

# XXXIII Canary Islands Winter School of Astrophysics

## Fundamental Physics with Galaxies (lecture II)

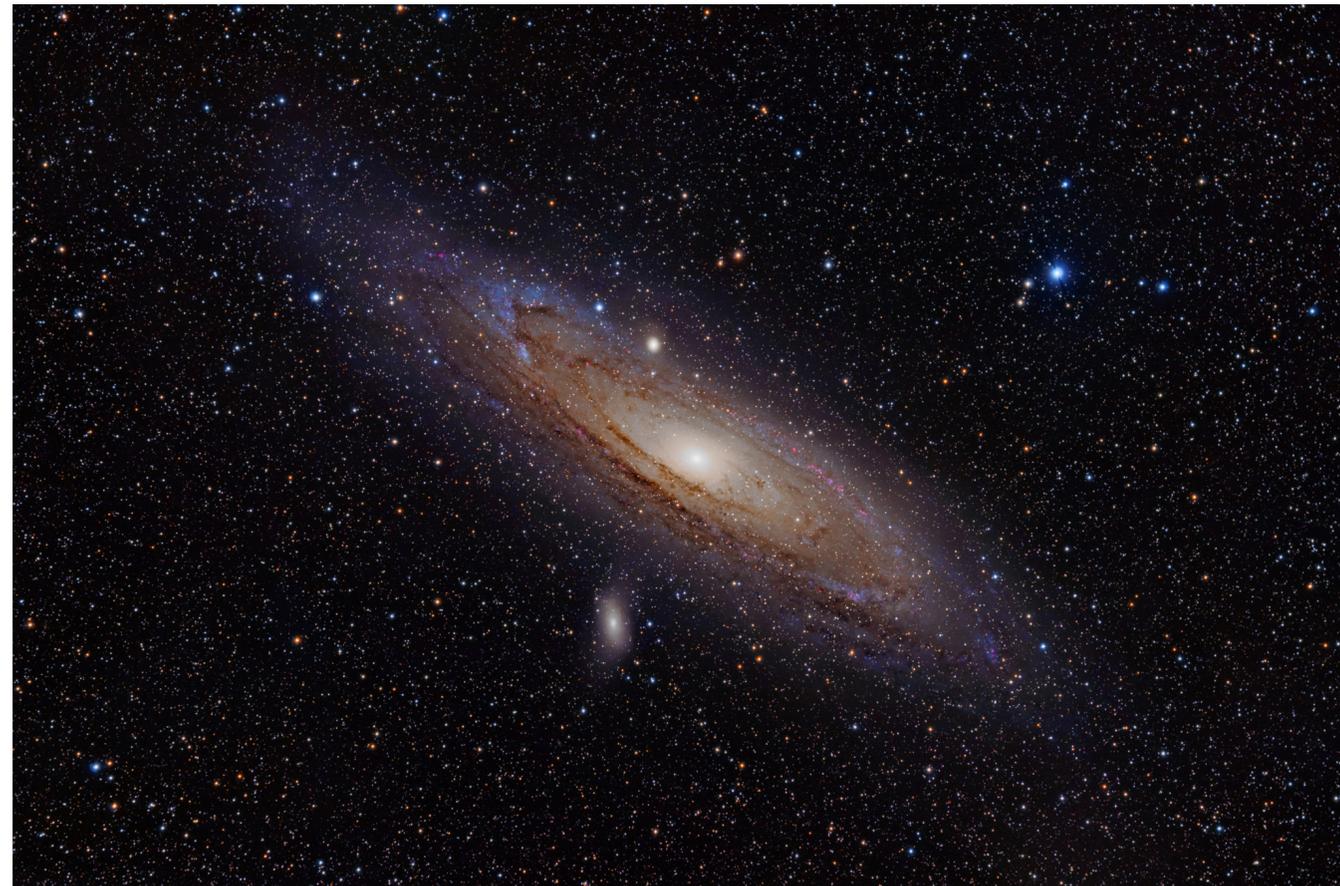


## Dark matter

$m_X$

$10^{-21}$  eV

$10^{-10} M_{\odot} \sim 10^{56}$  eV

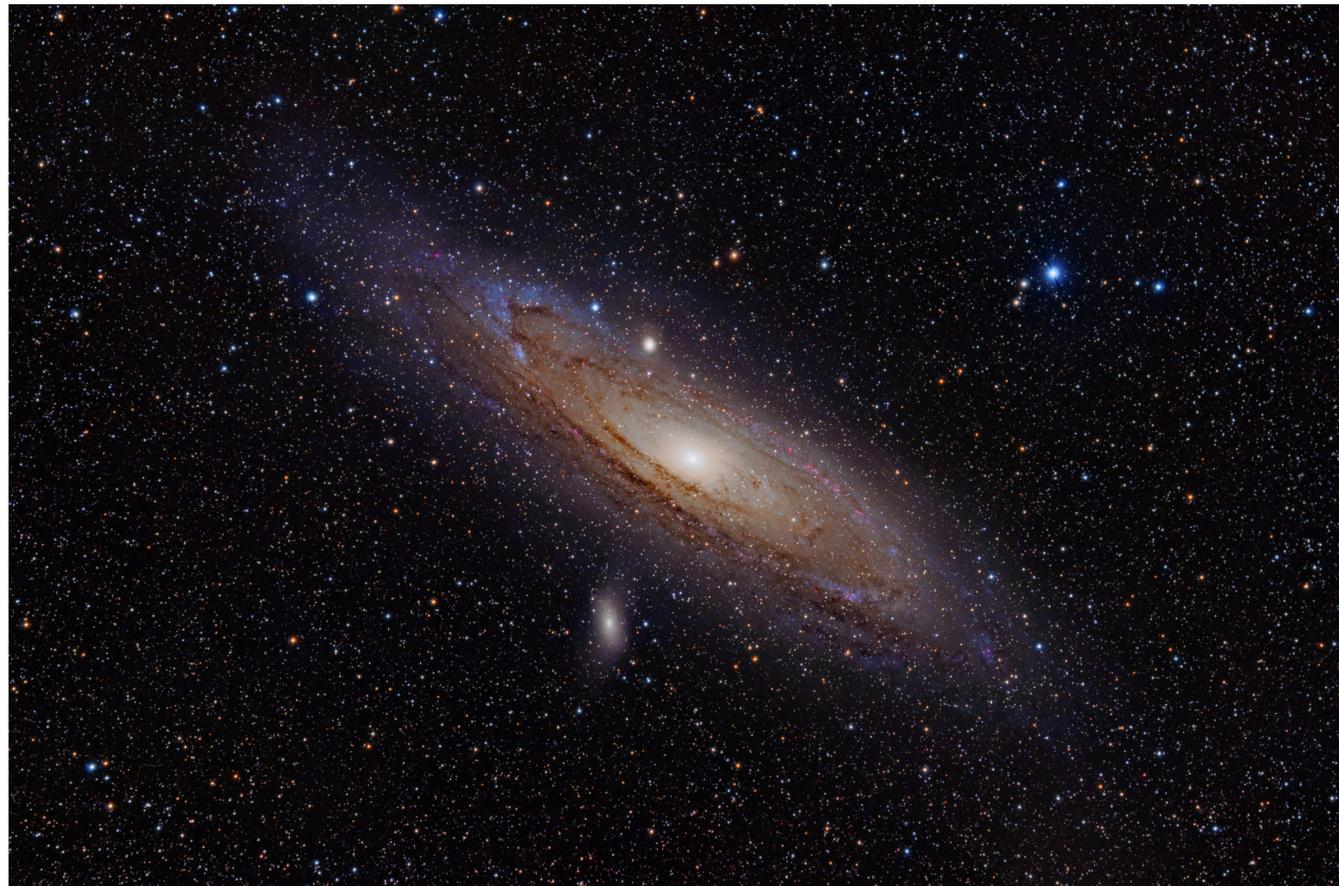


# Gravity alone

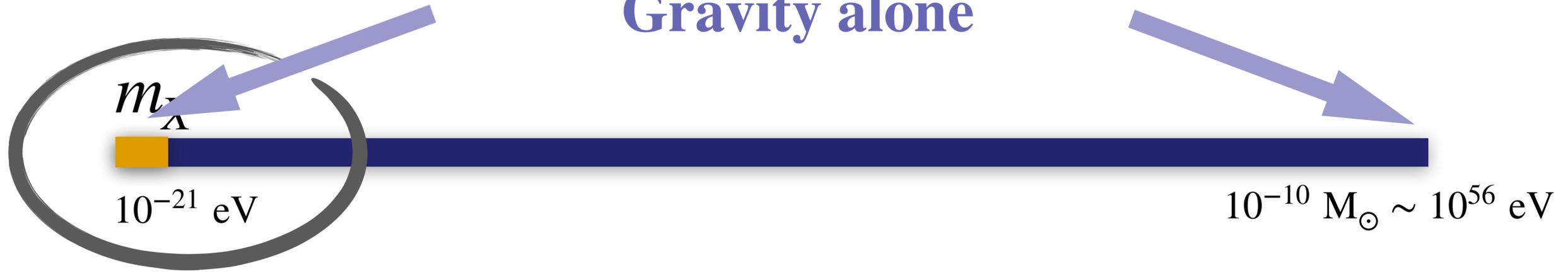
$m_{\Lambda}$

$10^{-21}$  eV

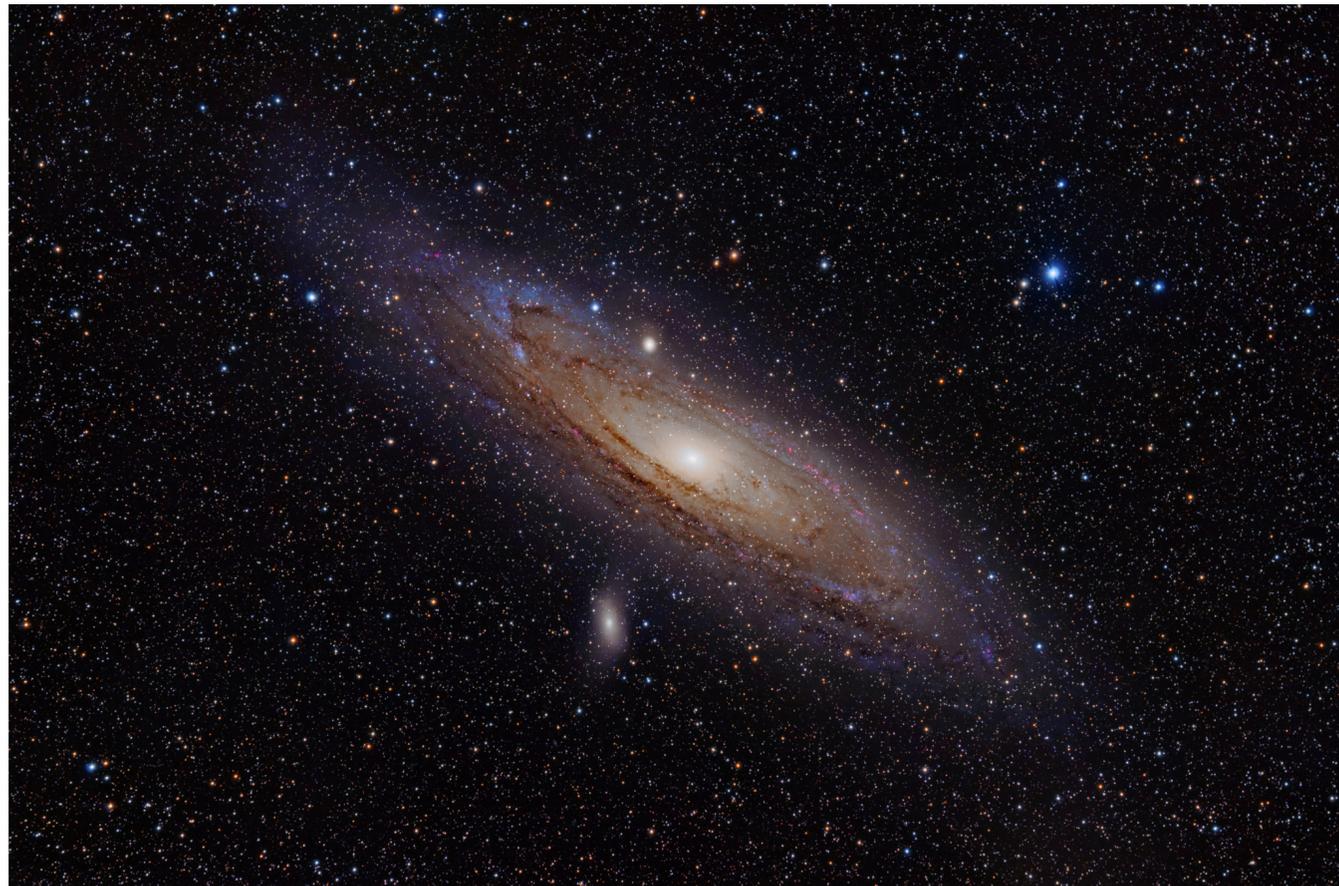
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# Gravity alone



## Ultralight dark matter



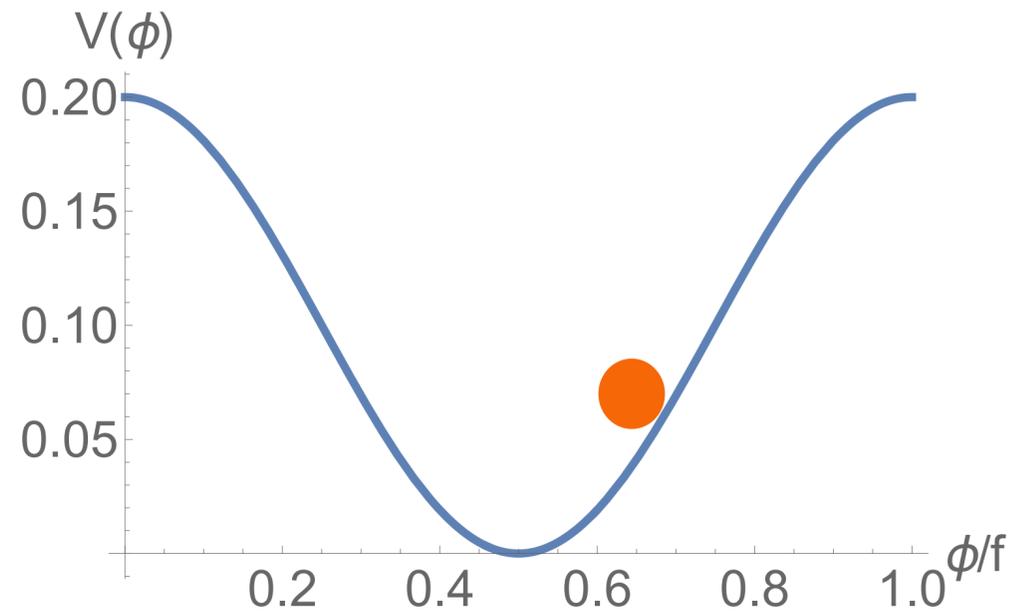
## Ultralight dark matter (ULDM)

Light fields (Goldstone bosons) feature in many models. Svrcek & Witten 2006; Arvanitaki et al 2010

Cosmology: field initially displaced from minimum of the potential (before/during inflation)

$$\frac{1}{\sqrt{-g}} \mathcal{L} = \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi + m^2 f^2 \cos \frac{\phi}{f}$$

$$\ddot{\phi} - \frac{\nabla^2}{a^2} \phi + m^2 \phi + 3H\dot{\phi} = 0$$



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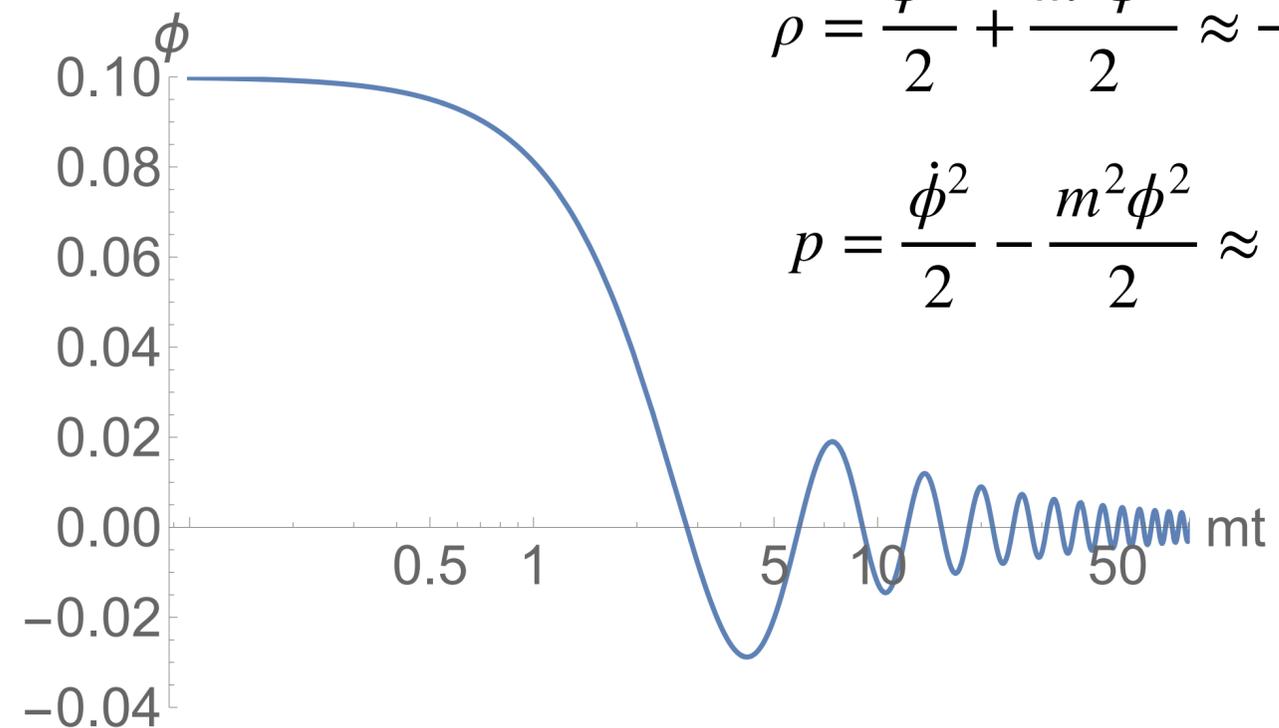
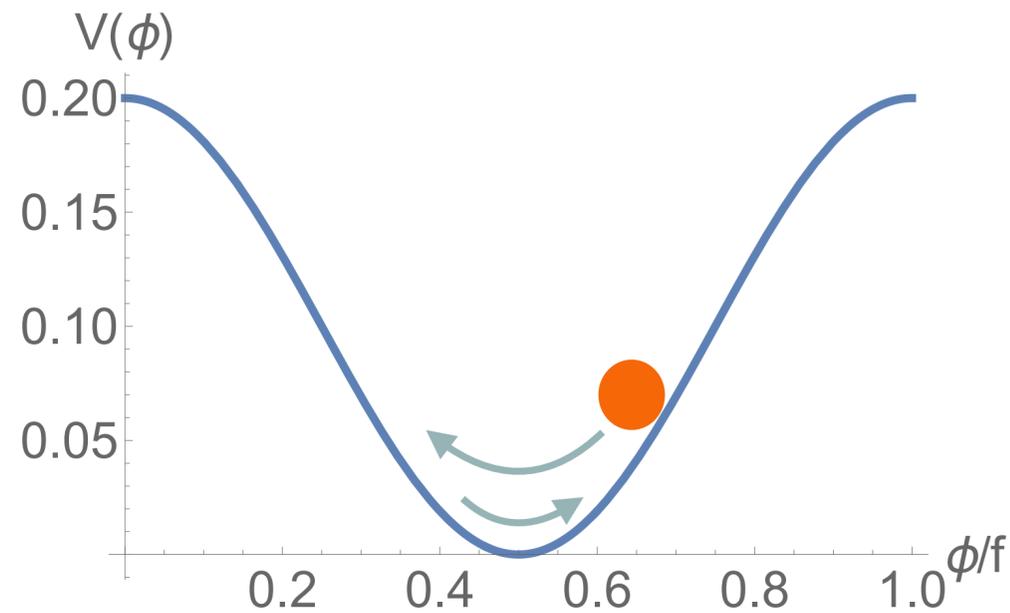
Cosmology: field initially displaced from minimum of the potential

Starts to oscillate when  $t \sim 1/m$ .

When  $t \gg 1/m$ , correct equation of state for dark matter.

$$\ddot{\phi} - \frac{\nabla^2}{a^2}\phi + m^2\phi + 3H\dot{\phi} = 0$$

$$\text{When } m \gg H: \quad \phi \approx (1+z)^{\frac{3}{2}} \phi_0 \cos mt$$



$$\rho = \frac{\dot{\phi}^2}{2} + \frac{m^2\phi^2}{2} \approx \frac{m^2\phi_0^2}{2}(1+z)^3$$

$$p = \frac{\dot{\phi}^2}{2} - \frac{m^2\phi^2}{2} \approx -\rho \cos 2mt$$

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Svrcek & Witten 2006; Arvanitaki et al 2010

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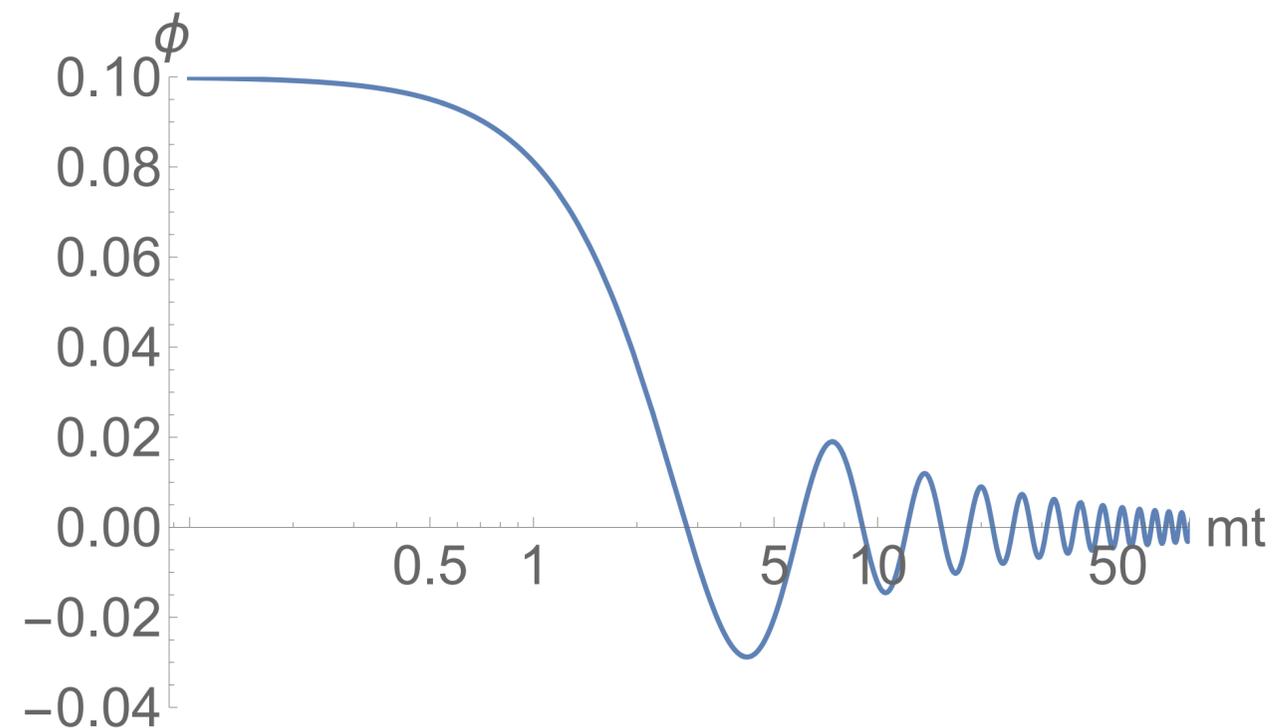
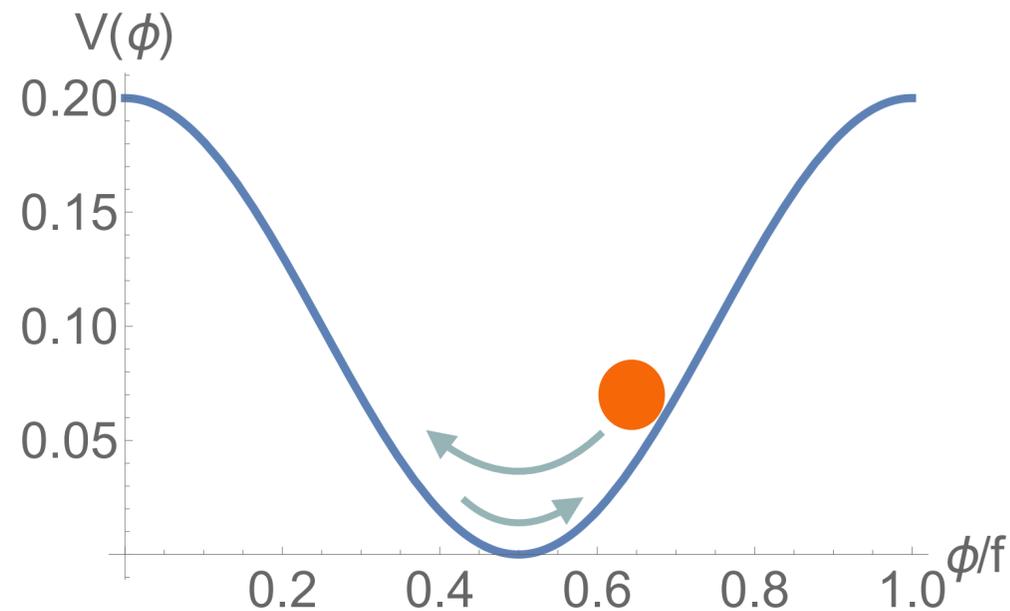
$$H_m \approx \frac{T_m^2}{M_{pl}} \approx m, \quad \phi(z_m) = \alpha f, \quad \alpha = \mathcal{O}(1)$$

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$$\rho(z=0) \approx \frac{\rho(z_m)}{(1+z_m)^3} \approx \frac{\alpha^2 m^2 f^2}{2(1+z_m)^3}$$

Contribution to energy density today:



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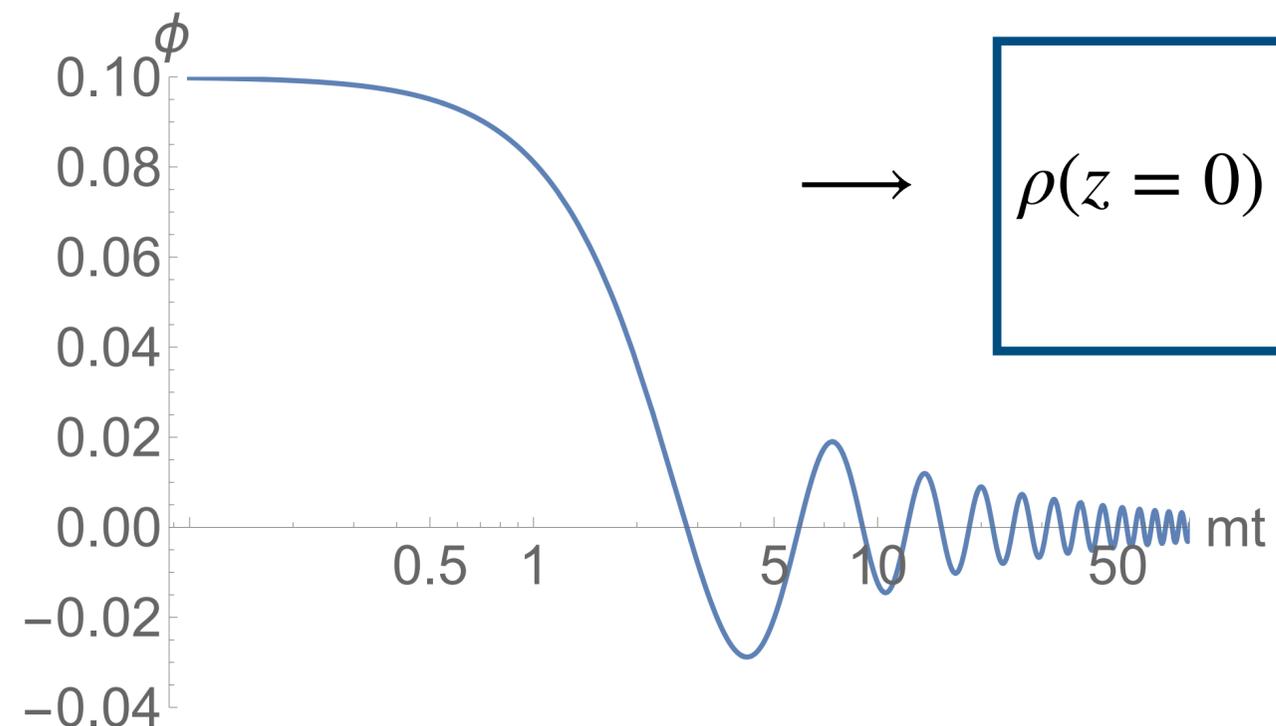
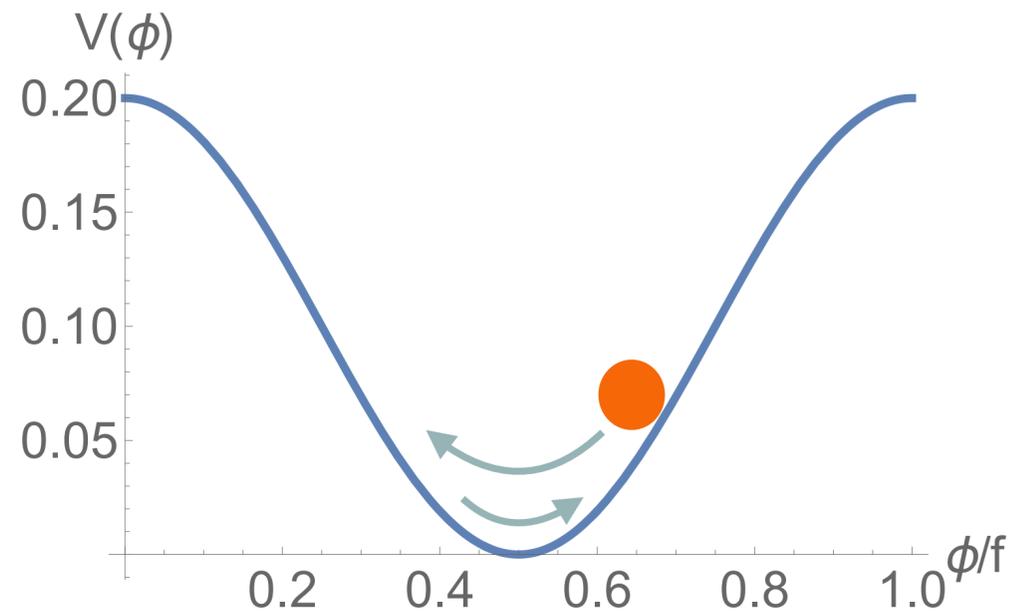
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Contribution to energy density today:

$$1+z_m \approx \frac{T_m}{T_0} \approx \frac{\sqrt{m M_{pl}}}{T_0}$$



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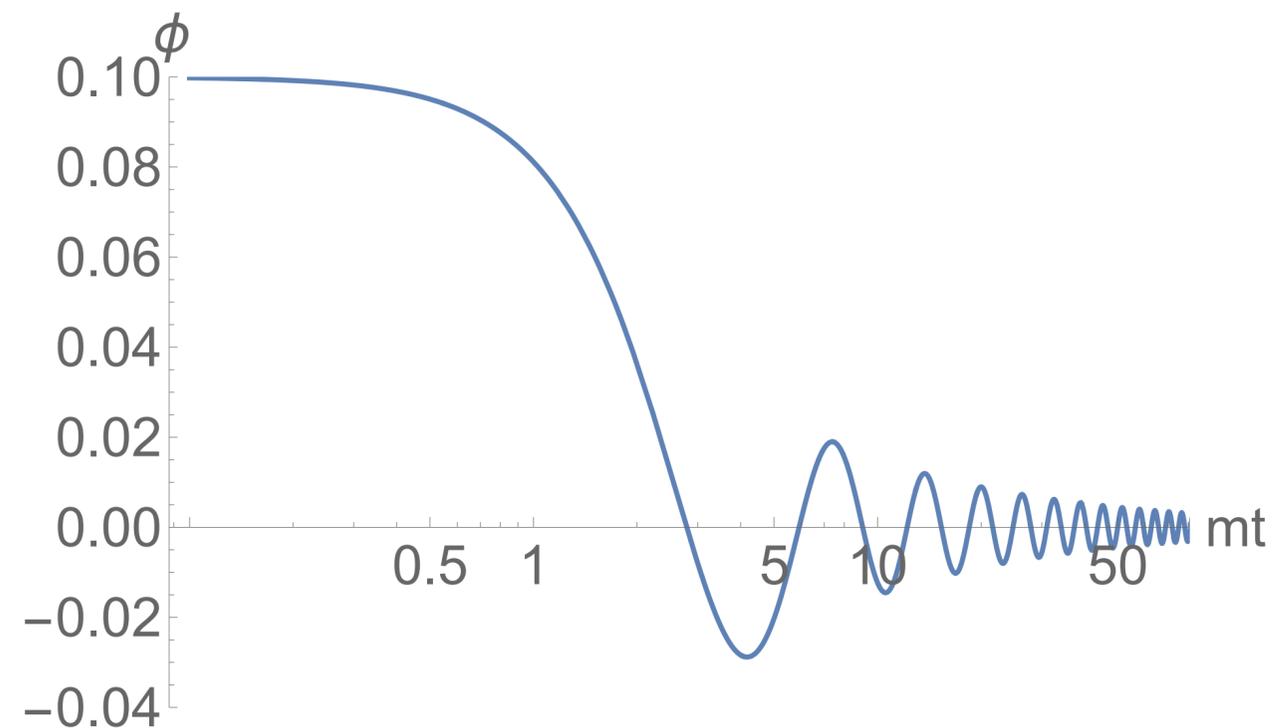
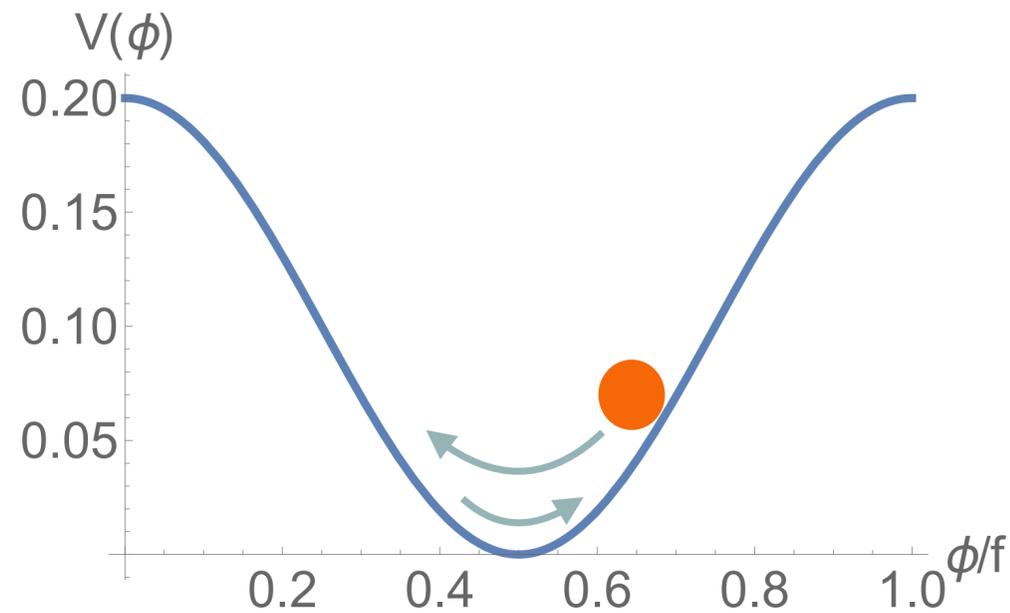
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Contribution to energy density today:

$$\Omega_m \approx 0.3 \left( \frac{m}{10^{-21} \text{ eV}} \right)^{\frac{1}{2}} \left( \frac{f}{10^{17} \text{ GeV}} \right)^2$$

e.g. Hui et al, 1610.08297



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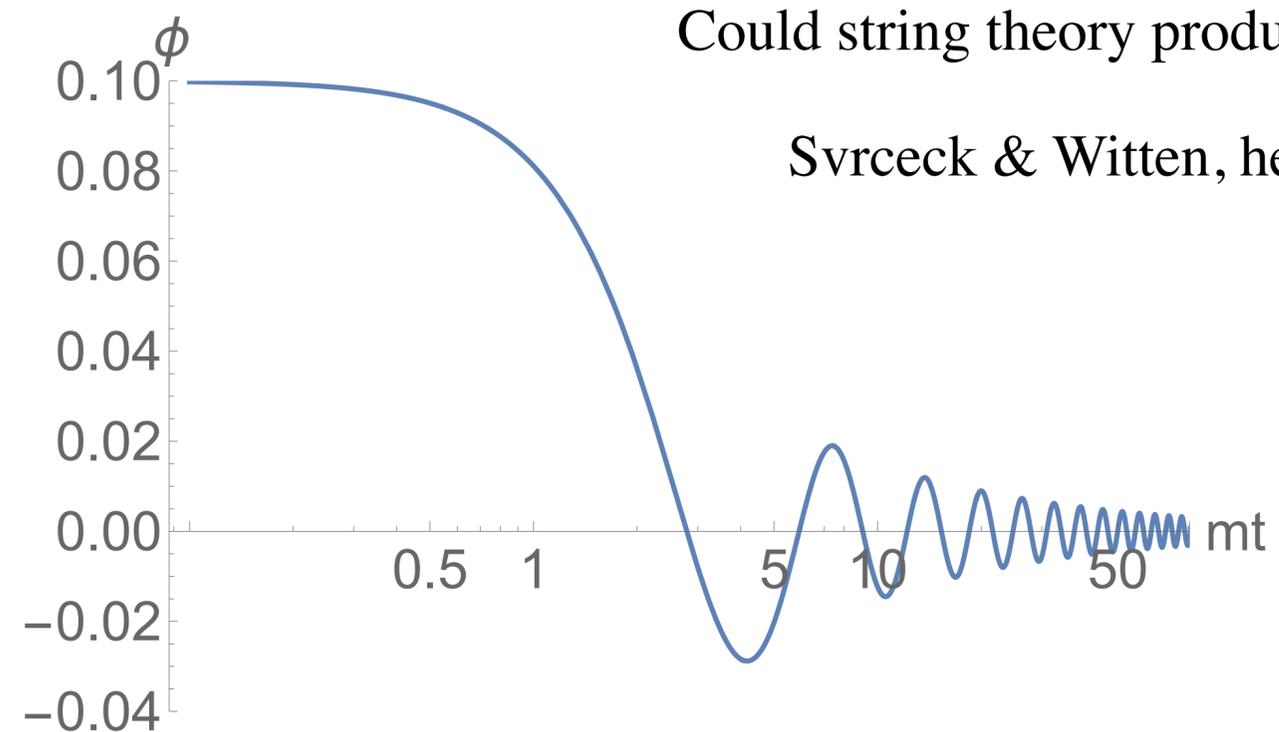
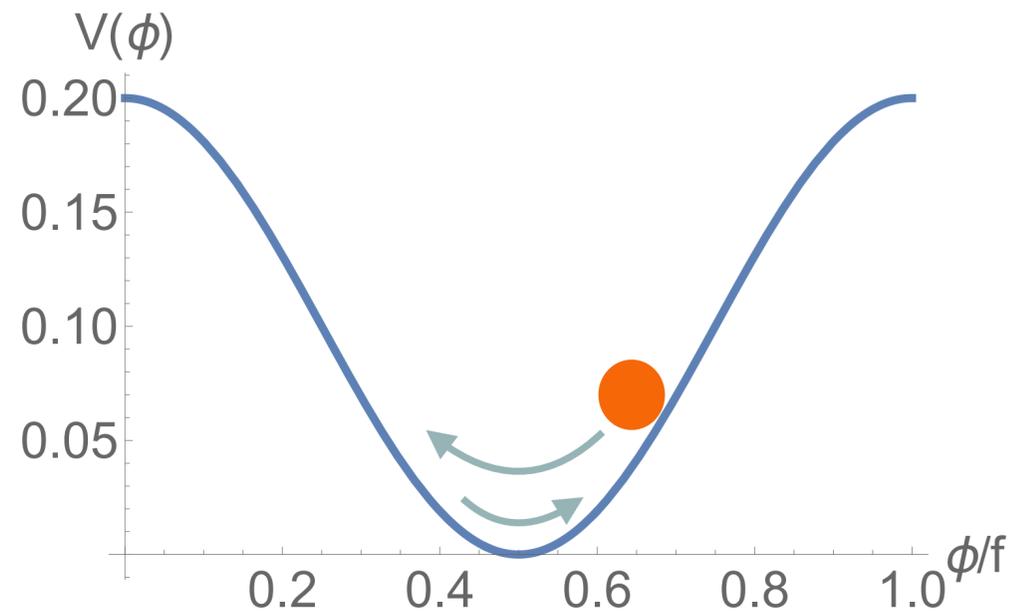
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Could string theory produce such states?

Svrcek & Witten, hep-th/0605206

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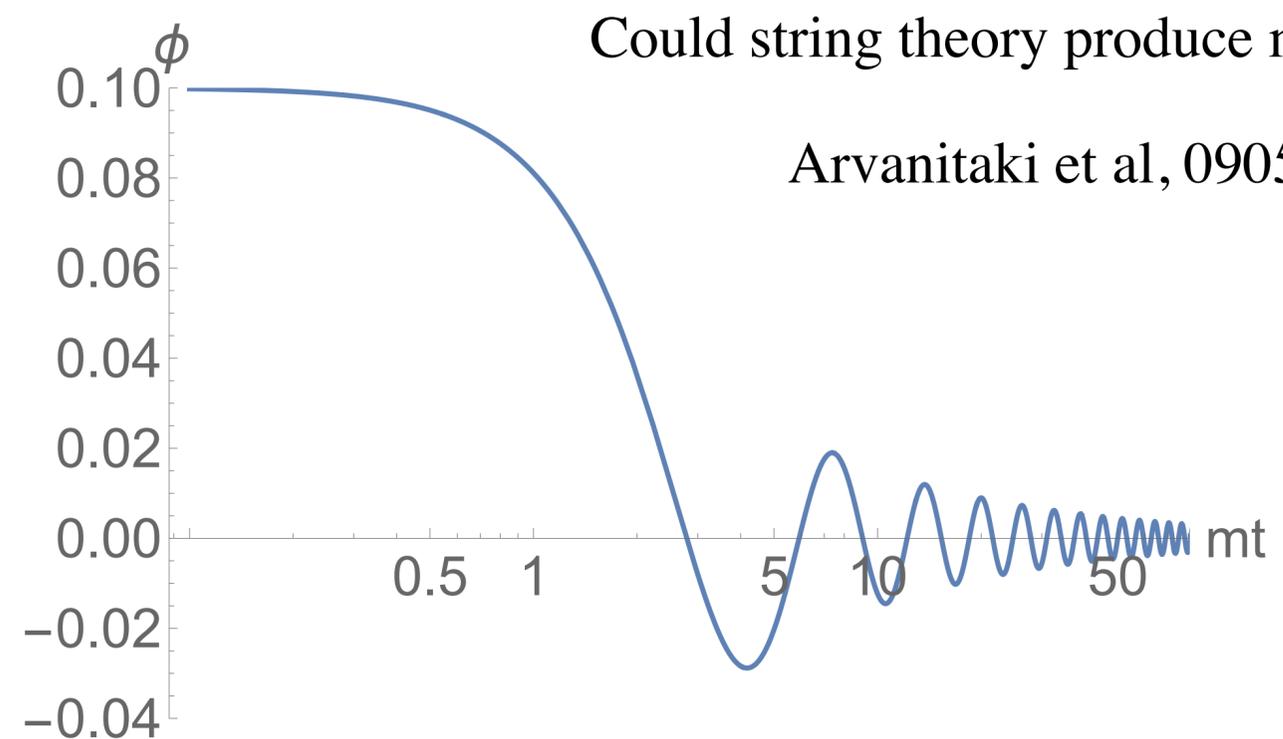
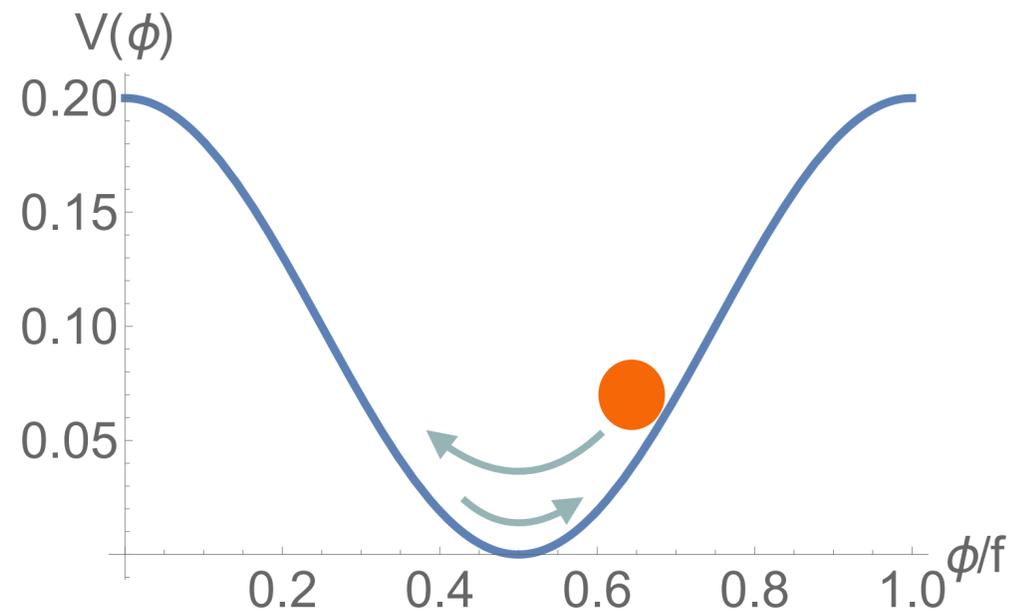
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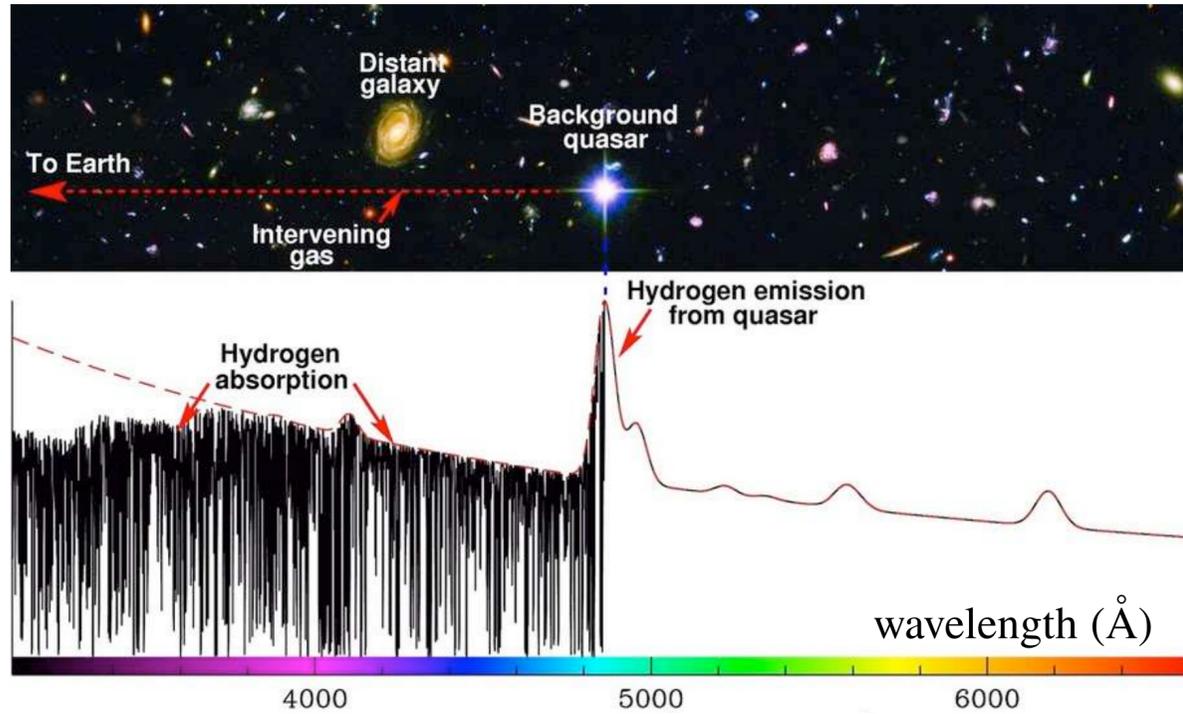
e.g. Hui et al, 1610.08297



Could string theory produce many such states?

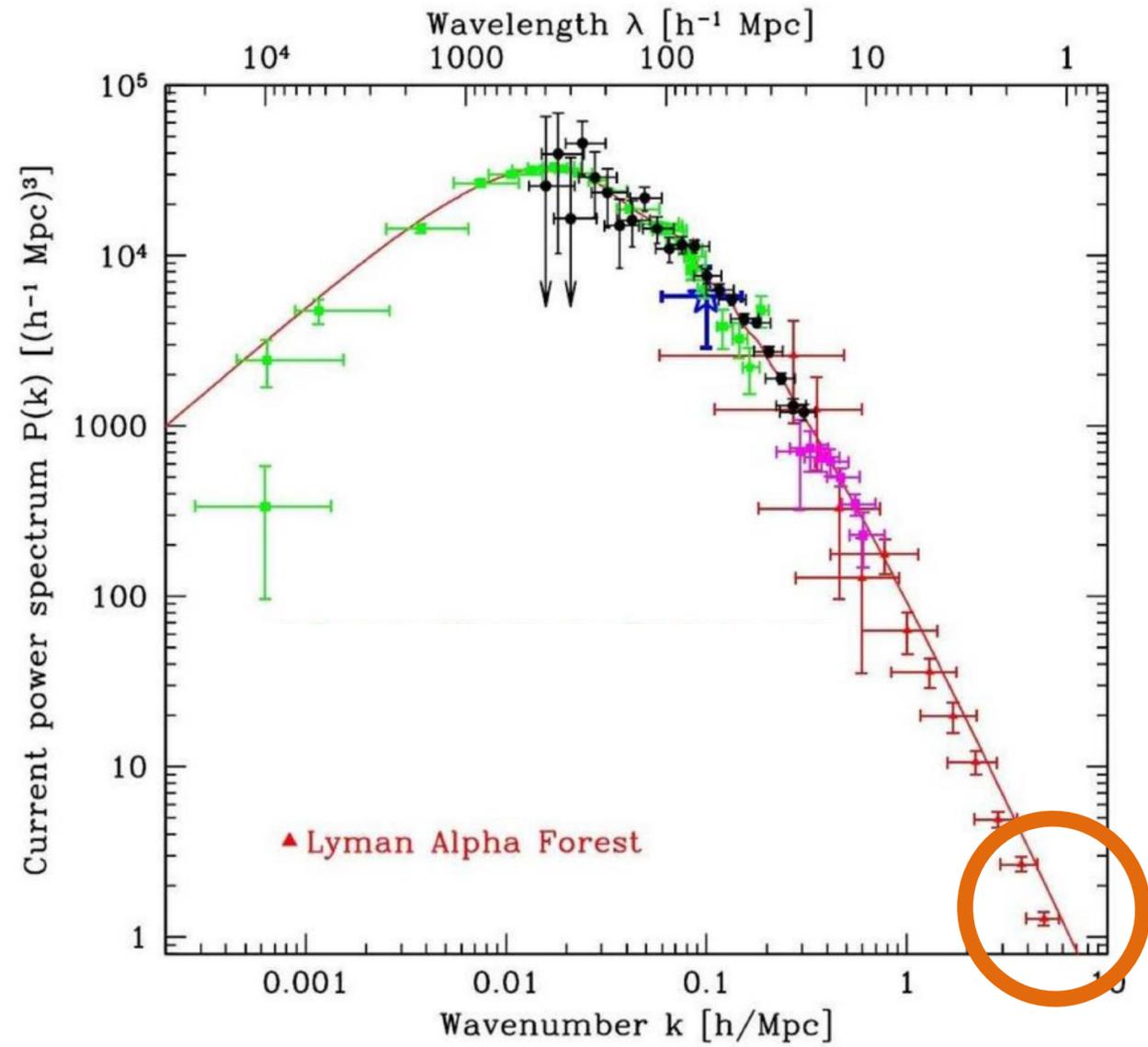
Arvanitaki et al, 0905.4720

On scales smaller than the de Broglie wavelength, **ULDM is different** than massive particle DM.



$$m \gtrsim 10^{-21} \text{ eV}$$

Armengaud et al 2017



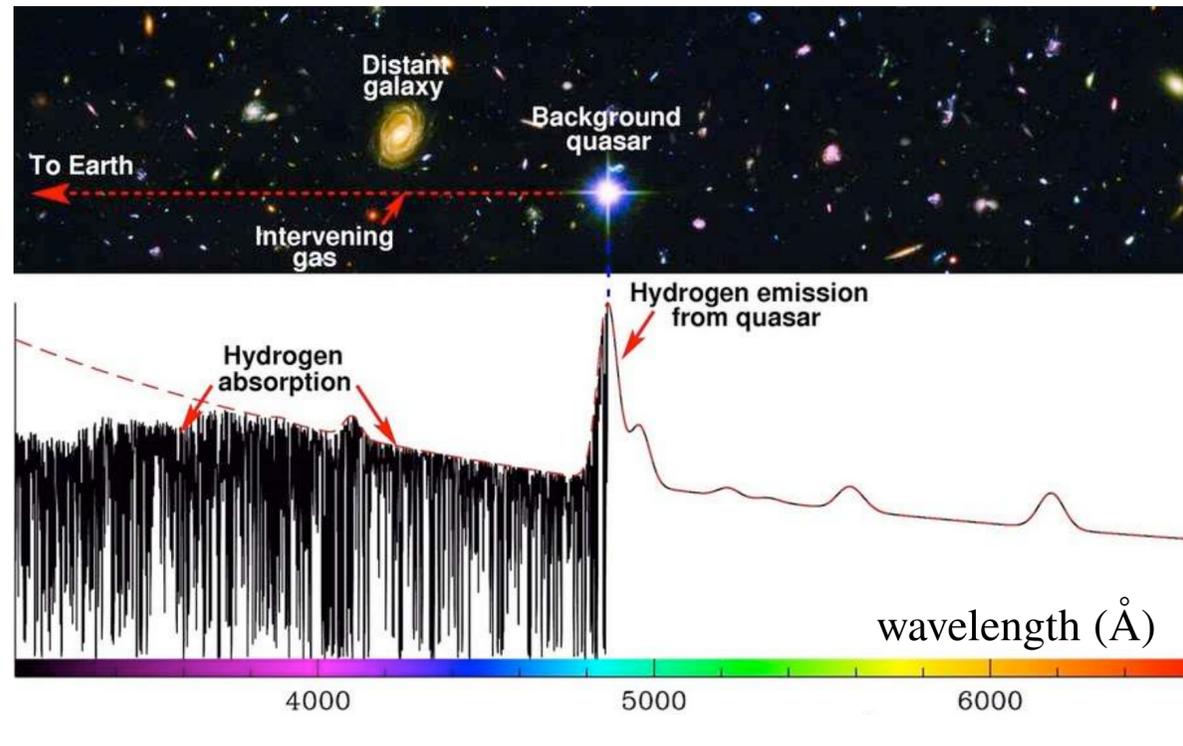
self-gravitating perturbation:

$$v^2 = \frac{GM}{R} = \frac{4\pi}{3} G \delta\rho \left( \frac{\lambda_{\text{dB}}}{2} \right)^2$$

$$\frac{\lambda_{\text{dB}}}{a} = \frac{2\pi}{amv}$$

$$\approx 6 \left( \frac{10^{-22} \text{ eV}}{m} \right)^{\frac{1}{2}} \left( \frac{10^{-5}}{\delta\rho/\rho} \right)^{\frac{1}{4}} \left( \frac{1+z}{10^3} \right)^{\frac{1}{4}} \text{ Mpc}$$

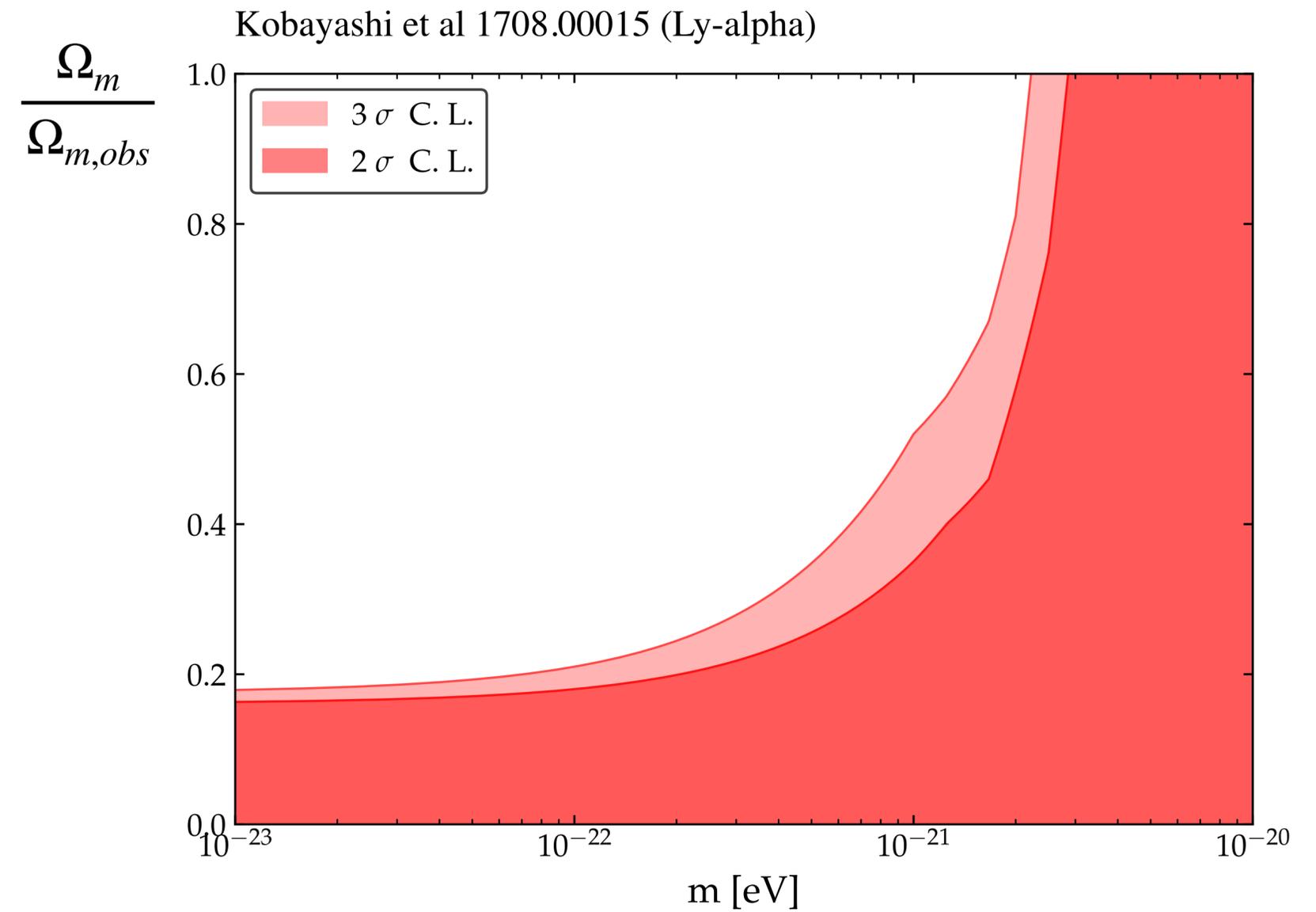
On scales smaller than the de Broglie wavelength, **ULDM is different** than massive particle DM.



See also:

Hlozek, Marsh, Grin 1708.05681 (CMB)

Lague et al, 2104.07802 (CMB+LSS)



# On scales smaller than the de Broglie wavelength, ULDM is “fuzzy”

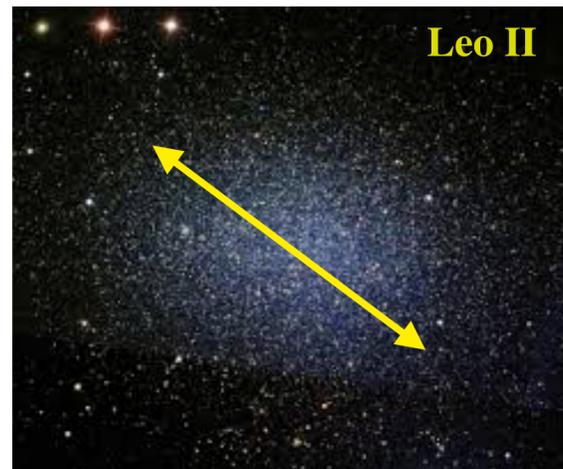
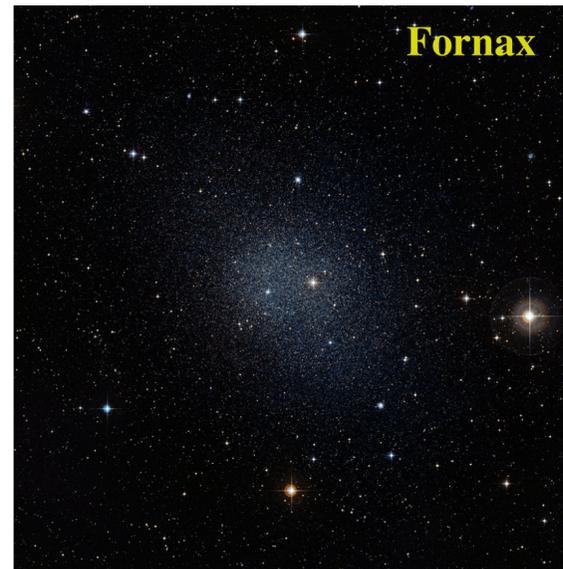
W. Hu, R. Barkana, and A. Gruzinov, Cold and Fuzzy Dark Matter, *Phys. Rev. Lett.* **85**, 1158 (2000).

$$\lambda_{\text{dB}} \sim \frac{2\pi}{mv}$$

$$v^2 \sim \frac{GM}{R}$$

$$R \sim \lambda_{\text{dB}} \quad \longrightarrow \quad MR \lesssim \frac{4\pi^2}{Gm^2}$$

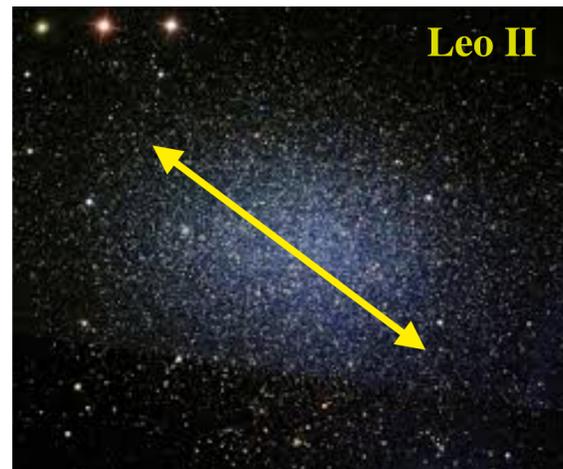
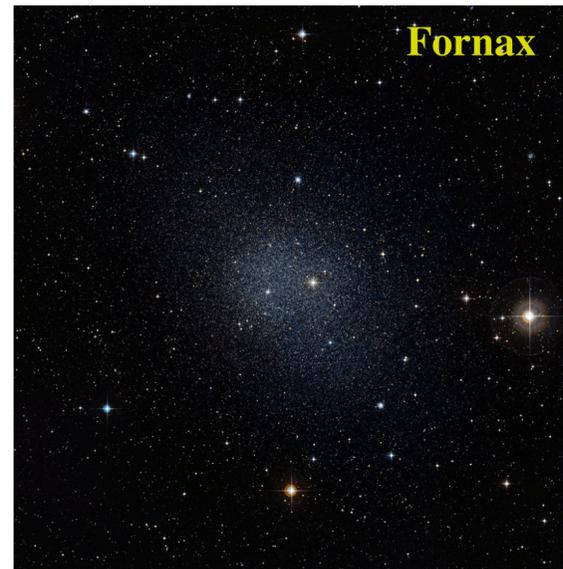
$$\left( \frac{M}{3 \times 10^9 M_{\odot}} \right) \left( \frac{R}{1 \text{ kpc}} \right) \lesssim \left( \frac{10^{-22} \text{ eV}}{m} \right)^2$$



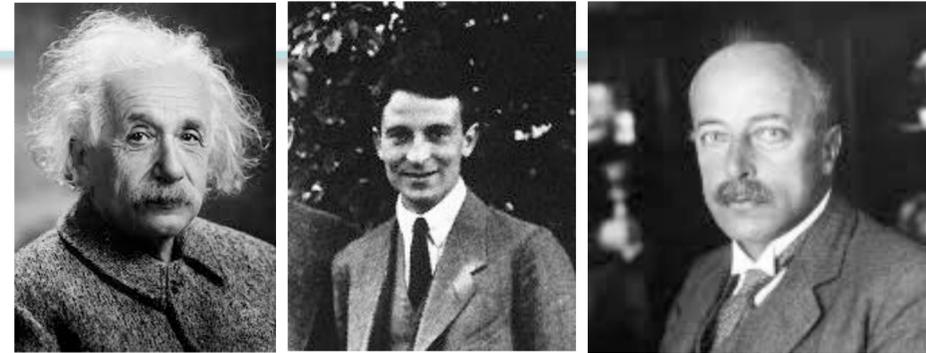
## On scales smaller than the de Broglie wavelength, ULDM is “fuzzy”

- [1] W. Hu, R. Barkana, and A. Gruzinov, Cold and Fuzzy Dark Matter, *Phys. Rev. Lett.* **85**, 1158 (2000).
- [2] A. Arbey, J. Lesgourgues, and P. Salati, Quintessential haloes around galaxies, *Phys. Rev. D* **64**, 123528 (2001).
- [3] J. Lesgourgues, A. Arbey, and P. Salati, A light scalar field at the origin of galaxy rotation curves, *New Astron. Rev.* **46**, 791 (2002).
- [4] P.-H. Chavanis, Mass-radius relation of Newtonian self-gravitating Bose-Einstein condensates with short-range interactions: I. Analytical results, *Phys. Rev. D* **84**, 043531 (2011).
- [5] P. H. Chavanis and L. Delfini, Mass-radius relation of Newtonian self-gravitating Bose-Einstein condensates with short-range interactions: II. Numerical results, *Phys. Rev. D* **84**, 043532 (2011).
- [6] H.-Y. Schive, T. Chiueh, and T. Broadhurst, Cosmic structure as the quantum interference of a coherent dark wave, *Nat. Phys.* **10**, 496 (2014).
- [7] H.-Y. Schive, M.-H. Liao, T.-P. Woo, S.-K. Wong, T. Chiueh, T. Broadhurst, and W. Y. P. Hwang, Understanding the Core-Halo Relation of Quantum Wave Dark Matter from 3D Simulations, *Phys. Rev. Lett.* **113**, 261302 (2014).
- [8] D. J. E. Marsh and A.-R. Pop, Axion dark matter, solitons and the cusp-core problem, *Mon. Not. R. Astron. Soc.* **451**, 2479 (2015).
- [9] S.-R. Chen, H.-Y. Schive, and T. Chiueh, Jeans analysis for Dwarf Spheroidal Galaxies in wave dark matter, *Mon. Not. R. Astron. Soc.* **468**, 1338 (2017).

It was suggested that Milky Way dwarf satellite galaxies may point to  $m \sim 10^{-22}$  eV



Free scalar field  $\phi(x, t)$



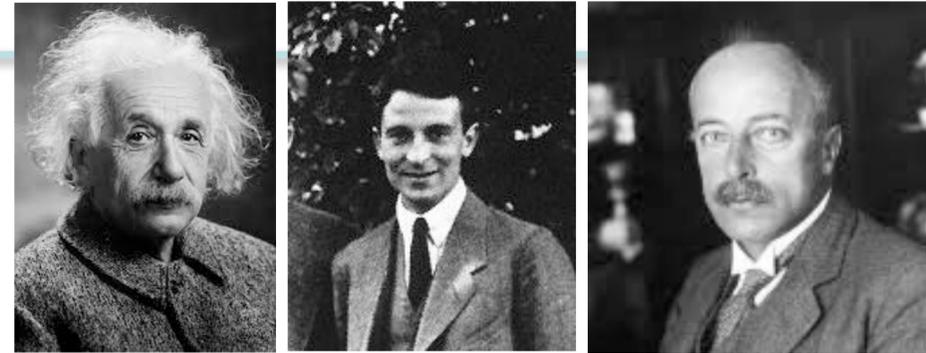
$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu} \right) + m^2 \phi = 0$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

Einstein-Klein-Gordon equations

## Nonrelativistic limit

Free scalar field  $\phi(x, t) = \frac{1}{\sqrt{2m}} e^{-imt} \psi(x, t) + cc$



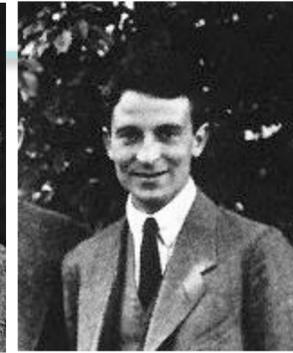
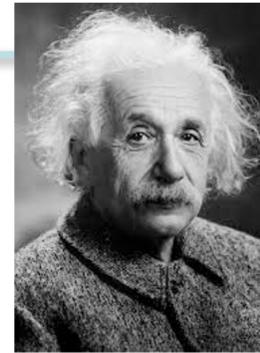
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$$i\partial_t\psi = -\frac{1}{2m}\nabla^2\psi + m\Phi\psi$$

$$\nabla^2\Phi = 4\pi G|\psi|^2$$

Schrödinger-Poisson equations



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Einstein-Klein-Gordon equations

$\text{In}[\ast]:= \eta_{\mu\nu} = \{\{1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, -1\}\};$

$x_{\mu} = \{t, x, y, z\};$

$$g_{\mu\nu\text{Upper}} = \begin{pmatrix} \frac{1}{1+2\alpha_{GR}\Psi[t,x,y,z]} & 0 & 0 & 0 \\ 0 & \frac{-1}{a[t]^2(1-2\alpha_{GR}\Psi[t,x,y,z])} & 0 & 0 \\ 0 & 0 & \frac{-1}{a[t]^2(1-2\alpha_{GR}\Psi[t,x,y,z])} & 0 \\ 0 & 0 & 0 & \frac{-1}{a[t]^2(1-2\alpha_{GR}\Psi[t,x,y,z])} \end{pmatrix};$$

$$g_{\mu\nu\text{Lower}} = \begin{pmatrix} 1+2\alpha_{GR}\Psi[t,x,y,z] & 0 & 0 & 0 \\ 0 & -a[t]^2(1-2\alpha_{GR}\Psi[t,x,y,z]) & 0 & 0 \\ 0 & 0 & -a[t]^2(1-2\alpha_{GR}\Psi[t,x,y,z]) & 0 \\ 0 & 0 & 0 & -a[t]^2(1-2\alpha_{GR}\Psi[t,x,y,z]) \end{pmatrix};$$

$\text{Dmg} = \text{Det}[-g_{\mu\nu\text{Lower}}];$

$$i\partial_t\psi = -\frac{1}{2m}\nabla^2\psi + m\Phi\psi$$

$$\nabla^2\Phi = 4\pi G|\psi|^2$$

Schrödinger-Poisson equations



$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\phi}{\partial x^\nu}\right) + m^2\phi = 0$$

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Einstein-Klein-Gordon equations

$$\text{In[*]}:= \phi[t, x, y, z] = \frac{e^{-i m t}}{\sqrt{2} m} (\psi_r[t, x, y, z] + i \psi_i[t, x, y, z]) + \frac{e^{i m t}}{\sqrt{2} m} (\psi_r[t, x, y, z] - i \psi_i[t, x, y, z]);$$

$$\text{EOM}\phi = \frac{1}{\sqrt{\text{Dmg}}} \text{Sum}\left[\text{D}\left[\sqrt{\text{Dmg}} g_{\mu\nu} \text{Upper}[[\mu, \nu]] \times \text{D}[\phi[t, x, y, z], x_{\mu}[[\nu]]], x_{\mu}[[\mu]]\right], \{\mu, 1, 4\}, \{\nu, 1, 4\}\right] + m^2 \phi[t, x, y, z];$$

$$\text{In[*]}:= \text{FullSimplify}\left[\frac{-m}{\sqrt{2}} \text{EOM}\phi_{\text{Newt}} /. \{\Psi \rightarrow \Phi\}\right]$$

$$\text{Out[*]}= \frac{1}{a[t]^2} (1 + 2\Phi[t, x, y, z]) (\text{Sin}[m t] (\psi_i^{(0,0,0,2)}[t, x, y, z] + \psi_i^{(0,0,2,0)}[t, x, y, z] + \psi_i^{(0,2,0,0)}[t, x, y, z]) + \text{Cos}[m t] (\psi_r^{(0,0,0,2)}[t, x, y, z] + \psi_r^{(0,0,2,0)}[t, x, y, z] + \psi_r^{(0,2,0,0)}[t, x, y, z])) + \dots$$

$$i\partial_t \psi = -\frac{1}{2m} \nabla^2 \psi + m\Phi\psi$$

$$\nabla^2 \Phi = 4\pi G |\psi|^2$$

Schrödinger-Poisson equations



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adding  $i$  x (Sin part) + (Cos part):

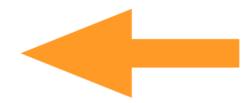
$$i \partial_t \psi = -\frac{1}{2m} \nabla^2 \psi + m \Phi \psi + \frac{1}{2m} \partial_t^2 \psi - 2 \psi_i \partial_t \Phi - \frac{2}{m} \partial_t \Phi \partial_t \psi_r + 2 i \partial_t \Phi \left( \psi_r - \frac{1}{m} \partial_t \psi_i \right) - \frac{1}{m} \Phi \nabla^2 \psi + \dots$$

subleading corrections

$$i \partial_t \psi = -\frac{1}{2m} \nabla^2 \psi + m \Phi \psi$$

$$\nabla^2 \Phi = 4\pi G |\psi|^2$$

Schrödinger-Poisson equations



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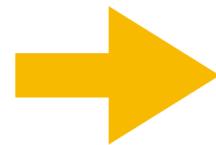
Einstein-Klein-Gordon equations

An aside: Madelung representation

$$\psi = \sqrt{\rho} e^{i\theta}$$

$$\nabla \theta = m\mathbf{v}$$

$$i\partial_t \psi = -\frac{1}{2m} \nabla^2 \psi + m\Phi \psi$$

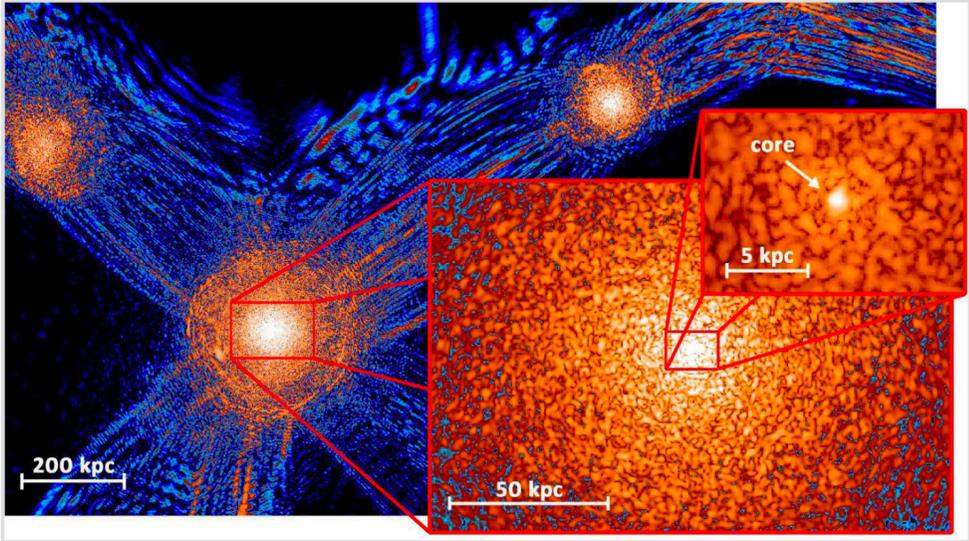


$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$$

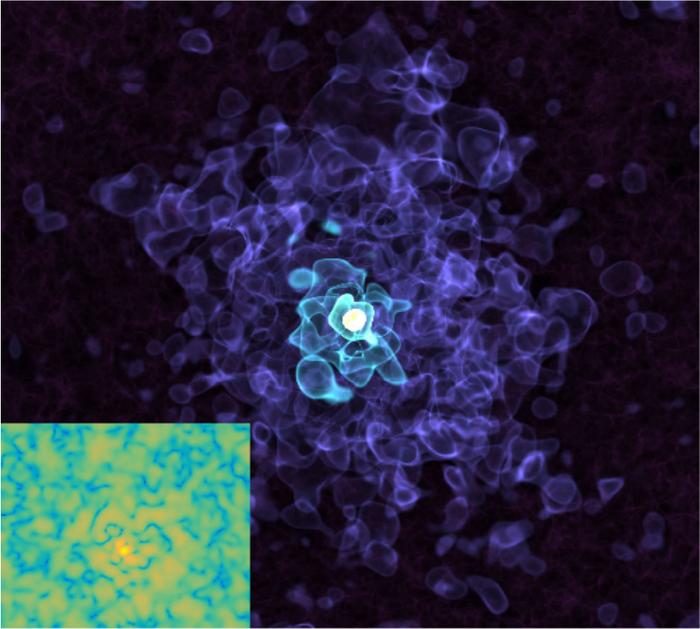
$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \Phi + \frac{1}{2m^2} \nabla \left( \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right)$$

# Numerical simulations

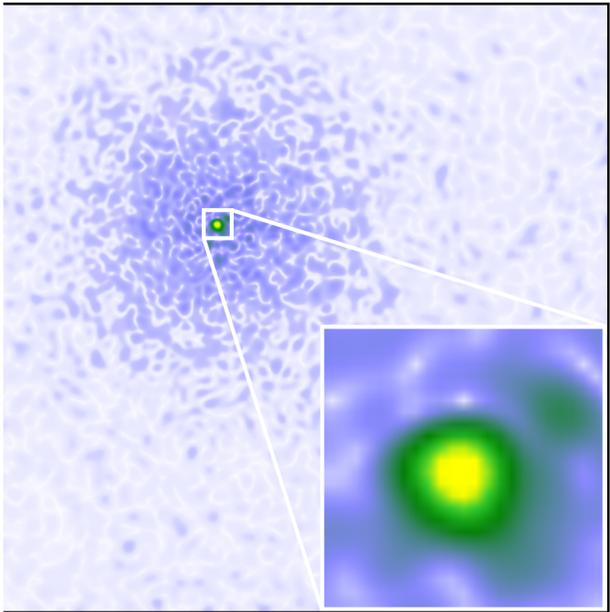
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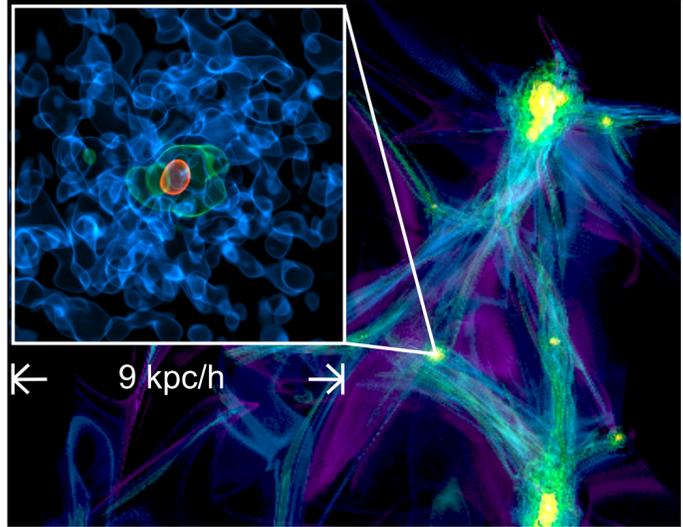
Schive et al 2014



Mocz et al 2017



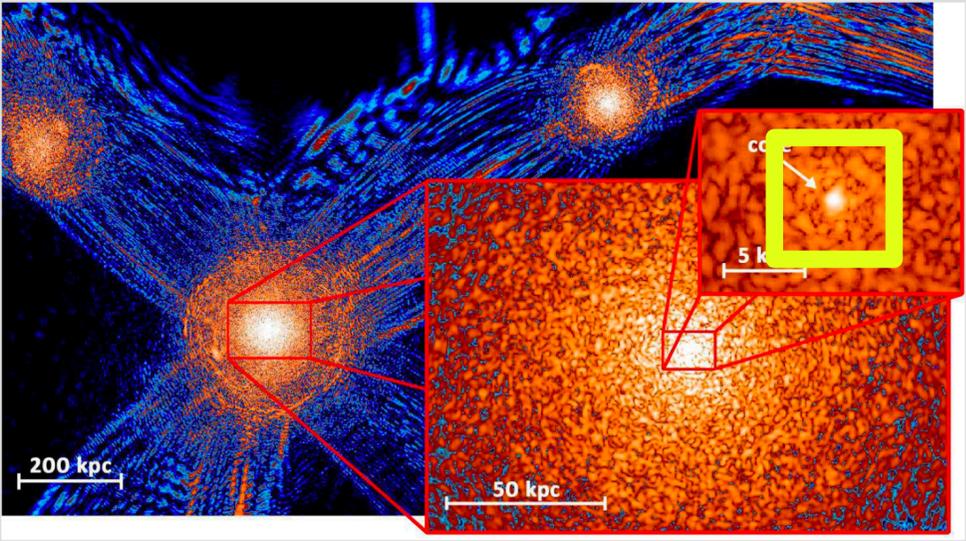
Levkov et al 2018



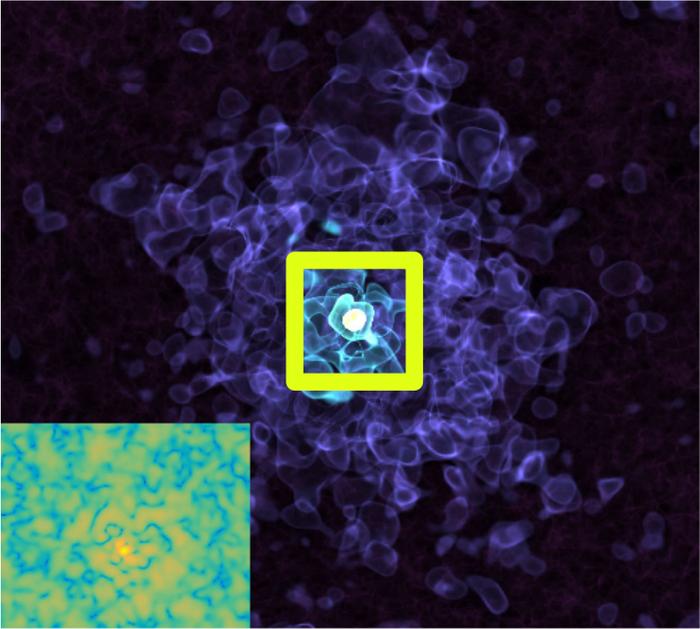
Veltmaat et al 2018

Inner part of simulated galaxies forms a **core**

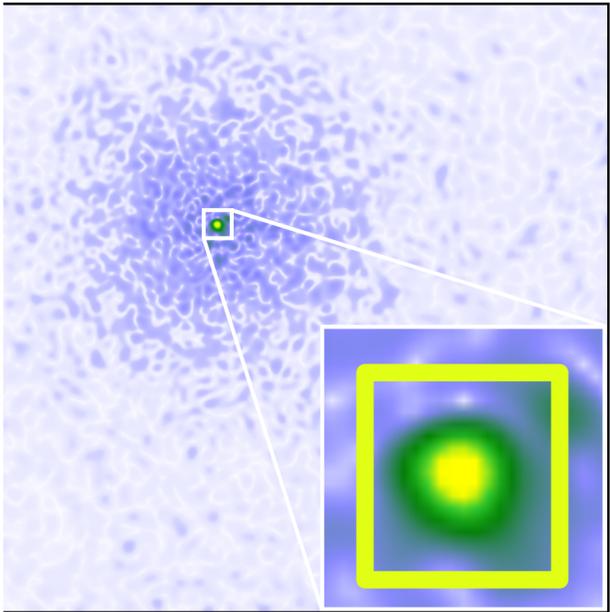
$$i\partial_t\psi = -\frac{1}{2m}\nabla^2\psi + m\Phi\psi$$
$$\nabla^2\Phi = 4\pi G|\psi|^2$$



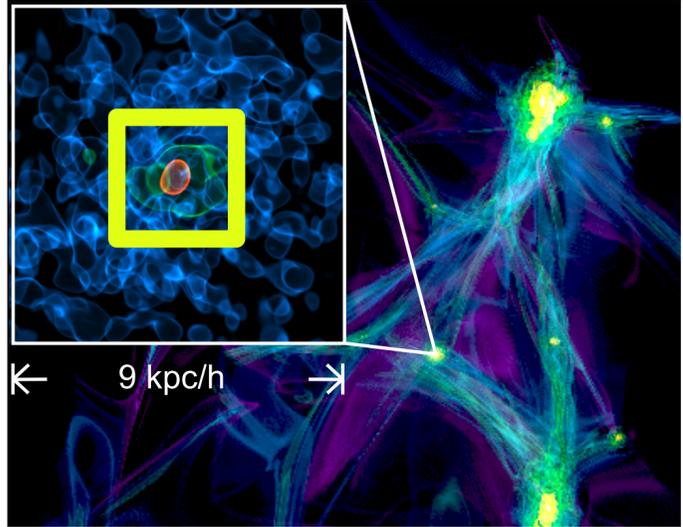
Schive et al 2014



Mocz et al 2017



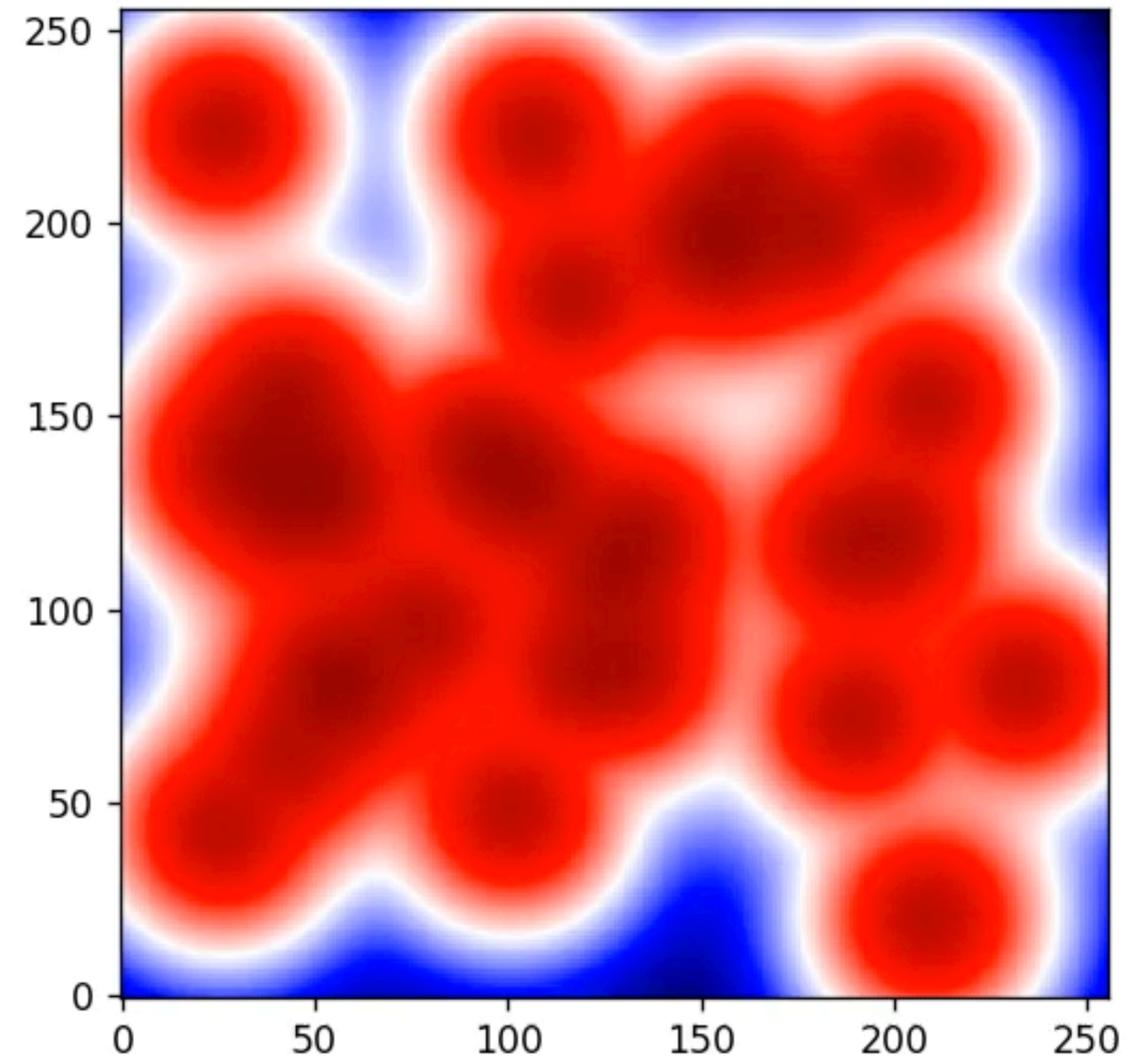
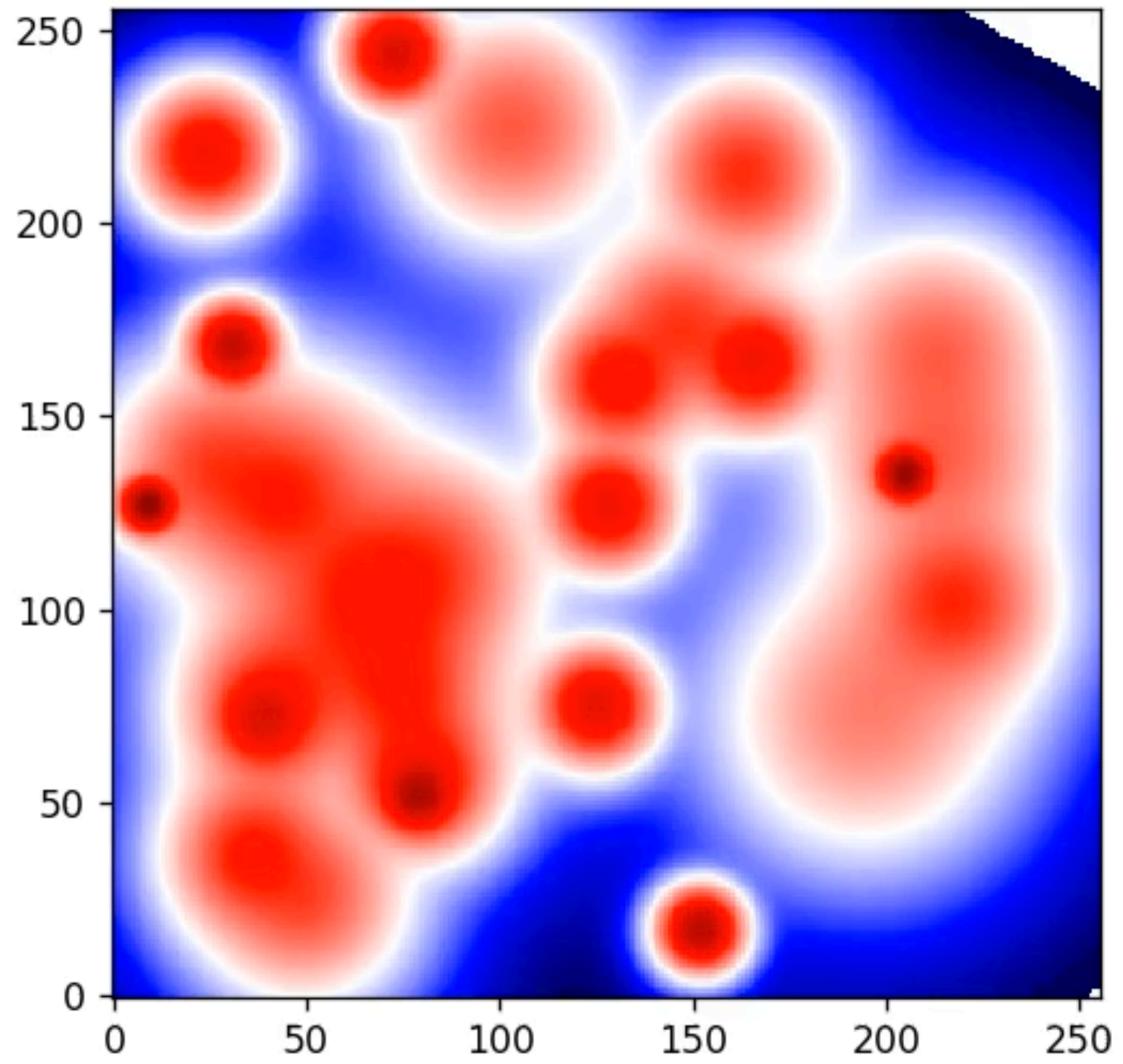
Levkov et al 2018



Veltmaat et al 2018

# Inner part of simulated galaxies forms a **core**

*Simulation: L. Teodori, M. Gorghetto*



Coherent ground state:  $\psi(x, t) = \left( \frac{mM_{pl}}{\sqrt{4\pi}} \right) e^{-i\gamma mt} \chi(r)$

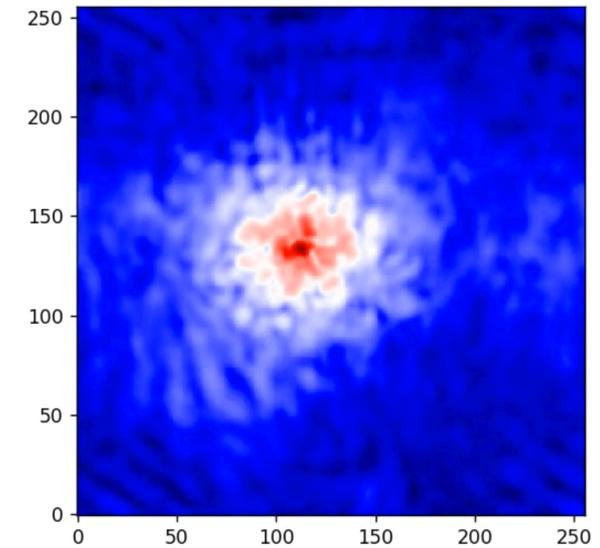
$$i\partial_t \psi = -\frac{1}{2m} \nabla^2 \psi + m\Phi \psi$$

$$\nabla^2 \Phi = 4\pi G |\psi|^2$$

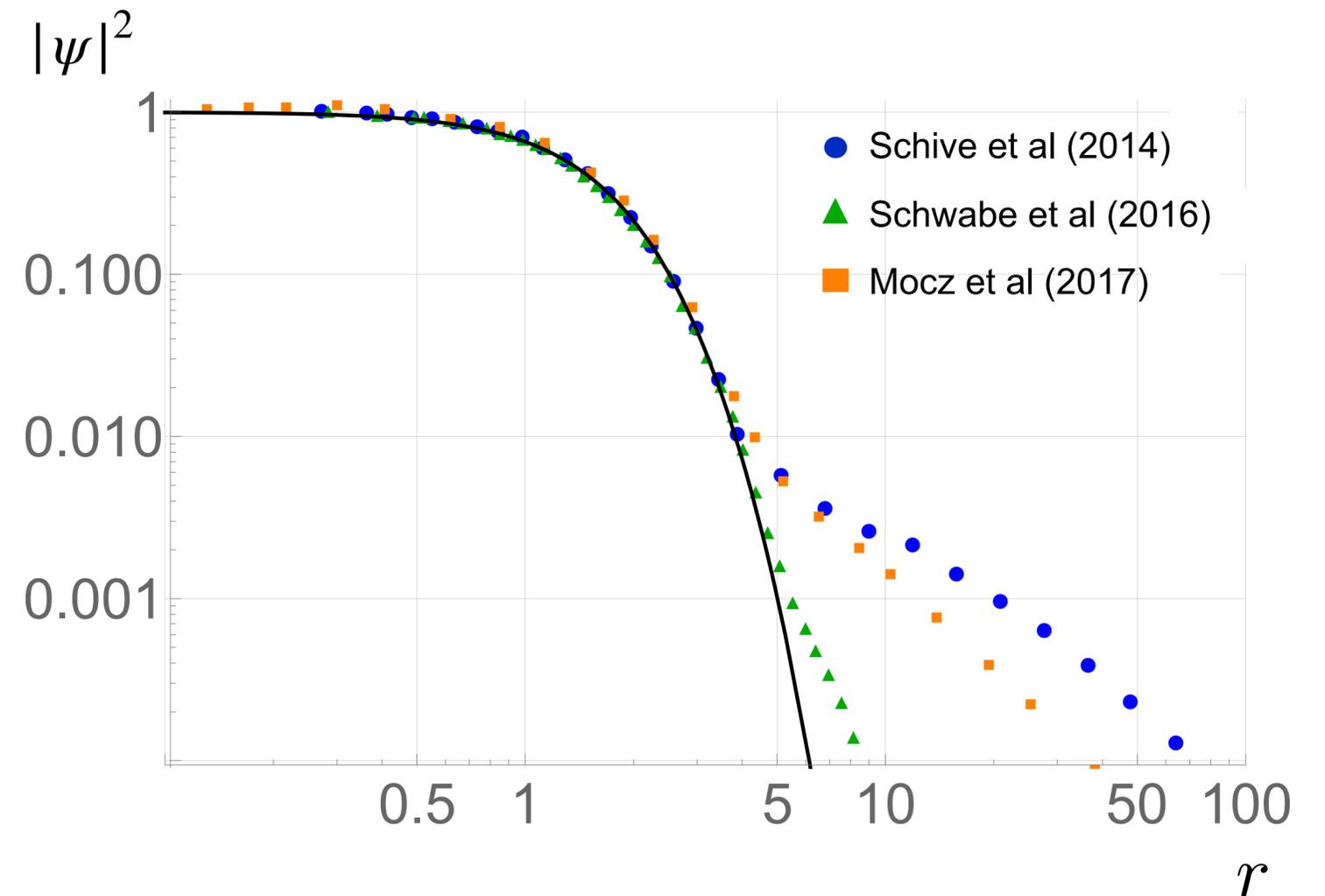


$$\partial_r^2 (r\chi) = 2r(\Phi - \gamma)\chi$$

$$\partial_r^2 (r\Phi) = r\chi^2$$



Numerical simulations find the ground state.



Coherent ground state:  $\psi(x, t) = \left( \frac{mM_{pl}}{\sqrt{4\pi}} \right) e^{-i\gamma mt} \chi(r)$

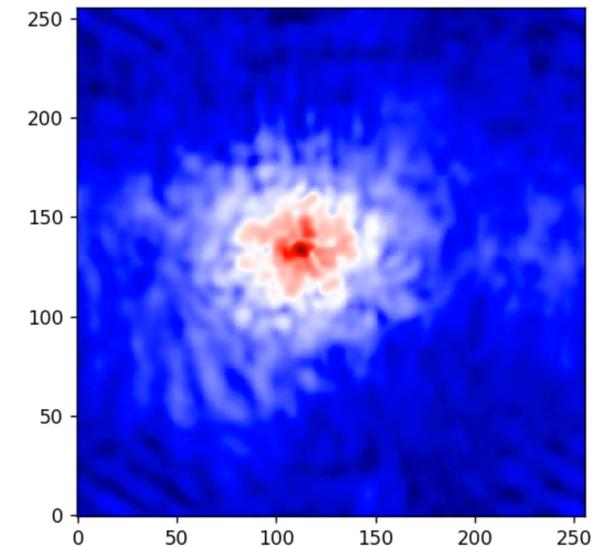
$$i\partial_t \psi = -\frac{1}{2m} \nabla^2 \psi + m\Phi \psi$$

$$\nabla^2 \Phi = 4\pi G |\psi|^2$$



$$\partial_r^2 (r\chi) = 2r(\Phi - \gamma)\chi$$

$$\partial_r^2 (r\Phi) = r\chi^2$$

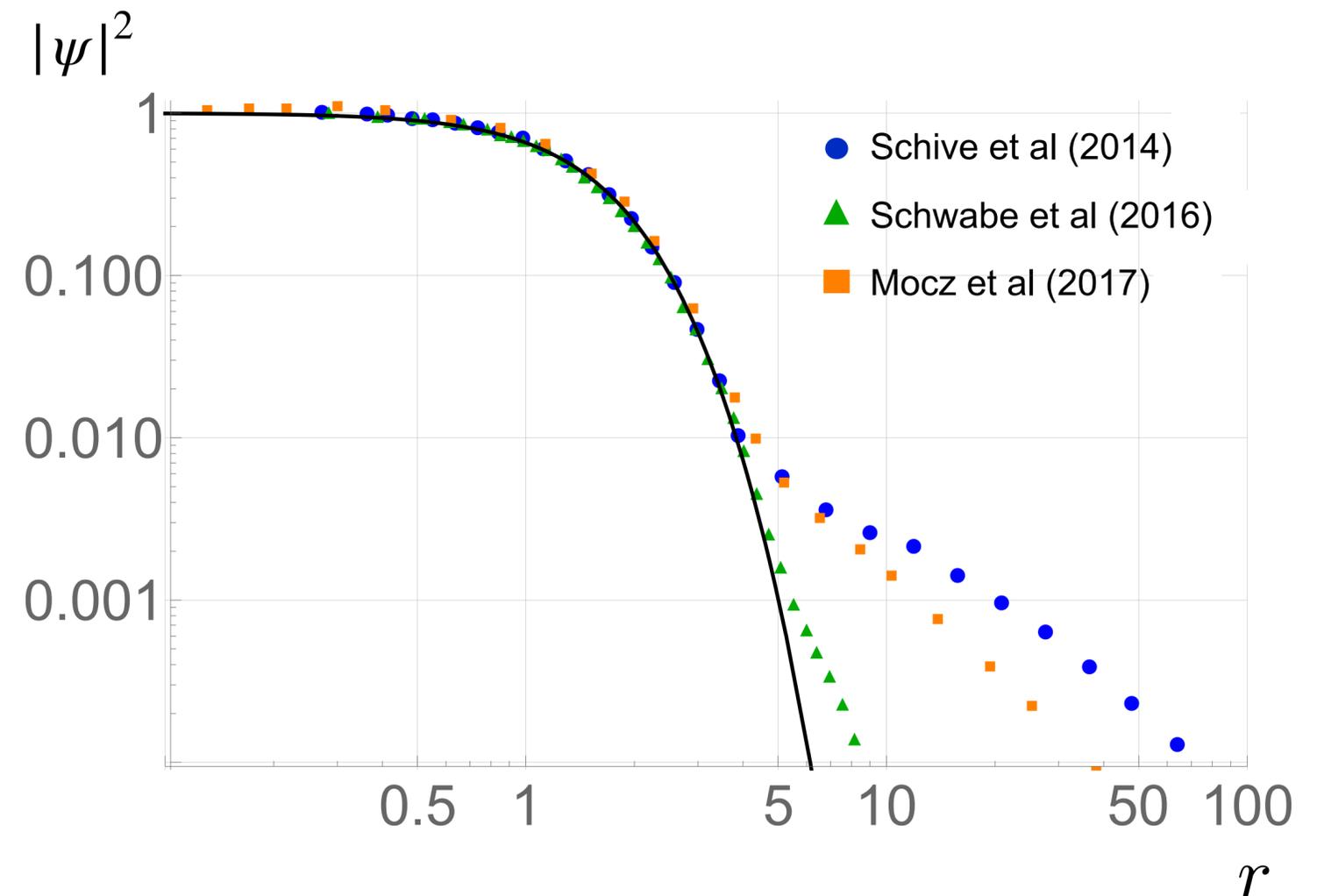


Numerical simulations find the ground state.

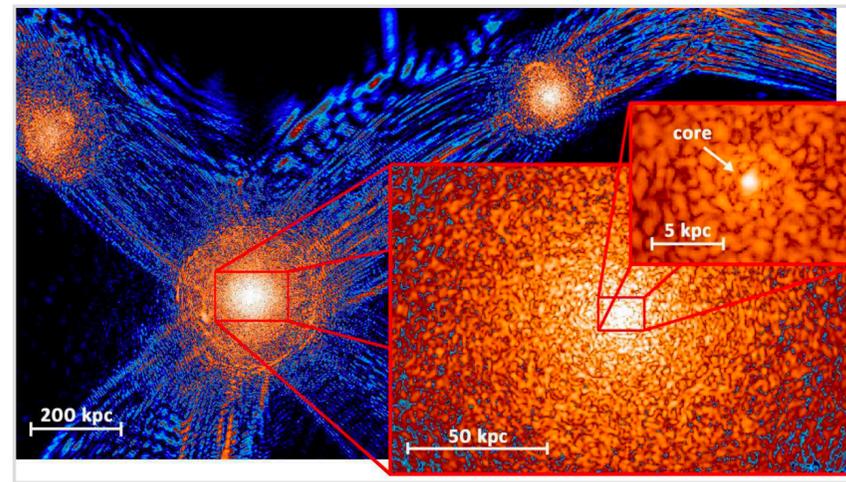
This is reminiscent of BEC:

Gross-Pitaevskii with nonlocal interaction:

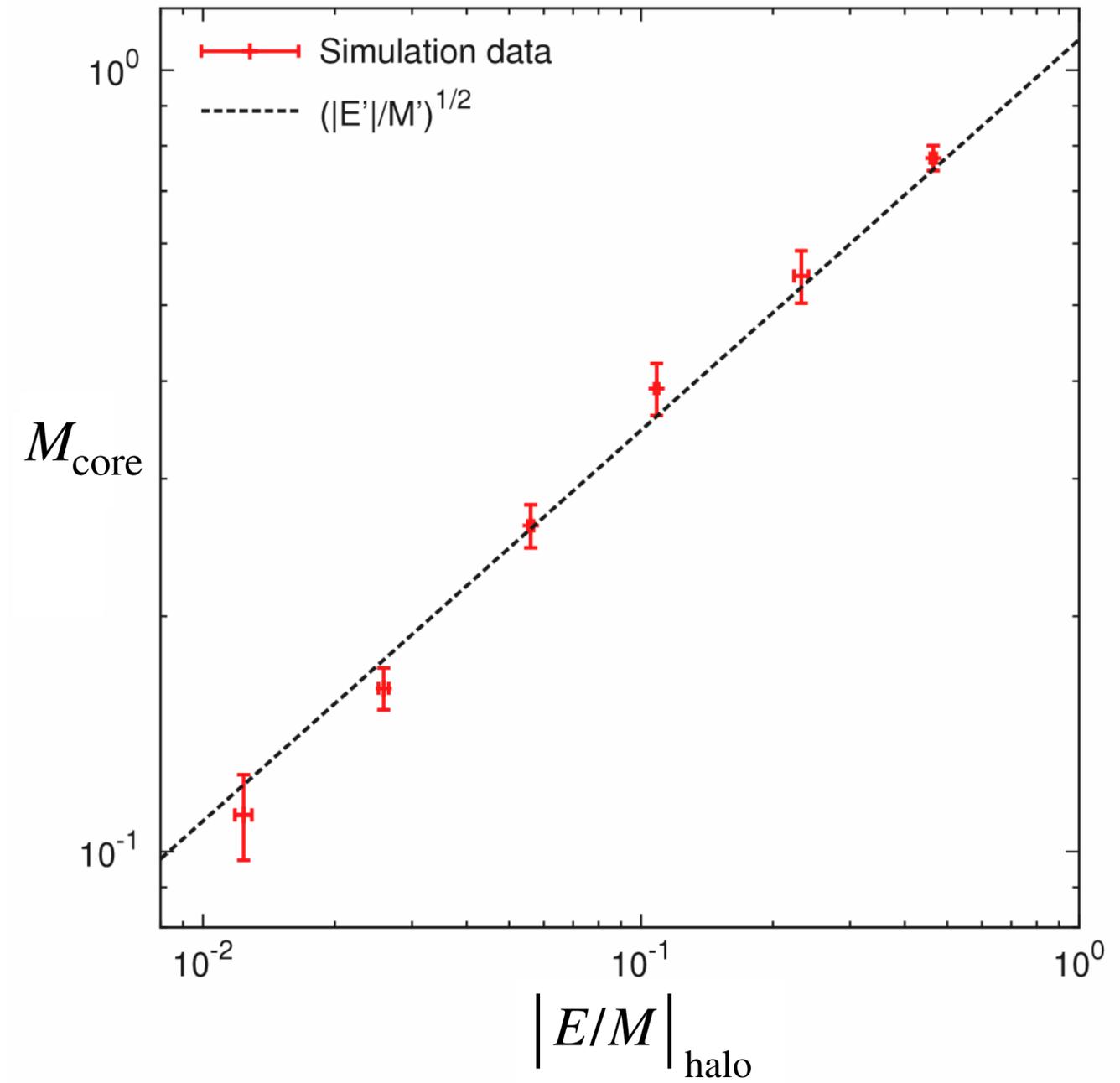
$$i\partial_t \psi(x) = -\left( \frac{1}{2m} \nabla^2 + Gm \int d^3y \frac{|\psi(y)|^2}{|x-y|} \right) \psi(x)$$



# Core — halo relation



Schive et al  
1406.6586, 1407.7762



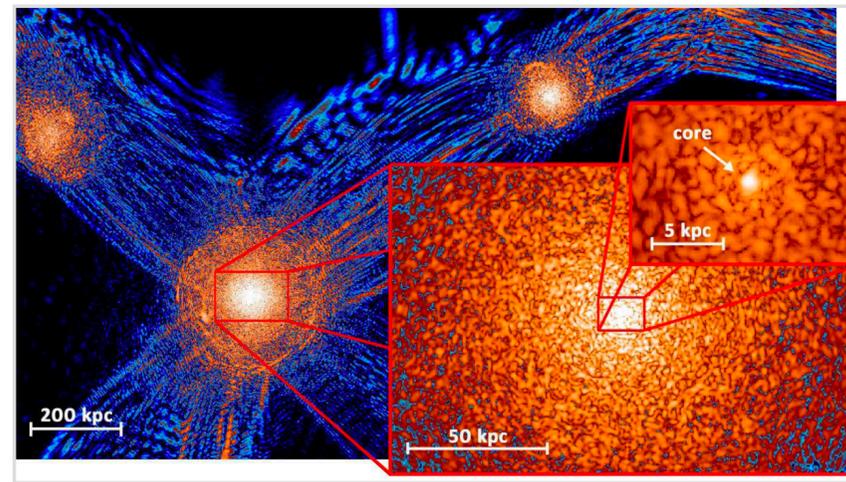
# Core — halo relation

Bar, et al 1805.00122

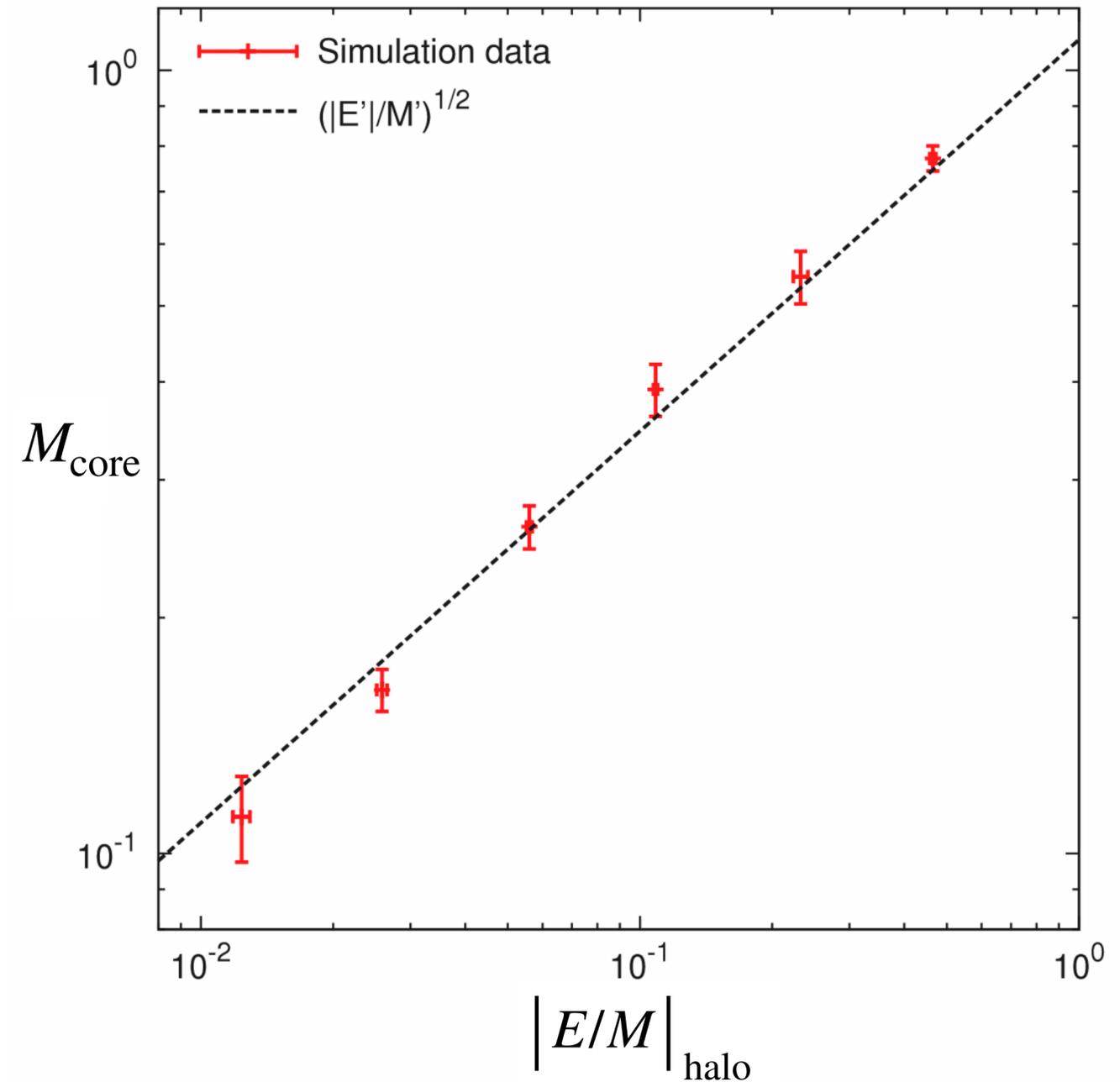
$$\frac{E}{M} \Big|_{\text{core}} = \frac{E}{M} \Big|_{\text{halo}}$$

$$M = \int d^3x |\psi|^2$$

$$E = \int d^3x \left( \frac{|\nabla\psi|^2}{2m^2} + \frac{\Phi |\psi|^2}{2} \right)$$



Schive et al  
1406.6586, 1407.7762



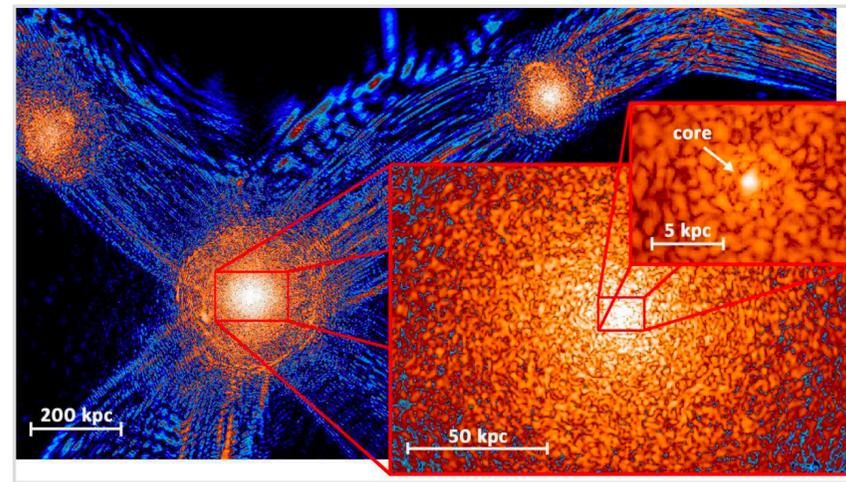
# Core — halo relation

Bar, et al 1805.00122

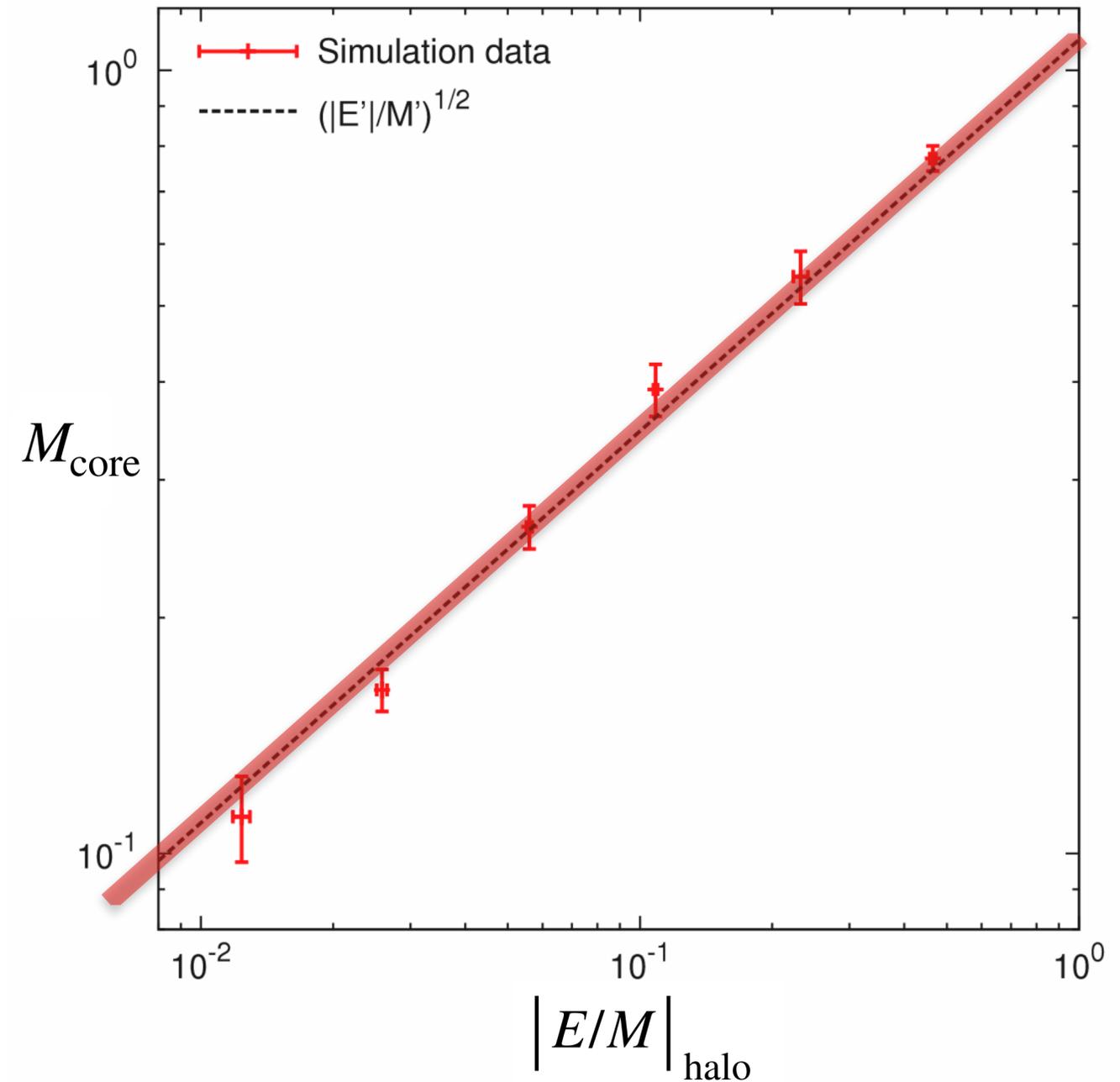
$$\frac{E}{M} \Big|_{\text{core}} = \frac{E}{M} \Big|_{\text{halo}}$$

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Schive et al  
1406.6586, 1407.7762



## Core — halo relation

Bar, et al 1805.00122, 1903.03402

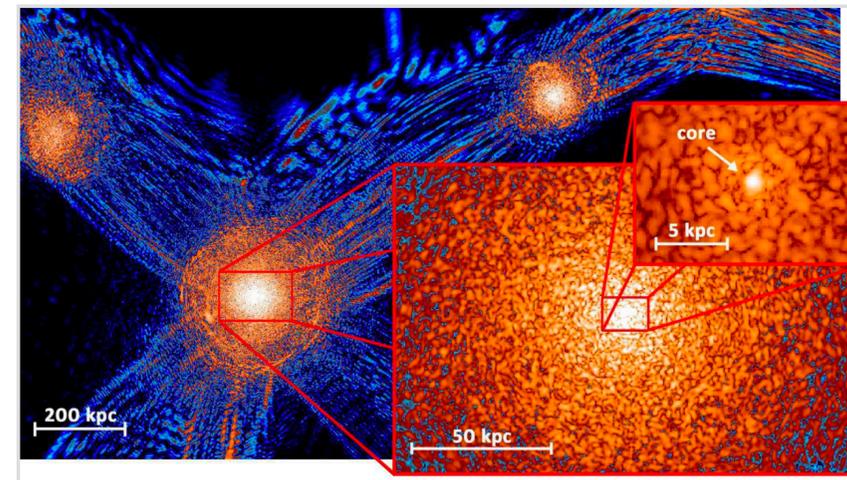
$$\frac{K}{M} \Big|_{\text{core}} = \frac{K}{M} \Big|_{\text{halo}}$$

K/M: kinetic energy/mass.

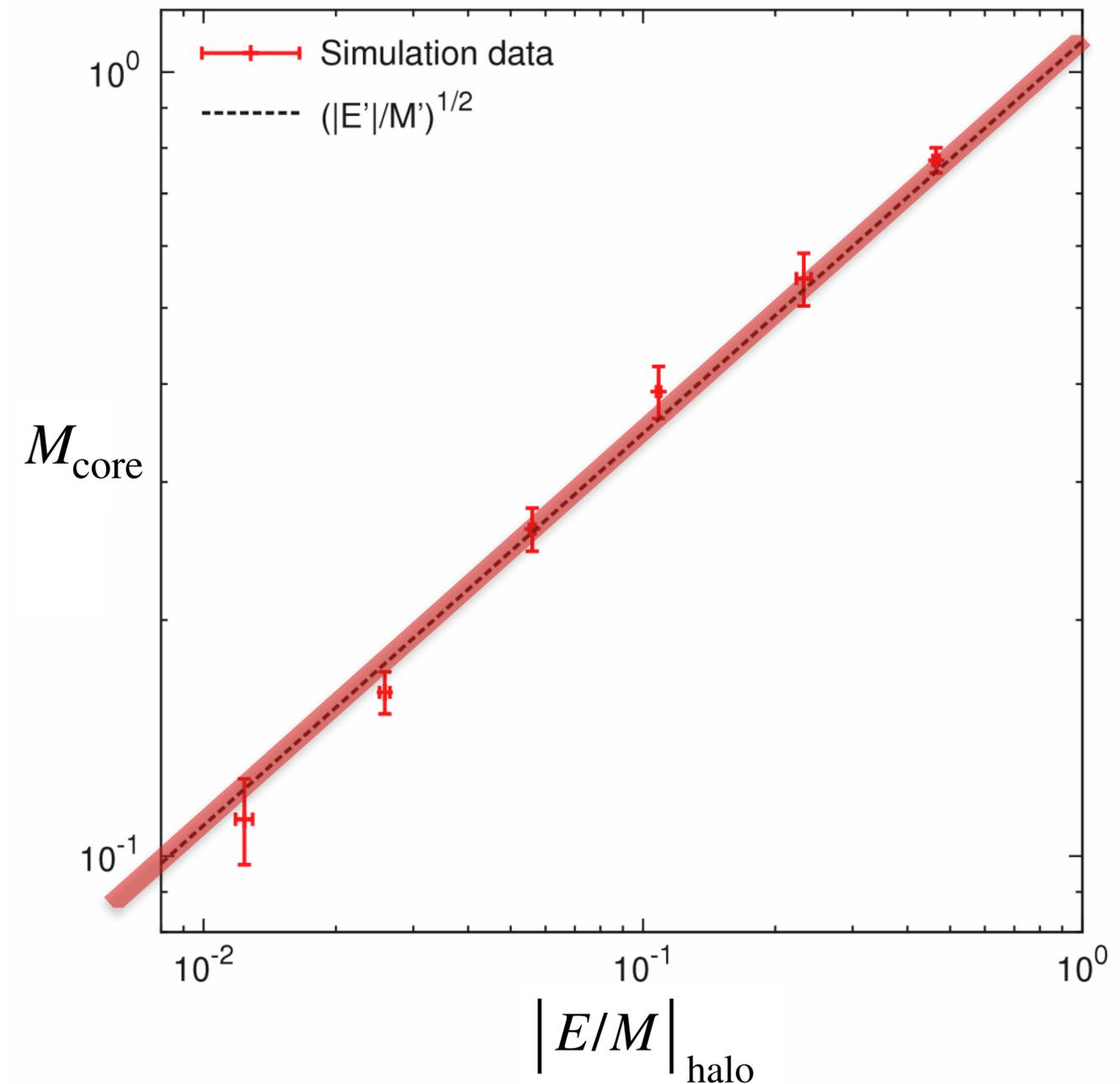
$$M = \int d^3x |\psi|^2$$

$$E = \int d^3x \left( \frac{|\nabla\psi|^2}{2m^2} + \frac{\Phi |\psi|^2}{2} \right)$$

$$K = \int d^3x \frac{|\nabla\psi|^2}{2m^2} = -E$$



Schive et al  
1406.6586, 1407.7762



# Core — halo relation

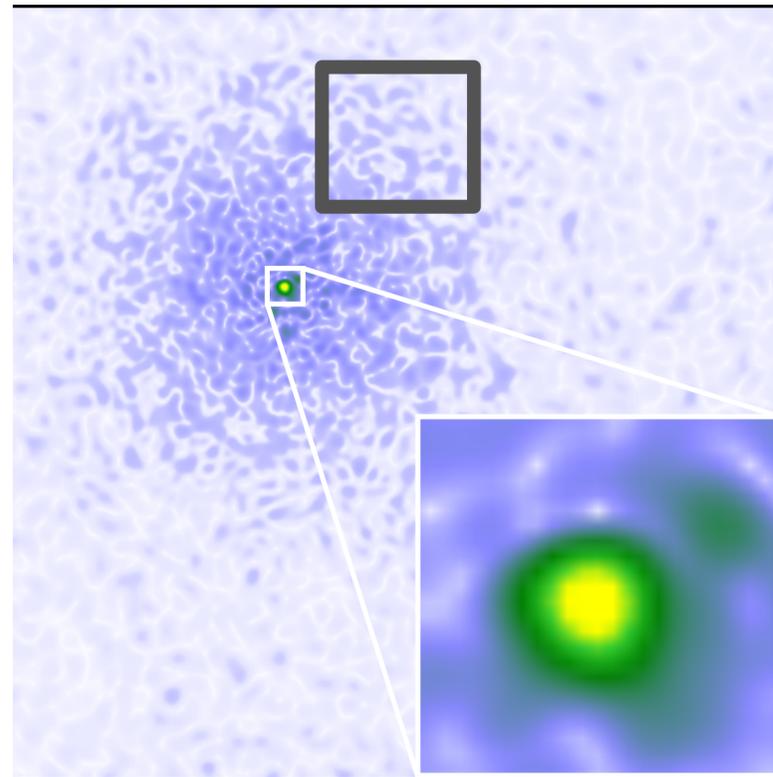
Bar, et al 1805.00122, 1903.03402

$$\frac{K}{M} \Big|_{\text{core}} = \frac{K}{M} \Big|_{\text{halo}}$$

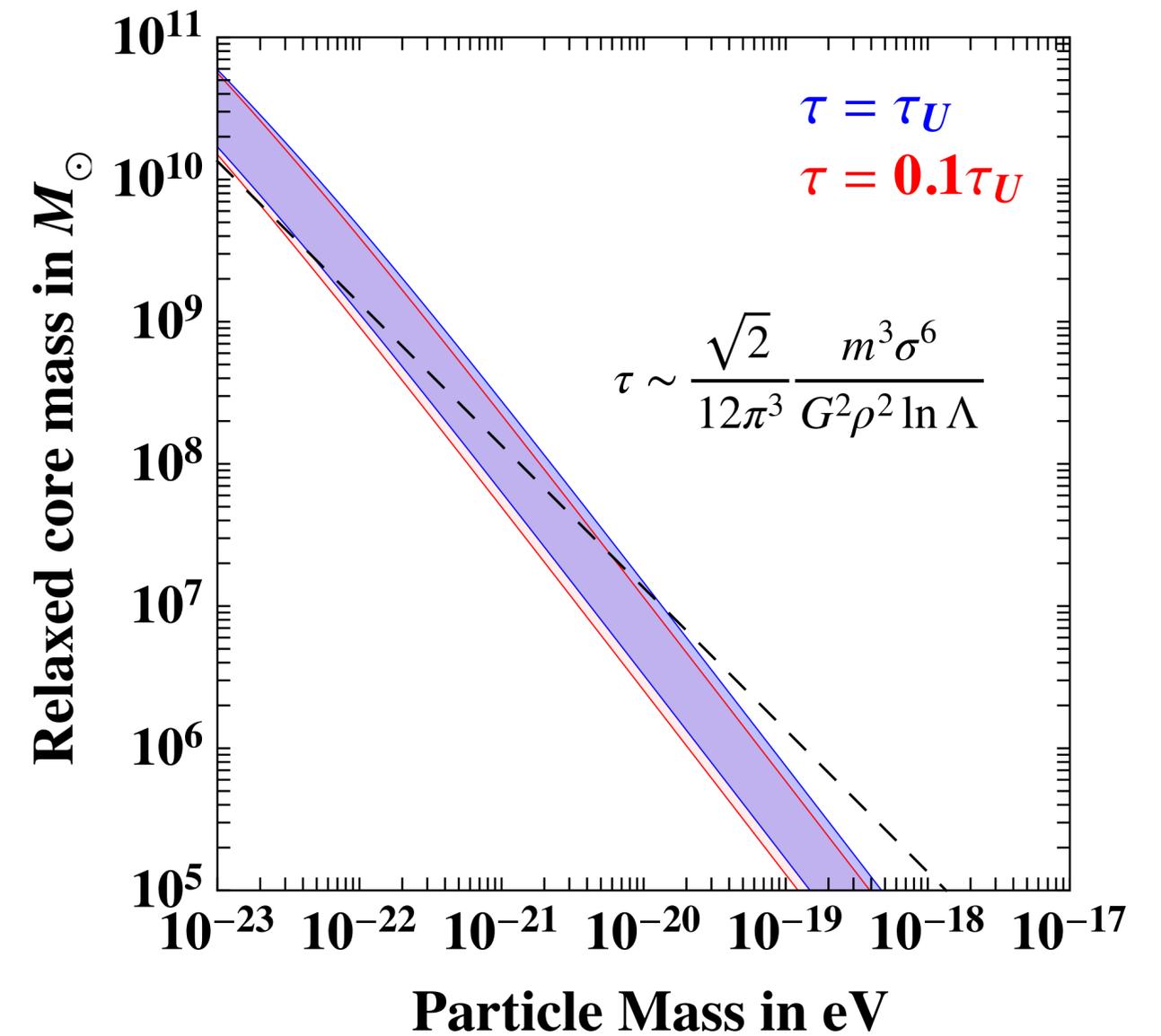
K/M: kinetic energy/mass.

Levkov et al 2018

Hui et al 2017,  
Bar-Or, Tremaine 2018,  
Chen et al 2021



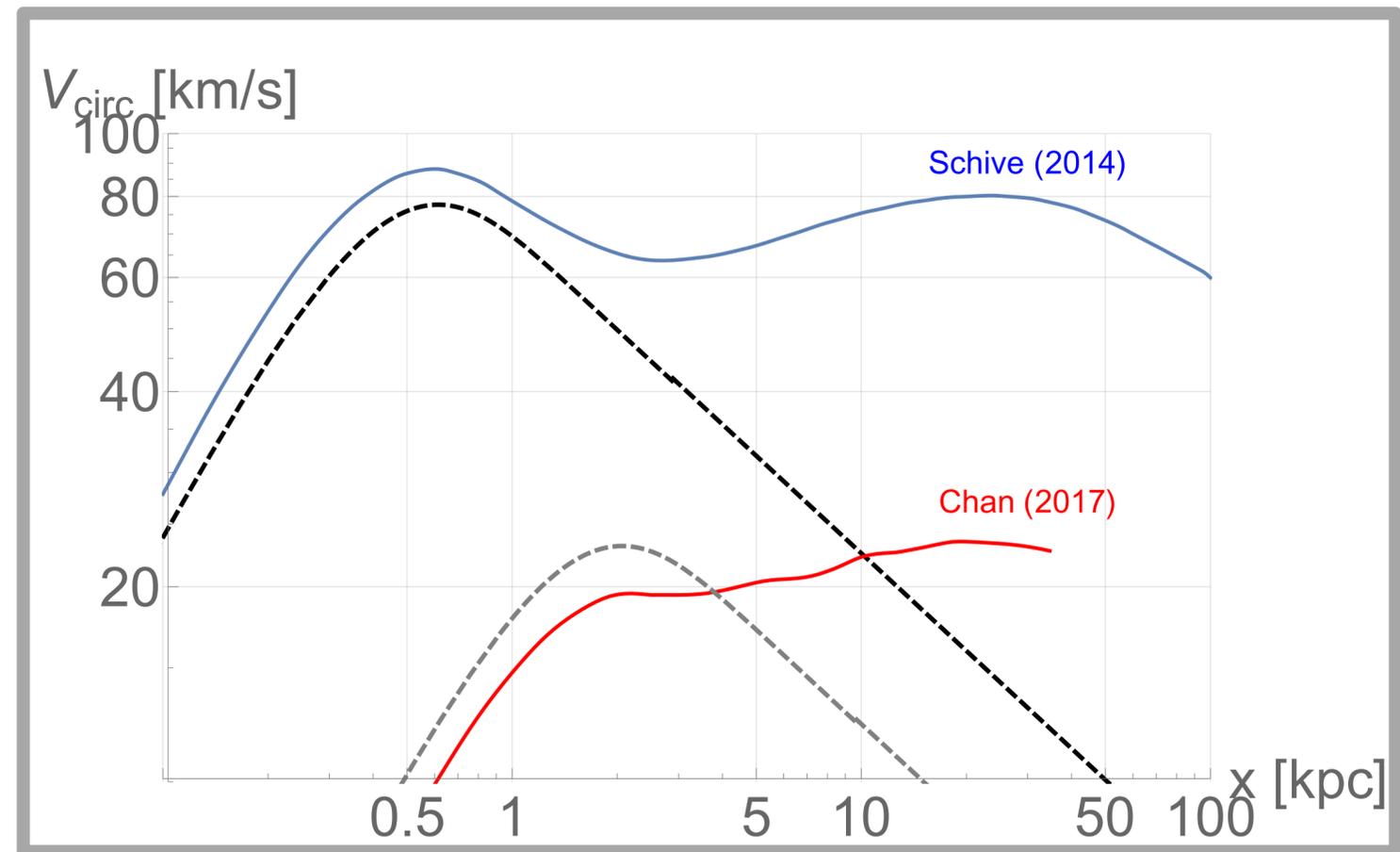
Dynamical relaxation is fast enough for  $m \lesssim 10^{-21}$  eV.



## Core — halo relation

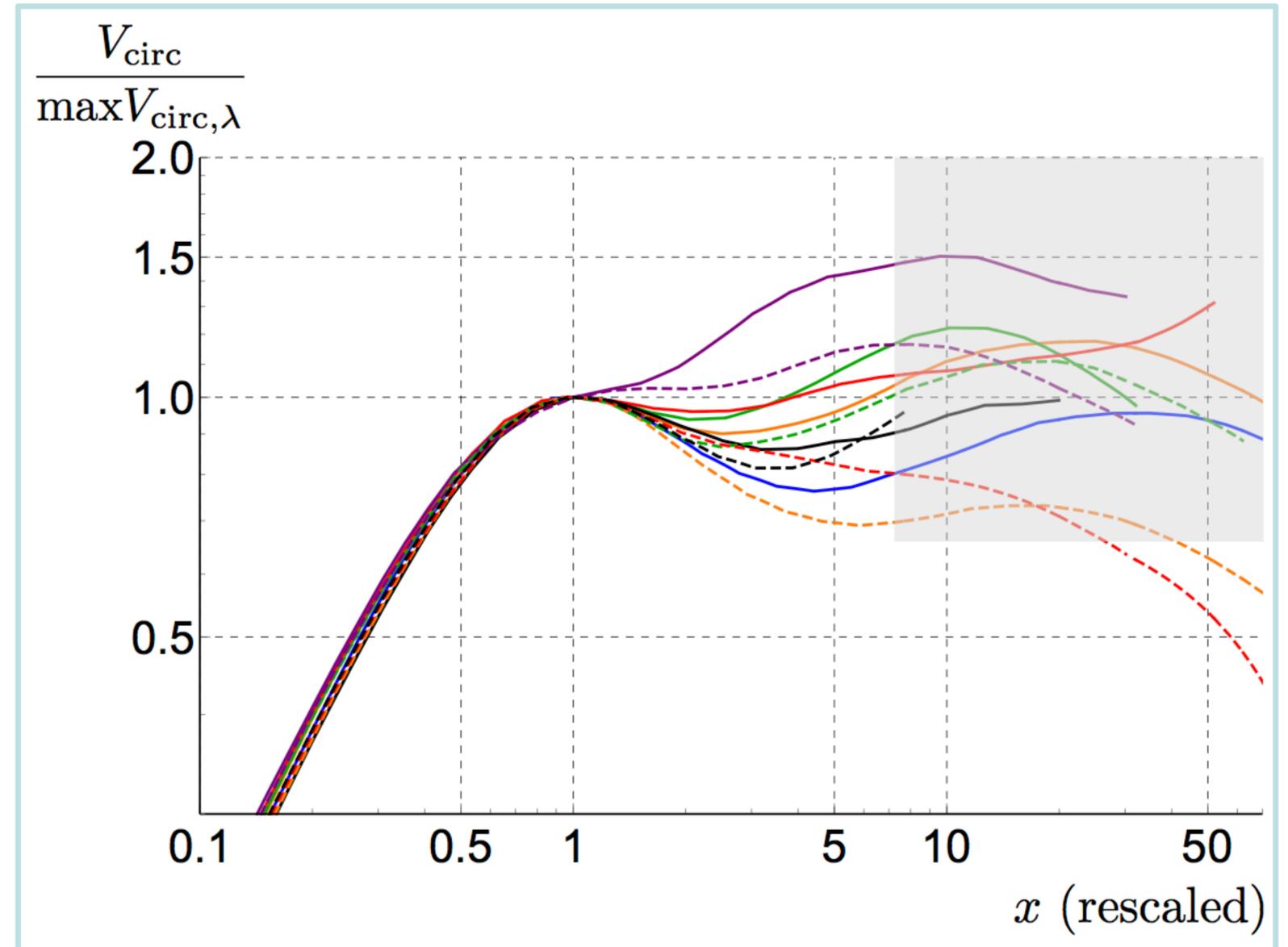
$$\left. \frac{K}{M} \right|_{\text{core}} = \left. \frac{K}{M} \right|_{\text{halo}}$$

## Rotation curves from simulations:

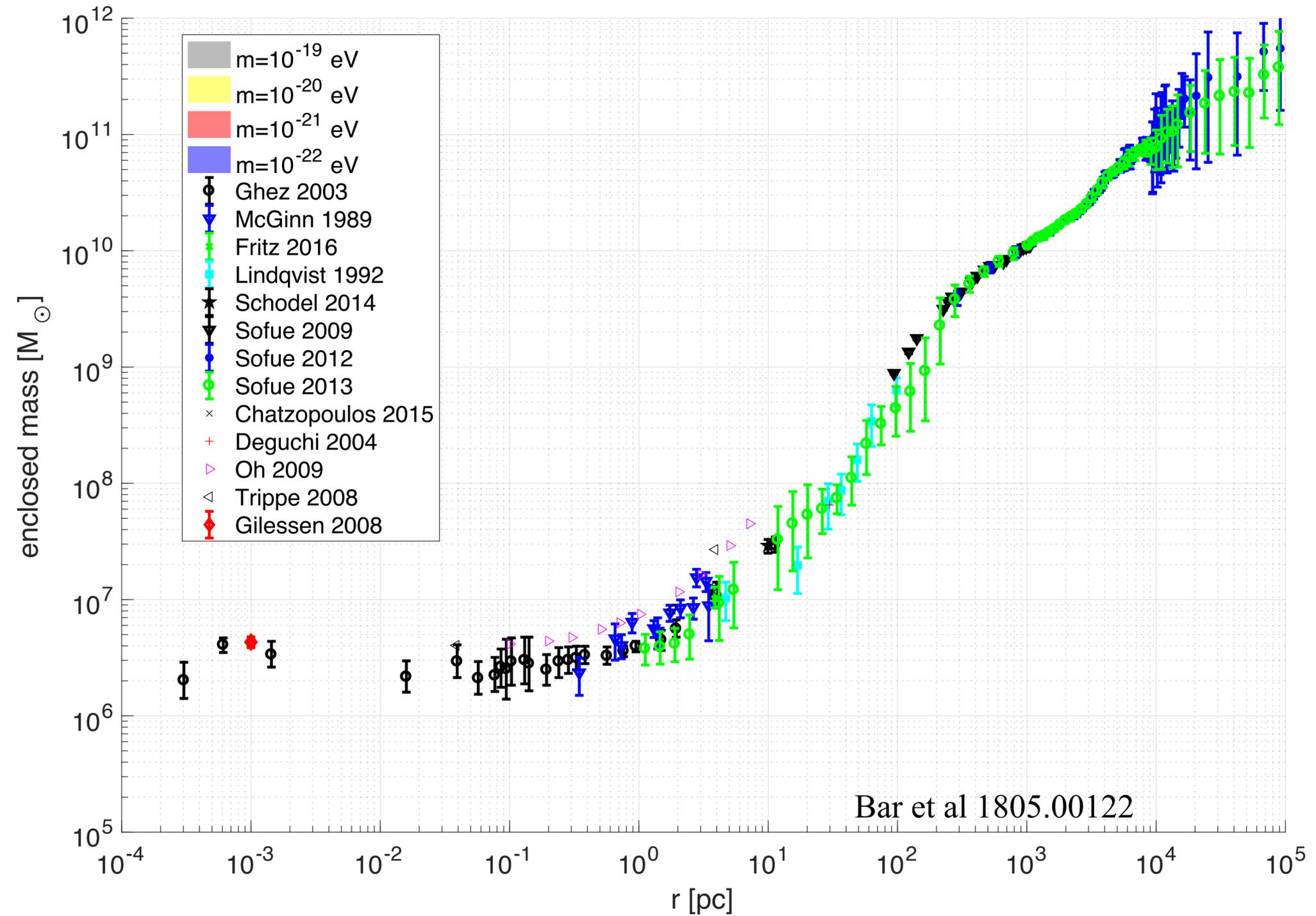


# Core — halo relation

$$\left. \frac{K}{M} \right|_{\text{core}} = \left. \frac{K}{M} \right|_{\text{halo}}$$

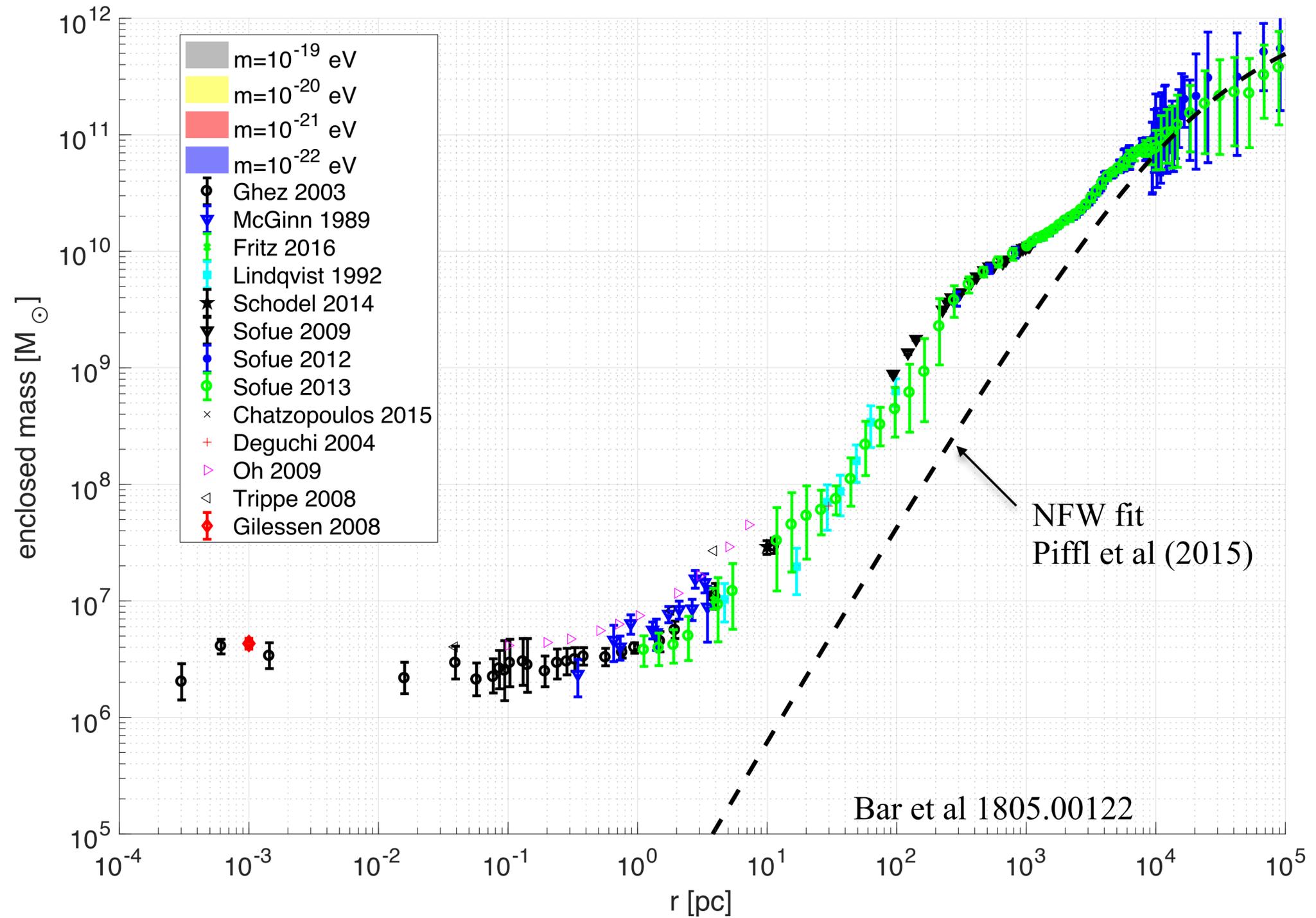


(Estimated) radially-averaged mass profile of the Milky Way



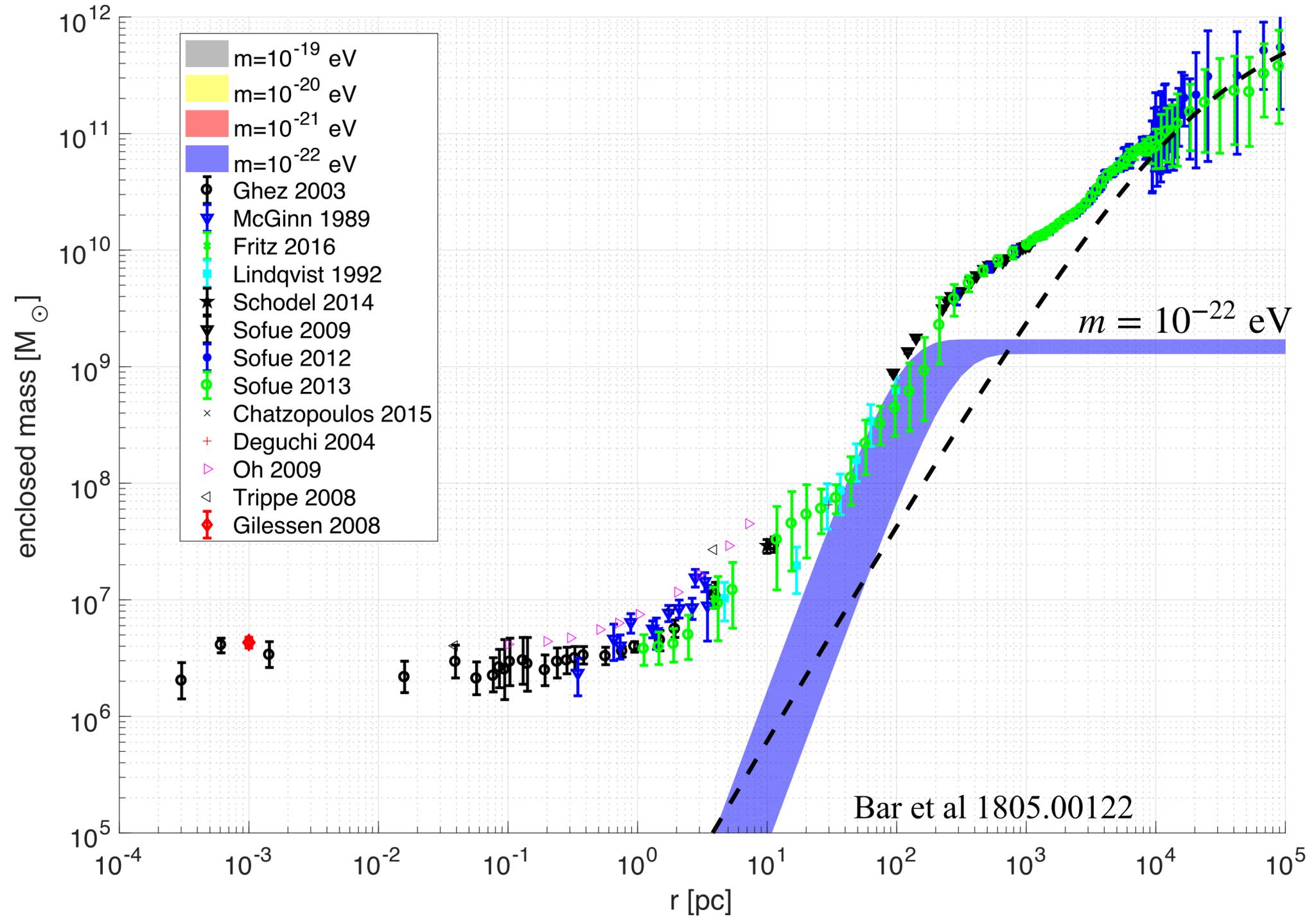
Massive cold dark matter:

thought to affect outer part of rotation curve



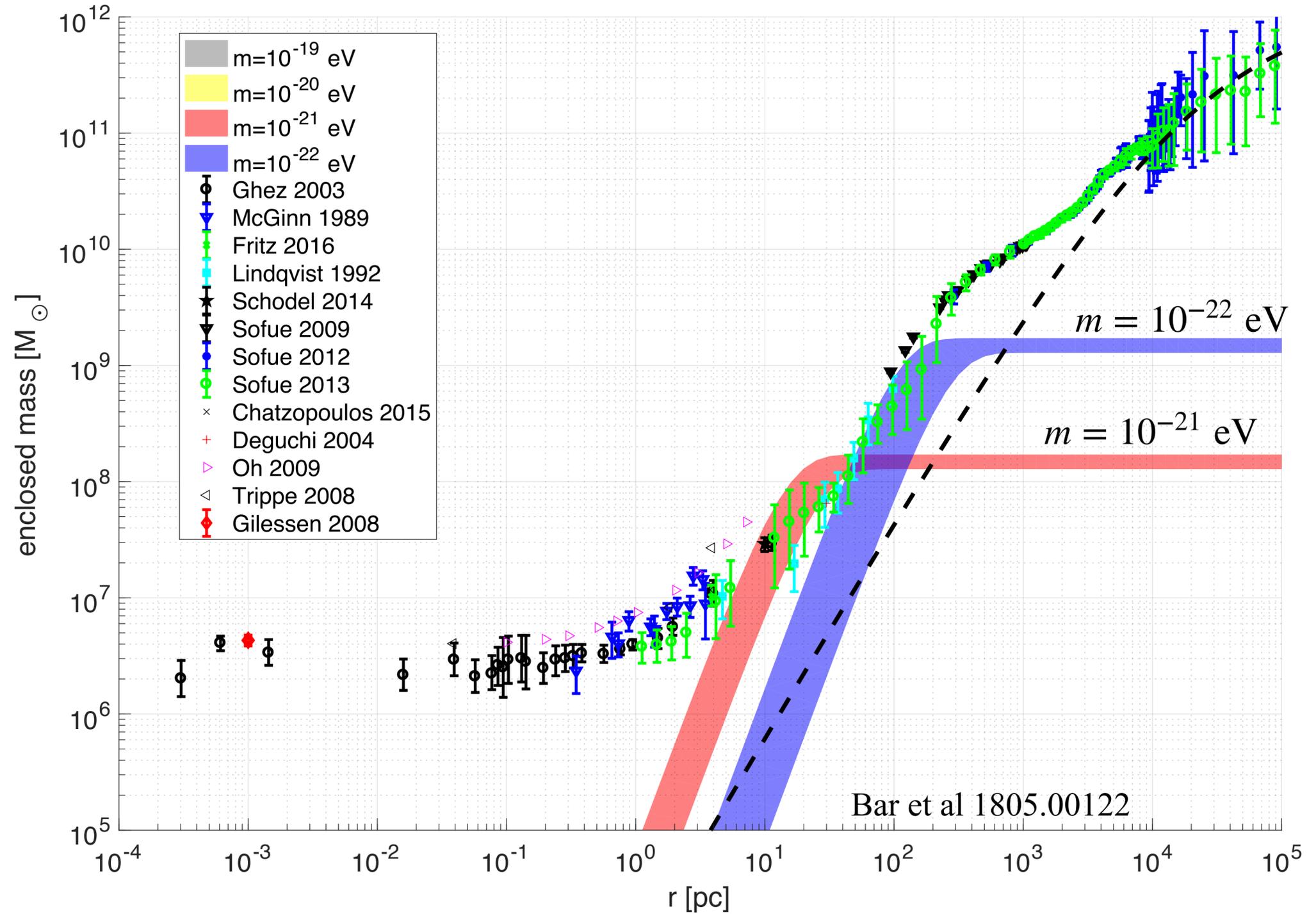
ULDM:

Affects the inner part of rotation curve



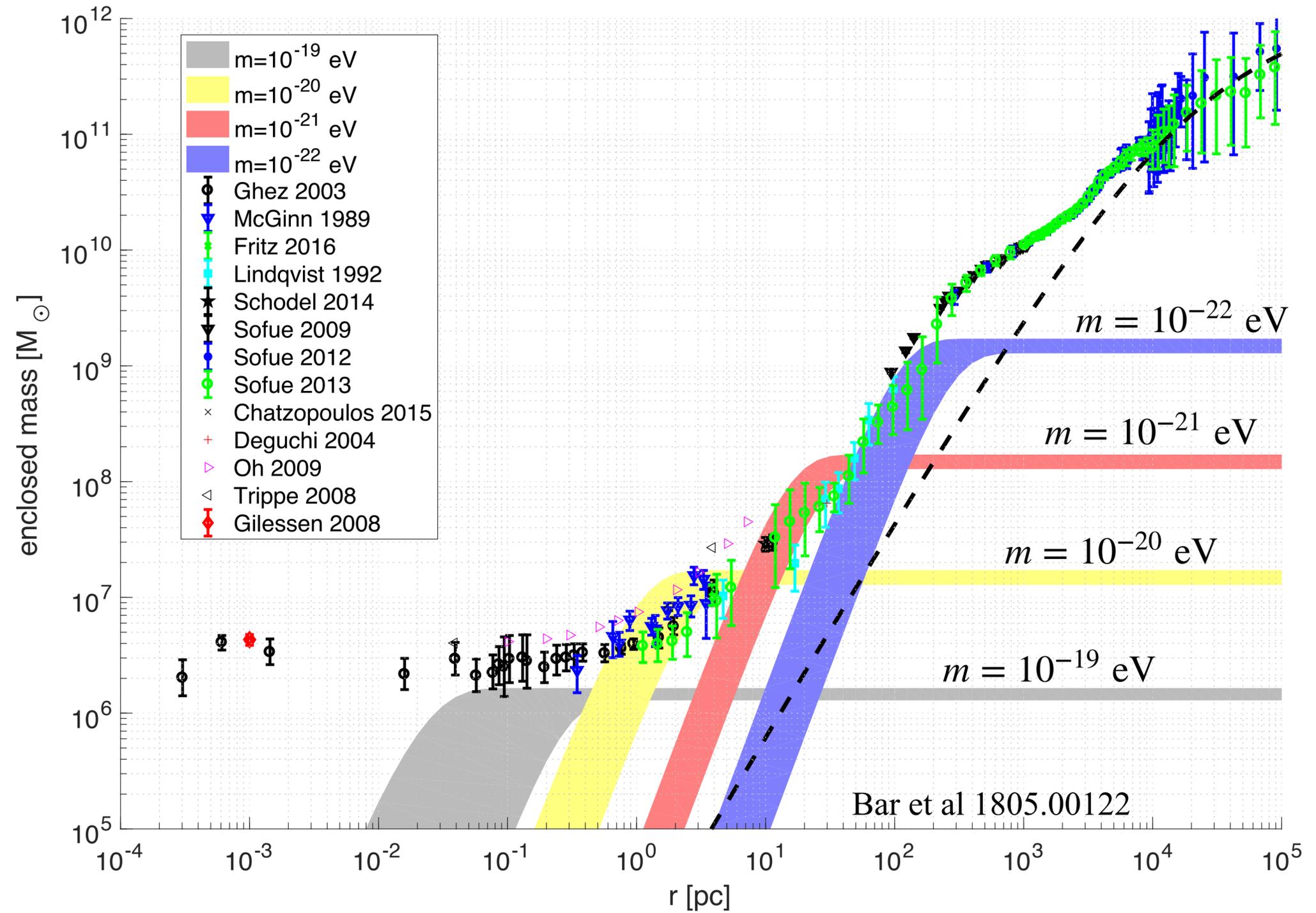
ULDM:

Affects the inner part of rotation curve



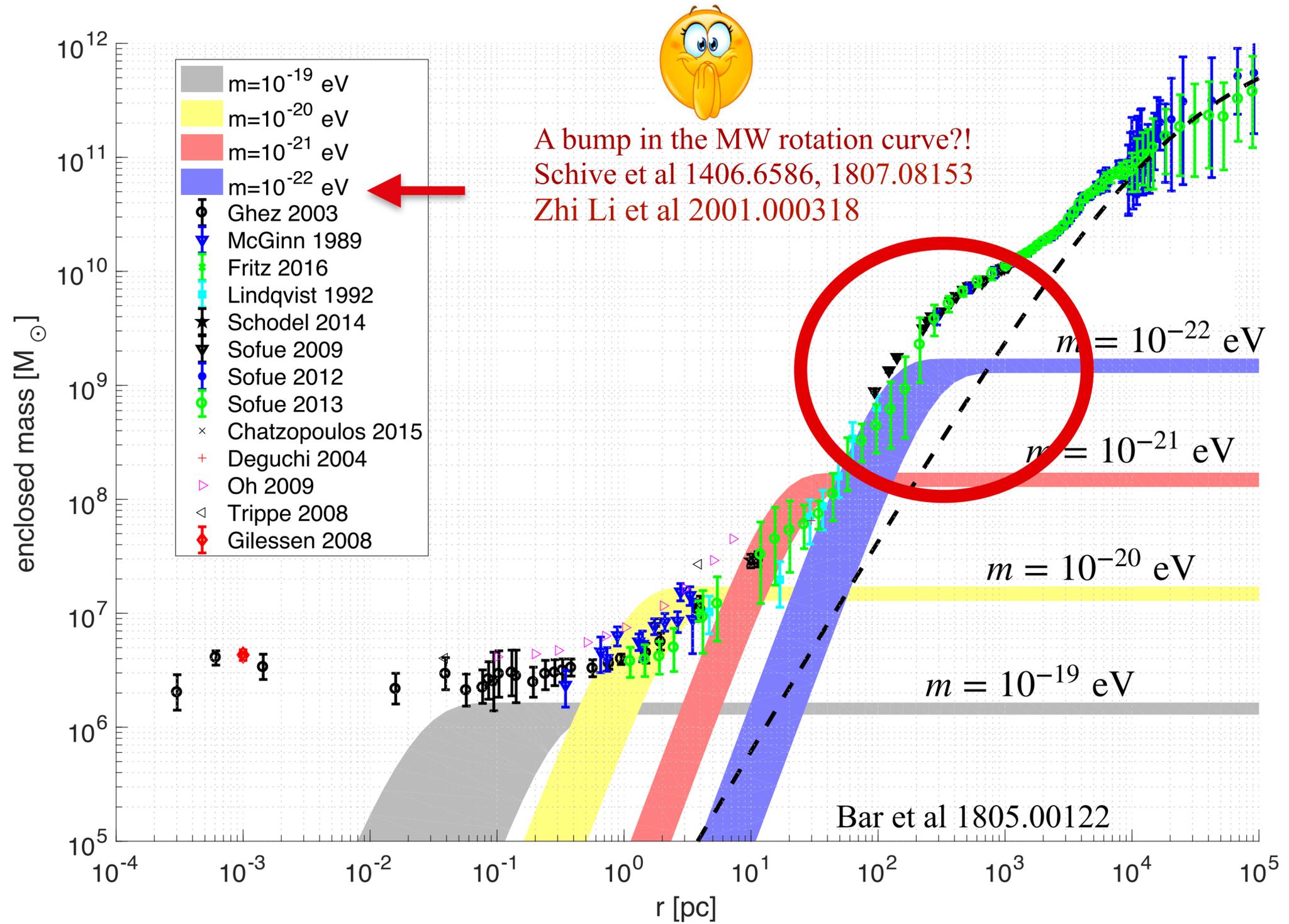
ULDM:

Affects the inner part of rotation curve



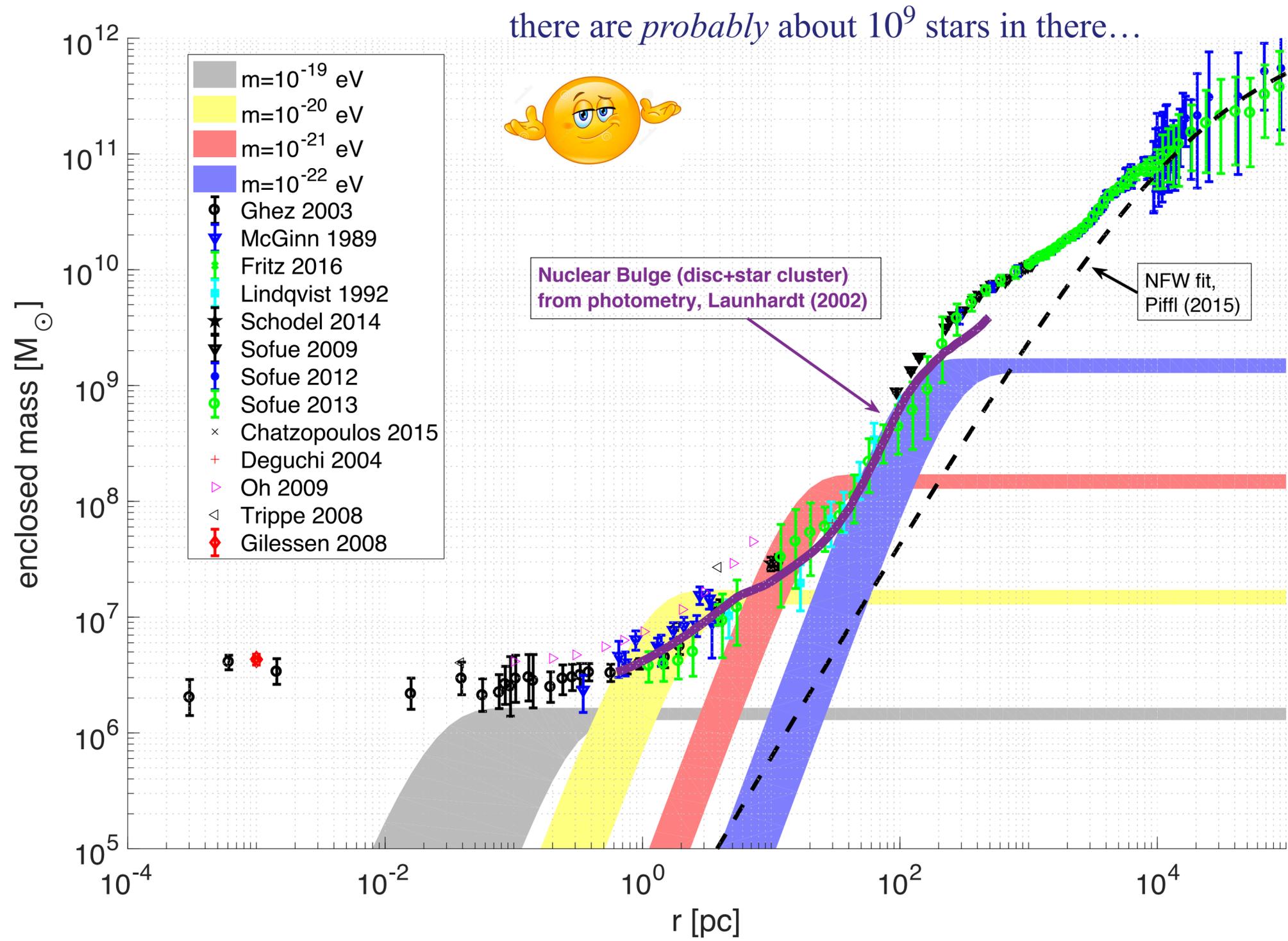
ULDM:

Affects the inner part of rotation curve

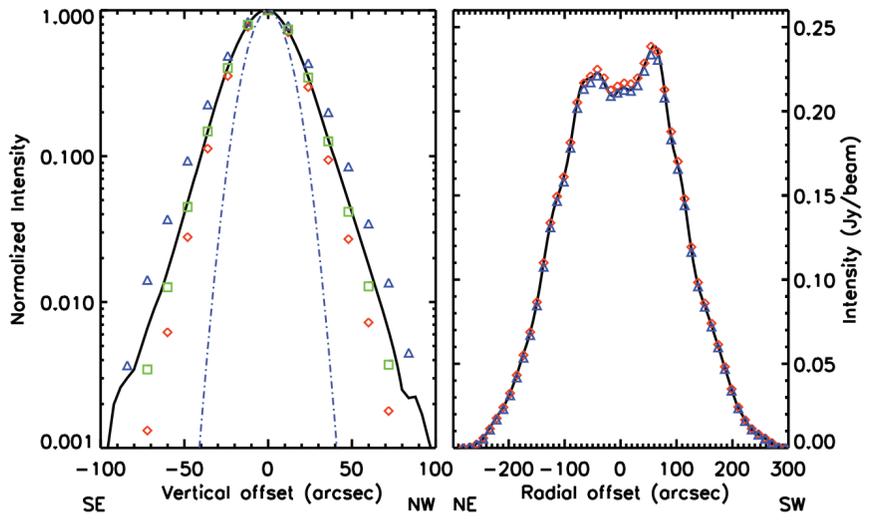
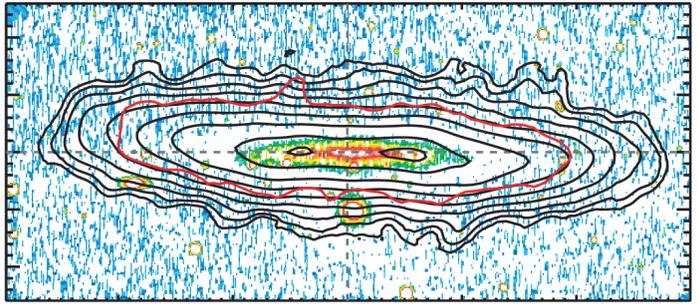
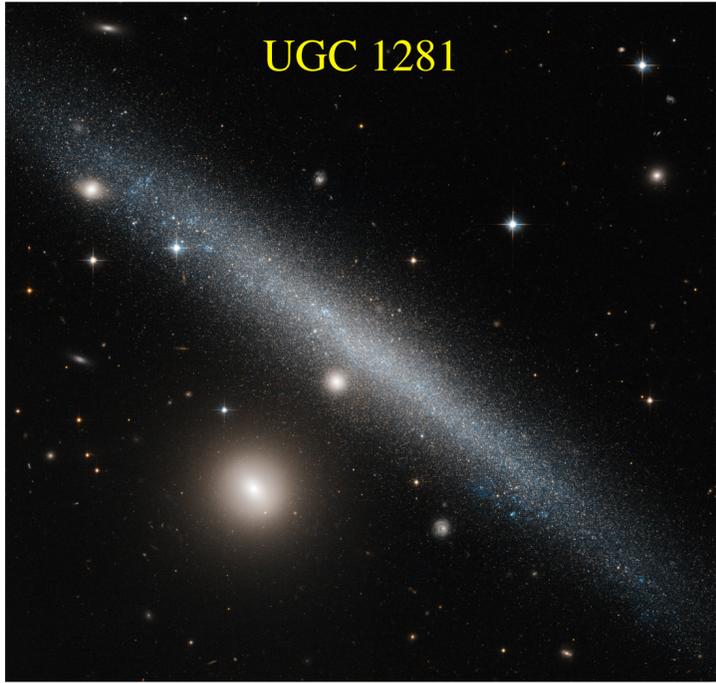
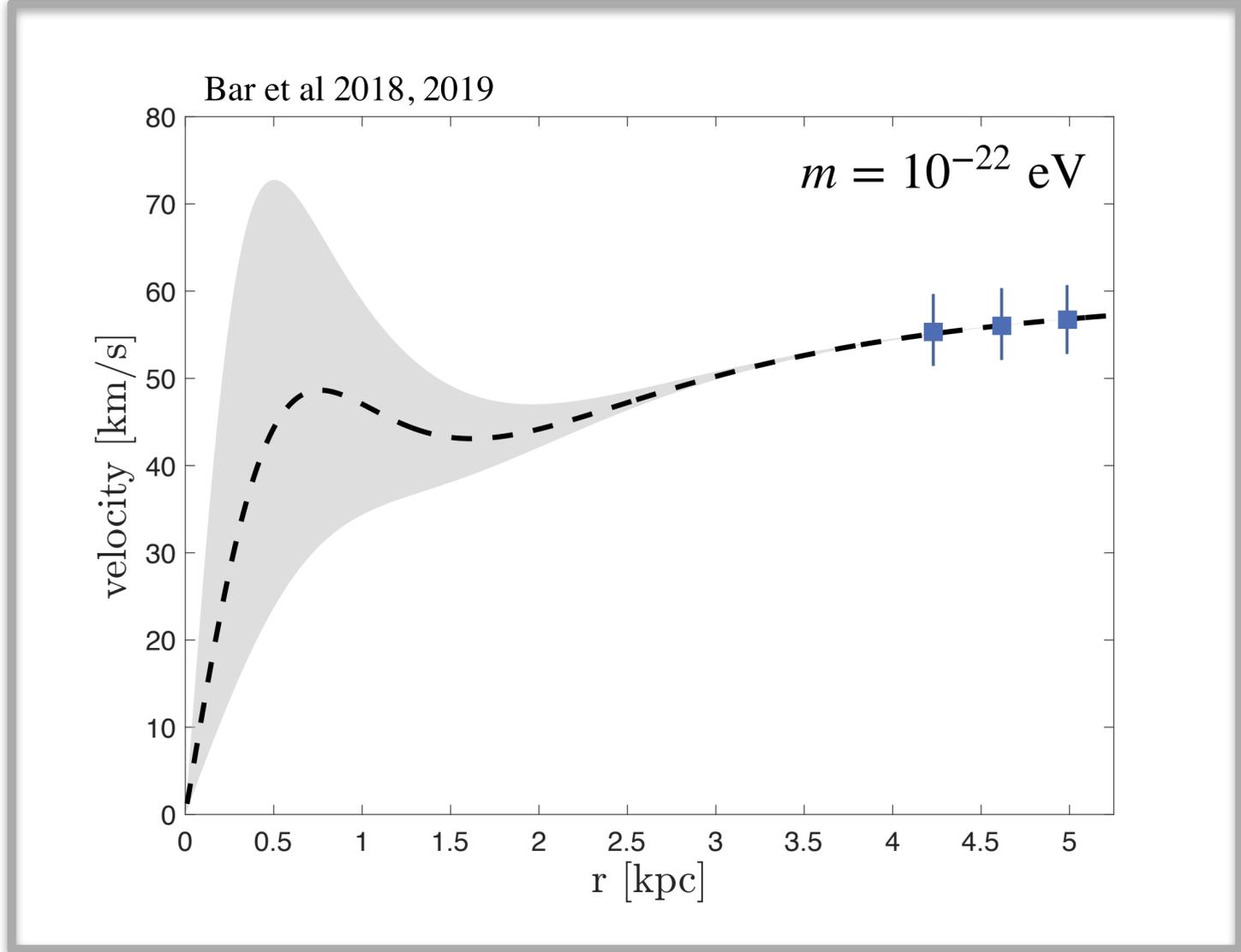


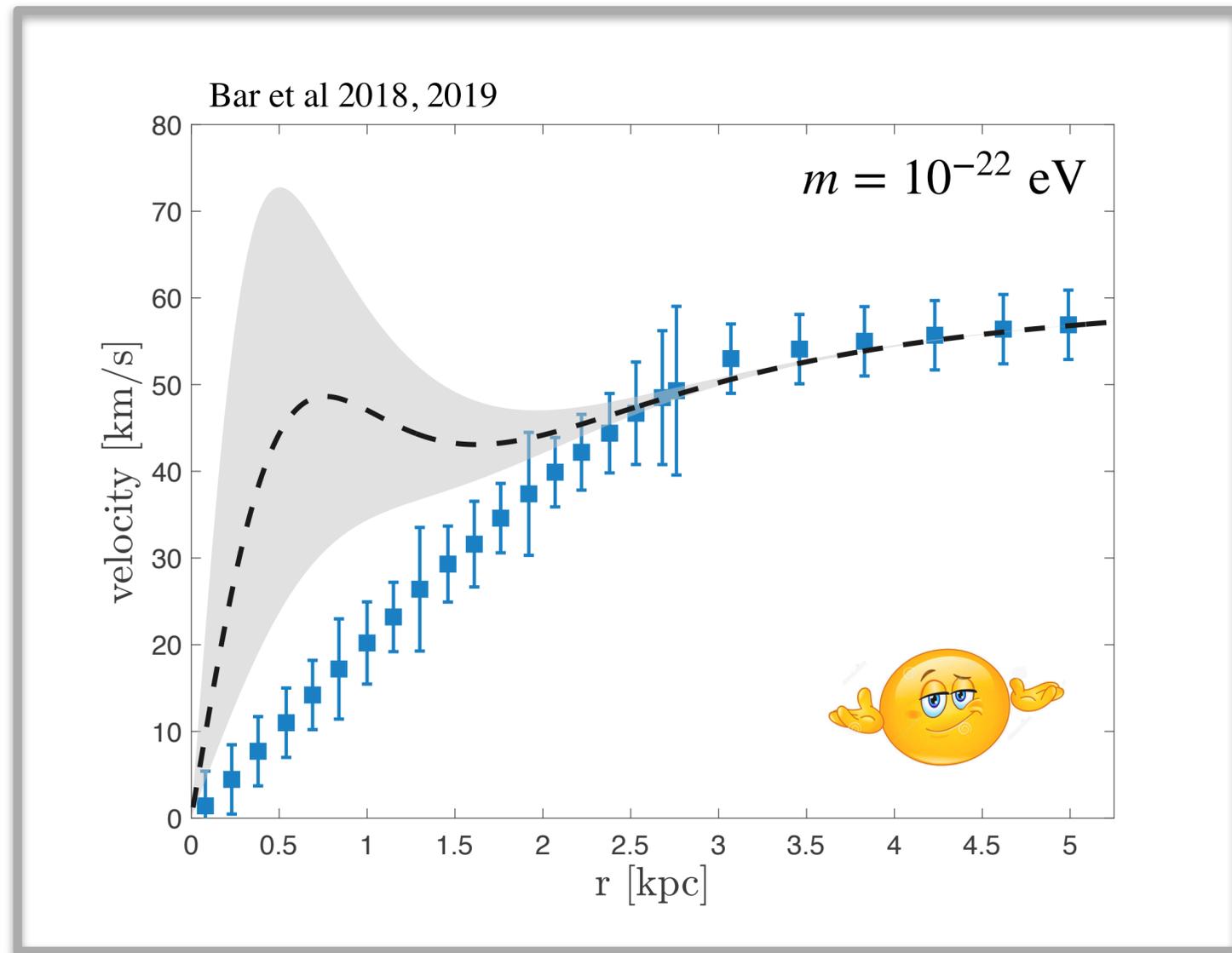
ULDM:

Affects the inner part of rotation curve

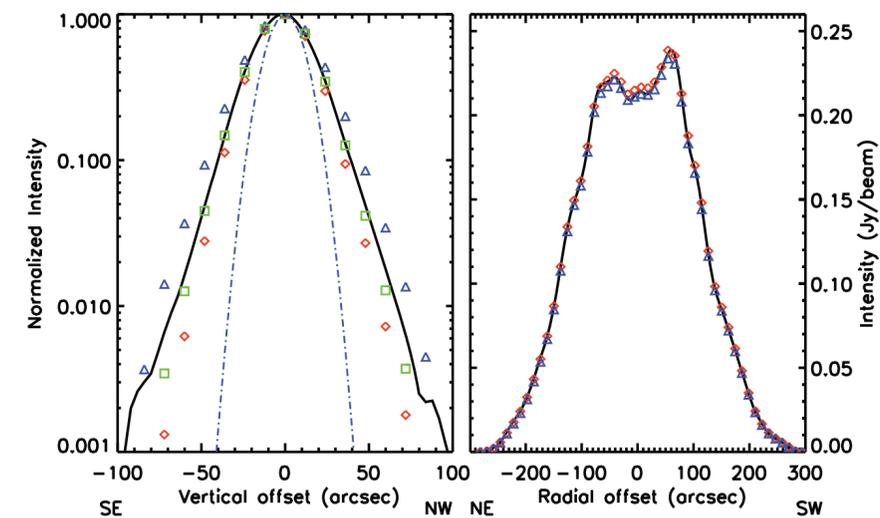
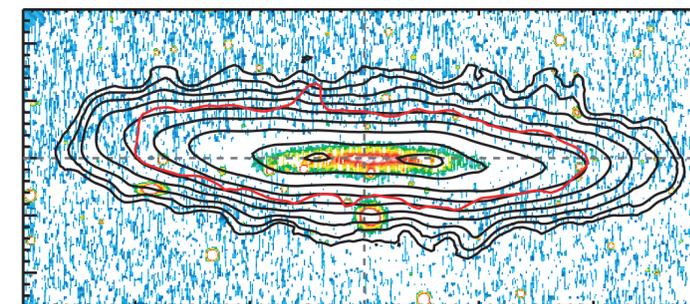
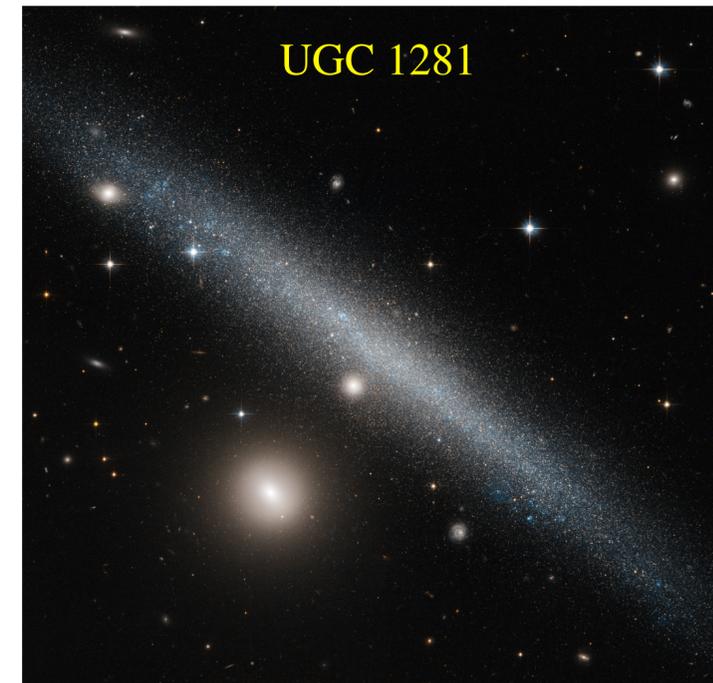








- Where is the core?
- This *could* have been spectacular...

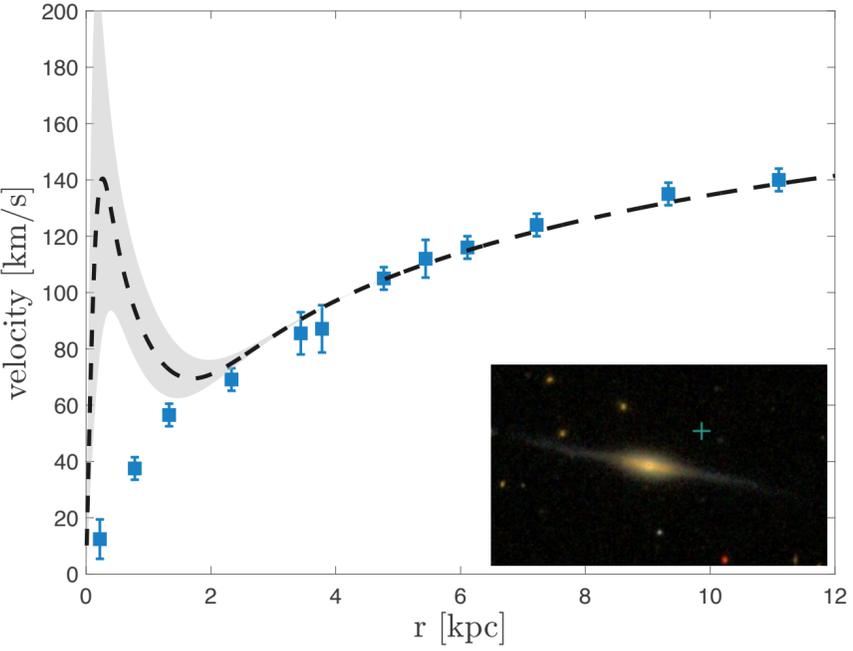
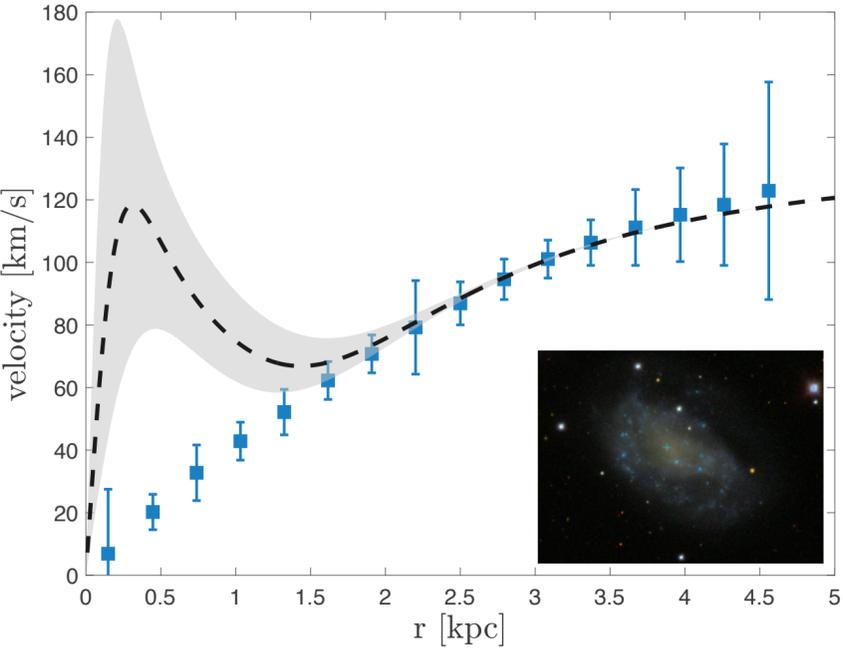
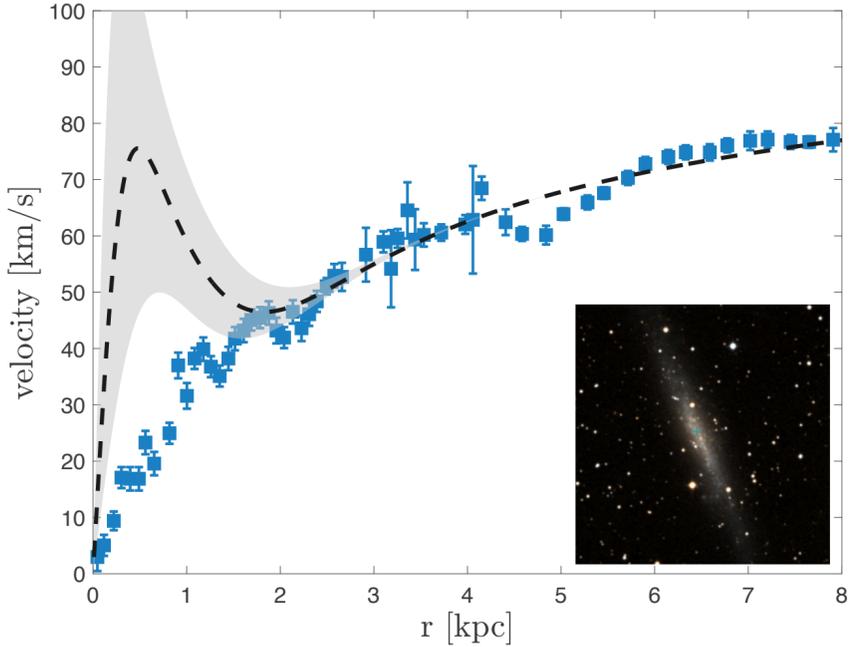
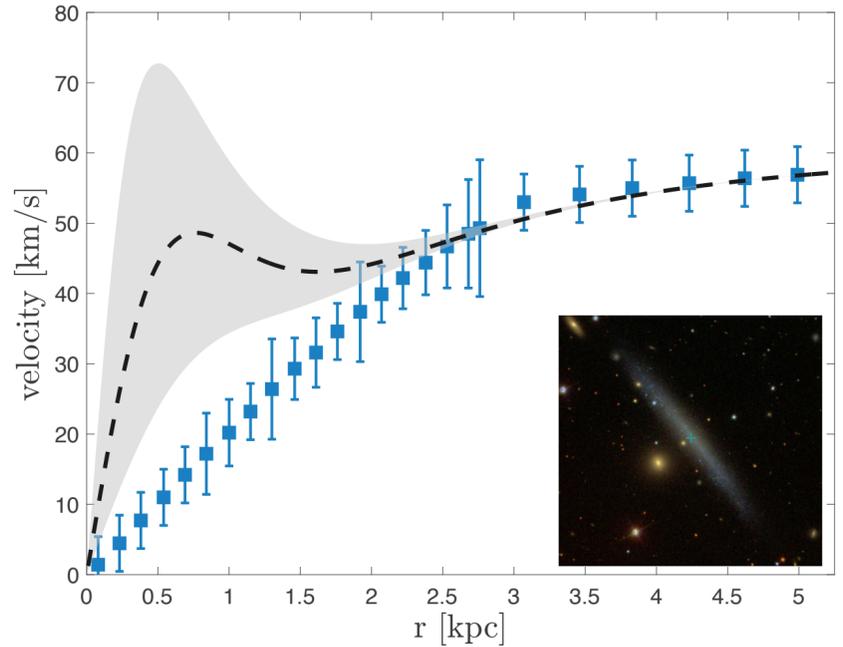


A lower limit on the mass of dark matter, with **gravity alone**

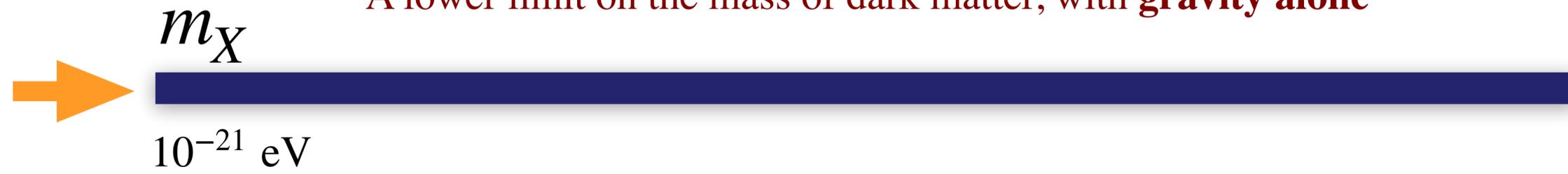
$$m_X$$



$$10^{-21} \text{ eV}$$



A lower limit on the mass of dark matter, with **gravity alone**

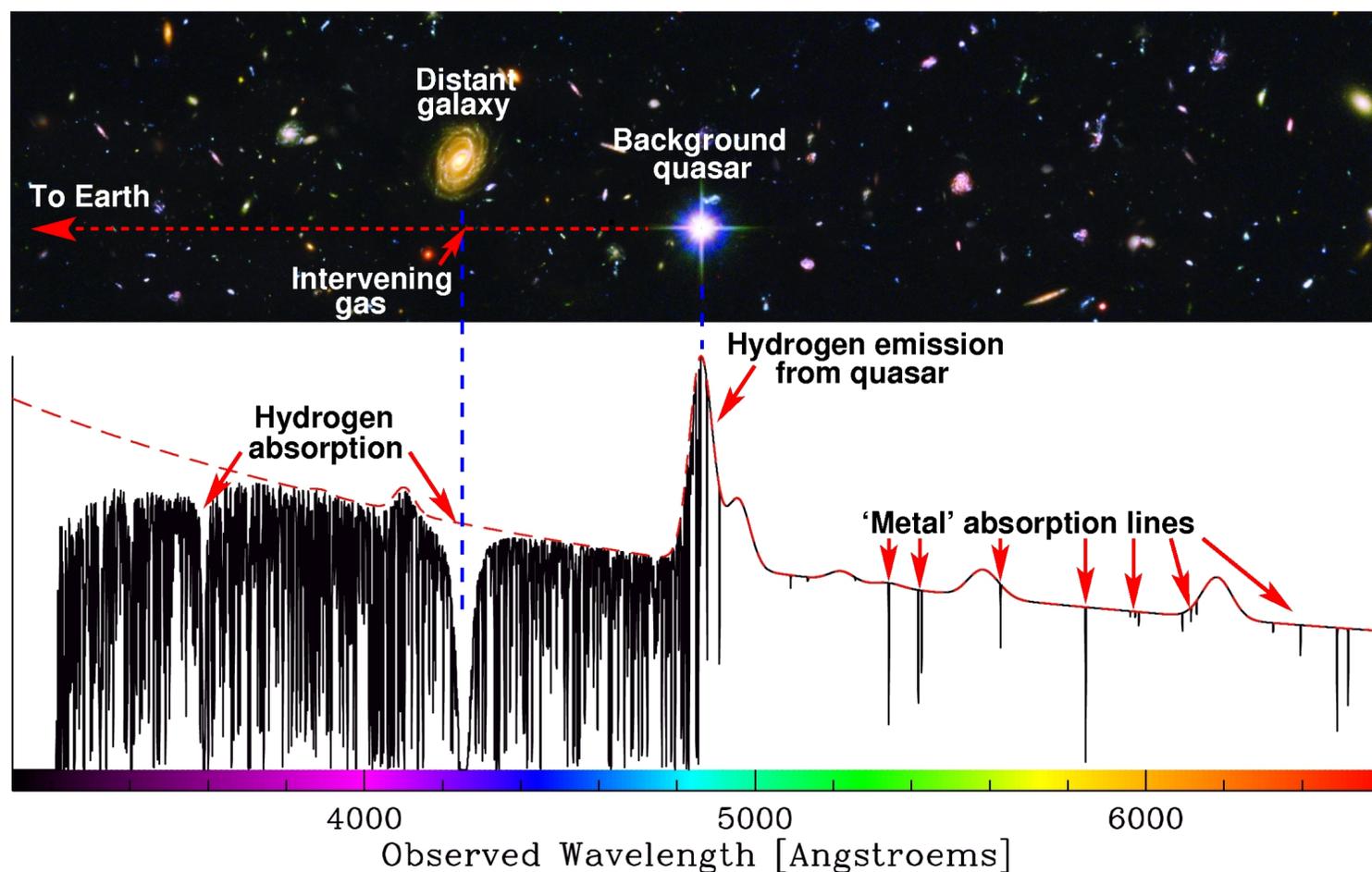


There are complimentary observables

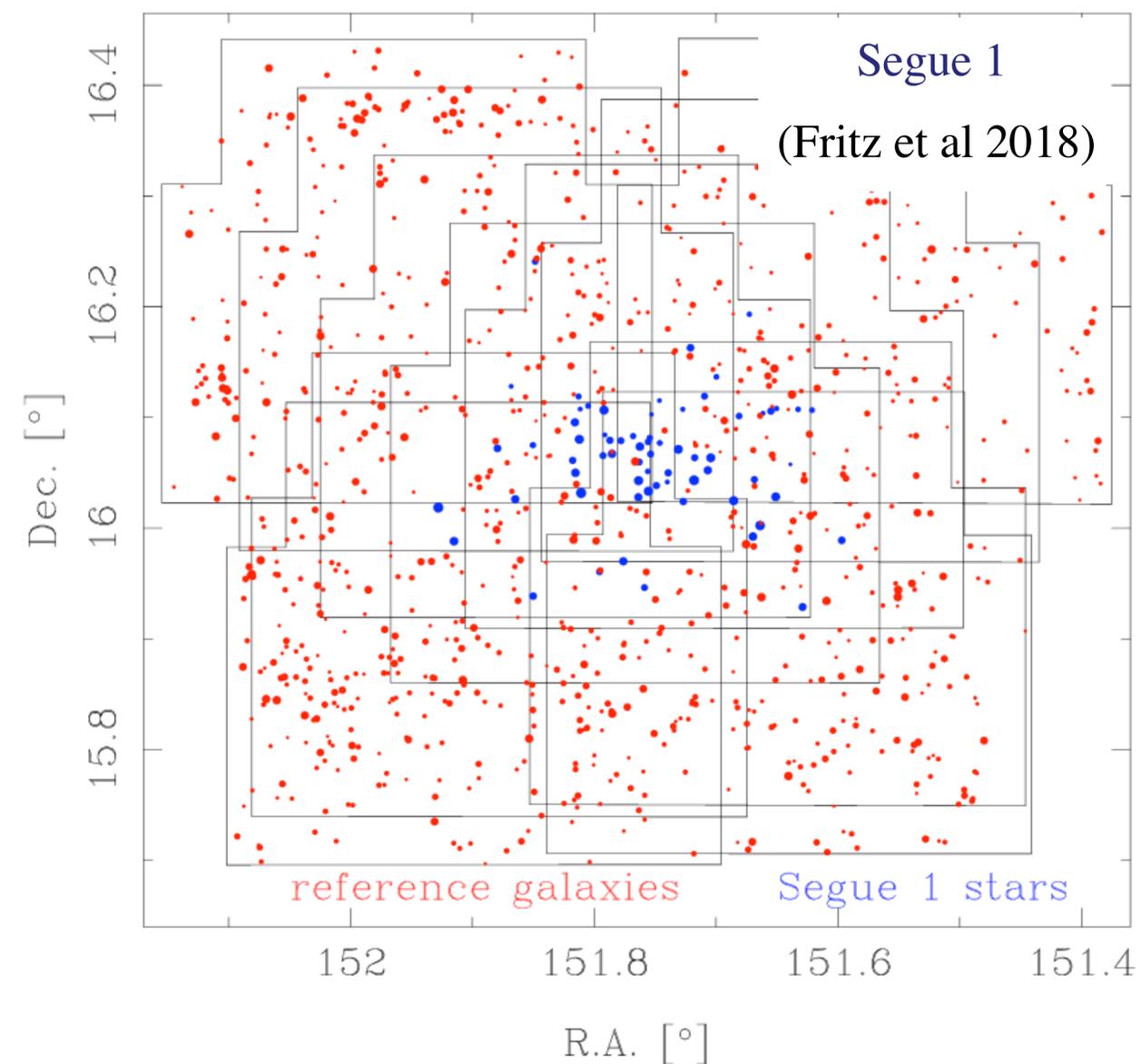


There are complimentary observables

Kobayashi et al 2017; Armengaud et al 2017; ...



Bar-Or, Fouvry, Tremaine 2018, 2020;  
Dalal, Kravstov 2022, ...





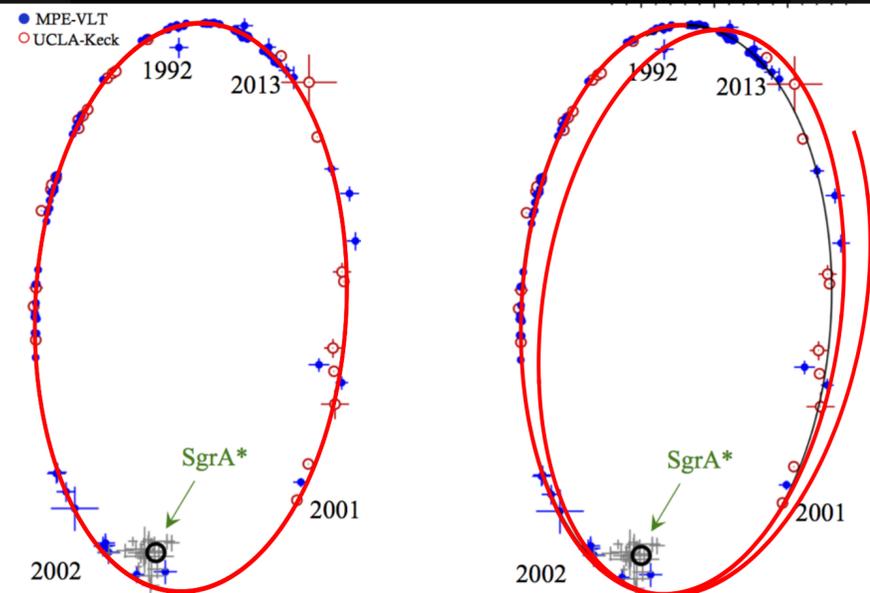
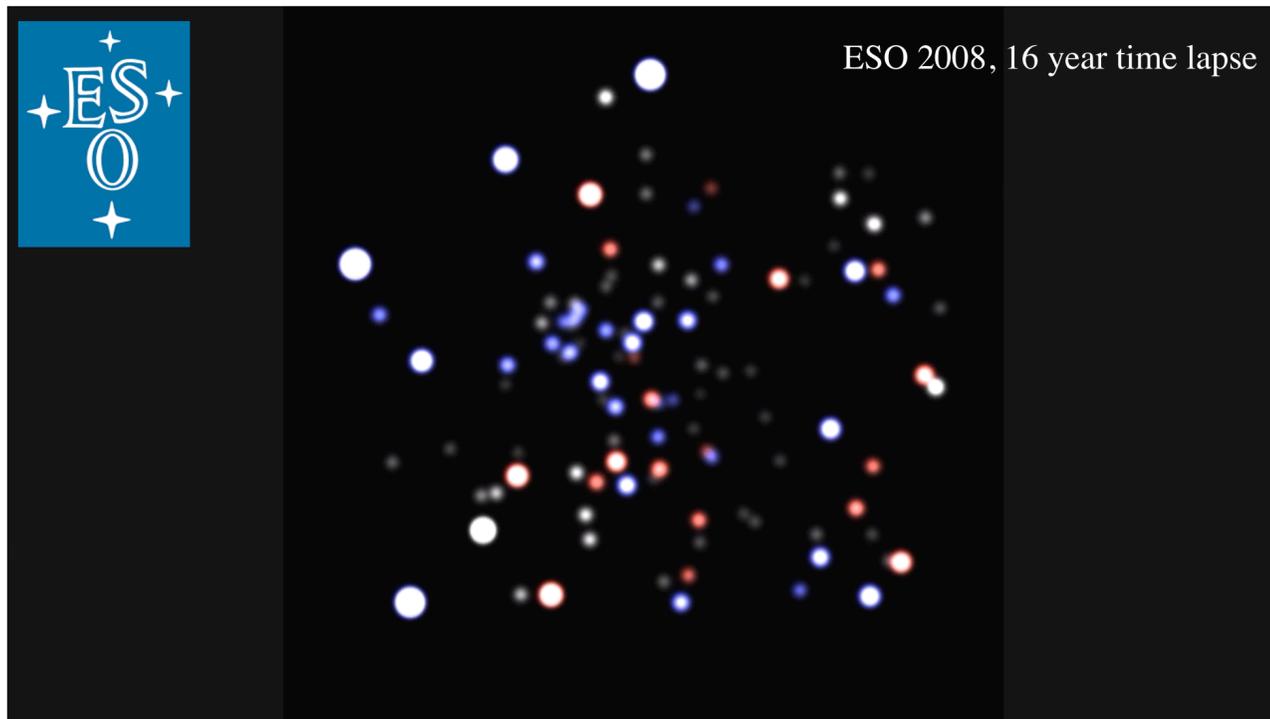
There are complimentary observables

Khmelnitsky & Rubakov 1309.5888,  
Poryako et al 1810.03227 (Parkes pulsar timing array)



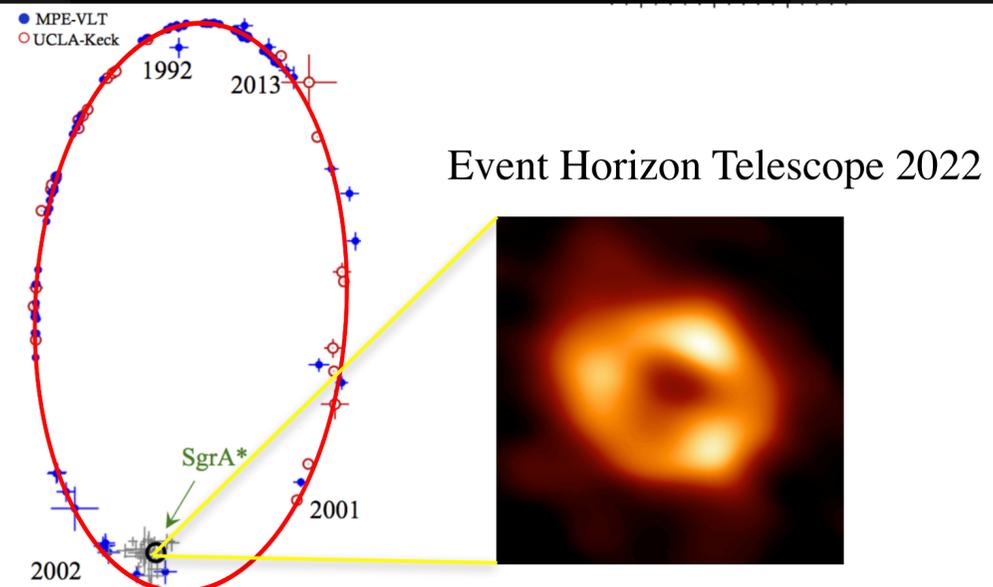
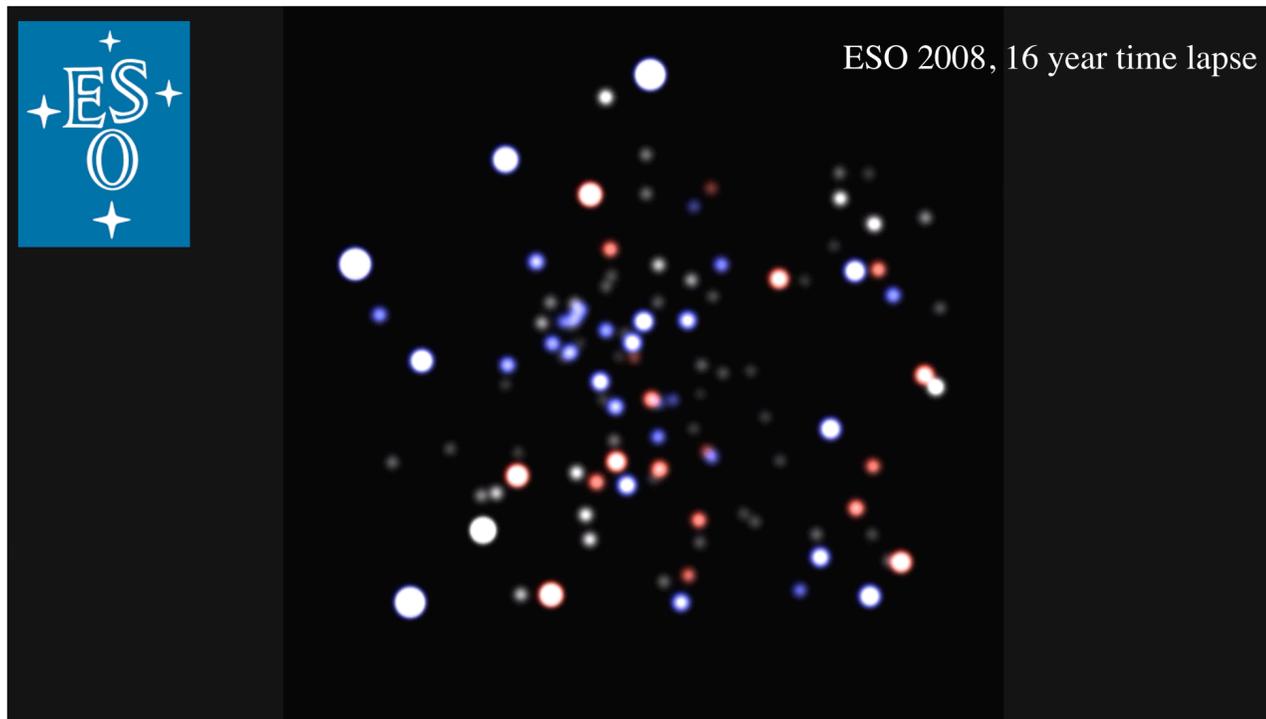


There are complimentary observables      Bar et al 2019; Amorim et al 2019 (GRAVITY),...



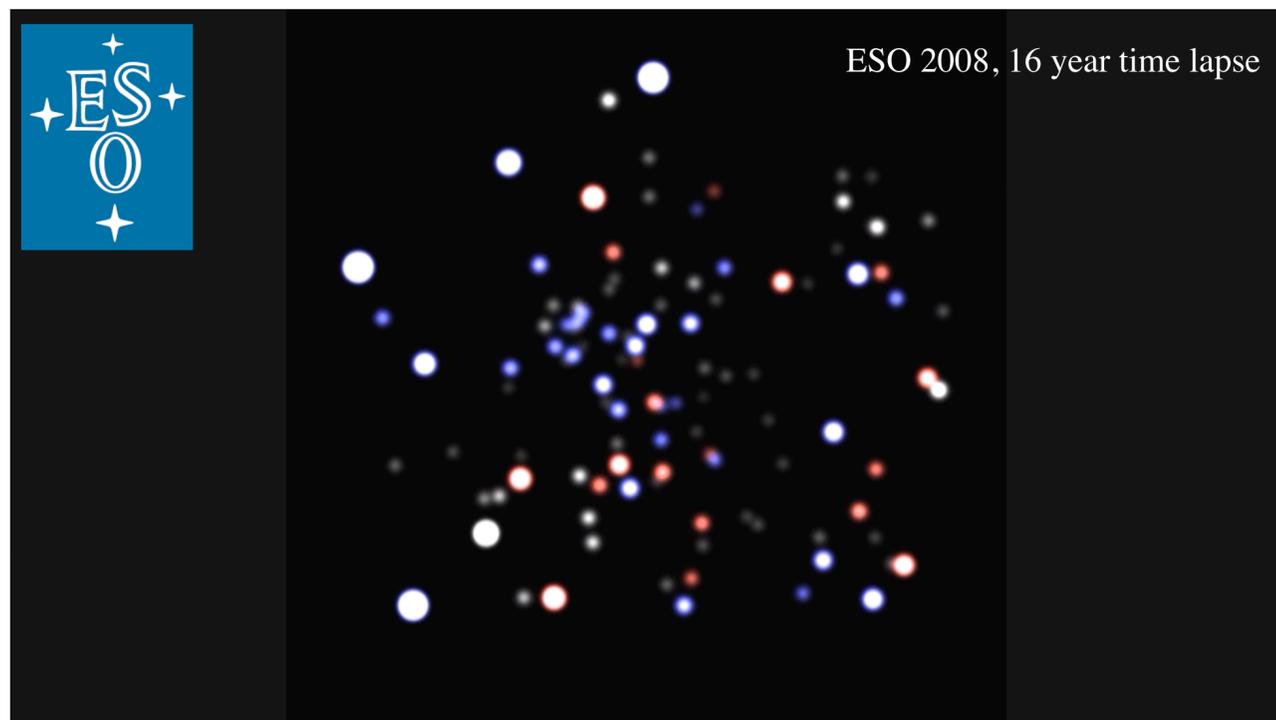


There are complimentary observables      Bar et al 2019; Amorim et al 2019 (GRAVITY),...





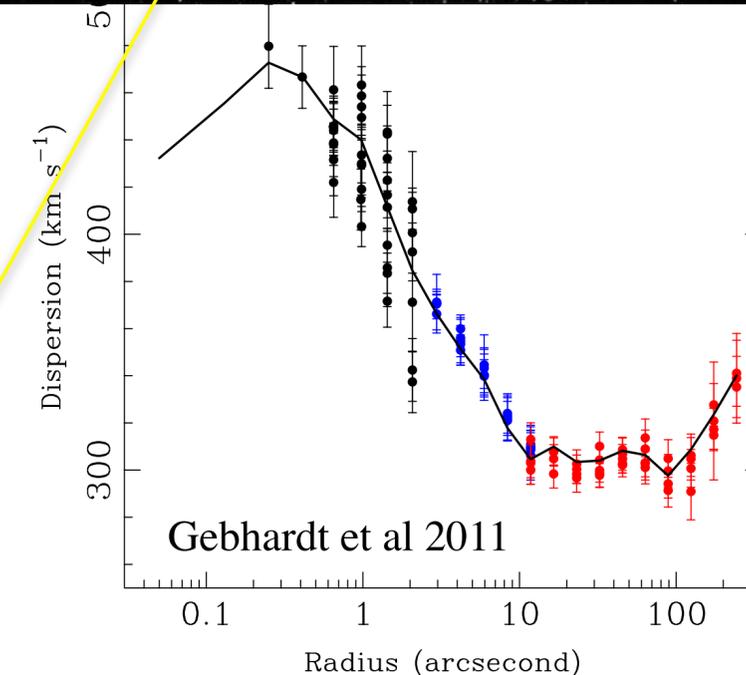
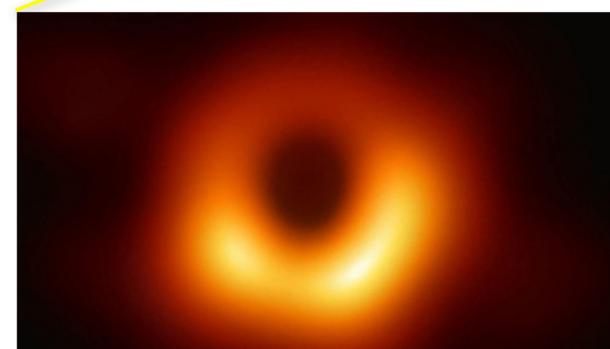
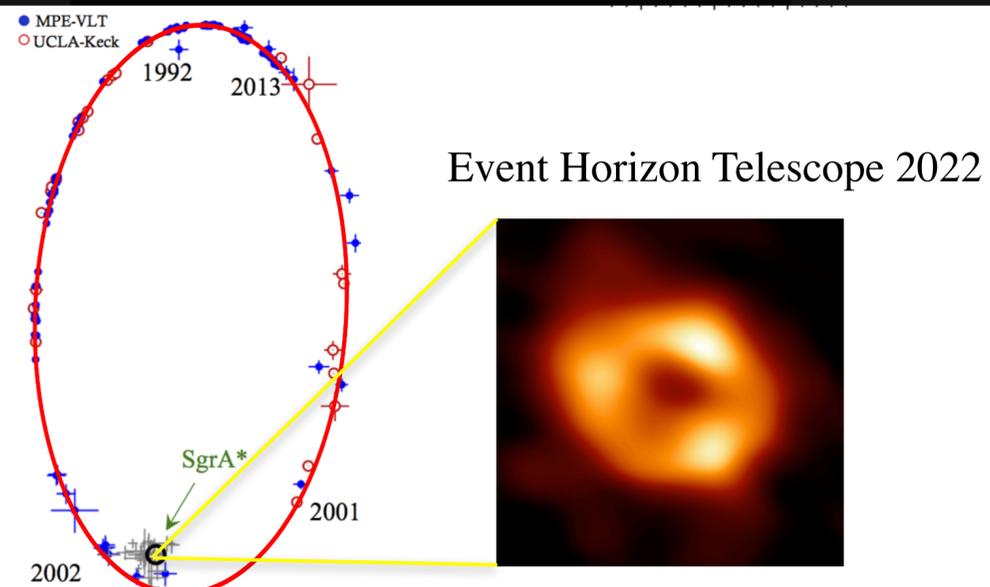
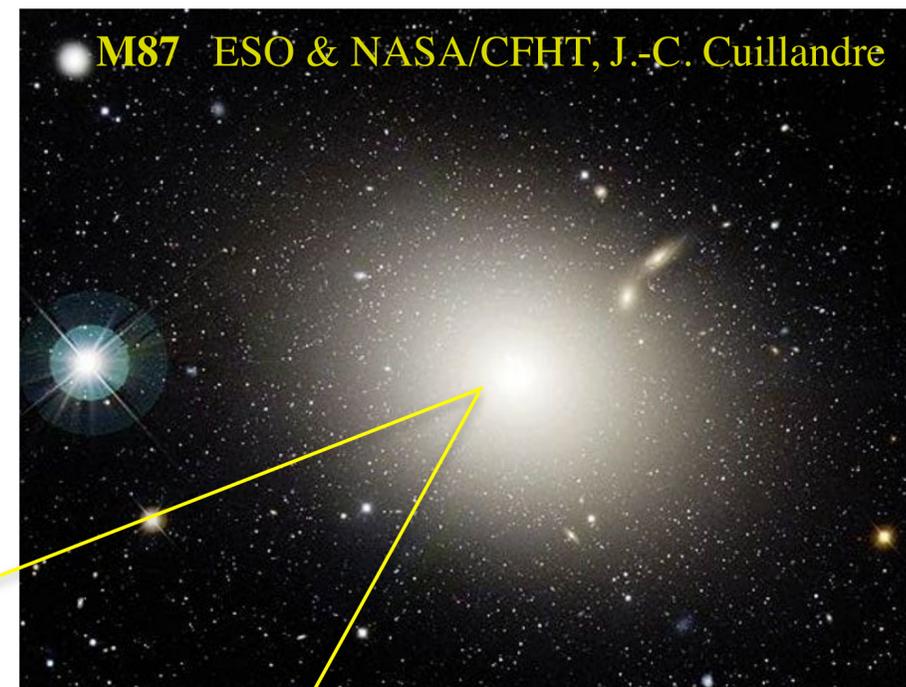
There are complimentary observables Bar et al 2019; Amorim et al 2019 (GRAVITY),...



BH shadow  $\sim 10^{10} \text{ km}$

Stellar velocities  $\sim 10^{16} \text{ km}$

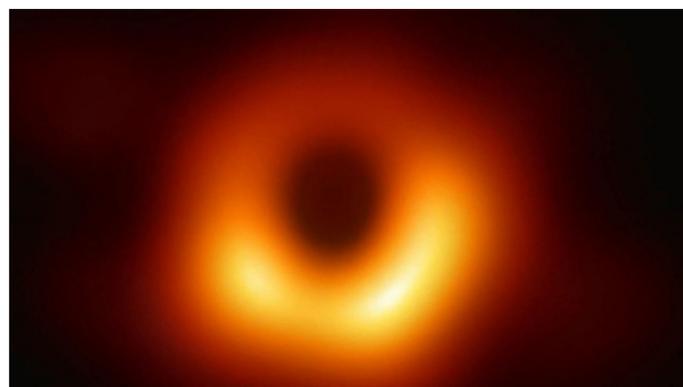
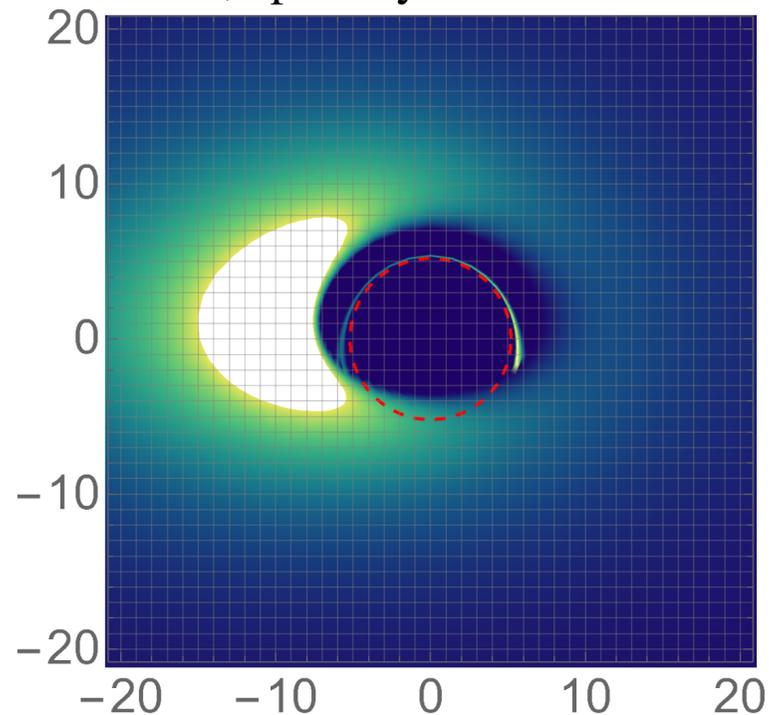
...Nothing in between.





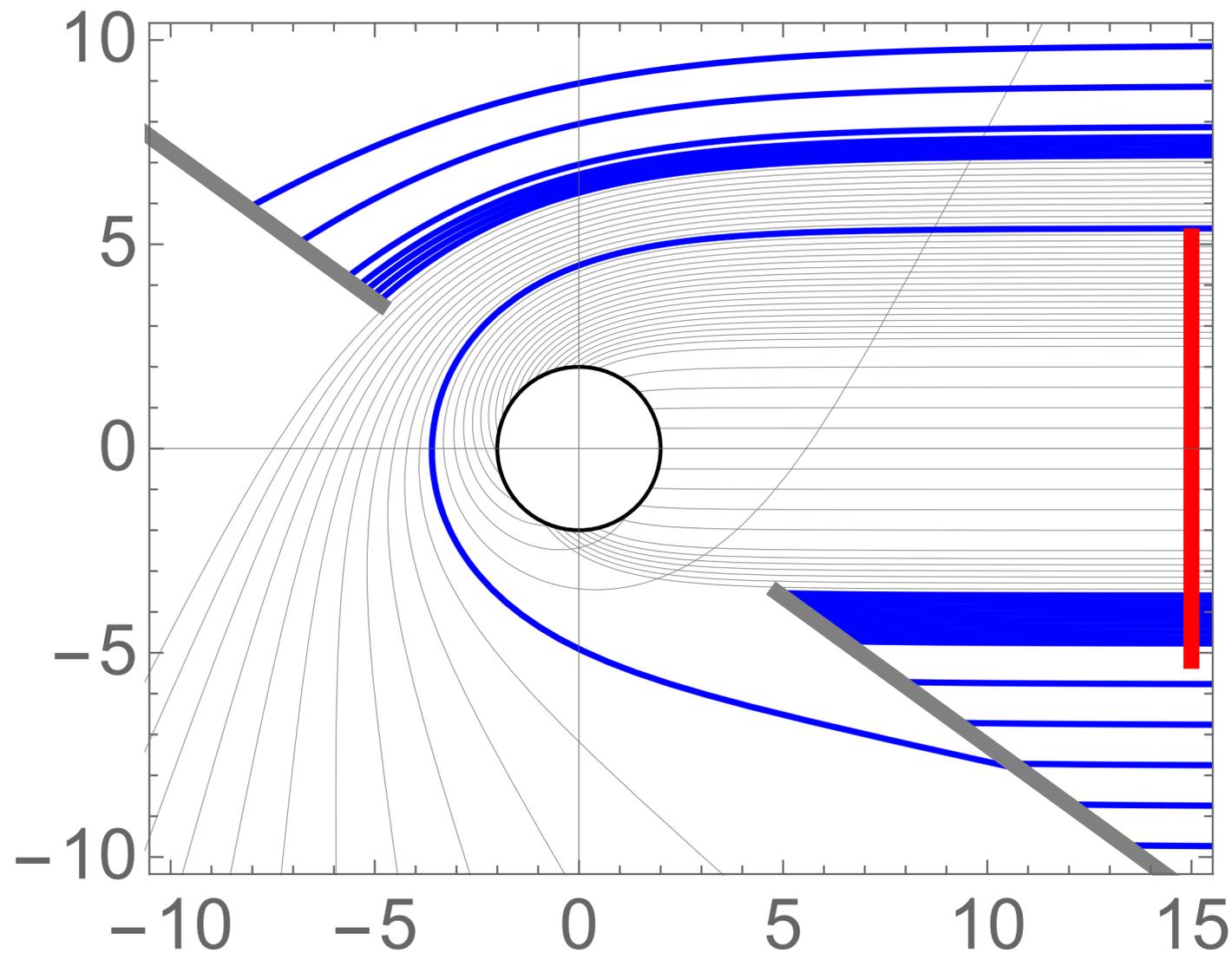
(Luminet 1978)

Thin, optically-thick disk



Model-dependence in accurate interpretation:

e.g. Gralla, Holz, Wald 1906.00873



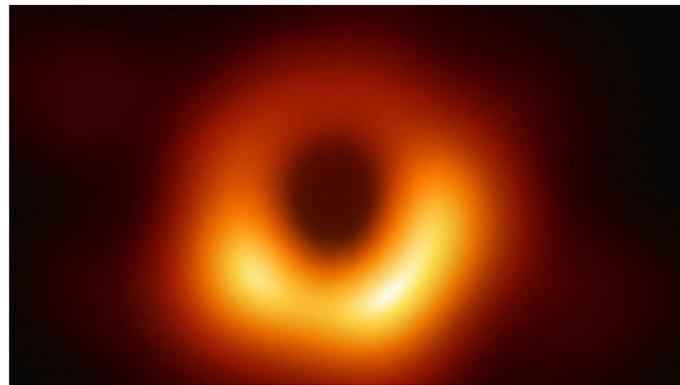
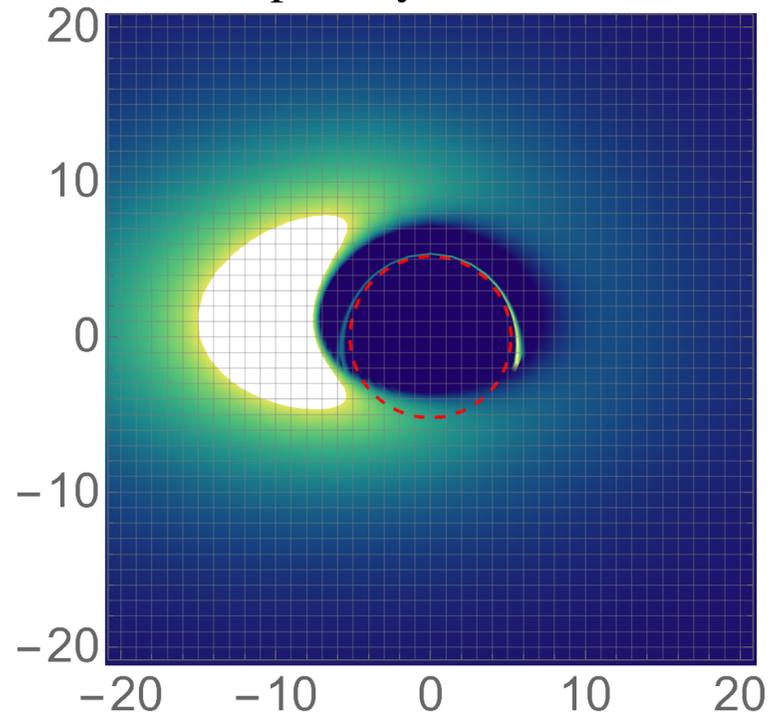
$$\left(\frac{du}{d\phi}\right)^2 + u^2(1 - 2u) = \frac{1}{b^2}$$

$$u = \frac{1}{r} \quad \text{units of GM}$$

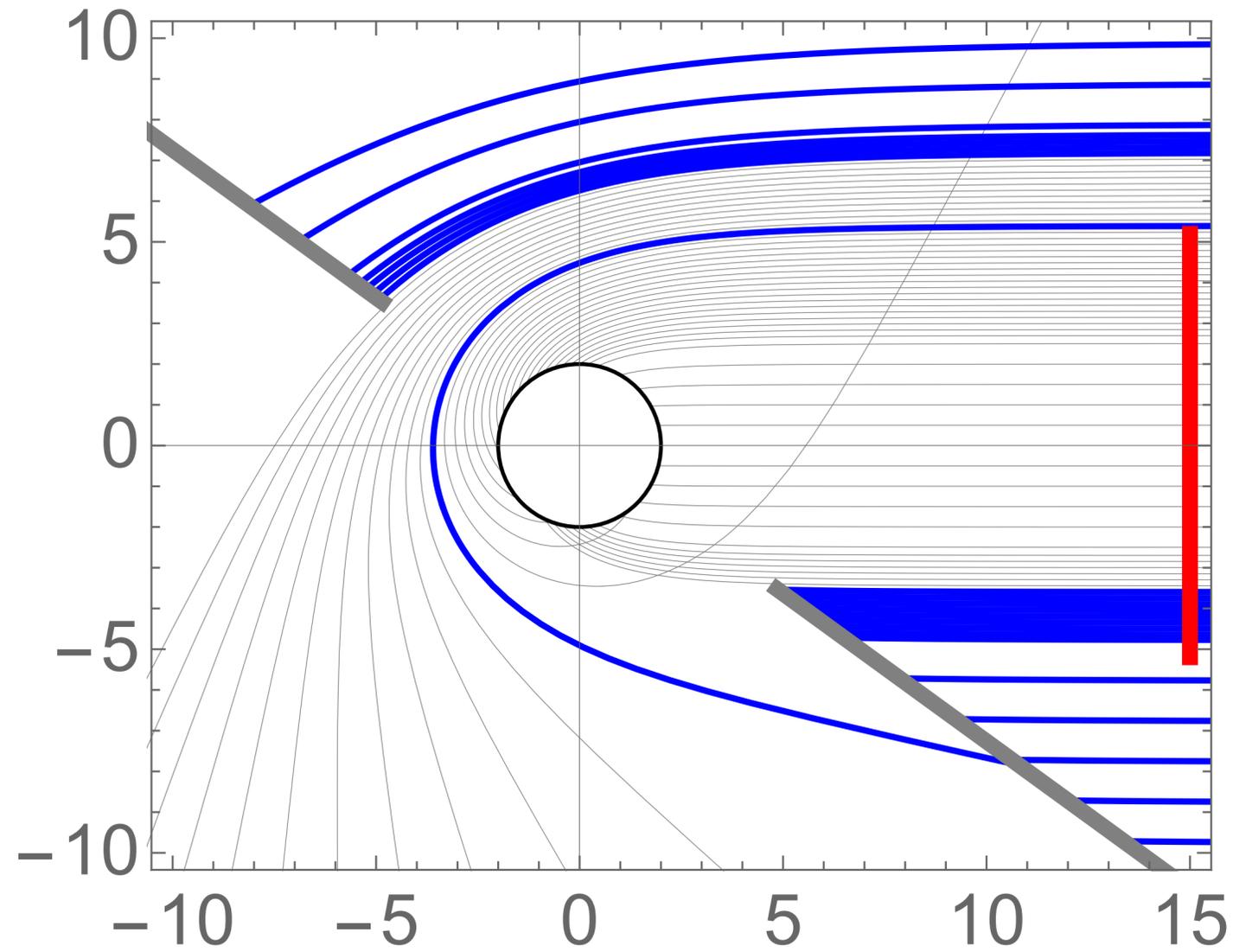


(Luminet 1978)

Thin, optically-thick disk

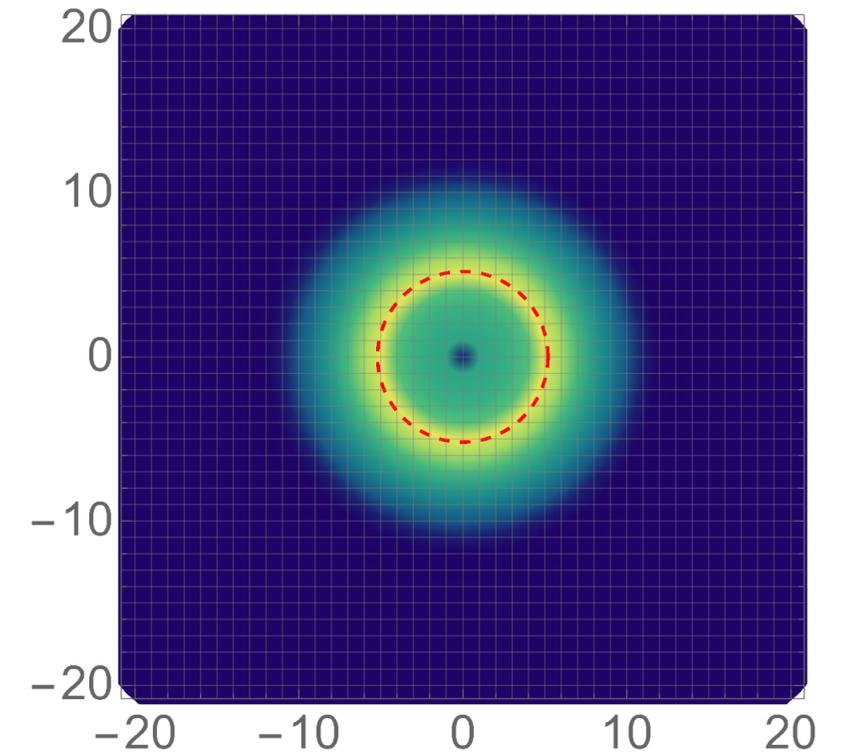


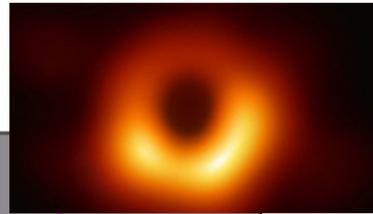
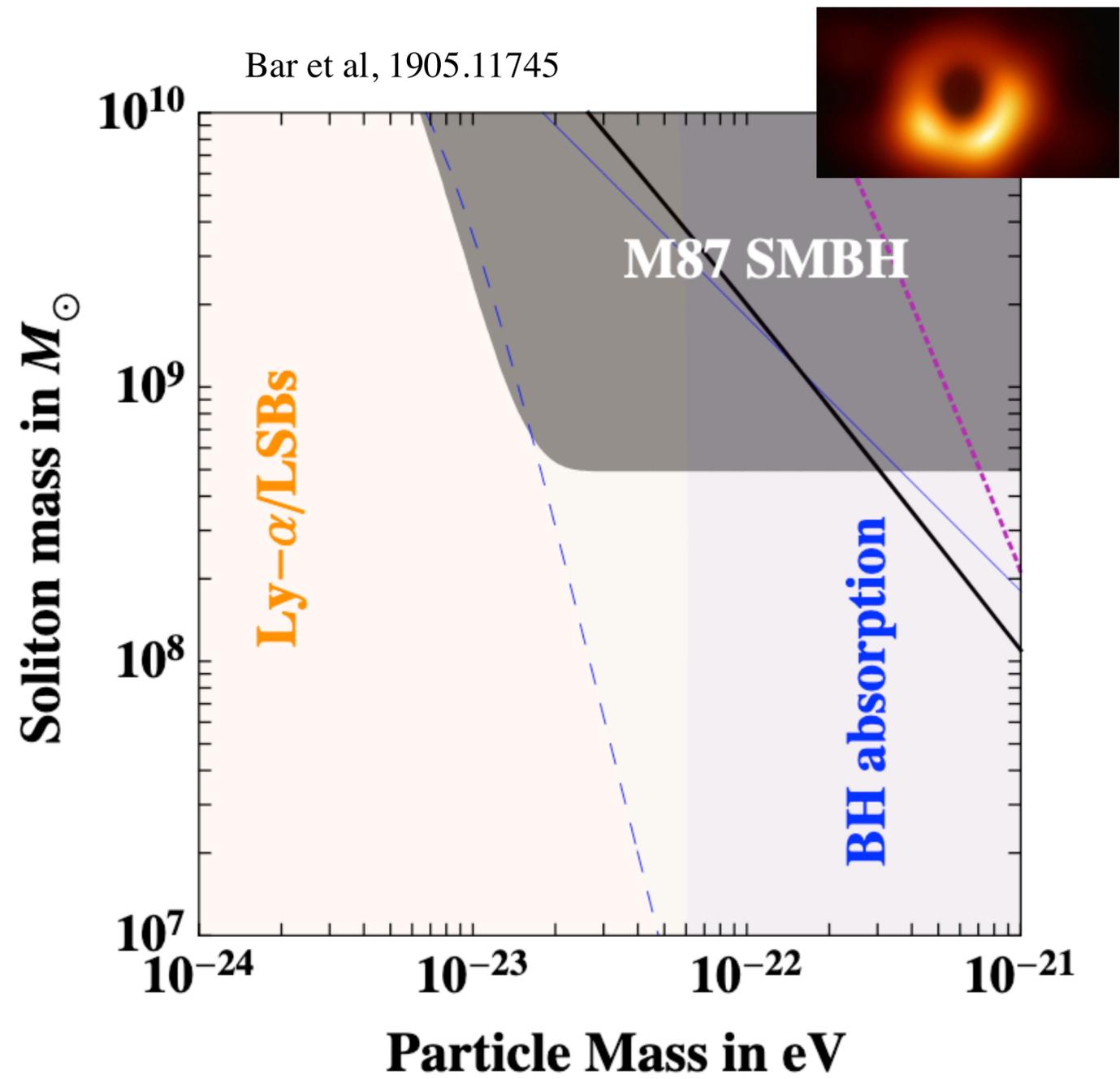
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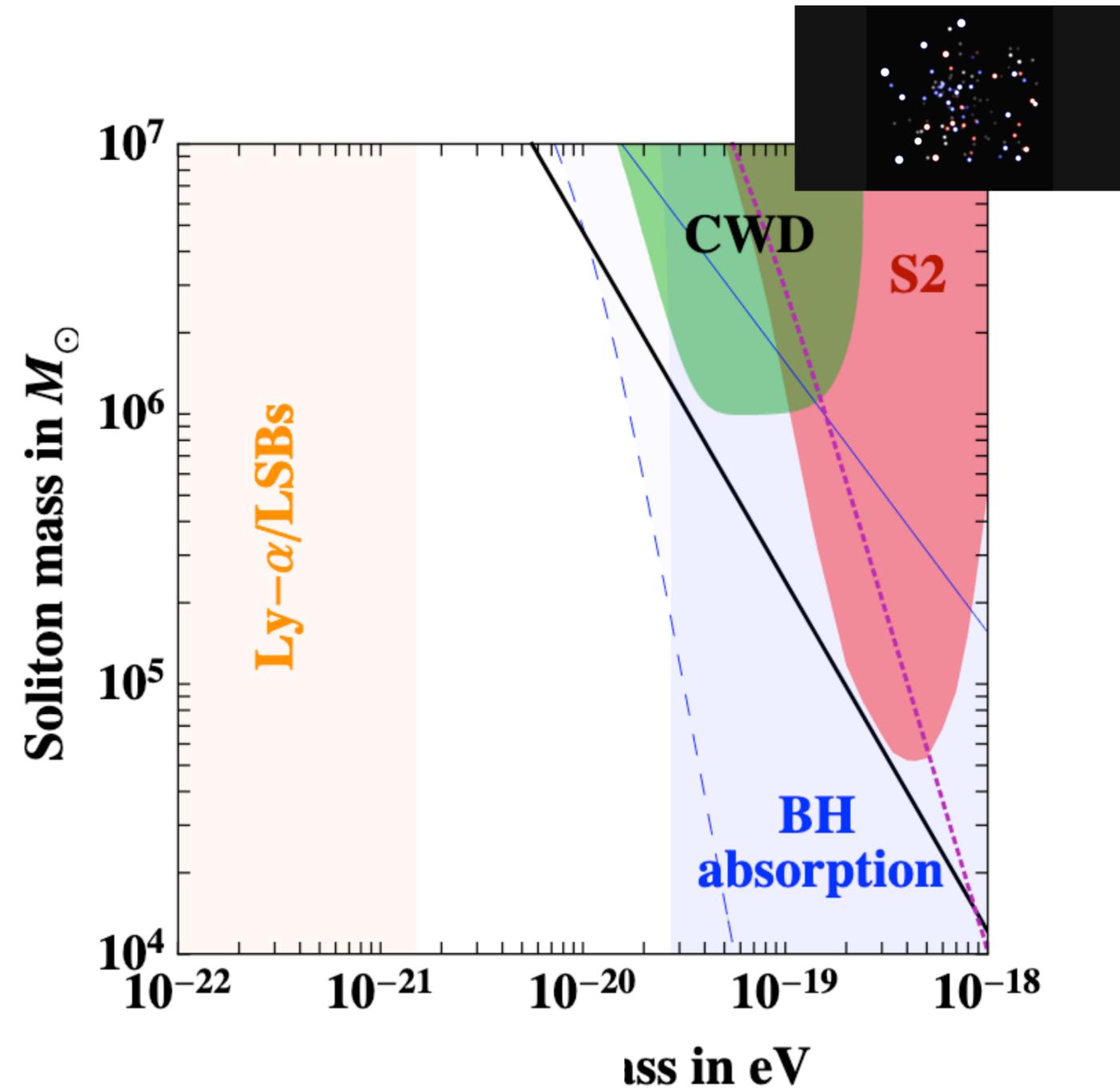
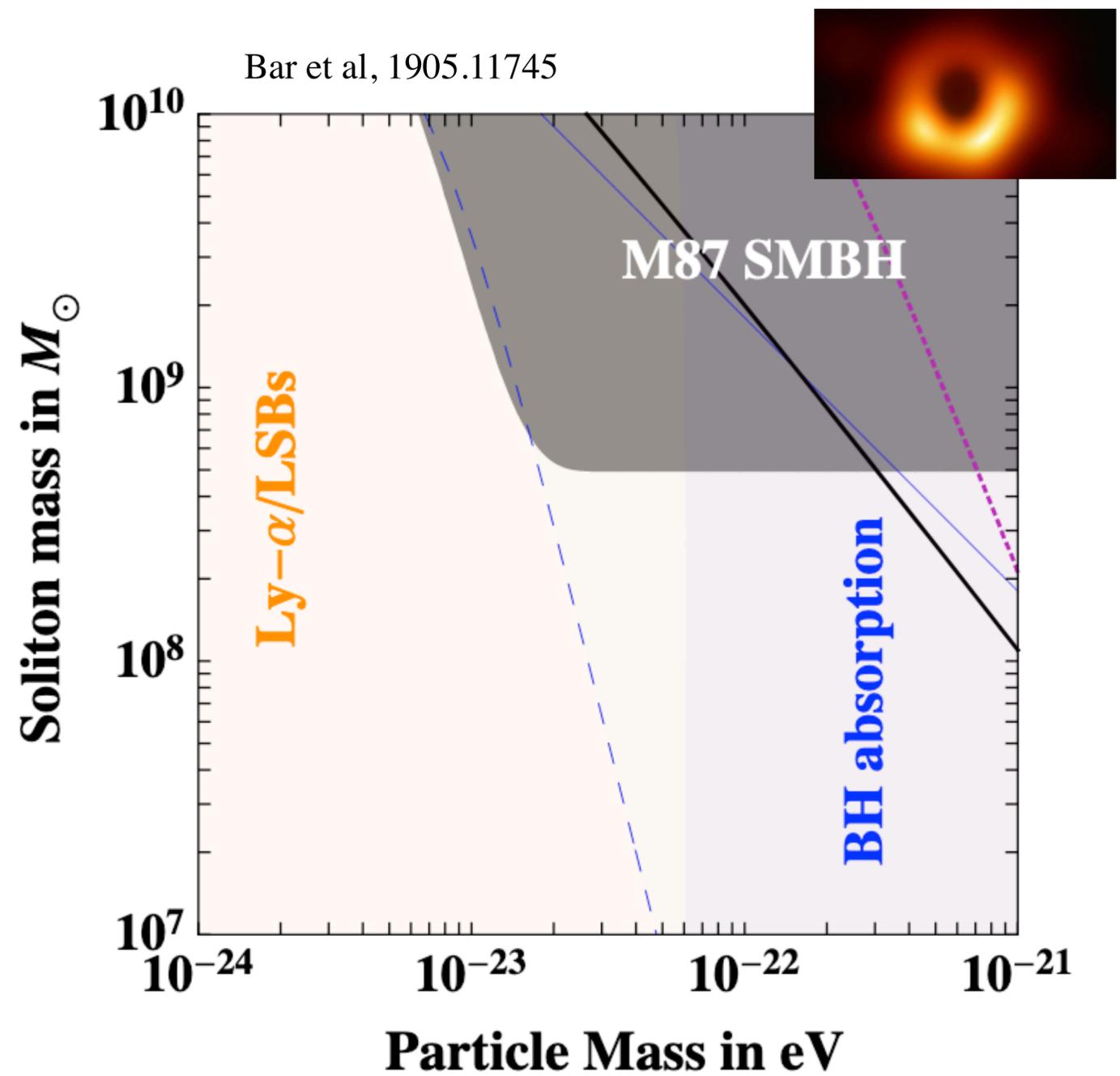


(Flacke, Melia, Agol 2000)

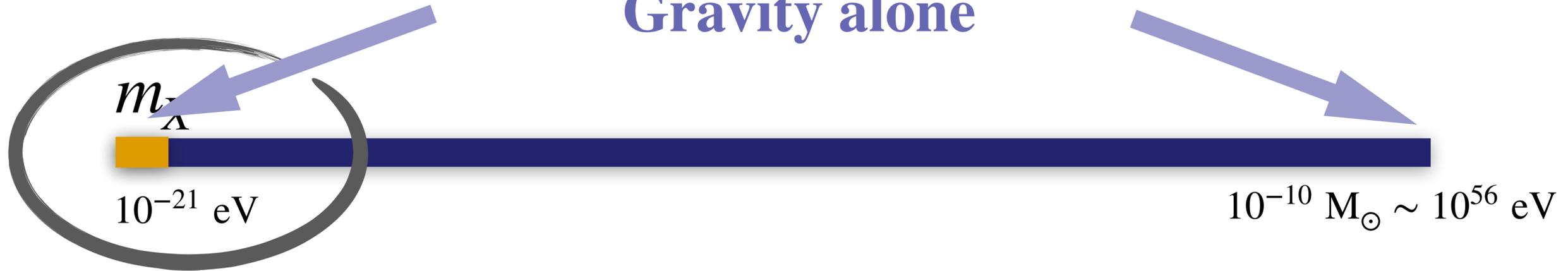
Optically-thin spherical gas



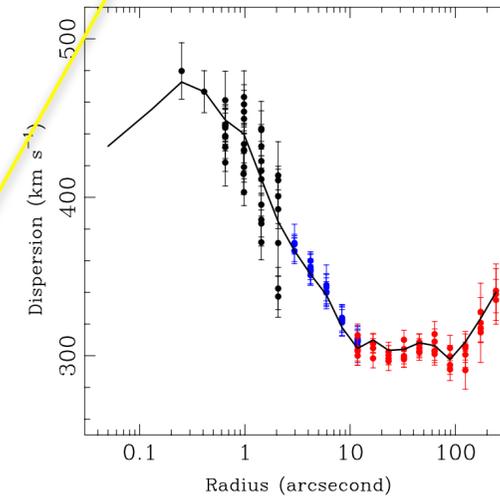
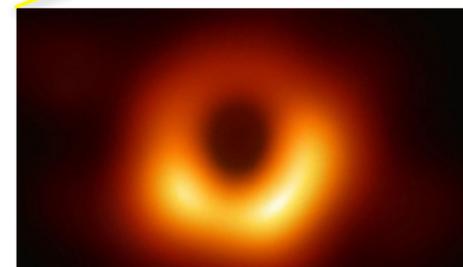
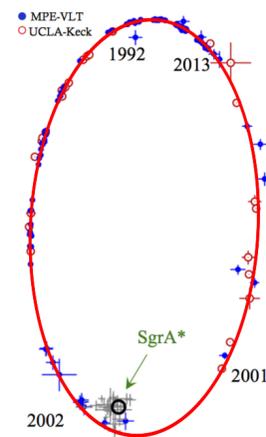
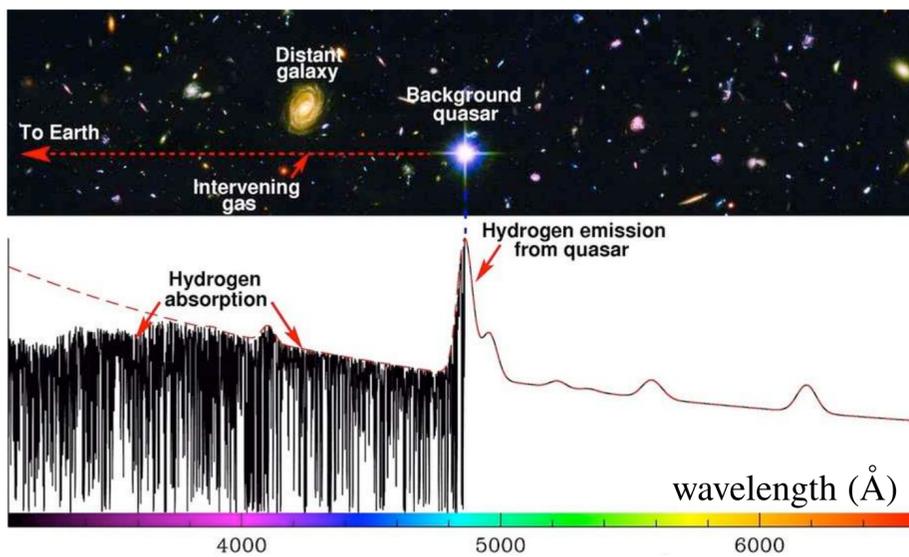
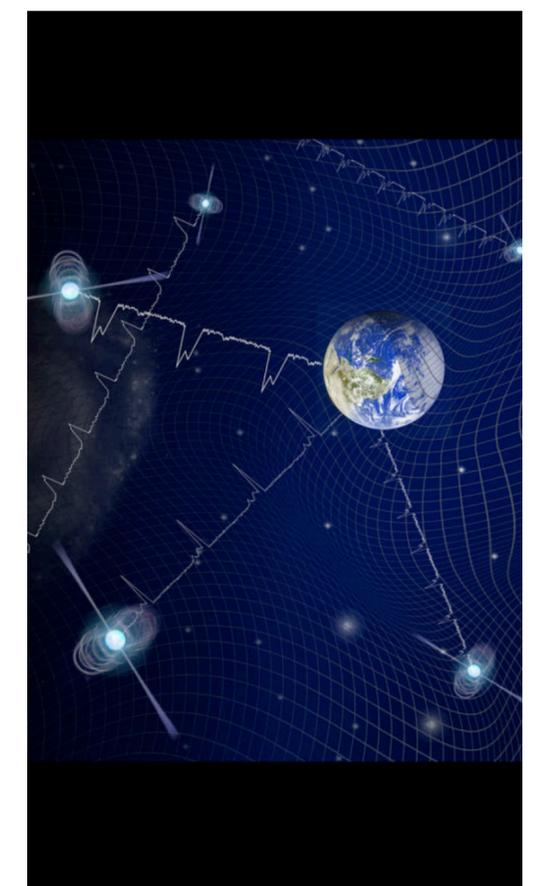
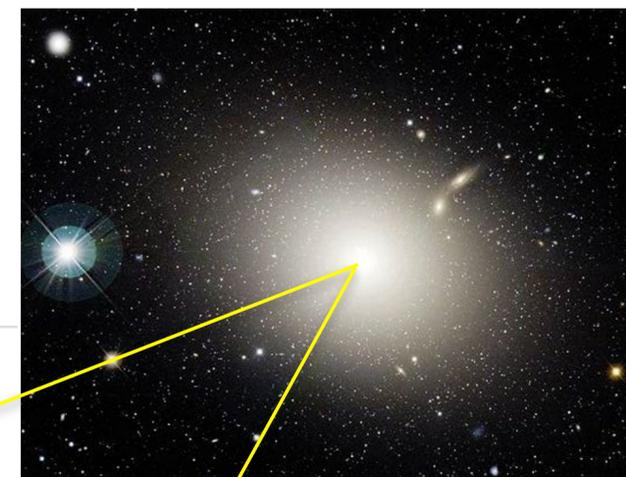
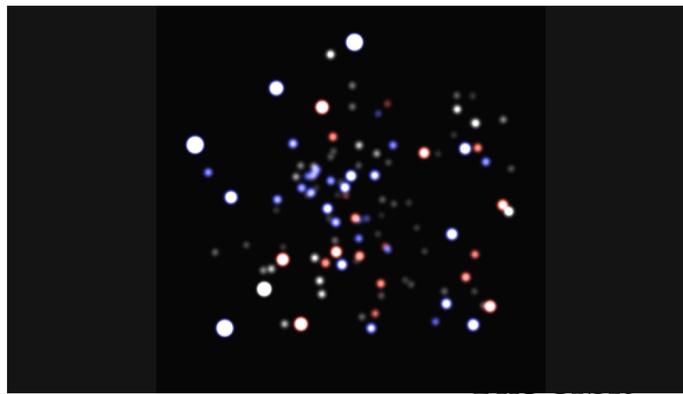
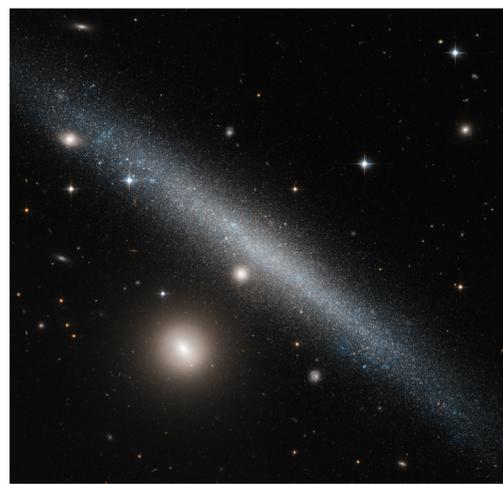
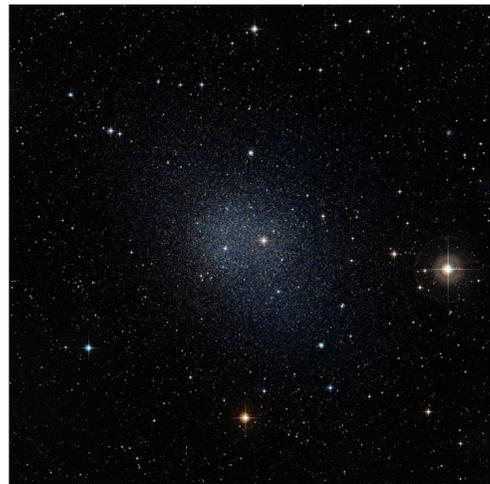




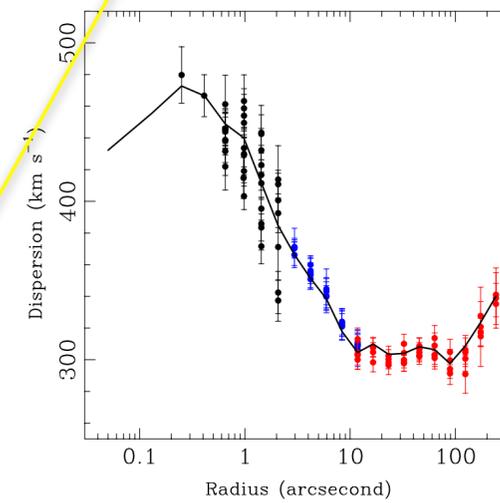
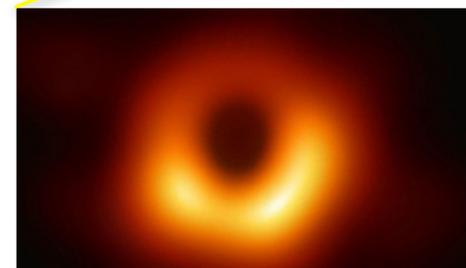
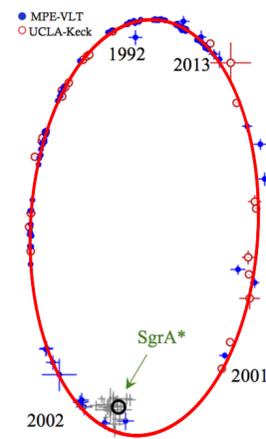
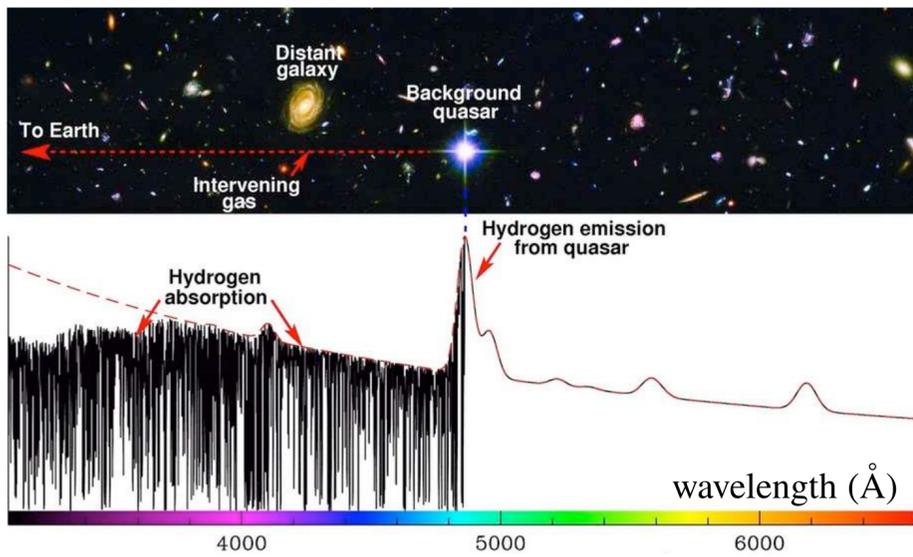
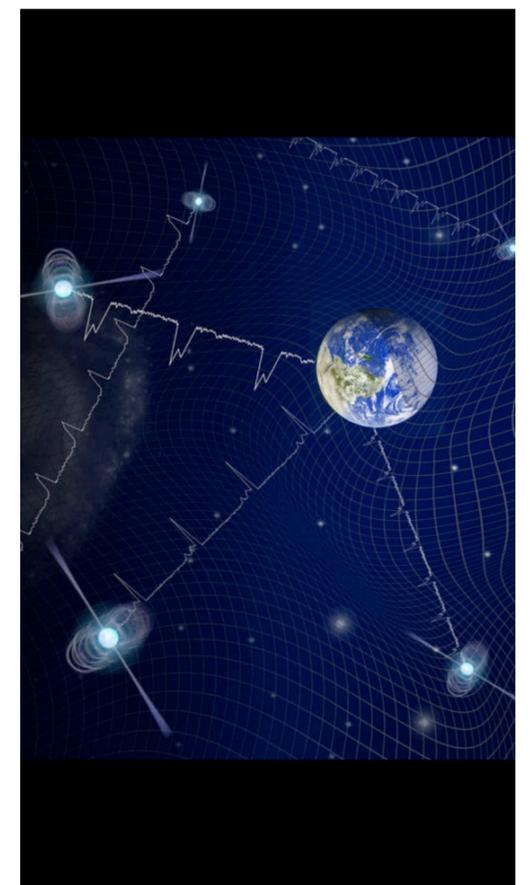
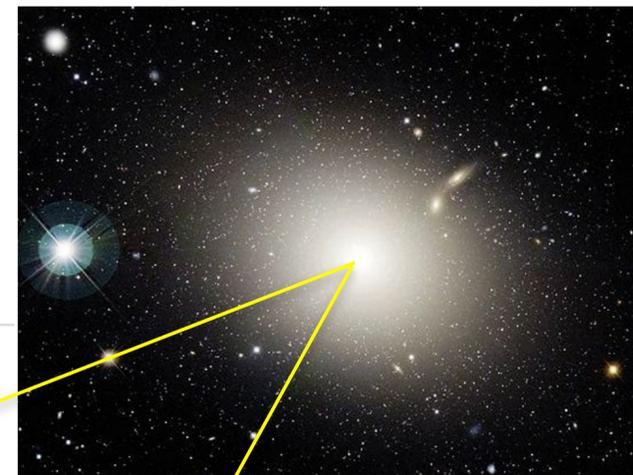
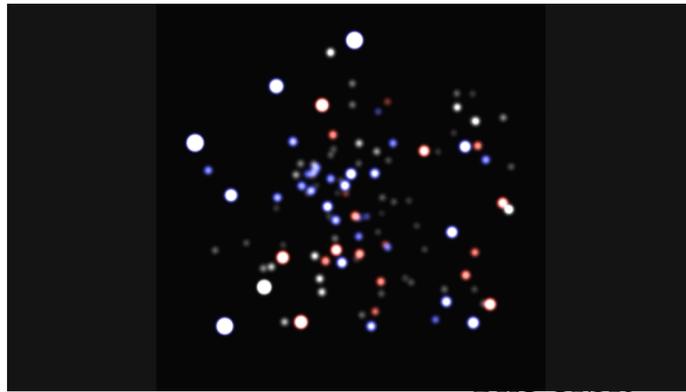
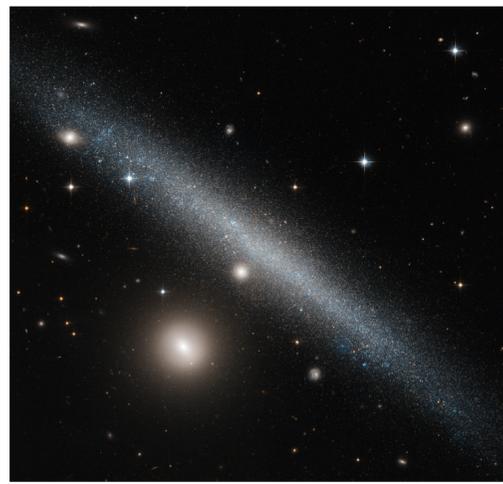
# Gravity alone



## Ultralight dark matter



# Thank you!



## Some facts about solitons

Continuous family of ground state solutions,  
characterised by one parameter

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Let  $\chi_1(r)$  be defined to satisfy  $\chi(0) = 1$ , vanishing at infinity w/ no nodes.

$$M_1 = \frac{M_{pl}^2}{m} \int_0^\infty dr r^2 \chi_1^2(r)$$
$$\approx 2.79 \times 10^{12} \left( \frac{m}{10^{-22} \text{ eV}} \right)^{-1} M_\odot$$

---

Other solutions obtained by scaling

$$\chi_\lambda(r) = \lambda^2 \chi_1(\lambda r),$$
$$\Phi_\lambda(r) = \lambda^2 \Phi_1(\lambda r),$$
$$\gamma_\lambda = \lambda^2 \gamma_1,$$

$$M_\lambda = \lambda M_1,$$
$$x_{c\lambda} = \lambda^{-1} x_{c1}$$