Pinning down the properties of black hole mergers with gravitational-wave memory

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What’s the gravitational wave memory?

\[ \partial^\mu \partial_\mu \tilde{h}^{jk} = 16\pi (T^{jk}_{\text{matter}} + T^{jk}_{\text{GW}}) \]

The GW itself sources GW!

Result of the non-linear effect of gravity!

\[ T^{jk}_{\text{GW}} = \frac{1}{R^2} \frac{dE_{GW}}{dt d\Omega} n_j n_k \sim \mathcal{O}(\dot{h}^2) \]

Small effect

The wave no longer returns to the zero-point of its oscillation, this growing-offset is called MEMORY.
The Memory Effect

**Thorne Formula:**

\[
\delta \tilde{h}_{ij}^{TT}(T_R) = \frac{4}{R} \int_{-\infty}^{T_R} dt' \left[ \frac{\frac{dE_{GW}}{dt'd\Omega'} n'_jn'_k}{1 - n' \cdot N} \right]^{TT} d\Omega'
\]

- Hereditary/Cumulative effect
- All sources of GW
- Permanent deformation

\[
\frac{d^2l}{dt^2} = \frac{1}{2} h_{ij}^{TT} l^j \quad \frac{\Delta l}{l} \approx \frac{1}{2} \Delta h^{TT}
\]

GW passing through the screen

Radiated gravitons

Different angular dependence

Interruption from past infinity

Christodoulou ‘91, Blanchet & Damour ’92
Wiseman & Will ‘91, Marc Favata ‘09-’11

Credit: Favata

Residual effect

Credit: Favata
PN and Numerical Results

\[ h_+ , 0_{PN} = \left[ - (1 + \cos^2 t) \cos 2\Phi + \frac{1}{96} \sin^2 t (17 + \cos^2 t) + O(x^{1/2}) \right] \frac{2\eta M x}{R} \]

- **Sensitive to the merger phase**
- **Numerical results needed** (inclusion of higher modes)
- **Step function approximation** \( \text{FT}[\Theta] \sim i/f \)
  \[ h_c(f) = 2f |\tilde{h}(f)| \sim \text{const} \text{ for } f \ll f_c \]

Memory extends the signal to low frequencies!
Is the memory helpful for parameter estimation?

\[ h(\hat{\theta}, t) = h_0(\hat{\theta}, t) + \delta h(\hat{\theta}, t) \]

**Forecasts for LISA:**

Fisher and covariance matrix:

\[ \Gamma_{ij} = \left( \frac{dh}{d\theta_i} \right) \left( \frac{dh}{d\theta_j} \right), \quad \Sigma_{ij} = \Gamma_{ij}^{-1} \]

\[ (a|b) = 4 \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{\Re[a^*(f)b(f)]}{S_n(f)} df \]

Signal-to-noise-ratio SNR:

\[ \rho = \sqrt{(h|h)} \]

Q: Can it break the distance-inclination degeneracy?

Credit: Abbott et al., 2016d

\[ M_{\text{tot}} = 2 \times 10^4 M_\odot, \]
\[ z = 0.5, \, t = 40 \text{ deg} \]
The memory helps for “short” and “light” signals.

Dependence on the mass and the duration of the signal pre-merger → the SNR-ratio $\frac{\rho_m}{\rho_0}$ changes!
**Observability and mean improvement**

<table>
<thead>
<tr>
<th>Astrophysical Catalogs</th>
<th>Light Seeds</th>
<th>Heavy Seeds</th>
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<tbody>
<tr>
<td><strong>SN-delays</strong></td>
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<td>$N_{\text{tot}}$ = 39</td>
<td>$N_{\text{tot}} = 25$</td>
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<td>$N_{\text{th}} = 0.1$</td>
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<td>$\rho_{\text{max}} = 1.4$</td>
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<td><strong>NoSN-delay</strong></td>
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<td>$N_{\text{tot}} = 196$</td>
<td>$N_{\text{tot}} = 10.5$</td>
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<td>$N_{\text{th}} = 2$</td>
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<td>$\langle \rho \rangle = 0.1$</td>
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<tr>
<td>$\rho_{\text{max}} = 12.1$</td>
<td>$\rho_{\text{max}} = 67$</td>
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<td><strong>SN-short Delays</strong></td>
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<td>$N_{\text{tot}} = 1155$</td>
<td>$N_{\text{tot}} = 814$</td>
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<tr>
<td>$N_{\text{th}} = 12$</td>
<td>$N_{\text{th}} = 197.15$</td>
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<td>$\langle \rho \rangle = 0.05$</td>
<td>$\langle \rho \rangle = 0.8$</td>
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<td>$\rho_{\text{max}} = 5.16$</td>
<td>$\rho_{\text{max}} = 14.7$</td>
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<tr>
<td><strong>noSN-short Delays</strong></td>
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<td>$N_{\text{tot}} = 1181$</td>
<td>$N_{\text{tot}} = 1254$</td>
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<td>$\langle \rho \rangle = 0.04$</td>
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<td>$\rho_{\text{max}} = 7.5$</td>
<td>$\rho_{\text{max}} = 17.8$</td>
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Assuming an observability window of 3 and 6 hours we find an **averaged improvement of 50%** on $d_L$ for light and closed-by sources.

Total number of events with SNR above threshold $\rho_{\text{th}} = 1(5)$ for **8 different astrophysical models of Massive Black Holes**.
Conclusions and Outlook

- We firstly studied the impact of the memory on the estimation of parameters of a binary GW.

- Memory can mainly help in pinning down the uncertainty on the luminosity distance and inclination.

- Interestingly: major impact for short and almost out of band sources.

- Outlook: “orphan” memory, test of GR, SGWB
Thank you for the attention!
How do we compute it?

To compute the waveform we need to solve this equation:

$$\bar{h}^{TT}_{ij}(t, x) = 4 \int \frac{(-g)[T^{jk}_{\text{matter}}(t', x') + T^{jk}_{\text{GW}}(t', x')]}{|x - x'|} \delta(t' - t - |x - x'|) dx'^4$$

The contribution from the energy-momentum tensor of the GW is:

$$\delta \bar{h}^{TT}_{ij} = \frac{4}{R} \int_{-\infty}^{T} dt' \left[ \int \frac{dE_{GW}}{dt'\,d\Omega'} \frac{n'_j n'_k}{|1 - n' \cdot N|} \, d\Omega' \right]^{TT}$$

The memory depends on the whole history of the binary

$$T^{jk}_{GW} \sim \frac{1}{R^2} \frac{dE_{GW}}{dt'\,d\Omega'} = \frac{c^3}{16\pi G} |\dot{h}(t, \Omega)|^2$$

Substituting post-Newtonian waveforms one finds:

$$h_+ = \frac{2\eta Mx}{R} \left[ -(1 + \cos^2 \iota \cos 2\Phi + \frac{1}{96} \sin^2 \iota (17 + \cos^2 \iota) + 0(x^{1/2}) \right]$$

The memory is present only in $h_+$
Inclination Dependence & Maximum Improvement
How many sources are we going to see in this mass range?

There is huge uncertainty on the merger rate of SMBHs because we don’t know their initial seed mass, the impact of the SN feedback, the time delay between the formation of the binary and the merger.... 8 models with different distribution in mass and redshift

White points
SNR ≥ 1;
Red stars:
SNR ≥ 5