The Mass Sheet Degeneracy in Cosmography Analyses

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Mostly based on LT, K. Blum, E. Castorina, M. Simonović and Y. Soreq (2022) [2201.05111]

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Measuring $H_0$ with lensing

- TDCOSMO collaboration (COSMOGRAIL, HoLiCOW, STRIDES, SHARP, COSMICLENS)
- Unaccounted galactic density profile features (core?): internal mass sheet degeneracy
- Large scale structures “on the way”: external mass sheet degeneracy (weak lensing)

credit: NASA

Wong et al. 2019
Strong Gravitational Lensing in Elliptical Galaxies

- \( \hat{\alpha} = 2 \int \nabla_\perp \Phi \, d\lambda \implies \vec{\beta} = \vec{\theta} - \frac{D_{ds}}{D_s} \hat{\alpha}(\vec{\theta}) \)

- Lens model + time delay measurement
  \[ \Delta t_{ij} \propto \frac{1}{H_0} \]

- Degeneracies: source position and mass of galaxy unknown
  \( \vec{\beta} \rightarrow \lambda \vec{\beta} , \vec{\alpha} \rightarrow \lambda \vec{\alpha} + (1 - \lambda)\vec{\theta} \implies H_0 \rightarrow \lambda H_0 \)

from V. Bonvin et al (2016)
Effects of Large Scale Structure

- Stellar kinematics can constrain mass in the galaxy. Bias:
  \[
  \frac{H_0^{\text{inferred}}}{H_0^{\text{true}}} = 1 + \kappa^L
  \]

- Large unaccounted cores have similar effects to weak lensing \(\kappa^S\) (internal mass sheet degeneracy), but it can have larger effects, \(1 - \lambda = \kappa_{c1} \implies \kappa_{c1} \simeq \frac{\delta H_0}{H_0}\)

External convergences \(\kappa\) are unobservable, they can be removed from modeling (not completely true in multiple source systems!)
Multi-source systems

Cluster MACS J1149.5+2223. Credit: NASA

- Observable ($C_i$ involves ratio of angular diameter distances):
  \[
  \delta \kappa^{\text{obs}} = \delta \kappa_{1i}^S - \delta \kappa_{1i}^L + \kappa_c (1 - C_i)
  \]

- Differential external convergences are measurable! Help characterizing the density field on the line of sight, or spot an unaccounted feature of the lens model.
Summary

Can we measure $H_0$ with Gravitational Lensing?

- Challenge: modeling degeneracies; important to point out all the shortcomings to lensing collaborations, in order to achieve an *accurate* $H_0$ measurement.
- Promising! New data arriving, including precise stellar kinematics, new lensed systems etc.
- Effects from observer-lens and lens-source sightlines *must* be taken into account for a non-biased inference of time delays and non-biased stellar kinematics constraint.
- With multi-source systems, you might be able to spot a (large) core component, and/or better characterize the density field on the line of sight.
- With an $H_0$ prior, we can measure galactic features like large cores, difficult to spot otherwise, and we can have a better characterization of the weak lensing field.
The multilens equation
The tidal approximation

\[
M_S = \kappa^S \mathbb{I} - \Gamma^S
\]

from McCully et al (2014)

\[
M_L = \kappa^L \mathbb{I} - \Gamma^L
\]

\[
M_{LS} = \kappa^{LS} \mathbb{I} - \Gamma^{LS}
\]

Lens equation in tidal approximation

\[
\vec{\beta} = (\mathbb{I} - M^S) \vec{\theta} - (\mathbb{I} - M^{LS}) \vec{\alpha}((\mathbb{I} - M^L) \vec{\theta})
\]

Degeneracy ("revised" MSD)

\[
1 - M^R \mapsto \lambda_R (1 - M^R),
\]

\[
\vec{\beta} \mapsto \lambda_S \vec{\beta},
\]

\[
\vec{\alpha}(\vec{\theta}) \mapsto \lambda_S \lambda_{LS}^{-1} \vec{\alpha}(\lambda_L^{-1} \vec{\theta}),
\]

\[
\Psi(\vec{\theta}) \mapsto \lambda_S \lambda_{LS}^{-1} \lambda_L \Psi(\lambda_L^{-1} \vec{\theta}).
\]

Choose

\[
\lambda_S = \frac{1}{1 - \kappa_S}, \quad \lambda_{LS} = \frac{1}{1 - \kappa_{LS}}, \quad \lambda_L = \frac{1}{1 - \kappa_L},
\]

external convergence removed from the modeling
Interpreting the Mass Sheet Degeneracy

Springer et al 2005

• Change $\lambda_S$: changing one’s mind about the true $\kappa^S$

$$\kappa^R \rightarrow \lambda_R \kappa^R + (1 - \lambda_R)$$

$$\Gamma^R \rightarrow \lambda_R \Gamma^R$$

• Time delays do change!

$$\Delta \tau \rightarrow \lambda_S \lambda^{-1}_{LS} \lambda_L \Delta \tau$$

$$H_0 \rightarrow \lambda_S \lambda^{-1}_{LS} \lambda_L H_0$$

• Estimate $\kappa^S$ via ray-tracing through Millennium Simulation and characterization of the lens field

• Degeneracy is limited by priors on weak lensing quantities and constraints on mass of lens galaxy (stellar kinematics)
Multi-source systems and MSD

- New lens equation (ignoring lens-lens coupling)

\[
\vec{\beta}_i = (\vec{\text{I}} - M_i^S)\vec{\theta} - (\vec{\text{I}} - M_i^{LS})C_i\vec{\alpha}_1 ((\vec{\text{I}} - M^L)\vec{\theta})
\]

\[
C_i := \frac{D_{S1}D_{LS_i}}{D_{LS1}D_{S1}}, \quad \left( \frac{\vec{\alpha}D_{LS}}{D_S} =: \vec{\tilde{\alpha}} \right)
\]

- MSD untouched! But I cannot rescale away all external convergences \(\implies\) differential external convergences

- Observable:

\[
C_i \frac{\left| \vec{\alpha}_1^{\text{model}}(\vec{\theta}) \right|}{\left| \vec{\alpha}_2^{\text{model}}(\vec{\theta}) \right|} \approx \left( 1 + \delta\kappa_{12}^S - \delta\kappa_{12}^{LS} \right)
\]
External convergence reinterpretation?

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Cluster MACS J1149.5+2223

• With \( \delta \kappa^{\text{ext}} := \delta \kappa_{21}^{S} - \delta \kappa_{21}^{LS} \),
  \( \delta \kappa^{\text{ext}} \rightarrow \delta \kappa^{\text{ext}, \lambda} = \delta \kappa^{\text{ext}} + \kappa_{c1} (C_2 - 1) \)

• "Good systems" we have now cannot say much; clusters have still a lot of modeling uncertainties.

• With future surveys, many more multisource systems \( \Rightarrow \) we could spot the core!