Pinning down the properties of black hole mergers with gravitational-wave memory

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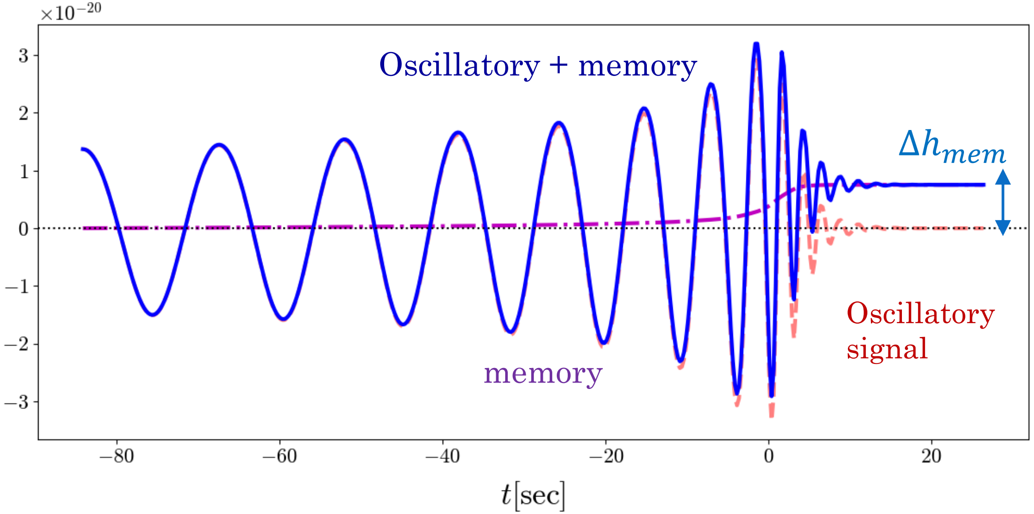
Institut de Física d'Altes Energies

Christodoulou ‘91,
Blanchet & Damour ’92
Wiseman & Will ‘91

What’s the gravitational wave memory?



$$\partial^{\mu}\partial_{\mu}\bar{h}^{\,j,k} = 16\pi\left(T^{jk}_{matter} + T^{jk}_{GW}\right)$$

The GW itself sources GW!

Result of the non-linear effect of gravity!

$$T^{jk}_{GW} = \frac{1}{R^2}\frac{dE_{GW}}{dt d\Omega} n_j n_k \sim \mathcal{O}(h^2)$$

Small effect

The memory slowly builds up during the inspiral, grows rapidly during the merger and saturates to its final value during the ringdown

The wave no longer returns to the zero-point of its oscillation, this growing-offset is called MEMORY

The Memory Effect

Christodoulou '91, Blanchet & Damour '92
Wiseman & Will '91, Marc Favata '09-'11

Thorne Formula:

$$\delta \bar{h}^{TT}_{ij}(T_R) = \frac{4}{R}\int_{-\infty}^{T_R} dt' \left[\int \frac{dE_{GW}}{dt'd\Omega'} \frac{n'_j n'_k}{|1 - n' \cdot N|} d\Omega' \right]^{TT}$$

Integration from past infinity

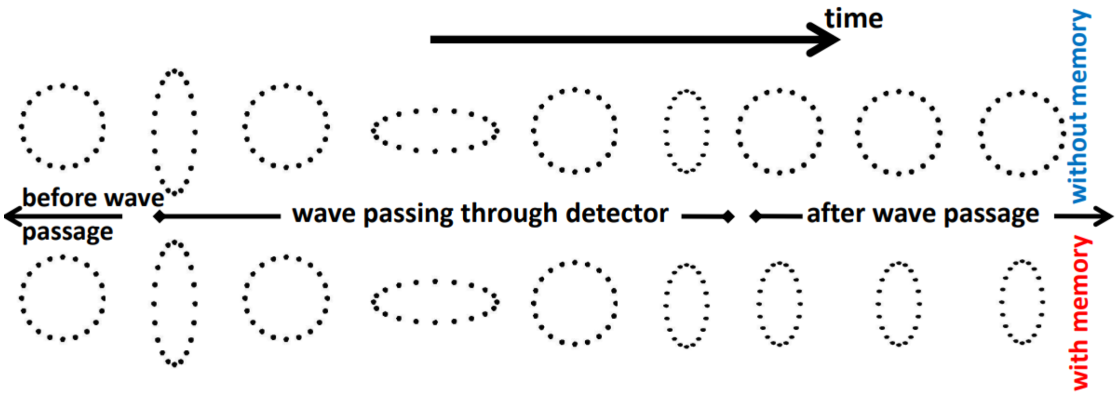
Radiated gravitons

Different angular dependence

- Hereditary/Cumulative effect
- All sources of GW
- Permanent deformation

$$\frac{d^2 l}{dt^2} = \frac{1}{2}\ddot{h}^{TT}_{ij} l^j \qquad \frac{\Delta l}{l} \approx \frac{1}{2}\Delta h^{TT}$$

GW passing through the screen



Residual effect

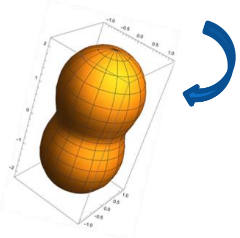
Credit:Favata

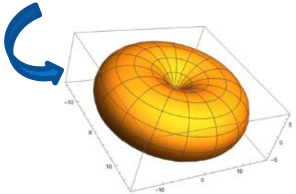
PN and Numerical Results

$$h_{+,0PN} = \left[-(1+\cos^2\iota)\cos 2\Phi + \frac{1}{96}\sin^2\iota(17+\cos^2\iota) + O(x^{1/2})\right]\frac{2\eta M x}{R}$$

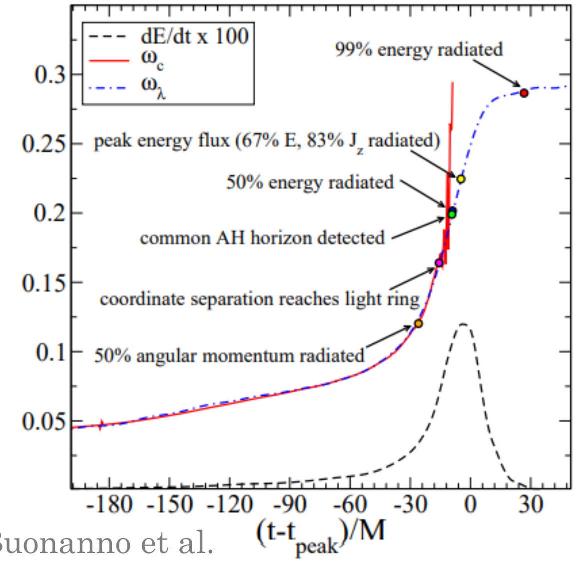
Dominant mode (2,2)

Dominant mode (2,0)

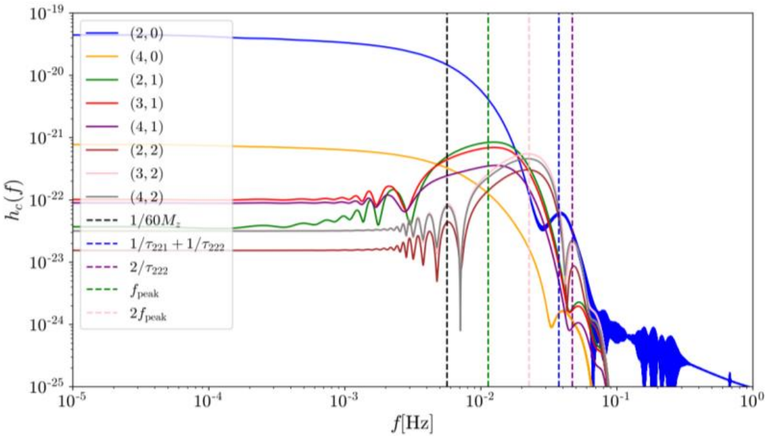




$$\eta = \frac{m_1 m_2}{M^2}$$
$$x = (M\omega)^{2/3}$$



Credit:Buonanno et al.



•**Sensitive to the merger phase**

•**Numerical results needed** (inclusion of higher modes)

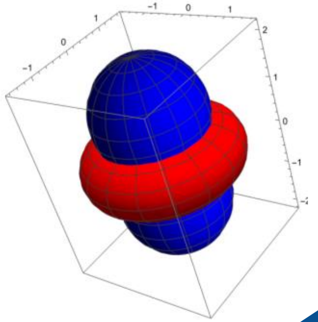
•Step function approximation FT[Θ]~ i/f
$h_c(f) = 2f|\tilde{h}(f)|$~const for $f \ll f_c$

Memory extends the signal to low frequencies!

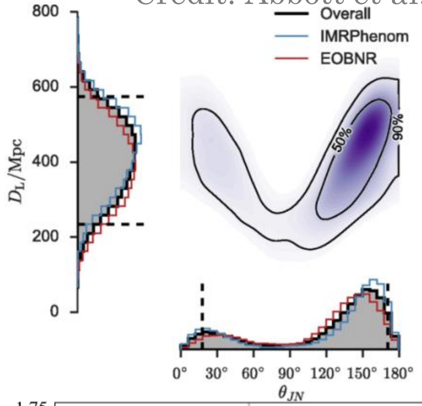
Is the memory helpful for parameter estimation?

$$h(\vec{\theta},t) = h_0(\vec{\theta},t) + \delta h(\vec{\theta},t)$$

Opposite inclination dependence



Credit: Abbott et al., 2016d



<u>Forecasts for LISA:</u>

Fisher and covariance matrix:

$$\Gamma_{ij} = \left(\frac{dh}{d\theta_j}\middle|\frac{dh}{d\theta_j}\right), \qquad \Sigma_{ij} = \Gamma_{ij}^{-1}$$

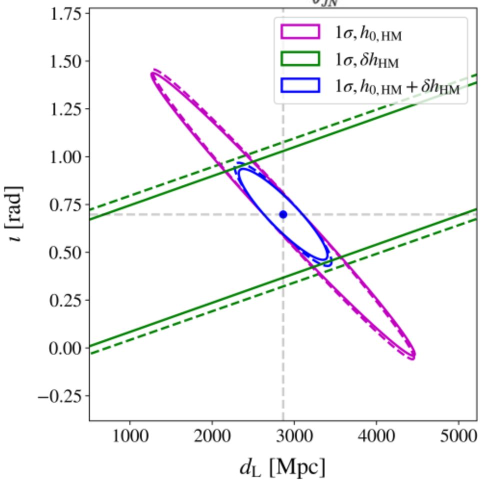
$$(a|b) = 4\int_{f_{min}}^{f_{max}} \frac{\mathcal{R}[a^*(f)b(f)]}{S_n(f)}\,df$$

Signal-to-noise-ratio SNR:

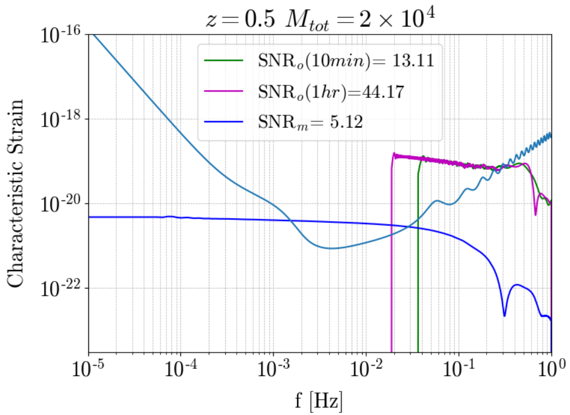
$$\rho = \sqrt{(h|h)}$$

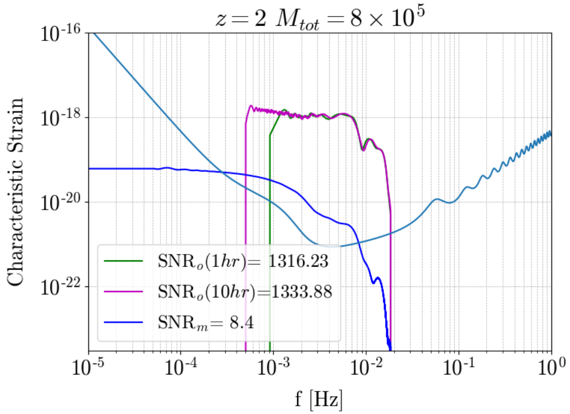
Q: Can it break the distance-inclination degeneracy?

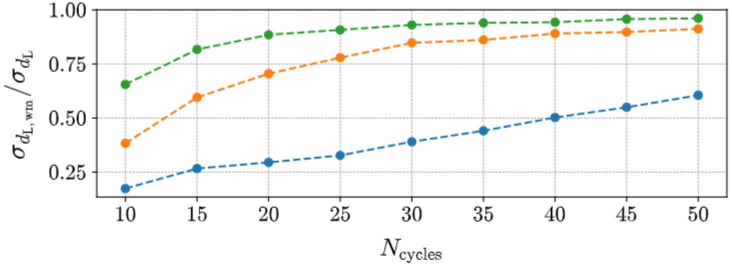
$M_{tot} = 2\times10^4\, M_\odot$,
$z = 0.5$, $\iota = 40$ deg

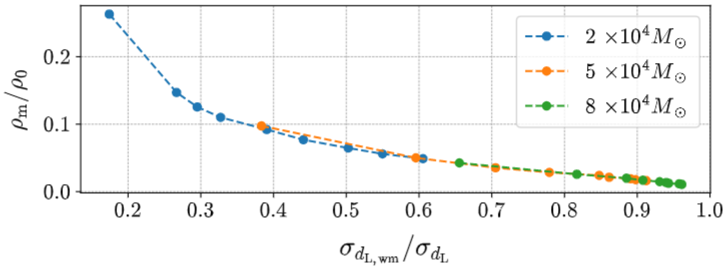


The memory helps for "short" and "light" signals









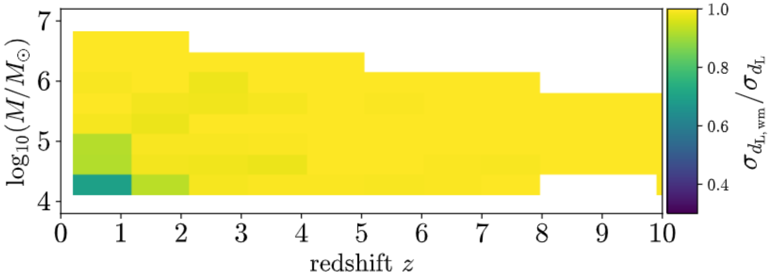
Dependence on the **mass** and the **duration** of the signal **pre-merger** → the **SNR-ratio** $\frac{\rho_m}{\rho_0}$ **changes!**

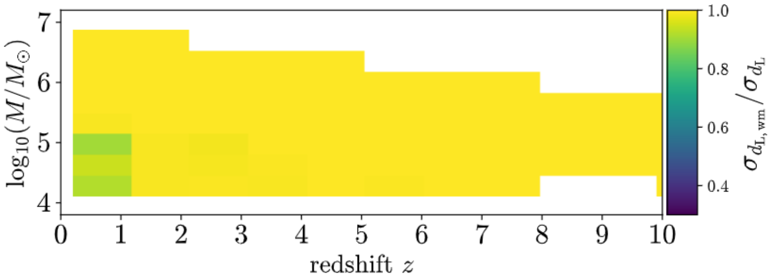
Observability and mean improvement

	Astrophysical Catalogs	
	Light Seeds	**Heavy Seeds**
SN-delays	$N_{\rm tot} = 39$ $N_{\rm th} = 0.1\,(0)$ $\langle\rho\rangle = 0.01$ $\rho_{\max} = 1.4$	$N_{\rm tot} = 25$ $N_{\rm th} = 16\,(6.5)$ $\langle\rho\rangle = 4.5$ $\rho_{\max} = 65$
NoSN-delay	$N_{\rm tot} = 196$ $N_{\rm th} = 2\,(1)$ $\langle\rho\rangle = 0.1$ $\rho_{\max} = 12.1$	$N_{\rm tot} = 10.5$ $N_{\rm th} = 6.4\,(3)$ $\langle\rho\rangle = 5.3$ $\rho_{\max} = 67$
SN-short Delays	$N_{\rm tot} = 1155$ $N_{\rm th} = 12\,(2)$ $\langle\rho\rangle = 0.05$ $\rho_{\max} = 5.16$	$N_{\rm tot} = 814$ $N_{\rm th} = 197\,(15)$ $\langle\rho\rangle = 0.8$ $\rho_{\max} = 14.7$
noSN-short Delays	$N_{\rm tot} = 1181$ $N_{\rm th} = 11\,(1)$ $\langle\rho\rangle = 0.04$ $\rho_{\max} = 7.5$	$N_{\rm tot} = 1254$ $N_{\rm th} = 300\,(24)$ $\langle\rho\rangle = 0.8$ $\rho_{\max} = 17.8$

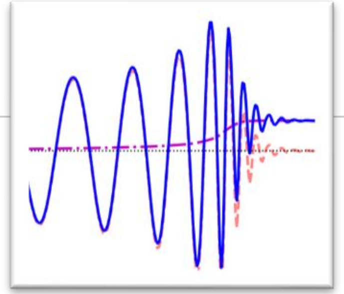
Total number of events with SNR above threshold ρ_{th} = 1(5) for **8 different astrophysical models of Massive Black Holes**

Assuming an observability window of 3 and 6 hours we find an **averaged improvement of 50 % on d_L** for light and closed-by sources





Conclusions and Outlook



- We firstly studied the impact of the memory on the estimation of parameters of a binary GW.
- Memory can mainly help in pinning down the uncertainty on the luminosity distance and inclination.
- Interestingly: major impact for short and almost out of band sources.
- Outlook: “orphan” memory, test of GR, SGWB

Thank you for the attention!

How do we compute it?

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To compute the waveform we need to solve this equation:

$$\bar{h}_{ij}^{TT}(t,\boldsymbol{x}) = 4\int \frac{(-g)\left[T^{jk}_{matter}(t',\boldsymbol{x}') + T^{jk}_{GW}(t',\boldsymbol{x}')\right]}{|\boldsymbol{x}-\boldsymbol{x}'|}\delta(t'-t-|\boldsymbol{x}-\boldsymbol{x}'|)dx'^4$$

The contribution from the energy-momentum tensor of the GW is:

$$\delta\bar{h}_{ij}^{TT} = \frac{4}{R}\int_{-\infty}^{T_R} dt' \left[\int \frac{dE_{GW}}{dt'd\Omega'}\frac{n'_j n'_k}{|1-n'\cdot N|}d\Omega'\right]^{TT}$$

The memory depends on the whole history of the binary

$$T^{jk}_{GW} \sim \frac{1}{R^2}\frac{dE_{GW}}{dt'd\Omega'} = \frac{c^3}{16\pi G}|\dot{h}(t,\Omega)|^2$$

Substituting post-Newtonian waveforms one finds:

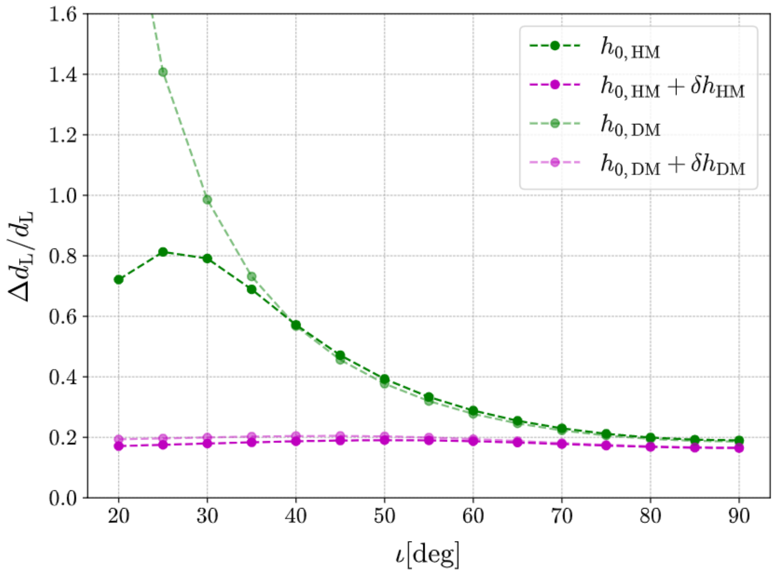
$$h_+ = \frac{2\eta M x}{R}\left[-(1+\cos^2\iota)\cos 2\Phi + \frac{1}{96}\sin^2\iota(17+\cos^2\iota) + O(x^{1/2})\right]$$

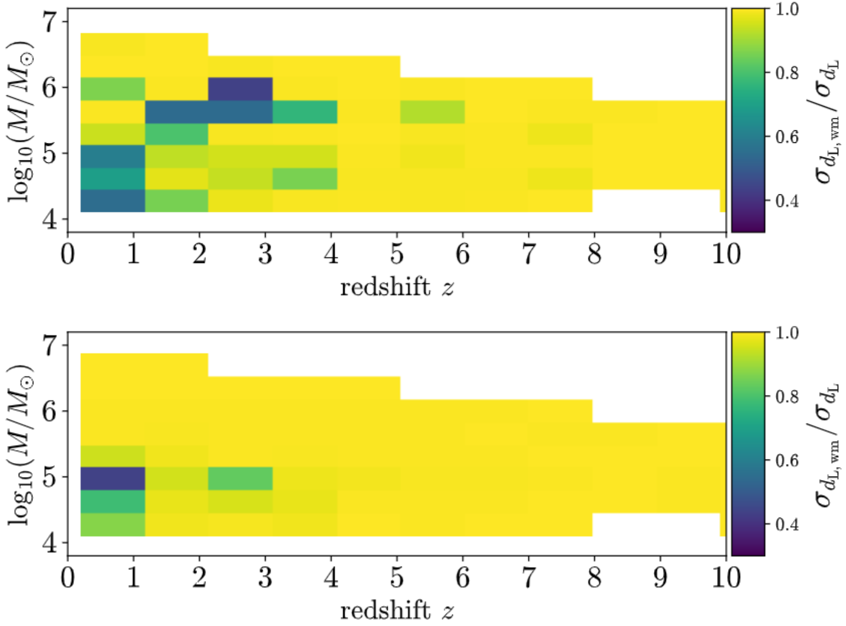
The memory is present only in h_+

Oscillatory

Memory

Inclination Dependence & Maximum Improvement





How many sources are we going to see in this mass range?

There is huge uncertainty on the merger rate of SMBHs because we don't know their initial seed mass, the impact of the SN feedback, the time delay between the formation of the binary and the merger…. 8 models with different distribution in mass and redshift

White points $SNR \geq 1$;
Red stars: $SNR \geq 5$