Fundamental Physics with Galaxies (lecture III)
Gravity alone

Beyond mean-field effects

$m_X$

$10^{-21}$ eV $10^{-10} M_\odot \sim 10^{56}$ eV
Gravity alone

$m_X$

$10^{-21}$ eV

$10^{-10} M_\odot \sim 10^{56}$ eV

Beyond mean-field effects

A whole field of research that I will not discuss now: dark matter substructure

Dalal & Kochaneck astro-ph/0111456,
Vegetti & Vogelsberger 1406.1170,
Hezaveh et al 1601.01388,
Minor et al 2011.19627,

Bovy 1512.00452,
Banik et al 1911.02663,

Nacib, Lisanti, Belokurov 1807.02519,
Ravi et al 1812.07578,

...
Beyond mean-field effects

Fornax dwarf galaxy

Fornax: data (2018) and models, $\beta=\text{const.}$

\[ \sigma_{\text{sys}} \text{ [km/s]} \]

- M19 NFW
- M19 ISO
- DDM
- SIDM
- CoreNFW

\[ \rho \left[ M_\odot/\text{kpc}^2 \right] \]

- Stars

$r \text{ [pc]}$
Beyond mean-field effects

Fornax: data (2018) and models, $\beta=$const.

Fornax dwarf galaxy

Globular Clusters (GC3: $\sim 5 \times 10^5 \, M_\odot$)
Beyond mean-field effects

Fornax dwarf galaxy

Globular Clusters (GC3: $\sim 5 \times 10^5 M_\odot$)

People also ask:

What is the weight of 1 apple?
An apple's weight depends on the variety and the size of the fruit. On average, an apple weighs between 150 g and 250 g.

People also ask:

What's heavier a rhino or a hippo?
The Indian Rhino is from 3–4 metres (10 – 14 feet) long. The record-sized specimen of this rhino was approximately 3,800 kg (8,377 lb). The Indian Rhino has a single horn that reaches a length of between 20 and 100 cm (8 – 39 inches).
Beyond mean-field effects

Globular Clusters (GC3: \( \sim 5 \times 10^5 M_\odot \))

People also ask:

What is the weight of 1 apple?

An apple's weight depends on the variety and the size of the fruit. On average, an apple weighs between 150 g and 250 g.

People also ask:

What's heavier a rhino or a hippo?

What is the heaviest rhino on record?

The Indian Rhino is from 3–4 metres (10 – 14 feet) long. The record-sized specimen of this rhino was approximately 3,800 kg (8,377 lb). The Indian Rhino has a single horn that reaches a length of between 20 and 100 cm (8 – 39 inches).

\[ \gg 3800e3/150 \]
\[ \text{ans} = 2.5333e04 \]
NGC5846-UDG1: ultradiffuse galaxy

The paper is organized as follows. In Sec. 1, we set up and study N-body simulations, in which some dynamical effects (notably effects that shape the GC population in a galactic halo is roughly proportional to the GC mass naturally provided by dynamical friction. The deceleration experienced by a GC due to dynamical friction in a nuclear cluster. As we will demonstrate, using more detailed analytic estimates as well as a suite of numerical simulations, UDG1 as we view it today may indeed be in this intermediate stage.

In this paper we show that this explanation can be a chance fluctuation and that there is no mass segregation. This luminosity or mass of about 84 to the center of the galaxy. The data shows a clear trend: more luminous GCs are on average closer to the right panel of Fig. 2. To explore this further, in Fig. 2 we show the distribution of objects from the left panel, divided into magnitude bins. The magnitude bins for objects at r<r have a p-value of 2 x 10^{-3}, which has a background contamination of about 1 object, estimated by comparing to the nearby field (post-selection criteria described in Danieli et al., 2022).

In this work we focus on a low contamination sample of GC candidates, selected as GC candidates based on the photometric selection criteria in Danieli et al. (2022). Spectroscopic information is available for 11 of these bright GCs (twice the S´ersic half-light radius of the stellar body). In this work we discuss dynamical effects (notably dynamical friction and GC mass loss) are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-analytically, in which some dynamical effects are modeled semi-
The data shows a possible trend: more luminous GCs are on average closer to the center of the galaxy. The right panel of Fig. 1 shows all compact sources that were selected as GC candidates based on the photometric selection criteria in Danieli et al. (2022). Spectroscopic information is available for 11 of these bright GCs and its nearby field, adapted from Danieli et al. (2022). In this work we focus on a low contamination sample of GC candidates, consisting of the 33 faint objects also exhibit radial clustering above the background, comparable to the stellar body. In this paper we show that this explanation can be naturally provided by dynamical friction. The deceleration experienced by a GC due to dynamical friction in a galactic halo is roughly proportional to the GC mass times the logarithm of the ratio of the GC mass to the total mass of the galaxy. It is noteworthy that most of the brighter GCs in UDG1 and similar galaxies are concentrated in the region within 2 times the Sersic radius of the stellar light profile, with contamination of about 1 object, estimated by comparison to the nearby field (post-selection criteria described in Danieli et al. 2022). To explore this further, in Fig. 2 we recapitulate the observations of UDG1, and define benchmark mass models. In Sec. 4 we set up and study N-body simulations, in which some dynamical effects (notably those due to molecular clouds) are modeled semi-analytically. The simulations, UDG1 as we view it today may indeed be in this intermediate stage. As we will demonstrate, using more detailed analytic estimates as well as a suite of numerical simulations, in which some dynamical effects (notably those due to molecular clouds) were included and some were neglected, we disprove the hypothesis that the data is a chance fluctuation and that there is no mass segregation. This luminosity or mass segregation calls for a quantitative dynamical explanation. In this work we consider that the mass segregation is due to dynamical friction and GC mass loss) are modeled semi-analytically. In the simulations, in which some dynamical effects (notably those due to molecular clouds) were included and some were neglected, we disprove the hypothesis that the data is a chance fluctuation and that there is no mass segregation. This luminosity or mass segregation calls for a quantitative dynamical explanation. The paper is organized as follows. In Sec. 4 we set up and study N-body simulations, in which some dynamical effects (notably those due to molecular clouds) were included and some were neglected, we disprove the hypothesis that the data is a chance fluctuation and that there is no mass segregation. This luminosity or mass segregation calls for a quantitative dynamical explanation. The paper is organized as follows. In Sec. 4 we set up and study N-body simulations, in which some dynamical effects (notably those due to molecular clouds) were included and some were neglected, we disprove the hypothesis that the data is a chance fluctuation and that there is no mass segregation. This luminosity or mass segregation calls for a quantitative dynamical explanation.
This is a beyond-mean-field effect. Qualitatively different window on dark matter, compared to stellar kinematics.
Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2). The phase space distribution of type 1 particles follows the continuity eq.: \( \frac{df_1}{dt} = C[f_1] \).

In the absence of collisions (C=0), for a nonrelativistic particle (p=mv) we have the usual mean-field dynamics:

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}_i \frac{\partial f}{\partial x_i} + \dot{p}_i \frac{\partial f}{\partial p_i} = \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} = 0
\]
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\]

\[
\int d^3p \; v_j \left( \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} \right) = \frac{\partial}{\partial t} \left( n \bar{v}_j \right) + \frac{\partial}{\partial x_i} \left( n \bar{v}_i v_j \right) + n \frac{\partial \Phi}{\partial x_j} = 0
\]

(assuming that f vanishes on the v-boundary, and noting \( n = \int d^3p \; f \), \( n \bar{v}_i = \int d^3p \; v_i \; f \), \( n \bar{v}_i v_j = \int d^3p \; v_i v_j \; f \))
Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2). The phase space distribution of type 1 particles follows the continuity eq.: \( \frac{df_1}{dt} = C[f_1] \).

Apply this formalism to a single massive particle flying around (velocity \( V = \frac{P}{M} \)):

\[
\begin{align*}
f &= \frac{1}{M^3} \delta^{(3)}(\mathbf{x} - \mathbf{X}(t)) \delta^{(3)}(\mathbf{v} - \mathbf{V}(t)), \\
n(\mathbf{x}) &= \int d^3p f = M^3 \int d^3vf = \delta^{(3)}(\mathbf{x} - \mathbf{X}), \\
n\bar{v} = \delta^{(3)}(\mathbf{x} - \mathbf{X})\bar{v} = M^3 \int d^3v vf = \delta^{(3)}(\mathbf{x} - \mathbf{X})\mathbf{V}, \text{ that is, } \bar{v} = \mathbf{V} = \int d^3x n\bar{v}, \\
\left[ \frac{\partial}{\partial t} (n\bar{v}_j) + \frac{\partial}{\partial x_j} (n\bar{v}_i\bar{v}_j) + n \frac{\partial \Phi}{\partial x_j} \right]
\end{align*}
\]
Basic physics view of dynamical friction:

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\[
f = \frac{1}{M^3} \delta^{(3)}(x - X(t)) \delta^{(3)}(v - V(t)), \quad n(x) = \int d^3p f = M^3 \int d^3v f = \delta^{(3)}(x - X),
\]

\[
n\bar{v} = \delta^{(3)}(x - X)\bar{v} = M^3 \int d^3v vf = \delta^{(3)}(x - X)V, \text{ that is, } \bar{v} = V = \int d^3x n\bar{v}, \]

\[
\int d^3x \left[ \frac{\partial}{\partial t} (n\bar{v}_j) + \frac{\partial}{\partial x_j} (n\bar{v}_i\bar{v}_j) + n \frac{\partial \Phi}{\partial x_j} \right]
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\]

\[
\int d^3x \left[ \frac{\partial}{\partial t} (n\vec{v}_j) + \frac{\partial}{\partial x_j} (n\vec{v}_i\vec{v}_j) + n \frac{\partial \Phi}{\partial x_j} \right] = \frac{\partial V_j}{\partial t} + \frac{\partial \Phi}{\partial x_j} = 0, \quad \text{or} \quad \dot{\vec{V}} = -\nabla \Phi
\]

(assuming that \( n\vec{v}_i\vec{v}_j \) vanishes on the spatial boundary)
Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2).

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\]

\[
\int d^3x \left[ \frac{\partial}{\partial t} (n\ddot{v}_j) + \frac{\partial}{\partial x_j} (n\ddot{v}_j) + n \frac{\partial \Phi}{\partial x_j} \right] = \frac{\partial V_j}{\partial t} + \frac{\partial \Phi}{\partial x_j} = 0, \quad \text{or} \quad \ddot{V} = -\nabla \Phi
\]

Collisions change this to: \( \ddot{V} = -\nabla \Phi + M^3 \int d^3x \int d^3v v C[f] \).
Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2).

The phase space distribution of type 1 particles follows the continuity eq.: \( \frac{df_1}{dt} = C[f_1] \).

The collision operator:

\[
C[f_1] = \frac{(2\pi)^4}{2E_p} \int d\Pi_k d\Pi_{p'} d\Pi_{k'} \delta^{(4)}(p + k - p' - k') |\mathcal{M}|^2
\]

\[
d\Pi_k = \frac{d^3k}{(2\pi)^3 2E_k}
\times \left[ f_1(p')f_2(k')(1 \pm f_1(p))(1 \pm f_2(k))
- f_1(p)f_2(k')(1 \pm f_1(p'))(1 \pm f_2(k')) \right]
\]

The process under discussion is graviton exchange:

\[
|\mathcal{M}|^2 = \frac{1}{2s + 1} \frac{(16\pi G)^2 m^4 M^4}{[(q^0)^2 - q^2]^2}
\]
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\]

\[
d\Pi_k = \frac{d^3k}{(2\pi)^3 2E_k} \times \left[ f_1(p')f_2(k')(1 \pm f_1(p))(1 \pm f_2(k)) \right] \quad \text{p' scatters into p} \\
\]

\[
- f_1(p)f_2(k)(1 \pm f_1(p'))(1 \pm f_2(k')) \quad \text{p scatters into p'}
\]

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The process under discussion is graviton exchange:

This analysis uses free asymptotic states: ignores back reaction, nontrivial boundary conditions…

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The collision operator:

\[
C[f_1] = \int \frac{d^3p'}{(2\pi)^3} \left[ S(p', p) f_1(p') (1 \pm f_1(p)) \
- S(p, p') f_1(p) (1 \pm f_1(p')) \right],
\]

Response function S:

\[
S(p, p') \equiv \frac{(2\pi)^4}{2E_p 2E_{p'}} \int d\Pi_k d\Pi_{k'} \delta^{(4)}(p + k - p' - k') |\mathcal{M}|^2 f_2(k) (1 \pm f_2(k'))
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The collision operator:

\[
C[f_1] = -\frac{\partial}{\partial p^i} \left[ f_1 (1 \pm f_1) D_i \right] + \frac{1}{2} \frac{\partial}{\partial p^i} \left[ \frac{\partial}{\partial p^j} (D_{ij} f_1) \pm f_1^2 \frac{\partial}{\partial p^j} D_{ij} \right]
\]

Fokker-Planck expansion in small \( q = p - p' \):

\[
D_i(p) = \int \frac{d^3q}{(2\pi)^3} q^i S(p, p + q)
\]

Diffusion coefficients:

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D_{ij}(p) = \int \frac{d^3q}{(2\pi)^3} q^i q^j S(p, p + q)
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Fokker-Planck expansion in small \( q = p - p' \)

We let type 1 have mass \( M \) and type 2 have mass \( m \). For classical background gas \( (f_2 \ll 1) \):

\[
D_i(p) = \int \frac{d^3q}{(2\pi)^3} q^i S(p, p + q) = 4\pi G^2 m^2 M^2 \left(1 + \frac{M}{m}\right) \ln \Lambda \frac{\partial}{\partial p^i} h(p; f_2)
\]

Diffusion coefficients:

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\]

Rosenbluth potentials:

\[
h(p; f) = \int \frac{d^3k}{(2\pi)^3} \frac{f(k)}{|\frac{k}{m} - \frac{p}{M}|}, \quad g(p; f) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{|\frac{k}{m} - \frac{p}{M}|} f(k)
\]
Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2).

The phase space distribution of type 1 particles follows the continuity eq.: \( \frac{df_1}{dt} = C[f_1] \).

The collision operator:

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Coulomb logarithm: \( \ln \Lambda \equiv \int_{q_{\text{min}}}^{q_{\text{max}}} \frac{dq}{q} \)
Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2). The phase space distribution of type 1 particles follows the continuity eq.: $\frac{df_1}{dt} = C[f_1]$.

The collision operator:

$$C[f_1] = -\frac{\partial}{\partial p^i} [f_1(1 \pm f_1) D_{i}] + \frac{1}{2} \frac{\partial}{\partial p^i} \left[ \frac{\partial}{\partial p^j} (D_{ij} f_1) \pm f_1^2 \frac{\partial}{\partial p^j} D_{ij} \right]$$

Back to the EOM: $\dot{V} = -\nabla \Phi + M^3 \int d^3x \int d^3\nu \nu C[f]$

For us, type 1 particles will be GCs.
Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2).

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\]

Back to the EOM:  \[
\dot{V} = - \nabla \Phi + M^3 \int d^3x \int d^3v \, v \, C[f]
\]

For us, type 1 particles will be GCs.

\[
M^3 \int d^3x \int d^3v_i \left[ - \frac{1}{M} \frac{\partial}{\partial v_j} \left( D_{j} f_1 \right) \right] = \frac{D_i}{M} = 4\pi G^2 m^2 M \left( 1 + \frac{M}{m} \right) \ln \Lambda \frac{\partial h}{\partial p_i} \quad \text{(mean momentum drift } \langle \Delta v_i \rangle / \Delta t) \]

\[
M^3 \int d^3x \int d^3v_i \left[ \frac{1}{2M^2} \frac{\partial^2}{\partial v_j \partial v_k} \left( D_{jk} f_1 \right) \right] = 0 \quad \text{(would produce mean energy / dispersion drift } \langle \Delta v_i \Delta v_j \rangle / \Delta t) \]
Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2).

The phase space distribution of type 1 particles follows the continuity eq.: \[ \frac{df_1}{dt} = C[f_1]. \]

The collision operator:

\[ C[f_1] = - \frac{\partial}{\partial p^i} [f_1 (1 \pm f_1) D_i] + \frac{1}{2} \frac{\partial}{\partial p^i} \left[ \frac{\partial}{\partial p^j} (D_{ij} f_1) \pm f_1 \frac{\partial}{\partial p^j} D_{ij} \right] \]

Back to the EOM: \[ \dot{V} = - \nabla \Phi + M^3 \int d^3x \int d^3v \mathbf{v} C[f] \]

For us, type 1 particles will be GCs (M>>m).

\[ M^3 \int d^3x \int d^3v \mathbf{v}_i \left[ - \frac{1}{M} \frac{\partial}{\partial v_j} \left( D_j f_1 \right) \right] = - 16\pi^2 G^2 \rho M \ln \Lambda \frac{V_i}{V^3} C_{df}(V), \quad C_{df}(V) = \frac{\int_0^V d^3v f_2(v')}{\int_0^\infty d^3v f_2(v')} \]
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The collision operator:
\[
C[f_1] = -\frac{\partial}{\partial p^i} \left[ f_1 (1 \pm f_1) D_i \right] + \frac{1}{2} \frac{\partial}{\partial p^i} \left[ \frac{\partial}{\partial p^j} (D_{ij} f_1) \pm f_1^2 \frac{\partial}{\partial p^j} D_{ij} \right]
\]

Back to the EOM:
\[
\dot{V} = -\nabla \Phi - \frac{1}{\tau} V
\]

For us, type 1 particles will be GCs ($M \gg m$).

\[
M^3 \int d^3x \int d^3v \left[ -\frac{1}{M} \frac{\partial}{\partial v_j} \left( D_j f_1 \right) \right] = -16\pi^2 G^2 \rho M \ln \Lambda \frac{V_i}{V^3} C_{df}(V), \quad C_{df}(V) = \frac{\int_0^V d^3v' f_2(v')}{\int_0^\infty d^3v' f_2(v')}
\]
Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2).

The phase space distribution of type 1 particles follows the continuity eq.: \( \frac{df_1}{dt} = C[f_1] \).

The collision operator:
\[
C[f_1] = - \frac{\partial}{\partial p^i} [f_1 (1 \pm f_1) D_i] + \frac{1}{2} \frac{\partial}{\partial p^i} \left[ \frac{\partial}{\partial p^j} (D_{ij} f_1) \pm f_1^2 \frac{\partial}{\partial p^j} D_{ij} \right]
\]

Back to the EOM: \( \dot{V} = - \nabla \Phi - \frac{1}{\tau} V \)

For us, type 1 particles will be GCs (M>>m).

\[
M^3 \int d^3x \int d^3v_v_i \left[ -\frac{1}{M} \frac{\partial}{\partial v_j} \left( D_j f_1 \right) \right] = - 16\pi^2 G^2 \rho M \ln \Lambda \frac{V_i}{V^3} C_{df}(V), \quad C_{df}(V) = \frac{\int_0^V d^3v f_2(v')}{\int_0^\infty d^3v f_2(v')}
\]

Chandrasekhar 1943
Basic physics view of dynamical friction:

$$\dot{V} = -\nabla \Phi - \frac{1}{\tau} V$$

![Dynamical friction simulation](image)

**Burkert**

- 5K, Rmx=1kpc (1)
- 5K, Rmx=1kpc (2)
- 5K, Rmx=1kpc (3)
- 20K, Rmx=1kpc (1)
- 20K, Rmx=1kpc (2)
- Semianalytic
Basic physics view of dynamical friction:

\[
\dot{V} = - \nabla \Phi - \frac{1}{\tau} V, \quad \tau = \frac{V^3}{16 \pi^2 G^2 \rho M C_{\text{df}} \ln \Lambda} \approx \frac{2.7 \text{ Gyr}}{C_{\text{df}} \ln \Lambda} \left( \frac{V}{20 \text{ km/s}} \right)^3 \left( \frac{10^7 M_\odot/\text{kpc}^3}{\rho} \right) \left( \frac{10^5 M_\odot}{M} \right)
\]
Basic physics view of dynamical friction:

\[ \dot{V} = - \nabla \Phi - \frac{1}{\tau} V, \quad \tau = \frac{V^3}{16\pi^2 G^2 \rho M C_{df} \ln \Lambda} \approx \frac{2.7 \text{ Gyr}}{C_{df} \ln \Lambda} \left( \frac{V}{20 \text{ km/s}} \right)^3 \left( \frac{10^7 \text{ M}_\odot/\text{kpc}^3}{\rho} \right) \left( \frac{10^5 \text{ M}_\odot}{M} \right) \]

Lack of nuclear star cluster in Fornax?

Need more statistics…

Would like to see the effect in action in dark matter-dominated galaxies

Tremaine 1976,
Oh, Lin, Richer 2000,
Petts, Gualandris, Read 2015,
Hui et al 2017, Lancaster et al 2019,
Meadows et al 2020, Bar et al 2021,
Shao et al 2021,…
DF in dwarf elliptical galaxies, HST survey. Many of these dEs are dark matter-dominated. Stacked GC radial distribution, 51 dEs in Virgo and Fornax clusters. 0-20 GCs/galaxy. Did not analyze luminosities beyond isolating most luminous and subtracting NSC.

(1) assumed GCs on circular orbits, (2) assumed GCs started on same distribution as stars, (3) deduced velocity dispersion from V magnitude, (4) scaled all radii to host 1/2 light radius, (5) defined GC joining NSC if (circular orbit) R reached 0 following analytical DF formula.

Noted deficit in high lumi GCs in inner region: fall into NSC? (Fig.5) Difficulty: faint dEs predicted to acquire more luminous NSCs than observed (Fig.8).

…Where are the missing GCs of Fig.5…?

…Similar to Fornax dSph?
modak, danieli, greene 2022
bar, danieli, KB 2022,
danieli et al 2022,
muller et al 2020, 2021,
forbes et al 2019, 2020,
What can we learn from radial profile of GCs?

EOM in radial coordinate:

\[
\begin{align*}
\left( r v_\varphi \right) &= -\frac{1}{\tau} \left( r v_\varphi \right) \quad (\varphi \text{ equation with } v_\varphi = r \dot{\varphi}) \\
v_\varphi^2 - v_{\text{circ}}^2 &= r \left( \ddot{r} + \frac{\dot{r}}{\tau} \right) \quad (r \text{ equation})
\end{align*}
\]

Consider: — circular orbit, — effect is slow, \( v_{\text{circ}} \gg r/\tau \)

\[
\rightarrow \quad v_\varphi \approx v_{\text{circ}} = \sqrt{GM(r)/r} \quad \rightarrow \quad \frac{\dot{r}}{r} \approx -\frac{2}{\left( 1 + \frac{d \ln M}{d \ln r} \right) \tau}
\]
\[
\frac{\dot{r}}{r} \approx -\frac{2}{\left(1 + \frac{d\ln M}{d\ln r}\right) \tau}
\]

This can be solved:

Define \( \alpha(r) = \frac{d\ln M(r)}{d\ln r} \), then \( \Delta t(r; r_0) \approx \int_{r}^{r_0} \frac{dr'}{2r'} \left(1 + \alpha(r')\right) \tau \left(r', v_{\text{circ}}(r')\right) \).
\[
\frac{\dot{r}}{r} \approx -\frac{2}{\left(1 + \frac{d \ln M}{d \ln \rho}\right) \tau}
\]

This can be solved:
Define \( \alpha(r) = \frac{d \ln M(r)}{d \ln r} \), then \( \Delta t(r; r_0) \approx \int_r^{r_0} \frac{dr'}{2r'} \left(1 + \alpha(r')\right) \tau \left(r', v_{\text{circ}}(r')\right) \)

Suppose that the halo is cored, \( \alpha \approx 3, \quad \tau \approx \text{Const} \)
then: \( \Delta t \approx \frac{1 + \alpha}{2} \tau \ln \frac{r_0}{r} \approx 2 \tau \ln \frac{r_0}{r}, \quad \text{or} \quad r_0 = e^{\frac{\Delta t}{2\tau}} r \)

A radial CDF today, \( N_{\Delta t}(r) \),
relates to an initial CDF via \( N_{\Delta t}(r) \approx N_0 \left(r e^{\frac{\Delta t}{2\tau}}\right) \)
\[ \frac{\dot{r}}{r} \approx -\frac{2}{\left(1 + \frac{d \ln M}{d \ln r}\right) \tau} \]

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A radial CDF today, \( N_{\Delta r}(r) \),
relates to an initial CDF via \( N_{\Delta r}(r) \approx N_0 \left( r e^{\frac{\Delta t}{2\tau}} \right) \)

\[ \langle r_{\perp} \rangle_{\Delta t} = \int \frac{d\Omega}{4\pi} \sin \theta \int dr \left( \frac{d}{dr} N_{\Delta r}(r) \right) \]
\[ \frac{\dot{r}}{r} \approx -\frac{2}{\left(1 + \frac{d \ln M}{d \ln r}\right) \tau} \]

This can be solved:
Define \( \alpha(r) = \frac{d \ln M(r)}{d \ln r} \), then \( \Delta t(r; r_0) \approx \int_{r_0}^{r} \frac{dr'}{2r'} \left(1 + \alpha(r')\right) \tau \left(r', v_{\text{circ}}(r')\right) \)

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\[ \ln \frac{\langle r_{\perp} \rangle}{\langle r_{\perp} \rangle_0} \approx -\frac{\Delta t}{2\tau} \frac{m_{\text{GC}}}{\bar{m}_{\text{GC}}} \]
Evidence for dynamical friction in dark matter-dominated, GC-rich ultra diffuse galaxy?

Bar, Danieli, KB 2202.10179

$$\ln \frac{\langle r_\perp \rangle}{\langle r_\perp \rangle_0} \approx -\frac{\Delta t}{2\bar{\tau}} \frac{m_{GC}}{\bar{m}_{GC}}$$

Related:
Saifollahi et al 2201.11750, Dutta Chowdhury, van den Bosch, van Dokkum 2019, 2020, Modak, Danieli, Greene 2211.01384
Evidence for dynamical friction in dark matter-dominated, GC-rich ultra diffuse galaxy?

Bar, Danieli, KB 2202.10179

Dynamical friction in a massive dark matter halo naturally produces observed mass segregation.

Lack of dark matter, or a low mass halo, comes with small velocity dispersion, and overshoots friction.

Consistent with, and independent of stellar and GC kinematics (Forbes et al 2021).
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Consistent with, and independent of stellar and GC kinematics (Forbes et al 2021).
II. PHENOMENOLOGY

Removing of GCs that were not spectroscopically confirmed still yields a large total GC mass of \( \sim 10^6 \text{M}_\odot \) for the left bins. Errors on mass are estimated either by bin size or standard deviation of the masses in the bin. Different binning yield slightly different results, though retaining a similar decreasing trend. Using only spectroscopy is inconclusive yet in determining the dynamics of the galaxy, yielding a line-of-sight velocity dispersion \( \sigma \). It is noteworthy that most of the brighter GCs in the right panel of Fig. 1 are concentrated in the region close to the center of the galaxy, resulting in mass segregation. This would naively be expected to hold over an intermediate range of \( r \), and in fact, the simple picture can be expected to hold over an intermediate range of \( r \).

The right panel of Fig. 2 shows all compact sources that were not confirmed as GCs. Different binning yield slightly different results, though retaining a similar decreasing trend. Using only spectroscopy is inconclusive yet in determining the dynamics of the galaxy, yielding a line-of-sight velocity dispersion \( \sigma \). It is noteworthy that most of the brighter GCs in the right panel of Fig. 1 are concentrated in the region close to the center of the galaxy, resulting in mass segregation. This would naively be expected to hold over an intermediate range of \( r \), and in fact, the simple picture can be expected to hold over an intermediate range of \( r \).

In this paper we show that this explanation can be relatively-reliably converted to mass (perhaps to an overall fudge factor), the projected distance is an unimportant parameter.

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In this paper we show that this explanation can be relatively-reliably converted to mass (perhaps to an overall fudge factor), the projected distance is an unimportant parameter.
Many more UDGs/dwarfs to investigate.

e.g Saifollahi et al, 2201.11750: Coma cluster UDGs
Examples how it could become very interesting — ultralight dark matter
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It was suggested that Milky Way dwarf satellite galaxies may point to \( m \sim 10^{-22} \) eV

Examples how it could become very interesting — ultralight dark matter

The collision operator:

\[
C[f_1] = \int \frac{d^3p'}{(2\pi)^3} \left[ S(p', p)f_1(p')(1 \pm f_1(p)) - S(p, p')f_1(p)(1 \pm f_1(p')) \right],
\]

Transfer function S:

\[
S(p, p') \equiv \frac{(2\pi)^4}{2E_p 2E_{p'}} \int d\Pi_k d\Pi_{k'} \delta^{(4)}(p + k - p' - k') \sqrt{M}^2 f_2(k)(1 + f_2(k'))
\]

\[
D_i(p) = \frac{4\pi G^2 m^2 M^3}{\mu_r} \ln \Lambda \frac{\partial}{\partial p^i} \left[ h(p; f_2) + \frac{\mu_r}{M} h(p; f_2^2) \right]
\]

\[
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Examples how it could become very interesting — ultralight dark matter

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Transfer function \( S \):

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Dynamical heating
Examples how it could become very interesting — **ultralight dark matter**

**Effective quasi-particles**
(Bar-Or, Fouvry, Tremaine 1809.07673)

\[
m_{\text{eff}} = \frac{\pi^{3/2} \hbar^3 \rho_b}{m_b^3 \sigma^3} = \rho_b \left( f \lambda_\sigma \right)^3.
\]

\[
\lambda_\sigma = \frac{\hbar}{(m_b \sigma)}
\]

\[
f = \frac{1}{2\sqrt{\pi}} = 0.282.
\]

Dynamical *heating*
Examples how it could become very interesting — **ultralight dark matter**

Dalal, Kravstov 2203.05750:
would have dispersed star cluster in Segue-I?

\[ m_{\text{eff}} \approx 430 \, M_\odot \left( \frac{10 \, \text{km/s}}{\sigma} \right)^3 \left( \frac{\rho}{10^7 \, M_\odot/\text{kpc}^3} \right)^3 \left( \frac{10^{-20} \, \text{eV}}{m} \right)^3 \]

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**Dynamical heating**
Examples how it could become very interesting — ultralight dark matter

Amusing fact:
Dynamical heating constraints on ultralight dark matter come from the same mechanism that constrains MACHO or PBH dark matter.
Examples how it could become very interesting — ultralight dark matter

**Amusing fact:**
Dynamical heating constraints on ultralight dark matter come from the same mechanism that constrains MACHO or PBH dark matter

**Hypothesis:**
If you go far enough in the extreme right, you end up in the extreme left.
Examples how it could become very interesting — **ultralight dark matter**

Dalal, Kravstov 2203.05750: would have dispersed star cluster in Segue-I?

\[ m_{\text{eff}} \approx 430 \, M_\odot \left( \frac{10 \, \text{km/s}}{\sigma} \right)^3 \left( \frac{\rho}{10^7 \, M_\odot / \text{kpc}^3} \right) \left( \frac{10^{-20} \, \text{eV}}{m} \right)^3 \]

\[ \lambda \approx 120 \, \text{pc} \left( \frac{10^{-20} \, \text{eV}}{m} \right) \left( \frac{10 \, \text{km/s}}{\sigma} \right) \]

**Effective quasi-particles**
(Bar-Or, Fouvry, Tremaine 1809.07673)

\[ m_{\text{eff}} = \frac{\pi^{3/2} \hbar^3 \rho_b}{m_b^3 \sigma^3} = \rho_b \left( f \lambda_\sigma \right)^3 \]

\[ \lambda_\sigma = \hbar / (m_b \sigma) \]

\[ f = 1 / (2\sqrt{\pi}) = 0.282. \]
Examples how it could become very interesting — ultralight dark matter

If the system is entirely inside the coherent region (the soliton), dynamical friction is suppressed

Hui et al, 1610.08297; Bar-Or, Fouvry, Tremaine, 1809.07673; 2010.10212

Lancaster et al, 1909.06381

Proposed for Fornax GC timing puzzle (Hui et al 2016).
Examples how it could become very interesting — ultralight dark matter

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Lancaster et al, 1909.06381

Proposed for Fornax GC timing puzzle (Hui et al 2016). But only works for $m < 10^{-21}$ eV (Lancaster et al 2019) in tension w/ LSBGs, Ly-$\alpha$ which suggest $m > 10^{-21}$ eV
Examples how it could become very interesting — light fermion dark matter
Examples how it could become very interesting — light fermion dark matter

It was suggested that Milky Way dwarf satellite galaxies may point to degenerate fermion dark matter with $m \sim 200$ eV

Domcke, Urbano, 1409.3167
Randall, Scholtz, Unwin, 1611.04590
Examples how it could become very interesting — light fermion dark matter

The collision operator:
\[
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- S(p, p')f_1(p)(1 \pm f_1(p')) \right],
\]

Transfer function S:
\[
S(p, p') \equiv \frac{(2\pi)^4}{2E_p 2E_{p'}} \int d\Pi_k d\Pi_{k'} \delta^{(4)}(p + k - p' - k')|\mathcal{M}|^2 f_2(k)(1 - f_2(k'))
\]

Reddy, Prakash, Lattimer, astro-ph/9710115
Bertoni, Nelson, Reddy, 1309.1721
Bar et al, 2102.11522

\[
C_{df} \rightarrow \frac{V^3}{\sqrt{3} \sqrt{\pi \sigma^3}}
\]

instead of the classical gas result:  \[
C_{df} \rightarrow \frac{\sqrt{2} V^3}{3 \sqrt{\pi \sigma^3}}
\]
Examples how it could become very interesting — light fermion dark matter (degenerate dark matter — DDM)

Bar et al, 2102.11522

DDM must be hot at high redshift due to unavoidable degeneracy pressure.

The minimal possible velocity dispersion can be compared with “standard” hot dark matter.

Ly-α limit $m > 2.96$ eV (Baur et al, 1512.01981) rules out dwarf galaxy cores as proposed in Domcke, Urbano, 1409.3167; Randall, Scholtz, Unwin, 1611.04590
Examples how it could become very interesting — light fermion dark matter (degenerate dark matter — DDM)

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Gravity alone

$m_X$

$10^{-21}$ eV  $10^{-10}$ $M_\odot \sim 10^{56}$ eV

Thank you!
What we hope to learn
part of the analysis, we also initiate the GC mass function (GCMF) by copying the current observed GCMF. (We explore changes to this set-up further below).

Histogram plots in Fig. 2 show projected cumulative luminosity profiles, normalized to the total GC luminosity at \( R = 2 R_e \) and presented vs. projected radial distance normalized to \( R_e \), where \( R_e = 1.9 \) kpc is the S´ersic radius of the observed stellar distribution of the galaxy. Cyan, magenta, and green curves show distributions computed using the semianalytic method, N-body with 5K halo particles, and N-body with 10K halo particles, respectively. Thick black curve shows the observed GC distribution. The three histogram panels correspond to an NFW DM halo model, a Burkert DM halo model, and a model lacking DM altogether. In all three cases, the contribution of stars to the halo total density profile is included based on the observed light profile with stellar \( M/L \) = 2 in solar units. The bottom-right panel shows the line of sight velocity dispersion (LOSVD) corresponding to the three halo models, compared with the measurement.

Our next step is to explore the dependence on the DM halo. Figs. 3 and 4 present GC luminosity histograms calculated for different NFW and Burkert halo model parameters. Here, we still hold the GC initial spatial distribution to match the current observed distribution of stars, and the GCMF to match the current observed GCMF.

The problem of initial conditions
For approximately circular orbit:

\[
\frac{\dot{r}}{r} \approx \frac{2}{\left(1 + \frac{d \ln M}{d \ln r}\right)} \tau
\]

\[\alpha(r) = \frac{d \ln M(r)}{d \ln r}\]

\[
\Delta t = \int_r^{r_0} \frac{dr'}{2r'} \left(1 + \alpha(r')\right) \tau(r')
\]

For approximately power law density profile:
(e.g. NFW $\beta \approx 2$, $\alpha \approx 2$)

\[
\tau = \bar{\tau} \left(\frac{r}{\bar{r}}\right)^\beta
\]

There is a critical radius (kill circle):

\[
r_{cr} = \bar{r} \left(\frac{2 \beta \Delta t}{1 + \alpha \bar{\tau}}\right)^{1/\beta}
\]

GCs that start at $r<r_{cr}$ at $t=0$, arrive at $r=0$ by $t=\Delta t$
Should we expect to see GCs inside the kill circle?

We may, but not many: GCs that are now inside critical radius (but not in nuclear cluster) come from a small sliver of space:

\[
\begin{align*}
    r_0(r; \Delta t) &= r_{\text{cr}} \left( 1 + \left( \frac{r}{r_{\text{cr}}} \right)^\beta \right)^{1/\beta} \\
    &= r_{\text{cr}} \left( 1 + \frac{1}{2\beta^2} \frac{\alpha \tau(r)}{\Delta t} + \cdots \right)
\end{align*}
\]

Cumulative count of GCs (CDF):

\[
N_{\Delta t}(r) \approx N_0(r_{\text{cr}}) + \frac{1}{2\beta^2} N'_0(r_{\text{cr}}) r_{\text{cr}} \frac{\tau(r)}{\Delta t} + \cdots
\]

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\[ r_0(r; \Delta t) = r_{cr} \left( 1 + \left( \frac{r}{r_{cr}} \right)^\beta \right)^{1/\beta} = r_{cr} \left( 1 + \frac{1 + \alpha \tau(r)}{2\beta^2} \frac{\Delta t}{r_{cr}} + \cdots \right) \]

Cumulative count of GCs (CDF):

\[ N_{\Delta t}(r) \approx N_0(r_{cr}) + \frac{1 + \alpha}{2\beta^2} N_0'(r_{cr}) r_{cr} \tau(r) \frac{\Delta t}{r_{cr}} + \cdots \]

Nuclear cluster?!
Fornax does not seem to have one…

Tremaine 1976,
Oh, Lin, Richer 2000,
Petts, Gualandris, Read 2015,
Hui et al 2017, Lancaster et al 2019,
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\[ = r_{cr} \left( 1 + \frac{1 + \alpha \tau(r)}{2\beta^2 \Delta t} + \cdots \right) \]

Cumulative count of GCs (CDF):

\[ N_{\Delta t}(r) \approx N_0(r_{cr}) + \frac{1 + \alpha \tau(r)}{2\beta^2} r_{cr} \frac{\tau(r)}{\Delta t} + \cdots \]

Nuclear cluster?!
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Shao et al 2021,…
Lack of nuclear star cluster in Fornax?

Bar et al (Shao et al) find mild (~null) statistical timing problem, ignoring the question of NSC.

Bar et al (Shao et al) find ~50% (~30%) GCs “tidally disrupt” —> arrive at r~0.

In either case, we might have expected an NSC of ~$10^6$ solar mass…

Tremaine 1976, 
Oh, Lin, Richer 2000, 
Petts, Gualandris, Read 2015, 
Hui et al 2017, Lancaster et al 2019, 
Meadows et al 2020, Bar et al 2021, 
Shao et al 2021,…