XXXIII Canary Islands Winter School of Astrophysics

Fundamental Physics with Galaxies

Kfir Blum | Weizmann Institute of Science
The Standard Model of Particle Physics

- No gravity
- Does not include neutrino mass
- Does not include dark matter
- Cosmic inflation (origin of Universe)
- Baryon asymmetry (origin of matter)

- Why vacuum energy just (not) zero?
- Why weak scale << Planck scale?
- Why no strong CP violation?
- Almost grand unification?
- Why Higgs vacuum just metastable?
The Standard Model of Particle Physics and Astrophysics

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Why vacuum energy just (not) zero?
Why weak scale << Planck scale?
Why no strong CP violation?
Almost grand unification?
Why Higgs vacuum just metastable?

- Observed first in astrophysics
- Observed only in astrophysics
- Constrained by astrophysics
Al-Sufi, around 1000 AD
Harlow Shapley

The Great Debate 1920

Heber Curtis
...a good friendly “scrap” is an excellent thing... sort of clears out the atmosphere.
To shake hands at the beginning and conclusion, but use our shillelaghs in the interim to the best of our ability.
...a good friendly “scrap” is an excellent thing... sort of clears out the atmosphere. To shake hands at the beginning and conclusion, but use our *shillelaghs* in the interim to the best of our ability.
Edwin Hubble

1923
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Dark matter
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ROTATION OF THE ANDROMEDA NEBULA FROM A SPECTROSCOPIC SURVEY OF EMISSION REGIONS

VERA C. RUBIN† AND W. KENFORD, Jr."†
Department of Terrestrial Magnetism, Carnegie Institution of Washington and Lowell Observatory, and Kitt Peak National Observatory

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\[ V^2 = \frac{GM}{R} \]

\[ M_{200} = \frac{V^2 R_{200}}{G} \approx \left( 150 \times 10^5 \text{ cm/s} \right)^2 \times (200 \times 3 \times 10^{21} \text{ cm}) \approx 2.07 \times 10^{45} \text{ g} \approx 1.04 \times 10^{12} \text{ M}_\odot \]

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\[ \frac{1}{200} \times \frac{M_{200}}{\frac{4\pi}{3} R_{200}^3} \approx \frac{1}{200} \times \frac{10^{12} \times 2 \times 10^{33} \text{g}}{\frac{4\pi}{3} \left(200 \times 3 \times 10^{21} \text{ cm} \right)^3} \approx 1.1 \times 10^{-29} \frac{\text{g/cm}^3}{\text{M}_\odot \text{kpc}^3} \sim \rho_c \]
Where does it end?
Where does it end?

**Time to make 1 revolution:**

\[ T = \frac{2\pi R}{V} \]

\[ T = \frac{2\pi \times 200 \text{ kpc}}{150 \text{ km/s}} \approx 8.2 \text{ Gyr} \]

\[ T = \frac{2\pi \times 400 \text{ kpc}}{150 \text{ km/s}} \approx 16.4 \text{ Gyr} \]
Where does it end?

Cosmic neighbors: \[ \rho_m \approx 0.3 \rho_c \approx \frac{10^{12} \text{ M}_\odot}{(3 \text{ Mpc})^3} \]

MW-M31 distance \( \sim \) 770 kpc
Where does it end?

Cosmic expansion:


Figure 10

Graphical results of the Hubble Space Telescope Key Project (Freedman et al. 2001). (Top) The Hubble diagram of distance versus velocity for secondary distance indicators calibrated by Cepheids. Velocities are corrected using the nearby flow model of Mould et al. (2000). Dark yellow squares, Type Ia supernovae; filled red circles, Tully-Fisher (TF) clusters (I-band observations); blue triangles, fundamental plane clusters; purple diamonds, surface brightness fluctuation galaxies; open black squares, Type II supernovae. A slope of $H_0 = 72 \pm 7 \text{km s}^{-1} \text{Mpc}^{-1}$ is shown (solid and dotted gray lines). Beyond 5,000 km s$^{-1}$ (vertical dashed line), both numerical simulations and observations suggest that the effects of peculiar motions are small. The Type Ia supernovae extend to about 30,000 km s$^{-1}$, and the TF and fundamental plane clusters extend to velocities of about 9,000 and 15,000 km s$^{-1}$, respectively. However, the current limit for surface brightness fluctuations is about 5,000 km s$^{-1}$.

(Bottom) The galaxy-by-galaxy values of $H_0$ as a function of distance. We update this analysis using the new HST-parallax Galactic calibration of the Cepheid zero point (Benedict et al. 2007) and the new supernova data from Hicken et al. (2009). We find an interval value of $H_0$, but with reduced systematic uncertainty, of $H_0 = 73 \pm 2$ (random) $\pm 4$ (systematic) km s$^{-1} \text{Mpc}^{-1}$. The reduced systematic uncertainty, discussed further in Section 4.1 below, results from having a more robust zero-point calibration based on the Milky Way Galaxy with comparable metallicity to the spiral galaxies in the HST Key Project sample. Although, the new parallax calibration results in a shorter distance to the LMC (which is no longer used here as a calibrator), the difference in $H_0$ is nearly offset by the fact that no metallicity correction is needed to offset the difference in metallicity between the LMC and calibrating galaxies.
Expansion modifies law of rotation around a point mass $M$:

$$\Omega^2 = \frac{GM}{R^3} \left(1 - \frac{H_0^2 R^3}{GM}\right)$$

$$V = \Omega R \quad \rightarrow \quad V^2 = \frac{GM}{R} \left(1 - \frac{R^3}{R_c^3}\right)$$

$$R_c = \left(\frac{GM}{H_0^2}\right)^{\frac{1}{3}} \approx 0.9 \left(\frac{M}{10^{12} M_\odot}\right)^{\frac{1}{3}} \left(\frac{10^{-29} \text{ g/cm}^3}{\rho_c}\right)^{\frac{1}{3}} \text{ Mpc}$$
Stellar mass map and dark matter distribution in M 31

A. Toom3, E. Tempel1,2, P. Tenjes1,3, O. Tibtonova1,3, and T. Tupikova1
Fig. 2. Stellar mass-density map of M 31. The ellipses enclose 50%, 75%, 90%, and 95% of the total mass, respectively.

Table 1. Synthetic stellar populations used for SED fitting.

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<tr>
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<th>Age [Gyr]</th>
<th>[Fe/H]</th>
<th>$L/L_\odot$</th>
<th>$M_3/L_\odot$</th>
<th>$M_2/L_\odot$</th>
<th>$M_1/L_\odot$</th>
<th>Fract.</th>
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</thead>
<tbody>
<tr>
<td>B07-1</td>
<td>0.7</td>
<td>0.40</td>
<td>0.76</td>
<td>0.78</td>
<td>0.72</td>
<td>0.014</td>
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</tr>
<tr>
<td>B07-3</td>
<td>0.4–1</td>
<td>0.05</td>
<td>0.47</td>
<td>0.50</td>
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<td>0.003</td>
<td></td>
</tr>
<tr>
<td>B07-4</td>
<td>7–12</td>
<td>0.03</td>
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<td>0.983</td>
<td></td>
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<td>1</td>
<td>0.00</td>
<td>1.11</td>
<td>1.00</td>
<td>0.85</td>
<td>0.008</td>
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<td>2</td>
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<td>M05-3</td>
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<tr>
<td>GALEV-1</td>
<td>1, 10</td>
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### Dark matter

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<tr>
<td>M05-5</td>
<td>12</td>
<td>-0.33</td>
<td>9.00</td>
<td>6.60</td>
<td>5.37</td>
<td>0.009</td>
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<tr>
<td>GALEV-1</td>
<td>1, 10</td>
<td>0.04</td>
<td>2.88</td>
<td>3.14</td>
<td>2.92</td>
<td>0.004</td>
</tr>
<tr>
<td>GALEV-2</td>
<td>2, 11</td>
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<td>4.13</td>
<td>3.65</td>
<td>0.011</td>
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<tr>
<td>GALEV-3</td>
<td>4, 13</td>
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<td>7.58</td>
<td>6.20</td>
<td>5.23</td>
<td>0.089</td>
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<tr>
<td>GALEV-4</td>
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<td>4.63</td>
<td>4.55</td>
<td>4.05</td>
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<tr>
<td>GALEV-5</td>
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<td>10.9</td>
<td>8.33</td>
<td>6.86</td>
<td>0.881</td>
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</table>

**Notes.** The columns contain the following: (1) stellar population model; for B07 models the number is as in the original paper; (2) approximate age of the dominant star-formation epoch(s); (3) average metallicity of the stars; (4)–(6) mass-to-light ratio in the gri filters; (7) total stellar mass fraction in M 31 of the corresponding stellar population.

---

**Fig. 2.** Stellar mass-density map of M 31. The ellipses enclose 50%, 75%, 90%, and 95% of the total mass, respectively.
Fig. 2. Stellar mass-density map of M 31. The ellipses enclose 50%, 75%, 90%, and 95% of the total mass, respectively.

Fig. 7. Observed rotation curve together with the maximum-stellar model, in which the stellar masses are 1.5 times higher than in the B07 model.
At ~30 kpc:

$\frac{M(\text{DM})}{M(\text{stars})} \sim 1$

At ~300 kpc:

$\frac{M(\text{DM})}{M(\text{stars})} \sim 10$

Fig. 7. Observed rotation curve together with the maximum-stellar model, in which the stellar masses are 1.5 times higher than in the B07 model.
How does this fit into the big picture?
How does this fit into the big picture?
How does this fit into the big picture?

Map $d = 400 \text{ kpc}$ (today) into $\Delta z$ near $z \sim 0$:

$$
\Delta \eta = c \int_{t_1}^{t_2} \frac{dt'}{a(t')} = c \int_z^{z + \Delta z} \frac{dz'}{H(z')} \approx \frac{c}{H_0} \int_z^{z + \Delta z} \frac{dz'}{\sqrt{\Omega_\Lambda + (1 + z')^3 \Omega_m}}
$$

$$
\approx \frac{c \Delta z}{H_0 \sqrt{\Omega_\Lambda + (1 + z)^3 \Omega_m}}
$$

$$
\approx 400 \text{ kpc} \frac{\Delta z}{10^{-4}}
$$
Map \( d = 1 \) Mpc (today) into \( \Delta z \) near \( z \sim 0 \):

\[
\Delta \eta = c \int_{t}^{t'} \frac{dt'}{a(t')} = c \int_{z}^{z+\Delta z} \frac{dz'}{H(z')} \approx \frac{c}{H_0} \int_{z}^{z+\Delta z} \frac{dz'}{\sqrt{\Omega_\Lambda + (1 + z')^3 \Omega_m}}
\]

\[
\approx \frac{c \Delta z}{H_0 \sqrt{\Omega_\Lambda + (1 + z)^3 \Omega_m}}
\]

\[
\approx 1 \text{ Mpc} \frac{\Delta z}{2.5 \times 10^{-4}}
\]
Fornax dwarf spheroidal, satellite galaxy of the Milky Way
What is observed:
1. Line-of-sight velocities
2. Column density of stars
\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}_i \frac{\partial f}{\partial x_i} + \dot{p}_i \frac{\partial f}{\partial p_i} = 0
\]
\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}_i \frac{\partial f}{\partial x_i} + \dot{p}_i \frac{\partial f}{\partial p_i} = \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} = 0
\]
\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_i} \frac{\partial x_i}{\partial x_i} + \frac{\partial f}{\partial p_i} \frac{\partial p_i}{\partial x_i} = \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} = 0
\]

\[
\int d^3p v_j \left( \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} \right) = \frac{\partial}{\partial t} \left( n \bar{v}_j \right) + \frac{\partial}{\partial x_i} \left( n \bar{v}_i \bar{v}_j \right) + n \frac{\partial \Phi}{\partial x_j} = 0
\]

\[
n \bar{v}_i \bar{v}_j(x) = \int d^3p v_i v_j f(x, v)
\]
Assume steady state

\[
\frac{\partial}{\partial t} (n\bar{v}_j) + \frac{\partial}{\partial x_i} (n\bar{v}_i\bar{v}_j) + n \frac{\partial \Phi}{\partial x_j} = 0
\]
Assume steady state

\[
\frac{\partial}{\partial t} (n \overline{v}_j) + \frac{\partial}{\partial x_i} (n \overline{v}_i \overline{v}_j) + n \frac{\partial \Phi}{\partial x_j} = 0
\]

Spherical symmetry

\[
\frac{\partial}{\partial x_i} (n \overline{v}_i \overline{v}_j) \rightarrow \frac{\partial}{\partial r} (n \overline{v}_r^2) + n \frac{2 \overline{v}_r^2 - \overline{v}_i^2}{r} = \frac{\partial}{\partial r} \left( n \overline{v}_r^2 \right) + \frac{2n \beta \overline{v}_r^2}{r}
\]

\[
\frac{\partial \Phi}{\partial x_j} \rightarrow \frac{GM(r)}{r^2}
\]

\[
\beta = 1 - \frac{v_i^2}{2 \overline{v}_r^2}
\]

Data from: Read, Walker, Steger, 1808.06634

Fornax


Assume steady state
\[
\frac{\partial}{\partial t} \left( n \nabla_j \right) + \frac{\partial}{\partial x_i} \left( n \nabla_i \nabla_j \right) + n \frac{\partial \Phi}{\partial x_j} = 0
\]

Spherical symmetry
\[
\frac{\partial}{\partial x_i} \left( n \nabla_i \nabla_j \right) \rightarrow \frac{\partial}{\partial r} \left( n \nabla_r \right) + n \frac{2 \nabla_r^2 - \nabla_i^2}{r} = \frac{\partial}{\partial r} \left( n \nabla_r^2 \right) + \frac{2n \beta \nabla_r^2}{r}
\]
\[
\frac{\partial \Phi}{\partial x_j} \rightarrow \frac{GM(r)}{r^2}
\]

Jeans equation
\[
\frac{1}{n \, dr} \left( n \nabla_r^2 \right) + \frac{2 \beta \nabla_r^2}{r} = - \frac{GM(r)}{r^2}
\]

Data from: Read, Walker, Steger, 1808.06634

Fornax

\[\sigma_{\cos}[\text{km/s}]\]

\[\text{r [pc]}\]

Wang et al (DES), The Astrophys
Data from: Read, Walker, Steger, 1808.06634

\[ \frac{1}{n} \frac{d}{dr} \left( n v_r^2 \right) + \frac{2\beta}{r} v_r^2 = - \frac{GM(r)}{r^2} \]

If \( \beta \) is constant in \( r \):

\[ n v_r^2 = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta - 2} n(y) M(y) \]
Data from: Read, Walker, Steger, 1808.06634

Fornax

0 500 1000 1500 2000

0 5 10 15 20

σ LOS

\[
\frac{1}{n} \frac{d}{dr} \left( n \overline{v_r^2} \right) + \frac{2\beta}{r} \overline{v_r^2} = - \frac{GM(r)}{r^2}
\]

If β is constant in r:

\[
\overline{v_r^2} = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2}n(y)M(y)
\]

What is actually observed — line-of-sight velocity:

\[
\sigma^2_{\text{LOS}}(r) = \frac{2l}{I(r)} \int_r^\infty dy \left( 1 - \frac{\beta r^2}{y^2} \right) \frac{ynv_r^2(y)}{\sqrt{y^2 - r^2}}
\]

\[ n v_r^2 = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y) \]

\[ \sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left( 1 - \frac{\beta r^2}{y^2} \right) \frac{y n_v^2(y)}{\sqrt{y^2 - r^2}} \]
\[ \overline{\sigma_{r}^2} = \frac{G}{r^{2\beta}} \int_{r}^{\infty} dy y^{2\beta - 2} n(y) M(y) \]

\[ \sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_{r}^{\infty} dy \left( 1 - \frac{\beta r^2}{y^2} \right) \frac{y \overline{\sigma_{r}^2}(y)}{\sqrt{y^2 - r^2}} \]

E.g., Plummer profile:

\[ I(r) = \frac{L}{\pi r_p^2 \left( 1 + \frac{r^2}{r_p^2} \right)^{\frac{3}{2}}} \]

\[ n(r) = \frac{3(L/l)}{4\pi r_p^3 \left( 1 + \frac{r^2}{r_p^2} \right)^{\frac{5}{2}}} \]


Data from: Read, Walker, Steger, 1808.06634
\[
\frac{n v_r^2}{r} = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y)
\]

\[
\sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left( 1 - \frac{\beta r^2}{y^2} \right) \frac{y n v_r^2(y)}{\sqrt{y^2 - r^2}}
\]

E.g., Navarro-Frenk-White (NFW):

\[
\rho(r) = \frac{\rho_s}{r \left( \frac{r}{r_s} \left( 1 + \frac{r}{r_s} \right) \right)^2}
\]

![Figure 3](image-url) — Density profiles of four halos spanning 4 orders of magnitude in mass. The arrows indicate the gravitational softening, \(h_s\), of each simulation. Also shown are fits from eq. (3). The fits are good over two decades in radius, approximately from \(h_s\) out to the virial radius of each system.
\[ n v_r^2 = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta-2} n(y) M(y) \]

\[ \sigma_{\text{LOS}}^2(r) = \frac{2l}{L(r)} \int_r^\infty dy \left( 1 - \frac{\beta r^2}{y^2} \right) \frac{y n v_r^2(y)}{\sqrt{y^2 - r^2}} \]

E.g., Navarro-Frenk-White (NFW):

\[ \rho(r) = \frac{\rho_s}{\left( \frac{r}{r_s} \right)^2 \left( \frac{1 + \frac{r}{r_s}}{1 + \frac{r}{r_s}} \right)} \quad r_s = 2 \text{ kpc}, \quad \rho_s = 1.2 \times 10^7 \frac{\text{M}_\odot}{\text{kpc}^3} \]

\[ M(r) = 4\pi \rho_s r_s^3 \left( \ln \left( \frac{1 + \frac{r}{r_s}}{1 + \frac{r}{r_s}} \right) - \frac{r}{r + r_s} \right) \]

Data from: Read, Walker, Steger, 1808.06634

Fornax
\[
\bar{\sigma}_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left(1 - \frac{\beta r^2}{y^2}\right) \frac{y \bar{n}^2_r(y)}{\sqrt{y^2 - r^2}}
\]

E.g., Navarro-Frenk-White (NFW):

\[
\rho(r) = \frac{\rho_s}{r \left(1 + \frac{r}{r_s}\right)^2} \quad r_s = 2 \, \text{kpc}, \quad \rho_s = 1.2 \times 10^7 \, \frac{M_\odot}{\text{kpc}^3}
\]

\[
M(r) = 4\pi \rho_s r_s^3 \left(\ln \left(1 + \frac{r}{r_s}\right) - \frac{r}{r + r_s}\right)
\]
\[ n v_r^2 = \frac{G}{r^{2\beta}} \int_r^\infty dy y^{2\beta - 2} n(y) M(y) \]

\[ \sigma_{\text{LOS}}^2(r) = \frac{2l}{I(r)} \int_r^\infty dy \left( 1 - \frac{\beta r^2}{y^2} \right) \frac{y n v_r^2(y)}{\sqrt{y^2 - r^2}} \]

E.g., Navarro-Frenk-White (NFW):

\[ \rho(r) = \frac{\rho_s}{\frac{r}{r_s} \left( 1 + \frac{r}{r_s} \right)^2} \quad r_s = 2 \text{ kpc}, \quad \rho_s = 1.2 \times 10^7 \frac{M_\odot}{\text{kpc}^3} \]

\[ M(r) = 4\pi \rho_s r_s^3 \left( \ln \left( 1 + \frac{r}{r_s} \right) - \frac{r}{r + r_s} \right) \]

\[ R_{200} \approx r_s \left( \frac{\rho_s}{200 \rho_c} \right)^{\frac{1}{3}} \approx 16 \text{ kpc}, \quad M_{200} \approx 1.6 \times 10^9 \frac{M_\odot}{\text{kpc}^3} \]

\[ M(2 \text{ kpc}) \approx 2.3 \times 10^8 \frac{M_\odot}{\text{kpc}^3} \]
de Boer et al (A&A 544, A73 (2012)) — stellar population synthesis:

Total stellar mass \( M_* \approx 4.3 \times 10^7 \, M_\odot \)

This means:

\[
\frac{M(r_s \approx 2 \, \text{kpc})}{M_*} \approx 5.3
\]

\[
\frac{M(R_{200} \approx 16 \, \text{kpc})}{M_*} \approx 37
\]
core / cusp

McGaugh et al 2001,
Gored et al 2006,
de Blok 2010,
Teyssier et al 2013,
Del Popolo & Face 2015,
Bullock & Boylan-Kolchin, 2017,
Meadows et al, 2019,
Santos-Santos et al 2020,…

\[ \rho_{\text{NFW}} = \frac{\rho_s}{r / r_s \left(1 + r / r_s \right)^2} \]

\[ \rho_{\text{Burkert}} = \frac{\rho_0}{\left(1 + \frac{r}{r_0}\right) \left(1 + \frac{r^2}{r_0^2}\right)} \]
\[ \rho_{\text{NFW}} = \frac{\rho_s}{r_s \left(1 + \frac{r}{r_s}\right)^2} \]

\[ \rho_{\text{Burkert}} = \frac{\rho_0}{\left(1 + \frac{r}{r_0}\right) \left(1 + \frac{r^2}{r_0^2}\right)} \]
\( \rho_{\text{NFW}} = \frac{\rho_s}{r/r_s \left(1 + \frac{r}{r_s}\right)^2} \)

\( \rho_{\text{Burkert}} = \frac{\rho_0}{\left(1 + \frac{r}{r_0}\right) \left(1 + \frac{r^2}{r_0^2}\right)} \)
The Galactic Centre of the low surface brightness galaxy: UGC1281

**HI**

![HI Distribution](http://astroweb.cwru.edu/SPARC/)

Fig. 5 (left) shows a cut 10-arcsec wide of the velocity field along the major axis. Overplotted are the velocities obtained by Kuzio de Naray et al. (2006). The blue symbols are the data obtained by Kuzio de Naray et al. (2006). The red lines are the input velocity: black symbols are the PPAK data presented in this paper, and the blue symbols are the data obtained by Kuzio de Naray et al. (2006). The Gaussian fitting procedure, and therefore the mean velocity, was chosen because with a channel separation of 70 km s\(^{-1}\), it is impossible to confidently fit the intrinsic shape of the emission line. We find a scaleheight of 8.5 arcsec (0.22 kpc) which is similar to the extent of the stars. The gas is still detected in UGC 1281, and we find a scaleheight of 8.5 arcsec (0.22 kpc) which is similar to the extent of the stars.

**Hα**

![Hα Distribution](http://astroweb.cwru.edu/SPARC/)

From Fig. 2 we can see that the Hα distribution displays a central depression. This warp was already observed by García-Benito et al. (2010). When we compare the lowest contour in the integrated moment 0 map of the WHISP observations with our own (see Fig. 2, red contour), we see that in our observations more emission is detected in the low-resolution cube (see Table 2) was chosen because with a channel separation of 70 km s\(^{-1}\), it is impossible to confidently fit the intrinsic shape of the emission line. We find a scaleheight of 8.5 arcsec (0.22 kpc) which is similar to the extent of the stars.

**Kamphuis et al. 2011**


**Section 4.** Blue dotted–dashed line: beam.
At \( \sim 5 \) kpc:

\[
\frac{M(\text{DM})}{M(\text{stars}+\text{gas})} \sim \frac{55^2 - 2 \times 20^2}{2 \times 20^2} \sim 3
\]
Dark matter
Dark matter

$m_X$

$10^{-21}$ eV

$10^{-10}$ $M_\odot \sim 10^{56}$ eV
$m_X \quad 10^6 \text{ eV} \quad 10^{12} \text{ eV}$

$10^{-21} \text{ eV}$ \quad ? \quad $10^{-10} \text{ M}_\odot \sim 10^{56} \text{ eV}$

<table>
<thead>
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<th>3rd</th>
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</tr>
<tr>
<td>$Z^0$</td>
<td>$g$</td>
<td>$X$</td>
</tr>
</tbody>
</table>

Up quark, charm quark, top quark, photon, Higgs boson, W boson, Z boson, gluon.
The mass of the dark matter particle $m_X$ is uncertain, ranging from $10^{-21} \text{ eV}$ to $10^{12} \text{ eV}$.

**“Indirect”**

- AMS
- Fermi
- Super K
- HESS

**“Direct”**

- Xenon1T
- LUX
- SENSEI

The mass of the Sun $M_\odot$ is approximately $10^{33} \text{ eV}$.
$m_X$  

$10^{-21}$ eV  

$10^6$ eV  

$10^{12}$ eV  

$10^{-10} \text{ M}_\odot \sim 10^{56}$ eV  

"If we pull this off, we'll eat like kings."
$m_X$

$10^{-21}$ eV \quad \sim \quad 10^{-10} M_\odot \sim 10^{56}$ eV
Gravity alone

$m_\chi$

$10^{-21} \text{ eV}$

$10^{-10} M_\odot \sim 10^{56} \text{ eV}$
Gravity alone

1. Ultralight dark matter

2. Beyond mean-field: dynamical friction / heating

3. Gravitational lensing
Gravity alone

$m_{\chi}$

$10^{-21}$ eV

$10^{-10} \, M_\odot \sim 10^{56}$ eV

Thank you!
Where does it end?

Cosmic expansion:

\[
\begin{align*}
\chi^V &= \begin{pmatrix} t \\ r \\ \theta \\ \phi \end{pmatrix}; \\
g_{\mu\nu} &= \begin{pmatrix}
f[r] & 0 & 0 & 0 \\
0 & -\frac{1}{f[r]} & 0 & 0 \\
0 & 0 & -r^2 & 0 \\
0 & 0 & 0 & -r^2 \sin^2[\theta] 
\end{pmatrix}; \\
g_{\mu\nu} &= \text{Inverse}[g_{\mu\nu}]; \\
dgdxtab &= \text{Table}[\{D[g_{\mu\nu}, \chi^V[[i, 1]]], \{i, 1, 4\}]; \\
\Gamma &= \text{Table}[\{\text{Table}[\frac{1}{2} \text{Sum}[g_{\mu\nu}[[i, \mu]] \text{dgdxtab}[[k, 1]][[\mu, j]] + \text{dgdxtab}[[j, 1]][[\mu, k]] - \text{dgdxtab}[[\mu, 1]][[j, k]]], \{\mu, 1, 4\}], \{j, 1, 4\}, \{k, 1, 4\}\}, \{i, 1, 4\}]; \\
R_{\mu\nu} &= \{\text{Table}[\text{Sum}[D[\Gamma[[\alpha, 1]][[\mu, \nu]], \chi^V[[\alpha, 1]]], \{\alpha, 1, 4\}], \{\mu, 1, 4\}, \{\nu, 1, 4\}] - \text{Table}[\text{Sum}[D[\Gamma[[\mu, 1]][[\alpha, \nu]], \chi^V[[\nu, 1]]], \{\alpha, 1, 4\}], \{\mu, 1, 4\}, \{\nu, 1, 4\}] + \text{Table}[\text{Sum}[D[\Gamma[[\lambda, 1]][[\mu, \nu]], \chi^V[[\lambda, 1]]], \{\lambda, 1, 4\}], \{\mu, 1, 4\}, \{\nu, 1, 4\}] - \text{Table}[\text{Sum}[D[\Gamma[[\lambda, 1]][[\mu, \nu]], \chi^V[[\lambda, \nu]]], \{\lambda, 1, 4\}], \{\mu, 1, 4\}, \{\nu, 1, 4\}]\}, \{\mu, 1, 4\}, \{\nu, 1, 4\}]; \\
R &= \text{Sum}[g_{\mu\nu}[[\mu, \nu]] \times R_{\mu\nu}[[\mu, \nu]], \{\mu, 1, 4\}, \{\nu, 1, 4\}];
\end{align*}
\]
Where does it end?

Cosmic expansion:

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \]

\[ T_{\mu\nu} \approx g_{\mu\nu} \rho_{\Lambda}, \quad H_0^2 \approx \frac{8\pi}{3} G \rho_{\Lambda} \]

\[ \longrightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 3H_0^2 g_{\mu\nu} \]

Solved by: \[ f(r) = 1 - \frac{2}{r} - H_0^2 r^2 \]
Where does it end?

**Cosmic expansion:**

\[
\frac{d^2 x^\mu}{dp^2} + \frac{\Gamma^\mu_{\alpha\beta}}{\alpha\beta} \frac{dx^\alpha}{dp} \frac{dx^\beta}{dp} = 0
\]
Where does it end?

Cosmic expansion:

\[ \frac{d^2 x^\mu}{dp^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{dp} \frac{dx^\beta}{dp} = 0 \]

Looking for a circular orbit solution:

\[ \theta = \frac{\pi}{2} = \text{Const}, \quad r = R = \text{Const} \]

Leads to:

\[ \frac{d\varphi}{dp} = C \Omega = \text{Const} \]

\[ \frac{dt}{dp} = C = \text{Const} \]

The \( r \) equation becomes:

\[ (2 - R + H_0^2 R^3) \Omega^2 - \frac{(1 - H_0^2 R^3) (2 - R + H_0^2 R^3)}{R^3} = 0 \]

Restoring units:

\[ \Omega^2 = \frac{GM}{R^3} \left(1 - \frac{H_0^2 R^3}{GM}\right) \]
Expansion modifies law of rotation around a point mass $M$:

$$\Omega^2 = \frac{GM}{R^3} \left( 1 - \frac{H_0^2 R^3}{GM} \right)$$
Project AMIGA: The Circumgalactic Medium of Andromeda

Nicolas Lehner1, Samantha C. Berek2,3,†, J. Christopher Howk1, Bart P. Wakker1, Jason Tumlinson4,5, Edward B. Jenkins6,7, J. Xavier Prochaska7, Ramona Augustin8, Suqing Ji6, Claude-André Faucher-Giguère9, Zachary Hafen1, Molly S. Peebles10,†, Kat A. Barger10, Michelle A. Berg1, Rongmon Bordoloi11, Thomas M. Brown12, Andrew J. Fox13, Karoline M. Gilbert14, Puragra Guhathakurta15, Jason S. Kalirai15, Felix J. Lockman16, John M. O'Meara17, D. J. Pisano18,19, Joseph Ribaudo18,20, and Jessica K. Werk16

Figure 9. Logarithm of the column densities for the individual components of various ions (low to high ions from top to bottom) as a function of the projected distance from M31 of the background QSOs. Blue circles are detections, while red ones are upper limits.
The Hα field of view is still resembling a slow rising rotation curve.

The Hα flux map of UGC 1281 is at first glance quite symmetrically and evenly distributed. However, a closer look reveals asymmetries and peculiarities in the Hα distribution.

**Figure 2.** DSS 2 red image of UGC 1281 overlaid with the Hα image. The black contours are obtained through the NASA extragalactic data base.

**Figure 3.** Velocity field of the ionized gas. The field was constructed by taking the central position of the fitted Gaussian in all the binned spectra. The systemic velocity we are referring to this mean velocity unless otherwise noted.

**Figure 4.** Shows the velocity field of the PPAK observations. This velocity field was obtained by taking the peak position of the Gaussian in all the binned spectra. The arrow indicates the positions of the velocity cuts parallel to the minor axis.

**Figure 5.** Uncertainties in the measured velocity. The arrows indicate the positions of the velocity cuts parallel to the minor axis but also in inclination. Thus if this Hα region that is either somewhat offset from the plane or located in the outskirts of the galaxy, its position can be in the (warped) plane of the inner disc at larger radii. This behaviour is seen especially in the edge-on orientation. How-