XXXIII Canary Islands Winter School of Astrophysics

Fundamental Physics with Galaxies (lecture III)

Kfir Blum | Weizmann Institute of Science

Gravity alone

m_X

10^{-21} eV

$10^{-10}\ \mathrm{M}_\odot \sim 10^{56}$ eV

Beyond mean-field effects

Gravity alone

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10^{-21} eV — $10^{-10}\ M_\odot \sim 10^{56}$ eV

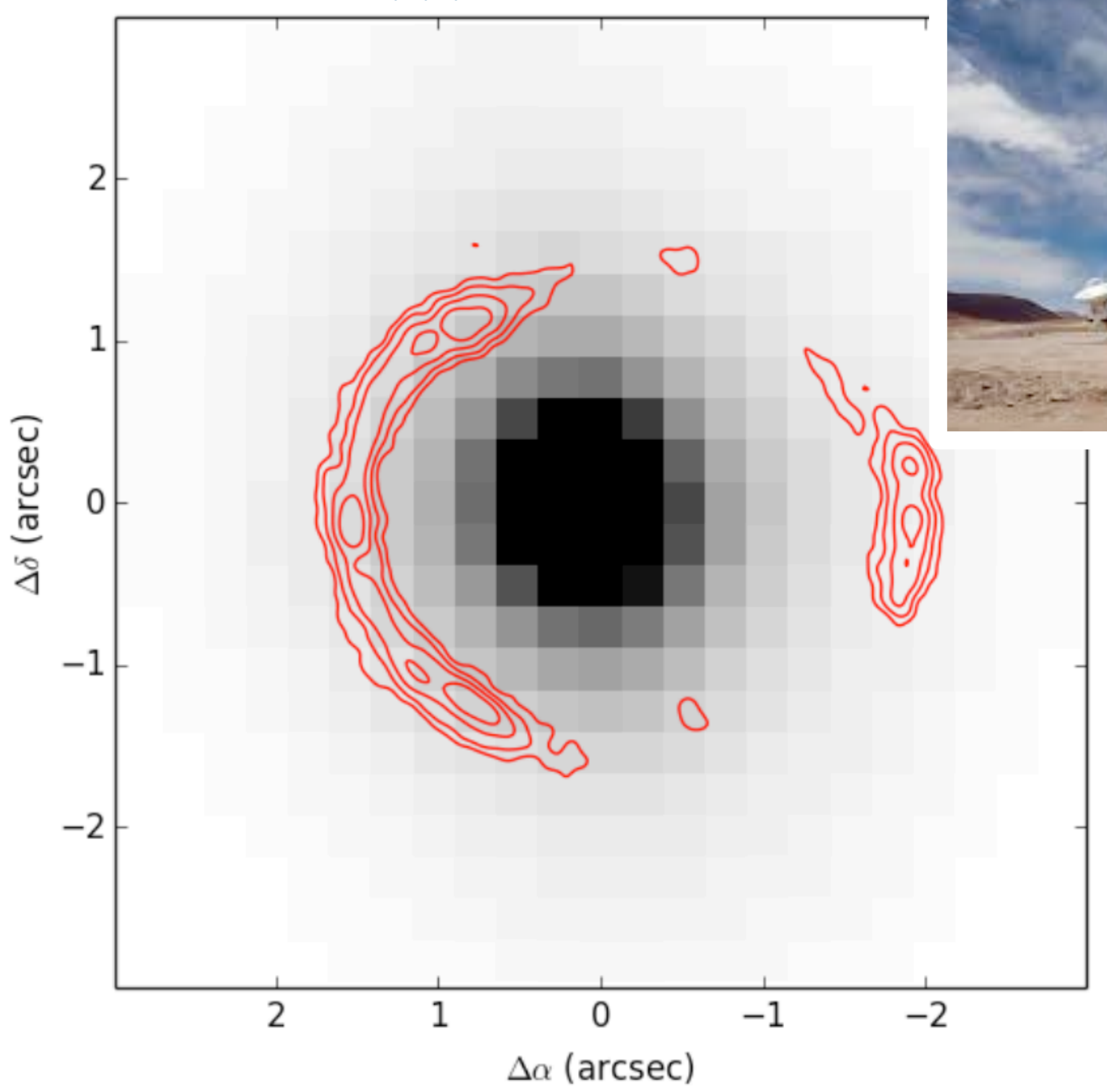
Beyond mean-field effects

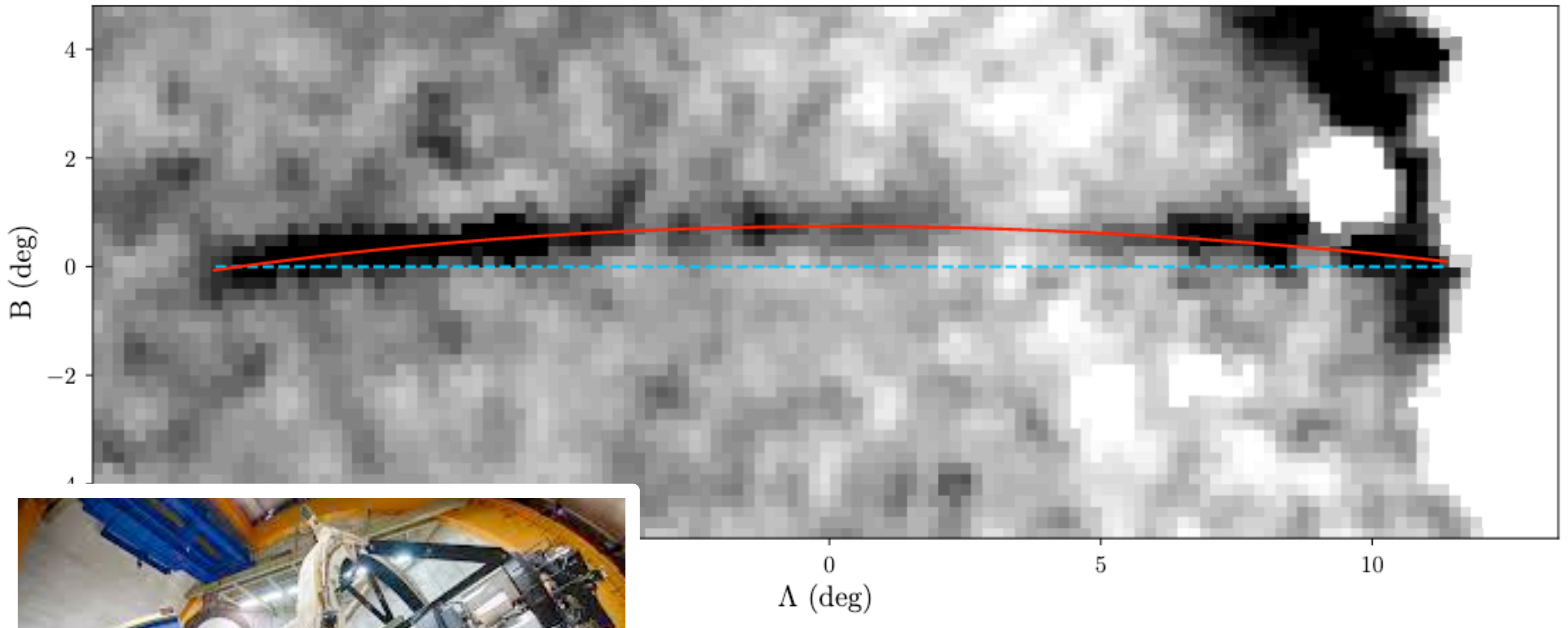
A whole field of research that I will *not* discuss now: dark matter substructure

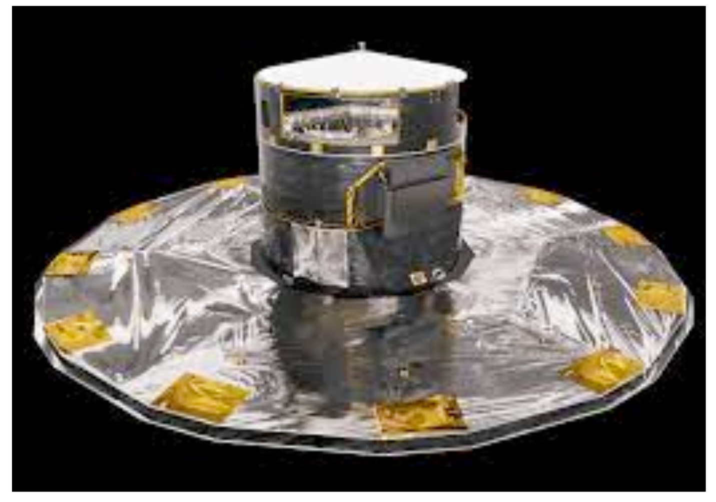
Dalal & Kochaneck astro-ph/0111456,
Vegetti & Vogelsberger 1406.1170,
Hezaveh et al 1601.01388,
Minor et al 2011.19627,
…

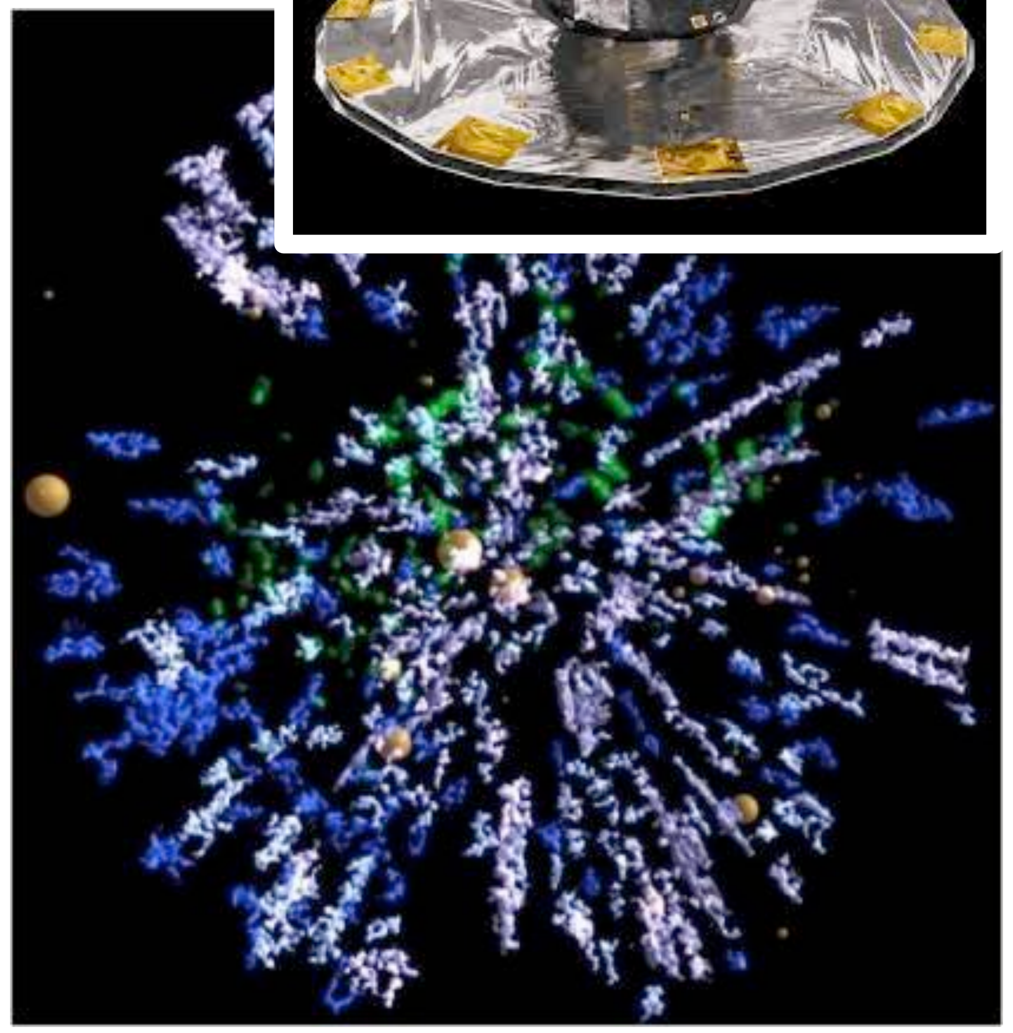
Bovy 1512.00452,
Banik et al 1911.02663,
…

Nacib, Lisanti, Belokurov 1807.02519,
Ravi et al 1812.07578,
…

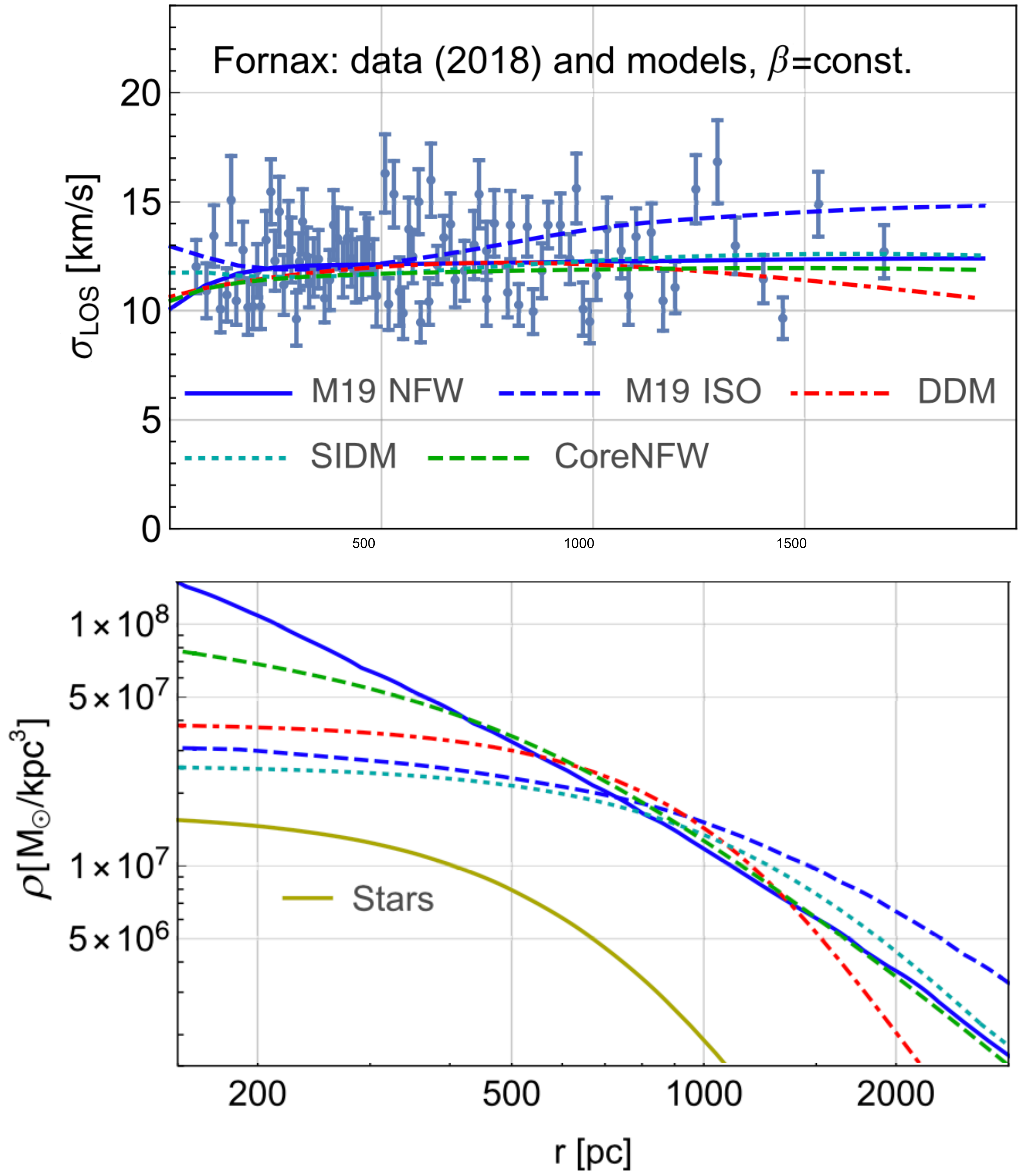






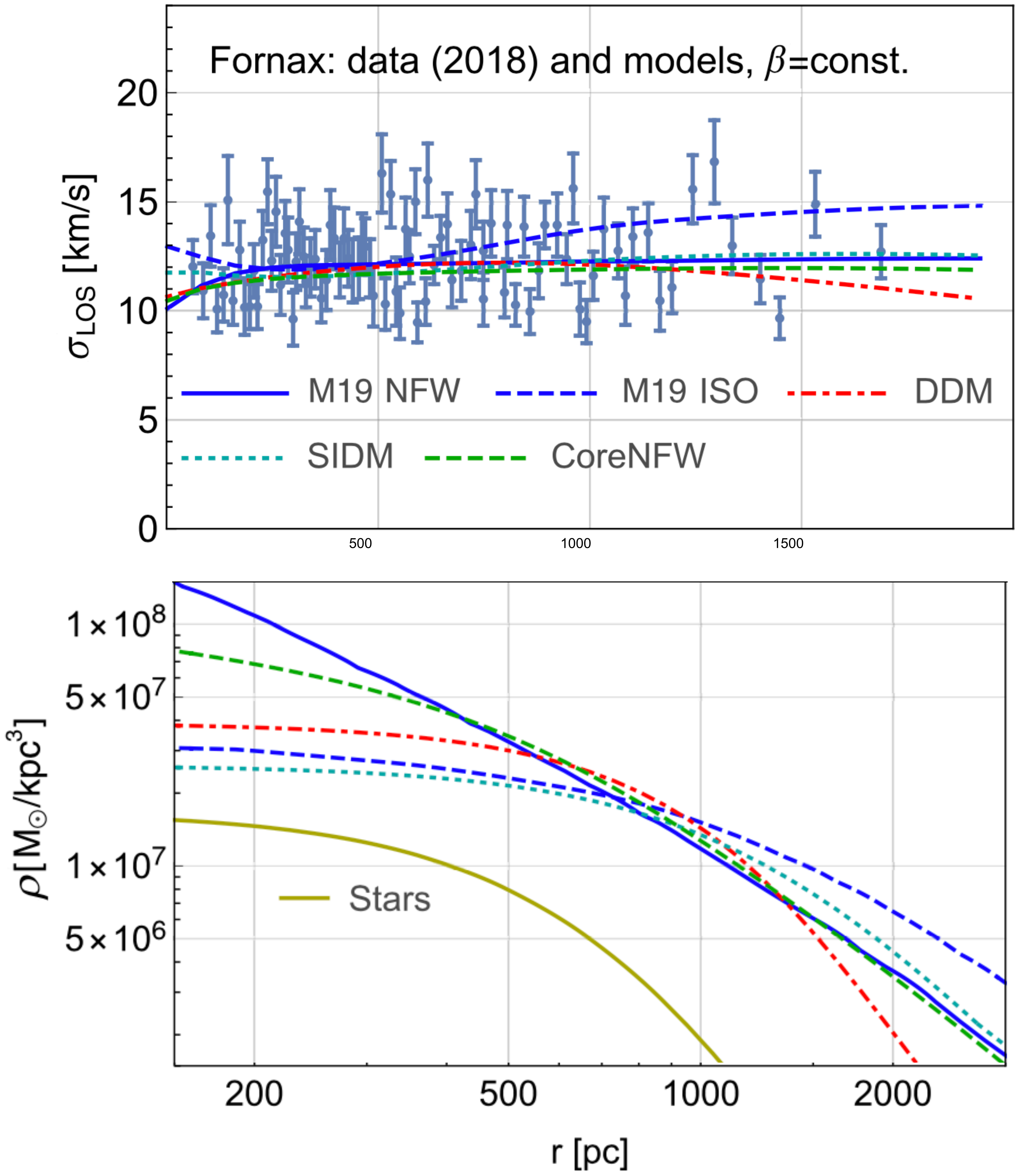


Beyond mean-field effects



Fornax dwarf galaxy

Beyond mean-field effects



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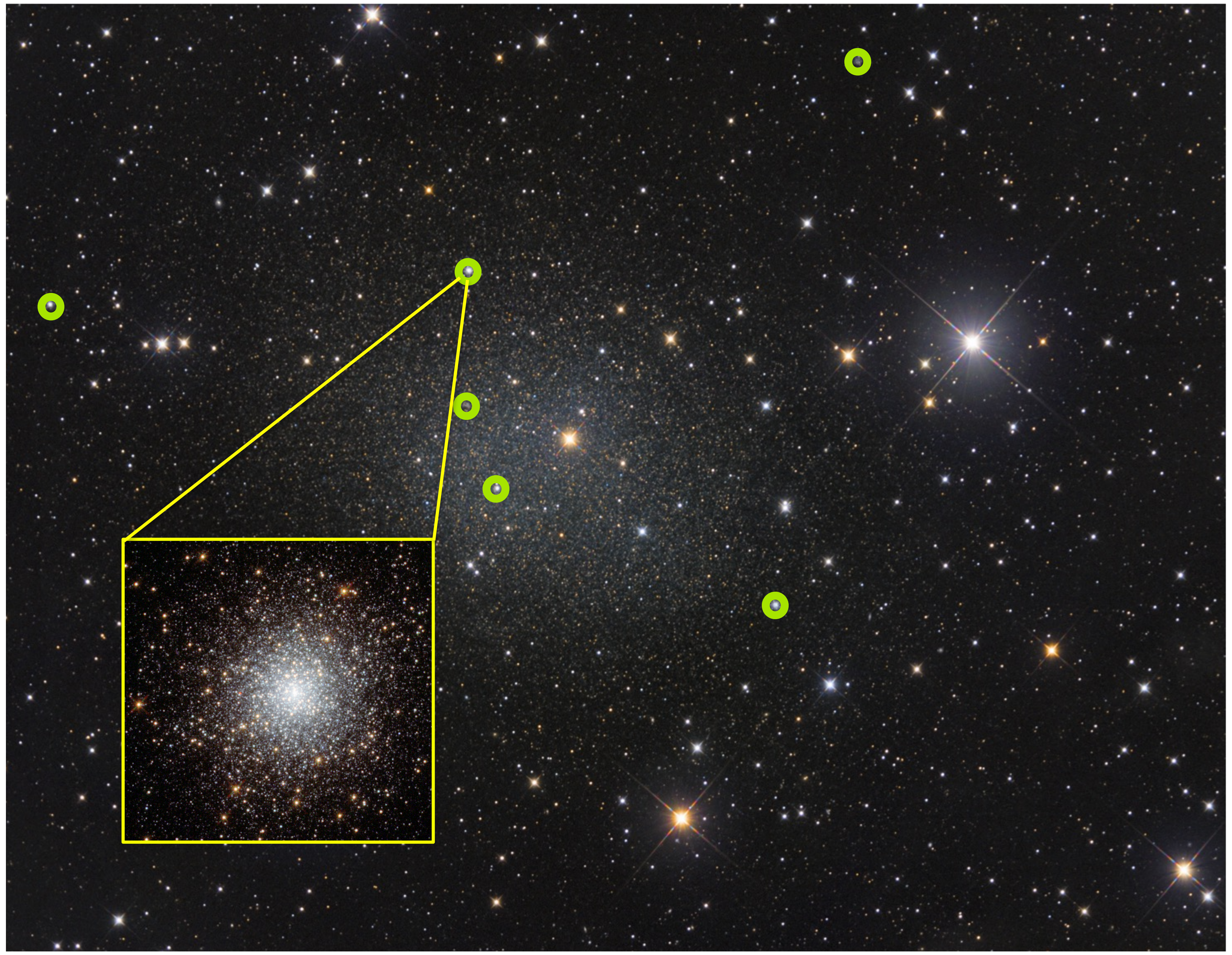


Globular Clusters (GC3: ~5×10^5 $M_\odot$)

Beyond mean-field effects

People also ask ⋮

What is the weight of 1 apple?

An apple's weight depends on the variety and the size of the fruit. On average, an apple weighs **between 150 g and 250 g**.

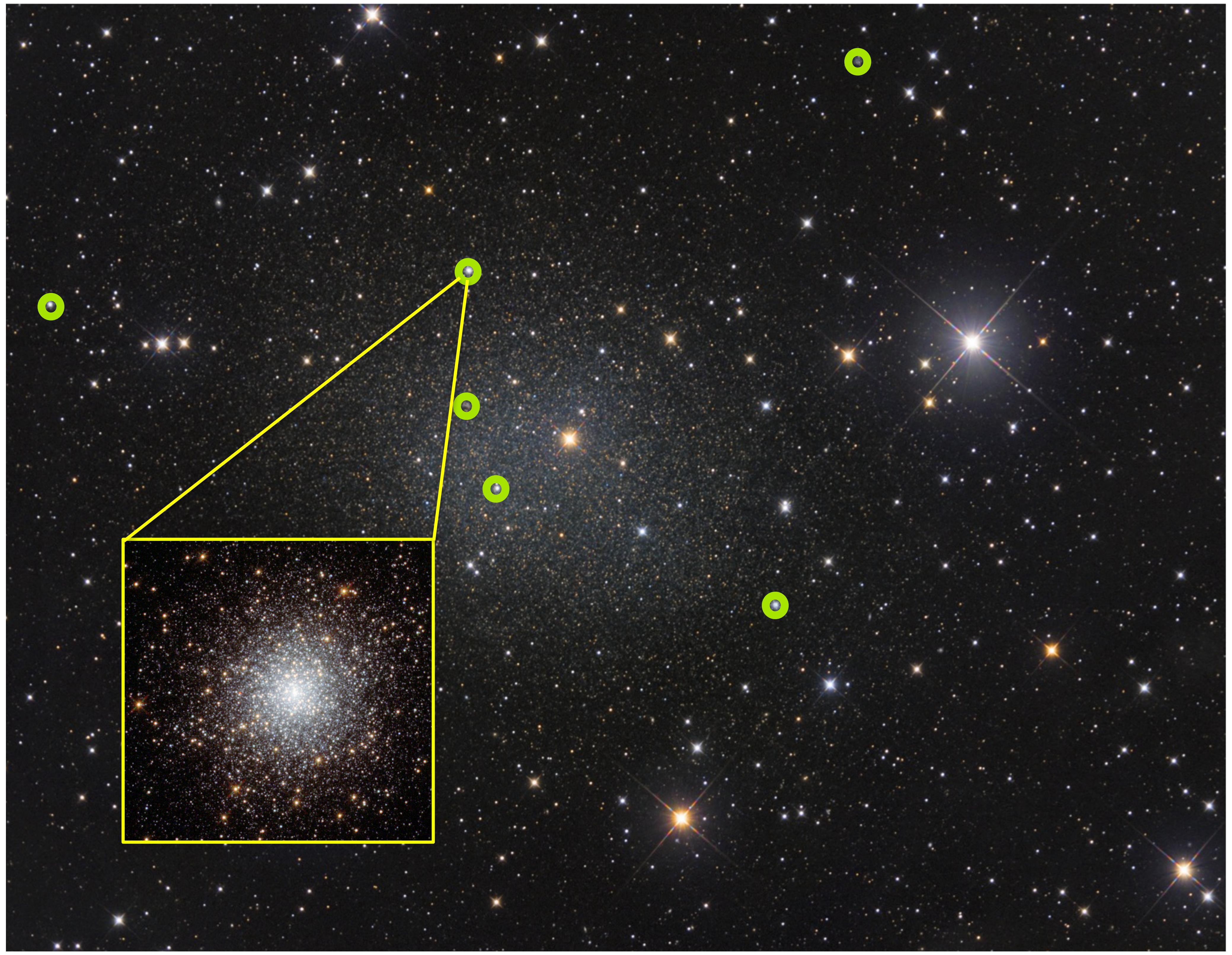
People also ask ⋮

What's heavier a rhino or a hippo?

What is the heaviest rhino on record?

The Indian Rhino is from 3–4 metres (10 – 14 feet) long. The record-sized specimen of this rhino was approximately **3,800 kg (8,377 lb)**. The Indian Rhino has a single horn that reaches a length of between 20 and 100 cm (8 – 39 inches).

Fornax dwarf galaxy



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```
>> 3800e3/150

ans =

   2.5333e+04
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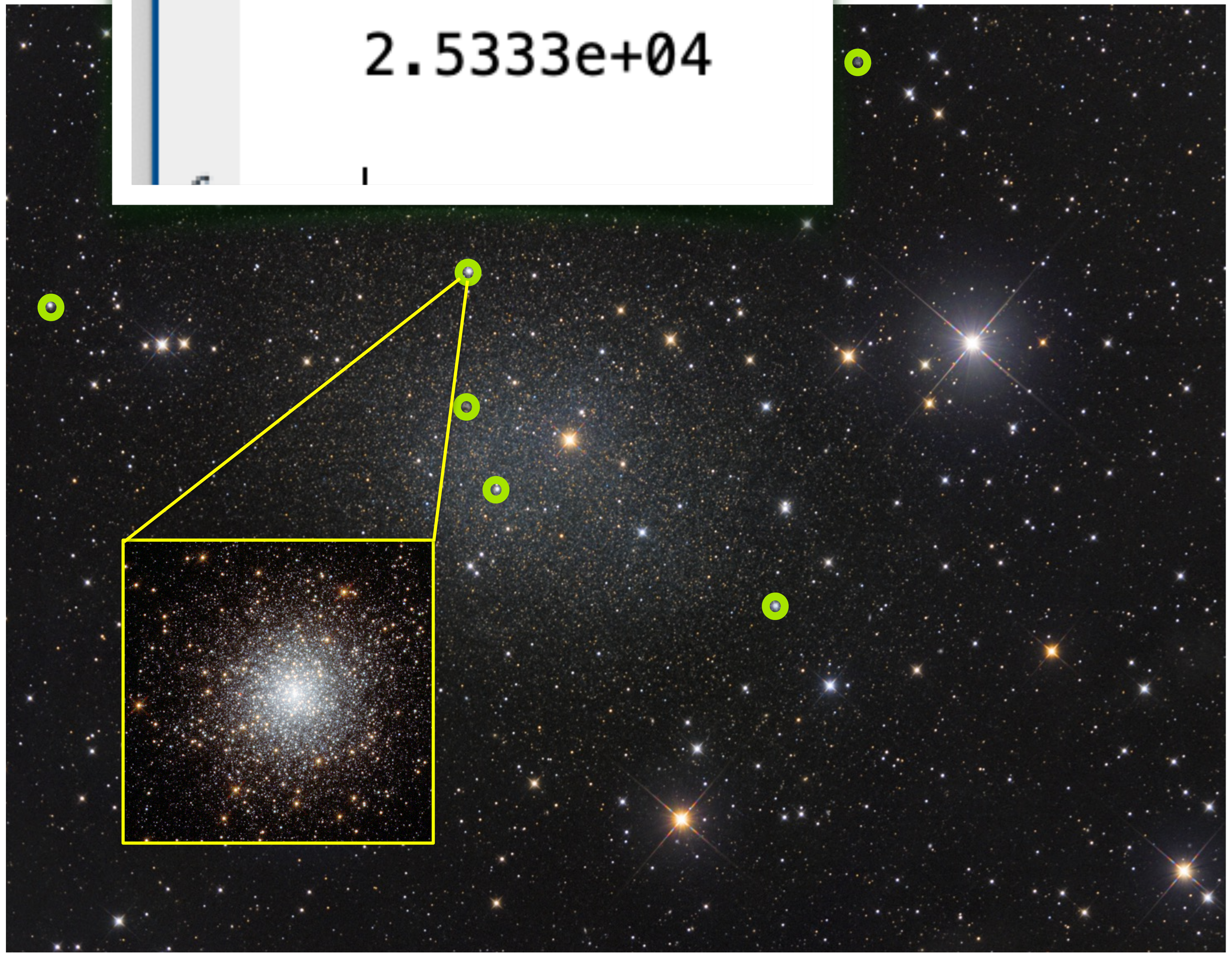
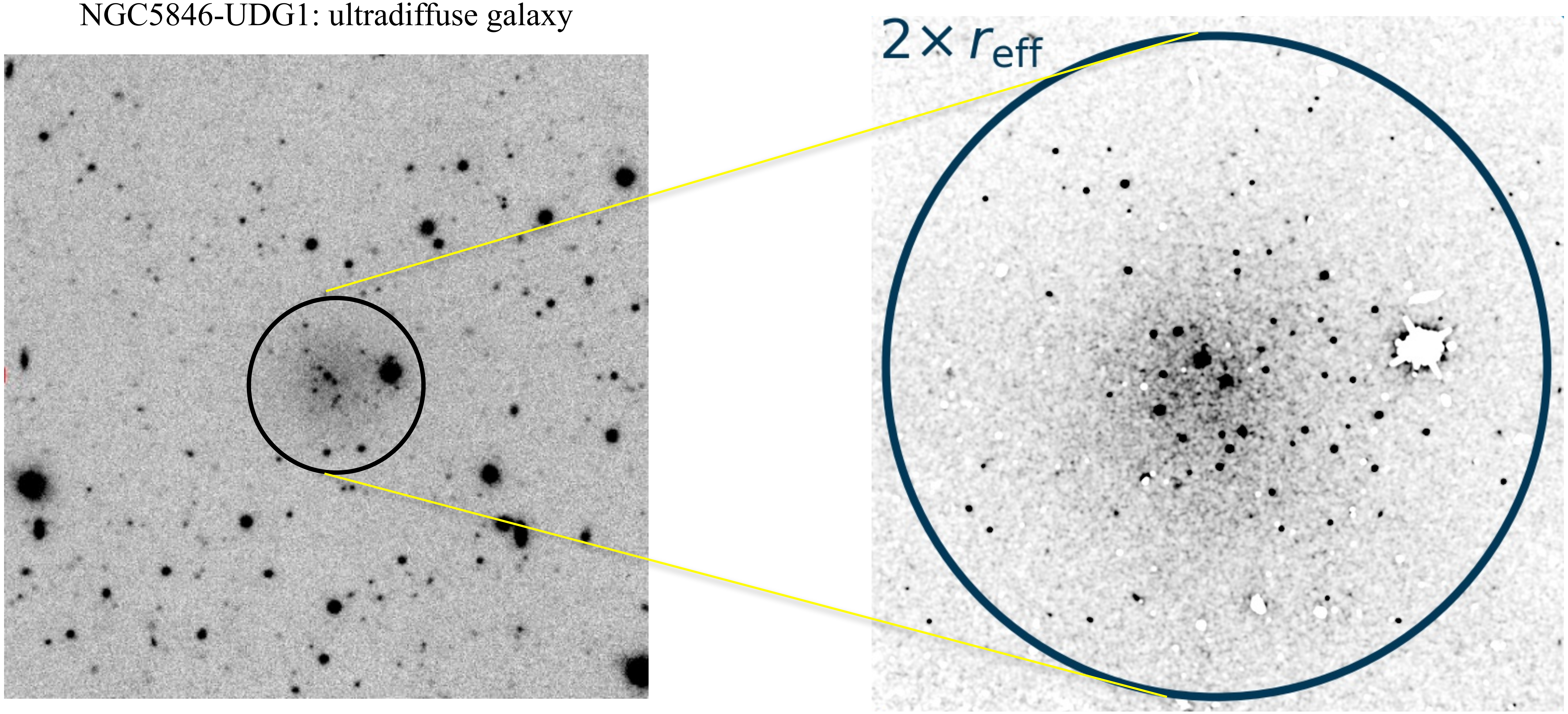
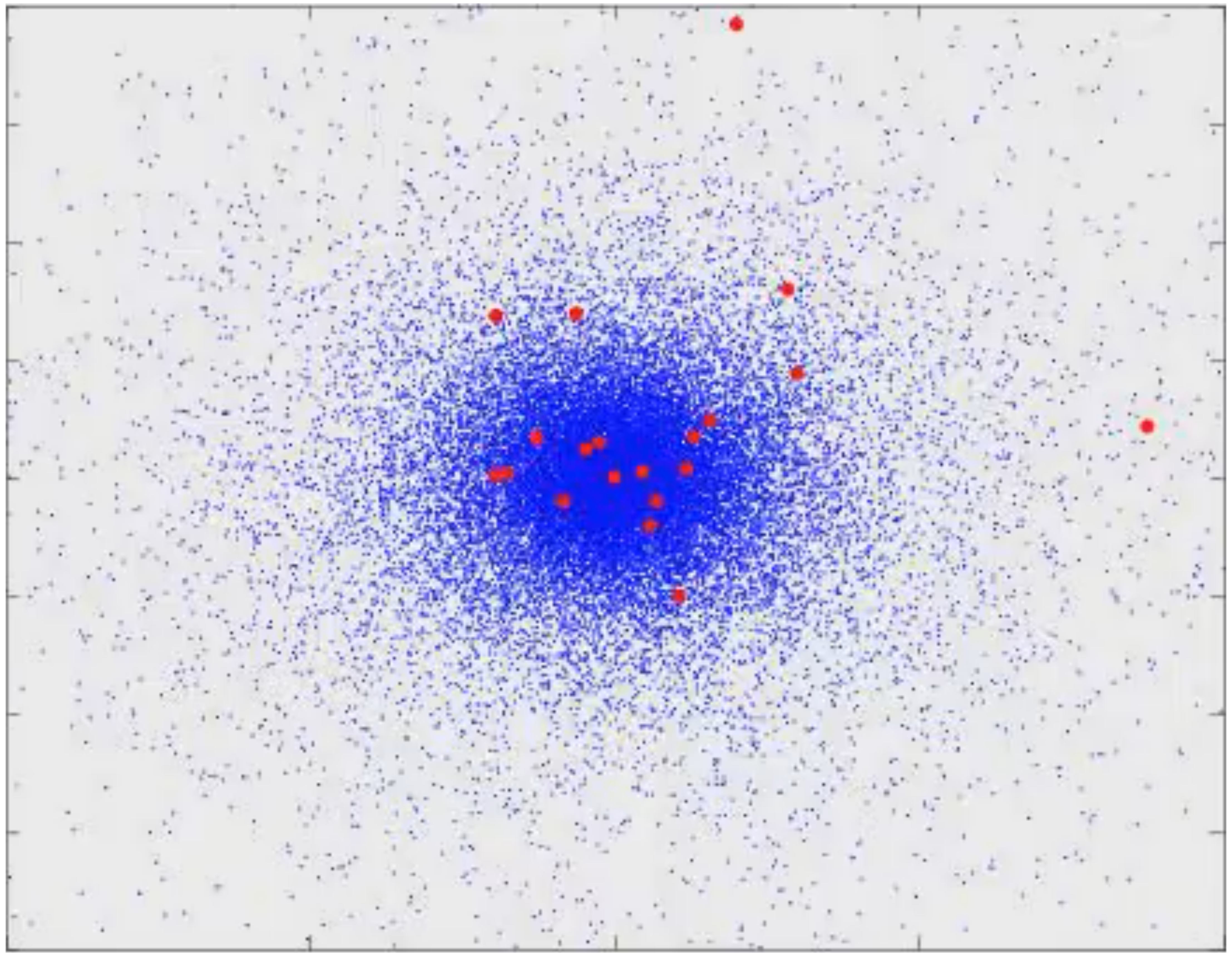
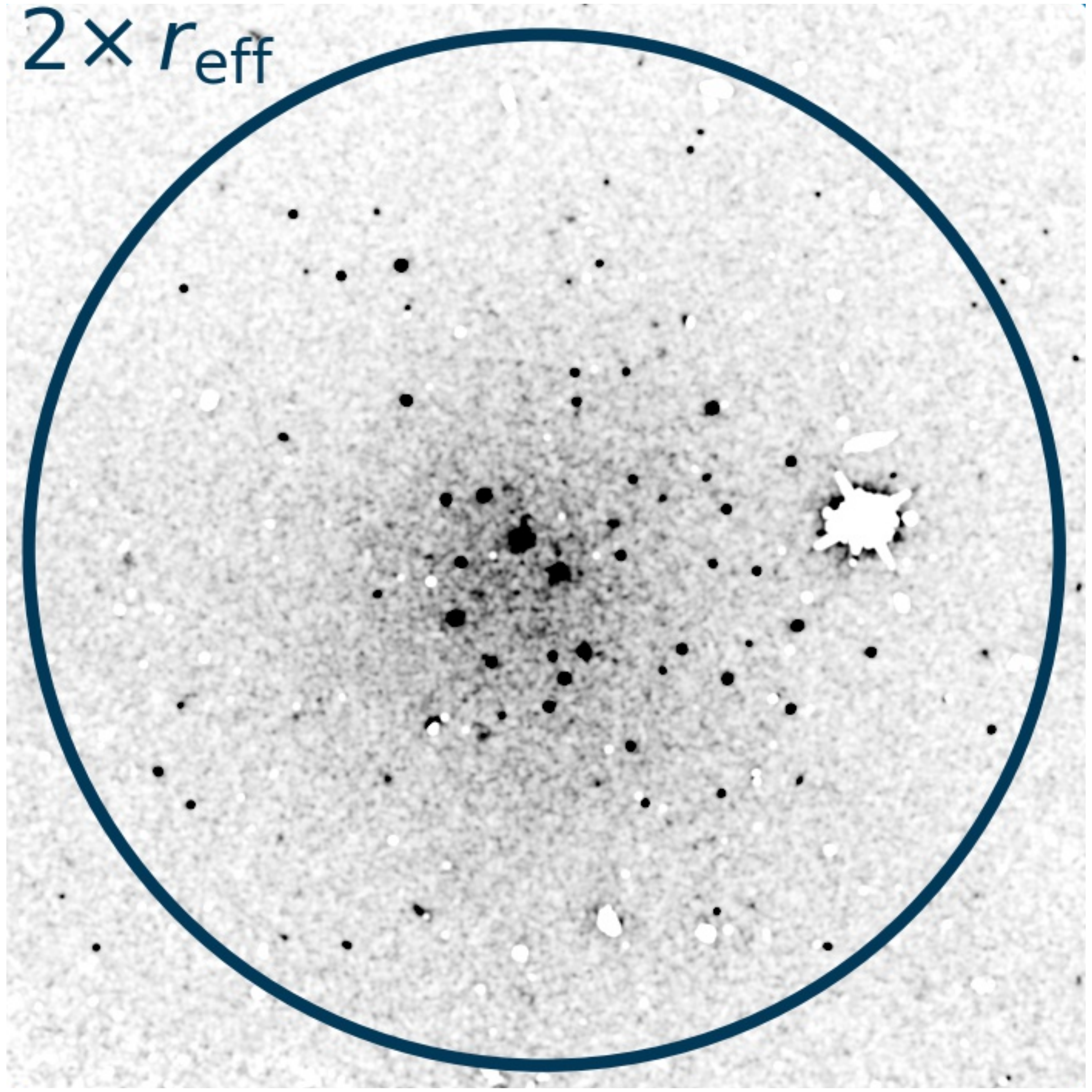


Globular Clusters (GC3: ~5×10^5 $M_\odot$)

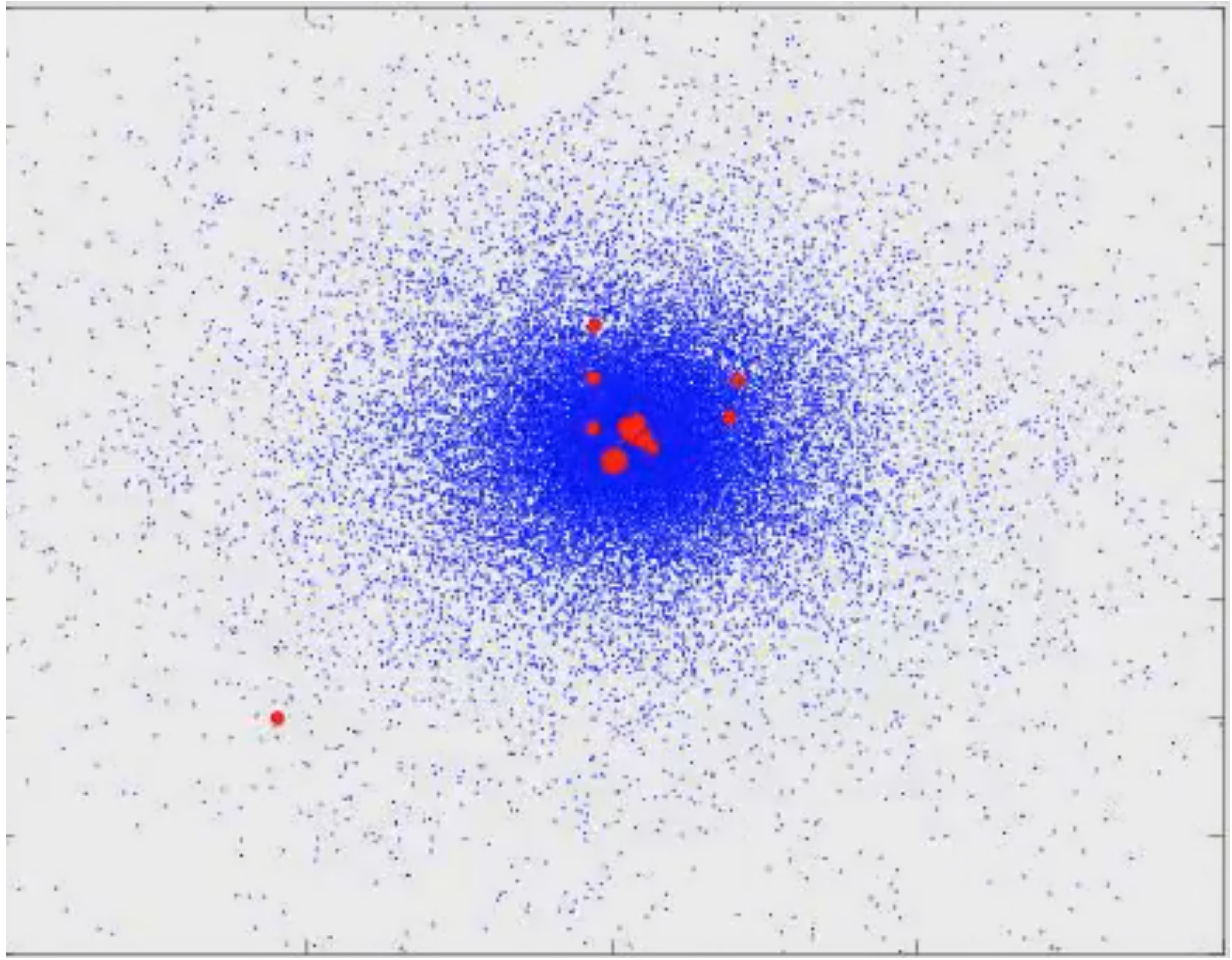
NGC5846-UDG1: ultradiffuse galaxy

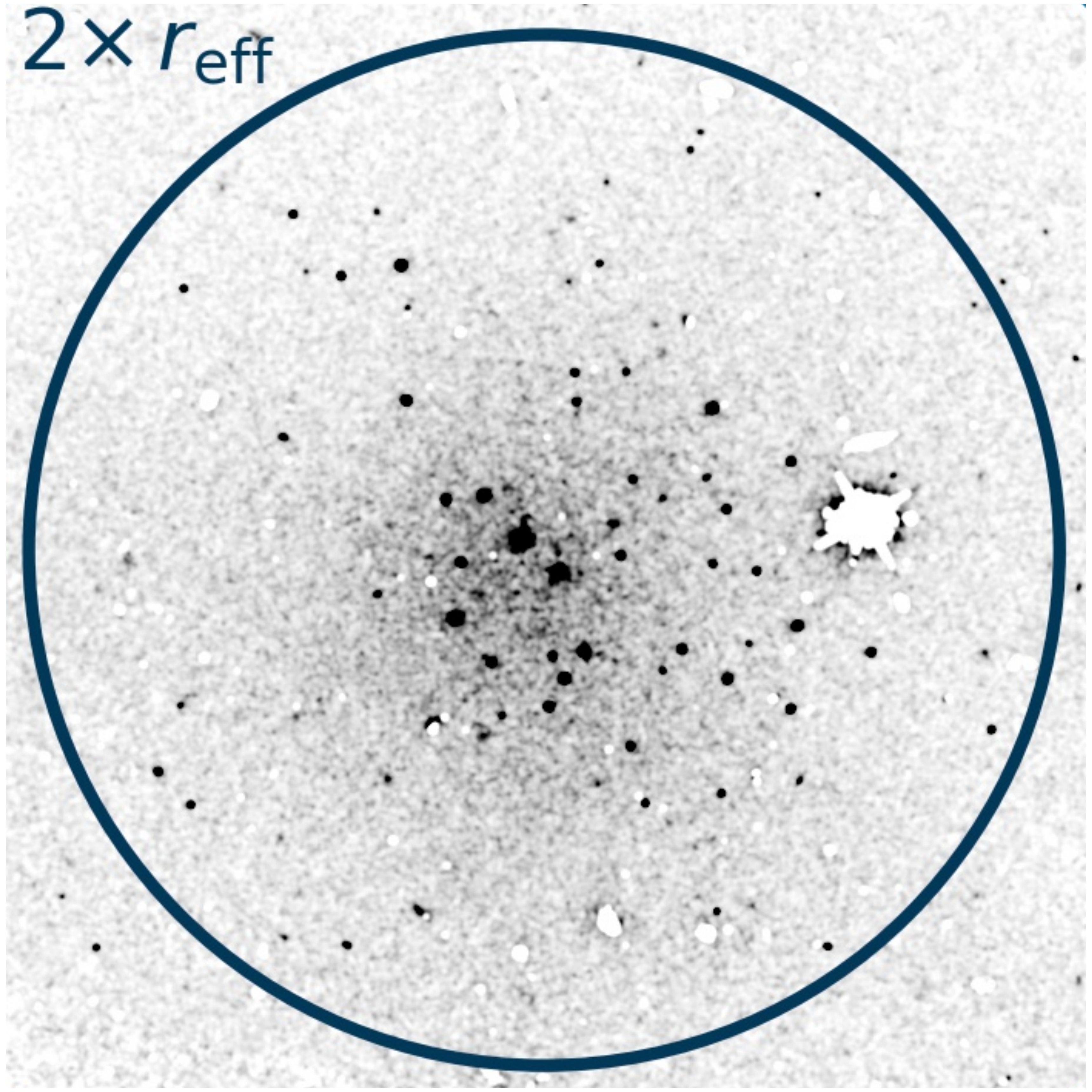





$2\times r_{\mathrm{eff}}$

This is a beyond-mean-field effect.
Qualitatively different window on dark matter, compared to stellar kinematics.





Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2).
The phase space distribution of type 1 particles follows the continuity eq.: $\frac{df_1}{dt} = C[f_1]$.

In the absence of collisions (C=0), for a nonrelativistic particle (p=mv) we have the usual mean-field dynamics:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}_i \frac{\partial f}{\partial x_i} + \dot{p}_i \frac{\partial f}{\partial p_i} \quad = \quad \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} \quad = \quad 0$$

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$$\int d^3p \, v_j \left(\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} \right) \quad = \quad \frac{\partial}{\partial t} \left(n \bar{v}_j \right) + \frac{\partial}{\partial x_i} \left(n \overline{v_i v_j} \right) + n \frac{\partial \Phi}{\partial x_j} \quad = \quad 0$$

(assuming that f vanishes on the v-boundary, and noting $n = \int d^3p f, \quad n\bar{v}_i = \int d^3p \, v_i \, f, \quad n\overline{v_i v_j} = \int d^3p \, v_i v_j \, f$)

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Apply this formalism to a single massive particle flying around (velocity V=P/M):

$$f = \frac{1}{M^3}\delta^{(3)}\left(\mathbf{x} - \mathbf{X(t)}\right)\delta^{(3)}\left(\mathbf{v} - \mathbf{V(t)}\right), \quad n(\mathbf{x}) = \int d^3p f = M^3\int d^3v f = \delta^{(3)}(\mathbf{x} - \mathbf{X}),$$

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$$\left[\frac{\partial}{\partial t}\left(n\bar{v}_j\right) + \frac{\partial}{\partial x_j}\left(n\overline{v_i v_j}\right) + n\frac{\partial \Phi}{\partial x_j}\right]$$

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(assuming that $n\overline{v_i v_j}$ vanishes on the spatial boundary)

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Collisions change this to: $\dot{\mathbf{V}} = -\nabla\Phi + M^3\int d^3x\int d^3v\,\mathbf{v}\,C[f]$

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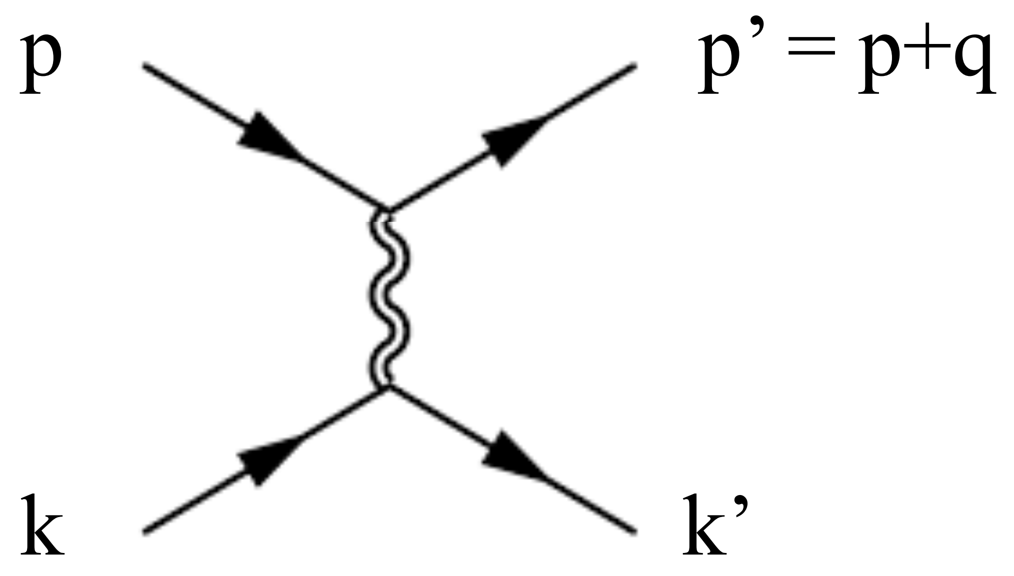
The collision operator:

$$C[f_1] = \frac{(2\pi)^4}{2E_p} \int d\Pi_k d\Pi_{p'} d\Pi_{k'}\, \delta^{(4)}(p+k-p'-k')|\overline{\mathcal{M}}|^2$$
$$\times\Big[f_1(p')f_2(k')(1\pm f_1(p))(1\pm f_2(k))$$
$$-f_1(p)f_2(k)(1\pm f_1(p'))(1\pm f_2(k'))\Big]$$

$$d\Pi_k = \frac{d^3k}{(2\pi)^3 2E_k}$$

The process under discussion is graviton exchange:

$$|\overline{\mathcal{M}}|^2 = \frac{1}{2s+1}\frac{(16\pi G)^2 m^4 M^4}{\left[(q^0)^2 - \mathbf{q}^2\right]^2}$$



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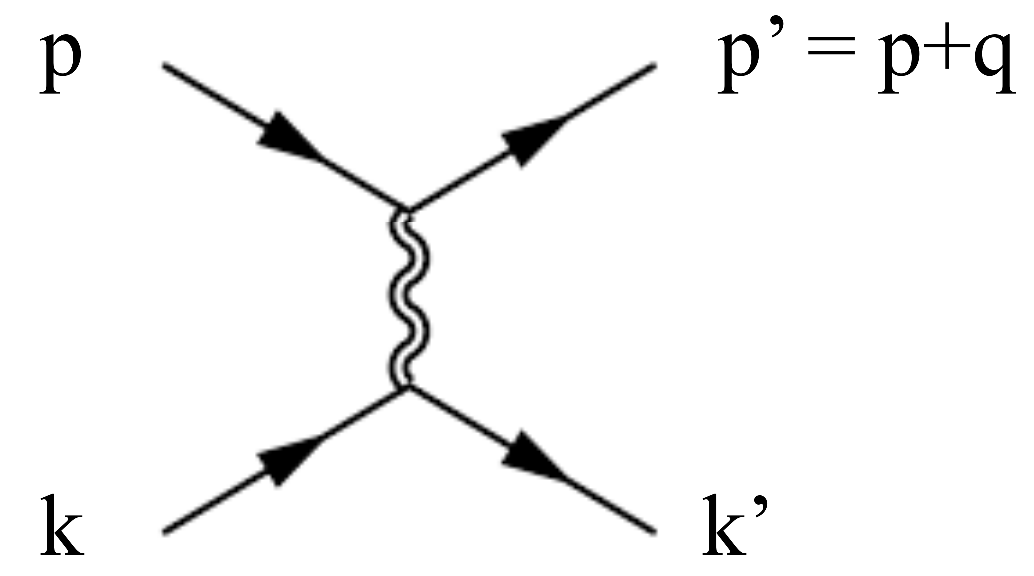

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The process under discussion is graviton exchange:

This analysis uses free asymptotic states: ignores back reaction, nontrivial boundary conditions…

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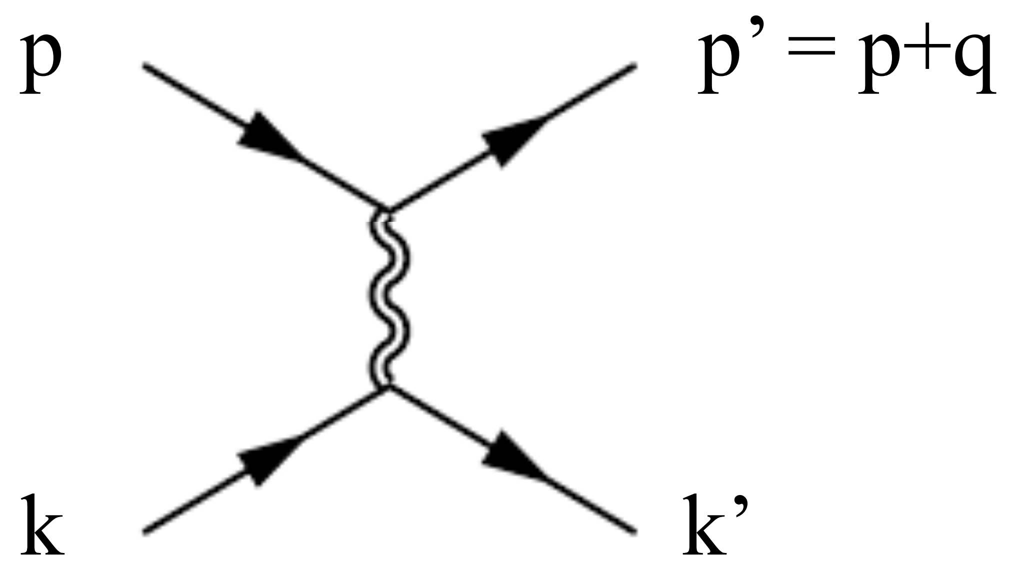

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Response function S:

$$S(\mathbf{p}, \mathbf{p}') \equiv \frac{(2\pi)^4}{2E_p 2E_{p'}} \int d\Pi_k d\Pi_{k'} \delta^{(4)}(p + k - p' - k') |\overline{\mathcal{M}}|^2 f_2(k)(1 \pm f_2(k'))$$

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Fokker-Planck expansion in small $q = p - p'$

Diffusion coefficients:

$$D_i(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} q^i S(\mathbf{p}, \mathbf{p}+\mathbf{q})$$

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We let type 1 have mass M and type 2 have mass m. For classical background gas ($f_2 \ll 1$):

Diffusion coefficients:
$$D_i(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} q^i S(\mathbf{p}, \mathbf{p}+\mathbf{q}) = 4\pi G^2 m^2 M^2 \left(1 + \frac{M}{m}\right) \ln\Lambda \frac{\partial}{\partial p^i} h(\mathbf{p};\, f_2)$$
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Rosenbluth potentials: $$h(\mathbf{p};\, f) = \int \frac{d^3k}{(2\pi)^3} \frac{f(k)}{\left|\frac{\mathbf{k}}{m} - \frac{\mathbf{p}}{M}\right|}, \qquad g(\mathbf{p};\, f) = \int \frac{d^3k}{(2\pi)^3} \left|\frac{\mathbf{k}}{m} - \frac{\mathbf{p}}{M}\right| f(k)$$

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Coulomb logarithm: $\ln\Lambda \equiv \int_{q_{\min}}^{q_{\max}} \frac{dq}{q}$

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Back to the EOM: $\dot{\mathbf{V}} = -\nabla\Phi + M^3 \int d^3x \int d^3v\, \mathbf{v}\, C[f]$

For us, type 1 particles will be GCs.

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$$M^3 \int d^3x \int d^3v\, v_i \left[-\frac{1}{M}\frac{\partial}{\partial v_j}\left(D_j f_1\right)\right] = \frac{D_i}{M} = 4\pi G^2 m^2 M \left(1 + \frac{M}{m}\right) \ln\Lambda \frac{\partial h}{\partial p_i}$$ (mean momentum drift $\langle \Delta v_i \rangle / \Delta t$)

$$M^3 \int d^3x \int d^3v\, v_i \left[\frac{1}{2M^2}\frac{\partial^2}{\partial v_j \partial v_k}\left(D_{jk} f_1\right)\right] = 0$$ (would produce mean energy / dispersion drift $\langle \Delta v_i \Delta v_j \rangle / \Delta t$)

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Back to the EOM: $\dot{\mathbf{V}} = -\nabla\Phi + M^3\int d^3x \int d^3v\, \mathbf{v}\, C[f]$

For us, type 1 particles will be GCs (M>>m).

$$M^3\int d^3x \int d^3v\, v_i\left[-\frac{1}{M}\frac{\partial}{\partial v_j}\left(D_j f_1\right)\right] = -16\pi^2 G^2 \rho M \ln\Lambda \frac{V_i}{V^3} C_{\rm df}(V), \qquad C_{\rm df}(V) = \frac{\int_0^V d^3v' f_2(v')}{\int_0^\infty d^3v' f_2(v')}$$

Basic physics view of dynamical friction:

Consider two types of particles, 1 and 2 (e.g., GCs being 1, and dark matter particles being 2).

The phase space distribution of type 1 particles follows the continuity eq.: $\frac{df_1}{dt} = C[f_1]$.

The collision operator:

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$$\boxed{\tau = \frac{V^3}{16\pi^2 G^2 \rho M C_{\rm df} \ln\Lambda} \qquad \text{Chandrasekhar 1943}}$$

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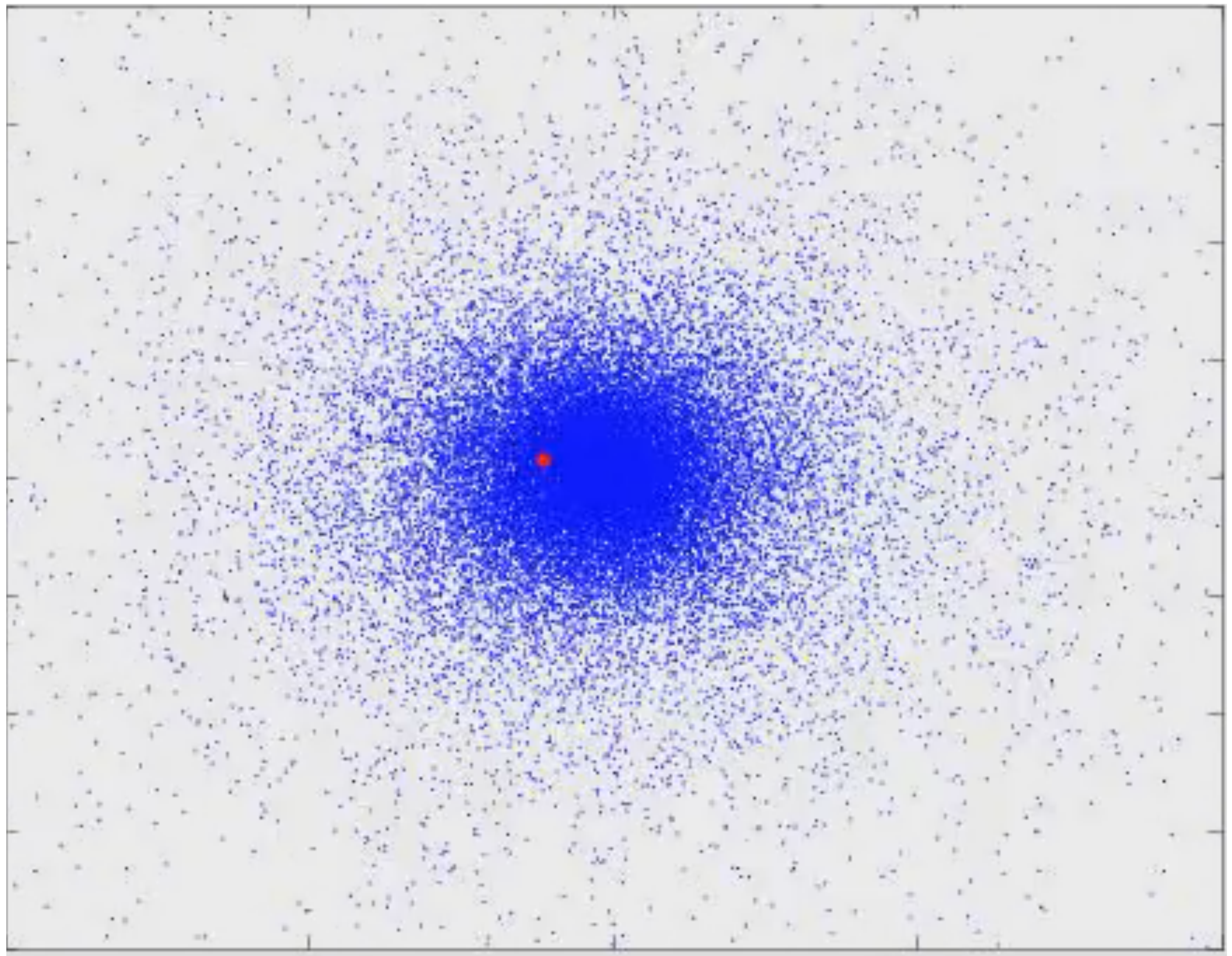
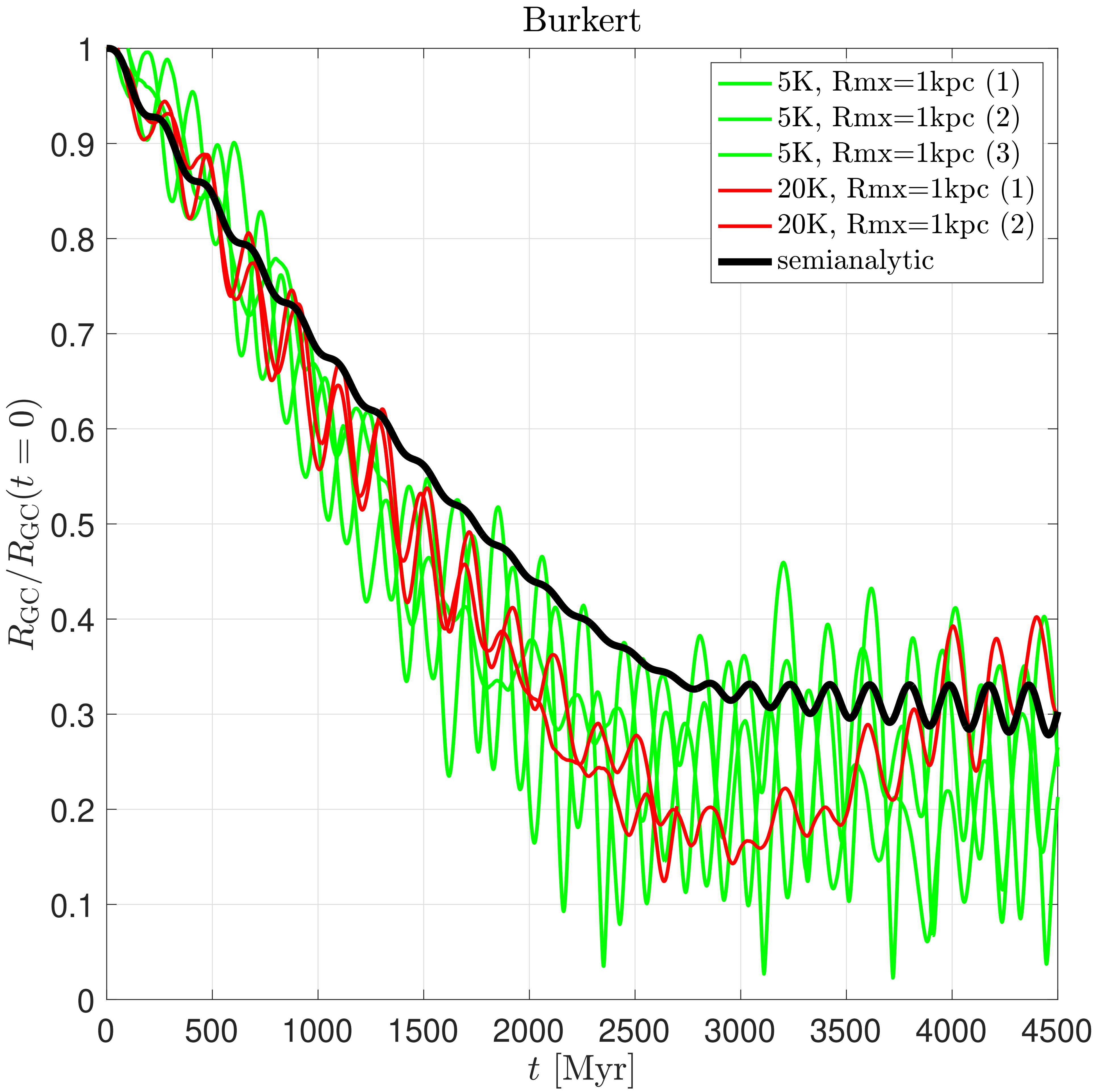
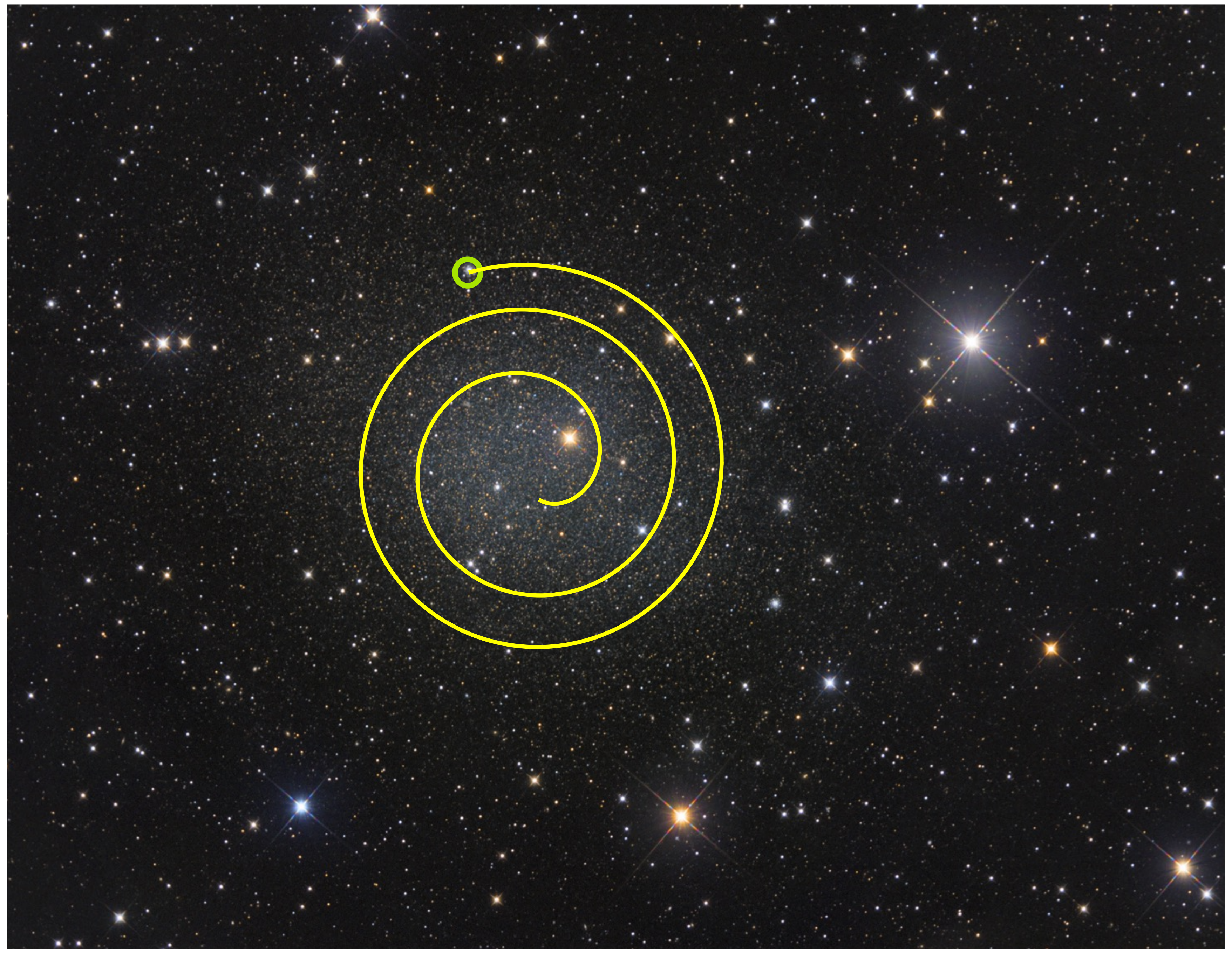
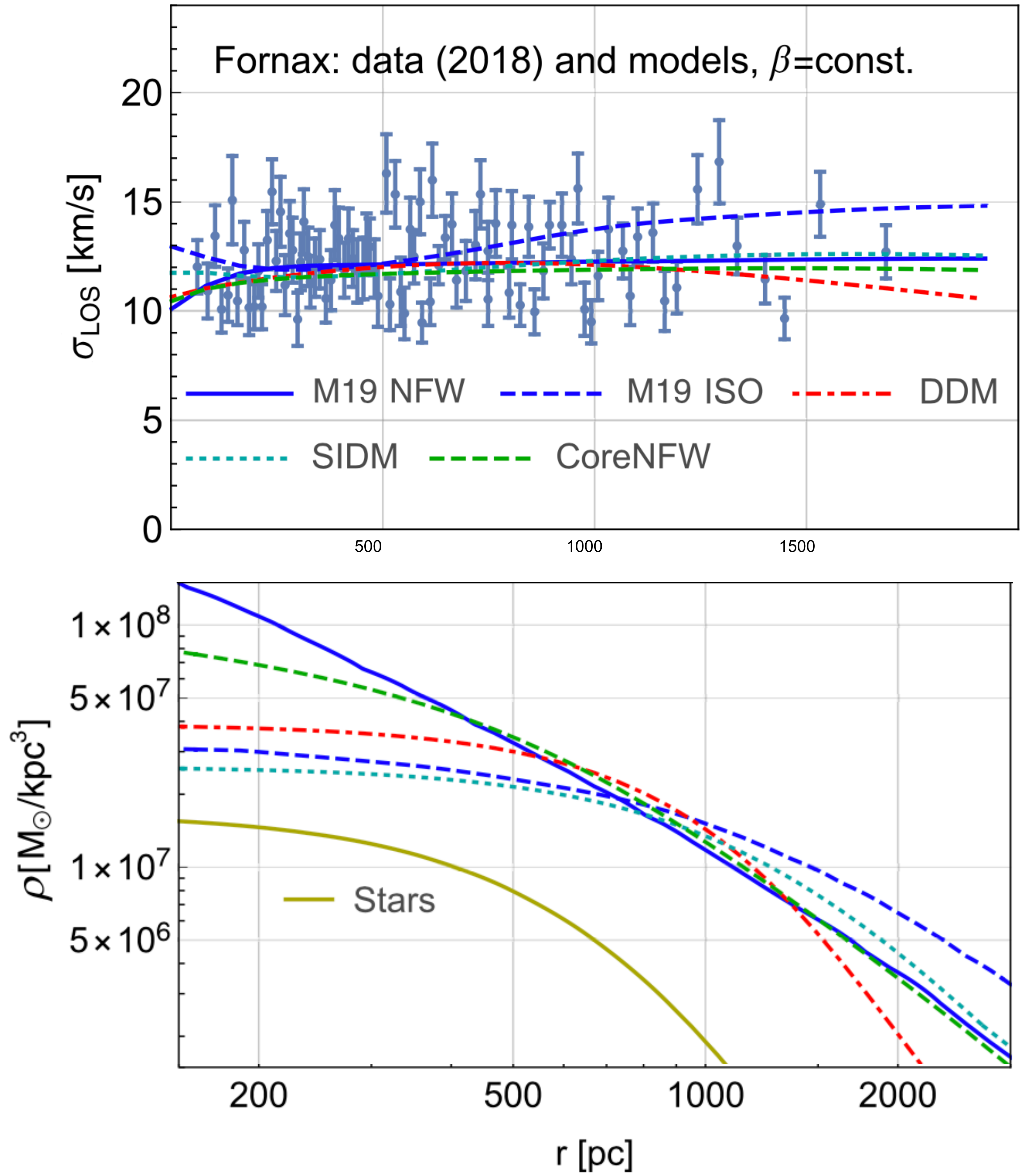




Basic physics view of dynamical friction:

$$\dot{\mathbf{V}} = -\nabla\Phi - \frac{1}{\tau}\mathbf{V}, \qquad \tau = \frac{V^3}{16\pi^2 G^2 \rho M C_{\rm df} \ln\Lambda} \approx \frac{2.7\ \mathrm{Gyr}}{C_{\rm df}\ln\Lambda}\left(\frac{V}{20\ \mathrm{km/s}}\right)^3 \left(\frac{10^7\ \mathrm{M_\odot/kpc^3}}{\rho}\right)\left(\frac{10^5\ \mathrm{M_\odot}}{M}\right)$$





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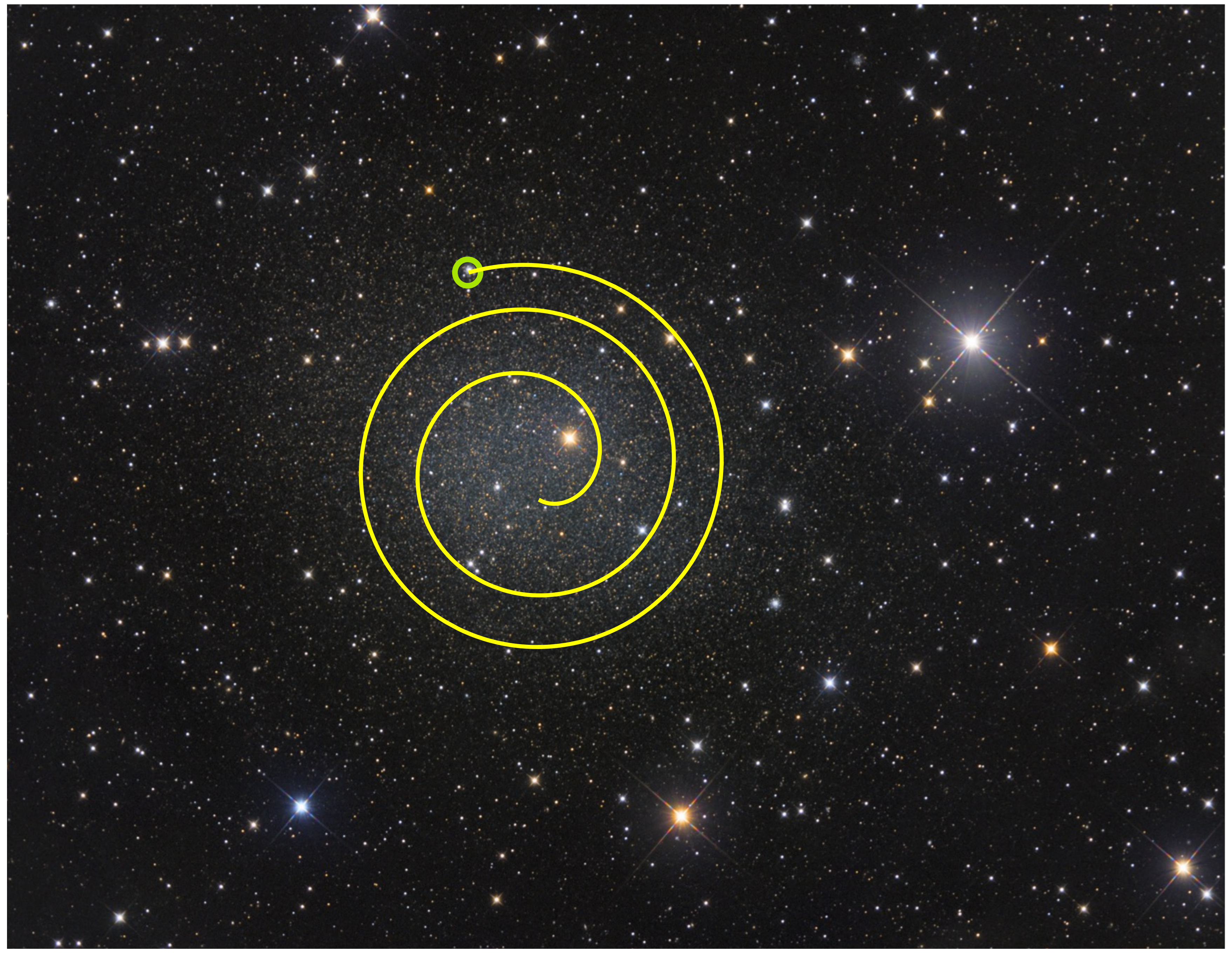


Lack of nuclear star cluster in Fornax?

Need more statistics…

Would like to see the effect in action
in dark matter-dominated galaxies

Tremaine 1976,
Oh, Lin, Richer 2000,
Petts, Gualandris, Read 2015,
Hui et al 2017, Lancaster et al 2019,
Meadows et al 2020, Bar et al 2021,
Shao et al 2021,…

Lotz et al ApJ 552 (2001)

DF in dwarf elliptical galaxies, HST survey. **Many of these dEs are dark matter-dominated.**
Stacked GC radial distribution, 51 dEs in Virgo and Fornax clusters.
0-20 GCs/galaxy. Did not analyze luminosities beyond isolating most luminous and subtracting NSC.

(1) assumed GCs on circular orbits, **(2)** assumed GCs started on same distribution as stars, **(3)** deduced velocity dispersion from V magnitude, **(4)** scaled all radii to host 1/2light radius, **(5)** defined GC joining NSC if (circular orbit) R reached 0 following analytical DF formula.

Noted deficit in high lumi GCs in inner region: fall into NSC? (Fig.5)
Difficulty: faint dEs predicted to acquire more luminous NSCs than observed (Fig.8).

…Where are the missing GCs of Fig.5…?

…Similar to Fornax dSph?

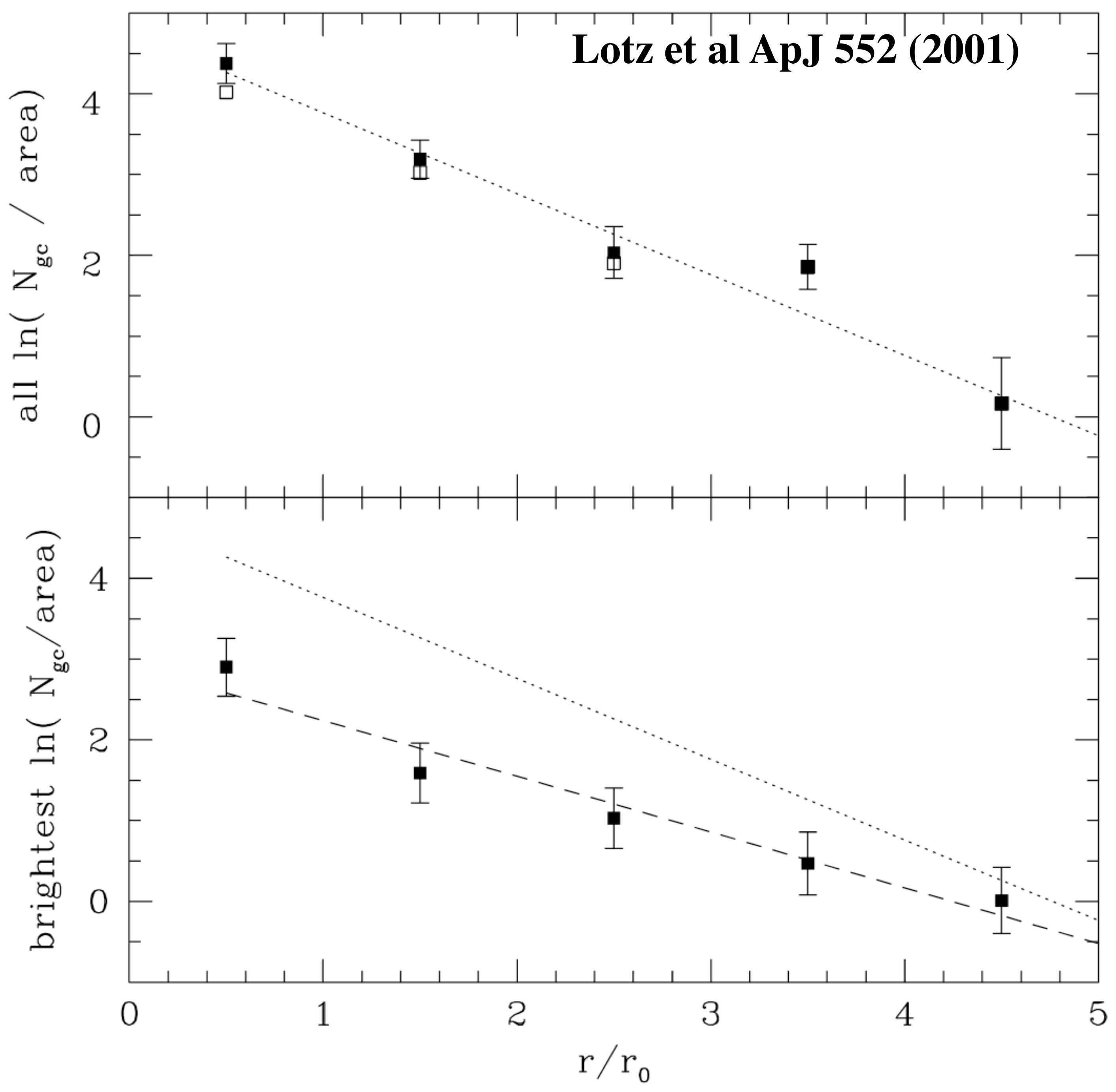


FIG. 5.—Summed radial distribution of the globular cluster candidates

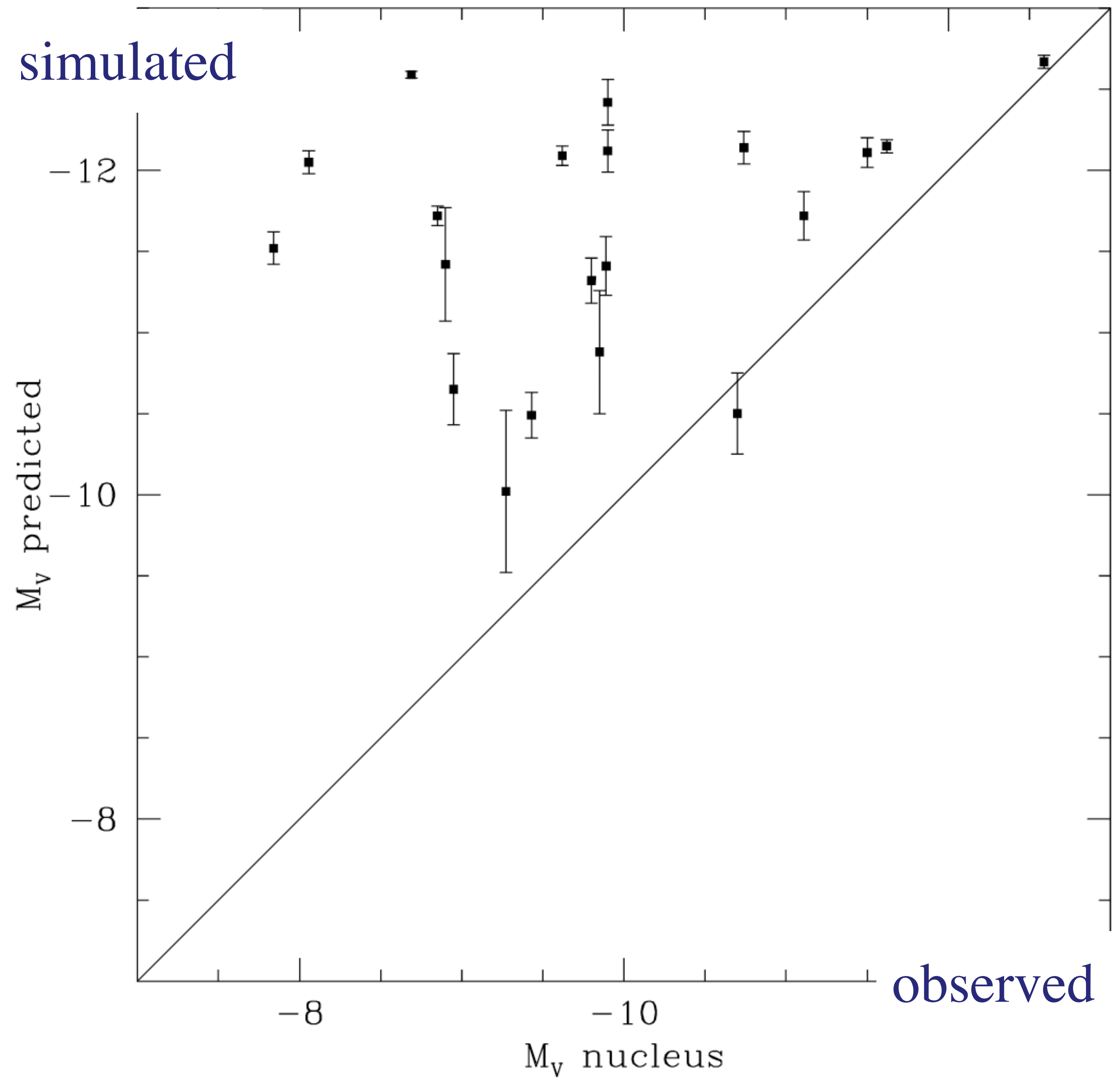
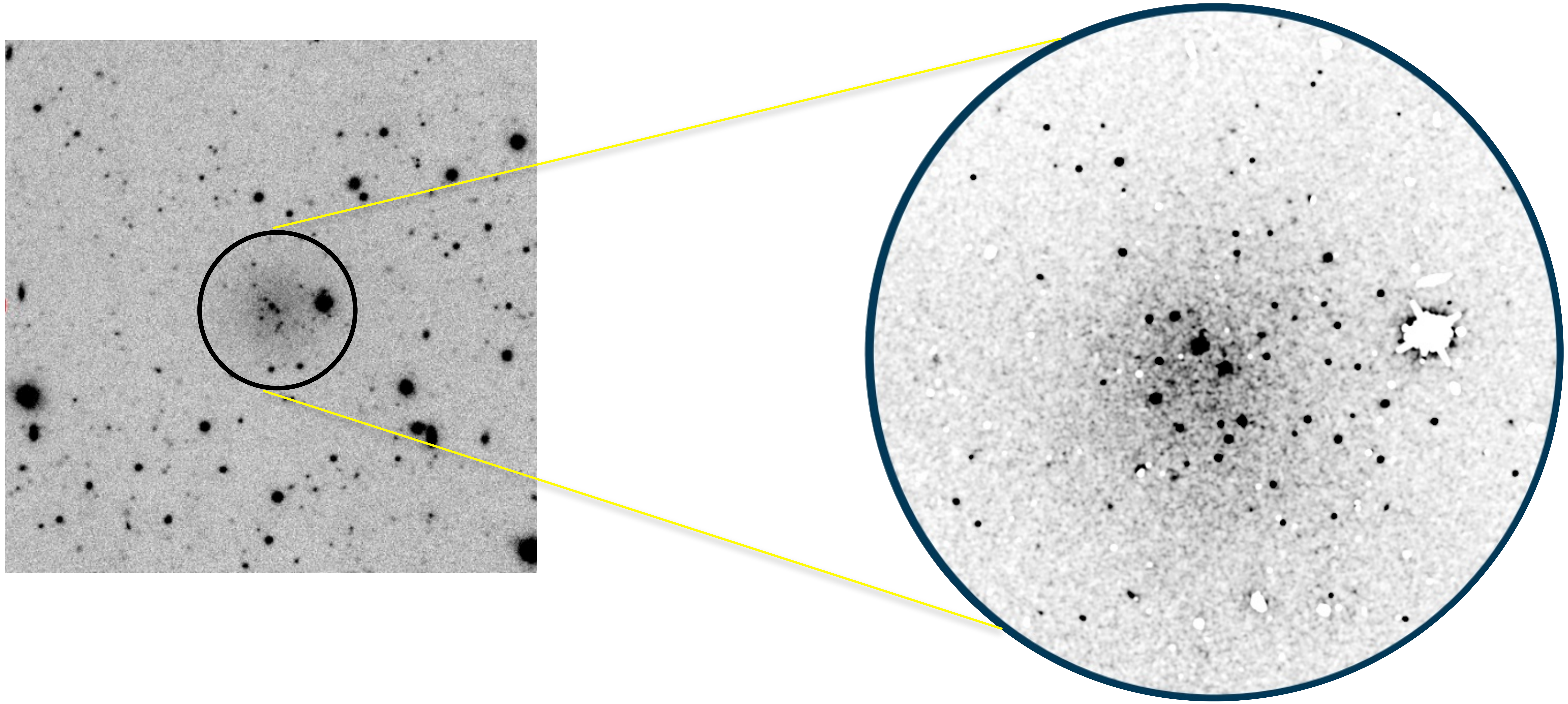


FIG. 8.—Predicted nuclear M_V from our Monte Carlo dynamical friction simulation vs. the observed nuclear M_V for each dE,N in our sample.

Forbes et al 2019, 2020,
Muller et al 2020, 2021,
Danieli et al 2022,
Bar, Danieli, KB 2022,
Modak, Danieli, Greene 2022



What can we learn from radial profile of GCs?

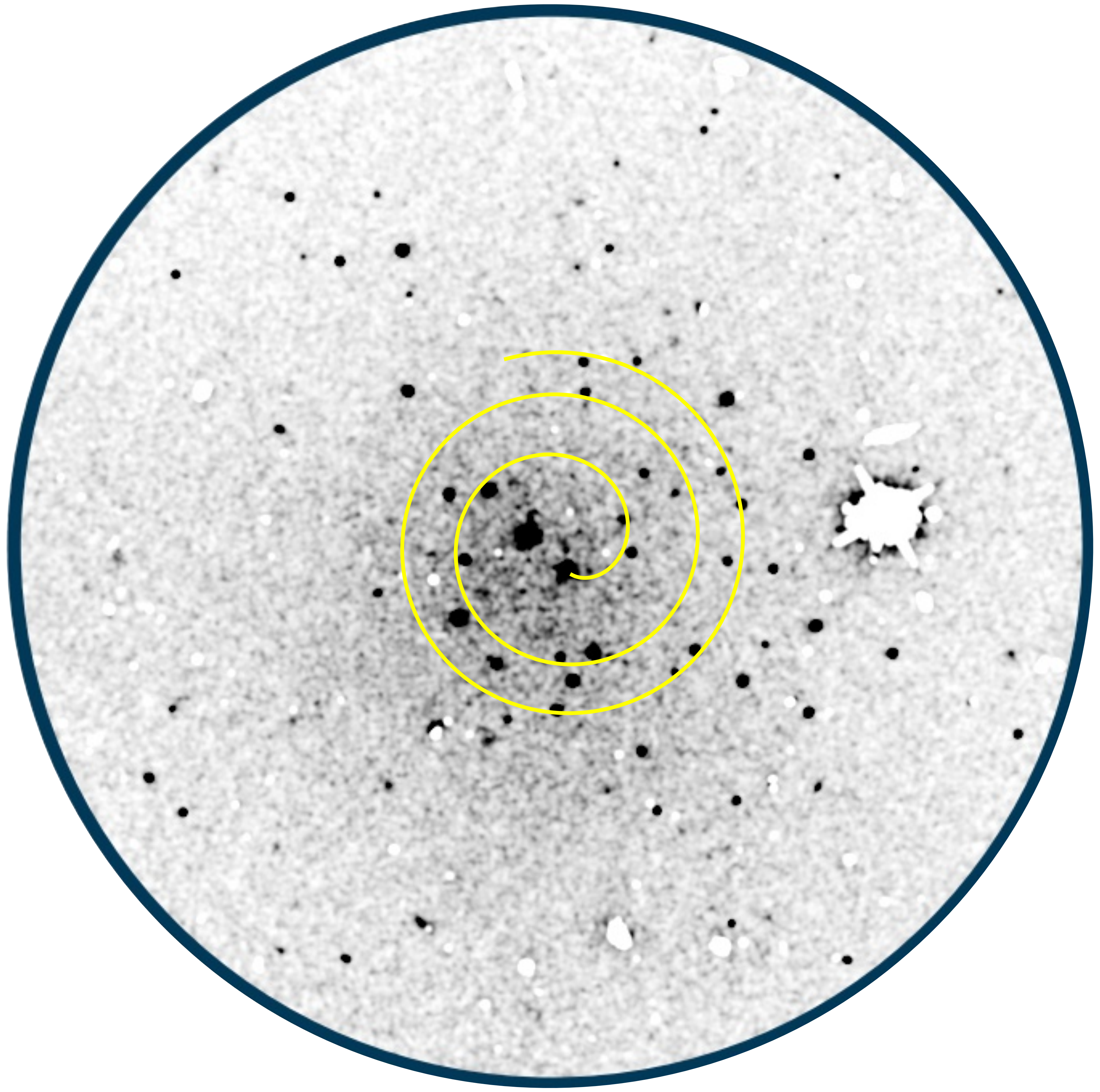
EOM in radial coordinate:

$$\dot{\left(r v_\varphi\right)} = -\frac{1}{\tau}\left(r v_\varphi\right) \qquad (\varphi \text{ equation with } v_\varphi = r\dot{\varphi})$$

$$v_\varphi^2 - v_{\rm circ}^2 = r\left(\ddot{r} + \frac{\dot{r}}{\tau}\right) \qquad (r \text{ equation})$$

Consider: — circular orbit, — effect is slow, $v_{\rm circ} \gg r/\tau$

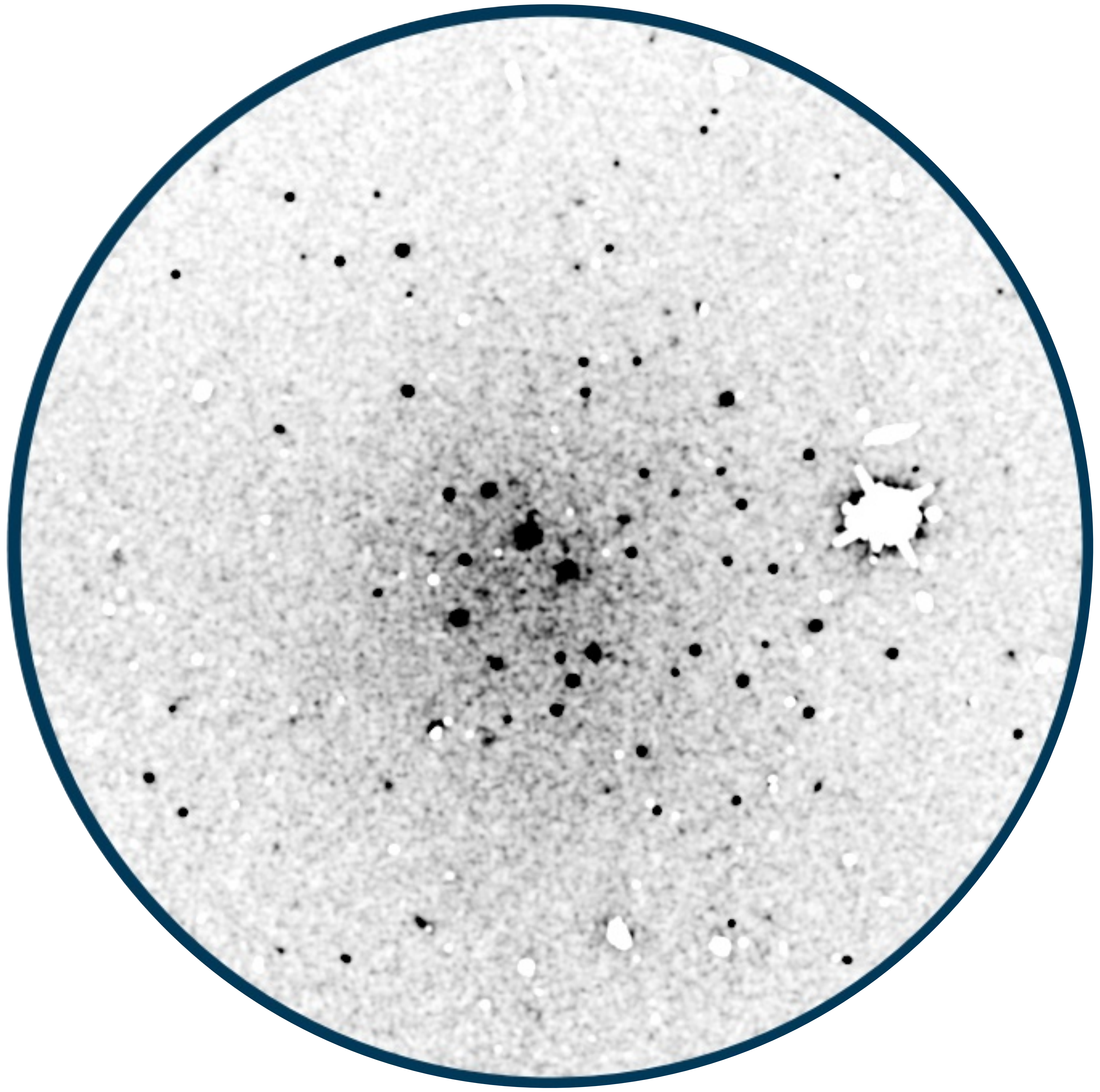
$$\longrightarrow\ v_\varphi \approx v_{\rm circ} = \sqrt{GM(r)/r}\ \longrightarrow\ \frac{\dot{r}}{r} \approx -\frac{2}{\left(1 + \frac{d\ln M}{d\ln r}\right)\tau}$$



$$\frac{\dot{r}}{r} \approx -\frac{2}{\left(1 + \frac{d\ln M}{d\ln r}\right)\tau}$$

This can be solved:

Define $\alpha(r) = \dfrac{d\ln M(r)}{d\ln r}$, then $\Delta t(r; r_0) \approx \displaystyle\int_r^{r_0} \frac{dr'}{2r'}\left(1 + \alpha(r')\right)\tau\left(r', v_{\rm circ}(r')\right)$



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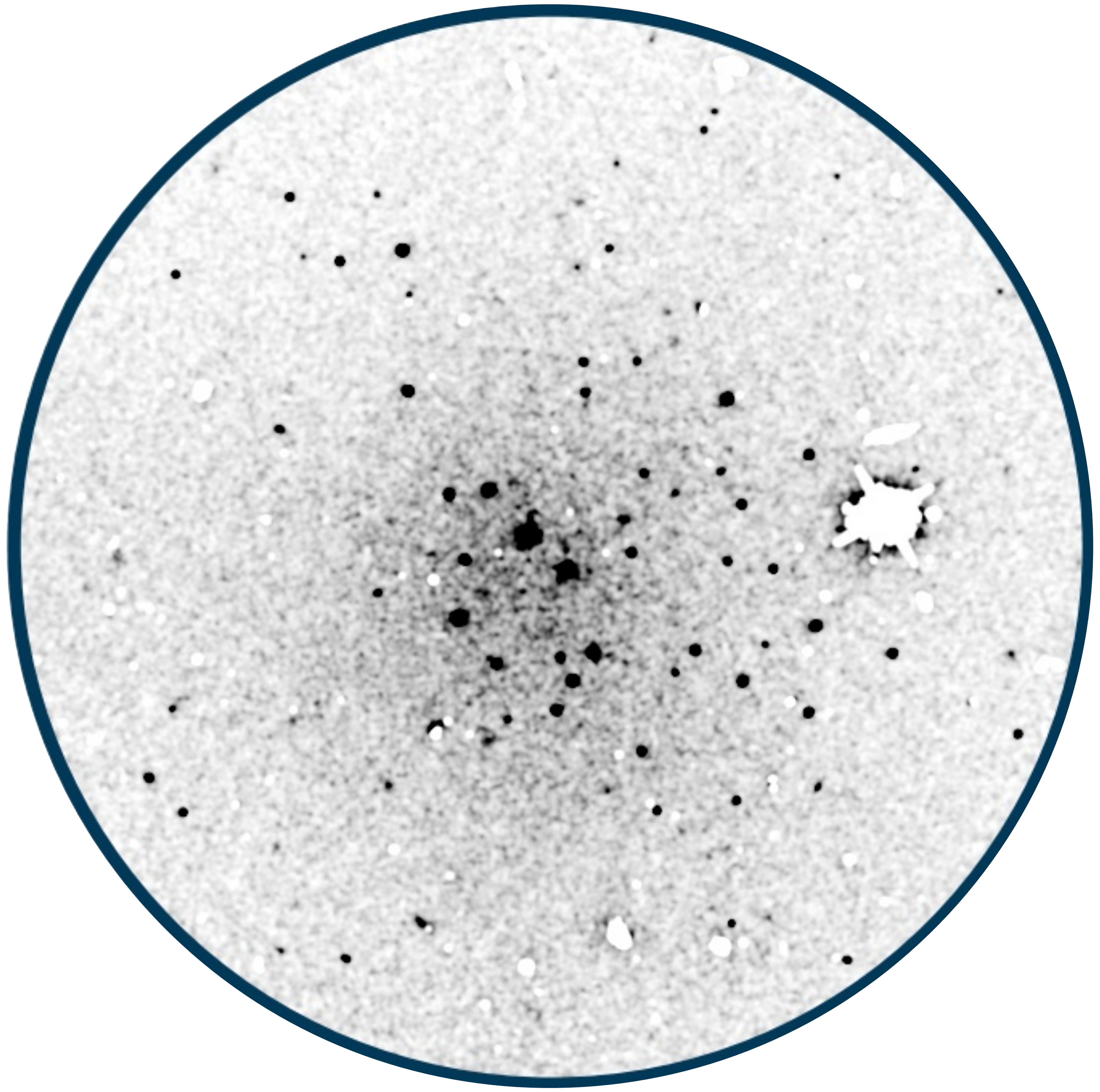
Suppose that the halo is cored, $\alpha \approx 3, \quad \tau \approx \mathrm{Const}$

then: $\Delta t \approx \dfrac{1+\alpha}{2} \tau \ln \dfrac{r_0}{r} \approx 2\tau \ln \dfrac{r_0}{r}$, or $r_0 = e^{\frac{\Delta t}{2\tau}} r$

A radial CDF today, $N_{\Delta t}(r)$,

relates to an initial CDF via $N_{\Delta \mathrm{t}}(r) \approx N_0 \left(r \, e^{\frac{\Delta t}{2\tau}}\right)$



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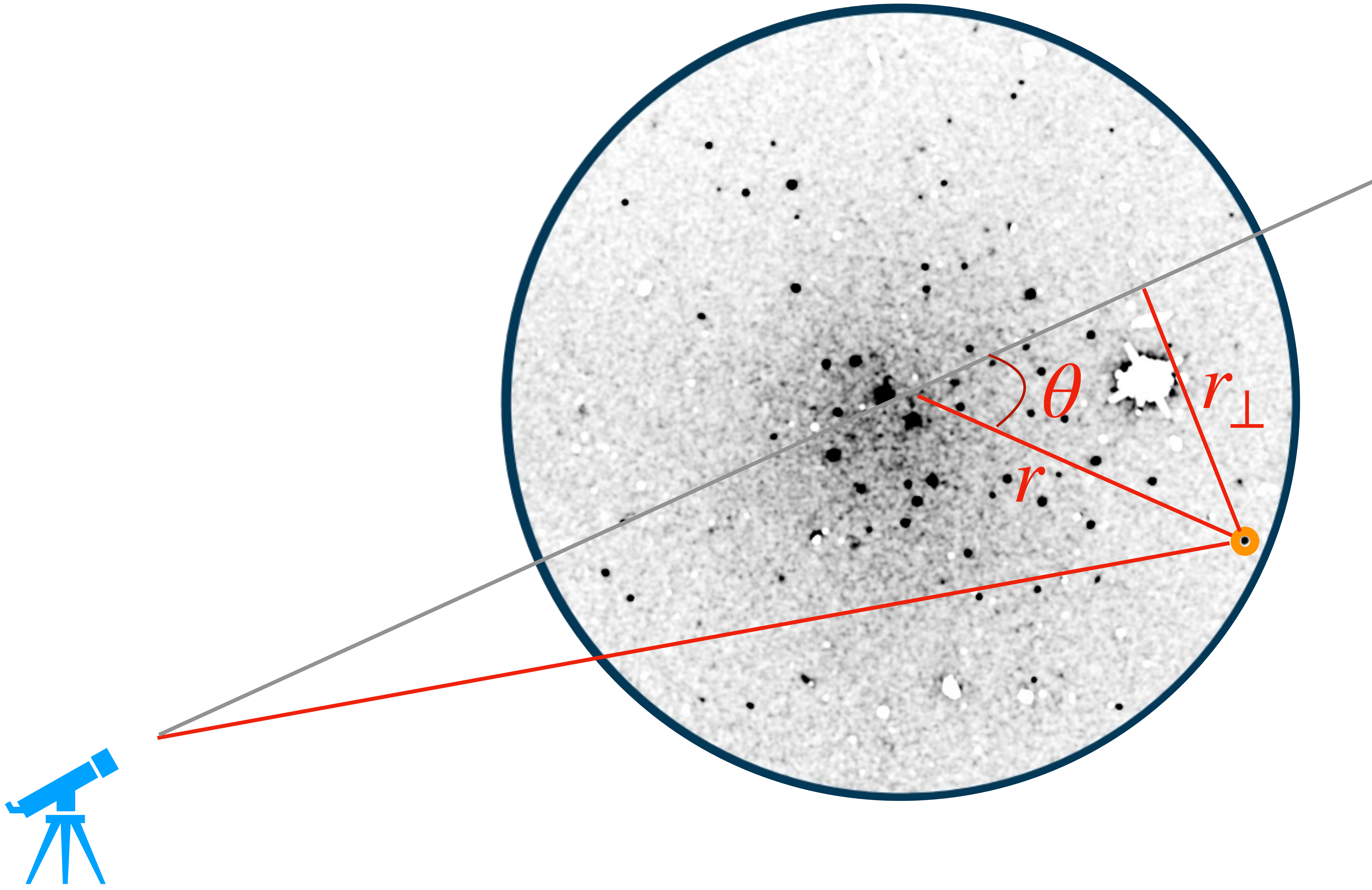
$$\langle r_\perp \rangle_{\Delta t} = \int \frac{d\Omega}{4\pi} \sin\theta \int dr\, r \frac{d}{dr} N_{\Delta t}(r)$$



$$\frac{\dot r}{r} \approx -\frac{2}{\left(1+\frac{d\ln M}{d\ln r}\right)\tau}$$

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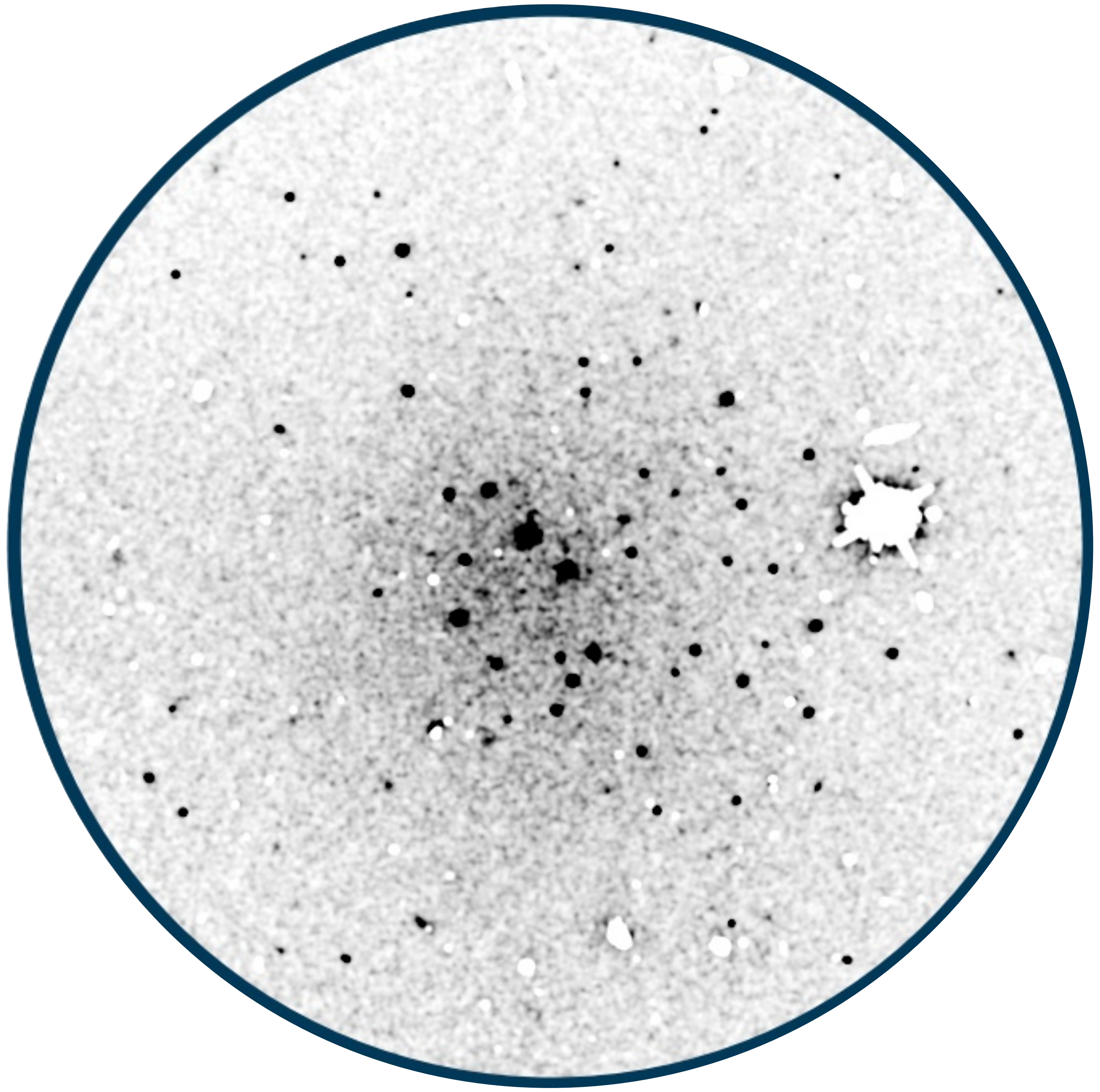
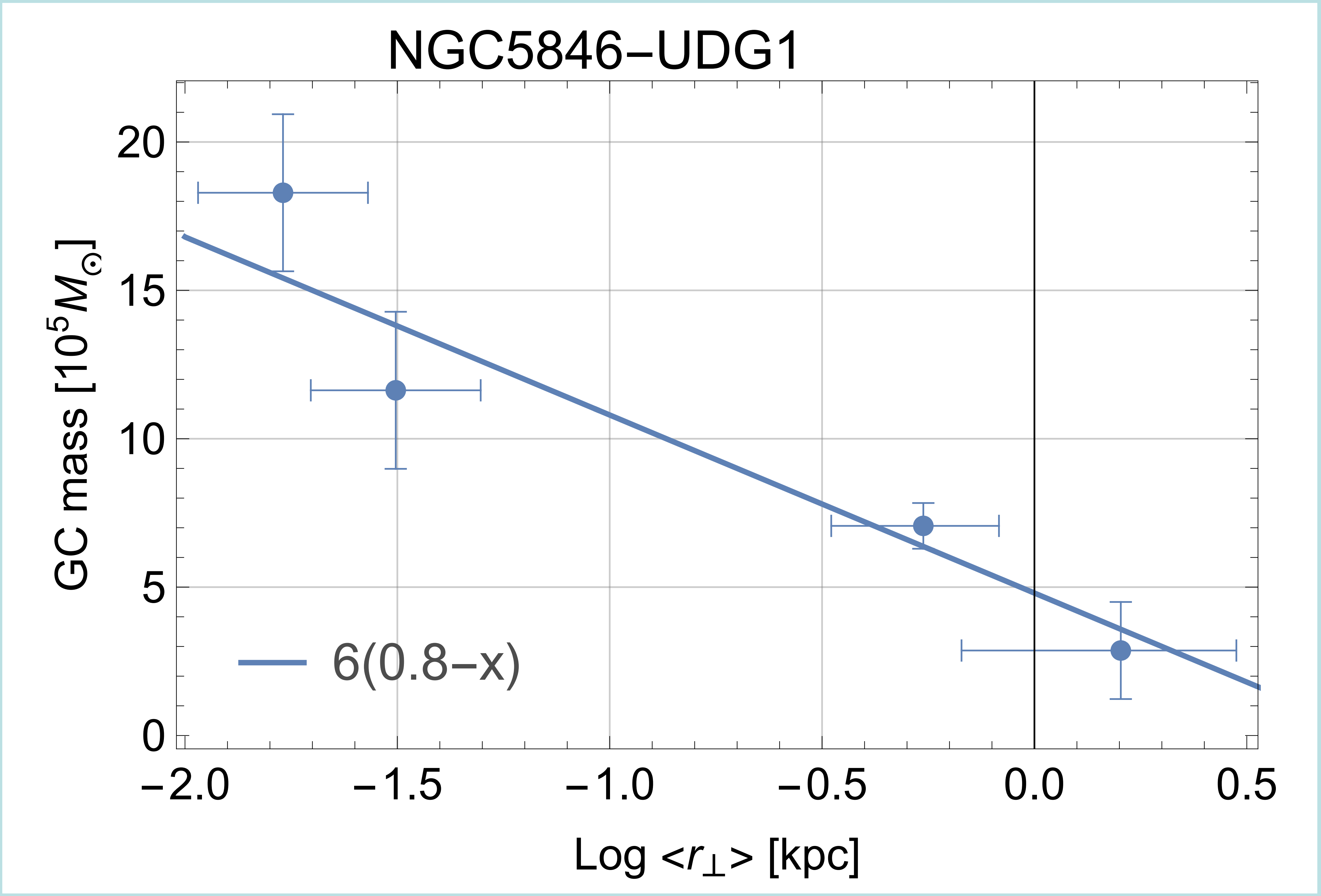
$$\ln\frac{\langle r_\perp\rangle}{\langle r_\perp\rangle_0} \approx -\frac{\Delta t}{2\bar\tau}\frac{m_{\rm GC}}{\bar m_{\rm GC}}$$



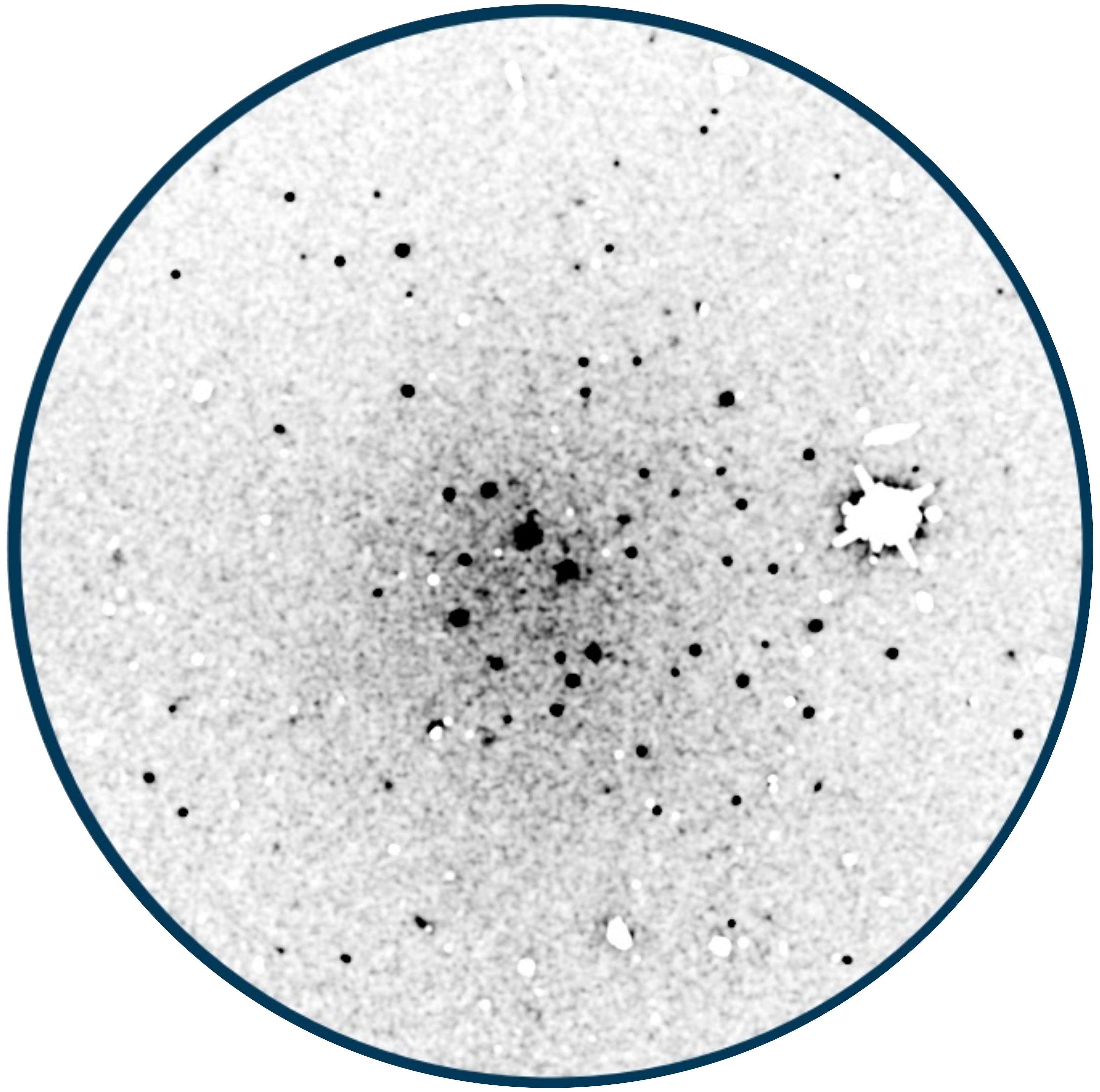
Evidence for dynamical friction
in dark matter-dominated, GC-rich ultra diffuse galaxy?

Bar, Danieli, KB 2202.10179

Related:
Saifollahi et al 2201.11750,
Dutta Chowdhury, van den Bosch, van Dokkum 2019, 2020,
Modak, Danieli, Greene 2211.01384



$$\ln \frac{\langle r_\perp \rangle}{\langle r_\perp \rangle_0} \approx - \frac{\Delta t}{2\bar{\tau}} \frac{m_{GC}}{\bar{m}_{GC}}$$



Evidence for dynamical friction
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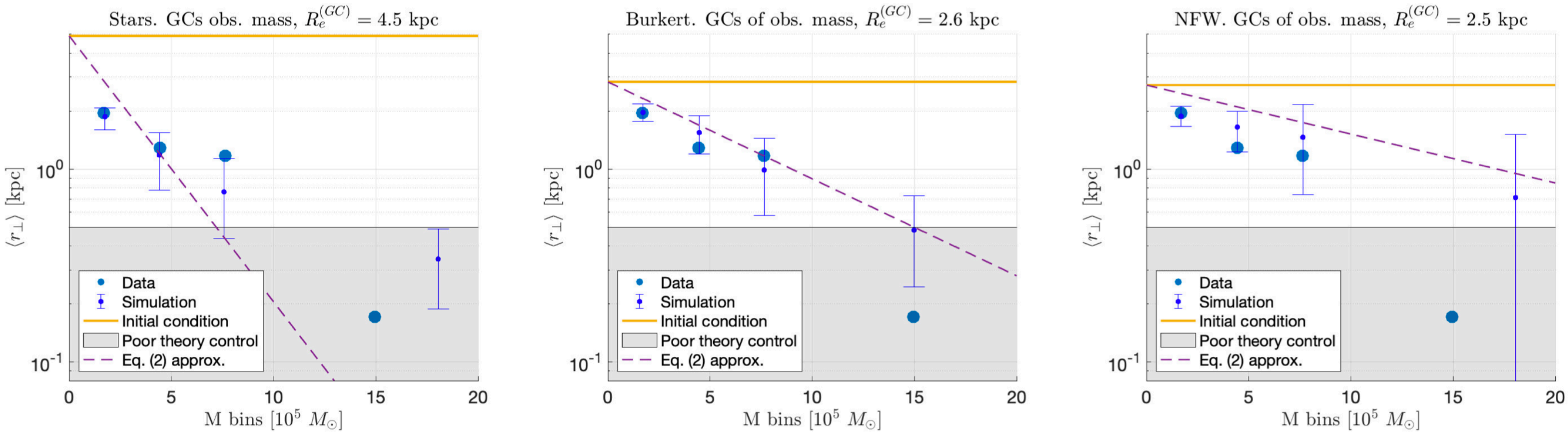


Dynamical friction in a massive dark matter halo naturally produces observed mass segregation.

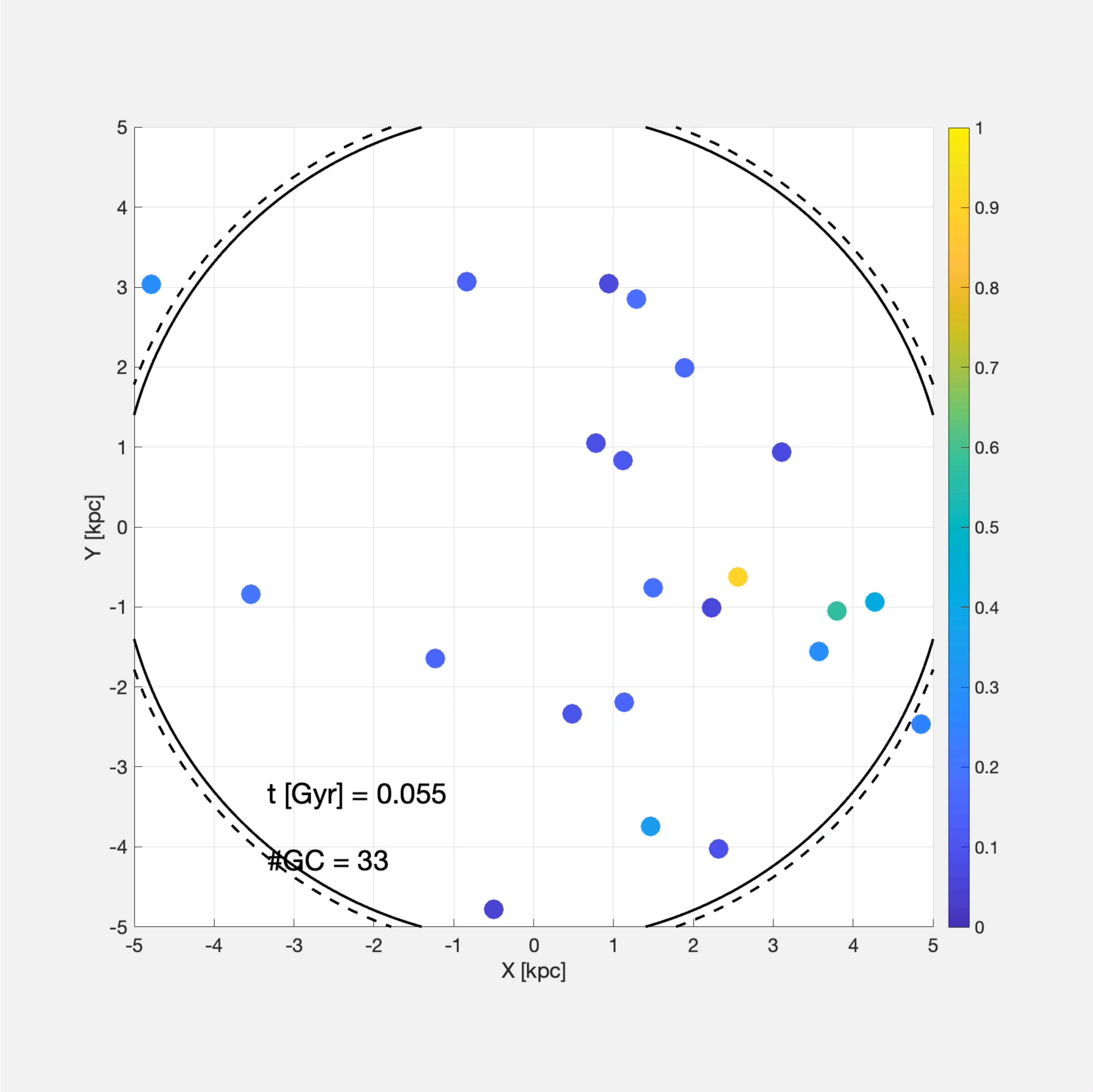
Lack of dark matter, or a low mass halo, comes with small velocity dispersion, and overshoots friction.

Consistent with, and **independent of** stellar and GC kinematics (Forbes et al 2021).

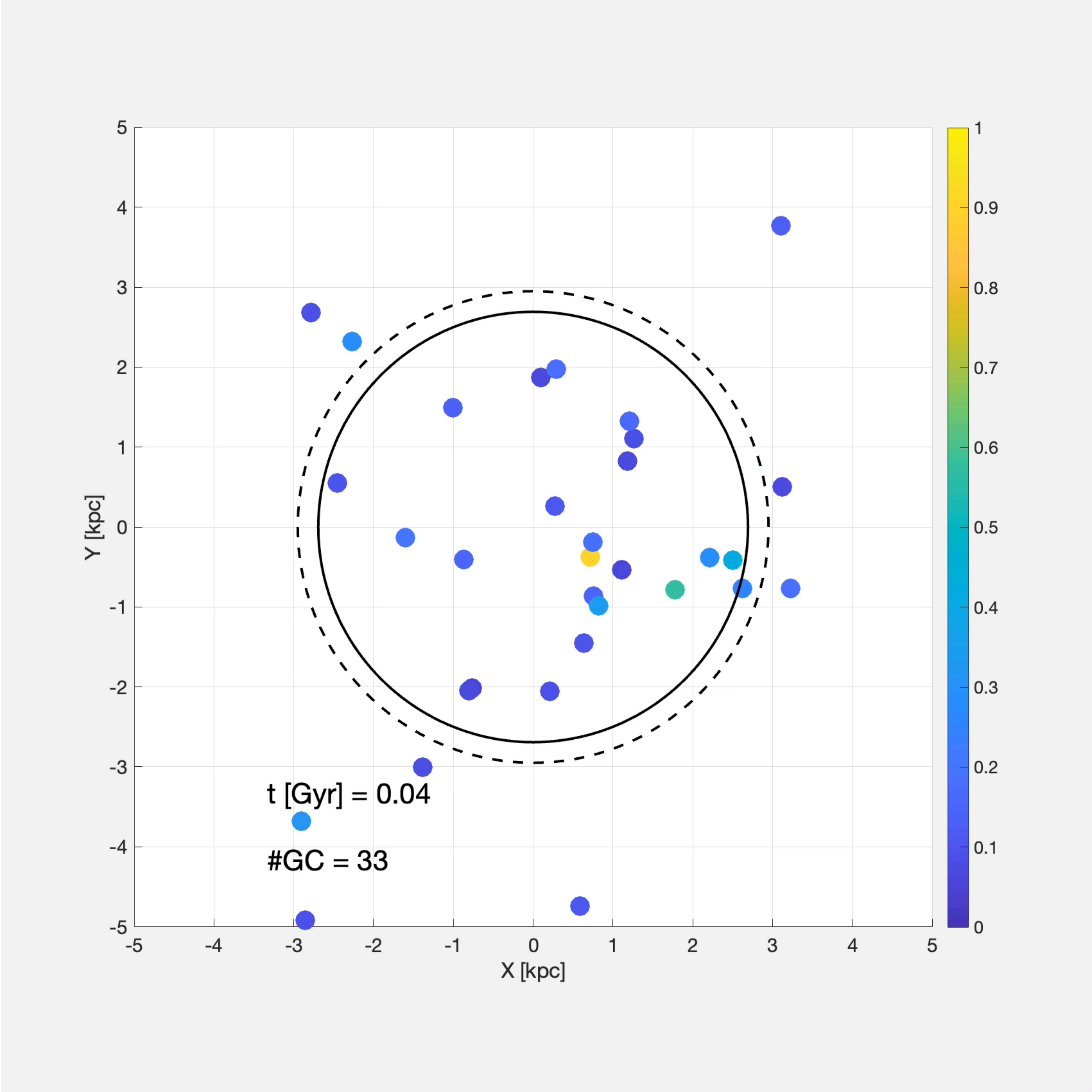
Evidence for dynamical friction
in dark matter-dominated, GC-rich ultra diffuse galaxy?

Bar, Danieli, KB 2202.10179

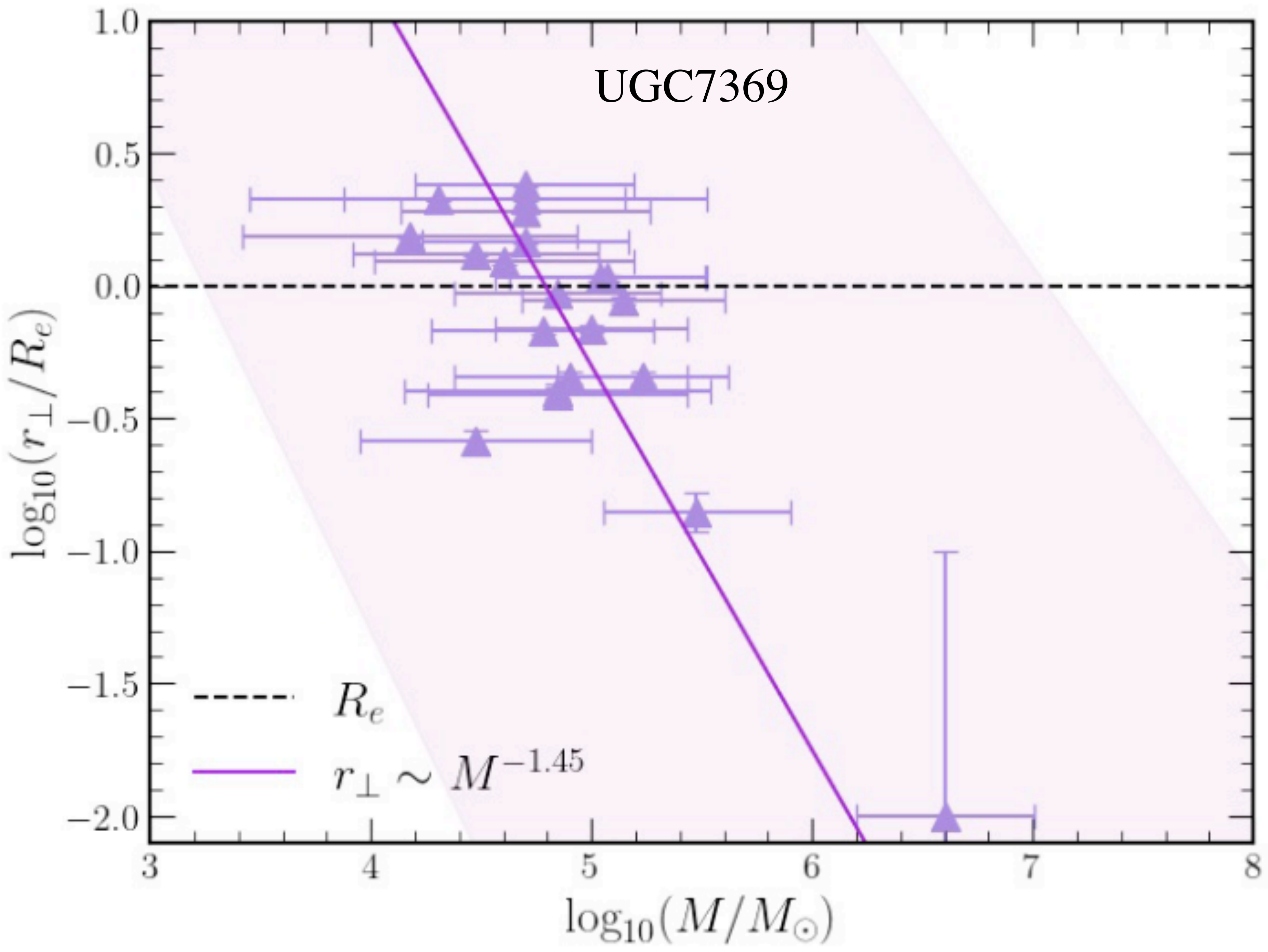
Stars. GCs obs. mass, $R_e^{(GC)} = 4.5$ kpc

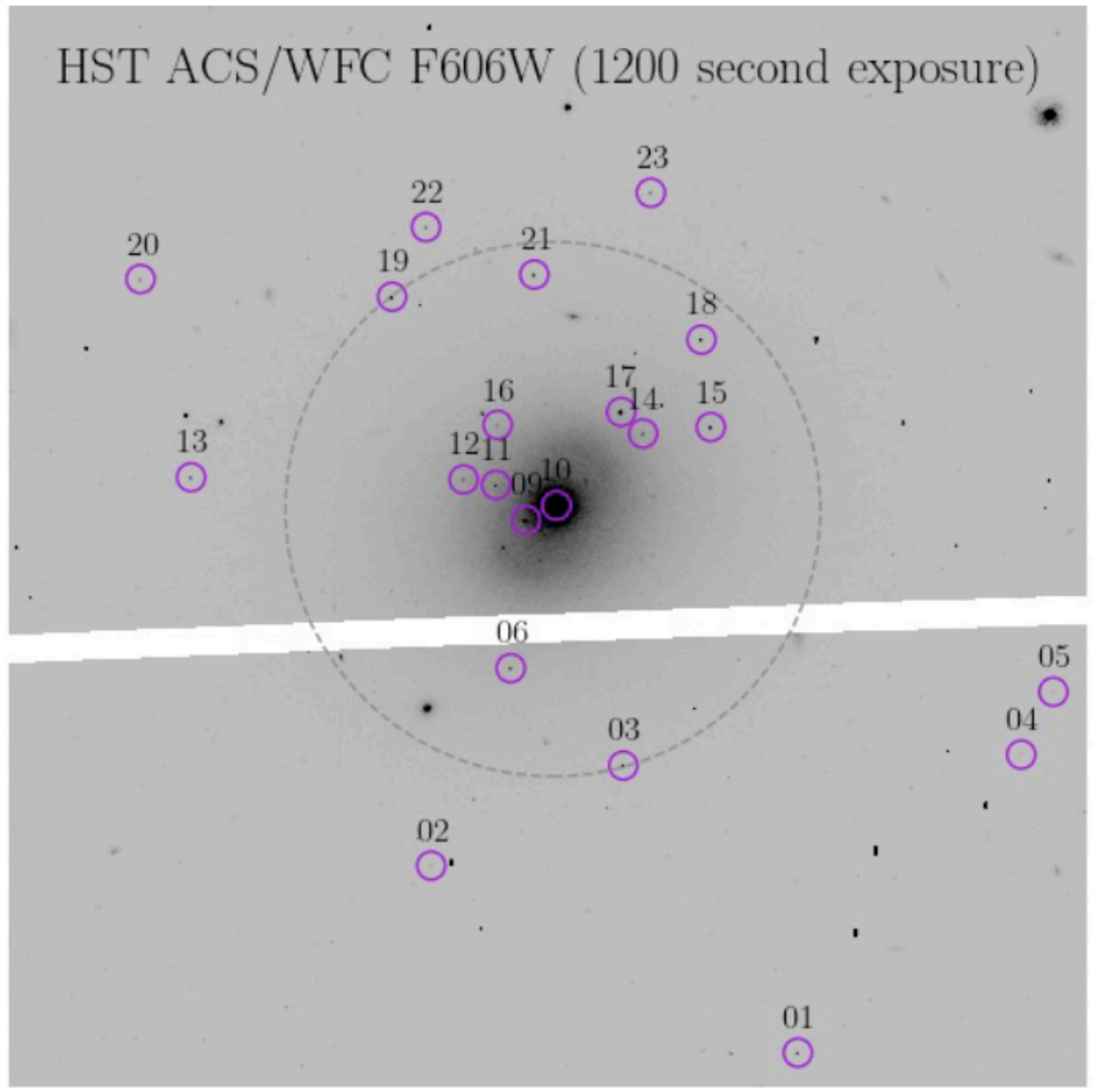


NFW. GCs of obs. mass, $R_e^{(GC)} = 2.5$ kpc



Modak, Danieli, Greene 2211.01384





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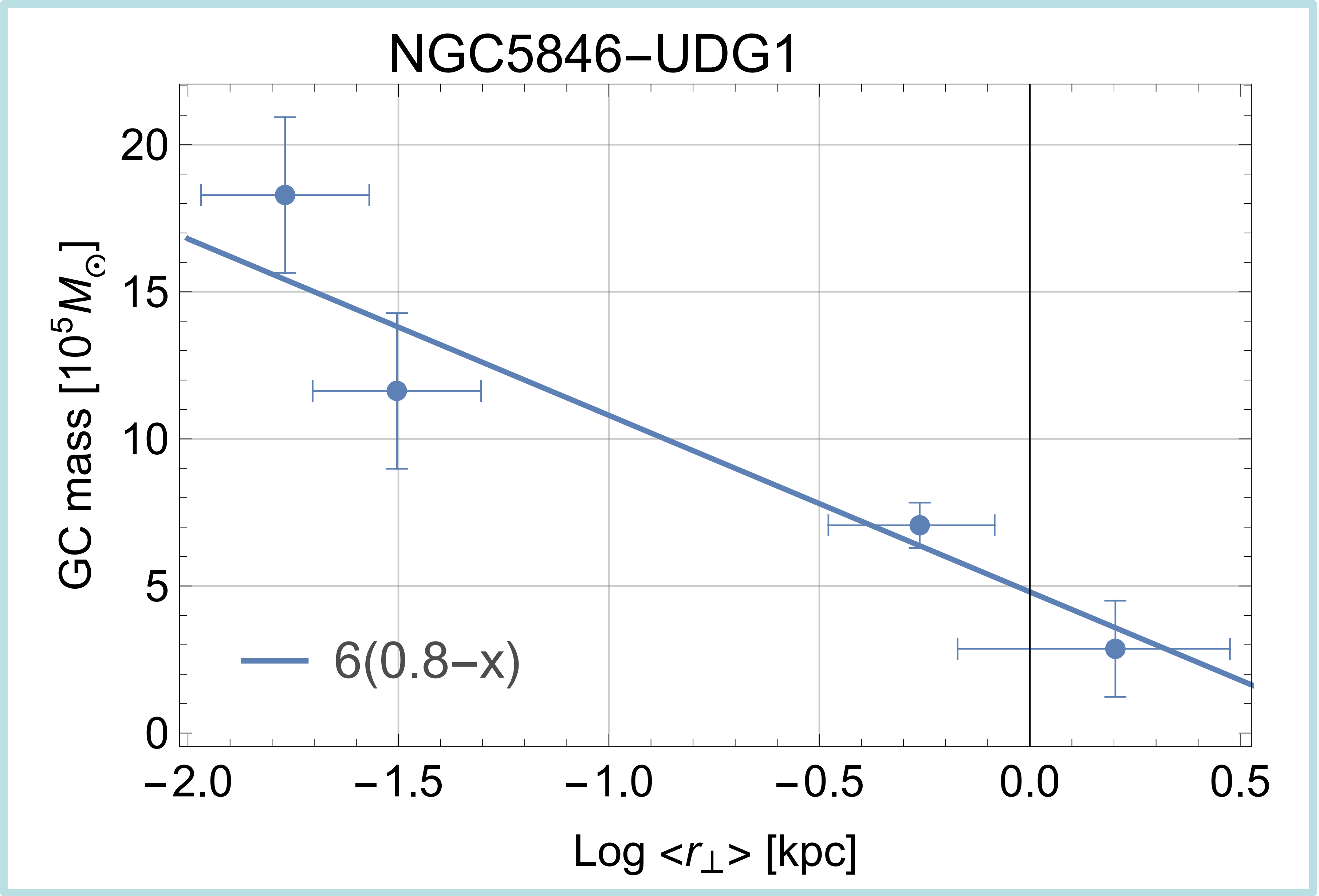
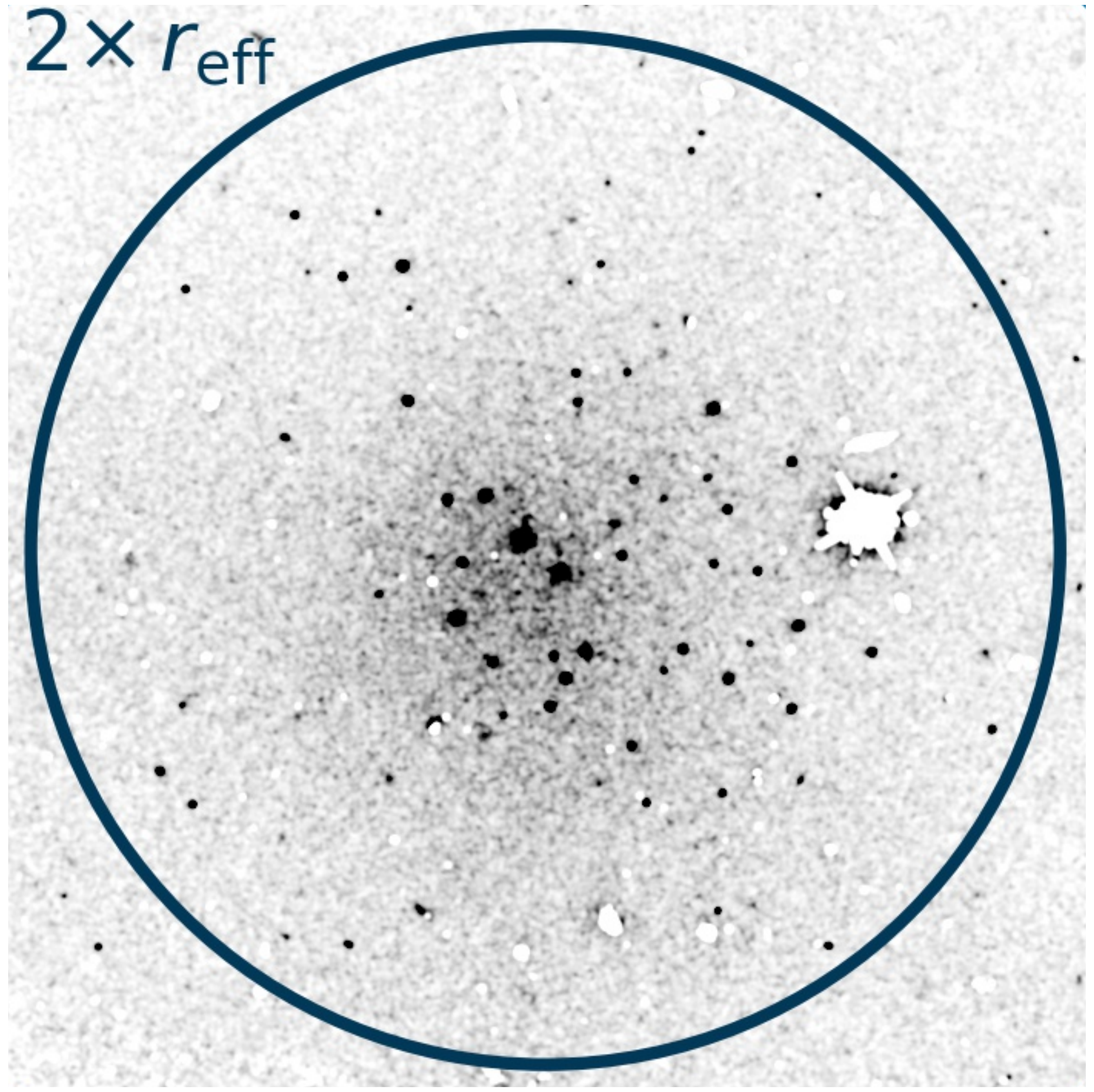
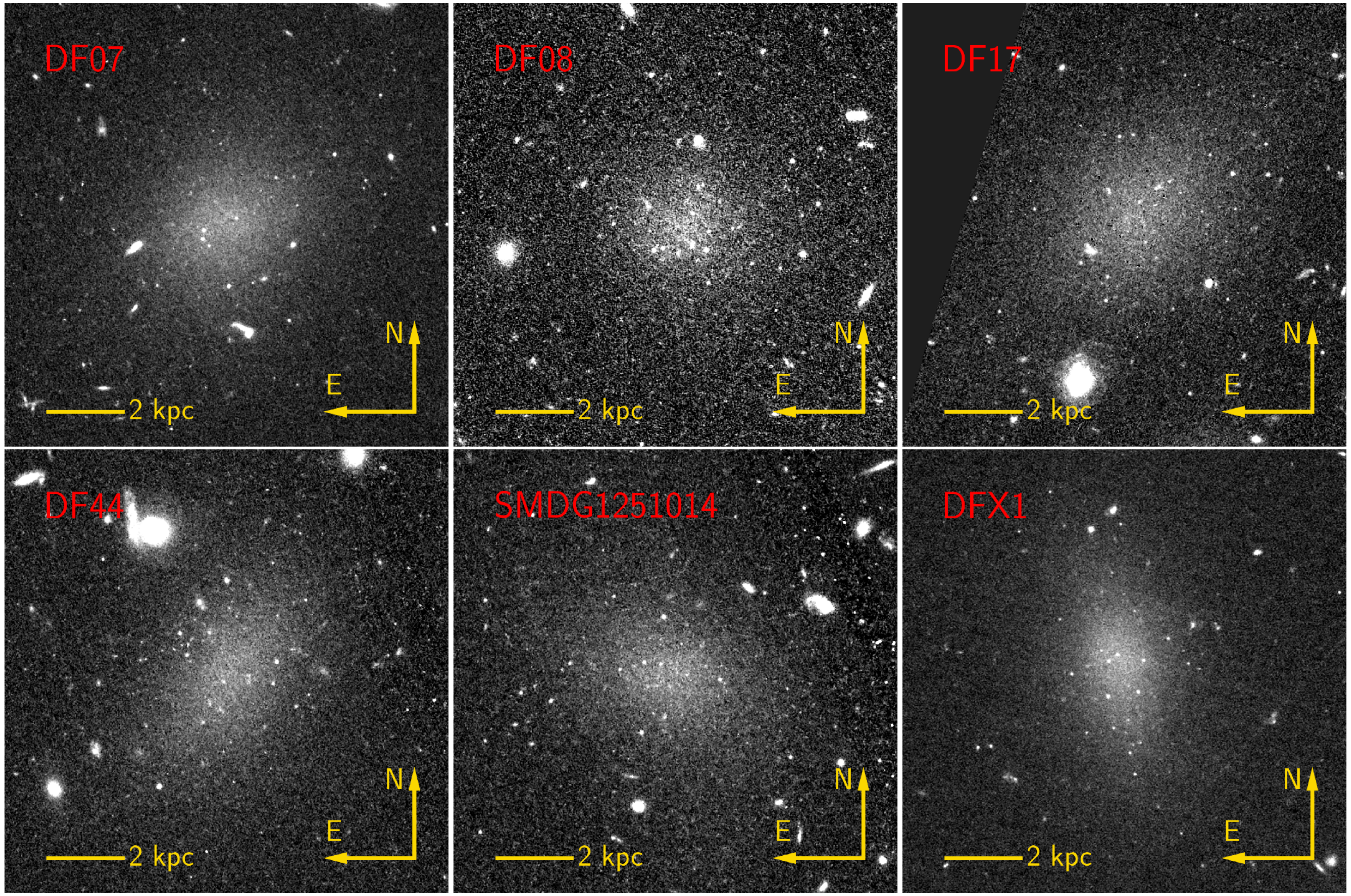




Many more UDGs/dwarfs to investigate.

e.g Saifollahi et al, 2201.11750: Coma cluster UDGs



Examples how it could become very interesting — ultralight dark matter

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It was suggested that Milky Way dwarf satellite galaxies may point to $m \sim 10^{-22}$ eV

[1] W. Hu, R. Barkana, and A. Gruzinov, Cold and Fuzzy Dark Matter, Phys. Rev. Lett. **85**, 1158 (2000).
[2] A. Arbey, J. Lesgourgues, and P. Salati, Quintessential haloes around galaxies, Phys. Rev. D **64**, 123528 (2001).
[3] J. Lesgourgues, A. Arbey, and P. Salati, A light scalar field at the origin of galaxy rotation curves, New Astron. Rev. **46**, 791 (2002).
[4] P.-H. Chavanis, Mass-radius relation of Newtonian self-gravitating Bose-Einstein condensates with short-range interactions: I. Analytical results, Phys. Rev. D **84**, 043531 (2011).
[5] P. H. Chavanis and L. Delfini, Mass-radius relation of Newtonian self-gravitating Bose-Einstein condensates with short-range interactions: II. Numerical results, Phys. Rev. D **84**, 043532 (2011).
[6] H.-Y. Schive, T. Chiueh, and T. Broadhurst, Cosmic structure as the quantum interference of a coherent dark wave, Nat. Phys. **10**, 496 (2014).
[7] H.-Y. Schive, M.-H. Liao, T.-P. Woo, S.-K. Wong, T. Chiueh, T. Broadhurst, and W. Y. P. Hwang, Understanding the Core-Halo Relation of Quantum Wave Dark Matter from 3D Simulations, Phys. Rev. Lett. **113**, 261302 (2014).
[8] D. J. E. Marsh and A.-R. Pop, Axion dark matter, solitons and the cusp-core problem, Mon. Not. R. Astron. Soc. **451**, 2479 (2015).
[9] S.-R. Chen, H.-Y. Schive, and T. Chiueh, Jeans analysis for Dwarf Spheroidal Galaxies in wave dark matter, Mon. Not. R. Astron. Soc. **468**, 1338 (2017).
[10] B. Schwabe, J. C. Niemeyer, and J. F. Engels, Simulations of solitonic core mergers in ultralight axion dark matter cosmologies, Phys. Rev. D **94**, 043513 (2016).
[11] J. Veltmaat and J. C. Niemeyer, Cosmological particle-in-cell simulations with ultralight axion dark matter, Phys. Rev. D **94**, 123523 (2016).
[12] L. Hui, J. P. Ostriker, S. Tremaine, and E. Witten, Ultralight scalars as cosmological dark matter, Phys. Rev. D **95**, 043541 (2017).
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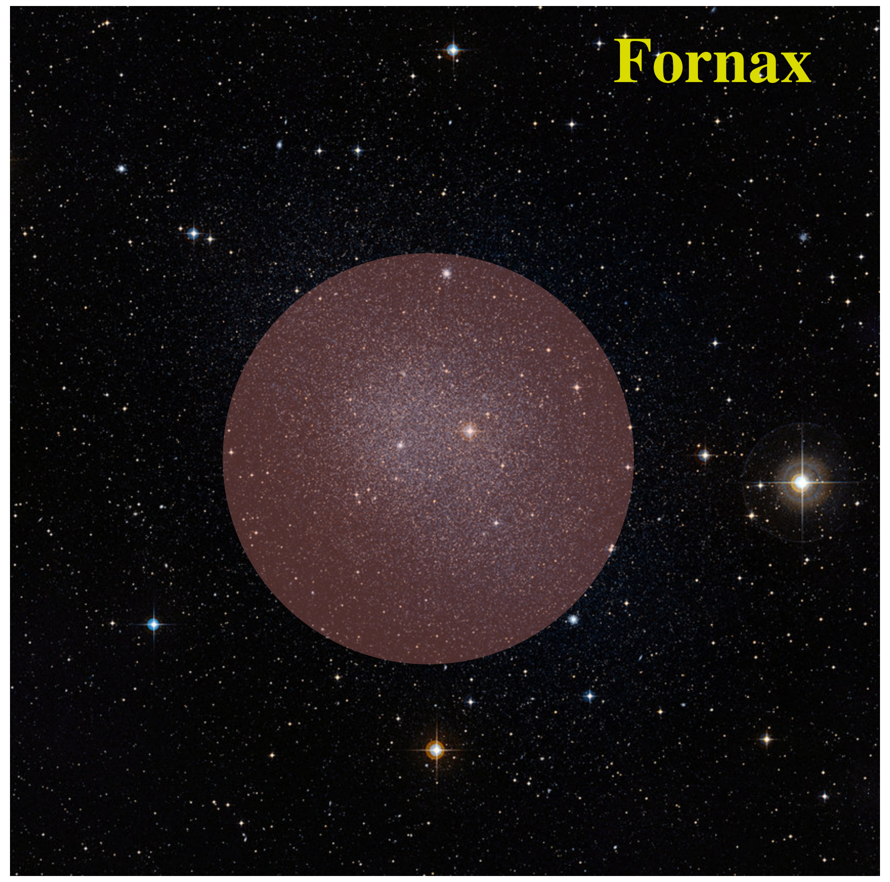

Examples how it could become very interesting — ultralight dark matter

The collision operator:

$$C[f_1] = \int \frac{d^3p'}{(2\pi)^3} \Big[S(\mathbf{p}', \mathbf{p}) f_1(p')(1 \pm f_1(p)) - S(\mathbf{p}, \mathbf{p}') f_1(p)(1 \pm f_1(p')) \Big],$$

Transfer function S:

$$S(\mathbf{p}, \mathbf{p}') \equiv \frac{(2\pi)^4}{2E_p 2E_{p'}} \int d\Pi_k d\Pi_{k'} \delta^{(4)}(p + k - p' - k') |\overline{\mathcal{M}}|^2 f_2(k) (1 + f_2(k'))$$

$$D_i(\mathbf{p}) = \frac{4\pi G^2 m^2 M^3 \ln \Lambda}{\mu_r} \frac{\partial}{\partial p^i} \left[h(\mathbf{p};\ f_2) + \frac{\mu_r}{M} h(\mathbf{p};\ f_2^2) \right]$$

$$D_{ij}(\mathbf{p}) = 4\pi G^2 m^2 M^4 \ln \Lambda \frac{\partial^2}{\partial p^i \partial p^j} \left[g(\mathbf{p};\ f_2) + g(\mathbf{p};\ f_2^2) \right]$$

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→ Dynamical *heating*

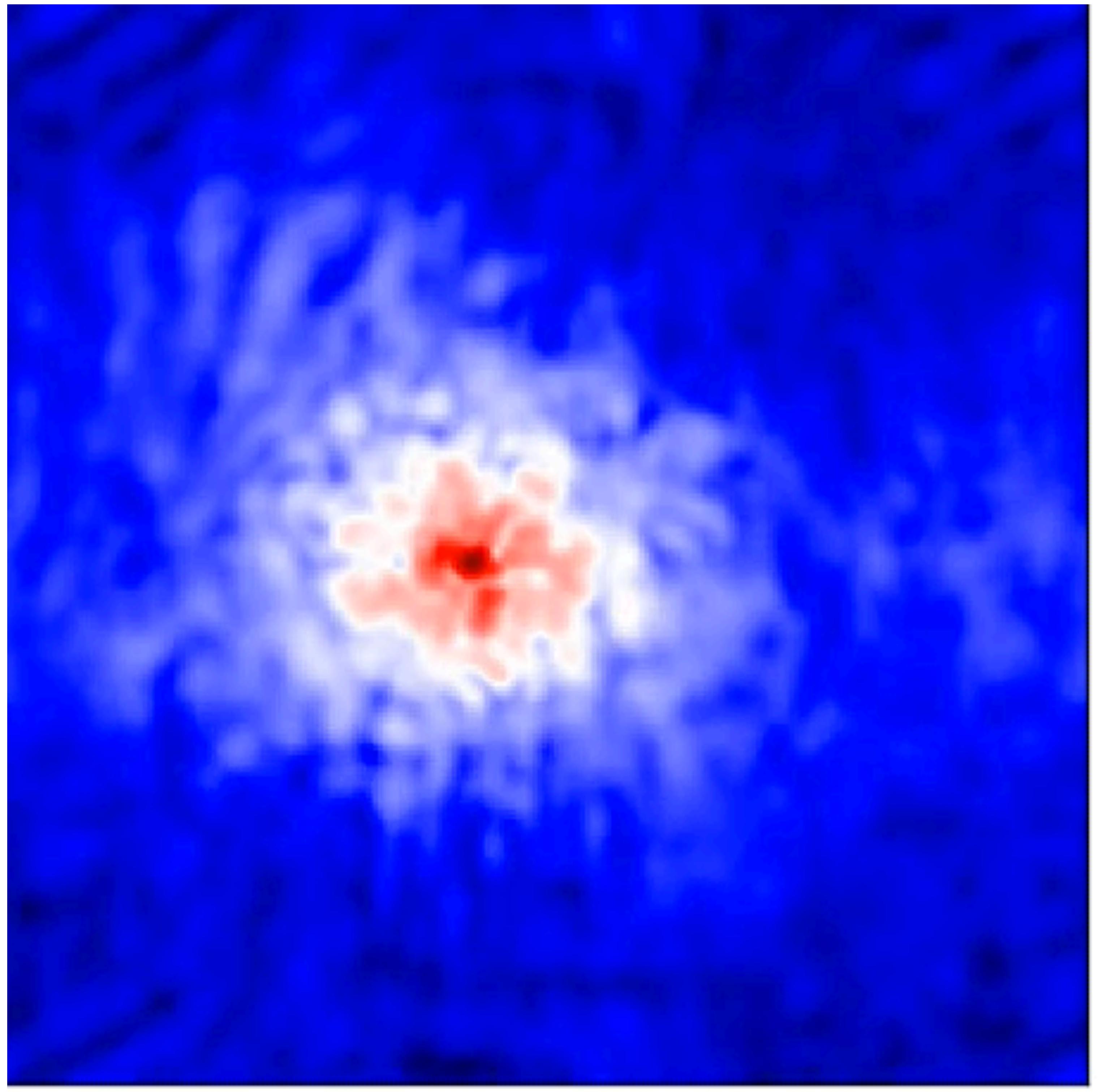
Examples how it could become very interesting — ultralight dark matter

Effective **quasi-particles**
(Bar-Or, Fouvry, Tremaine 1809.07673)

$$m_{\rm eff} = \frac{\pi^{3/2}\hbar^3\rho_{\rm b}}{m_{\rm b}^3\sigma^3} = \rho_{\rm b}\left(f\lambda_\sigma\right)^3$$

$$\lambda_\sigma = h/(m_{\rm b}\sigma)$$

$$f = 1/(2\sqrt{\pi}) = 0.282.$$

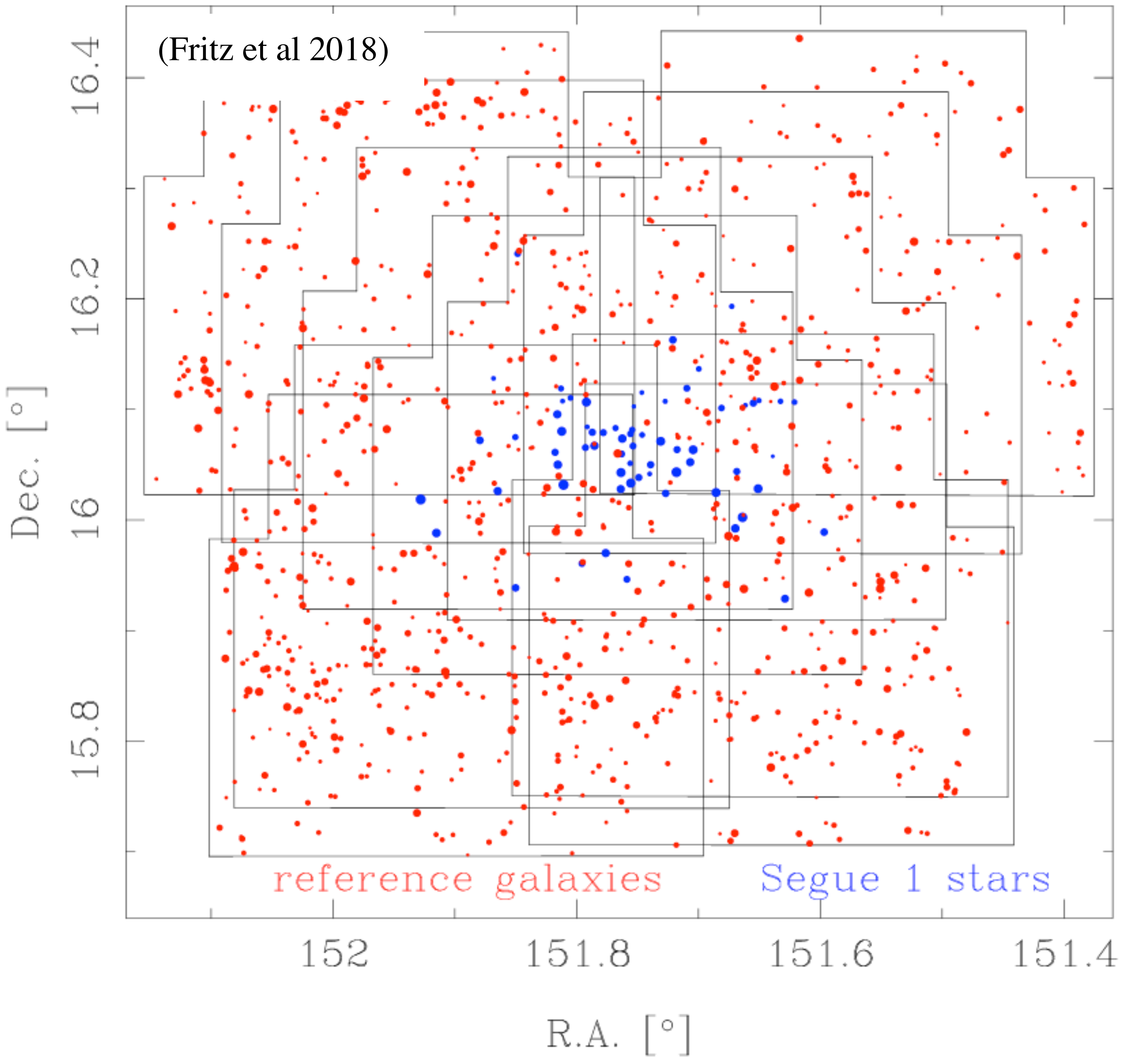


Dynamical *heating*

Examples how it could become very interesting — ultralight dark matter

Dalal, Kravstov 2203.05750:
would have dispersed star cluster in Segue-I?

$$m_{\rm eff} \approx 430\ \mathrm{M}_\odot \left(\frac{10\ \mathrm{km/s}}{\sigma}\right)^3 \left(\frac{\rho}{10^7\ \mathrm{M}_\odot/\mathrm{kpc}^3}\right)\left(\frac{10^{-20}\ \mathrm{eV}}{m}\right)^3$$



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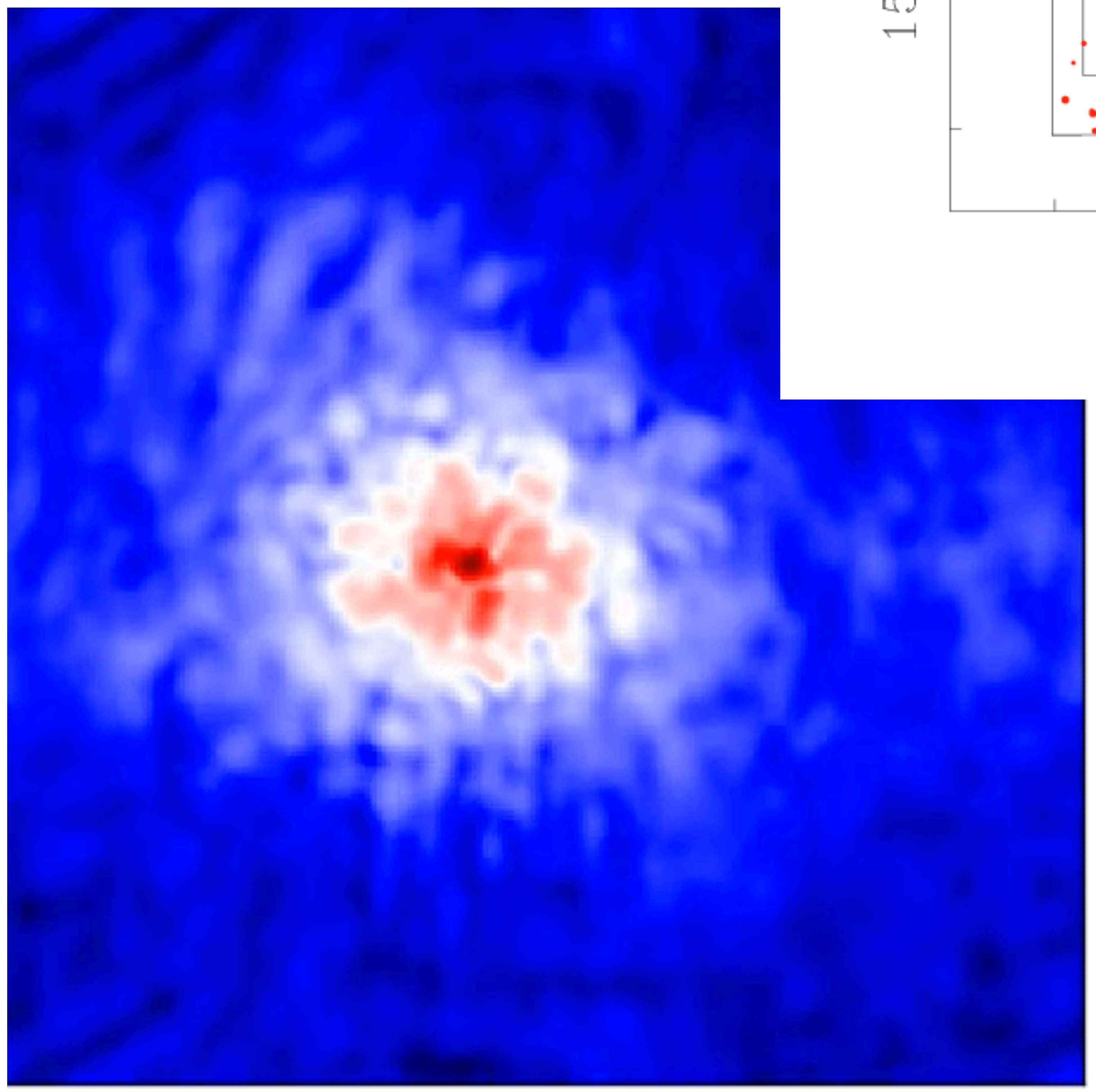
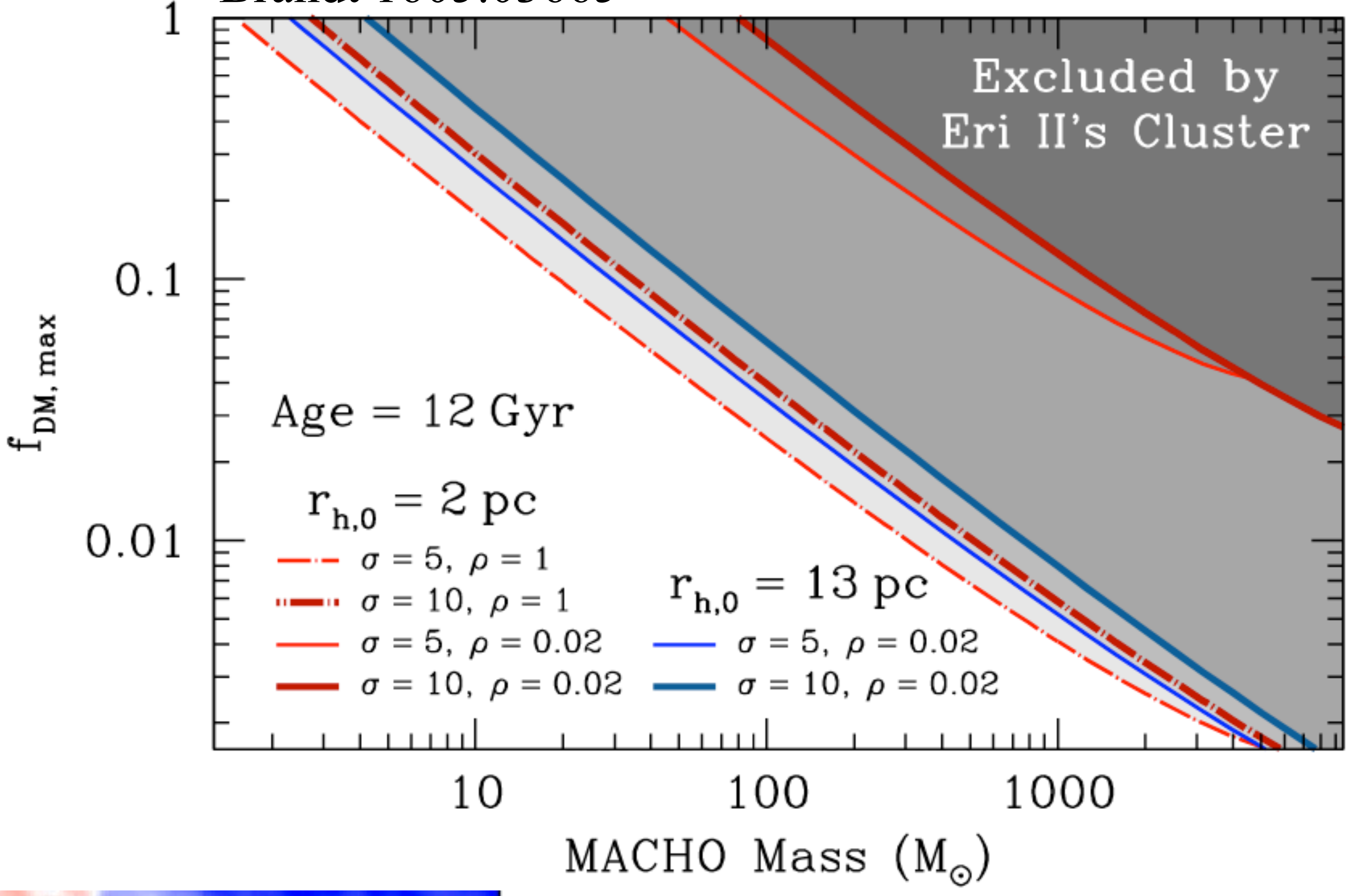


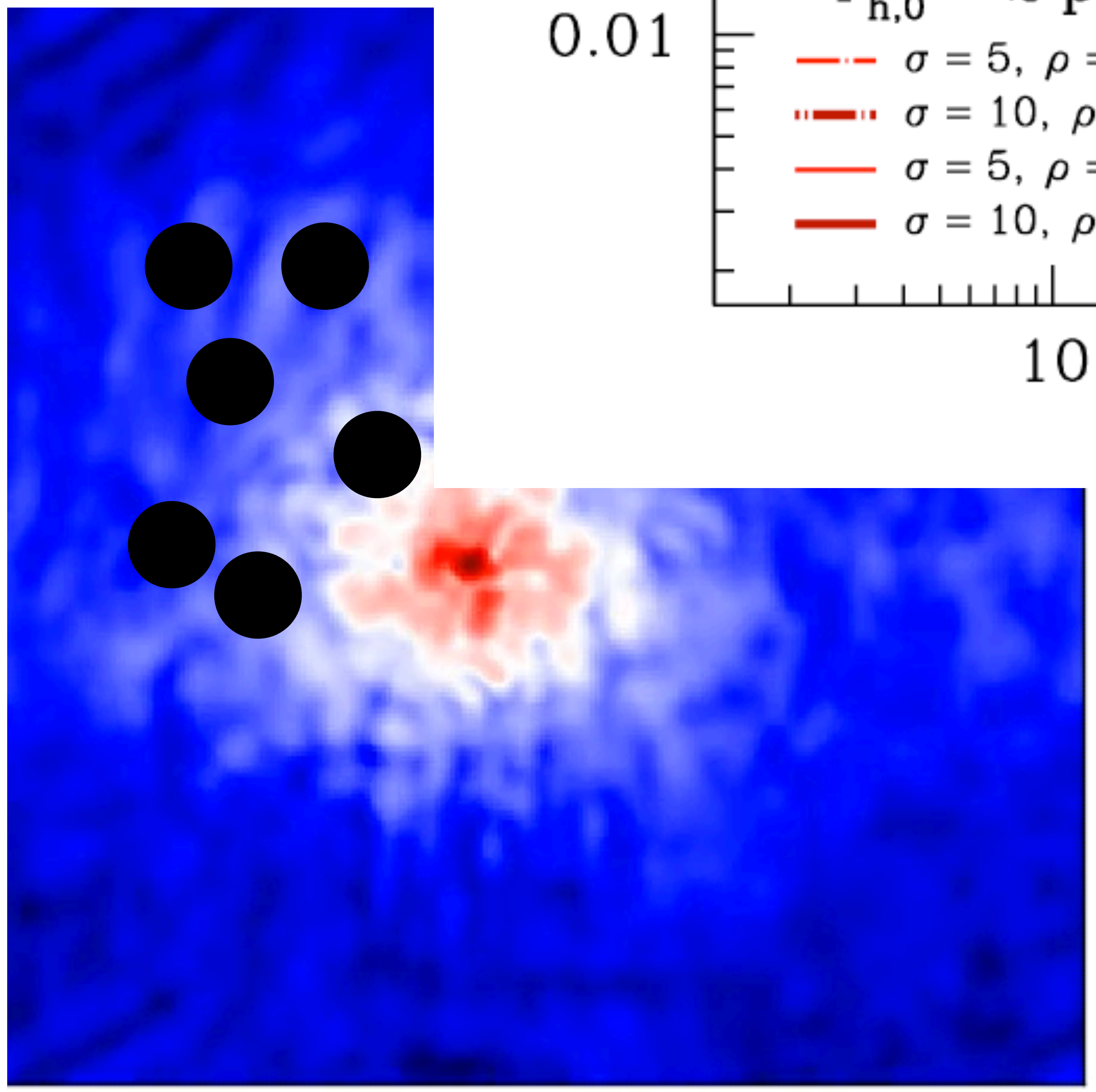
Dynamical *heating*

Examples how it could become very interesting — ultralight dark matter

Amusing fact:
Dynamical heating constraints on ultralight dark matter come from the same mechanism that constrains MACHO or PBH dark matter

Brandt 1605.03665





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Examples how it could become very interesting — ultralight dark matter

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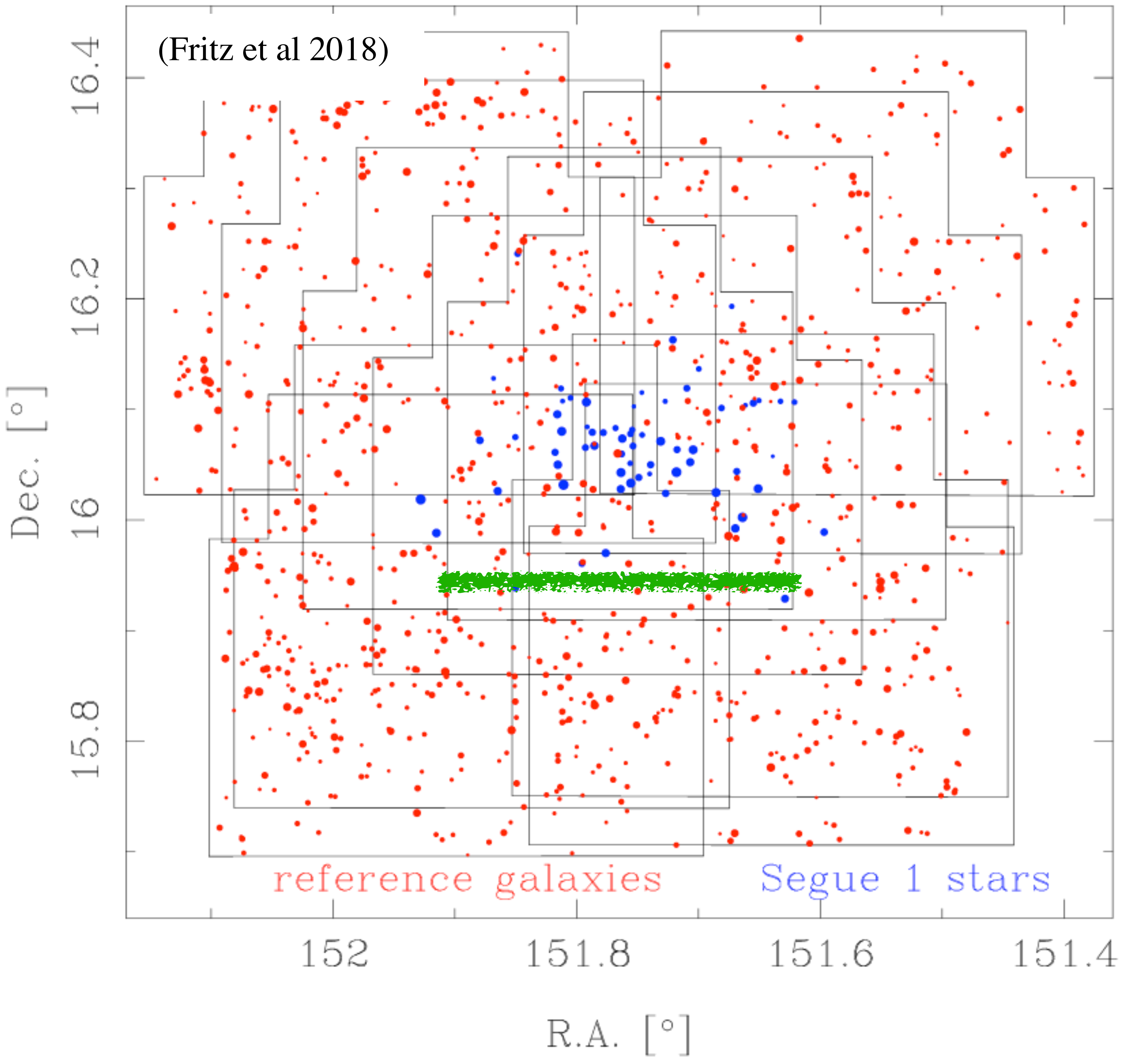
Hypothesis:
If you go far enough in the extreme right, you end up in the extreme left.

Examples how it could become very interesting — ultralight dark matter

Dalal, Kravstov 2203.05750:
would have dispersed star cluster in Segue-I?

$$m_{\rm eff} \approx 430\ {\rm M}_\odot \left(\frac{10\ {\rm km/s}}{\sigma}\right)^3 \left(\frac{\rho}{10^7\ {\rm M}_\odot/{\rm kpc}^3}\right) \left(\frac{10^{-20}\ {\rm eV}}{m}\right)^3$$

$$\lambda \approx 120\ {\rm pc} \left(\frac{10^{-20}\ {\rm eV}}{m}\right) \left(\frac{10\ {\rm km/s}}{\sigma}\right)$$

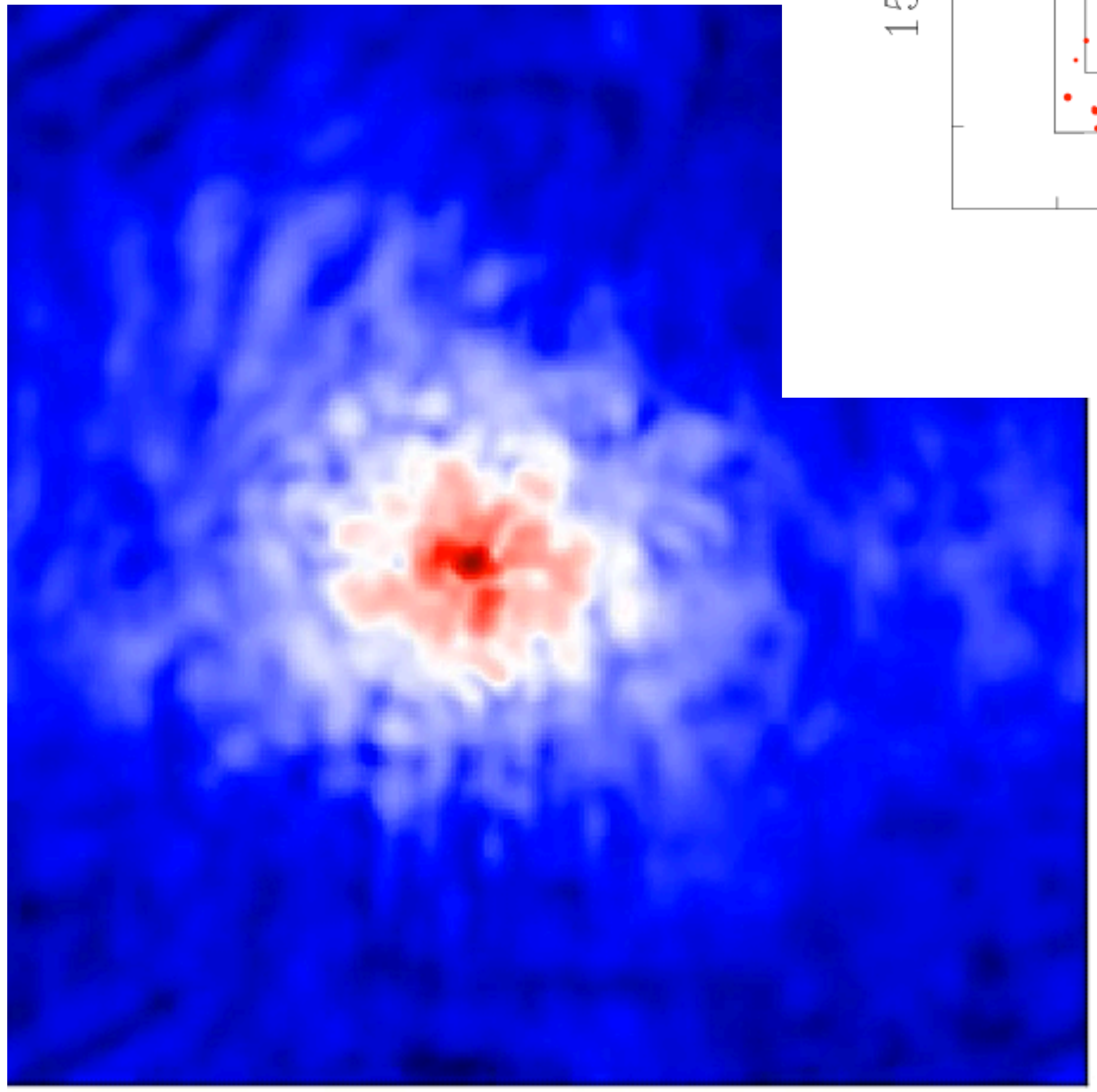


Effective **quasi-particles**
(Bar-Or, Fouvry, Tremaine 1809.07673)

$$m_{\rm eff} = \frac{\pi^{3/2}\hbar^3 \rho_{\rm b}}{m_{\rm b}^3 \sigma^3} = \rho_{\rm b} \left(f\lambda_\sigma\right)^3,$$

$$\lambda_\sigma = h/(m_{\rm b}\sigma)$$

$$f = 1/(2\sqrt{\pi}) = 0.282.$$



Dynamical *heating*

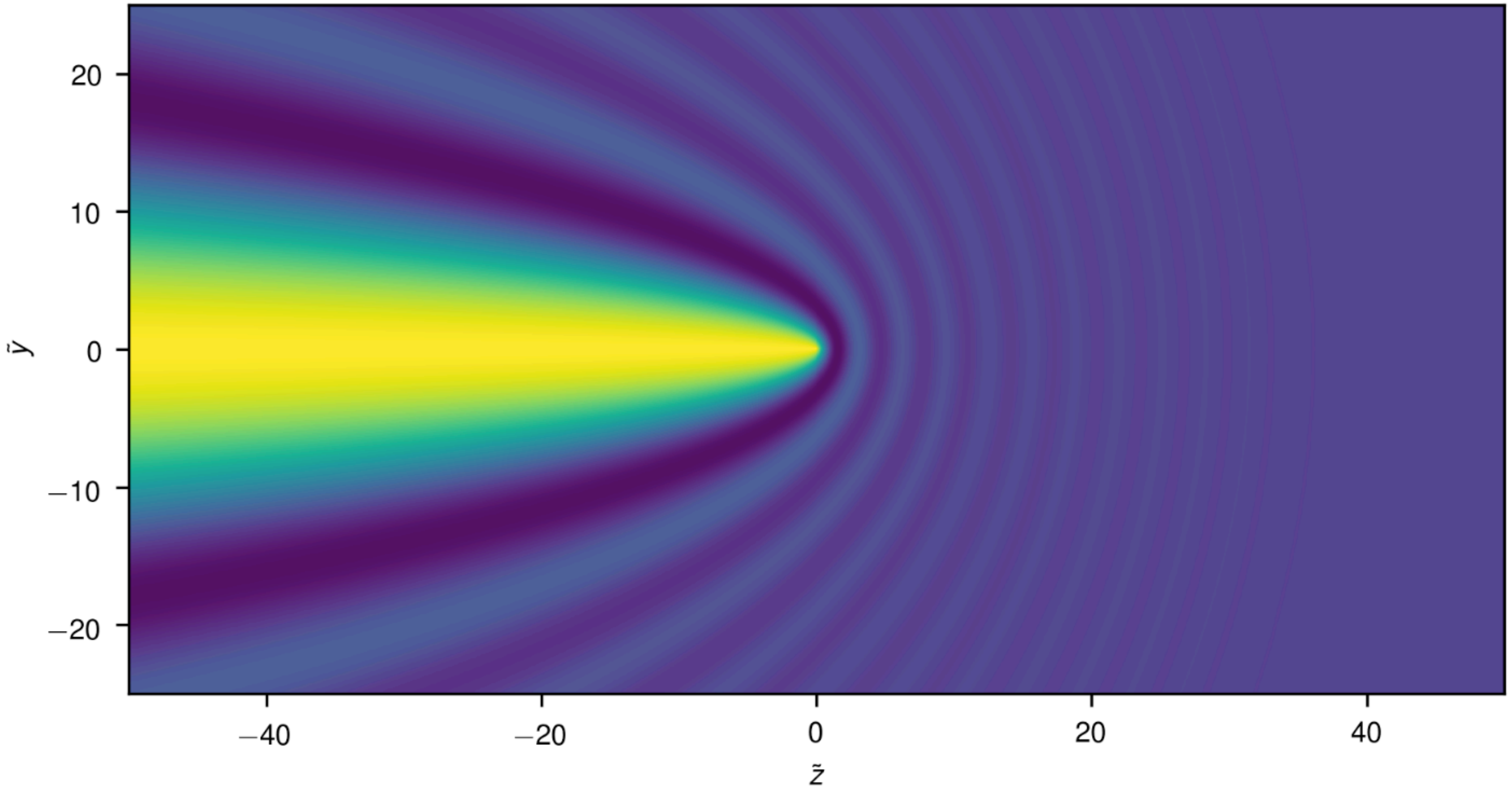
Examples how it could become very interesting — ultralight dark matter

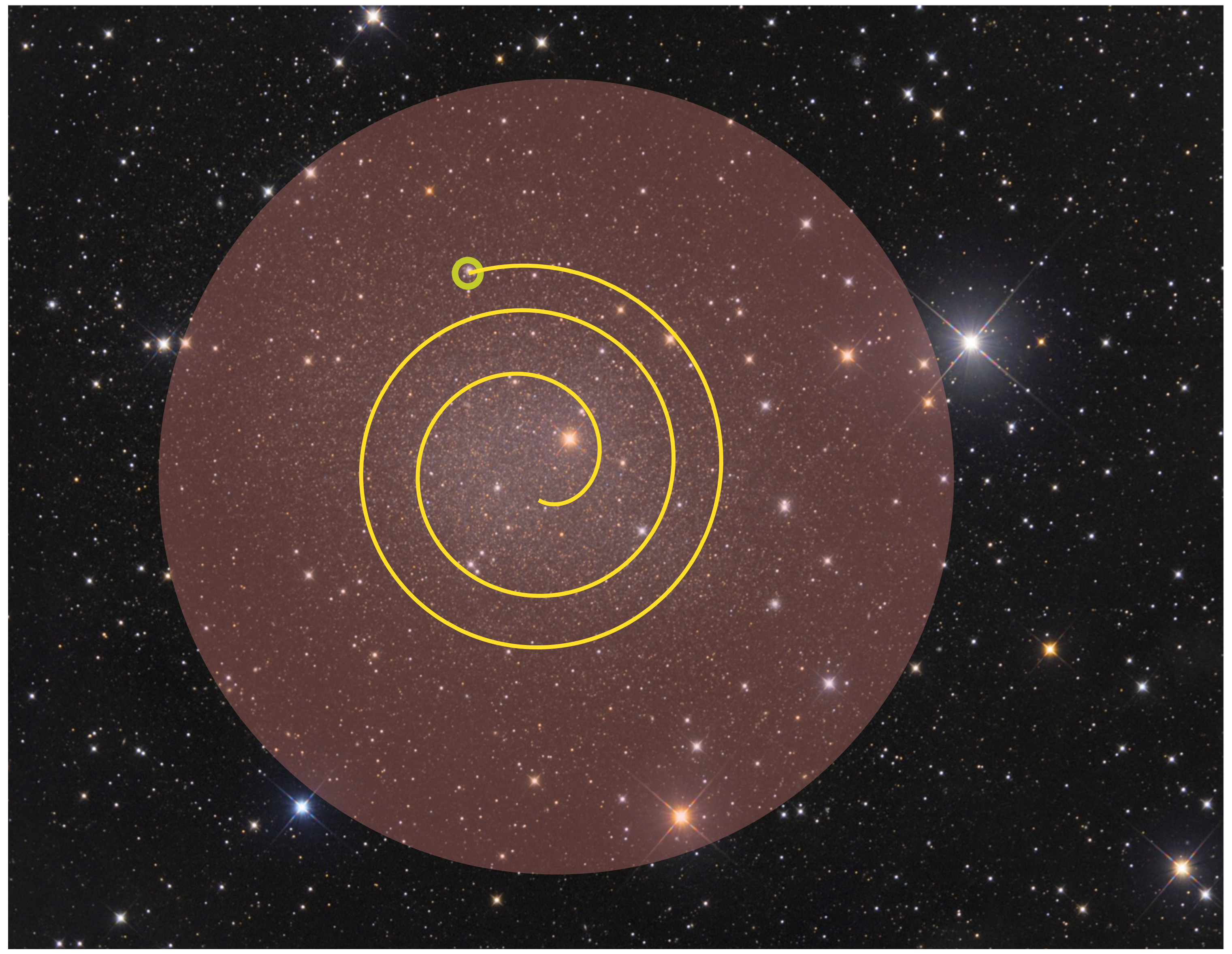
If the system is entirely inside the coherent region (the *soliton*), dynamical friction is suppressed

Hui et al, 1610.08297; Bar-Or, Fouvry, Tremaine, 1809.07673; 2010.10212

Proposed for Fornax GC timing puzzle (Hui et al 2016).

Lancaster et al, 1909.06381





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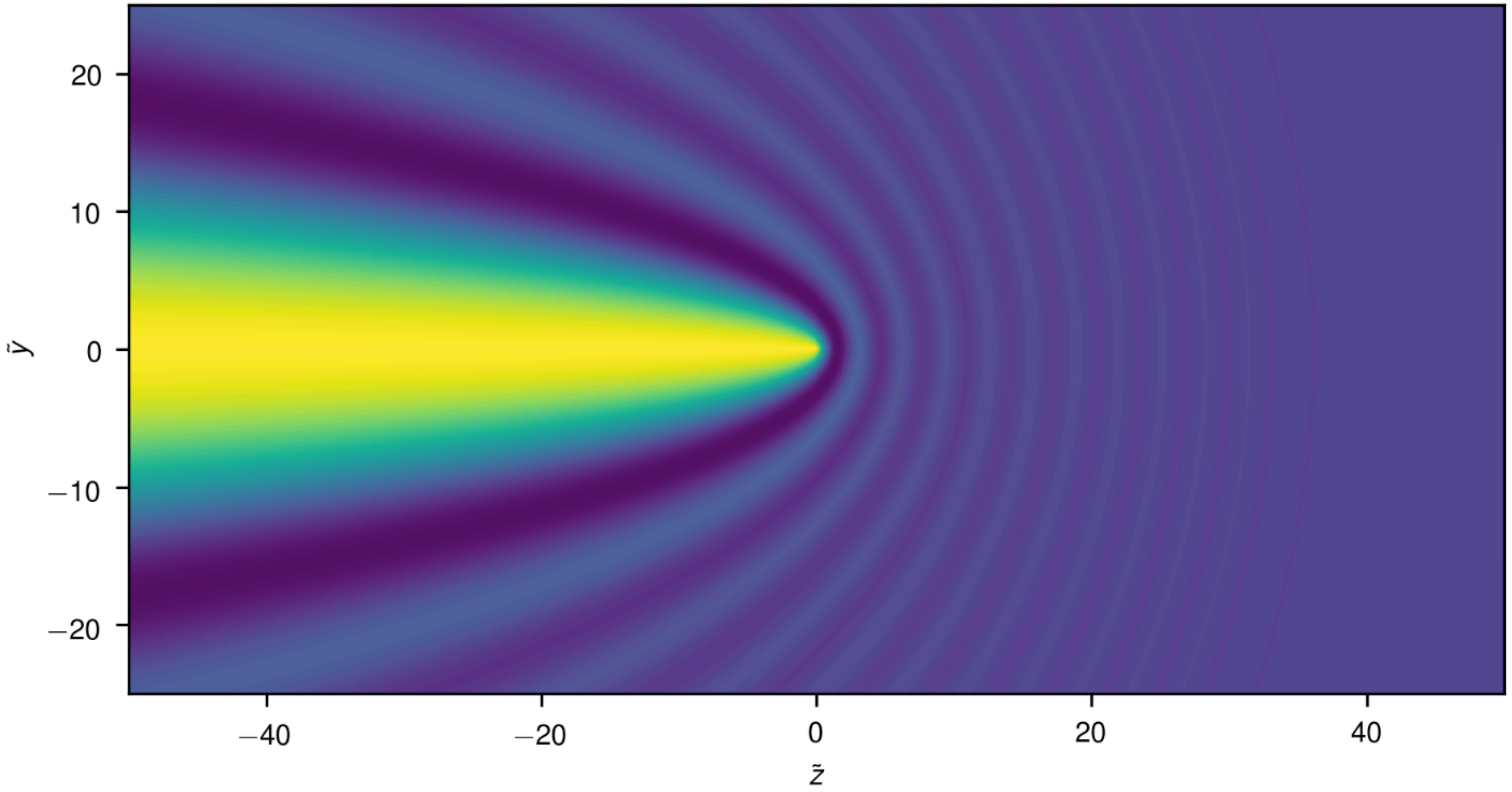
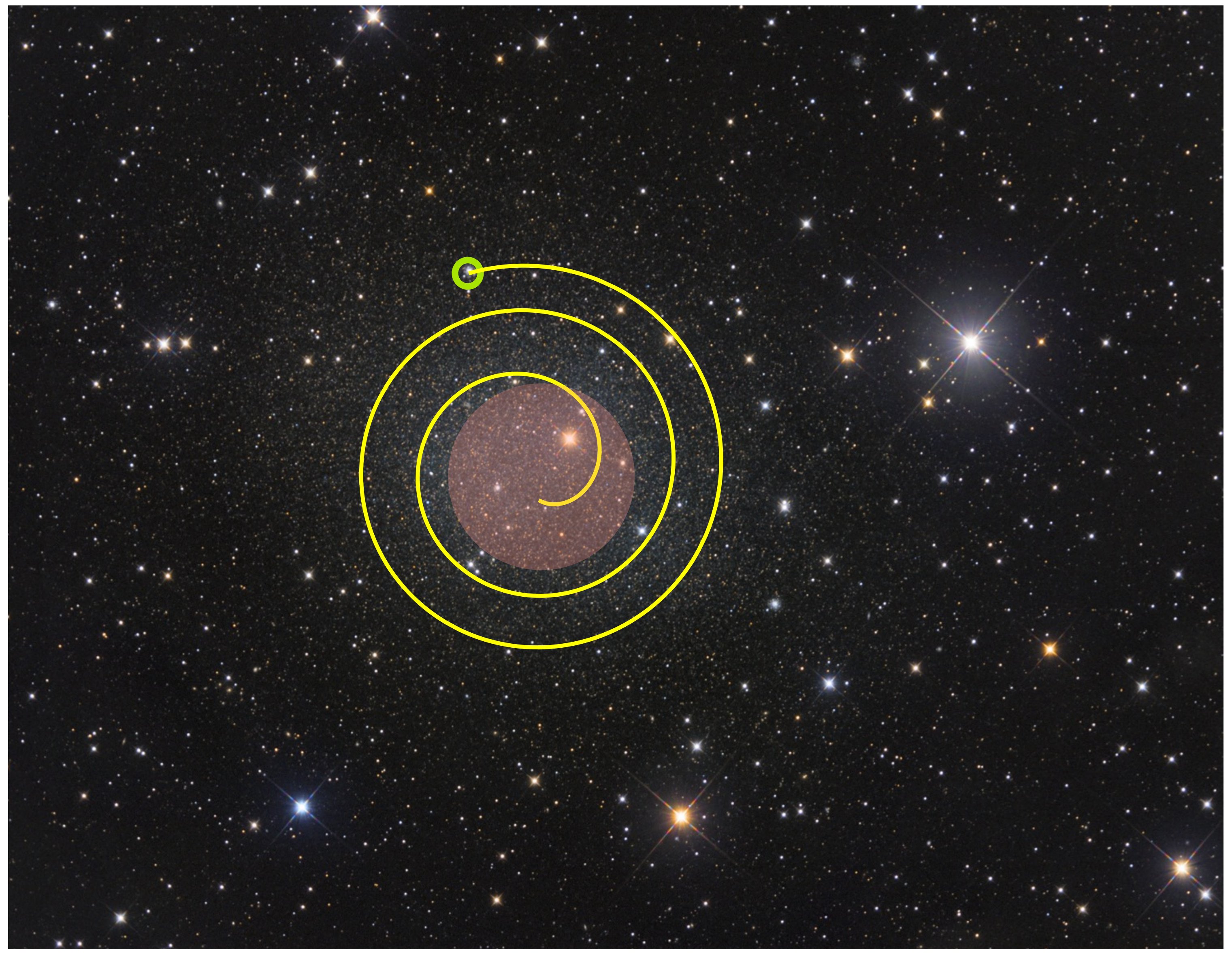
Proposed for Fornax GC timing puzzle (Hui et al 2016).
But only works for $m < 10^{-21}$ eV (Lancaster et al 2019)
in tension w/ LSBGs, Ly-α which suggest $m > 10^{-21}$ eV





Examples how it could become very interesting — light *fermion* dark matter

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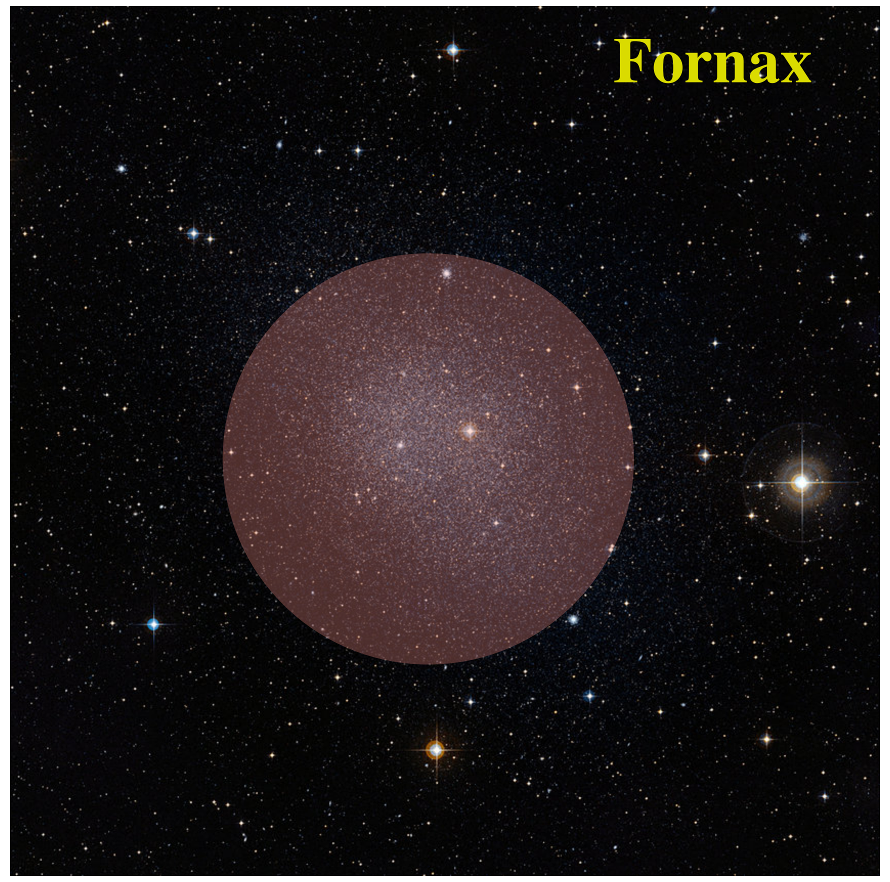
It was suggested that Milky Way dwarf satellite galaxies may point to degenerate fermion dark matter with $m \sim 200$ eV

Domcke, Urbano, 1409.3167
Randall, Scholtz, Unwin, 1611.04590



Examples how it could become very interesting — light *fermion* dark matter

The collision operator:

$$C[f_1] = \int \frac{d^3p'}{(2\pi)^3} \Big[S(\mathbf{p}', \mathbf{p}) f_1(p')(1 \pm f_1(p)) - S(\mathbf{p}, \mathbf{p}') f_1(p)(1 \pm f_1(p')) \Big],$$

Transfer function S:

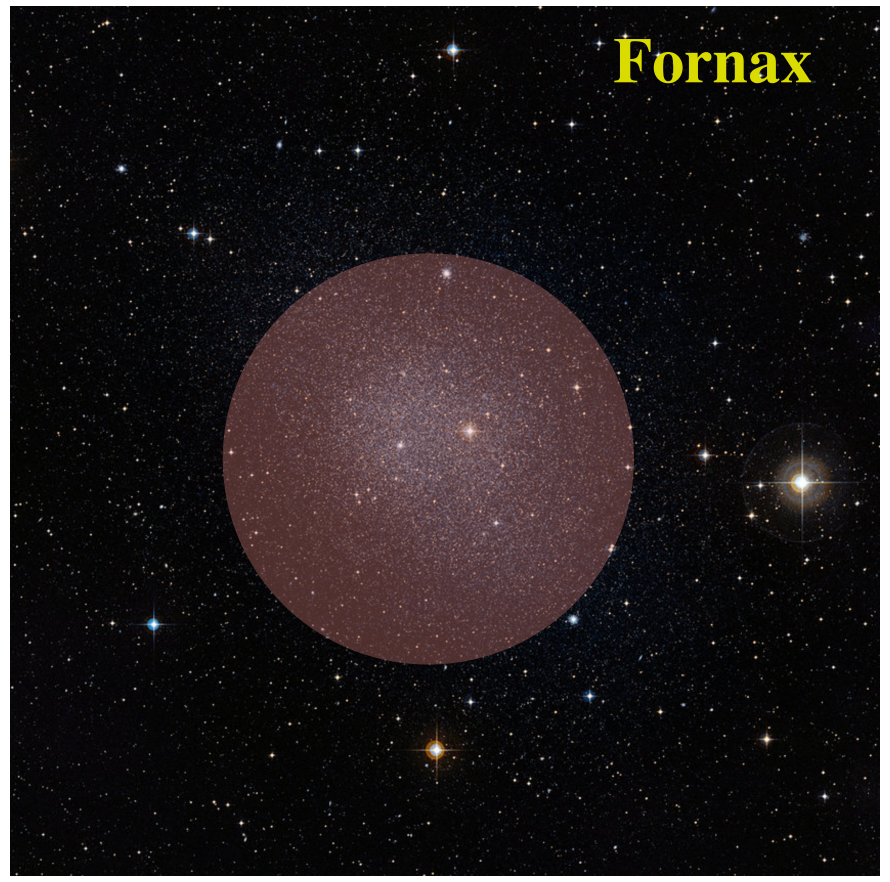
$$S(\mathbf{p}, \mathbf{p}') \equiv \frac{(2\pi)^4}{2E_p 2E_{p'}} \int d\Pi_k d\Pi_{k'} \delta^{(4)}(p + k - p' - k') |\overline{\mathcal{M}}|^2 f_2(k)(1 - f_2(k'))$$

Reddy, Prakash, Lattimer, astro-ph/9710115
Bertoni, Nelson, Reddy, 1309.1721
Bar et al, 2102.11522

$$C_{\mathrm{df}} \to \frac{V^3}{v_F^3}$$ instead of the classical gas result: $$C_{\mathrm{df}} \to \frac{\sqrt{2}V^3}{3\sqrt{\pi}\sigma^3}$$

Examples how it could become very interesting — light *fermion* dark matter (degenerate dark matter — DDM)

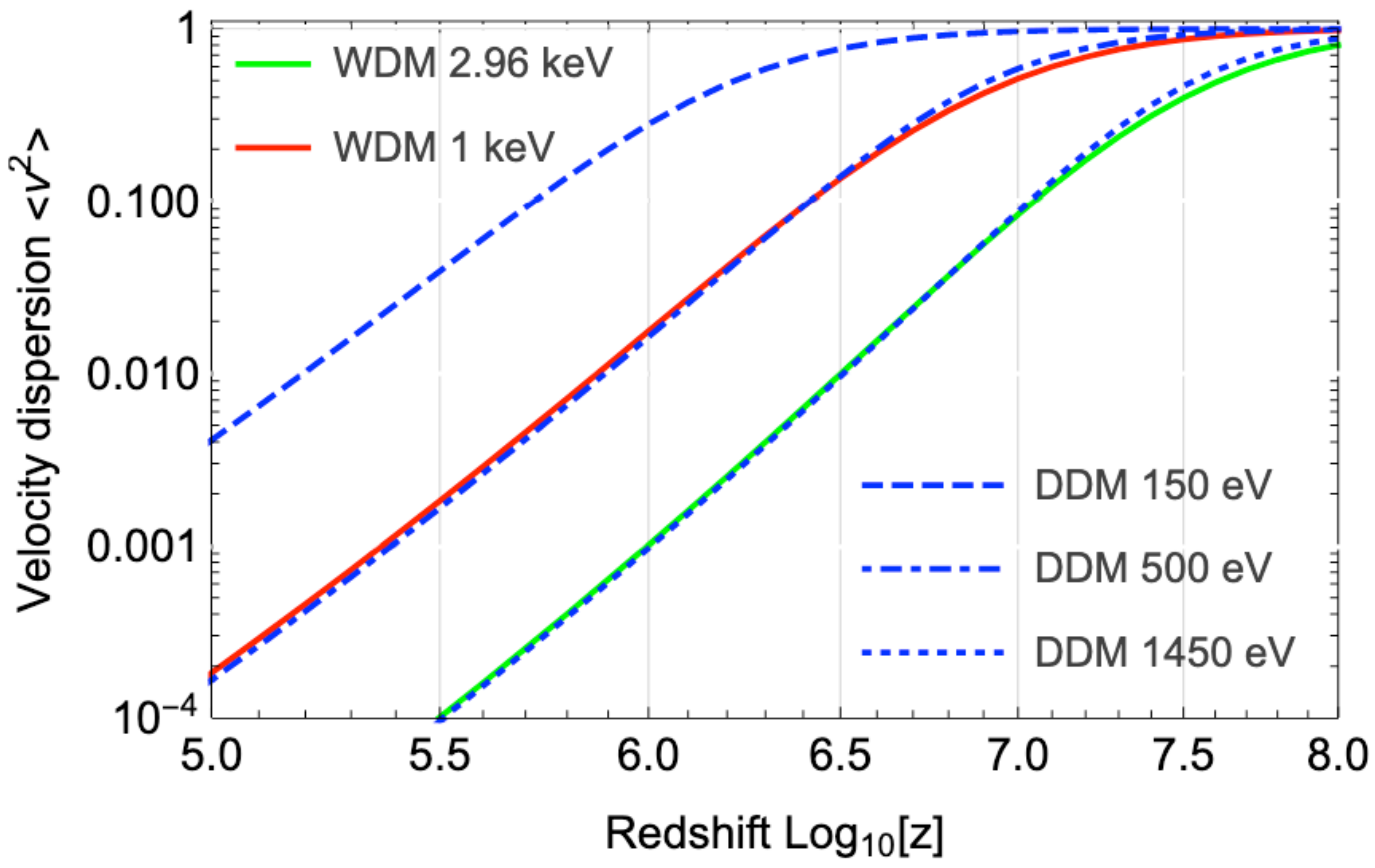
Bar et al, 2102.11522



DDM must be hot at high redshift due to unavoidable degeneracy pressure.

The minimal possible velocity dispersion can be compared with "standard" hot dark matter.

Ly-α limit $m > 2.96$ eV (Baur et al, 1512.01981) rules out dwarf galaxy cores as proposed in Domcke, Urbano, 1409.3167; Randall, Scholtz, Unwin, 1611.04590



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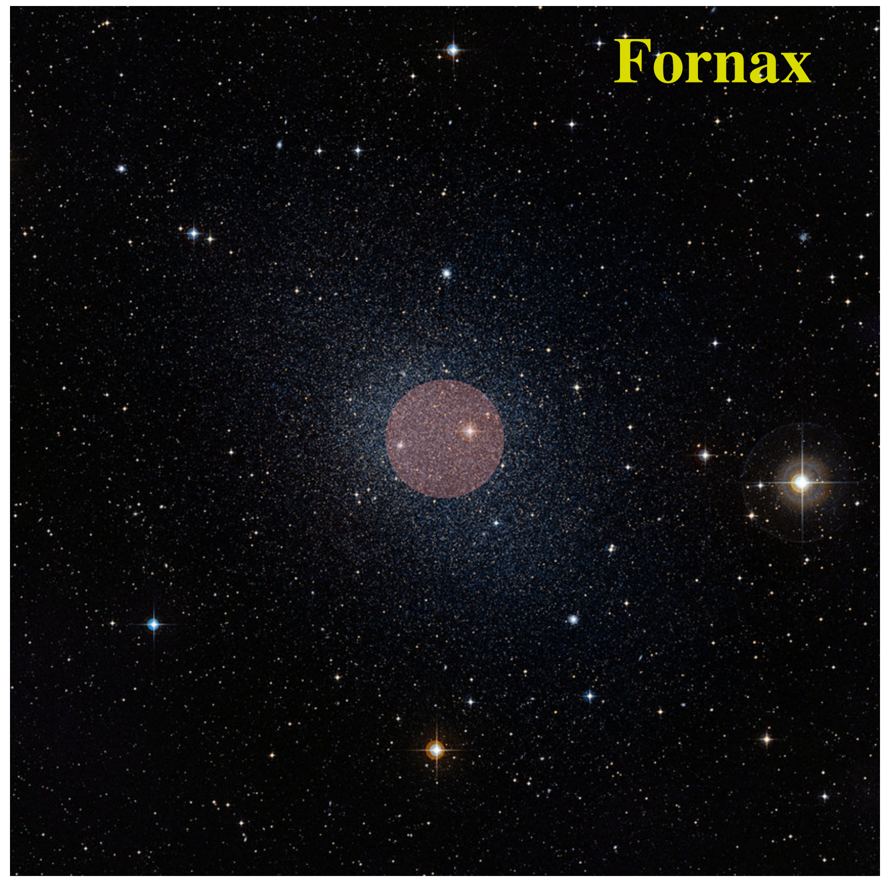

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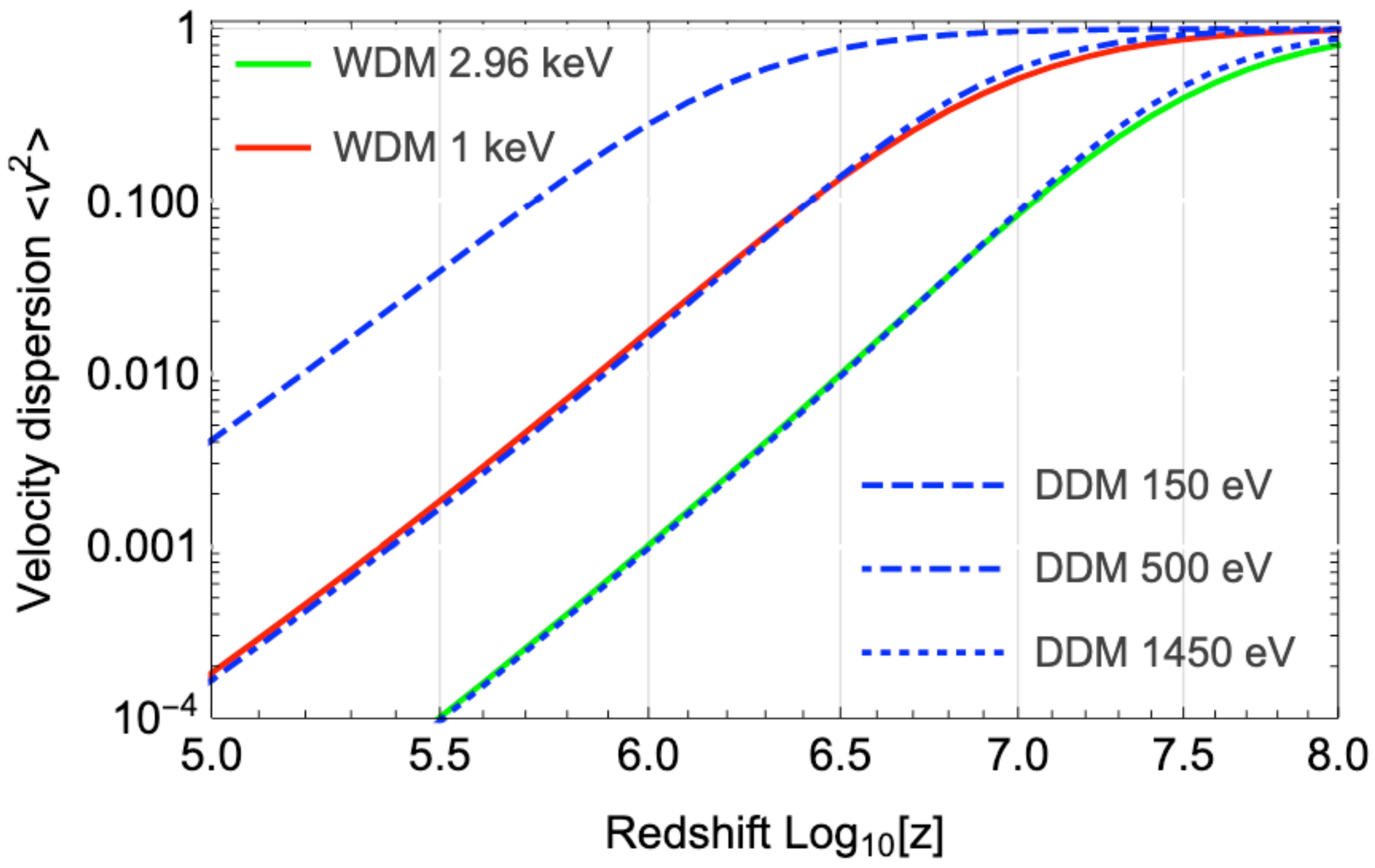
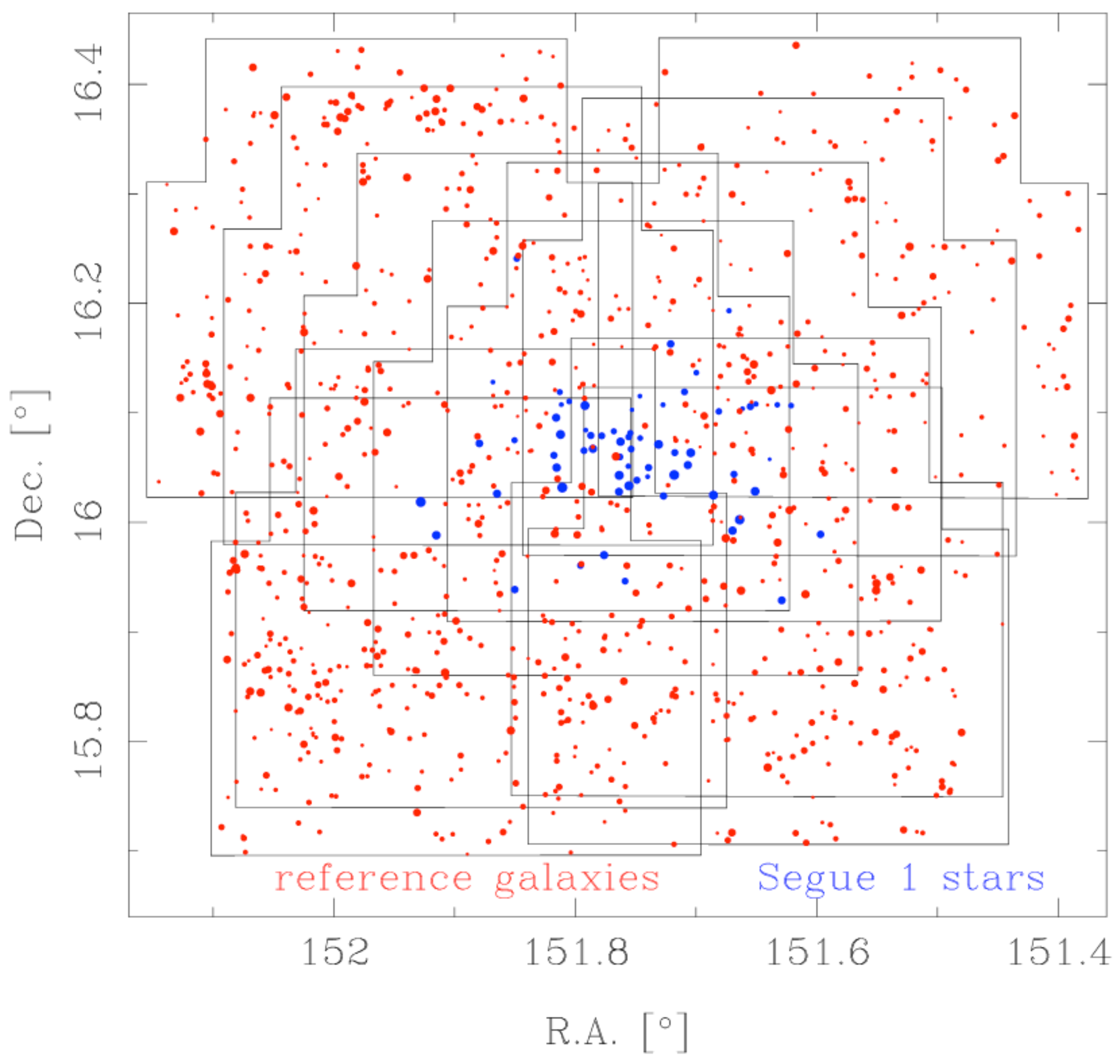


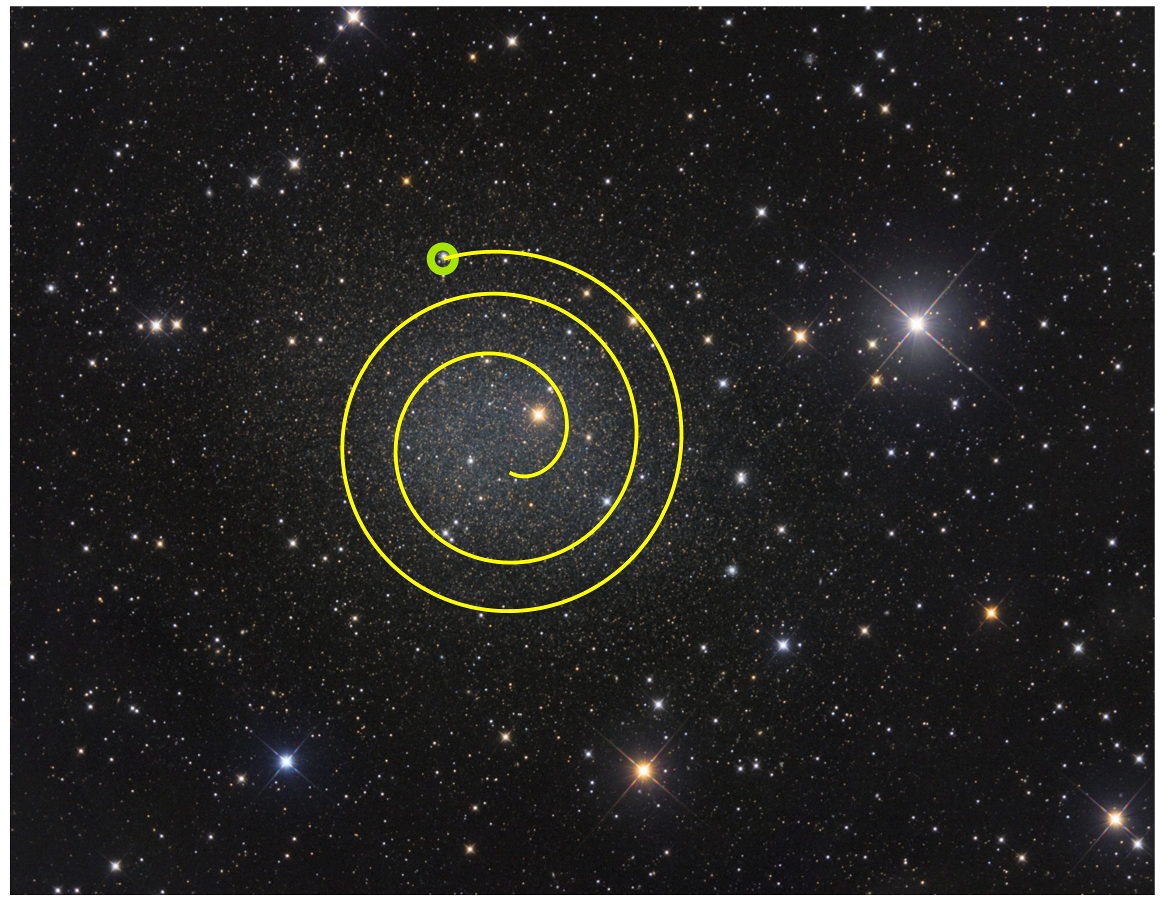
Gravity alone

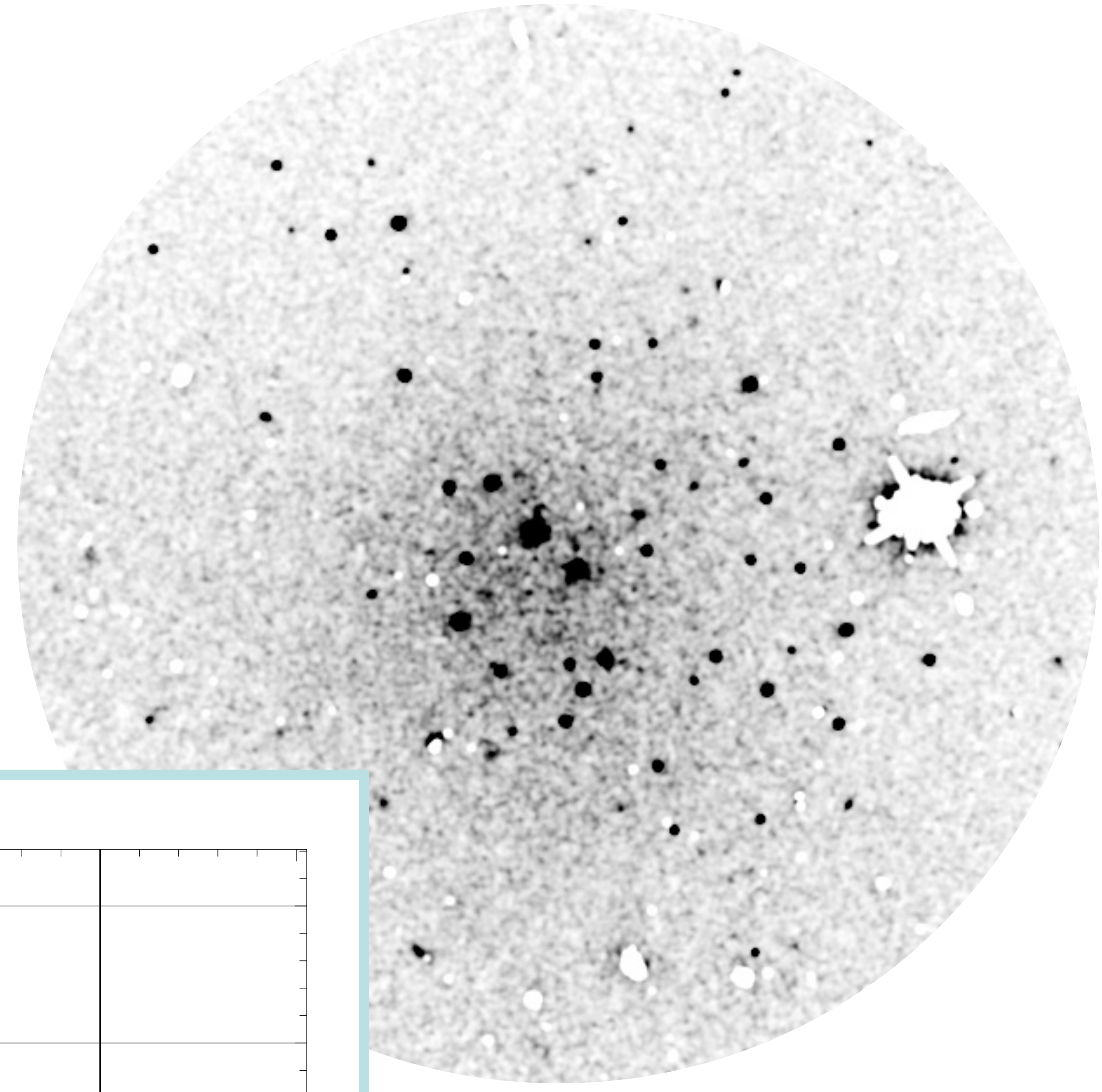
m_X

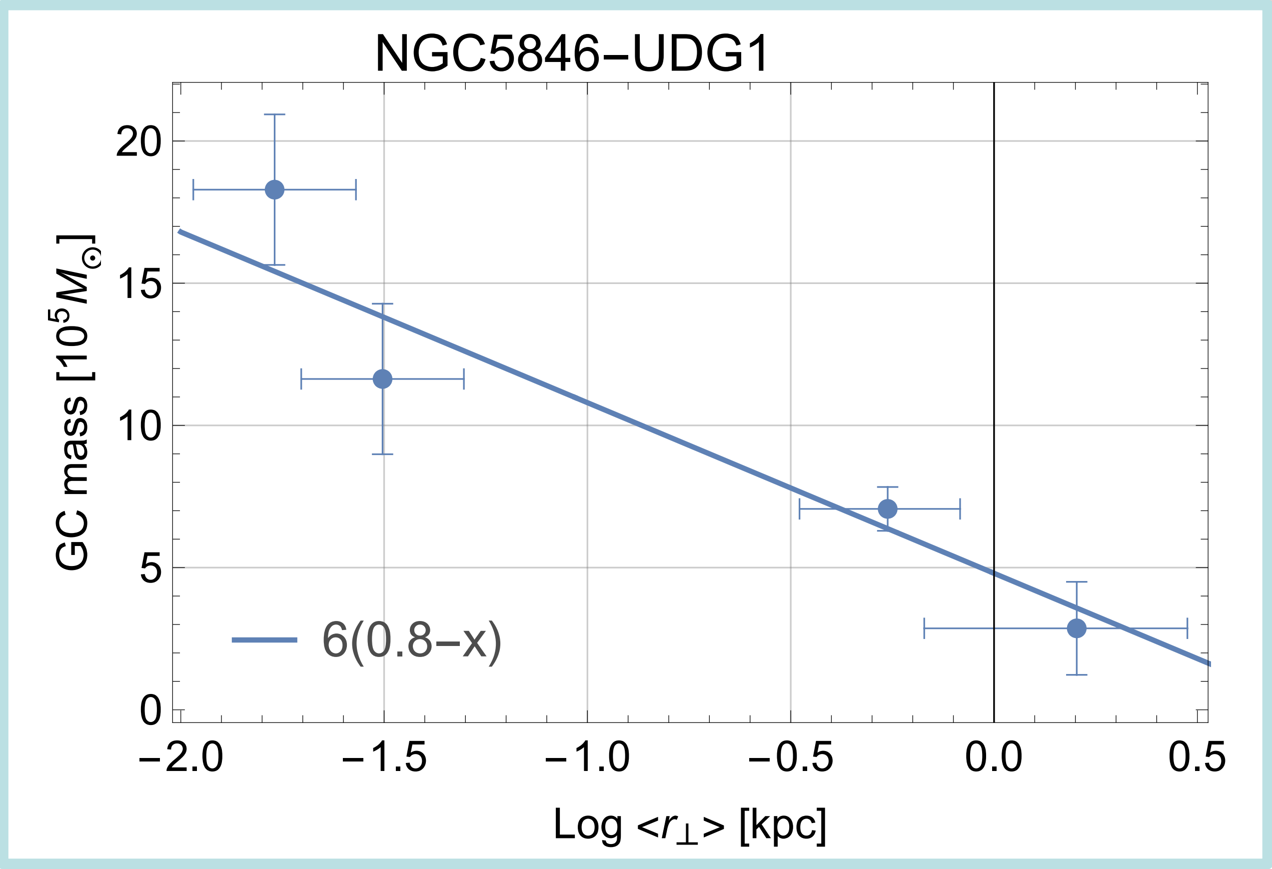
10^{-21} eV — $10^{-10}\ \mathrm{M}_\odot \sim 10^{56}$ eV

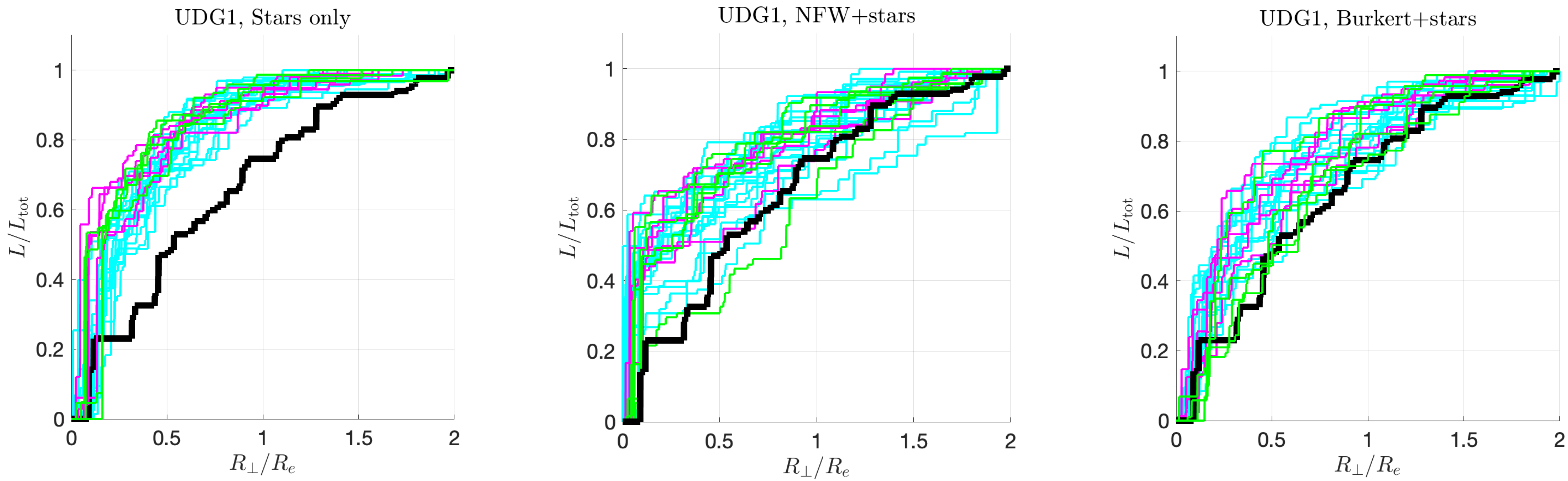
Thank you!



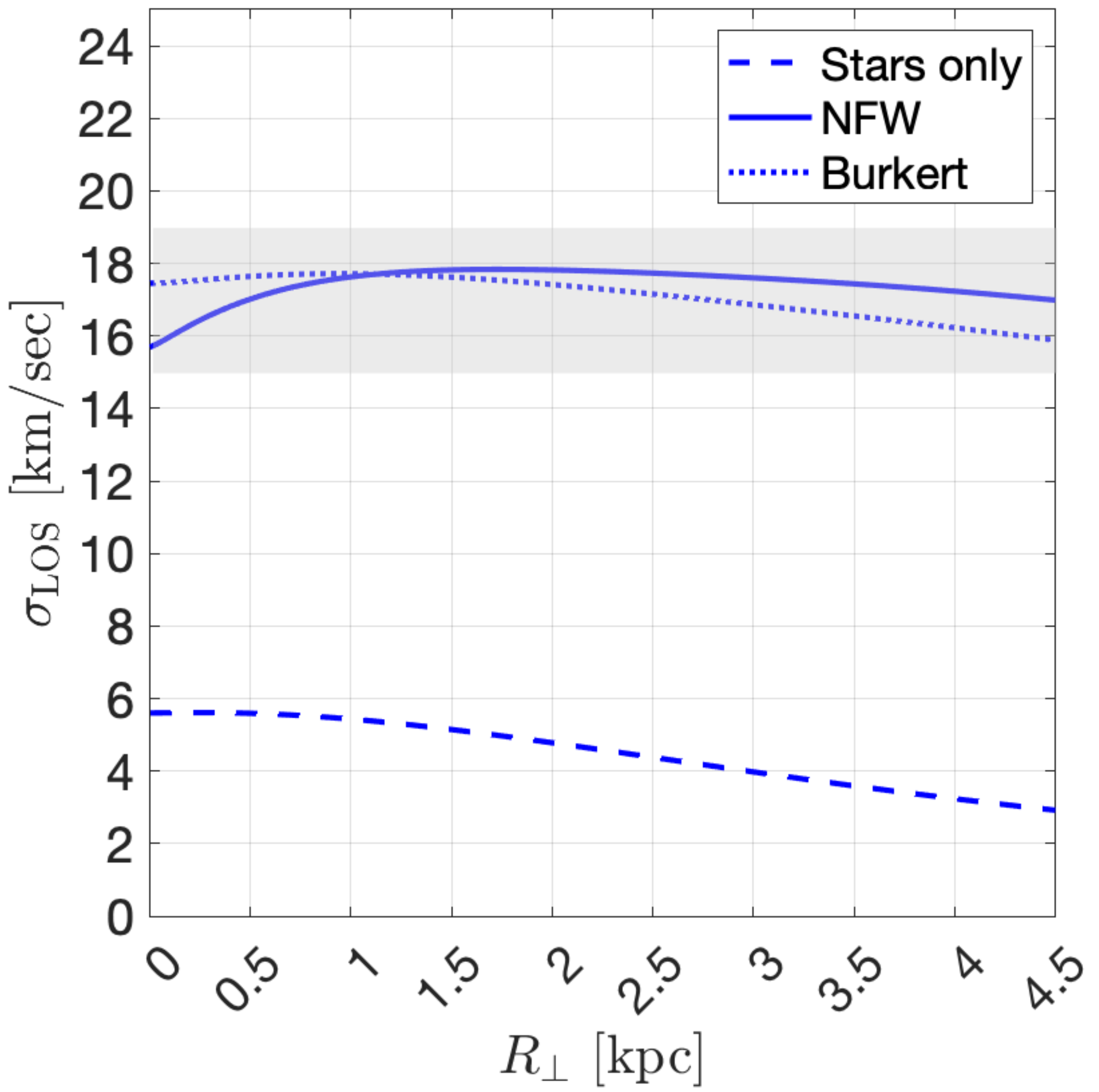


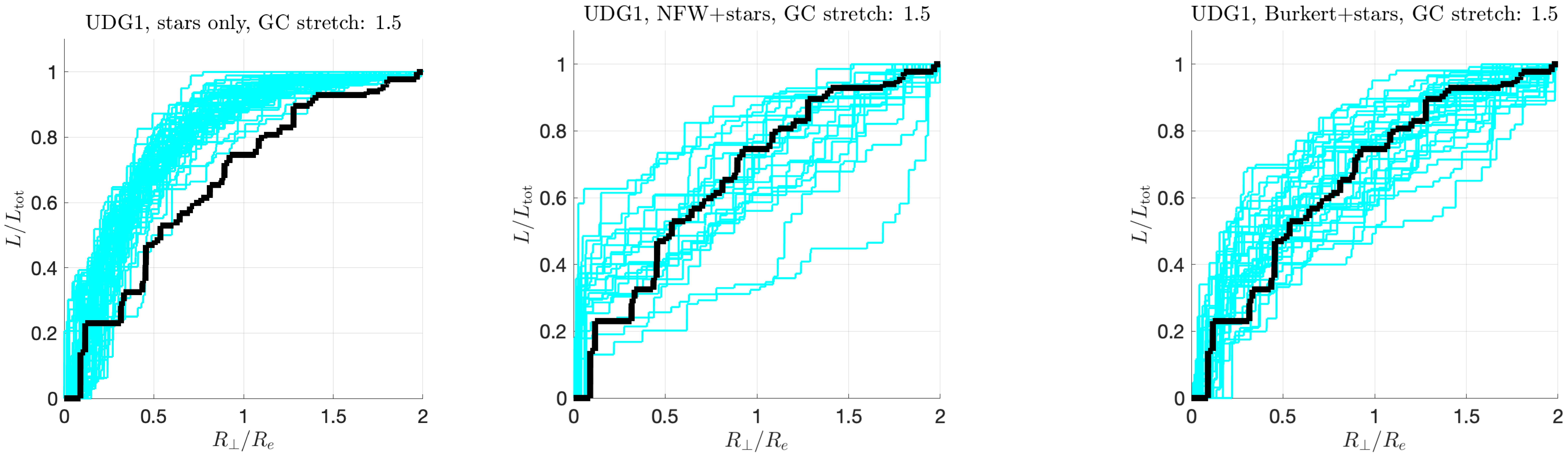




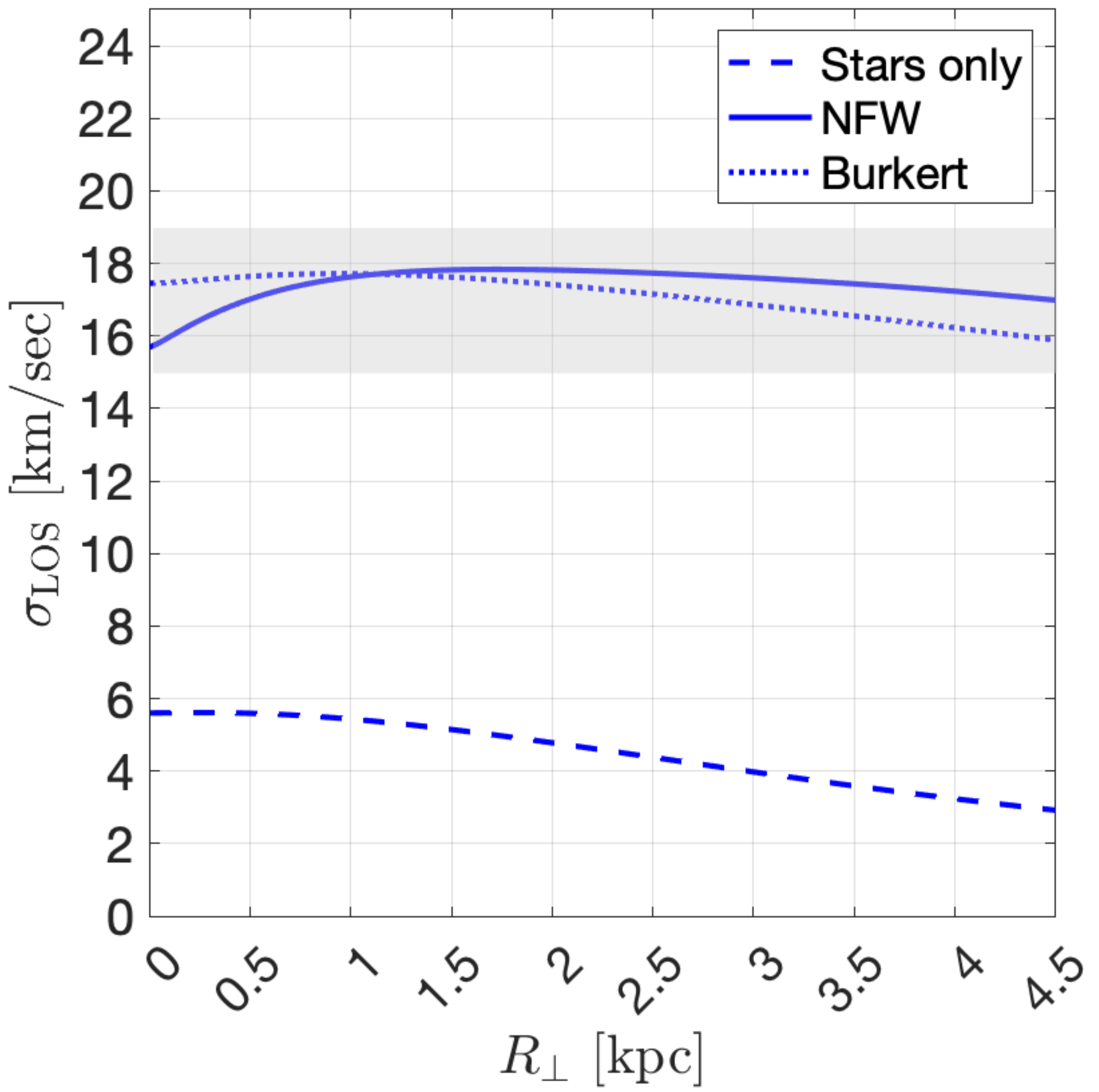


What we hope to learn





The problem of initial conditions



For approximately circular orbit:

$$\frac{\dot{r}}{r} \approx -\frac{2}{(1+\frac{d\ln M}{d\ln r})\tau}$$

$$\alpha(r) = \frac{d\ln M(r)}{d\ln r}$$

$$\Delta t = \int_r^{r_0} \frac{dr'}{2r'}(1+\alpha(r'))\tau(r')$$

Tremaine 1976,
Oh, Lin, Richer 2000,
Petts, Gualandris, Read 2015,
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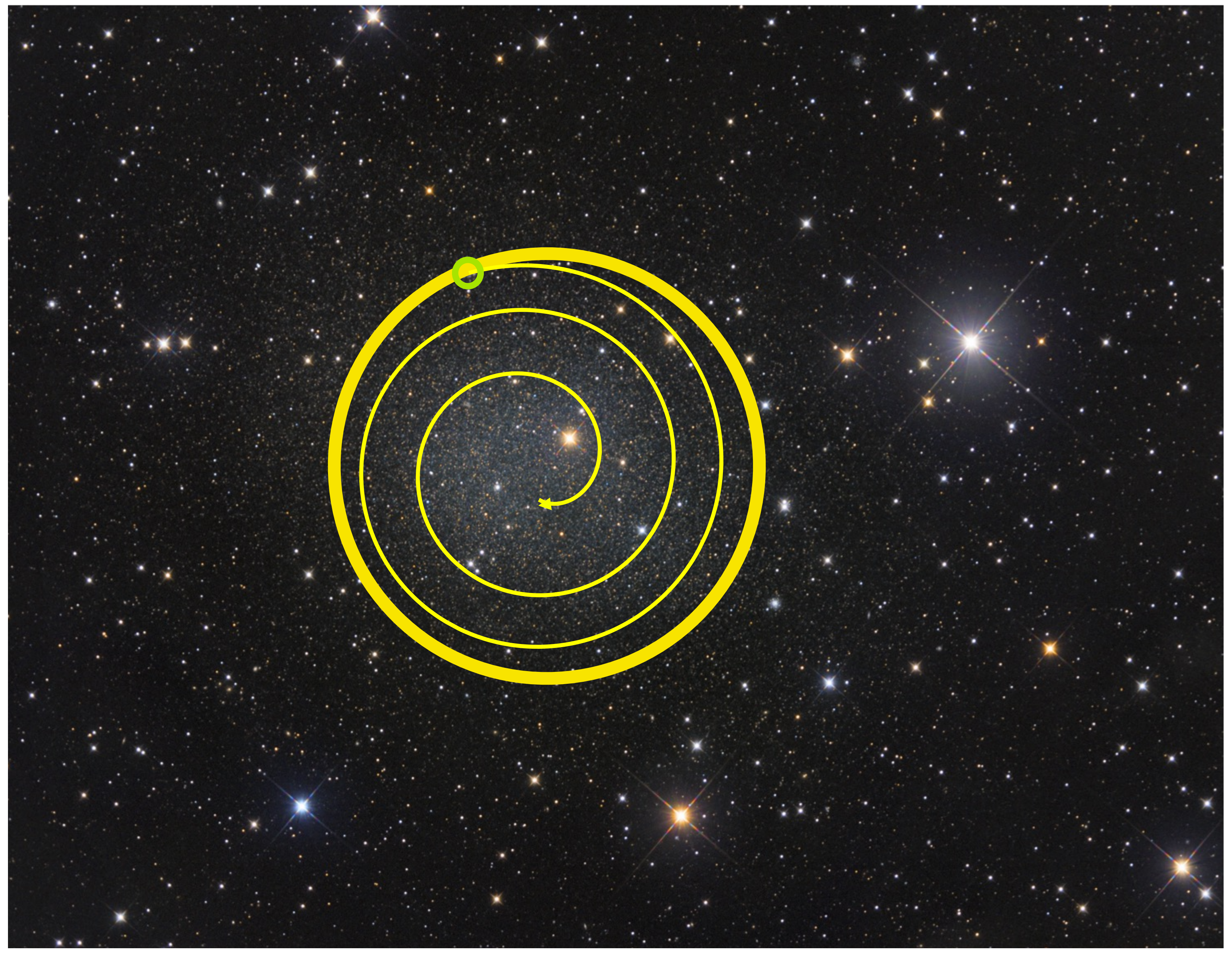

For approximately power law density profile:
(e.g. NFW $\beta \approx 2, \ \alpha \approx 2$)

$$\tau = \bar{\tau}(r/\bar{r})^{\beta}$$

There is a **critical radius** (kill circle):

$$\boxed{r_{\rm cr} = \bar{r}\left(\frac{2\beta}{1+\alpha}\frac{\Delta t}{\bar{\tau}}\right)^{1/\beta}}$$

GCs that start at $r<r_{cr}$ at t=0, arrive at r=0 by $t=\Delta t$

Should we expect to see GCs inside the kill circle?

We may, but not many: GCs that are now inside critical radius (*but not in nuclear cluster*) come from a small sliver of space:

$$r_0(r;\Delta t) = r_{\rm cr}\left(1+\left(\frac{r}{r_{\rm cr}}\right)^{\beta}\right)^{1/\beta}$$

$$= r_{\rm cr}\left(1+\frac{1+\alpha}{2\beta^2}\frac{\tau(r)}{\Delta t}+\cdots\right)$$

Cumulative count of GCs (CDF):

$$N_{\Delta t}(r) \approx N_0(r_{\rm cr}) + \frac{1+\alpha}{2\beta^2}N_0'(r_{\rm cr})r_{\rm cr}\frac{\tau(r)}{\Delta t} + \ldots$$

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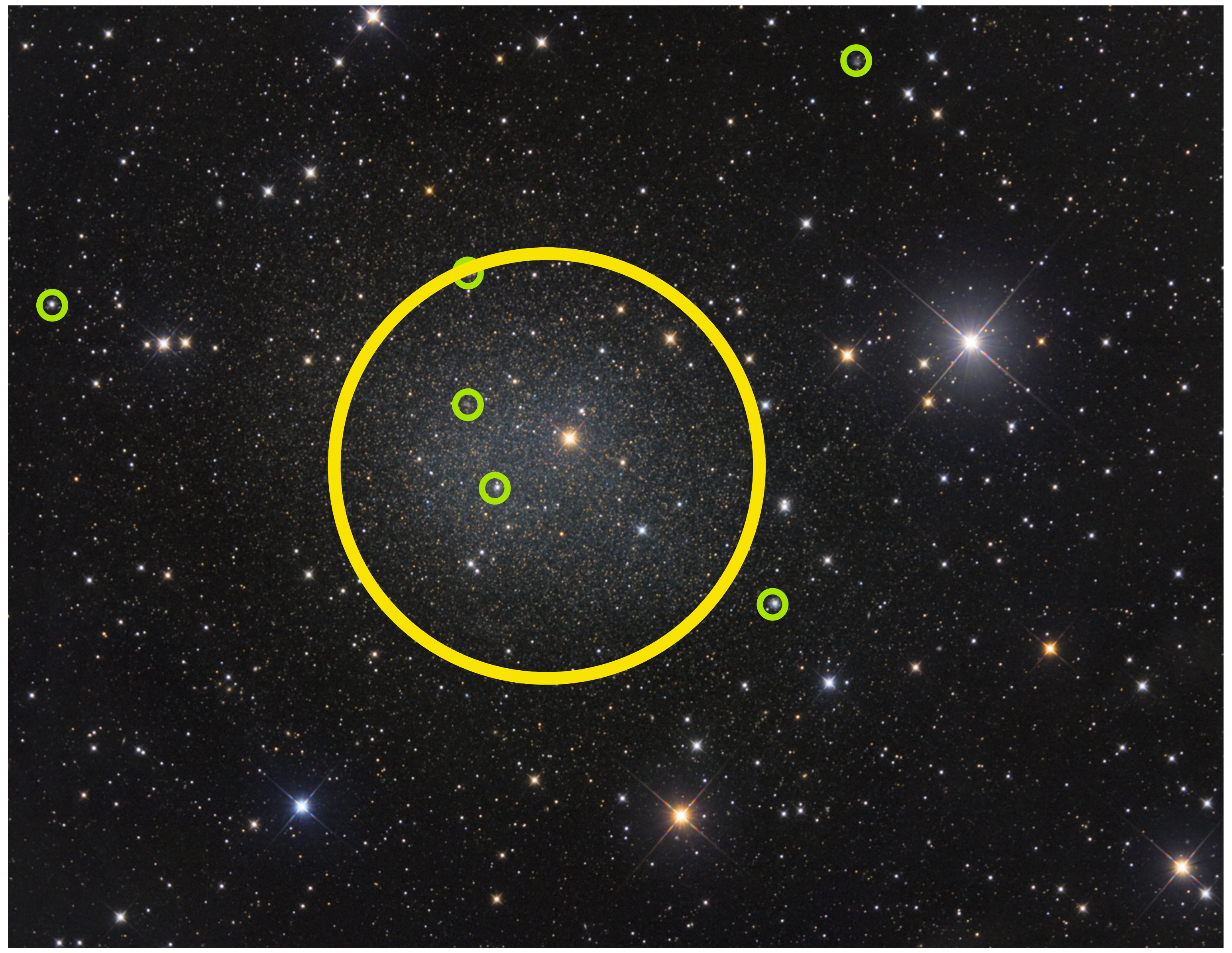

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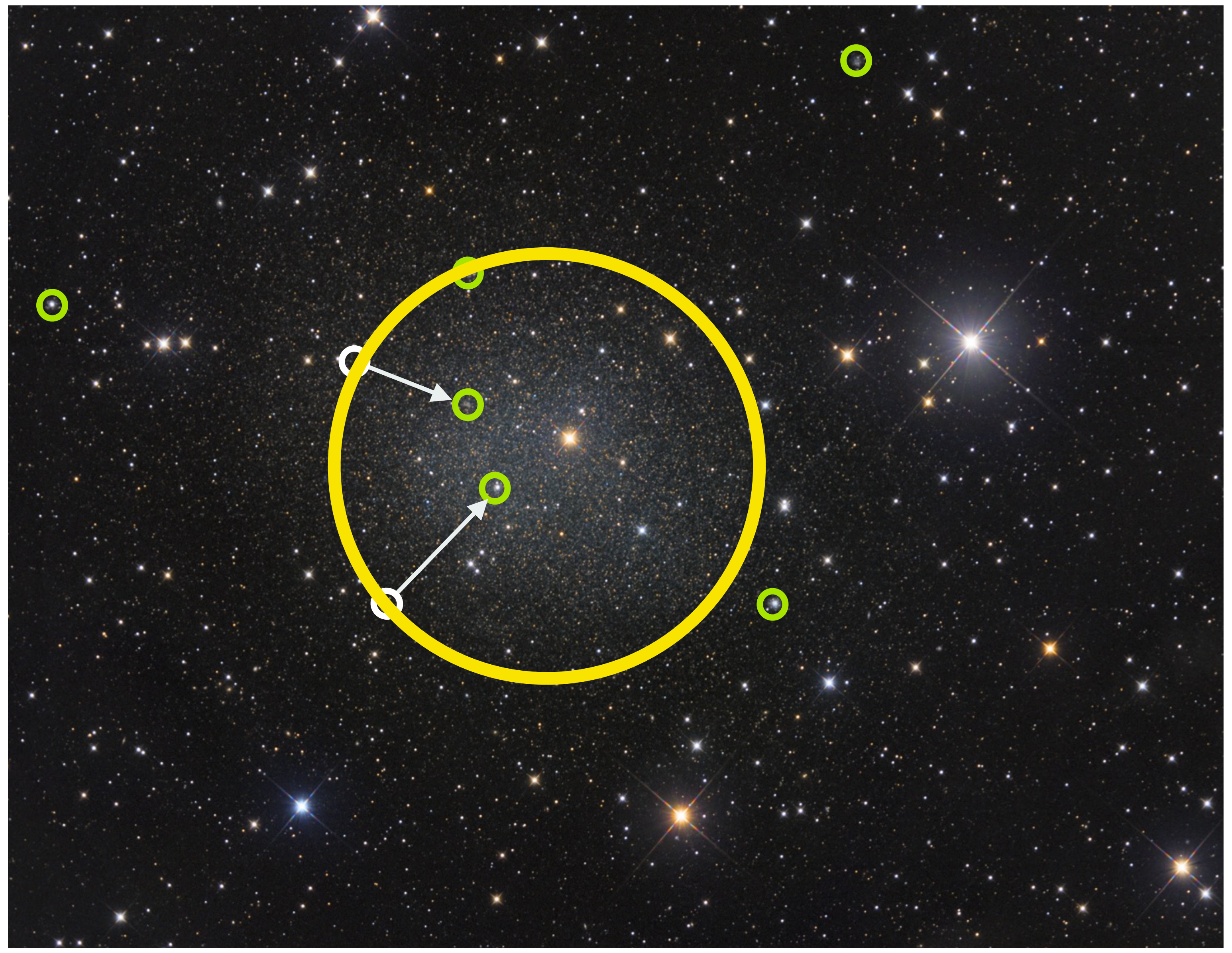

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Nuclear cluster?!
Fornax does not seem to have one…

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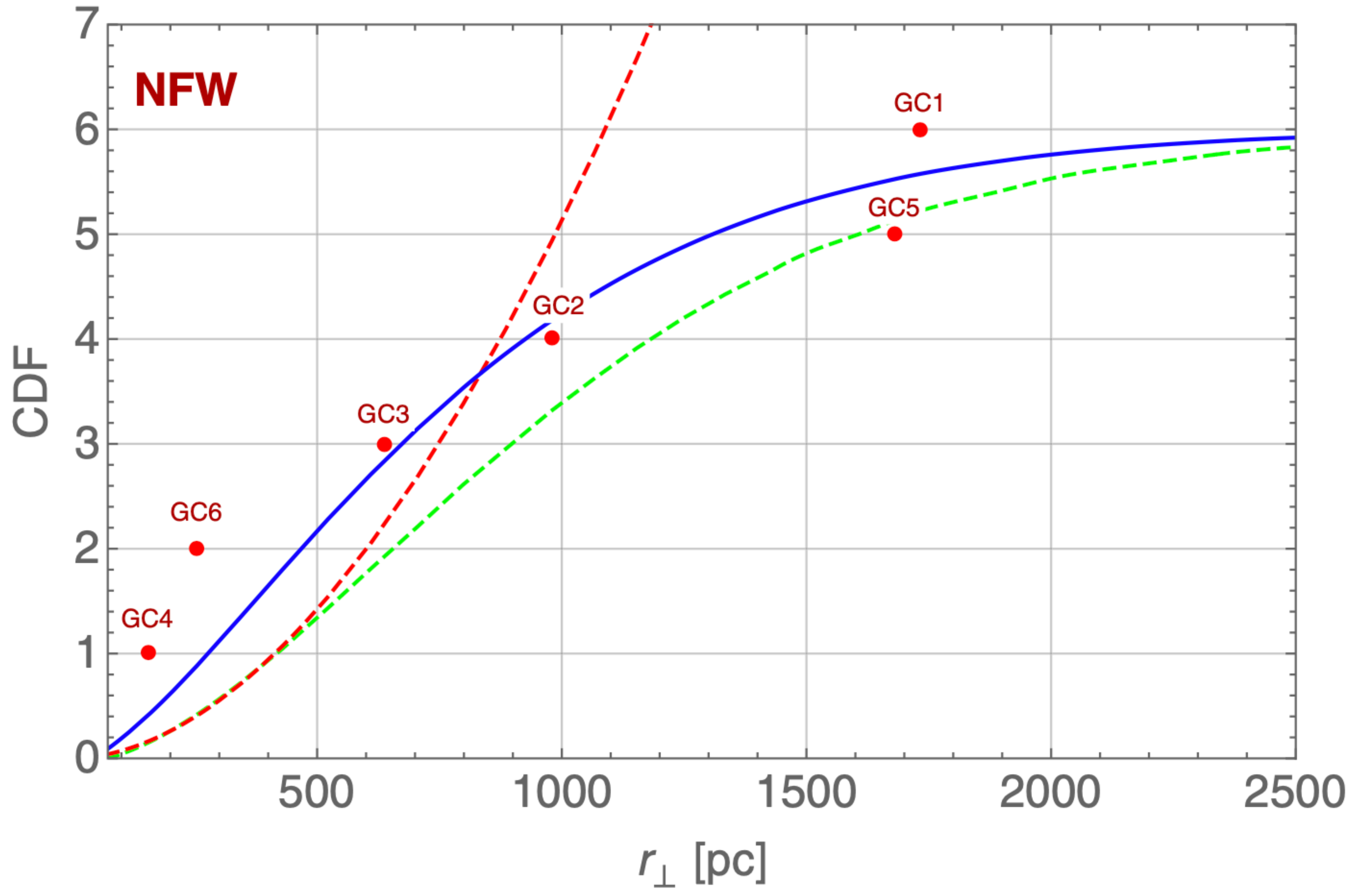

Lack of nuclear star cluster in Fornax?

Bar et al (Shao et al) find mild (~null) statistical timing problem, ignoring the question of NSC.

Bar et al (Shao et al) find ~50% (~30%) GCs “tidally disrupt” —> arrive at r~0.

In either case, we might have expected an NSC of ~10^6 solar mass…

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Oh, Lin, Richer 2000,
Petts, Gualandris, Read 2015,
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