Early Universe

Lectures at Canary Islands Winter School 2022

Outline:

1) CMB & BBN
2) Baryogenesis
3) Inflation (background)
4) Inflation (perturbations)

Literature

- Bailin, Love : Cosmology in gauge field theory and string theory
- Baumann: TASI Lecture Notes Inflation arxiv: 0907.5424

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Lecture 3 – Inflation – background evolution

1) Horizon problem

past light-cone, region of causal contact: $dt = a\, d\tau$

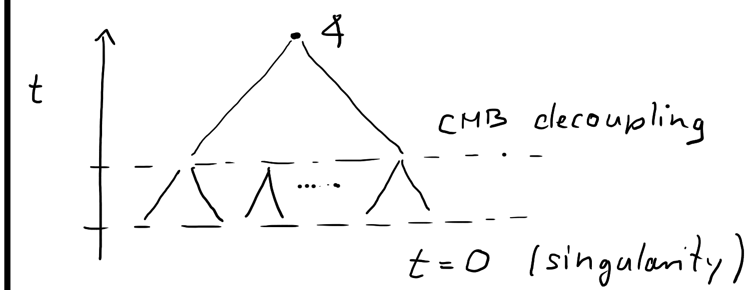
$$\tau = \int_0^t \frac{dt'}{a(t')} \underset{H = \frac{\dot a}{a}}{=} \int_0^a \frac{da}{H a^2} = \int_0^a d\ln a \, \underbrace{\frac{1}{aH}}_{1/\dot a}$$

$$(F2): \quad \ddot a = -\frac{1}{6} a \rho (1 + 3w) < 0 \quad \text{for } w > -1/3$$

$$\rightarrow \frac{1}{aH} \nearrow \quad a \nearrow t \qquad \text{for } w > -1/3$$

→ fraction of universe in causal contact increases with time

in particular, CMB:



- CMB as observed today consists of ~ 10^5 regions which were never in causal contact
- how can they all have the same temperature?

+ flatness problem, see exercise

2) Single field slow-roll inflation

consider a single real scalar field ϕ :

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) = S_{EH} + S_\phi$$

$\delta\sqrt{-g}/\delta g^{\mu\nu}$

$$\rightarrow T^{(\phi)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}} = \partial_\mu \phi \, \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \underbrace{\partial^\sigma \phi \partial_\sigma \phi}_{-\partial_0\phi\partial_0\phi + \partial_i\phi\partial_i\phi} + V(\phi) \right)$$

$(-,+,+,+)$

$$= \mathrm{diag}\,(\rho, -p, -p, -p)$$

assume $\phi(\vec{x},t) = \phi(t)$ homogeneous , $\partial_i \phi = 0$

$$\hookrightarrow \underline{\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)} \; , \quad \underline{P_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)}$$

$$\rightarrow \omega_\phi = \frac{P_\phi}{\rho_\phi} = \frac{1/2\, \dot{\phi}^2 - V(\phi)}{1/2\, \dot{\phi} + V(\phi)}$$

if $V(\phi) \gg 1/2\, \dot{\phi}^2 \Rightarrow \underline{\omega \rightarrow -1} < -1/3$ ■

equation of motion for ϕ :

$$\frac{\partial S}{\partial \phi} - \partial_\mu \frac{\partial S}{\partial \phi_{,\mu}} = \sqrt{-g} \left(-V_{,\phi} - \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, \partial^\mu \phi \right) \right) = 0$$

ϕ homogeneous , FRW metric , $k/a^2 \ll \rho$:

$$\boxed{\begin{aligned} &\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \\ &\text{with } H^2 = 1/3 \left(\tfrac{1}{2} \dot{\phi}^2 + V(\phi) \right) \end{aligned}}$$

eom for a(t), cosmic expansion :

(F1)

$$\rightarrow H = \frac{\dot{a}}{a} = \sqrt{\frac{1}{3} \rho} \; , \quad \rho = a^{-3(1+\omega)} \xrightarrow{\omega = -1} \text{const}$$

$$\rightarrow \text{for } \omega = -1 : \quad \underline{a(t) \propto e^{Ht}}$$

$$\rightarrow \underline{\tau = -\frac{1}{aH}} \qquad (-\infty < \tau < 0)$$

→ exponentially expanding Universe, dominated by potential energy of scalar field

3) Slow-roll approximation

slow-roll parameters:

F1 + F2

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2}\frac{\dot{\phi}^2}{H^2} < 1$$

for accelerated expansion = inflation

$$\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} < 1$$

for sustained slow-roll inflation

Often easier to work with 'potential slow-roll parameters' :

$$\epsilon_V = \frac{M_P^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 , \quad \eta_V = M_P^2 \frac{V_{,\phi\phi}}{V}$$

$$\epsilon_V, |\eta_V| \ll 1 \quad \leftrightarrow \quad \epsilon \approx \epsilon_V, \eta \approx \eta_V - \epsilon_V$$

Slow-roll approximation $\epsilon, \eta \ll 1$:

$$3H\dot{\phi} + V_{,\phi} = 0 \,,$$

$$H^2 = \tfrac{1}{3}V \simeq \text{const.}\,, \quad a(t) \propto e^{Ht}$$

a worked example $V(\phi) = \frac{1}{2}m^2\phi^2$

Introduce a new time-coordinate, 'e-folds':

$$dN = -H\,dt \quad \cdot \quad \frac{da}{a}$$

$$N(\phi) = \ln \frac{a_{end}}{a} = -\int_t^{t_{end}} H\,dt = -\int_\phi^{\phi_{end}} \frac{H}{\dot{\phi}}\,d\phi = \int_{\phi_{end}}^{\phi} \frac{1}{\sqrt{2\varepsilon}}\,d\phi$$

$\varepsilon = 1$

Solving horizon & flatness problem requires $N \gtrsim 60$

$$\varepsilon_V = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 = 2\left(\frac{M_p}{\phi}\right)^2 \overset{!}{<} 1$$

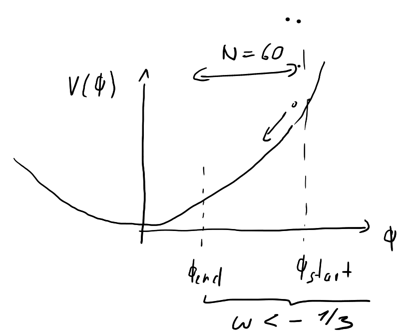
$\varepsilon = 1$

$\rightarrow \phi_{end} = \sqrt{2}\,M_p$

inflation for $\phi > \phi_{end}$

$$N(\phi) = \frac{1}{M_p}\int_{\phi_{end}}^{\phi} \frac{1}{\sqrt{2\varepsilon}}\,d\phi = \frac{\phi^2}{4M_p^2} - \frac{1}{2} \overset{!}{=} 60$$

$\rightarrow \phi_{start} \simeq 2\,N_{60}^{1/2}\,M_p \simeq 15\,M_p$



Simple case of massive scalar field does the job!

4) Cosmological perturbation theory

Consider small perturbation around homogeneous background,

$$X(t,\vec{x}) = \bar{X}(t) + \delta X(t,\vec{x})\,, \quad \delta X \ll \bar{X}\,, \quad X = \phi, g_{\mu\nu}$$

- linear order in δX
- SVT decomposition into scalars, vectors, tensors

counting degrees of freedom:

$\delta\phi$ 1 scalar

$\delta g_{\mu\nu}$ 4 scalars, 2 vectors, 1 tensor (10 dofs of symmetric tensor)

$\hookrightarrow$ GWs

$\downarrow$

only one physical (= dynamical) dof (non-gauge) ← definition of equal time hypersurfaces

$\hat{=}$ choice of equal time hypersurface:

$$\underbrace{\delta X(t,\vec{x})}_{\text{gauge dependent}} = \underbrace{X(t,\vec{x})}_{\text{locally unambig. defined}} - \underbrace{\bar{X}(t)}_{\text{depends on choice of equal time hypersurface}}$$

→ a) define gauge invariant scalar, e.g. comoving curvature perturbation

$$\mathcal{R} = \underset{\delta g_{ii}}{\psi} + \frac{H}{\dot{\phi}}\delta\phi$$

spatial curvature of const. - ϕ hypersurface (during slow roll inflation)

or b) choose a gauge