FUNDAMENTAL PHYSICS WITH LARGE/MEDIUM/SMALL SCALE STRUCTURES

LECTURE 3

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Tenerife (Spain)
INTRO

FUNDAMENTAL PHYSICAL TESTS "BEFORE" GALAXIES ARE FORMED IN THE POST-REIONIZATION UNIVERSE

IGM
INTENSITY MAPPING

GALAXY CLUSTERING: DYNAMICAL AND GEOMETRICAL PROBE

WEAK LENSING
GALAXY CLUSTERS

PLAN

CONNECTIONS

FABIO FINELLI: CMB x LSS

KFIR BLUM: SMALLER SCALES
PROPERTIES OF GALAXIES

LUCA AMENDOLA: MODIFICATION OF GRAVITY/DARK ENERGY

OLGA MENA: NEUTRINOS

TRACY SLATYER: DARK MATTER
GALAXY CLUSTERING

Baumann et al. 2012 EFT of the LSS
Philcox & Ivanov 2021 - Full Shape power spectrum analysis
Manzotti, Peloso, Pietroni, MV, Villaescusa-Navarro 14 - Coarse Grained perturbation theory
Galaxy Formation and Evolution - Mo, Van Den Bosch & White book
DESI Collaboration (2016), arXiv:1611.00037
Alam et al. 2020 - full analysis of BOSS galaxy clustering
Semenaitė+22, arXiv: 2210.07304 - Beyond LCDM with galaxies
Chapman+21 - Growth with galaxy clustering
The Sloan Digital Sky Survey

eBOSS + BOSS Lyman-α (2008-2019)
eBOSS + SDSS HI Quasars (1998-2019)
eBOSS Young Blue Galaxies (2014-2019)
eBOSS Old Red Galaxies (2014-2019)
BOSS Old Red Galaxies (2008-2014)
SDSS I-II Nearby Galaxies (1998-2008)
Galaxies biased tracers of the underlying density field
Galaxy Clustering: Basics - I

Statistical description of galaxy distribution should shed light on underlying density field

\[ \xi(x) = \langle \delta_1 \delta_2 \rangle \]

\[ \xi(x) = \frac{1}{V_\text{a}} \sum_k P(k) e^{i \mathbf{k} \cdot \mathbf{x}} = \frac{1}{(2\pi)^3} \int P(k) e^{i \mathbf{k} \cdot \mathbf{x}} d^3 \mathbf{k}, \]

\[ P(k) = V_\text{a} \langle |\delta_k|^2 \rangle \]

\[ \left\langle p^{(1)}(x_i) p^{(1)}(x_i + x) \right\rangle_{x_i} = (\bar{n} \Delta V)^2 \left[ 1 + \xi(x) \right] \]

\[ dN(r) = 4\pi r^2 \bar{n} \left[ 1 + \xi(r) \right] dr, \quad \text{Number of neighbours} \]

"Mean density profile" around each particle
Galaxy Clustering: Basics - II

Redshift space breaks the isotropy but can be used for constraining cosmological models

In linear theory...

\[ \beta \equiv f(\Omega_m)/b \]

\[ \delta^{(s)}(r) = \left[ 1 + \beta \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) V_r^2 \right] \delta(r) \]

\[ \delta_k^{(s)} = (1 + \beta \mu_k^2) \delta_k, \quad \text{where} \quad \mu_k \equiv k_z/k. \]

Plane-parallel approximation

\[ P^{(s)}(k) = \sum_{\ell} \omega(\mu_k) P^{(s)}_{\ell}(k), \quad P^{(s)}_{\ell}(k) = \frac{2\ell+1}{2} \int_{-1}^{1} P^{(s)}(k) P_{\ell}(\mu_k) \mathrm{d}\mu_k, \]

\[ P^{(s)}_0(k) \equiv \left( 1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2 \right) P(k) \]

\[ P^{(s)}_2(k) = \left( \frac{4}{3} \beta + \frac{4}{7} \beta^2 \right) P(k), \quad P^{(s)}_4(k) = \frac{8}{35} \beta^2 P(k). \]

Angular correlation function, projected correlation function, evolution of correlation function etc.
But galaxies cannot be treated just like points.

Selection effects: magnitude limited sample and/or volume limited sample

\[ b(k) = b_{\text{lim}} \sqrt{\frac{1 + Qk^2}{1 + Ak}} \]

On top of that window function not really easy to deal with

Cresswell & Percival 2009
Galaxy Clustering: Basics - IV

Galaxy (Halo) bias

\[ N(1|0) dM_1 = \frac{M_0}{M_1} f(1|0) \frac{dS_1}{dM_1} dM_1, \]

Number of haloes

M0 could be halo mass or mass of a Lagrangian region (not collapsed)

\[ f(1|0) dS_1 \equiv \frac{1}{\sqrt{2\pi}} \frac{\delta_1 - \delta_0}{(S_1 - S_0)^{3/2}} \exp \left[-\frac{(\delta_1 - \delta_0)^2}{2(S_1 - S_0)}\right] dS_1. \]

\[ \delta_h^L(1|0) = \frac{N(1|0)}{n(M_1,z_i)V_L} - 1, \quad \text{where} \quad V_L = \frac{4\pi}{3} R_0^3. \]

\[ \delta_h^L(1|0) = \frac{\nu^2 - 1}{\delta_1} \delta_0, \quad \text{where} \quad \nu_1 = \frac{\delta_1}{\sqrt{S_1}}. \]

\[ \delta_h(1|0) = \frac{N(1|0)}{n(M_1,z_i)V_L/V_E} - 1, \quad V_L/V_E = 1 + \delta(t). \]

\[ \delta_h(1|0) = \delta(t) + \frac{\nu^2 - 1}{\delta_1} \delta_0 + \frac{\nu^2 - 1}{\delta_1} \delta_0 \delta(t) \]

\[ \delta_h(1|0) = b_h(M_1,\delta_1;t) \delta(t), \]

Simple (linear) bias model

\[ b_h(M_1,\delta_1;t) = 1 + \frac{1}{D(t)} \left( \frac{\nu^2 - 1}{\delta_1} \right) \]
The 20% of halos with the latest half-mass assembly redshifts in a 30 Mpc/h thick slice

\[ M_{\text{halo}} \sim 10^{11} M_\odot \]
The 20% of halos with the *earliest* half-mass assembly redshifts in a 30 Mpc/h thick slice

\[ M_{\text{halo}} \sim 10^{11} M_\odot \]
Galaxy Clustering: Assembly bias - III

The dependence of bias on formation redshift is strongest at low mass

This behaviour is inconsistent with simple versions of excursion set theory and of HOD and halo abundance matching models

Halo bias will not depend only on halo mass but also:
- formation time
- concentration
- substructure content
- spin
- shape
- saddle density
- velocity anisotropy

Problems for HOD
- Halo Occupation Distribution Approaches
  - including Intensity Mapping mocks-like approaches
Baryonic Acoustic Oscillations

Combined effect of:
1) BAO
2) RSDs
3) for theory modelling (Alcock-Paczynski test)
The acoustic wave

Start with a single perturbation. The plasma is totally uniform except for an excess of matter at the origin. High pressure drives the gas+photon fluid outward at speeds approaching the speed of light.

Baryons  Photons  Mass profile

Eisenstein, Seo & White (2006)
The acoustic wave

Initially both the photons and the baryons move outward together, the radius of the shell moving at over half the speed of light.
The acoustic wave

This expansion continues for $10^5$ years
The acoustic wave

After $10^5$ years the universe has cooled enough the protons capture the electrons to form neutral Hydrogen. This decouples the photons from the baryons. The former quickly stream away, leaving the baryon peak stalled.
The acoustic wave

The photons continue to stream away while the baryons, having lost their motive pressure, remain in place.
The acoustic wave

The photons have become almost completely uniform, but the baryons remain overdense in a shell 100Mpc in radius. In addition, the large gravitational potential well which we started with starts to draw material back into it.
The acoustic wave

As the perturbation grows by $\sim 10^3$ the baryons and DM reach equilibrium densities in the ratio $\Omega_b/\Omega_m$.

The final configuration is our original peak at the center (which we put in by hand) and an “echo” in a shell roughly 100Mpc in radius.

Further (non-linear) processing of the density field acts to broaden and very slightly shift the peak -- but galaxy formation is a local phenomenon with a length scale $\sim 10$Mpc, so the action at $r=0$ and $r\sim 100$Mpc are essentially decoupled. We will return to this …
Analysis of **anisotropic correlation function** focussed on the BAO signal

Two sources of anisotropies: Redshift Space Distortion (RSD)  
Geometrical induced anisotropy (AP)

\[
\xi(r, \mu) = \frac{DD(r, \mu) - 2DR(r, \mu) + RR(r, \mu)}{RR(r, \mu)}, \quad \xi_\ell(r) = \frac{2\ell + 1}{2} \int_{-1}^{+1} d\mu \xi(r, \mu) L_\ell(\mu)
\]

**Non-linear modelling of correlation function**

\[
P(k, \mu) = (1 + \beta \mu^2)^2 F(k, \mu, \Sigma_s) P_{NL}(k, \mu).
\]

\[
F(k, \mu, \Sigma_s) = \frac{1}{(1 + k^2 \mu^2 \Sigma_s^2)}
\]

\[
P_{\text{dw}}(k, \mu) = [P_{\text{lin}}(k) - P_{\text{nw}}(k)] \times \exp \left[ \frac{-k^2 \mu^2 \Sigma_s^2 + k^2(1-\mu^2)\Sigma_s^2}{2} \right] + P_{\text{nw}}
\]

\[
P_{\text{lin}}(k) = \frac{2l + 1}{2} \int_{-1}^{1} P_l(k, \mu) L_1(\mu) d\mu
\]

\[
\xi_{l,t}(r) = i^l \int \frac{k^3 d\log(k)}{2\pi^2} P_{l,t} j_l(kr)
\]

\[
\xi_0(r) = B_0^2 \xi_0(r) + A_0(r),
\]

\[
\xi_2(r) = \xi_2(r) + A_2(r),
\]

where

\[
A_\ell(r) = \frac{a_\ell,1}{r^2} + \frac{a_\ell,2}{r} + a_\ell,3; \ell = 0, 2, \perp, \parallel
\]
Galaxy Clustering - II: Theoretical Framework

\[ \alpha = \alpha_{\perp}^{2/3} \alpha_{\parallel}^{1/3}, \]

\[ 1 + \epsilon = \left( \frac{\alpha_{\parallel}}{\alpha_{\perp}} \right)^{1/3} \]

\[ \alpha_{\perp} = \frac{D_A(z)r_s^{\text{fid}}}{D_A^{\text{fid}}r_s}, \]

\[ \alpha_{\parallel} = \frac{H(z)r_s^{\text{fid}}}{H(z)r_s}. \]

\( \alpha, \epsilon \) usually appear when referring to systematics/mocks while distances and \( F_{AP} \) when quoting final cosmologically relevant numbers.

\[ D_V(z) = \left( \frac{D_M(z)^2 c z}{H(z)} \right)^{1/3} \]

\[ F_{AP}(z) = D_M(z) H(z)/c. \]
Galaxy clustering challenges - ca 2020

Measurement of galaxy clustering hampered by systematics and statistical errors.

Estimating the window function and selection function is not trivial.

Focus on:

1) optimization of codes to handle large number of objects
2) getting reliable mocks
3) quantifying systematic effects
4) covariance matrix estimation
5) improving reconstruction techniques

State-of-the-art provided by BOSS survey (e.g. Alam+18, Vargas-Magana+18)

1) Systematics are estimated and appear as weights in the selection function
2) Mock generation using several different methods - based on Perturbation theory or N-body simulations
   3) Estimation of the 2D correlation function using Landy-Szalay estimator
4) Analysis focused on BAO peak and in second instance on sub-BAO shape info
5) Different pipelines tested with estimation of systematic errors introduced in each step
6) Main conclusions: unlike naively expected latest BOSS results are dominated by statistical errors
Galaxy clustering: the data set

10,000 sq. deg. & 1.2 million galaxies in V=20 Gpc^3

medium-resolution spectra (R = 1500–2600) in the wavelength range from 3600 to 10000 Å through 2 arcsec fibres

LOWZ was designed to target luminous red galaxies up to z = 0.4, while CMASS was designed to target massive galaxies from 0.4 < z < 0.7

Seven data analyses performed with different methodologies (tested on mocks)
Galaxy clustering: the signal

Analysis performed in configuration and Fourier space and gives consistent results.
Galaxy clustering: constraints from BAOs

BAO measurement combined with CMB prior allows to measure $H(z)$ at 2.4% and $D_A(z)$ at 1.5% in each redshift bin. When combined, $D_A(z)$ measured at 1% and $H(z)$ at 1.6%.

Perfect agreement with Planck.

Remarkable measurements not much freedom to escape from LCDM.

Consensus constraints are obtained by combined up to 7 different methods with strong covariance (mock estimated) but they improve the overall constraint significantly.
Galaxy clustering: constraints from full shape

Full-shape measurement with a variety of methods, this allows to measure the $f\sigma_8$ combination with a 10% precision in each bin and overall a 6% measurement.

Perfect agreement with Planck.
$D_M(z)$ and $H(z)$ are more strongly correlated for the BAO-only analysis, so while the $D_V(z)$ constraints from postreconstruction BAO-only are appreciably tighter than those from pre-reconstruction FS, the marginalized constraints on $D_M(z)$ and $H(z)$ are not.

The constraints on $F_{AP}(z)$ from sub-BAO scales in the FS analyses help to break the degeneracy between $D_M$ and $H$, leading to rounder confidence contours and smaller errors on $F_{AP}$.

Combined BAO+FS contours take advantage of both the sharpening of the BAO feature by reconstruction and the improved degeneracy breaking from the sub-BAO Alcock–Paczynski effect.
## Constraints from post-reconstruction BAO measurements and pre-reconstruction full-shape

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Redshift</th>
<th>BAO only</th>
<th>Full shape</th>
<th>BAO + FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_M(r_d, \text{fid}/r_d) \text{ (Mpc)}$</td>
<td>$z = 0.38$</td>
<td>$1512 \pm 22 \pm 11$</td>
<td>$1529 \pm 24 \pm 11$</td>
<td>$1518 \pm 20 \pm 11$</td>
</tr>
<tr>
<td>$D_M(r_d, \text{fid}/r_d) \text{ (Mpc)}$</td>
<td>$z = 0.51$</td>
<td>$1975 \pm 27 \pm 14$</td>
<td>$2007 \pm 29 \pm 15$</td>
<td>$1977 \pm 23 \pm 14$</td>
</tr>
<tr>
<td>$D_M(r_d, \text{fid}/r_d) \text{ (Mpc)}$</td>
<td>$z = 0.61$</td>
<td>$2307 \pm 33 \pm 17$</td>
<td>$2274 \pm 36 \pm 17$</td>
<td>$2283 \pm 28 \pm 16$</td>
</tr>
<tr>
<td>$H(r_d/r_d, \text{fid}) \text{ (km s}^{-1}\text{Mpc}^{-1})$</td>
<td>$z = 0.38$</td>
<td>$81.2 \pm 2.2 \pm 1.0$</td>
<td>$81.2 \pm 2.0 \pm 1.0$</td>
<td>$81.5 \pm 1.7 \pm 0.9$</td>
</tr>
<tr>
<td>$H(r_d/r_d, \text{fid}) \text{ (km s}^{-1}\text{Mpc}^{-1})$</td>
<td>$z = 0.51$</td>
<td>$90.9 \pm 2.1 \pm 1.1$</td>
<td>$88.3 \pm 2.1 \pm 1.0$</td>
<td>$90.5 \pm 1.7 \pm 1.0$</td>
</tr>
<tr>
<td>$H(r_d/r_d, \text{fid}) \text{ (km s}^{-1}\text{Mpc}^{-1})$</td>
<td>$z = 0.61$</td>
<td>$99.0 \pm 2.2 \pm 1.2$</td>
<td>$95.6 \pm 2.4 \pm 1.1$</td>
<td>$97.3 \pm 1.8 \pm 1.1$</td>
</tr>
<tr>
<td>$f\sigma_8$</td>
<td>$z = 0.38$</td>
<td>–</td>
<td>$0.502 \pm 0.041 \pm 0.024$</td>
<td>$0.497 \pm 0.039 \pm 0.024$</td>
</tr>
<tr>
<td>$f\sigma_8$</td>
<td>$z = 0.51$</td>
<td>–</td>
<td>$0.459 \pm 0.037 \pm 0.015$</td>
<td>$0.458 \pm 0.035 \pm 0.015$</td>
</tr>
<tr>
<td>$f\sigma_8$</td>
<td>$z = 0.61$</td>
<td>–</td>
<td>$0.419 \pm 0.036 \pm 0.009$</td>
<td>$0.436 \pm 0.034 \pm 0.009$</td>
</tr>
</tbody>
</table>

**Statistical error**

**Systematic error**
BAO Hubble Diagram and Growth

Alam+20

[Graph showing BAO Hubble Diagram and Growth]
BAO constraints at high and low redshift

In general Early DE models can alleviate tensions of LCDM (low clustering amplitude and large $H_0$ are predicted at low $z$)

Tensions between Lya BAO and CMASS BAO
RSD measurements from BOSS
In the LCDM case $\Omega_m = 0.311 \pm 0.006$ and $H_0 = 67.6 \pm 0.5$ km/s/Mpc
If $\Omega_k$ and $w$ are varied $\Omega_k = 0.0003 \pm 0.0026$ and $w = -1.01 \pm 0.06$

Main results:
1) impressive agreement with LCDM even after opening a 2 parameter space ($w$, $\Omega_k$)
2) FOM for wCDM ~20–30
3) Adding z-bins helps
Planck + BOSS galaxy clustering - II

<table>
<thead>
<tr>
<th>Cosmological model</th>
<th>Data sets</th>
<th>$\Omega_m h^2$</th>
<th>$\Omega_m$</th>
<th>$H_0$ (km s$^{-1}$ Mpc$^{-1}$)</th>
<th>$\Omega_K$</th>
<th>$w_0$</th>
<th>$w_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_0 w_a$CDM</td>
<td>Planck + SN</td>
<td>0.1428 (14)</td>
<td>0.294 (16)</td>
<td>69.8 (18)</td>
<td>--</td>
<td>-0.85 (13)</td>
<td>-0.99 (63)</td>
</tr>
<tr>
<td>$w_0 w_a$CDM</td>
<td>Planck + BAO</td>
<td>0.1427 (11)</td>
<td>0.336 (21)</td>
<td>65.2 (21)</td>
<td>--</td>
<td>-0.63 (20)</td>
<td>-1.16 (55)</td>
</tr>
<tr>
<td>$w_0 w_a$CDM</td>
<td>Planck + BAO + FS</td>
<td>0.1427 (11)</td>
<td>0.334 (18)</td>
<td>65.5 (17)</td>
<td>--</td>
<td>-0.68 (18)</td>
<td>-0.98 (53)</td>
</tr>
<tr>
<td>$w_0 w_a$CDM</td>
<td>Planck + BAO + FS + SN</td>
<td>0.1426 (11)</td>
<td>0.313 (9)</td>
<td>67.5 (10)</td>
<td>--</td>
<td>-0.91 (10)</td>
<td>-0.39 (34)</td>
</tr>
<tr>
<td>$w_0 w_a$CDM</td>
<td>Planck + BAO</td>
<td>0.1422 (14)</td>
<td>0.331 (21)</td>
<td>65.6 (21)</td>
<td>-0.0022 (30)</td>
<td>-0.66 (19)</td>
<td>-1.22 (53)</td>
</tr>
<tr>
<td>$w_0 w_a$CDM</td>
<td>Planck + BAO + FS</td>
<td>0.1422 (14)</td>
<td>0.333 (16)</td>
<td>65.4 (16)</td>
<td>-0.0020 (28)</td>
<td>-0.67 (18)</td>
<td>-1.12 (59)</td>
</tr>
<tr>
<td>$w_0 w_a$CDM</td>
<td>Planck + BAO + FS + SN</td>
<td>0.1420 (14)</td>
<td>0.314 (10)</td>
<td>67.3 (10)</td>
<td>-0.0023 (28)</td>
<td>-0.87 (11)</td>
<td>-0.63 (45)</td>
</tr>
</tbody>
</table>
Planck + BOSS galaxy clustering - III

\[ f_{\sigma_8} \rightarrow f_{\sigma_8} \left[ A_{f_{\sigma_8}} + B_{f_{\sigma_8}}(z - z_p) \right] \]

- few percent offset from GR case - this is not significant
- B is different from zero at 1.5sigma
- 6% error on A
- Again very stable also when curvature and w are considered
How sensitive are we to $\sigma_8$?

Envelope bracketing $s8=0.53-0.85$

Philcox & Ivanov 22
Curvature
Dark Energy OR neutrinos
Dark Energy AND neutrinos

Semenaitė+22

$\nu_{CDM}$

M$\nu<0.211$ eV (2sigma C.L.)
\[ f \sigma_8 = \gamma f_{\Lambda CDM} \sigma_8 \]

\[ \nu_{bc} : \text{parameter that controls in the HOD how the central galaxy is distributed within the DM halo - non-linear velocity} \]

- Consistent with ΛCDM on largest scales (1.4σ) matches “standard” RSD model that agree with ΛCDM expectation
- Deviations on small scales (up to 4.5σ) - RSD equivalent of “lensing is low” is “non-linear velocity dispersion is low”
  - Insufficient mass in halos?
  - HOD model break down?
  - Simulations wrong?
  - Systematic errors?
BOSS: main conclusions

1) ~1% constraints on H(z) and DA(z) from BAO

2) amplitude of pec. vel. measured at ~10% level

3) No evidence for physics beyond LCDM

4) Agreement with Planck low values for H0, with limits remarkably stable also for owCDM or ow0waCDM models with 1sigma error bar of 1km/s/Mpc

5) Limits on neutrino mass are 0.16 eV, which become 0.25 when removing RSD and ~0.3 when opening the w parameter space

6) No support for Neff>3

OVERALL the stage is set and future seems promising for the next experiments like eBOSS, DESI, WFIRST/ROMAN, Euclid, LSST etc. it is expected that statistical errors will improve and a new level of systematics will be hit (sub-percent precision constraints)
The BOSS DR12 Full-Shape Cosmology:
\(\Lambda\)CDM Constraints from the Large-Scale Galaxy Power Spectrum and Bispectrum Monopole

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\(^1\)Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544, USA
\(^2\)School of Natural Sciences, Institute for Advanced Study, 1 Einstein Drive, Princeton, NJ 08544, USA

We present a full \(\Lambda\)CDM analysis of the BOSS DR12 dataset, including information from the power spectrum multipoles, the real-space power spectrum, the reconstructed power spectrum and the bispectrum monopole. This is the first analysis to feature a complete treatment of the galaxy bispectrum, including a consistent theoretical model and without large-scale cuts. Unlike previous works, the statistics are measured using window-free estimators; this greatly reduces computational costs by removing the need to window-convolve the theory model. Our pipeline is tested using a suite of high-resolution mocks and shown to be robust and precise, with systematic errors far below the statistical thresholds. Inclusion of the bispectrum yields consistent parameter constraints and shrinks the \(\sigma_8\) posterior by 13\% to reach < 5\% precision; less conservative analysis choices would reduce the error-bars further. Our constraints are broadly consistent with Planck; in particular, we find \(H_0 = 69.6^{+1.7}_{-1.4}\) km s\(^{-1}\) Mpc\(^{-1}\), \(\sigma_8 = 0.692^{+0.010}_{-0.007}\) and \(n_s = 0.967^{+0.007}_{-0.004}\) including a BBN prior on the baryon density. When \(n_s\) is set by Planck, we find \(H_0 = 68.3^{+1.3}_{-0.8}\) km s\(^{-1}\) Mpc\(^{-1}\) and \(\sigma_8 = 0.722^{+0.018}_{-0.012}\). Our \(S_8\) posterior, 0.751 \(\pm\) 0.039, is consistent with weak lensing studies, but lower than Planck. Constraints on the higher-order bias parameters are significantly strengthened from the inclusion of the bispectrum, and we find no evidence for deviation from the dark matter halo bias relations. These results represent the most complete full-shape analysis of BOSS DR12 to-date, and the corresponding spectra will enable a variety of beyond-\(\Lambda\)CDM analyses, probing phenomena such as the neutrino mass and primordial non-Gaussianity.

Note: BBN prior is used

Redshift Space \(P_0, P_2, P_4\) + Real Space \(P(k)\) computed from the above
+ BAO
+ Bispectrum

\[ Q_0(k) \equiv P_0(k) - \frac{1}{2} P_2(k) + \frac{3}{8} P_4(k) \]
The S8 tension

\[ S_8 \equiv \sigma_8 (\Omega_m/0.3)^{0.5} \]

\[ S_8^{\text{Planck}} = 0.832 \pm 0.012 \quad \text{CMB} \]

\[ S_8^{\text{DES}} = 0.776 \pm 0.017 \quad \text{DES (Weak lensing and clustering)} \]

\[ S_8 = 0.734^{+0.035}_{-0.041} \quad \text{Galaxy Clustering Full Shape} \]

Agreement with WL and 2.5sigma tension with CMB!
**Effective field theory of LSS**

\[
\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \frac{1}{m} \vec{p} \cdot \nabla f - m \nabla \Phi \cdot \frac{\partial f}{\partial \vec{p}} = 0
\]

VLASOV EQ.

\[
m \int_{p} f(\vec{x}, \vec{p}, \eta) \equiv \rho_m(\vec{x}, t),
\]

Moments of \( f \) to define density

\[
m \int_{p} \frac{\partial f}{\partial \vec{p}} f(\vec{x}, \vec{p}, t) \equiv \rho_m \nu(\vec{x}, t),
\]

velocity and stress energy tensor

\[
m \int_{p} \frac{\partial f}{\partial \vec{p}} f(\vec{x}, \vec{p}, t) \equiv \rho_m \nu(\vec{x}, t) + \kappa_i(\vec{x}, t).
\]

Continuity

\[
\partial_t \rho_m + \nabla_i (\rho_m \nu^i) = 0.
\]

Ansatz for a generic fluid with viscosity

\[
\kappa_{ij} = -p \delta_{ij} + \nu \left[ \nabla_i \nu_j + \nabla_j \nu_i - \frac{2}{3} \delta_{ij} \nabla \cdot \nu \right] + \zeta \delta_{ij} \nabla \cdot \nu.
\]

NOW: separate the scales between long modes (l) and short modes (s)

\[
\rho \left[ \dot{v}_l^i + v_l^j \nabla_j v_l^i \right] + \rho \nabla_i \phi = \nabla_j \left[ \tau_{ij}^l \right],
\]

\([
\ldots \] \) \( \lambda \) is spatial average over \( 1/\lambda \) scale

\[
\tau_{ij}^l \equiv \rho_m v_l^i v_l^j - \frac{\phi_k^i \phi_k^j \delta_{ij} - 2 \phi_k^i \phi_k^j}{8 \pi G}.
\]

Effective stress tensor in Euler eq. induced by small scale fluctuations

\[
3 p_{\text{eff}} = \left[ \tau_{ij}^l \right] \lambda - \frac{\left[ \phi_{ik}^l \phi_{jk}^l \right] \lambda}{8 \pi G}.
\]

\[
\rho_{\text{eff}} = \gamma(v_s) \rho_m \lambda + \frac{1}{2} \left[ \rho_m \phi^s \right] \lambda, \quad \rho_m = \gamma(v_s) e^{-3 \phi_s} \rho, \quad \gamma(v_s) \equiv \frac{1}{\sqrt{1 - v_s^2}},
\]

\[
\rho_{\text{eff}} = \left[ \rho_m \left( 1 + \frac{1}{2} v_s^2 + \frac{1}{2} \phi_s \right) \right] \lambda.
\]

Effective energy densities gets velocity and gravitational correction and can be derived in full GR as well

Note validity range:

\[
\frac{k}{k_{\text{NL}}} \ll 1 \quad \text{and} \quad k_{\text{NL}}^{-1} \sim 5 \text{Mpc}
\]
Effective field theory of LSS - II

Now the problem is to calculate the effective stress tensor

$$\frac{1}{a\rho} \partial_j T^{ij} = \int d\tau' \ K(\tau, \tau') \ \partial^j \delta(x_\theta(x, \tau; \tau'), \tau') + \ldots,$$

Time propagator Fluid element

Taylor expand the above

$$-\frac{1}{a\rho} \partial_j \tau^{ij} = -c^2 \partial^j \delta + \frac{c^2}{H} \partial^j \partial_v \delta - c_1 \partial^j \delta^2 - c_2 \partial^j (s_{\delta k}) - c_3 s_{ij} \partial_k \delta - \frac{1}{a\rho} \tau^{ij}_{\text{dmb.}}.$$ 

Insert the above in Euler equation --> and compute!

Convenient to set the stress energy to zero first and then look for corrections to this

$$\delta = [\delta_{(1)} + \delta_{(2)} + \delta_{(3)} + \ldots] + \delta_{(4)} + \ldots$$

Biased tracers (e.g. galaxies, DM haloes)

$$\delta_g = b_1 \delta + b_2 \delta^2 + b_{\nu_2} G_2 + b_{\nu_3} \nabla^2 \delta + b_{\gamma_3} \gamma_3 + \varepsilon + \ldots$$

$$G_2(\Phi) \equiv (\partial_\Phi \partial_\Phi)^2 - (\Delta \Phi)^2, \quad \gamma_3 \equiv G_2(\Phi) - G_2(\Phi_v)$$

+ redshift space distortions...
+ baryons....
+ neutrinos....

Things get non-trivial with many parameters... but...

from BOSS DR12
Future seems bright

DESI paper (2016) - stage IV experiment - Y1 DR in Fall 2023 / early data release in Jan 23
Future galaxy redshift surveys (e.g. DESI from the ground or Euclid from the sky) will continue an on-going effort to map the large-scale galaxy distribution.

Different features of the galaxy power spectrum provide different constraint on the cosmological model:

- BAO are a standard ruler, a geometrical probe of the expansion history
- The anisotropy of the galaxy power spectrum (Redshift-Space Distortions) measure instead the growth of structure
- The “shape” of the power spectrum provide an upper bound on neutrino masses

Current efforts are aimed at extracting all available information in 2-point and higher-order correlation functions and extend PT predictions beyond the Standard Cosmological Model ---> Bispectrum/Trispectrum.... etc
Challenges of GC studies for dark energy

- bias modelling at mildly non-linear scales to exploit also other smaller volume surveys

- neutrino mass measurement and modelling of neutrino induced non-linearities in GCs (scale dependence of the bias)

- higher order statistics and the search for non-Gaussianities

- Cross correlations

- Multi purpose experiments with high degree of complementarity

- Machine learning and data science (pixel-by-pixel analysis)

- High redshift regime/huge discovery potential
Euclid
definition study report 2011

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$w_p$</th>
<th>$w_z$</th>
<th>FoM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euclid Primary</td>
<td>0.015</td>
<td>0.150</td>
<td>430</td>
</tr>
<tr>
<td>Euclid All</td>
<td>0.013</td>
<td>0.048</td>
<td>1540</td>
</tr>
<tr>
<td>Euclid+Planck</td>
<td>0.007</td>
<td>0.035</td>
<td>4020</td>
</tr>
<tr>
<td>Current</td>
<td>0.100</td>
<td>1.500</td>
<td>~10</td>
</tr>
<tr>
<td>Improvement Factor</td>
<td>&gt;10</td>
<td>&gt;50</td>
<td>&gt;300</td>
</tr>
</tbody>
</table>
DESI constraints

Table 2.9: DETF Figures of Merit and uncertainties $\sigma_{w_p}$ and $\sigma_{\Omega_k}$. $\sigma_{w_p}$ is the error on $w$ at the pivot redshift, which also equal to the error on a constant $w$ holding $w_a = 0$. $\sigma_{\Omega_k}$ is the error on the curvature of the Universe, $\Omega_k$. All DESI lines contain the BGS, and BOSS in the range $0.45 < z < 0.6$ that does not substantially overlap with DESI. All cases include Planck CMB constraints. The pivot point, where $w(a)$ has minimal uncertainty is indicated by $a_p$. We note that a FoM of 110 is 10 times the Stage II level of [109], which we take to be the definition of Stage IV. DESI BAO galaxy exceeds this threshold even with a 9,000 square degree survey.

<table>
<thead>
<tr>
<th>Surveys</th>
<th>FoM</th>
<th>$a_p$</th>
<th>$\sigma_{w_p}$</th>
<th>$\sigma_{\Omega_k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOSS BAO</td>
<td>37</td>
<td>0.65</td>
<td>0.055</td>
<td>0.0026</td>
</tr>
<tr>
<td>DESI 14k galaxy BAO</td>
<td>133</td>
<td>0.69</td>
<td>0.023</td>
<td>0.0013</td>
</tr>
<tr>
<td>DESI 14k galaxy and Ly-α forest BAO</td>
<td>169</td>
<td>0.71</td>
<td>0.022</td>
<td>0.0011</td>
</tr>
<tr>
<td>DESI 14k BAO + gal. broadband to $k &lt; 0.1 \ h \ Mpc^{-1}$</td>
<td>332</td>
<td>0.74</td>
<td>0.015</td>
<td>0.0009</td>
</tr>
<tr>
<td>DESI 14k BAO + gal. broadband to $k &lt; 0.2 \ h \ Mpc^{-1}$</td>
<td>704</td>
<td>0.73</td>
<td>0.011</td>
<td>0.0007</td>
</tr>
<tr>
<td>DESI 9k galaxy BAO</td>
<td>95</td>
<td>0.69</td>
<td>0.027</td>
<td>0.0015</td>
</tr>
<tr>
<td>DESI 9k galaxy and Ly-α forest BAO</td>
<td>121</td>
<td>0.71</td>
<td>0.026</td>
<td>0.0012</td>
</tr>
<tr>
<td>DESI 9k BAO + gal. broadband to $k &lt; 0.1 \ h \ Mpc^{-1}$</td>
<td>229</td>
<td>0.73</td>
<td>0.018</td>
<td>0.0011</td>
</tr>
<tr>
<td>DESI 9k BAO + gal. broadband to $k &lt; 0.2 \ h \ Mpc^{-1}$</td>
<td>502</td>
<td>0.73</td>
<td>0.013</td>
<td>0.0009</td>
</tr>
</tbody>
</table>
Planck+BSH combination has a $\Delta \chi^2 = -0.8$ w.r.t. base LCDM.

WL is CFHTLens: prefers $w_0 - w_a$ at 2$\sigma$ w.r.t. LCDM, wants higher $\sigma_8$ and (not shown) high value of $H_0$.

Implications for the small scale crisis of LCDM
Principal Component Analysis reconstruction in 4 bins, increasing number of bins does lead to same conclusions.

\[ w(z) = p_{i-1} + \Delta w \left( \tanh \left( \frac{z - z_i}{s} \right) + 1 \right) \] for \( z < z_i, \ i \in \{1, 4\} \)
Note further that Planck is providing a measurement of the sound horizon at the drag epoch with an error of 0.2% ...
Remarkable results from Planck experiment in constraining DE properties

1) In \( (w_0 - w_a) \), Planck TT+lowP+BSH is compatible with LCDM, as well as BAO/RSD. When adding WL to Planck TT+lowP, both WL and CMB prefer the \( (w_0 - w_a) \) model with respect to LCDM at ~2\( \sigma \) (with preference for high values of \( H_0 \), excluded when including BSH). CMB lensing does not change the numbers.

2) Tests on time varying \( w(z) \) are compatible with LCDM for all data sets tested.

3) EDE model with constant fraction till recent. Constraints are incredibly tight: previous constraints improved by a factor 3-4, \( \Omega_e < 0.0036 \) for PlanckTT,TE,EE+lowP+BSH. Polarization improves limits by a factor 2.

4) \( \Omega_e(z) \) as a function of \( z_e \), the redshift starting from which a fraction is present. \( \Omega_e < 2\% \) (95\% C.L.) even for \( z_e \) as late as 50 (important results in the era of structure formation with implications for EDGES and other science). CMB lensing is important.
Lyman-alpha BAO: a tuned oscillation?

$\Delta \chi^2 = -6.6$ with 3 d.o.f. for this model
• Redshift covered $z=2.1-3.5$, $<z>=2.33$.

• 160,000 QSOs (DESI will have ~ 10 times more).

• Statistical improvement over DR9, DR11 (Delubac+14) hinted for a change in the sign of dark energy density to reconcile with Planck.

• Better physical modelling for high column density systems, UV fluctuations, broad band power (marginalized over).

• Complementarity with low redshift BAO, high redshift BAO provide a stronger support for $\Omega_{\Lambda} > 0$ (independent of CMB).
15,000 deg$^2$ Euclid

$f \geq 2 \times 10^{-16}$ erg s$^{-1}$ cm$^{-2}$

$0.9 \leq z \leq 1.8$

- Red line: Calzetti et al.
- Blue line: Charlot & Fall
- Dashed line: Ferrara et al. + MCs

$dN/dz$ [thousands of galaxies deg$^{-2}$]

Redshift