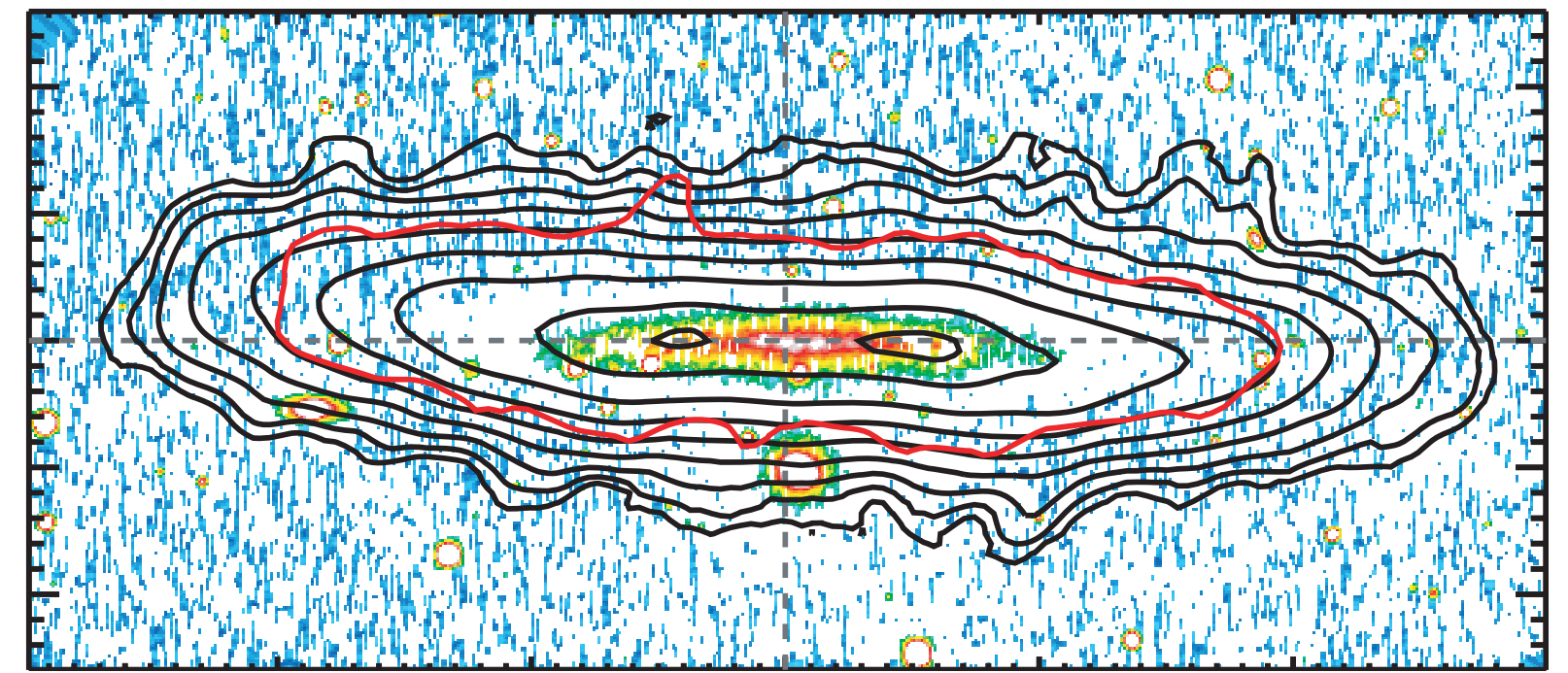
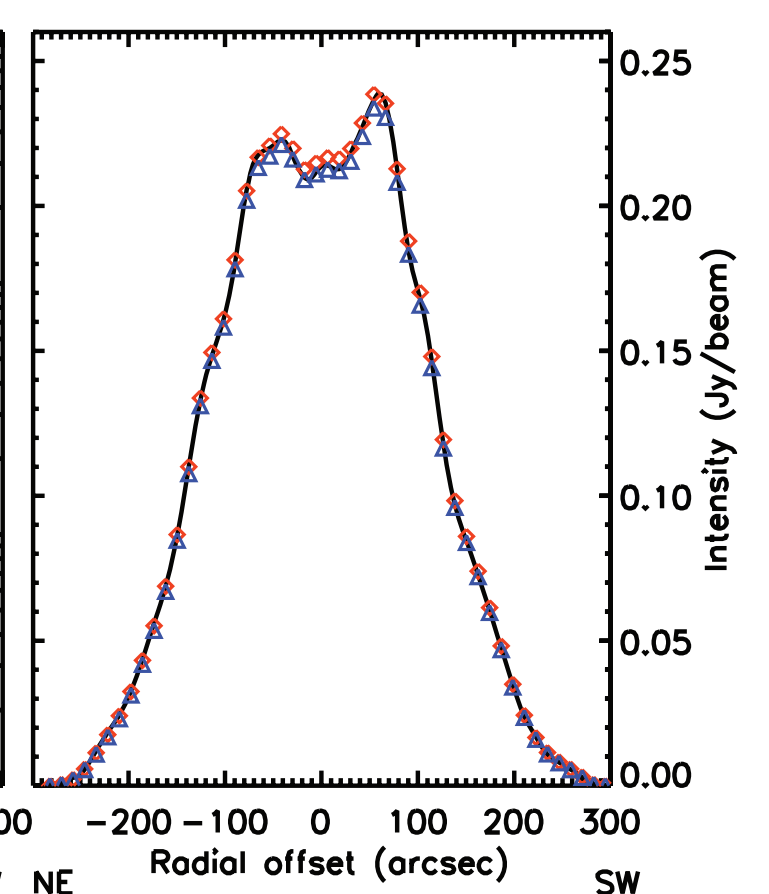
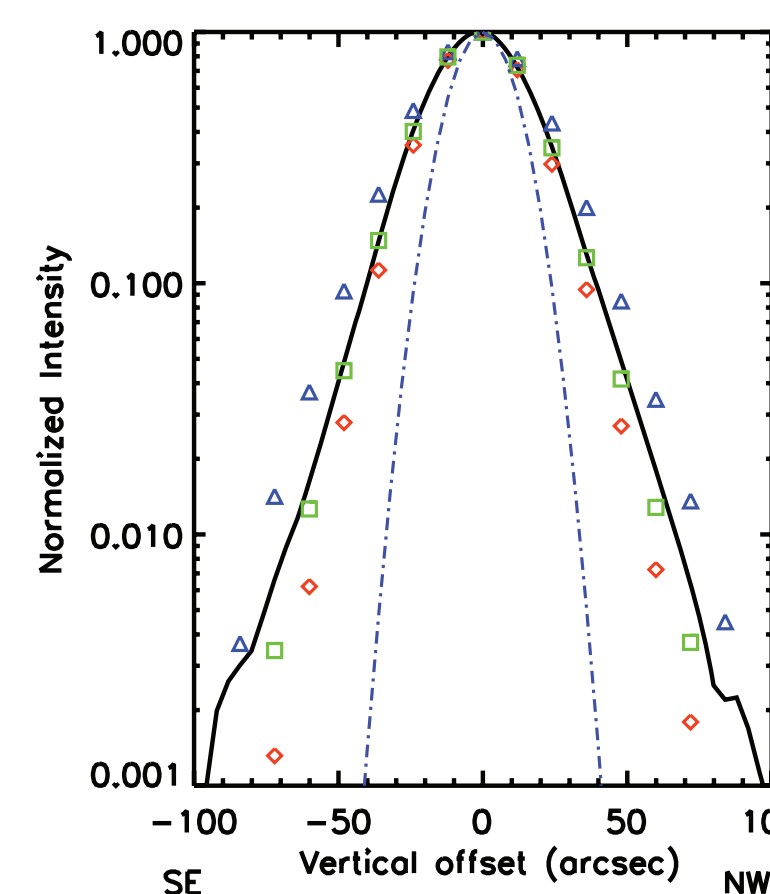
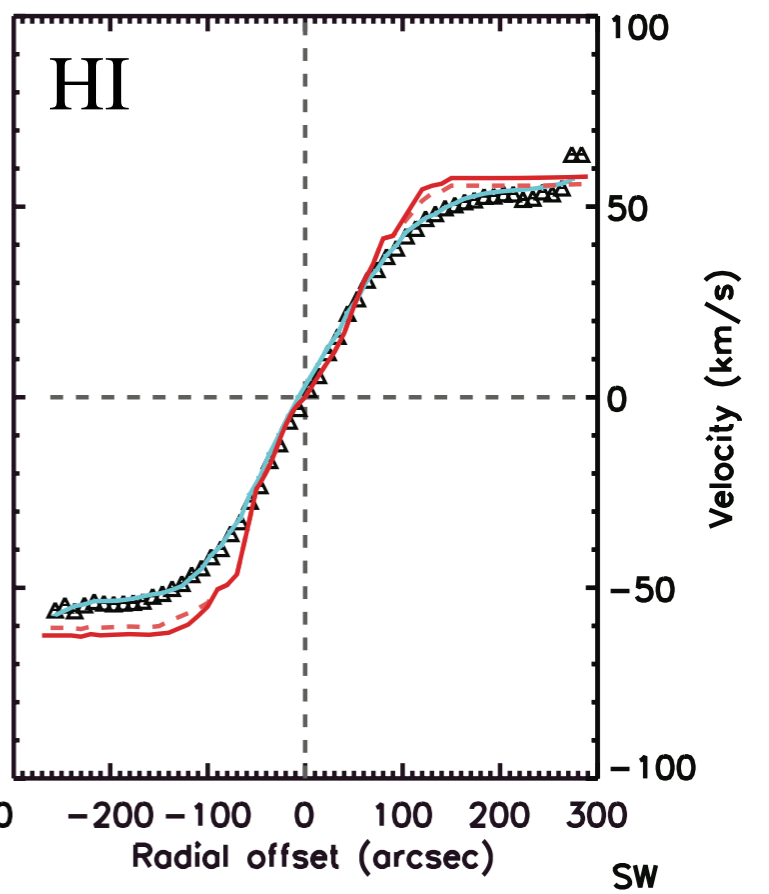
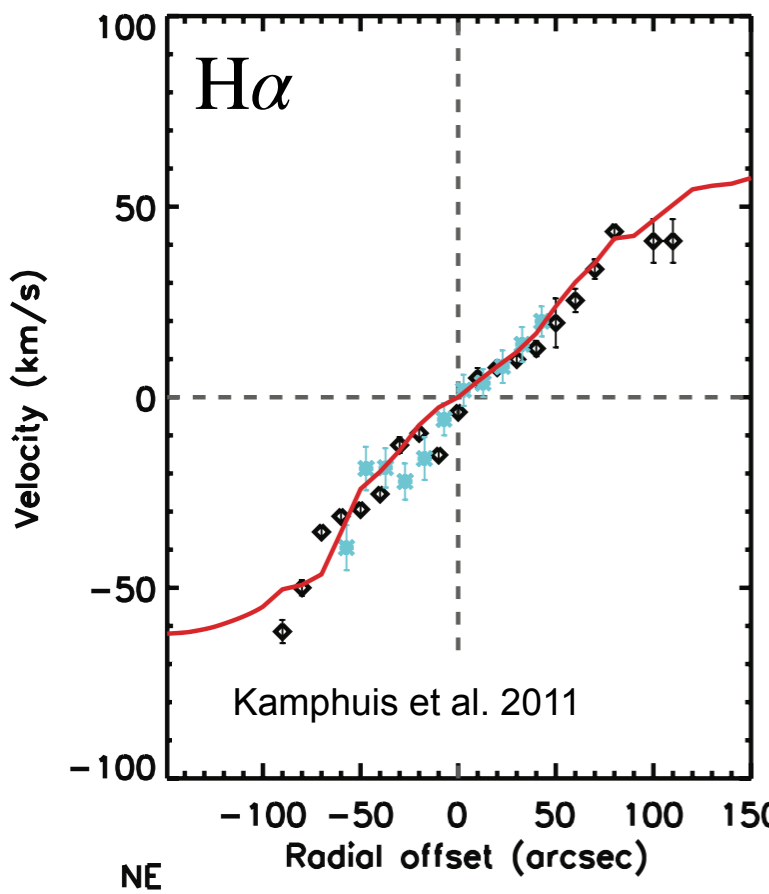
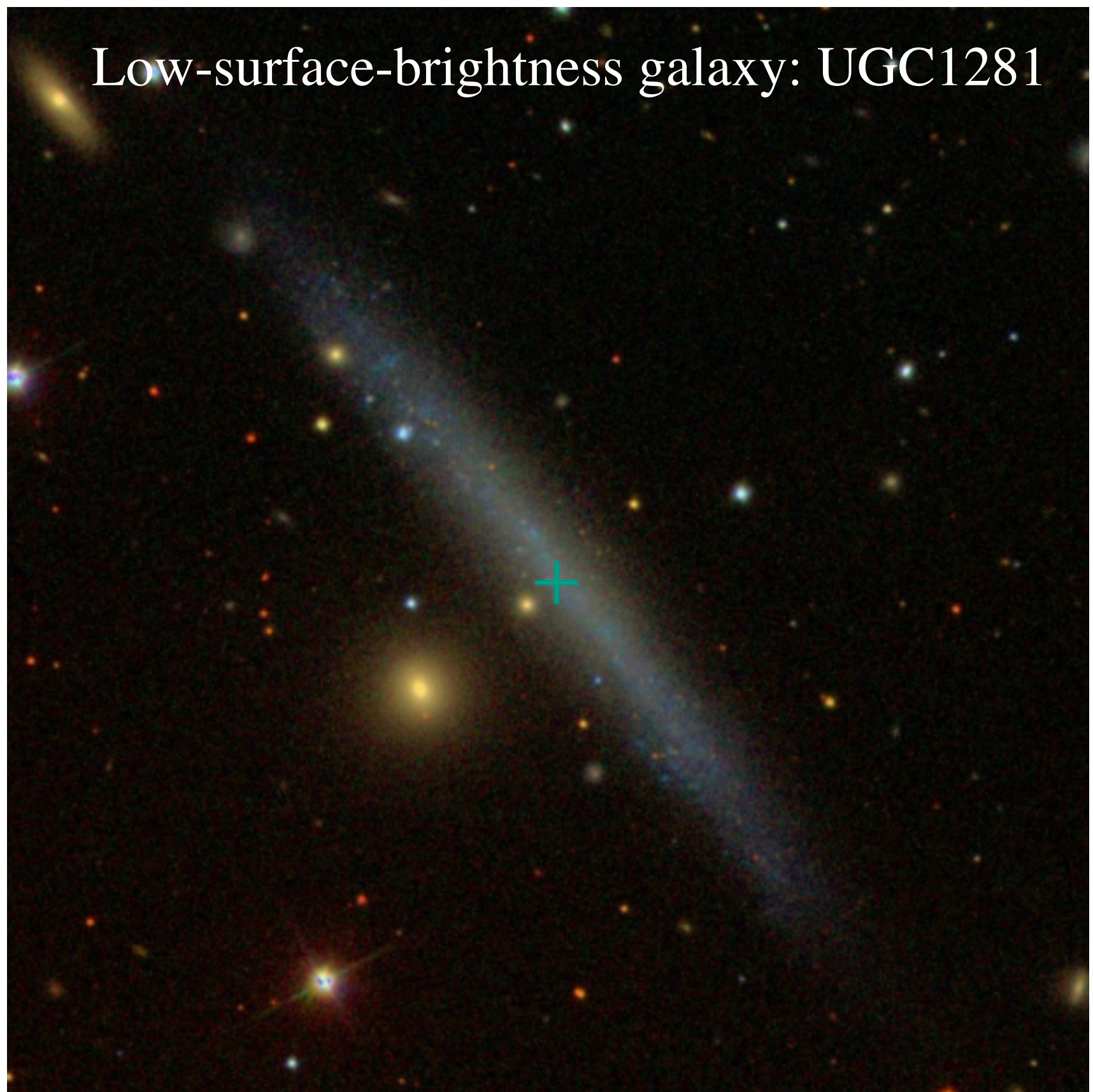
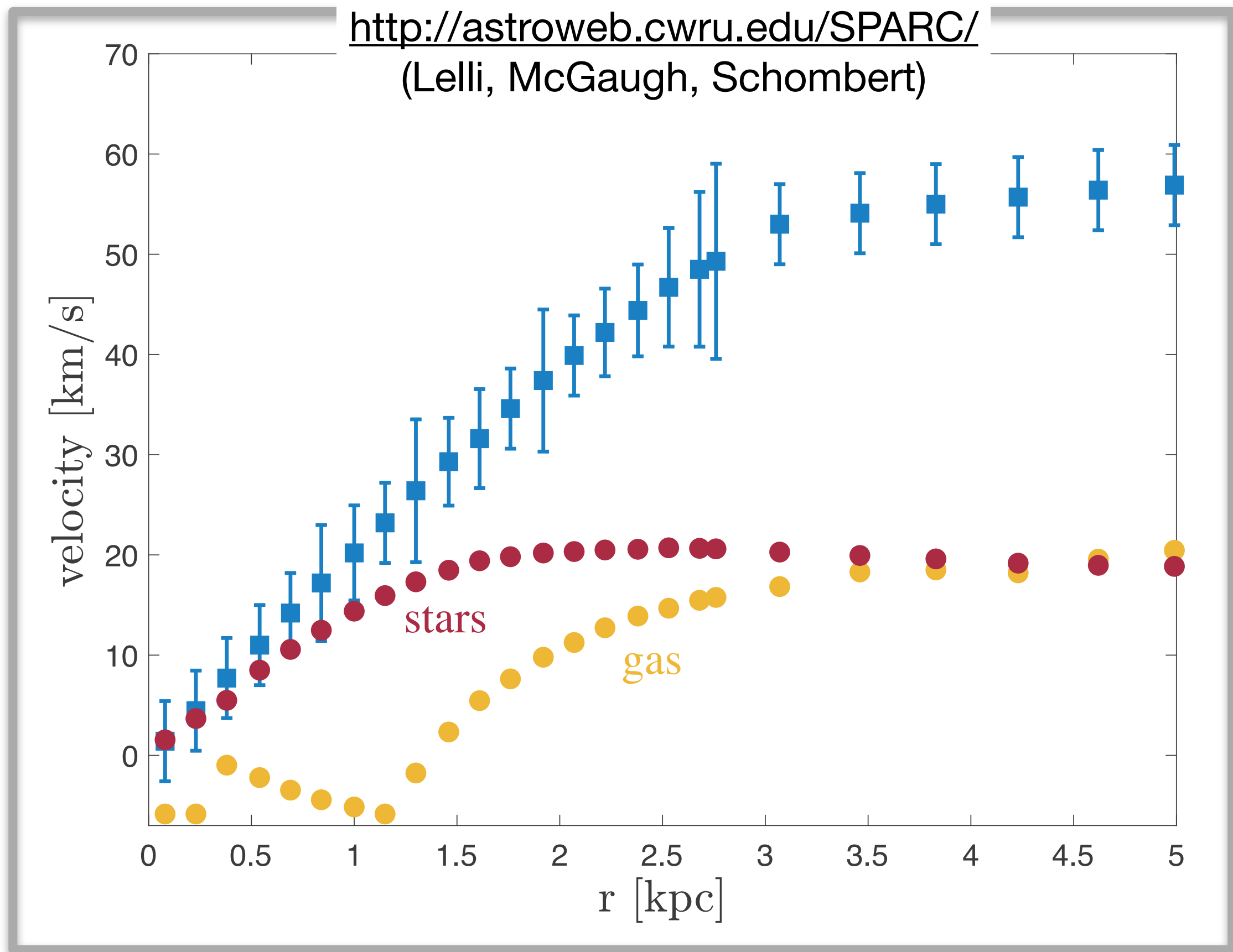


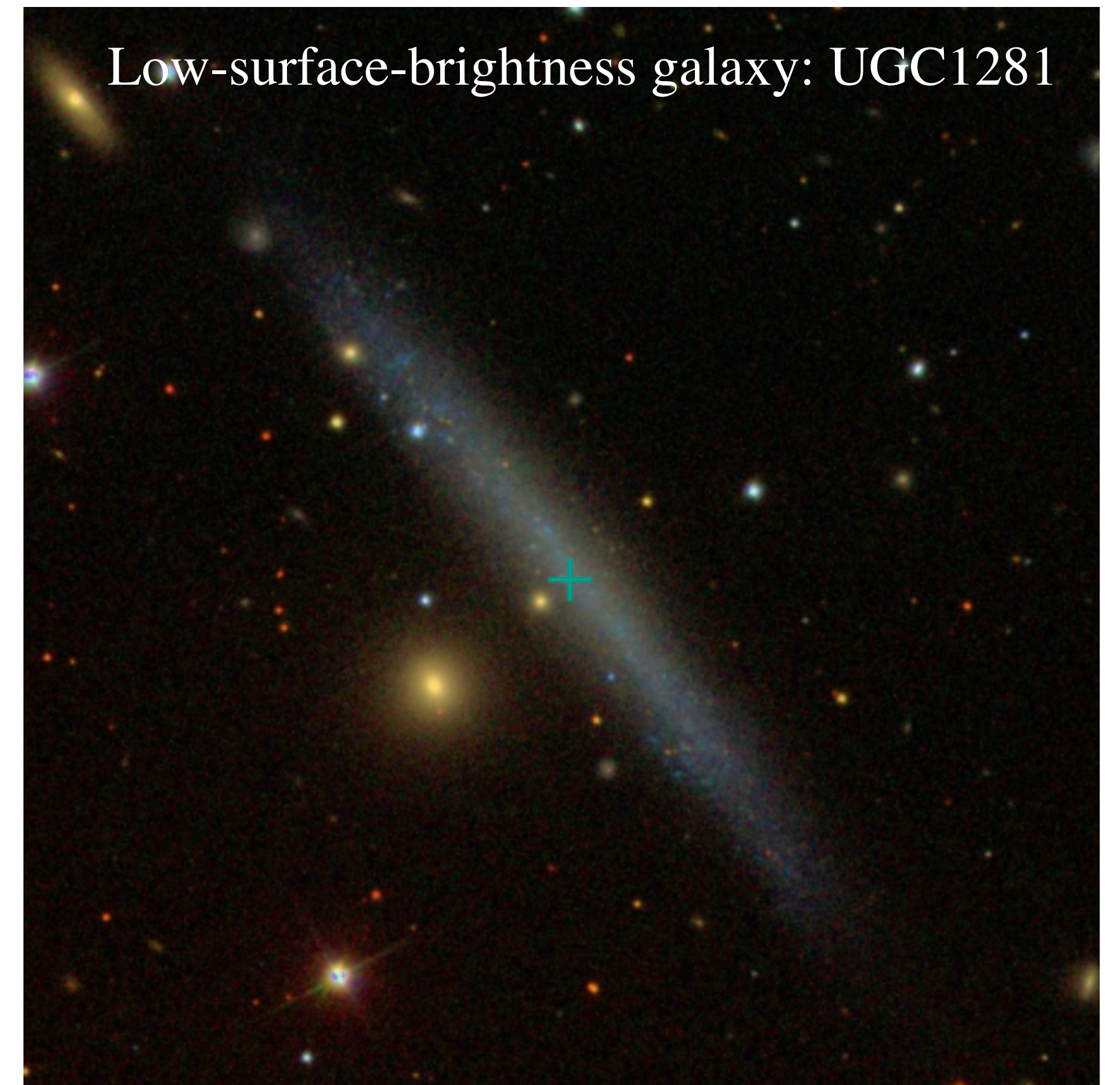
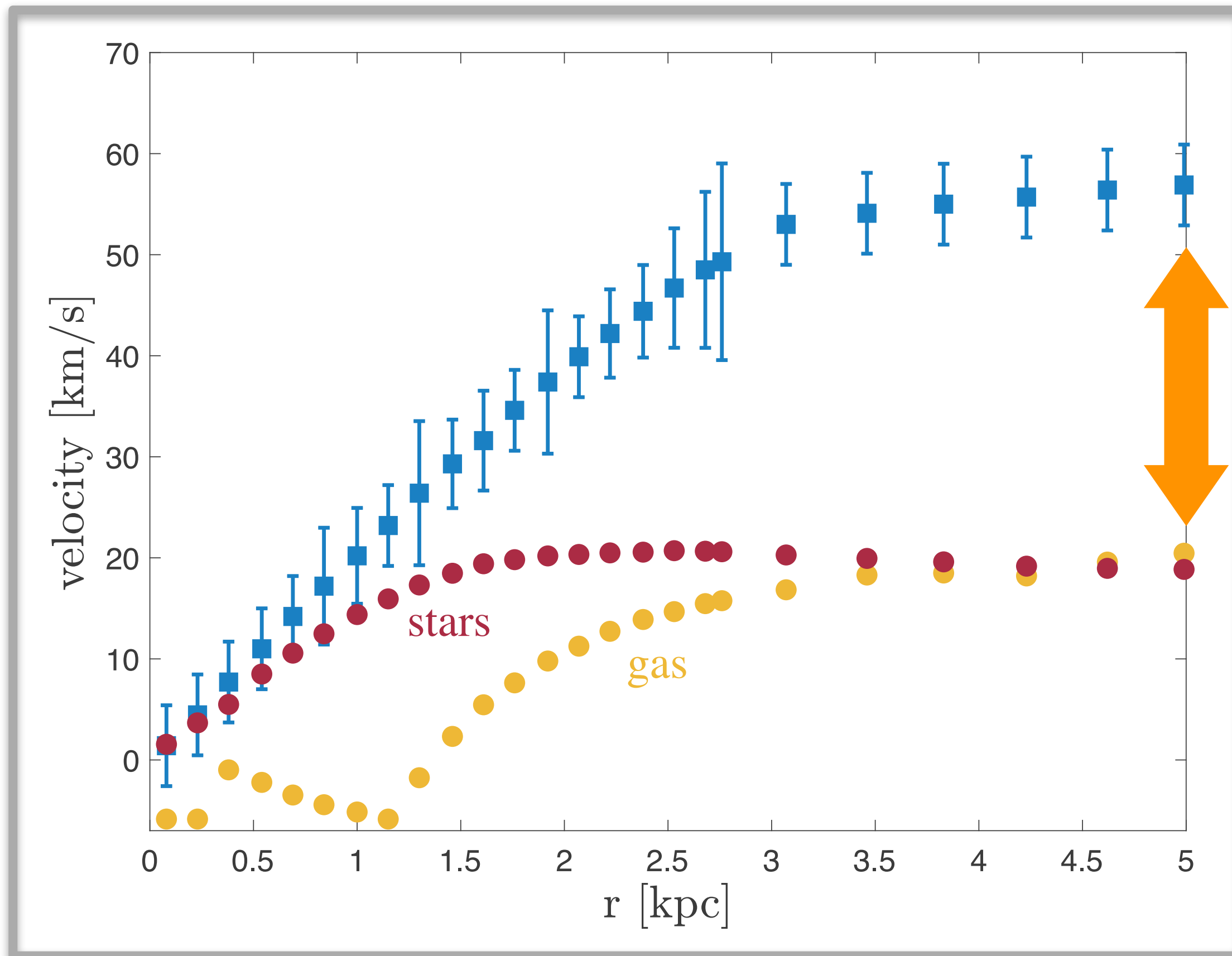
core / cusp

$$\rho_{\text{NFW}} = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

$$\rho_{\text{Burkert}} = \frac{\rho_0}{\left(1 + \frac{r}{r_0}\right) \left(1 + \frac{r^2}{r_0^2}\right)}$$

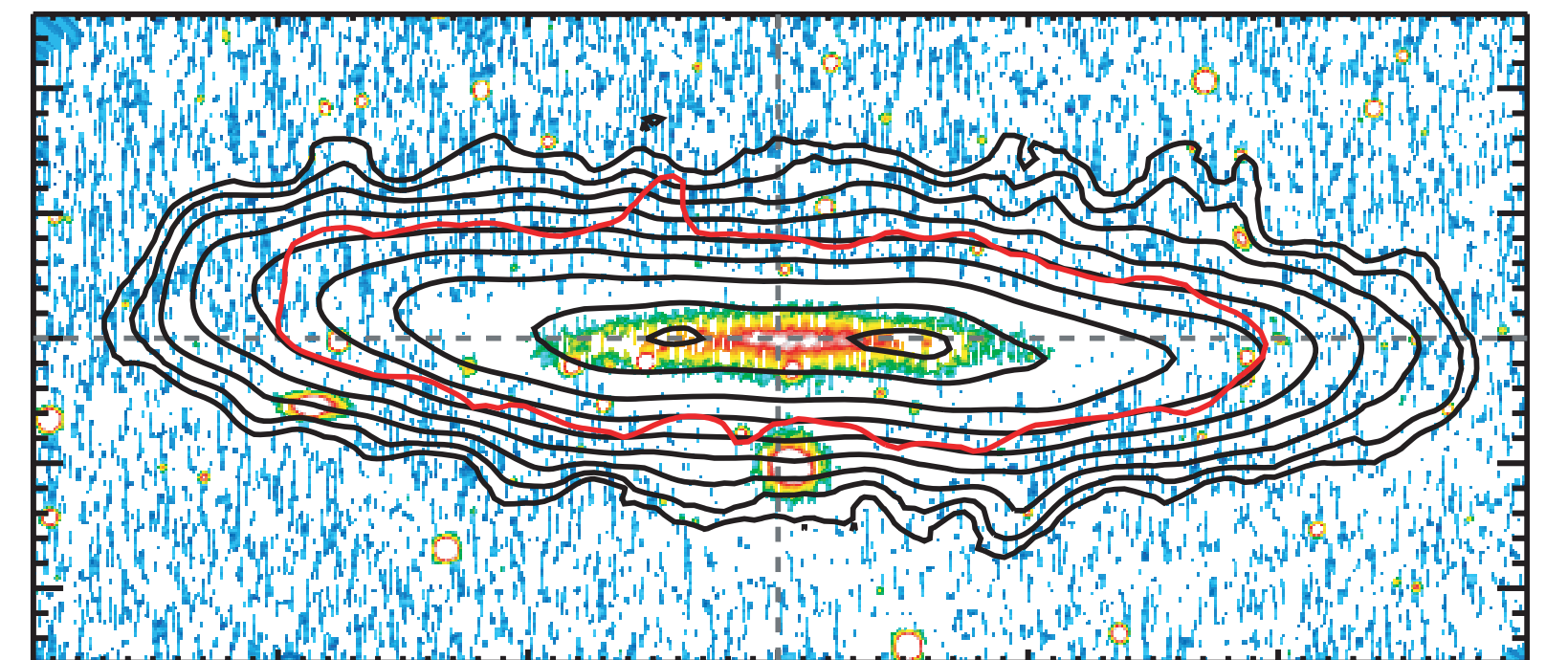




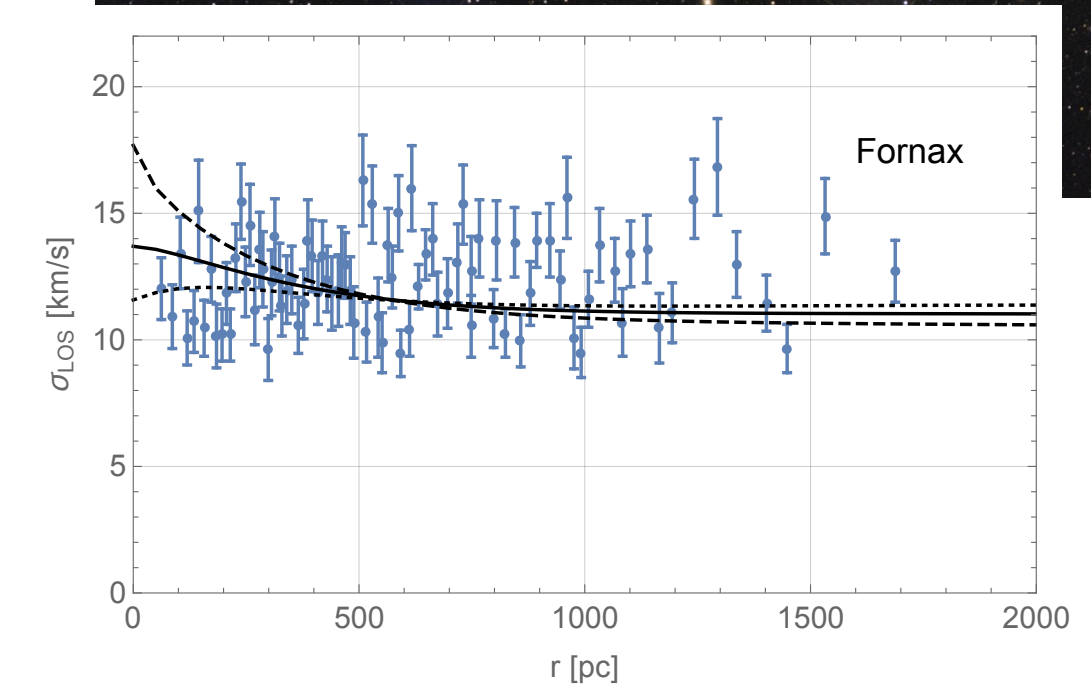
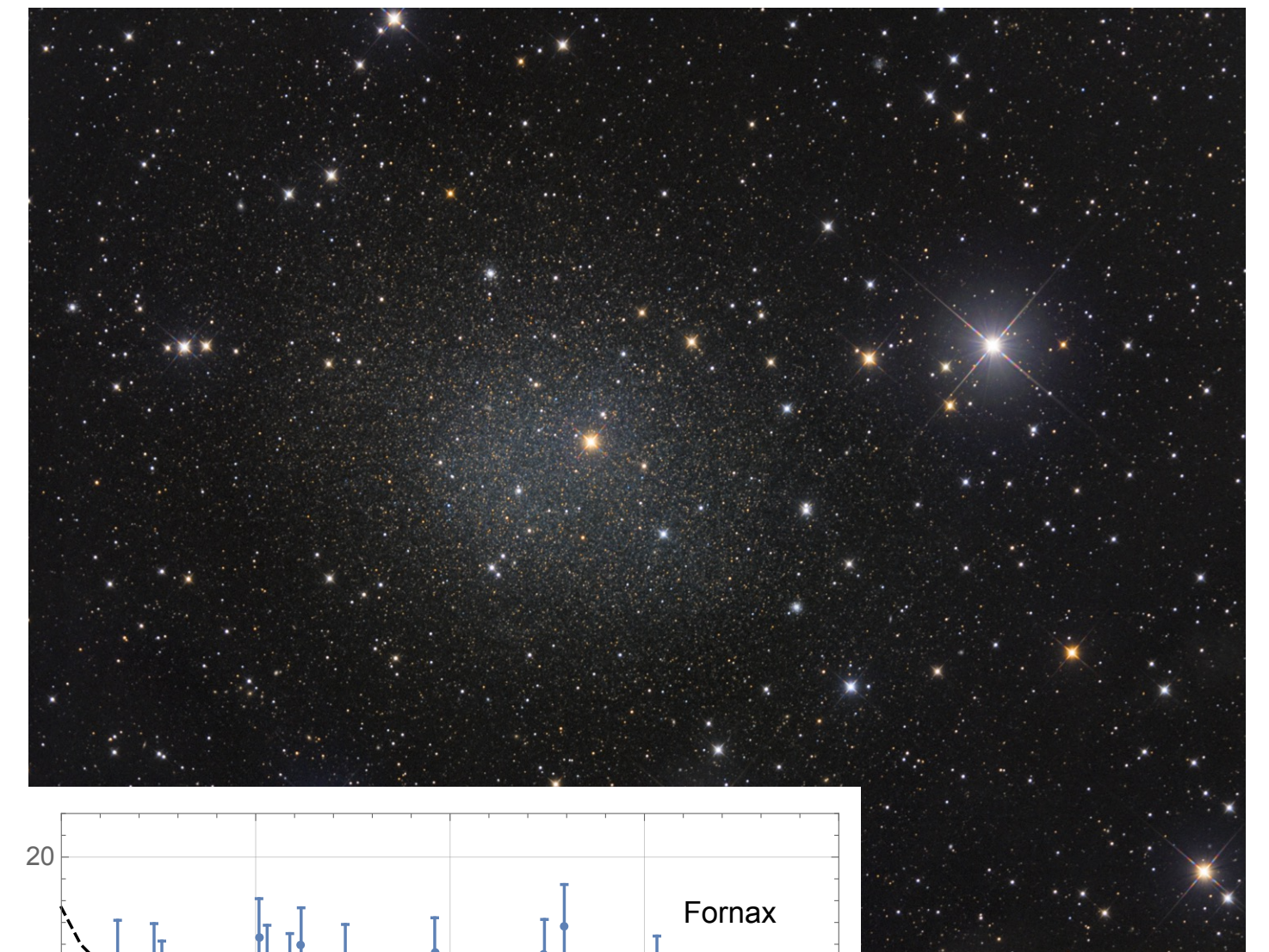
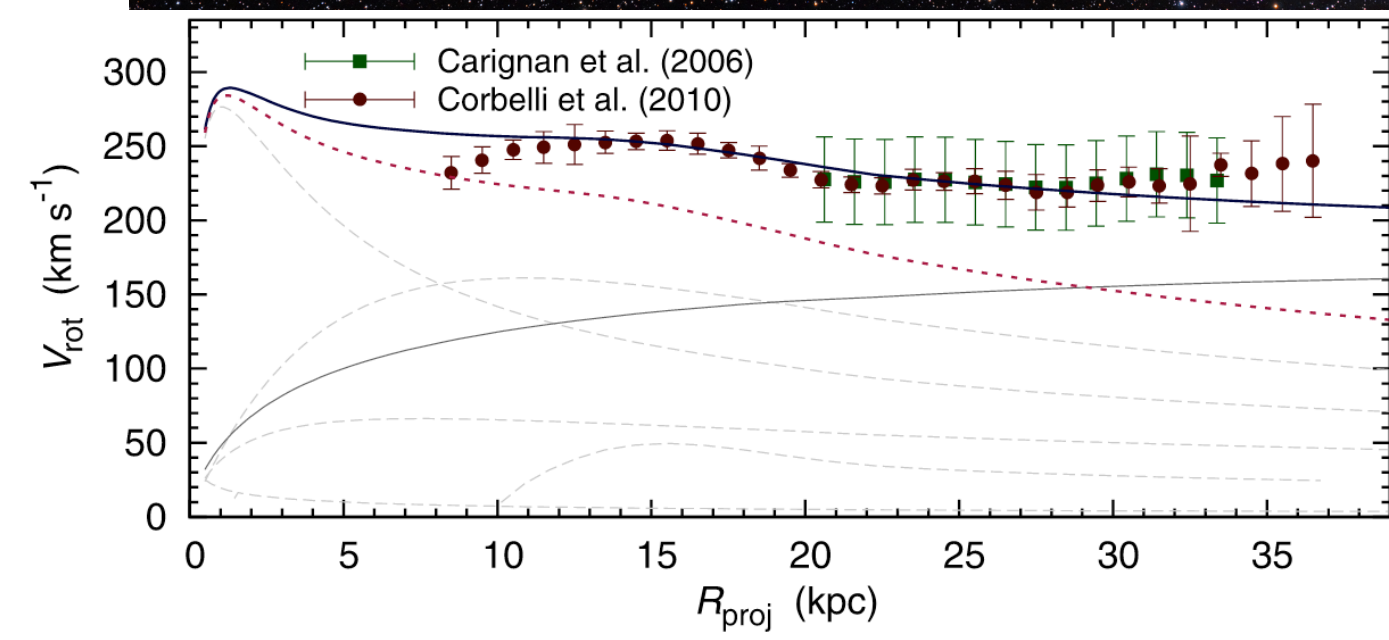
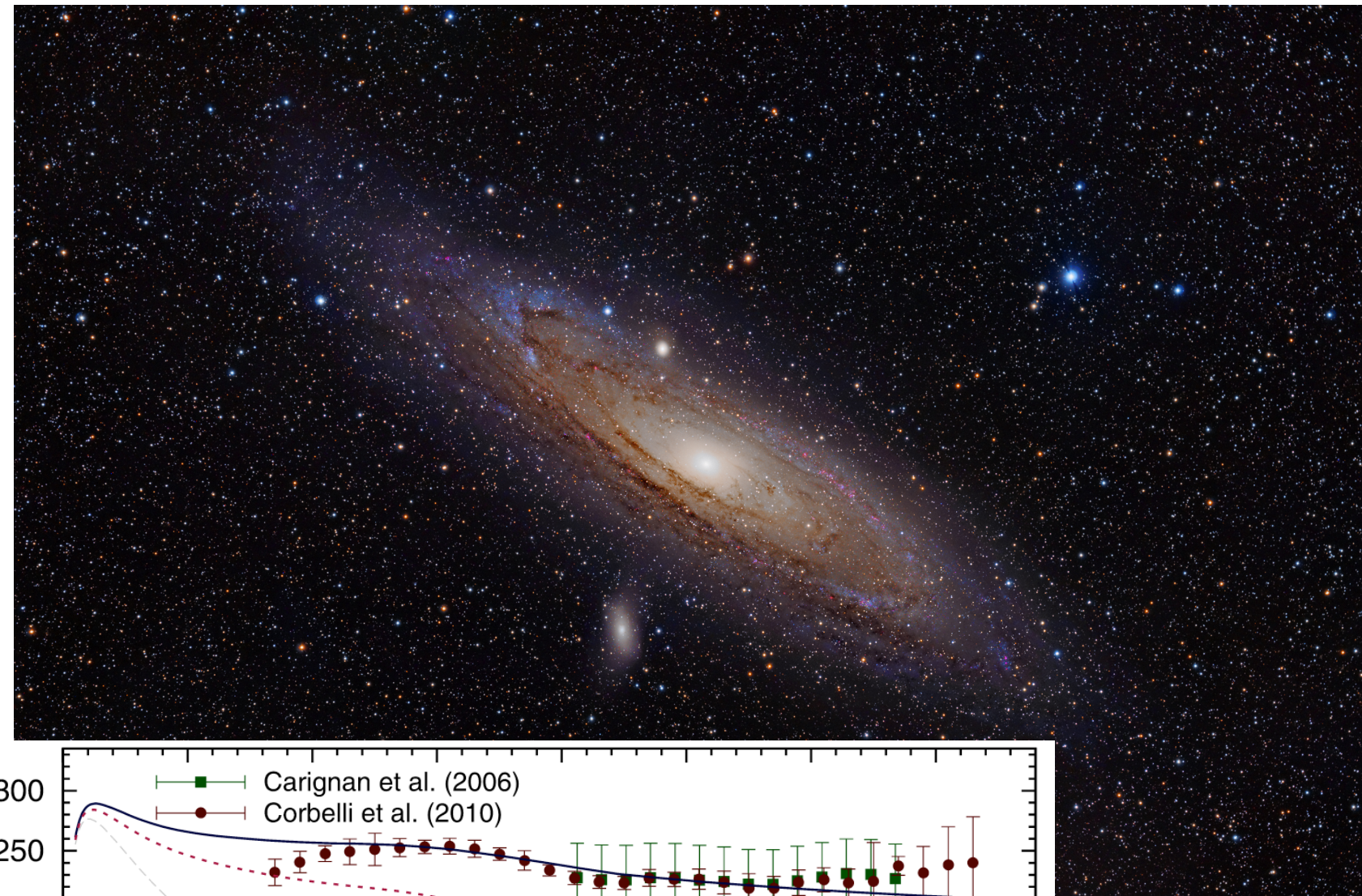
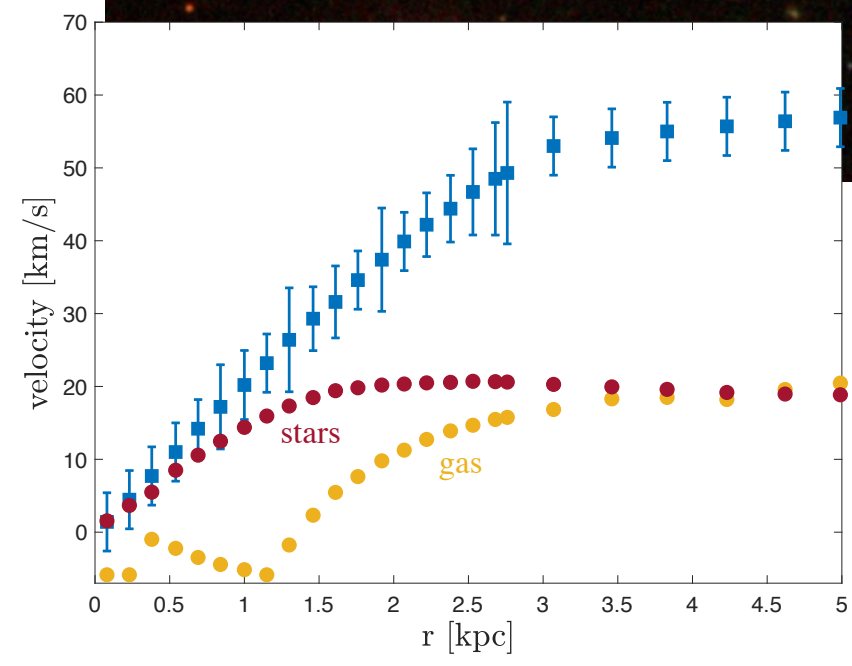
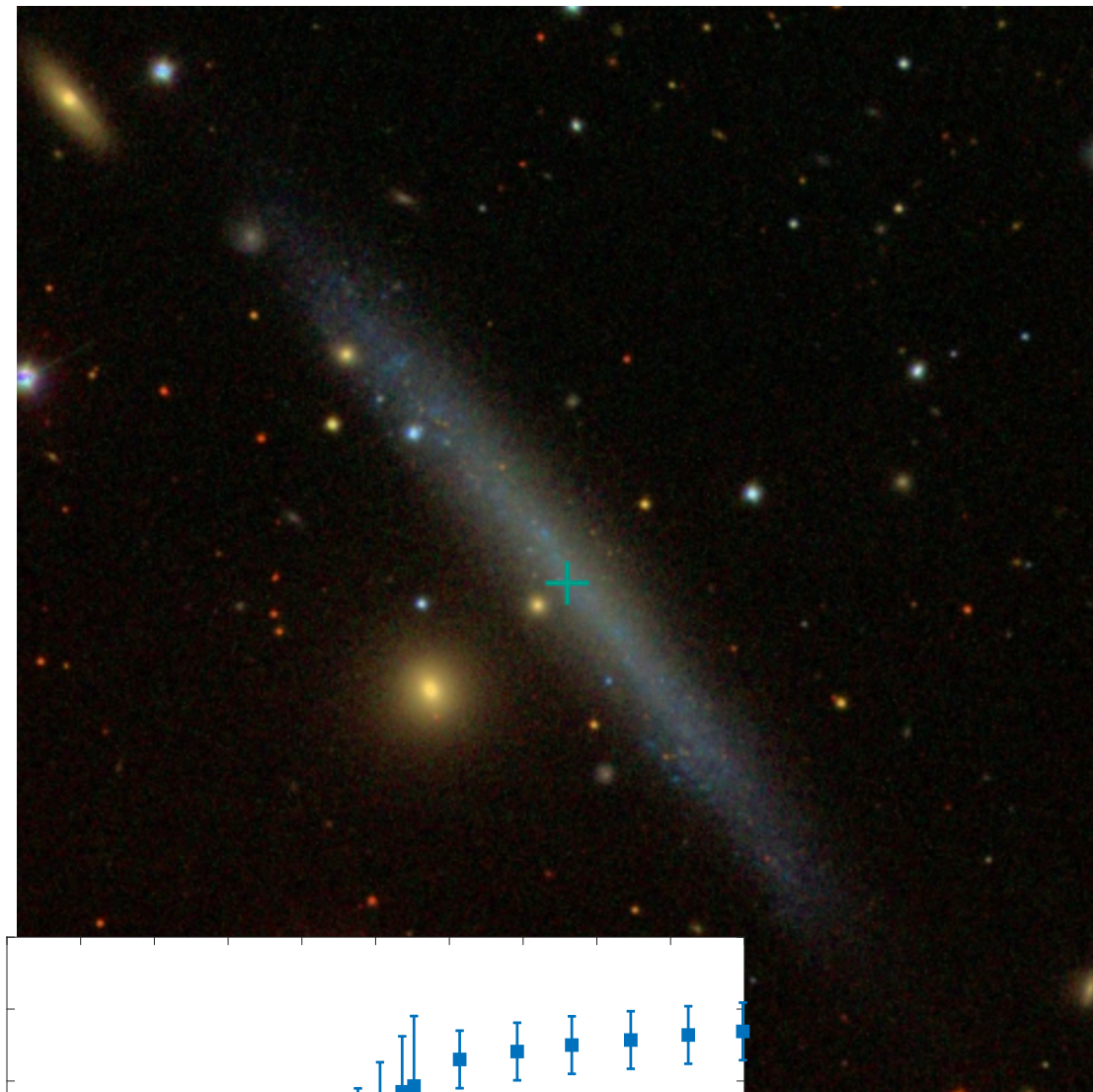


At ~5 kpc:

$$M(\text{DM})/M(\text{stars+gas}) \sim \frac{55^2 - 2 \times 20^2}{2 \times 20^2} \sim 3$$



Dark matter

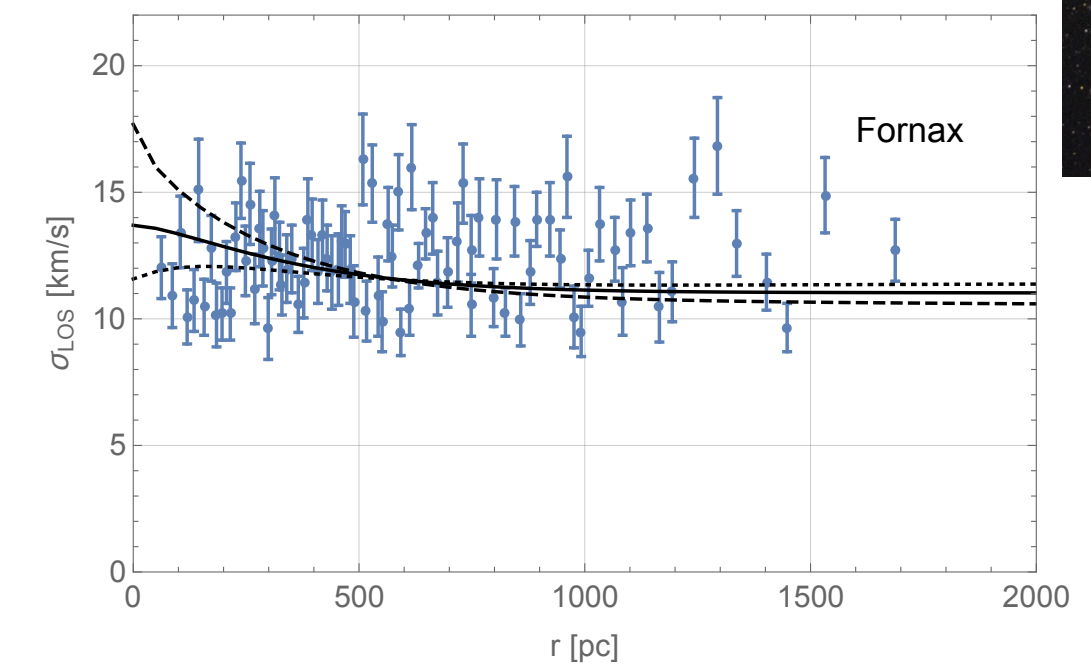
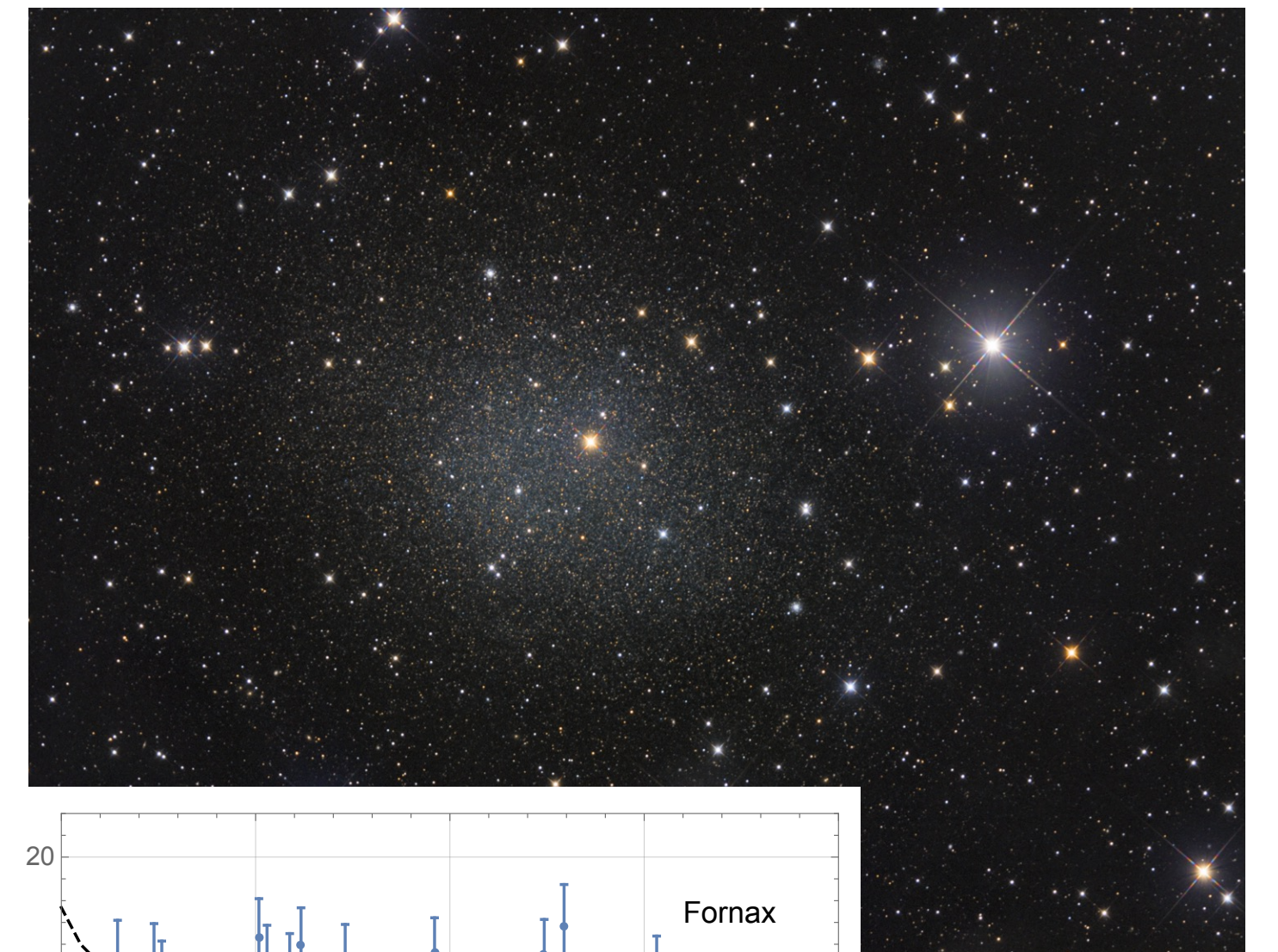
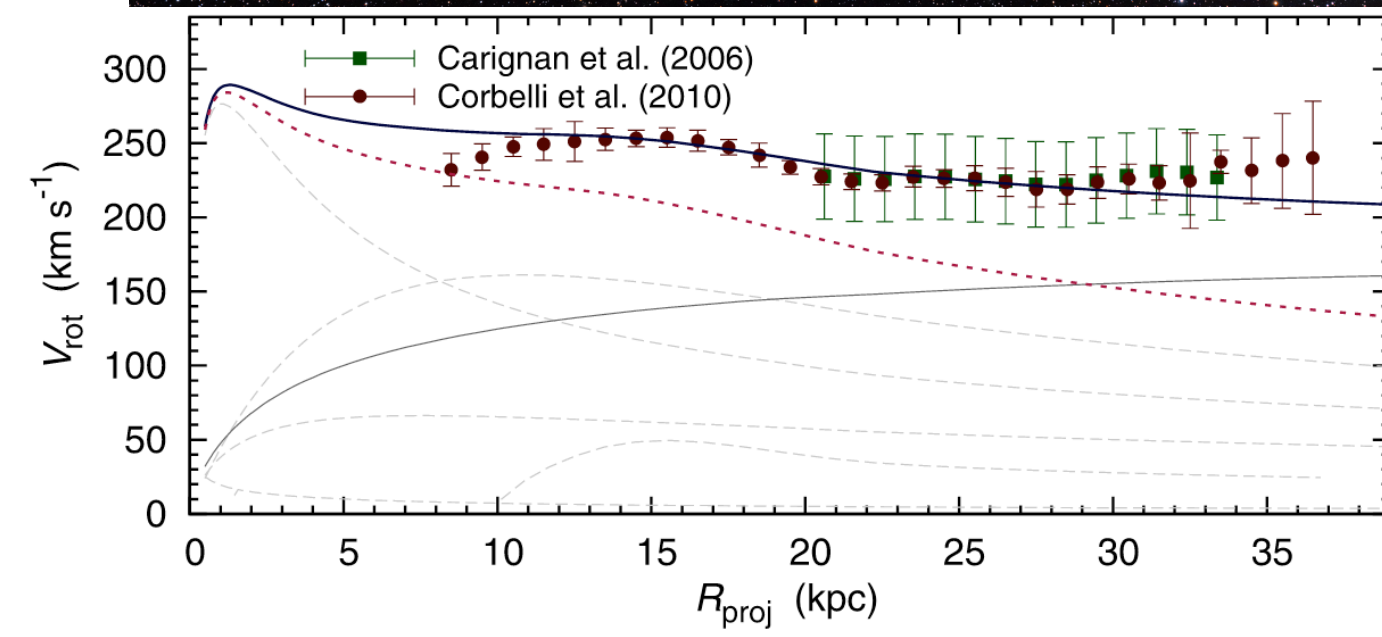
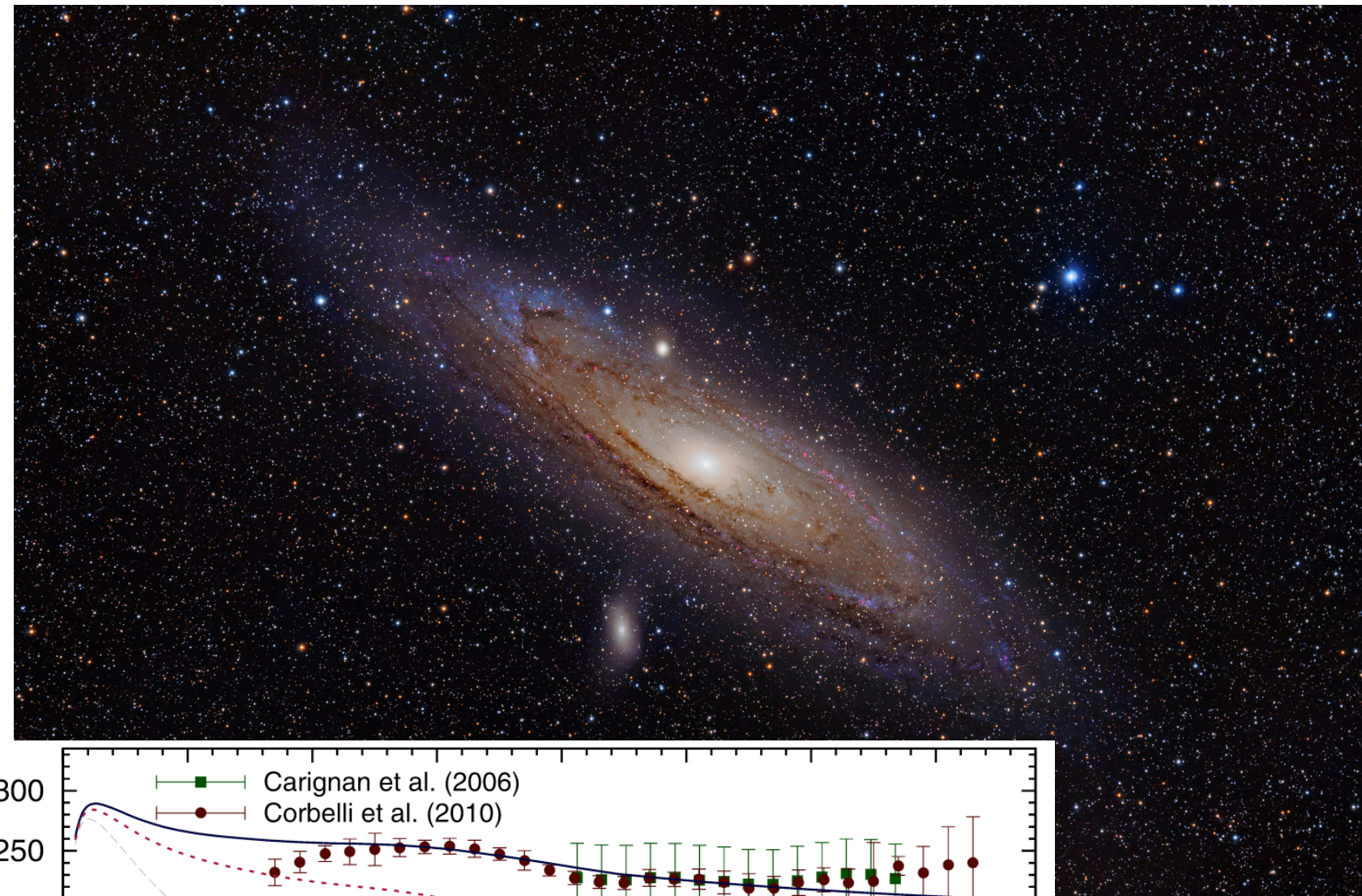
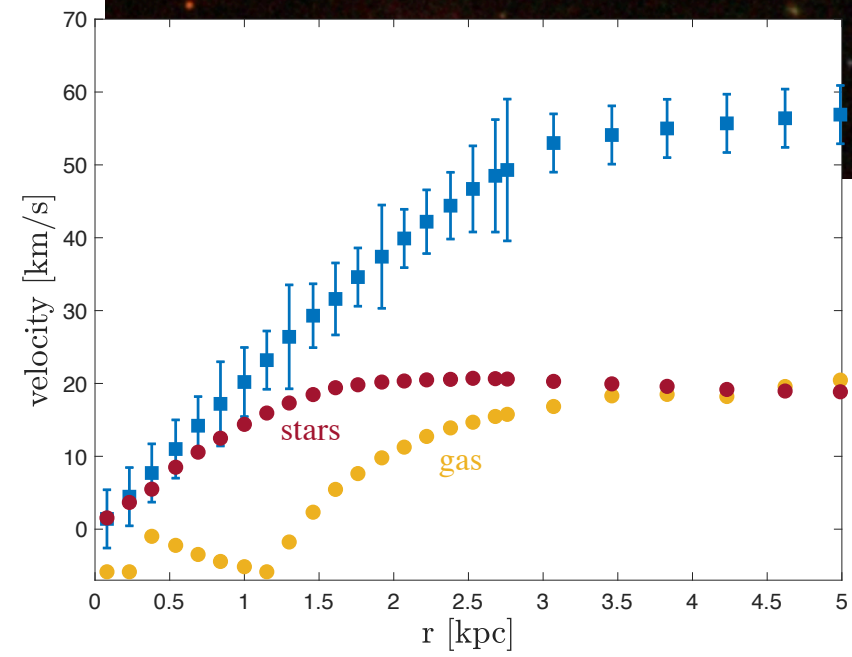
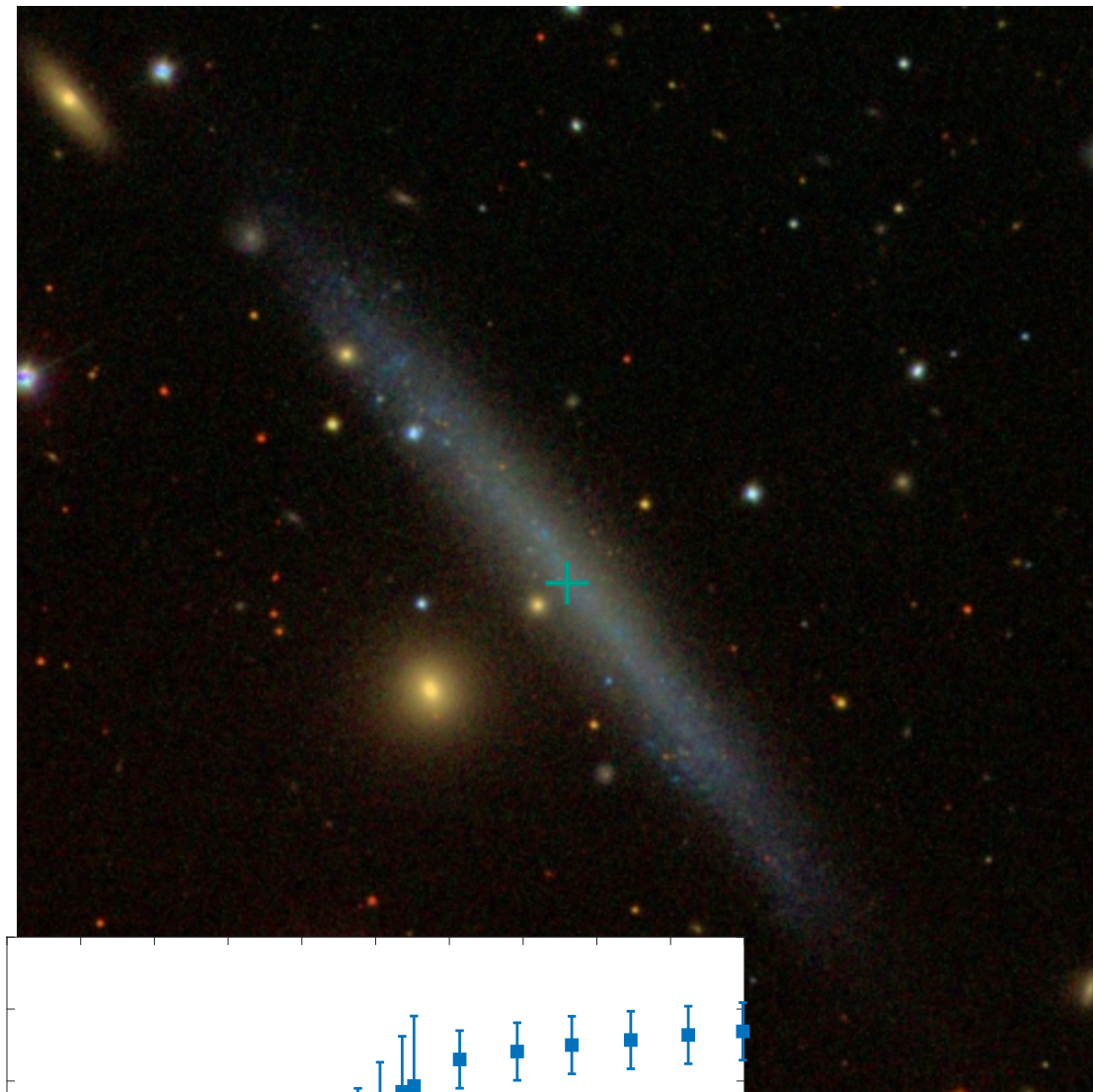


Dark matter

$$m_X$$

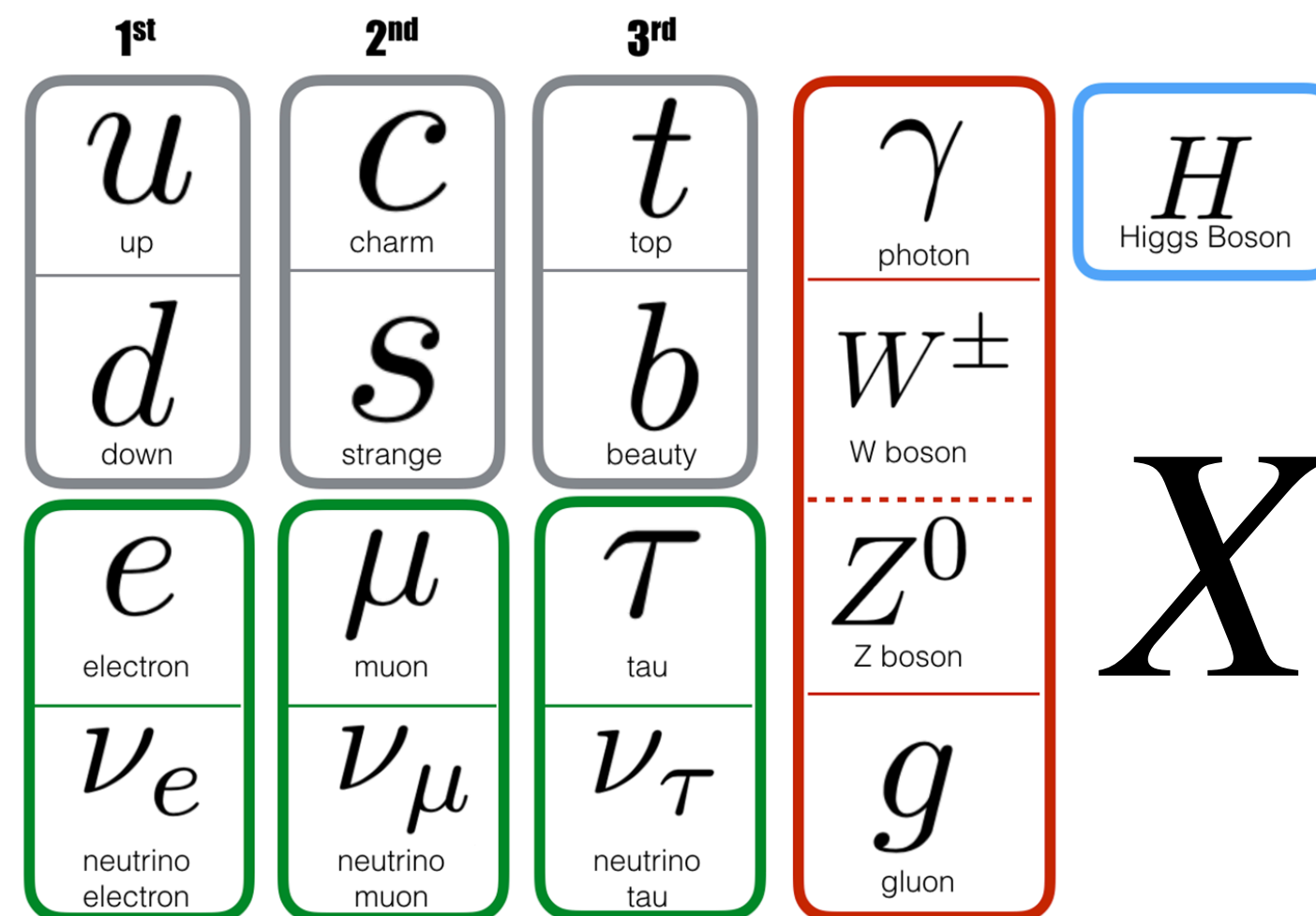
$$10^{-21} \text{ eV}$$

$$10^{-10} M_\odot \sim 10^{56} \text{ eV}$$



m_X 10^6 eV 10^{12} eV 10^{-21} eV

?

 $10^{-10} M_\odot \sim 10^{56} \text{ eV}$ 

m_X

10^6 eV 10^{12} eV

10^{-21} eV

?

$10^{-10} M_\odot \sim 10^{56} \text{ eV}$

“Indirect”

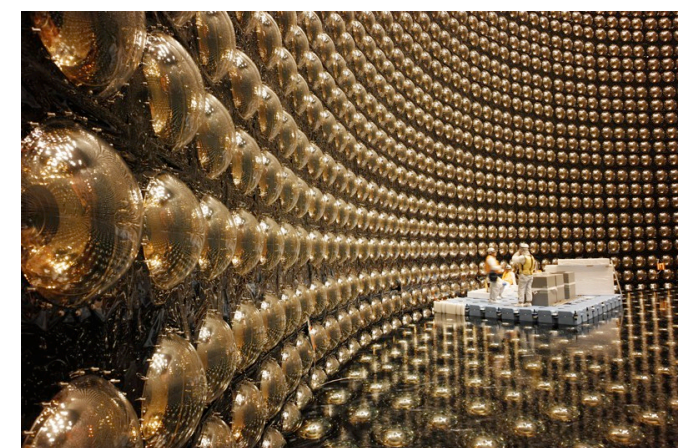
“Direct”



AMS



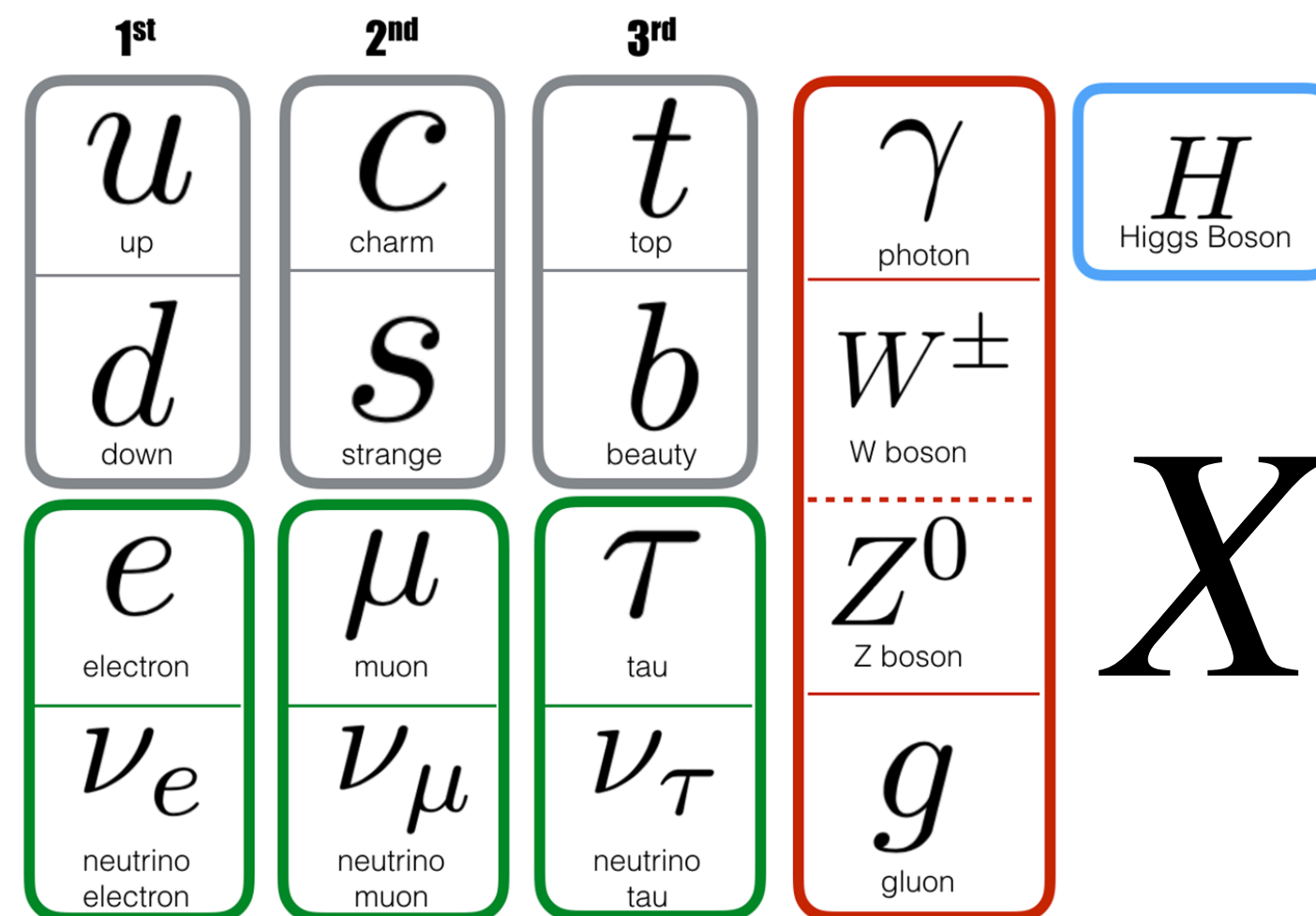
Fermi



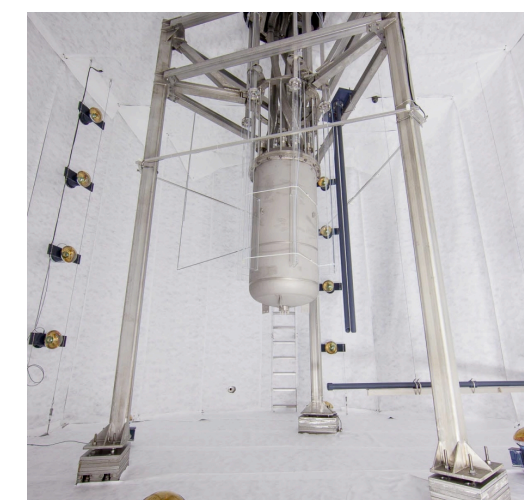
Super K



HESS



Xenon1T



LUX



SENSEI

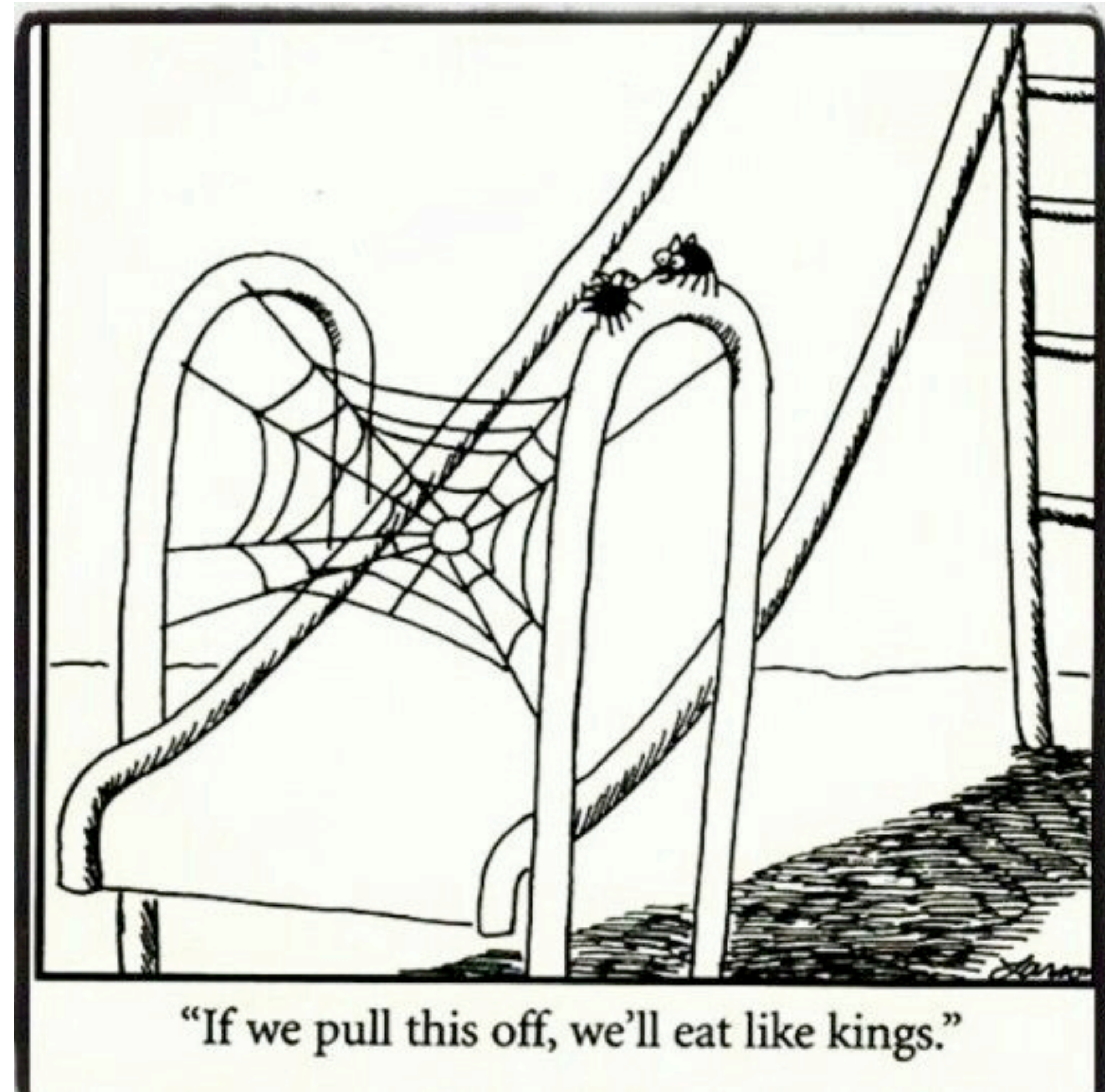
m_X

10^6 eV 10^{12} eV

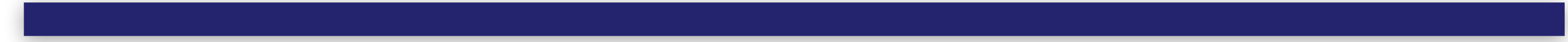
10^{-21} eV

?

$10^{-10} M_\odot \sim 10^{56} \text{ eV}$



m_X



10^{-21} eV

$10^{-10} M_\odot \sim 10^{56}$ eV

Gravity alone



Gravity alone



1. Ultralight dark matter

2. Beyond mean-field: dynamical friction / heating

3. Gravitational lensing

Gravity alone

m_{χ}

10^{-21} eV

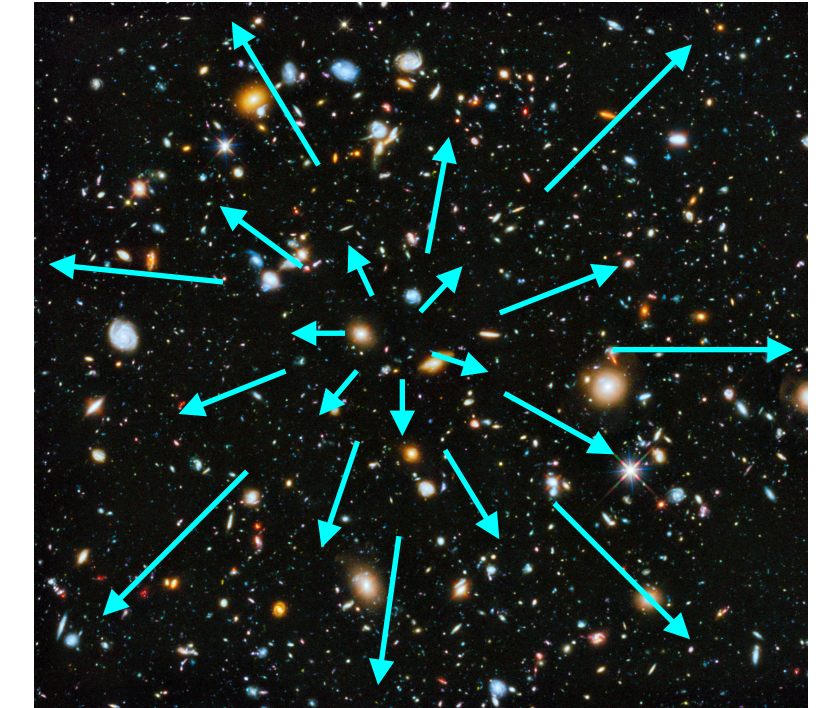
$10^{-10} M_{\odot} \sim 10^{56}$ eV



Thank you!

Where does it end?

Cosmic expansion:



$$\chi^{\nu} = \begin{pmatrix} t \\ r \\ \theta \\ \varphi \end{pmatrix}; \quad g_{\mu\nu} = \begin{pmatrix} f[r] & 0 & 0 & 0 \\ 0 & -\frac{1}{f[r]} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin[\theta]^2 \end{pmatrix};$$

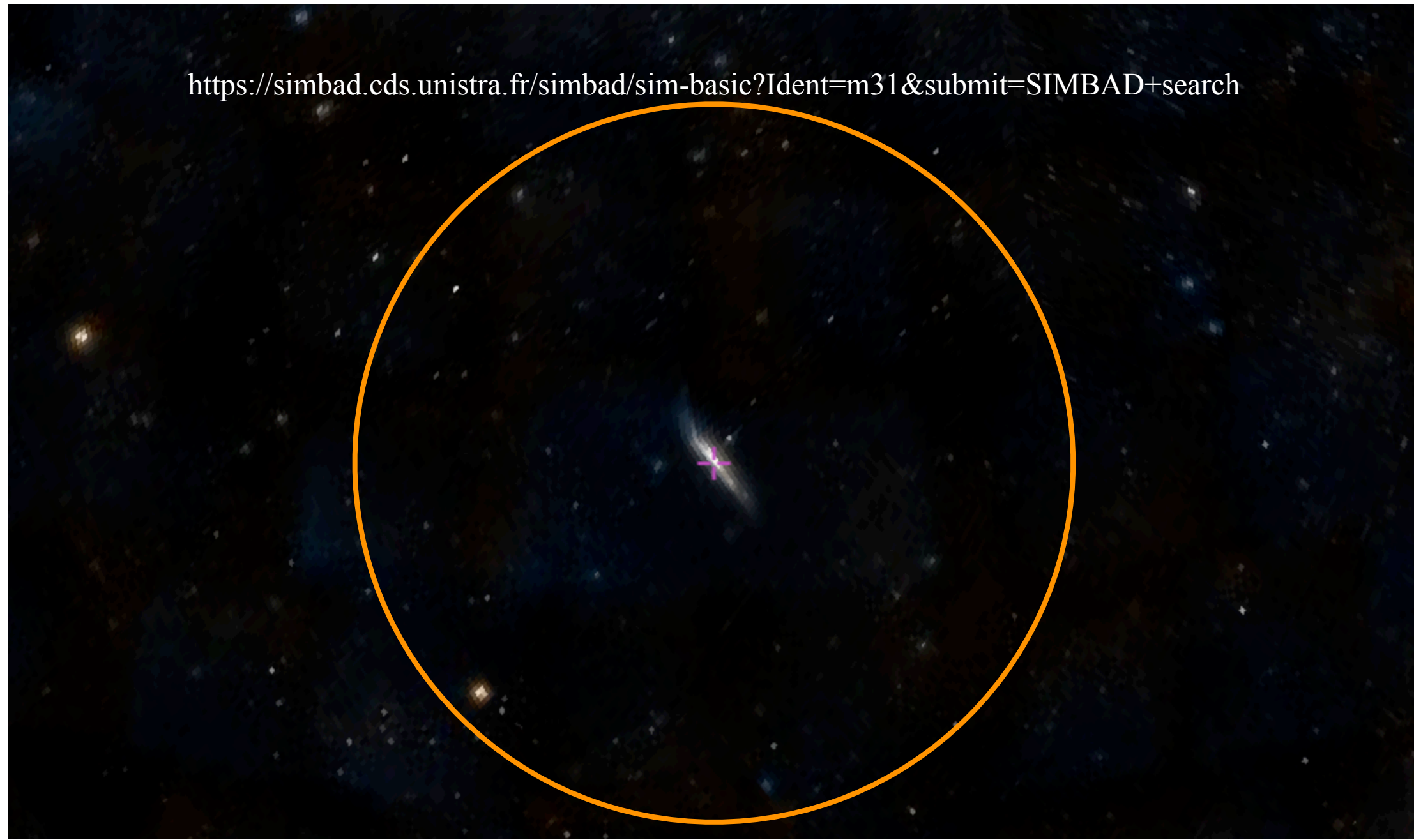
`gI $\mu\nu$ = Inverse[g $\mu\nu$];`

`dgdxtab = Table[{D[g $\mu\nu$, χ^{ν} [[i , 1]]]}, { i , 1, 4}];`

`Γ = Table[{Table[$\frac{1}{2}$ Sum[gI $\mu\nu$ [[i , μ]] (dgdxtab[[k , 1]][[μ , j]] + dgdxtab[[j , 1]][[μ , k]] - dgdxtab[[μ , 1]][[j , k]]), { μ , 1, 4}], { j , 1, 4}, { k , 1, 4}], { i , 1, 4}];`

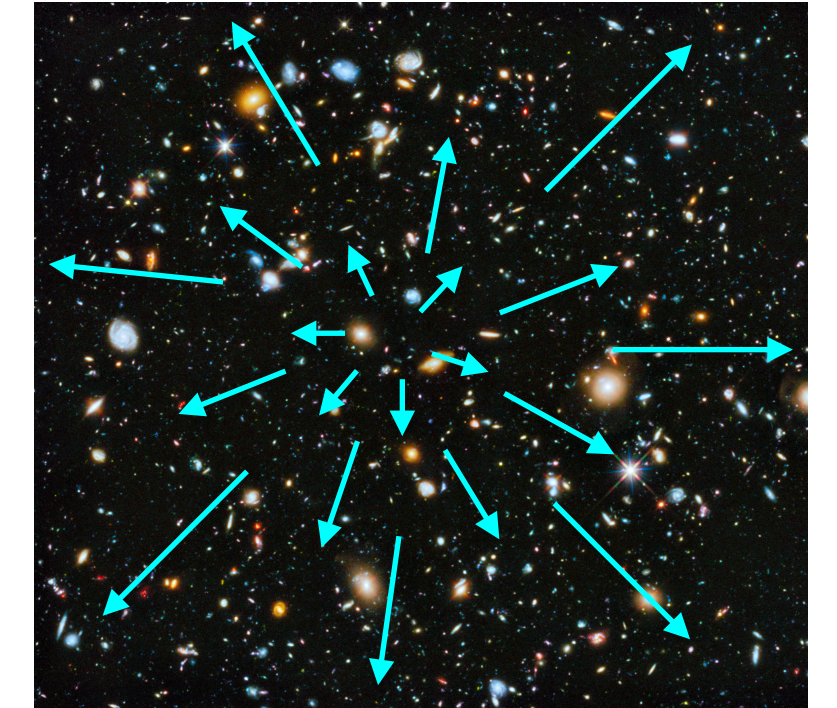
`R $\mu\nu$ = (Table[Sum[D[Γ [[α , 1]][[μ , ν]], χ^{ν} [[α , 1]]], { α , 1, 4}], { μ , 1, 4}, { ν , 1, 4}] - Table[Sum[D[Γ [[α , 1]][[μ , α]], χ^{ν} [[ν , 1]]], { α , 1, 4}], { μ , 1, 4}, { ν , 1, 4}] + Table[Sum[(Γ [[α , 1]][[λ , α]]) (Γ [[λ , 1]][[μ , ν]]), { α , 1, 4}, { λ , 1, 4}], { μ , 1, 4}, { ν , 1, 4}] - Table[Sum[(Γ [[λ , 1]][[μ , α]]) (Γ [[α , 1]][[λ , ν]]), { α , 1, 4}, { λ , 1, 4}], { μ , 1, 4}, { ν , 1, 4}]);`

`R = Sum[gI $\mu\nu$ [[μ , ν]] \times R $\mu\nu$ [[μ , ν]], { μ , 1, 4}, { ν , 1, 4}];`



Where does it end?

Cosmic expansion:



$$x^\nu = \begin{pmatrix} t \\ r \\ \theta \\ \phi \end{pmatrix}; \quad g_{\mu\nu} = \begin{pmatrix} f[r] & 0 & 0 & 0 \\ 0 & -\frac{1}{f[r]} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin[\theta]^2 \end{pmatrix};$$

$$\text{In[*]} := f[r_] := 1 - \frac{2}{r} - H^2 r^2;$$

$$\text{FullSimplify}\left[\left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 3 H^2 g_{\mu\nu}\right) // \text{MatrixForm}\right]$$

Out[*]//MatrixForm=

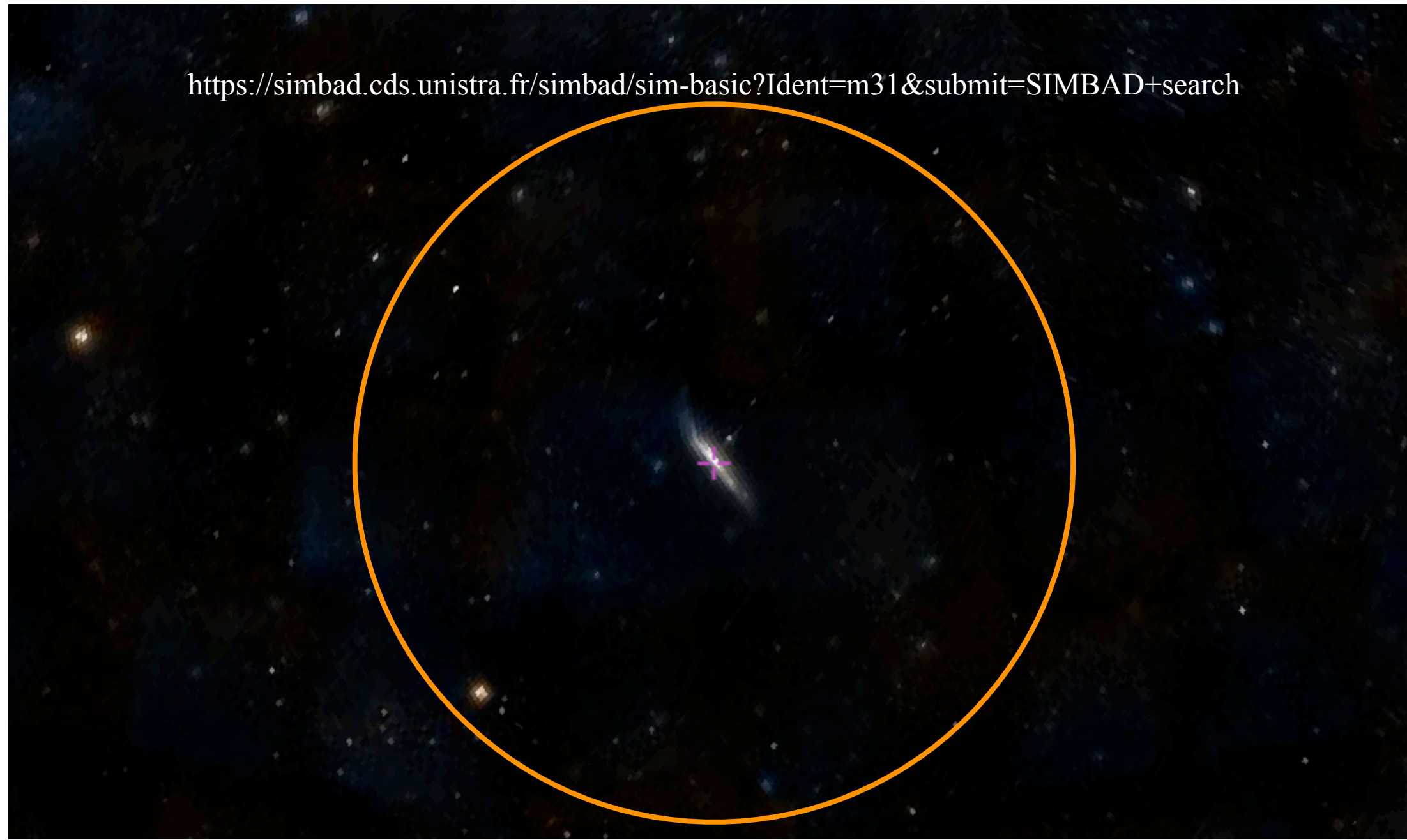
$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$T_{\mu\nu} \approx g_{\mu\nu} \rho_\Lambda, \quad H_0^2 \approx \frac{8\pi}{3} G \rho_\Lambda$$

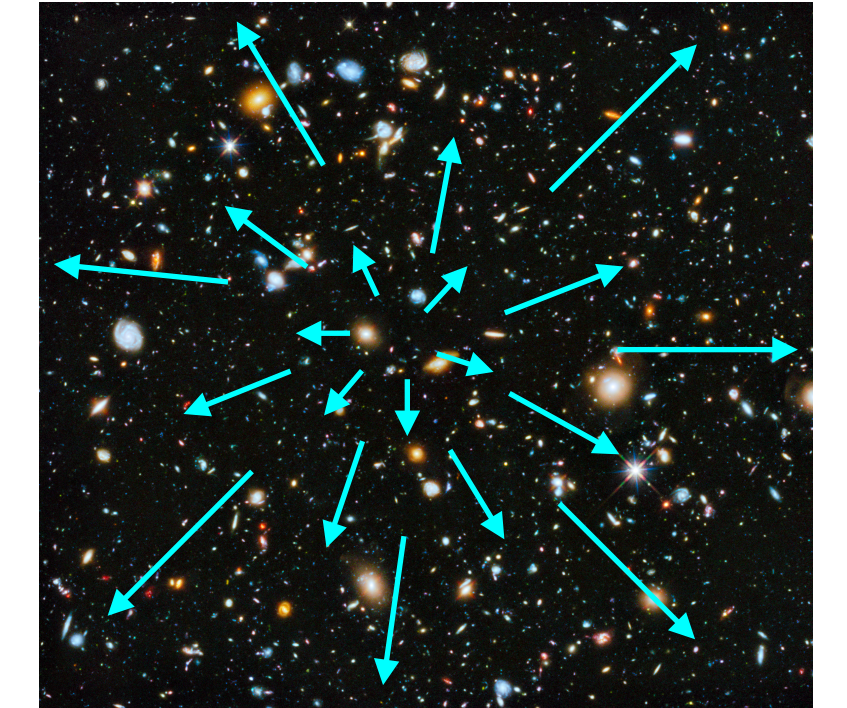
$$\longrightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 3H_0^2 g_{\mu\nu}$$

$$\text{Solved by: } f(r) = 1 - \frac{2}{r} - H_0^2 r^2$$



Where does it end?

Cosmic expansion:



$$\text{In[*]:= } \mathbf{xvp} = \begin{pmatrix} \mathbf{t}'[p] \\ \mathbf{r}'[p] \\ \theta'[p] \\ \varphi'[p] \end{pmatrix};$$

`Simplify[r''[p] + Sum[Simplify[Flatten[r[[2]], 1][[μ]][[ν]] × xvp[[μ]] × xvp[[ν]], {μ, 1, 4}, {ν, 1, 4}]] == 0`
`Simplify[θ''[p] + Sum[Simplify[Flatten[r[[3]], 1][[μ]][[ν]] × xvp[[μ]] × xvp[[ν]], {μ, 1, 4}, {ν, 1, 4}]] == 0`
`Simplify[φ''[p] + Sum[Simplify[Flatten[r[[4]], 1][[μ]][[ν]] × xvp[[μ]] × xvp[[ν]], {μ, 1, 4}, {ν, 1, 4}]] == 0`
`Simplify[t''[p] + Sum[Simplify[Flatten[r[[1]], 1][[μ]][[ν]] × xvp[[μ]] × xvp[[ν]], {μ, 1, 4}, {ν, 1, 4}]] == 0`

$$\text{Out[*]= } \left\{ \frac{(1 - H^2 r^3) r'[p]^2}{r (2 - r + H^2 r^3)} + \frac{(-1 + H^2 r^3) (2 - r + H^2 r^3) t'[p]^2}{r^3} + (2 - r + H^2 r^3) \theta'[p]^2 + (2 - r + H^2 r^3) \sin[\theta]^2 \varphi'[p]^2 + r''[p] \right\} == 0$$

$$\text{Out[*]= } \left\{ \frac{2 r'[p] \theta'[p]}{r} - \cos[\theta] \sin[\theta] \varphi'[p]^2 + \theta''[p] \right\} == 0$$

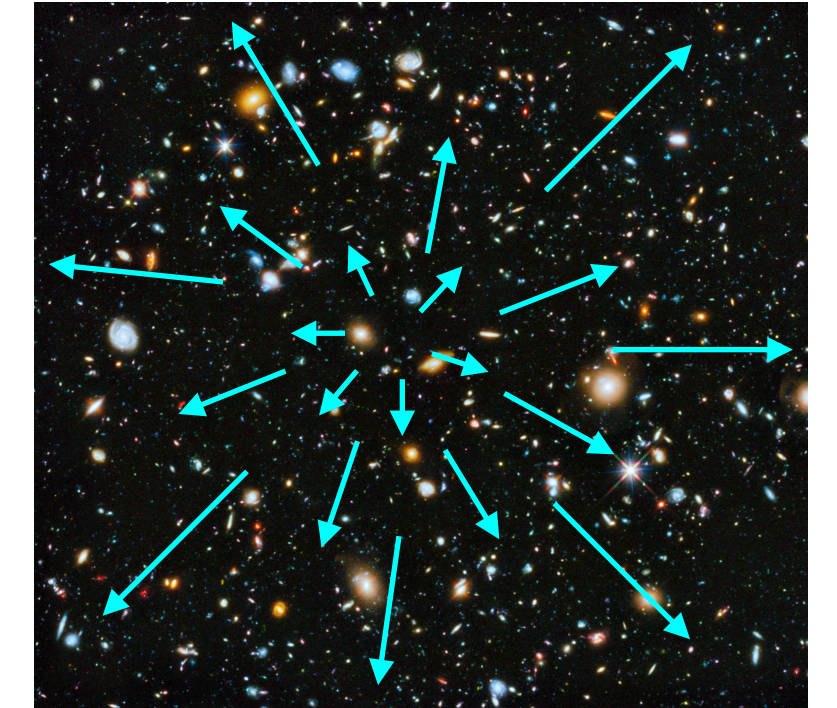
$$\text{Out[*]= } \left\{ \frac{2 r'[p] \varphi'[p]}{r} + 2 \cot[\theta] \theta'[p] \varphi'[p] + \varphi''[p] \right\} == 0$$

$$\text{Out[*]= } \left\{ \frac{2 (-1 + H^2 r^3) r'[p] t'[p]}{r (2 - r + H^2 r^3)} + t''[p] \right\} == 0$$

$$\frac{d^2 x^\mu}{dp^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{dp} \frac{dx^\beta}{dp} = 0$$

Where does it end?

Cosmic expansion:



$$\frac{d^2 x^\mu}{dp^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{dp} \frac{dx^\beta}{dp} = 0$$

Looking for a circular orbit solution:

$$\theta = \frac{\pi}{2} = \text{Const}, \quad r = R = \text{Const}$$

Leads to:

$$\frac{d\varphi}{dp} = C \Omega = \text{Const}$$

$$\frac{dt}{dp} = C = \text{Const}$$

The r equation becomes:

$$(2 - R + H_0^2 R^3) \Omega^2 - \frac{(1 - H_0^2 R^3) (2 - R + H_0^2 R^3)}{R^3} = 0$$

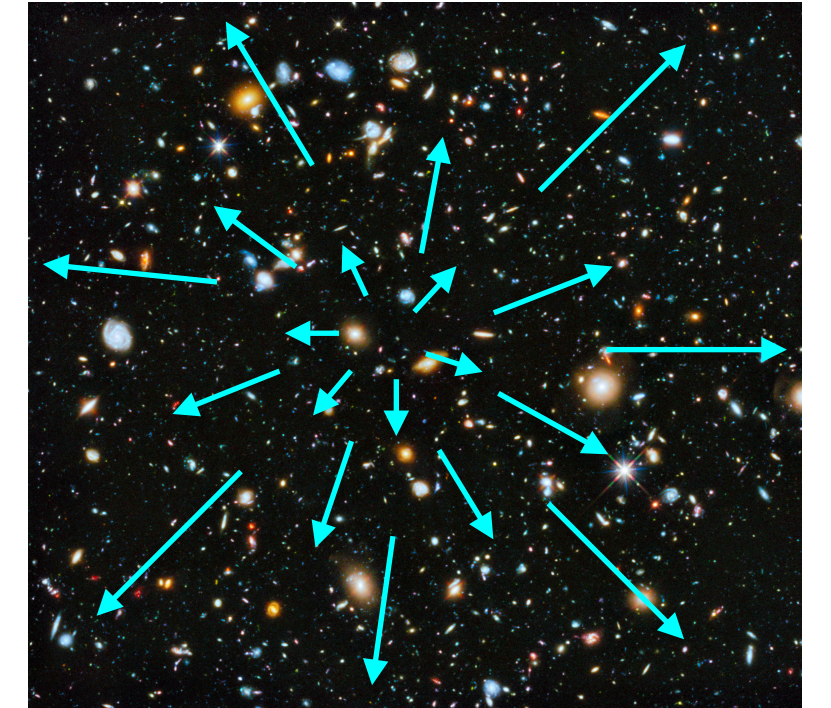
Restoring units:

$$\Omega^2 = \frac{GM}{R^3} \left(1 - \frac{H_0^2 R^3}{GM} \right)$$

<https://simbad.cds.unistra.fr/simbad/sim-basic?Ident=m31&submit=SIMBAD+search>

Where does it end?

Cosmic expansion:



Expansion modifies law of rotation around a point mass M :

$$\Omega^2 = \frac{GM}{R^3} \left(1 - \frac{H_0^2 R^3}{GM} \right)$$



Project AMIGA: The Circumgalactic Medium of Andromeda*

Nicolas Lehner¹ , Samantha C. Berek^{1,2,20} , J. Christopher Howk¹ , Bart P. Wakker³ , Jason Tumlinson^{4,5} ,
 Edward B. Jenkins⁶ , J. Xavier Prochaska⁷ , Ramona Augustin⁴ , Suoqing Ji⁸ , Claude-André Faucher-Giguère⁹ ,
 Zachary Hafen⁹ , Molly S. Peeples^{4,5} , Kat A. Barger¹⁰ , Michelle A. Berg¹ , Rongmon Bordoloi¹¹ , Thomas M. Brown⁴ ,
 Andrew J. Fox¹² , Karoline M. Gilbert^{4,5} , Puragra Guhathakurta⁷ , Jason S. Kalirai¹³ , Felix J. Lockman¹⁴ ,
 John M. O'Meara¹⁵ , D. J. Pisano^{16,17,21} , Joseph Ribaldo¹⁸ , and Jessica K. Werk¹⁹

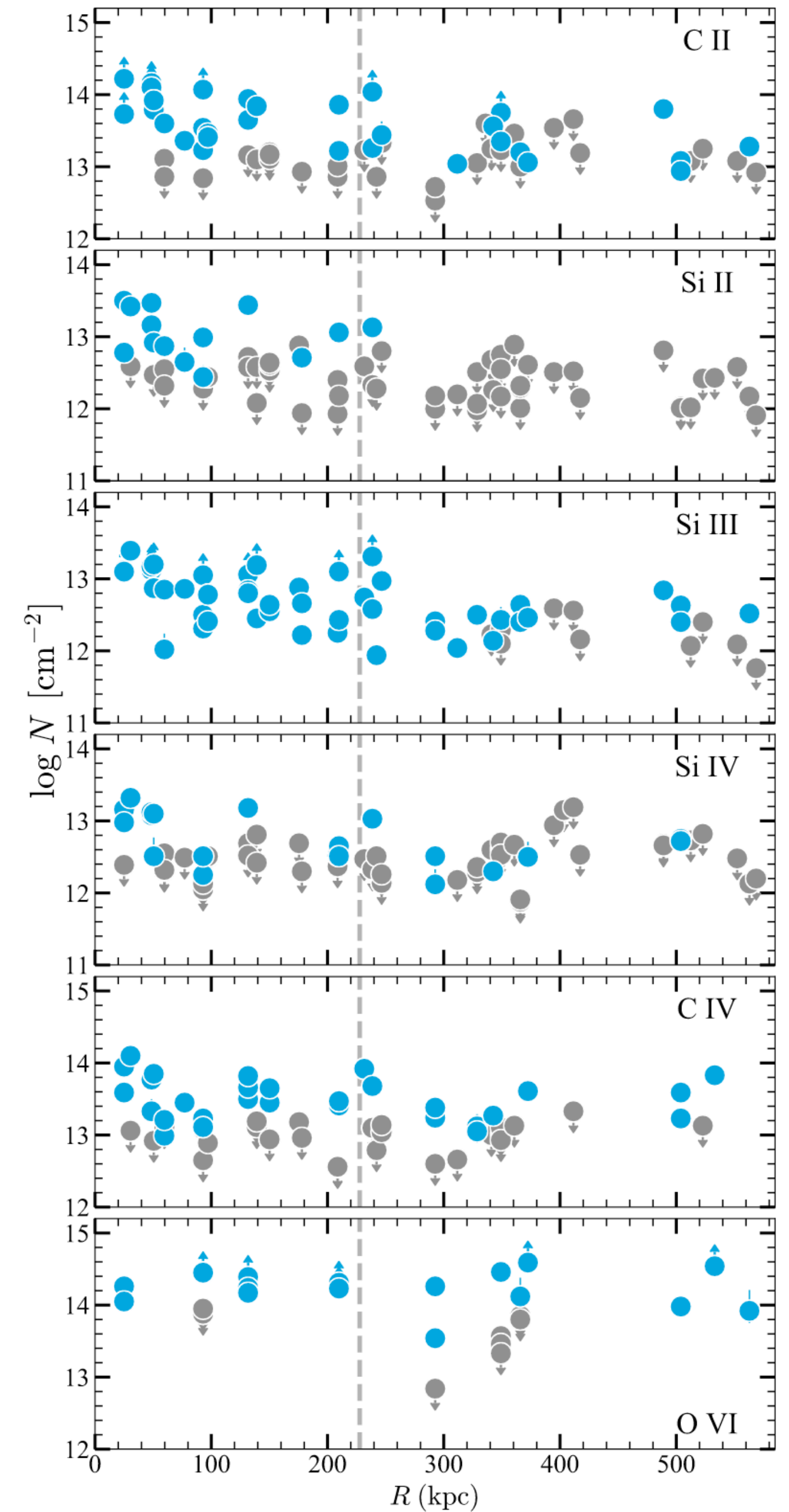
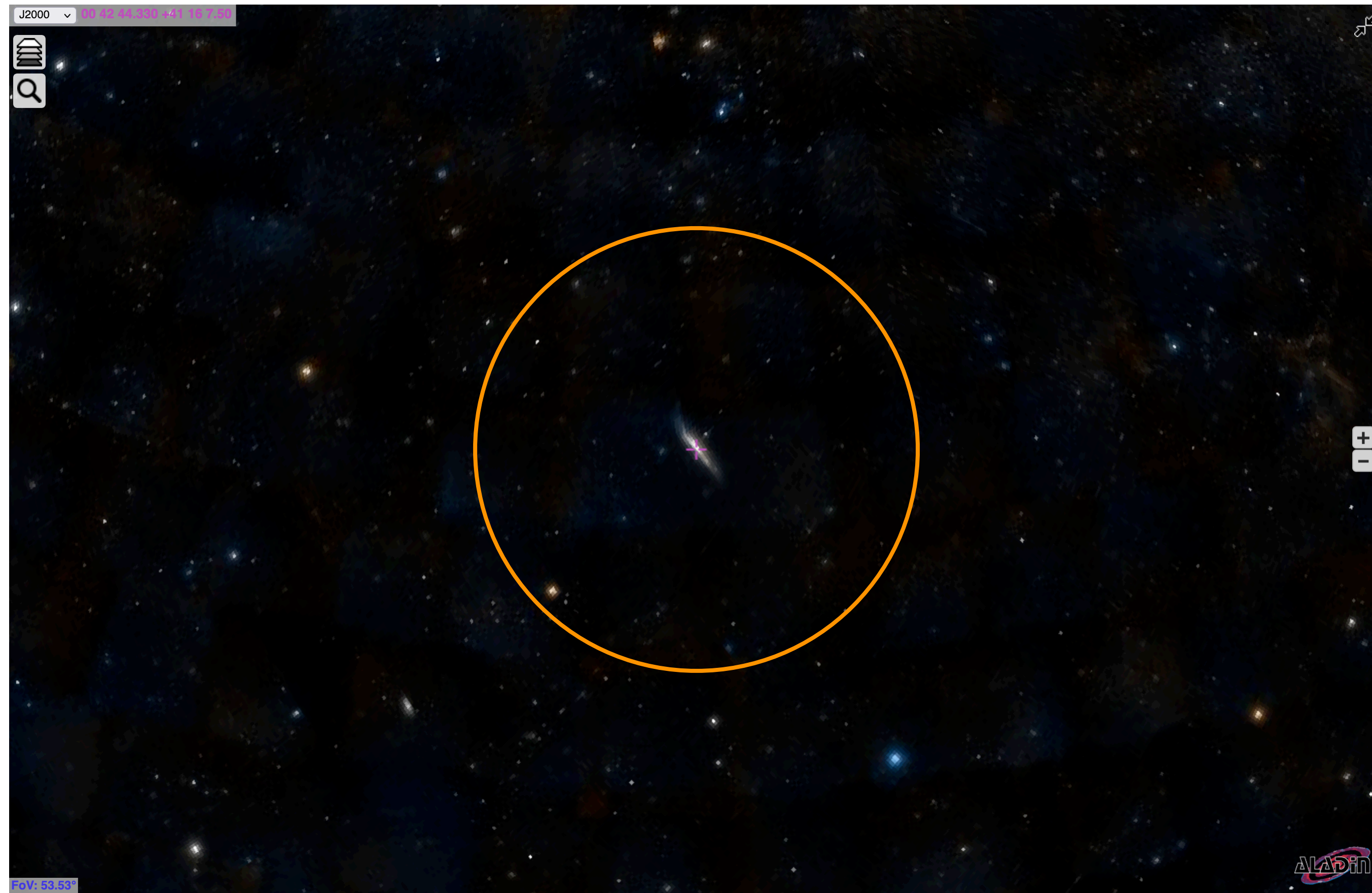


Figure 9. Logarithm of the column densities for the individual components of various ions (low to high ions from top to bottom) as a function of the projected distances from M31 of the background QSOs. Blue circles are detections, while

Low-surface-brightness galaxy: UGC1281

