

Dark Matter - Theoretical and Observational Status Lecture 2

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Goals (Lecture 2)

- Discuss the history of thermal relic DM post-freezeout, and constraints on the mass and interactions of thermal DM from cosmology
- Outline widely-used benchmark models both for heavy WIMPs and for light (sub-GeV) particle DM
- Briefly discuss a series of variations on thermal freezeout for generating particle DM abundance in the early universe

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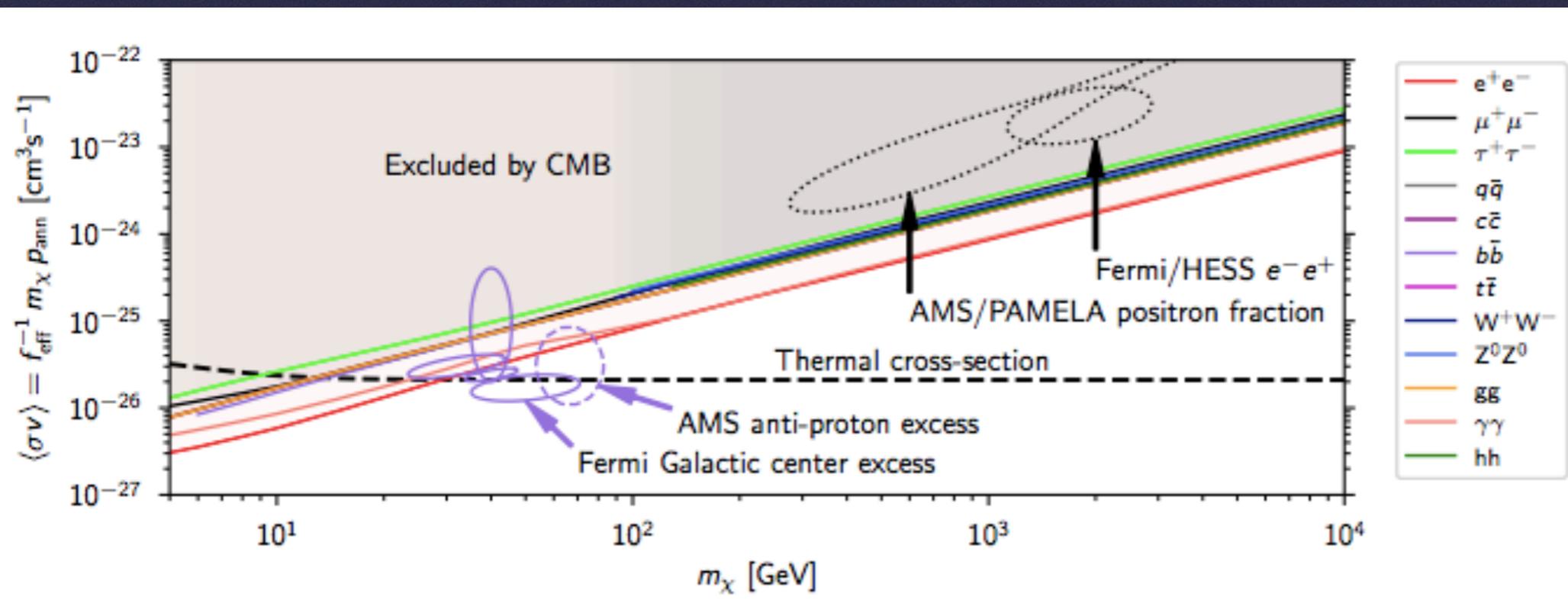
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 - In matter / dark energy domination the energy injection falls off more rapidly at low temperatures; when densities increase in galaxies, so does the annihilation rate \rightarrow potentially large indirect detection signals [\[see lectures by Prof. Calore\]](#)

The consequences of early energy injection

- After recombination, a significant fraction of this injected power can go into ionizing hydrogen
 - Temperature at recombination $T \sim 0.2$ eV
1 eV ($1 \text{ GeV}/T_f$) power injected per baryon over a Hubble time (note CMB is emitted not long after matter-radiation equality)
 $O(10)$ eV to ionize one H
 - Suggests that $T_f \sim 0.1$ GeV (so $m \sim 2$ GeV) provides power to ionize all hydrogen over a Hubble time
 - CMB constrains changes to the ionization level post-recombination of order $10^{-3} \Rightarrow$ potential sensitivity to TeV-scale and lighter thermal DM
- Detailed calculation finds this excludes DM annihilating to \sim all SM final states (except neutrinos) with thermal relic xsec after emission of the CMB, for masses below 10 GeV

The consequences of early energy injection

- Detailed calculation finds this excludes DM annihilating to \sim all SM final states (except neutrinos) with thermal relic xsec after emission of the CMB, for masses below 10 GeV
- Major loophole: suppress annihilation at late times/low velocities



Planck
Collaboration
'18 1807.06209
based on results
of TRS PRD '16

BBN limits

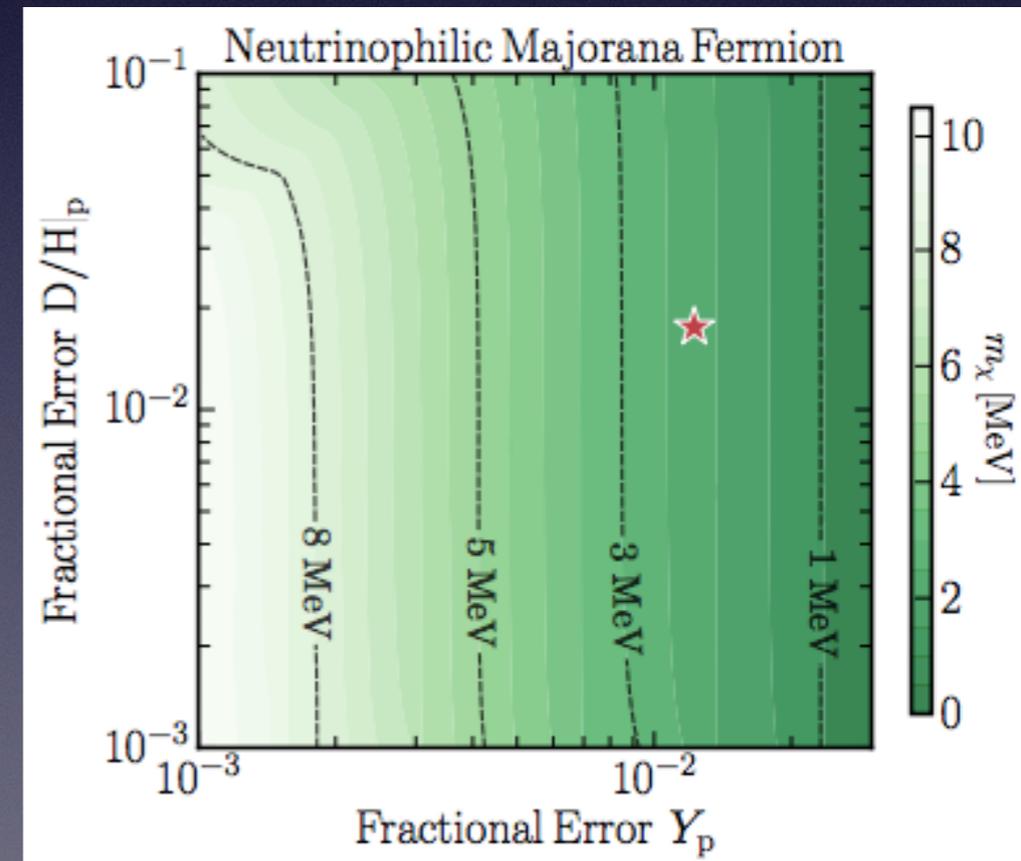
- Big Bang nucleosynthesis (BBN) occurs around $T \sim 0.1 - 1$ MeV
- Constrained by observations of light-element abundances
- Annihilation can produce energetic particles that catalyze or inhibit reaction network that produces nuclei
- BBN allows measurement of photon-to-baryon ratio and provides a “clock” for the cosmic expansion H at this time
 - Once T drops below the neutron-proton mass difference (~ 1.3 MeV), neutrons are depleted by decays/scatterings that convert them into protons, and must bind into helium to be stable
 - Final abundance of helium is sensitive to the relative timescale for neutron-proton interconversion via weak interactions and cosmic expansion
 \Rightarrow allows sensitive measurement of H
- BBN occurs during radiation domination: H is sensitive to the number and temperature of relativistic species

BBN limits from N_{eff}

- The contribution of relativistic species to the energy density is expressed in terms of the number of effective neutrino species, N_{eff}

$$\frac{\rho_i}{\rho_\gamma} = \frac{7}{8} N_{\text{eff}} \left(\frac{4}{11} \right)^{4/3}$$
 - Measured value of N_{eff} from BBN is 2.889 ± 0.229 [Yeh et al '22]
 - Expected SM value is 3.044
 - Changing the temperature of the neutrinos relative to the photons also affects their contribution to N_{eff}

$$\rho_i = \frac{7}{8} g_i \frac{\pi^2}{30} T^4$$
 - If DM is still relativistic at the start of BBN, it can itself contribute to N_{eff}
 - If the DM annihilates to some invisible “dark radiation”, that will contribute to N_{eff}
 - If the DM annihilates to neutrinos and/or other SM particles after neutrinos decouple from the main SM bath ($T \sim 2$ MeV), it will generically modify N_{eff} from neutrinos via changing $T_{\text{neutrinos}}/T_\gamma$
- All of these effects constrain thermal relic DM that becomes non-relativistic and freezes out around the BBN epoch
- Lower limits on DM mass vary depending on the DM spin and annihilation channels, but lie in the ~ 4 -14 MeV range [Sabti et al '19, An et al '22]



Kinetic decoupling

- Even after freezeout, interactions between the DM and SM may be sufficient to keep the DM at the same temperature as the SM - this kinetic equilibrium can be maintained by elastic scattering of DM off relativistic SM particles, which have a much higher number density than DM post-freezeout (for eV+ DM)
- Such a tight coupling damps DM density fluctuations - specifically, fluctuations that have “entered the horizon” (have characteristic length smaller than the horizon scale) at the time of kinetic decoupling are suppressed (review by [Bringmann '09](#)). Cuts off power on small scales.

$$M_{\text{ao}} \approx \frac{4\pi}{3} \frac{\rho_\chi}{H^3} \Big|_{T=T_{\text{kd}}} = 3.4 \times 10^{-6} \left(\frac{T_{\text{kd}} g_{\text{eff}}^{1/4}}{50 \text{ MeV}} \right)^{-3} M_\odot$$

T_{kd} is model-dependent, but typically ~ 100 keV or higher (for MeV+ DM) unless there is appreciable neutrino scattering

- Furthermore, even non-relativistic dark matter can free-stream after it is decoupled - it just doesn't go very far, so suppresses power only on very small scales.

Characteristic scale

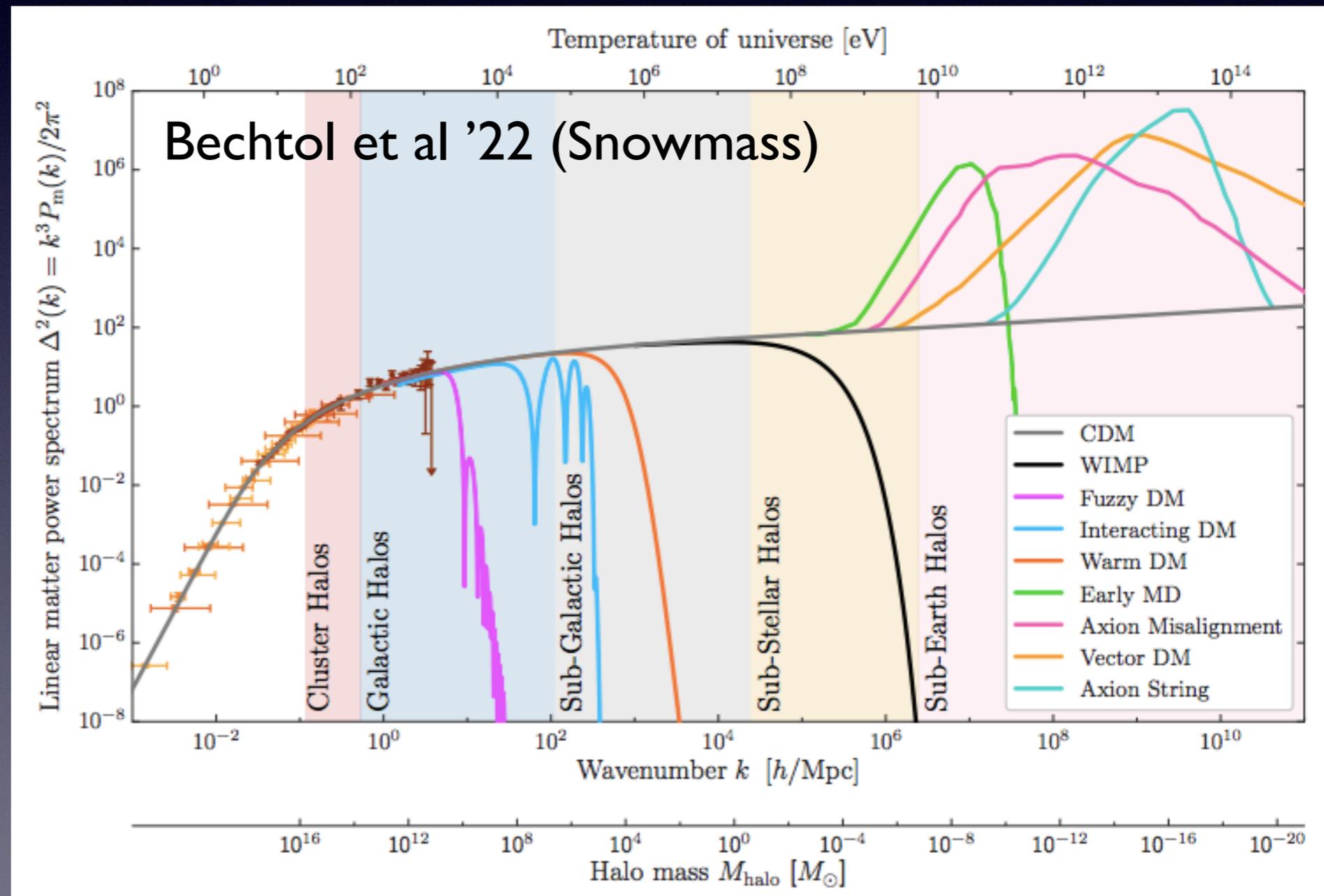
Resulting mass cutoff

$$k_{\text{fs}} \approx \left(\frac{m_\chi}{T_{\text{kd}}} \right)^{1/2} \frac{a_{\text{eq}}/a_{\text{kd}}}{\ln(4a_{\text{eq}}/a_{\text{kd}})} \frac{a_{\text{eq}}}{a_0} H_{\text{eq}} \quad M_{\text{fs}} \approx \frac{4\pi}{3} \rho_\chi \left(\frac{\pi}{k_{\text{fs}}} \right)^3 = 2.9 \times 10^{-6} \left(\frac{1 + \ln \left(g_{\text{eff}}^{1/4} T_{\text{kd}} / 50 \text{ MeV} \right) / 19.1}{(m_\chi / 100 \text{ GeV})^{1/2} g_{\text{eff}}^{1/4} (T_{\text{kd}} / 50 \text{ MeV})^{1/2}} \right)^3 M_\odot$$

The matter power spectrum

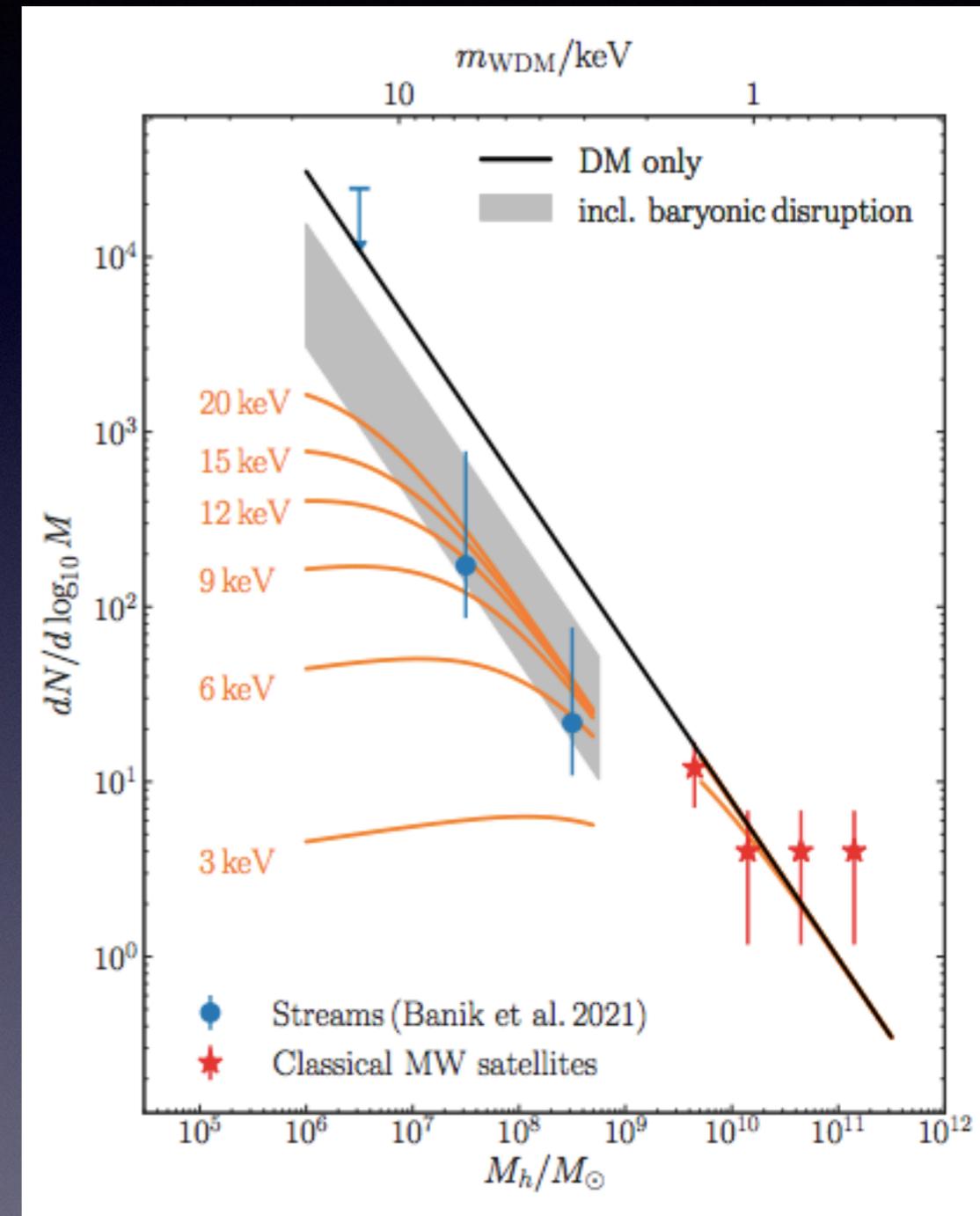
$$\delta(\mathbf{r}) = \frac{\rho(\mathbf{r}) - \bar{\rho}}{\bar{\rho}} \quad \delta(\mathbf{k}) = \frac{1}{(2\pi)^3} \int \delta(\mathbf{x}) e^{-i\mathbf{k}\mathbf{x}} d\mathbf{x} \quad P(k) = \langle |\delta(\mathbf{k})|^2 \rangle$$

- We have observational probes of the matter power spectrum from large-scale structure, down to halo mass scales around 10^{11-12} solar masses
- Corresponds to modes that entered the horizon around the keV mass scale



Testing light thermal DM

- We can study the properties of smaller DM halos, (currently) down to $\sim 10^{7-8}$ solar masses (note star formation becomes inefficient below this scale), using a range of probes:
 - Lyman- α forest (probes matter clumpiness at $z \sim 2-6$) [e.g. Armengaud et al '17, Irsic et al '17, Nori et al '19]
 - Fluctuations in the density of stellar streams (perturbed by DM subhalos) [e.g. Banik et al '21]
 - Strong gravitational lensing of quasars [e.g. Hsueh et al '19, Gilman et al '19, Nadler et al '21]
 - Observations of faint MW satellite galaxies [e.g. Nadler et al '19, '21]
- These constrain thermal DM below the ~ 10 keV scale, as well as ultralight “fuzzy” DM, self-interacting DM, and DM-baryon scattering (see Bechtol et al '22 (Snowmass) & references therein).
- Similar limits also apply to DM that has a high temperature for reasons unrelated to its coupling to the SM.
- Temperature is not generally the full story - for DM that is not in thermal equilibrium, the full phase space distribution must be tracked (see e.g. Dvorkin et al '19, '21; Bringmann et al '21 for examples)



Bounding the “thermal window”

- WDM and BBN limits constrain thermal relic dark matter lighter than the keV and MeV scales respectively
- What about the other end of the mass range?
- Unitarity (i.e. probability conservation) sets an upper limit on the contribution to the annihilation rate during freezeout from any partial wave,
$$(\sigma v_{\text{rel}})^J_{\text{total}} < (\sigma v)^J_{\text{max}} = \frac{4\pi(2J+1)}{M_{\text{DM}}^2 v_{\text{rel}}}$$
- Given a set of partial waves that contribute significantly to depletion + a velocity scale for freezeout + assumptions of standard cosmology, unitarity bound sets an upper limit on the DM mass in this thermal scenario

The unitarity mass bound

- Estimate: consider saturating the bound with $L=0$, i.e. take:
$$\sigma v_{\text{rel}} \sim 20/(100 \text{ TeV})^2 \sim 4\pi/m^2 v_{\text{rel}}$$
$$\Rightarrow m \sim 100 \text{ TeV} (\pi/5 v_{\text{rel}}) \sim 300 \text{ TeV} (\text{taking } v \sim (T/m)^{1/2} \sim 0.2)$$
- A more careful calculation finds an upper mass bound that is often quoted as 100-200 TeV, valid when only a few partial waves contribute, although:
 - for bound states / extended objects higher partial waves may contribute significantly,
 - argument in [Smirnov & Beacom '19](#) that shallowly-bound high- l states will be disrupted by plasma effects before they can annihilate \rightarrow upper bound on l_{max} depending on $T_{\text{freezeout}}$ \rightarrow upper mass limit of 1 PeV
- Saturating this unitarity bound typically requires long-range interactions and/or strong couplings [e.g. [von Harling & Petraki '14](#)]

Ultraheavy DM: beyond the unitarity bound

- Limit can be evaded in non-thermal scenarios or if cosmology is modified
- (Much) higher masses can be achievable for thermal relic DM when standard assumptions break down, e.g.:
 - modified cosmology: large entropy injections, or a first-order phase transition in the dark sector [e.g. Asadi, TRS et al '21]
 - formation of many-particle bound states after freezeout [e.g. Coskuner et al '19, Bai et al '19] - can lead to macroscopic DM candidates
- Non-thermal production mechanisms are also possible: e.g. out-of-equilibrium decay of a heavier state, inflationary gravitational particle production, black hole evaporation (see Asadi et al '22 (Snowmass) for a review)

Mass range for thermal relic DM

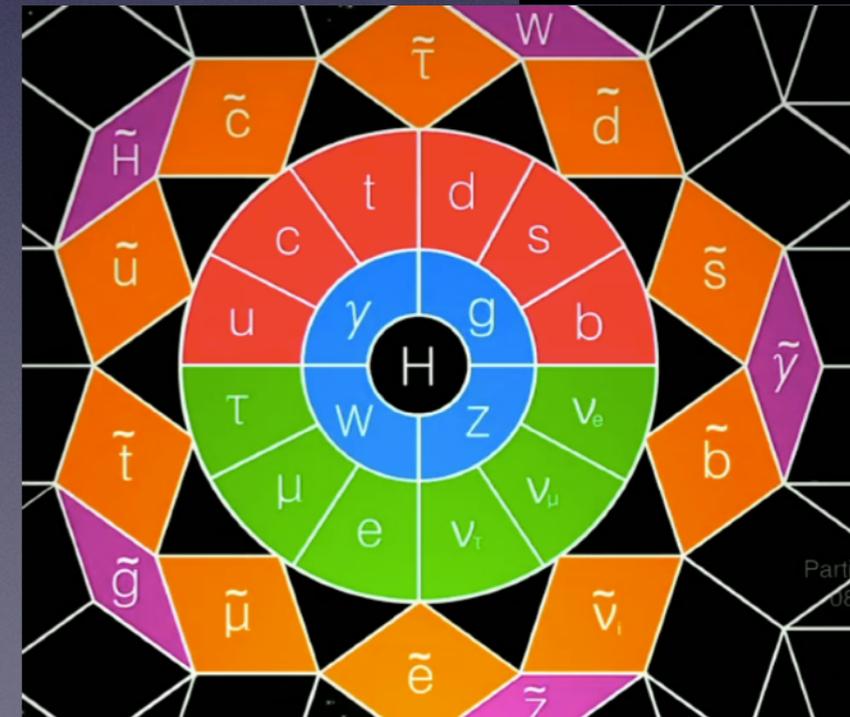
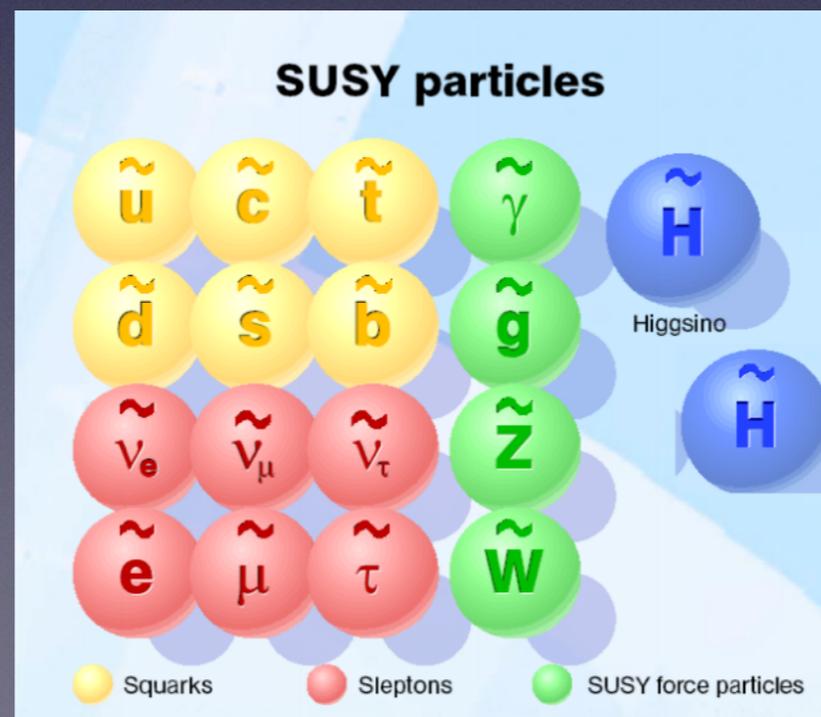
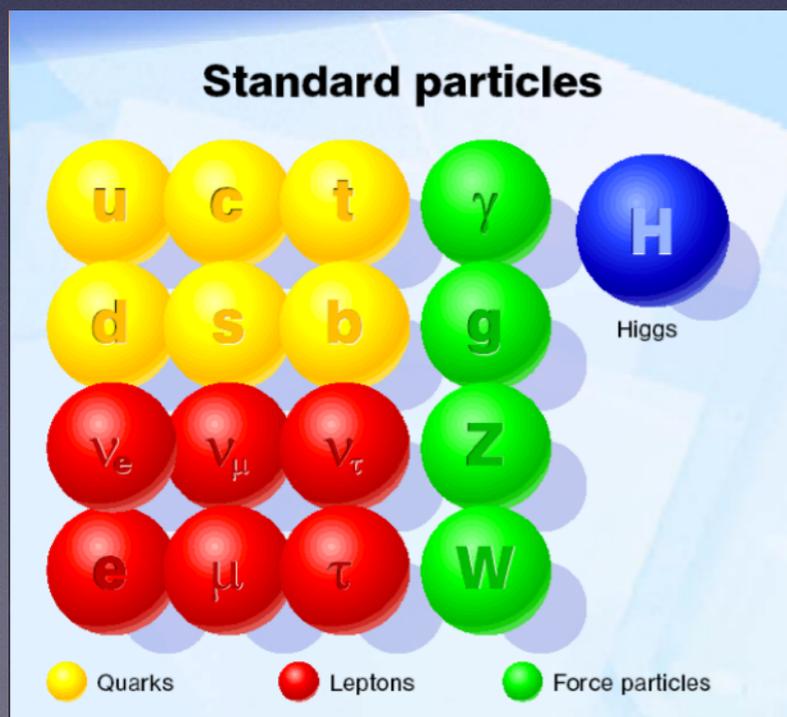
- We can summarize the previous results as:
 - For thermal relic DM below $\sim 5-10$ MeV, BBN can often be used to set stringent constraints
 - Where loopholes exist in this constraint, thermal relic DM below ~ 10 keV is independently constrained by warm dark matter bounds
 - For DM masses much heavier than 100 TeV, it is challenging to obtain a sufficiently large cross section to reproduce the correct relic abundance without additional ingredients in the cosmological history
- In the case of a weakly-coupled theory with $\alpha \sim 10^{-2}$, we find $m \sim 200$ GeV for standard thermal freezeout (WIMP miracle) - may hint at weak-scale physics, but the mechanism works over a much wider range

Classic WIMP candidates

Supersymmetry (SUSY)

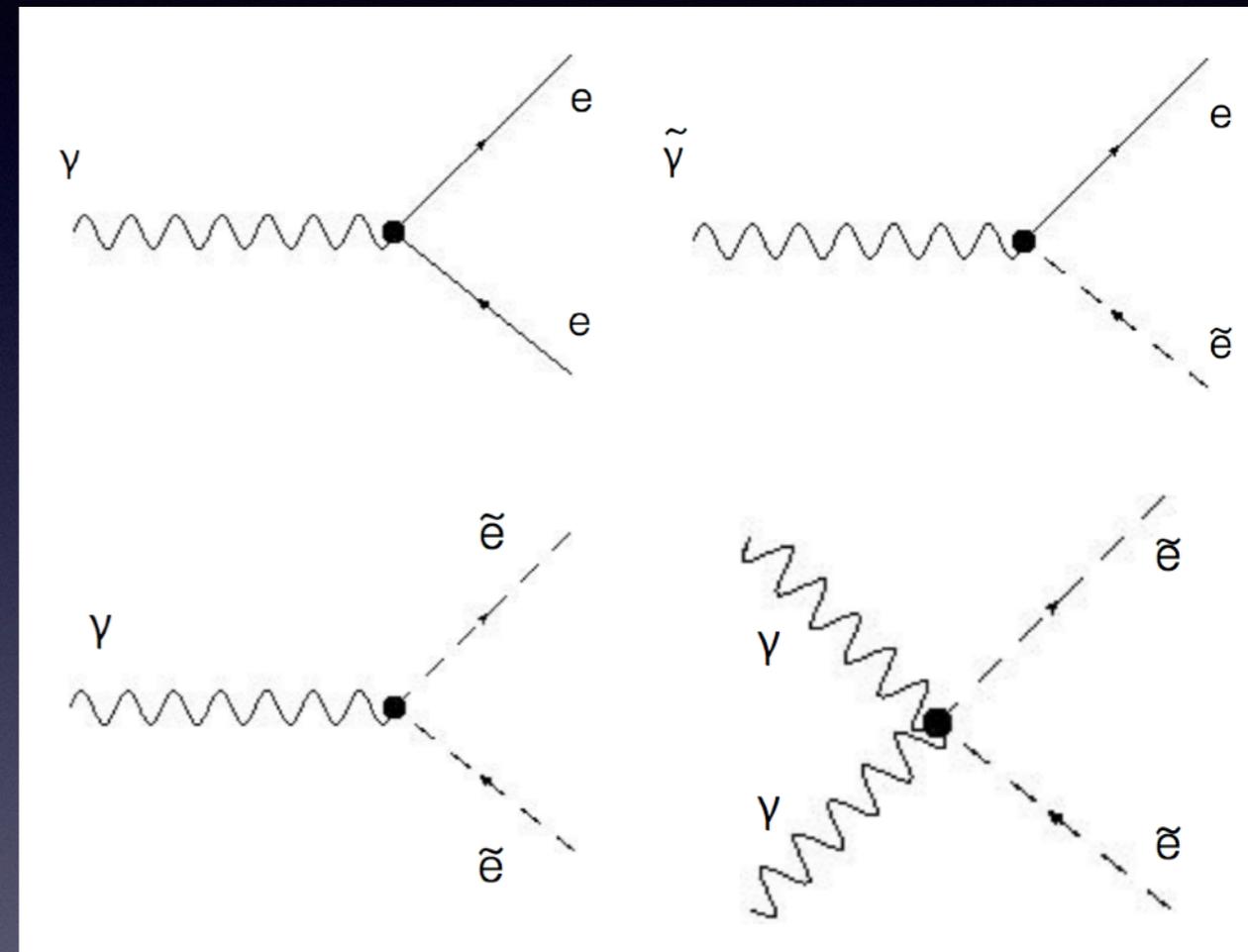
- Most famous dark matter candidate is the Lightest Supersymmetric Particle (LSP).
- In supersymmetric theories, every particle has a superpartner.
- Fermions have boson superpartners and vice versa.
- These additional particles cancel what would otherwise be very large predicted contributions to the Higgs mass - motivated independently of DM.
- For an in-depth introduction to SUSY, see e.g. [Martin hep-ph/9709356](#).

Particle Fever
D.E. Kaplan



Supersymmetry (II)

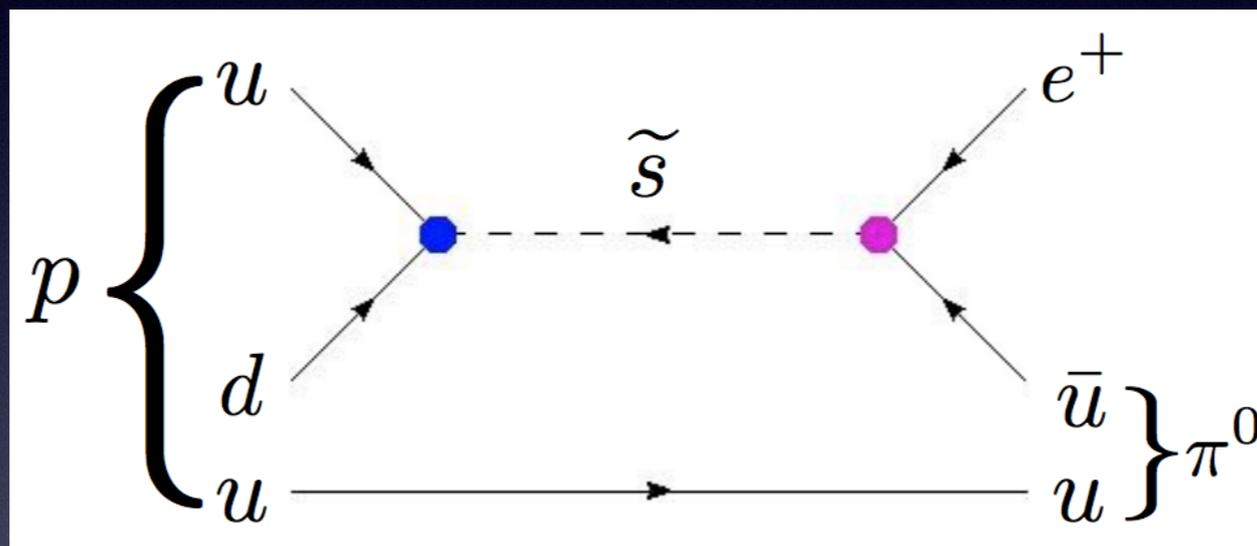
- In unbroken supersymmetry, particles and superpartners have same mass and closely related interactions.
- This symmetry must be broken as clearly superpartners do not have mass equal to their (known) counterparts!
- But if we break it “softly”, while masses are separated, interactions remain fixed by supersymmetry.
- SUSY theories also inherit huge structure from the Standard Model.
- Consequently many quantities in SUSY theories can be calculated from just the masses of the superpartners.



Example of interactions related by supersymmetry (credit: [Tim Tait '15](#))

R-parity

- These SUSY interactions naively imply some peculiar behavior!
- For example, they could make protons decay quickly:



- This is not observed (experimental limit: lifetime $> 10^{33}$ years).
- Usual approach is to impose a symmetry called R-parity, so the superpartners can only couple in pairs to the ordinary particles.
 - Superpartners have R-parity = -1
 - Ordinary particles have R-parity = +1
 - Product of R-parities before and after an interaction must be conserved

The LSP

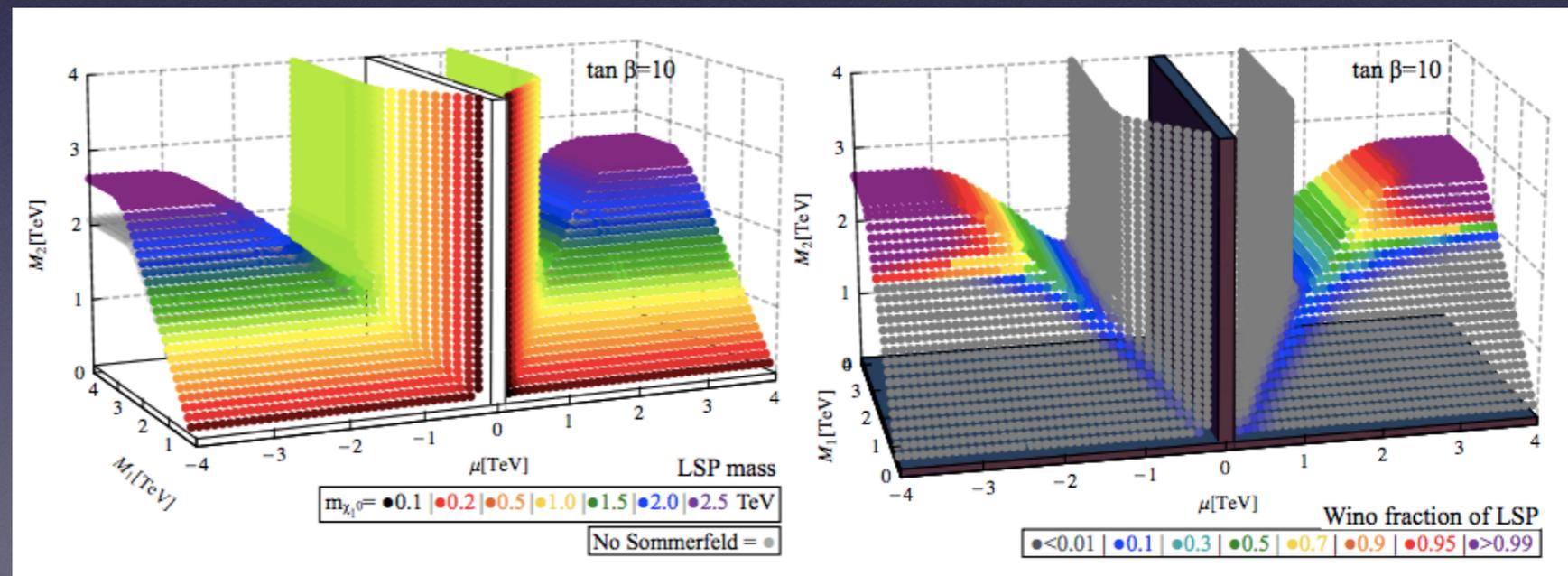
- But then lightest particle with R-parity odd (i.e. $= -1$) cannot decay
 - can't produce any other particles with R-parity -1 (kinematically forbidden)
 - can't decay just to SM particles (violates R-parity)
- Avoiding proton decay gives us a stable DM candidate!
- Furthermore, any R-parity-odd particles in the early universe must eventually produce stable R-parity-odd particles by decays.

Neutralino dark matter

- Neutral fermionic superpartners = superpartners of the neutral SM bosons (accompanied by “chargino” partners)
 - Higgsino = superpartner of the Higgs(es)
 - Wino = corresponds to electrically neutral gauge boson of electroweak SU(2) gauge group
 - Bino = corresponds to gauge boson of the electroweak U(1) gauge group
- In general the physical states (of definite mass) correspond to mixtures of these
- The lowest-mass such admixture may be a good DM candidate (if it is the LSP)

Notes:

- Even the “minimal supersymmetric Standard Model” (MSSM) has a very large number of parameters - often more restricted subsets are taken for purpose of parameter scans (e.g. cMSSM, pMSSM)
- Much of the naive parameter space is excluded due to null searches; much more does not yield a viable candidate for 100% of the DM

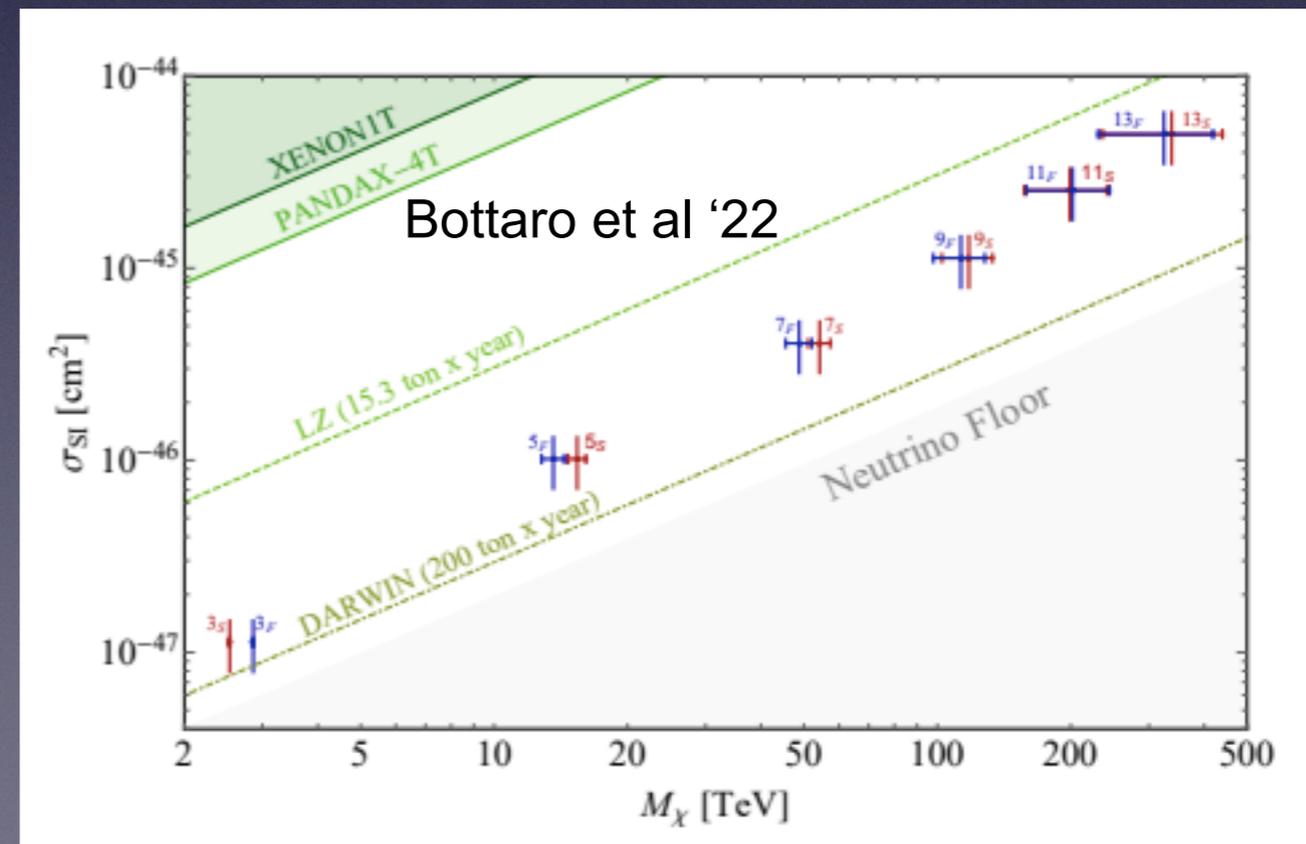


Bramante et al '16

Example of (part of) the “relic neutralino surface” with scalar superpartners decoupled; right-hand panel shows wino fraction of LSP

Minimal dark matter

- It turns out that some of the SUSY scenarios that are least constrained experimentally are those where the neutralino is a nearly-pure higgsino or gaugino [e.g. Bramante et al '16].
- More generally, we don't need supersymmetry - we can consider just adding more $SU(2)_L$ multiplets to the SM [Cirelli et al '05].
- Interactions are fully determined by gauge structure.
- Mass of new multiplet can be fixed if we require it to explain the full relic density (e.g. 1 TeV for higgsino, 3 TeV for wino, 14 TeV for $SU(2)$ quintuplet, etc)
- Yields discrete set of relatively heavy benchmark WIMP models



Light thermal DM

New mediators and the Lee-Weinberg bound

- It is a generic possibility that DM interacts with the SM through the known weak gauge bosons / Higgs (as for SUSY neutralinos and minimal DM models) - but not the only possibility
- Especially in older papers, you will sometimes see a statement that WIMPs must be above ~ 2 GeV in mass: this **Lee-Weinberg bound** assumes the DM obtained its abundance through freezeout and the mediator is the Z
- If the DM is relatively light but annihilates through the Z boson (which we can integrate out), we would parametrically expect the cross section to be bounded above by:
$$\sigma v_{\text{rel}} \sim 20 / (100 \text{ TeV})^2 < m^2 / m_Z^4$$
$$\Rightarrow m > 4 m_Z (m_Z / 100 \text{ TeV}) \sim \mathcal{O}(\text{GeV})$$
- DM below this mass scale is fine - but if it is a thermal relic, likely requires new (lighter) mediators between the DM and the SM

Portal dark matter

- Two very commonly-used benchmarks [e.g. Batell et al '22 (Snowmass)] are the “scalar” and “vector” portals (also known as the “Higgs portal” and “dark photon” / “kinetic mixing” portals, respectively)

$$\mathcal{L} \supset \begin{cases} -\frac{\epsilon}{2 \cos \theta_W} B_{\mu\nu} F'^{\mu\nu} & \text{vector portal} & \Rightarrow & g_f^V \approx \epsilon e q_f \\ (\mu\phi + \lambda\phi^2) H^\dagger H & \text{Higgs portal} & \Rightarrow & g_f^S = \mu m_f / m_h^2, \end{cases}$$

- Only two options that satisfy:
 - mediator is a SM-neutral boson (singlet under all SM symmetries)
 - has a renormalizable interaction with the SM (dimension 4)
 - interaction respects all SM symmetries
- Some studies also add the “neutrino portal”: $\mathcal{L} \supset -y^\alpha L_\alpha H N + \text{h.c.},$
Heavy gauge-singlet fermion mediator N couples to a gauge-invariant operator constructed from the SM lepton and Higgs SU(2) doublets; leads to mixing of N with SM neutrinos after EW symmetry breaking
- Also many more general possibilities - e.g. we can allow for higher-dimension interaction structures, or a mediator that carries SM quantum numbers, etc

Freezeout for portal dark matter

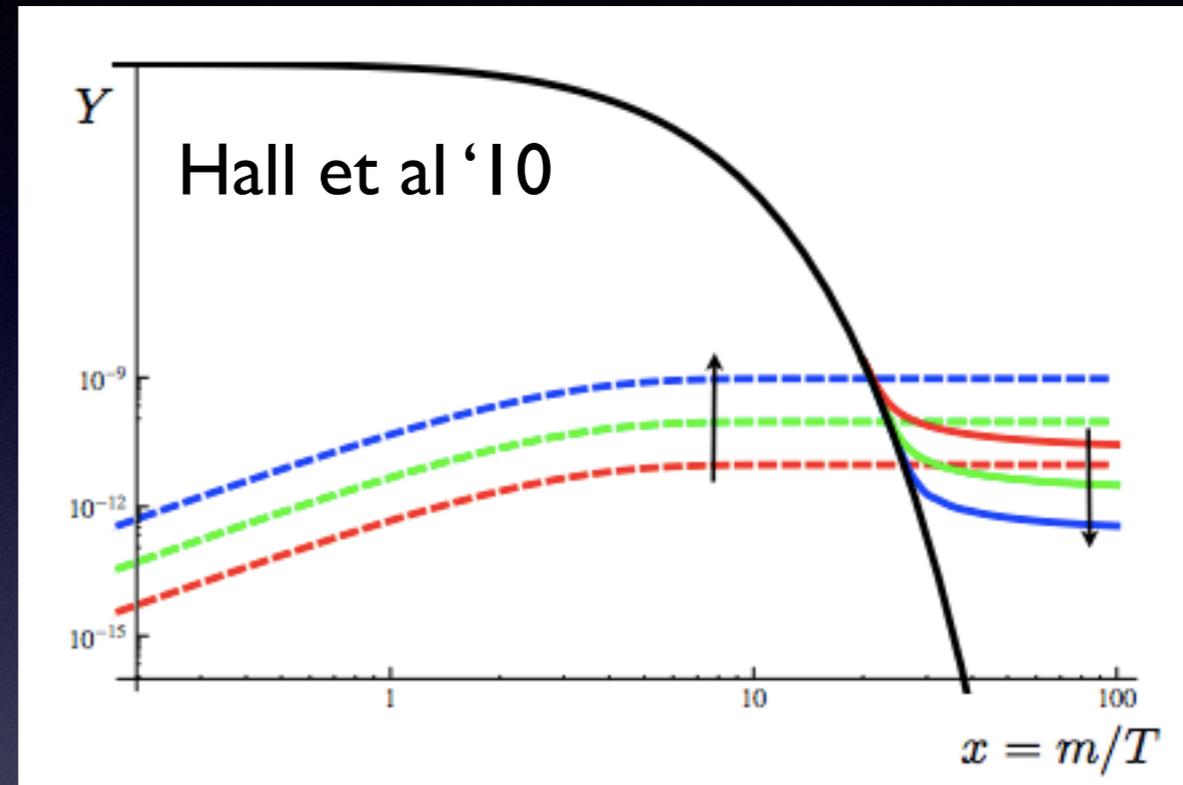
- Two general scenarios:
 - DM annihilates through off-shell mediators into SM particles
 - DM annihilates to on-shell mediators, which subsequently decay to SM particles (requires DM heavier than mediator, or only slightly lighter)
- In the first case the calculation proceeds as previously; the annihilation cross section is controlled by the coupling of the mediator to the DM and SM, + the mediator mass.
 - If the mediator mass is heavy, the same combination of parameters gives an effective DM-SM coupling that can also be probed in accelerators and direct detection
- In the second case the freezeout calculation is very similar if the mediators remain in equilibrium with the SM throughout freezeout (if not, see e.g. [Bringmann et al '21](#)), but the annihilation rate is insensitive to the SM-mediator coupling.

Variations on freezeout

- More than two DM particles in the initial state - e.g. Strongly Interacting Massive Particles (SIMPs) [Hochberg et al '14] - as discussed in lecture 1, “maximum” mass scale goes from $O(100)$ TeV to $(m_{\text{PI}} \times T_{\text{MRE}}^{N-1})^{1/N}$
- Annihilation suppressed at late times / low velocities - e.g. p-wave annihilation, asymmetric DM, coannihilation, forbidden DM [e.g. d’Agnolo et al '15] - often favored for light DM to avoid CMB bounds
- Annihilation enhanced at late times / low velocities - e.g. Sommerfeld enhancement [e.g. Hisano et al '03] - can be natural in the presence of mediators much lighter than the DM, e.g. it is automatic for minimal DM
- Fully or partially secluded dark sectors (mediator not in equilibrium with SM); cannibalism where number density falls while conserving entropy, increasing temperature [e.g. Carlson et al '92]
 - If decoupling happens during freezeout, transition from exponential depletion is triggered by kinetic decoupling - abundance controlled by scattering cross section. This is the ELastically DEcoupling Relic (ELDER) scenario [Kuflik et al '16].

Freeze-in

- Another particularly important benchmark is the case of “freeze-in”
- Basic picture [Hall et al '10]:
 - initial abundance of DM is negligible
 - either DM or a DM precursor is produced (with abundance \ll equilibrium) through rare interactions with the SM bath
 - production cuts off when $T < m$, due to kinematic suppression
 - if the production rate rises with decreasing T , most DM will be produced by interactions occurring around $T \sim m$: abundance will be insensitive to UV physics

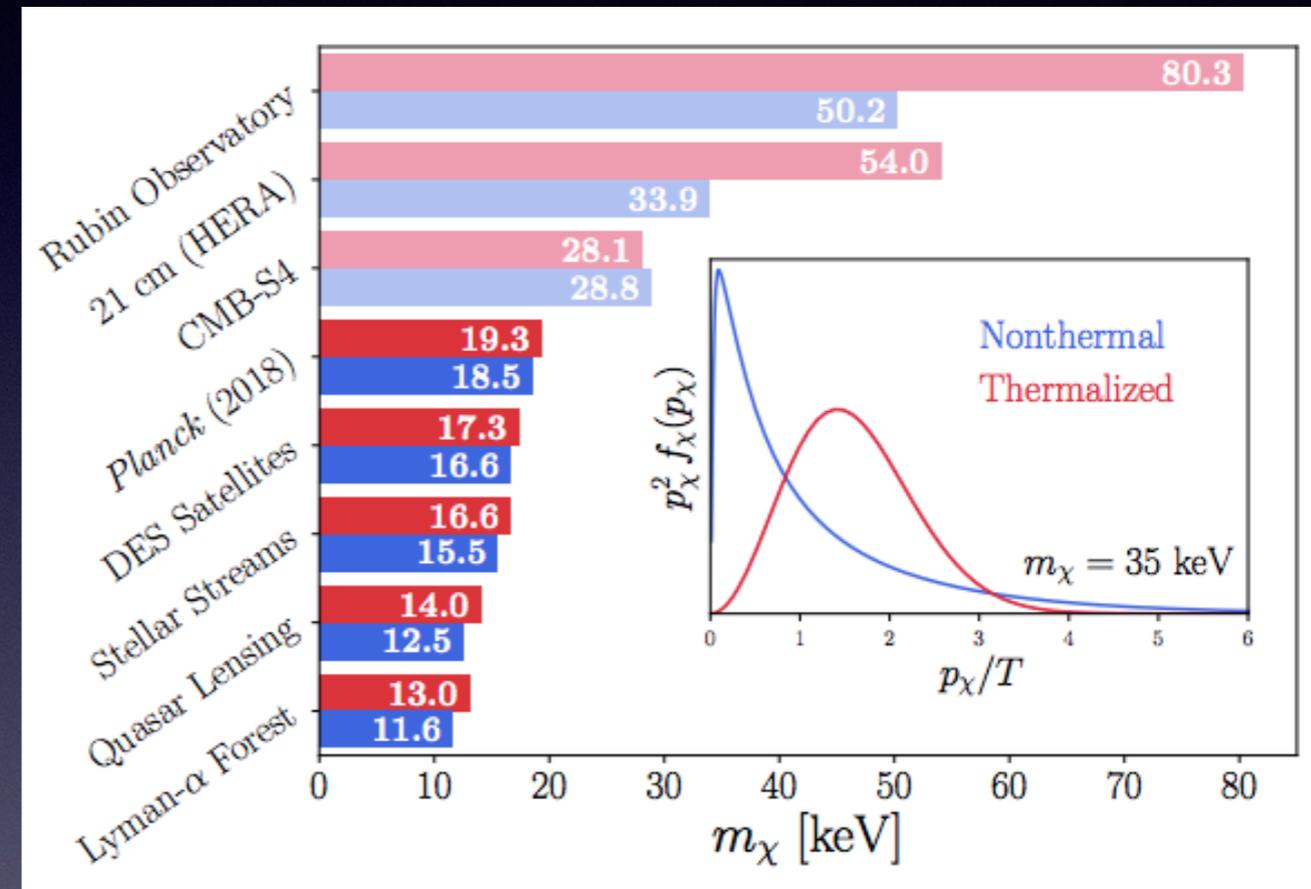


- Requires much smaller couplings than freeze-out (avoid equilibration) - can be natural for neutral mediators
- Can still be detectable especially if the mediator is sufficiently light

Cosmology of freeze-in DM

Dvorkin et al '21

- DM that either carries a tiny millicharge or couples to the SM through a kinetically mixed ultralight dark photon can obtain the correct relic abundance through freeze-in
- These scenarios lead to enhanced scattering at low velocities (proportional to v^{-4}): potentially observable signals in the CMB and direct-detection experiments
- Phase space distribution is non-thermal, with additional high-velocity component - strongly constrained by warm dark matter bounds



- Largely evades N_{eff} bounds due to suppressed production in the early universe - abundance is always \ll thermal while relativistic

Some limits on secluded dark sectors

- Bounds on N_{eff} are powerful at constraining new light dark sectors in general [e.g. [Dvorkin et al '22 \(Snowmass\)](#)]
- CMB limits set comparable bounds on N_{eff} to BBN, at a later redshift - can be more constraining than BBN limits, depending on the model [e.g. [An et al '22](#)].
- Future CMB experiments / LSS surveys aim to reduce the uncertainty on N_{eff} to the percent level - test dark sectors that were in thermal equilibrium with SM prior to electroweak phase transition
- CMB observations also test invisible decays of DM - upper limit of 4% reduction from CMB to present day [[Poulin et al '16](#)]
Note: much larger fraction is viable than $\sim T_{\text{CMB}}/T_f$ ratio we discussed for visibly annihilating DM!

