Weizmann Institute

The Mass Sheet Degeneracy in Cosmography Analyses

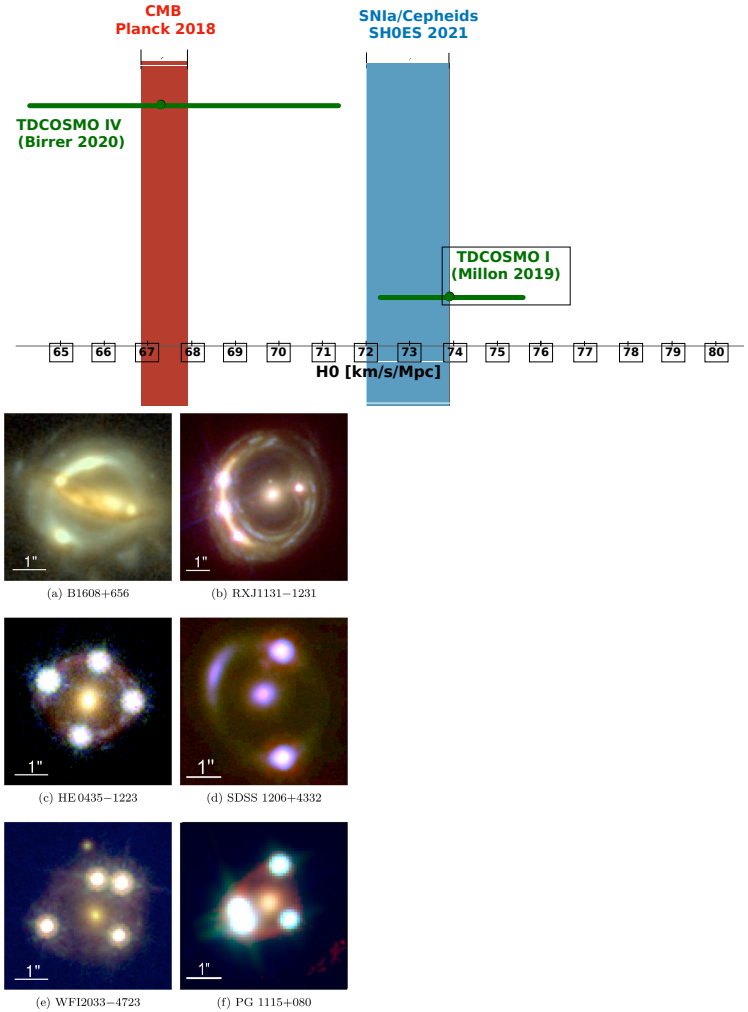
XXXIII Canary Island Winter School of Astrophysics

Teodori Luca

Mostly based on LT, K. Blum, E. Castorina, M. Simonović and Y. Soreq (2022) [2201.05111]

November/December 2022

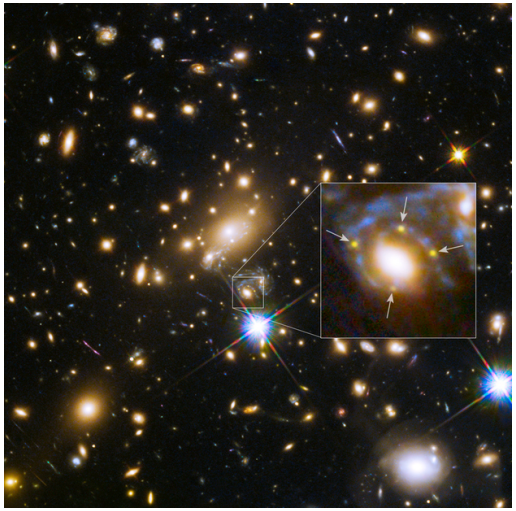
Measuring H_0 with lensing



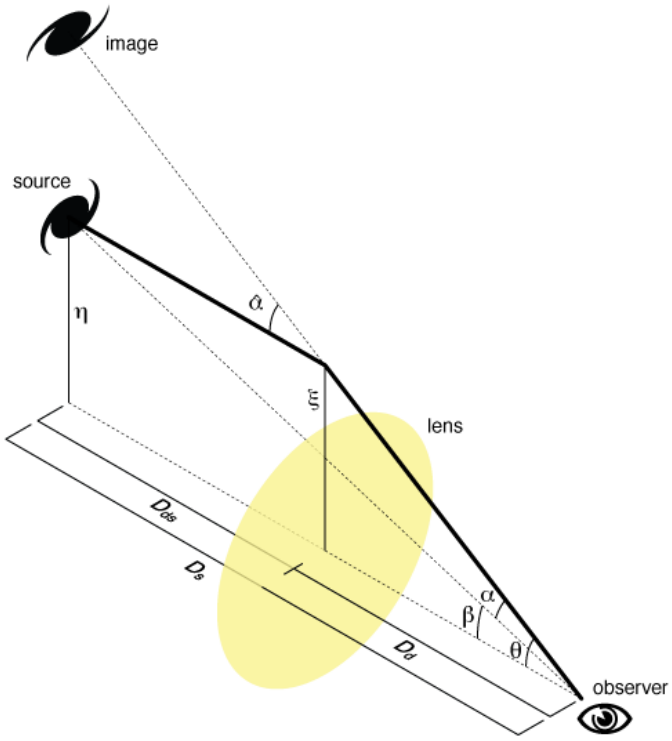
Wong et al. 2019

- TDCOSMO collaboration (COSMOGRAIL, HoLiCOW, STRIDES, SHARP, COSMICLENS)
- Unaccounted galactic density profile features (core?): internal mass sheet degeneracy
- Large scale structures "on the way": external mass sheet degeneracy (weak lensing)

credit: NASA



Strong Gravitational Lensing in Elliptical Galaxies



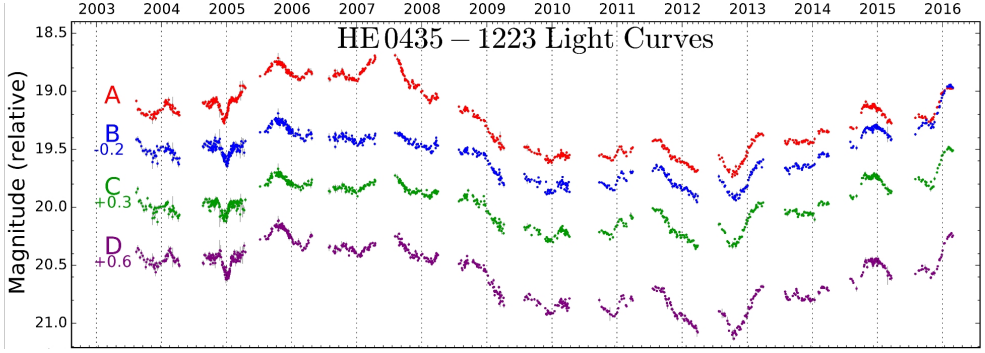
credit: Wikipedia

- $\hat{\alpha} = 2\int \nabla_\perp \Phi \, \mathrm{d}\lambda \implies \vec{\beta} = \vec{\theta} - \frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}}\hat{\alpha}(\vec{\theta})$
- Lens model + time delay measurement

$$\Delta t_{ij} \propto \frac{1}{H_0} \, .$$

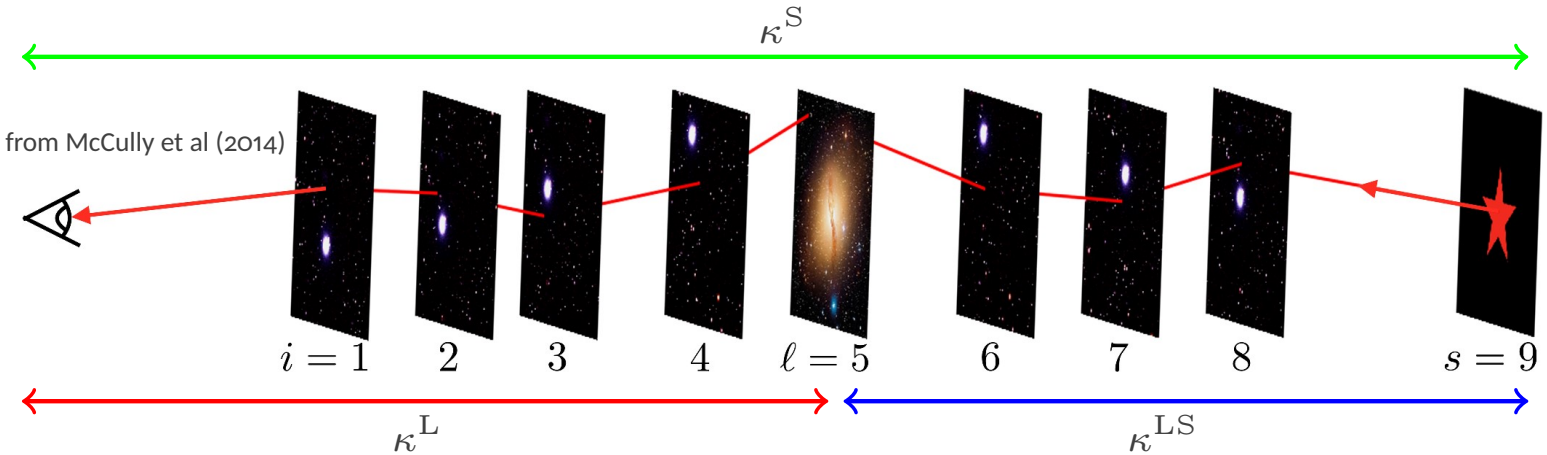
- Degeneracies: source position and mass of galaxy unknown

$$\vec{\beta} \to \lambda\vec{\beta} \, , \; \vec{\alpha} \to \lambda\vec{\alpha} + (1-\lambda)\vec{\theta} \implies H_0 \to \lambda H_0$$

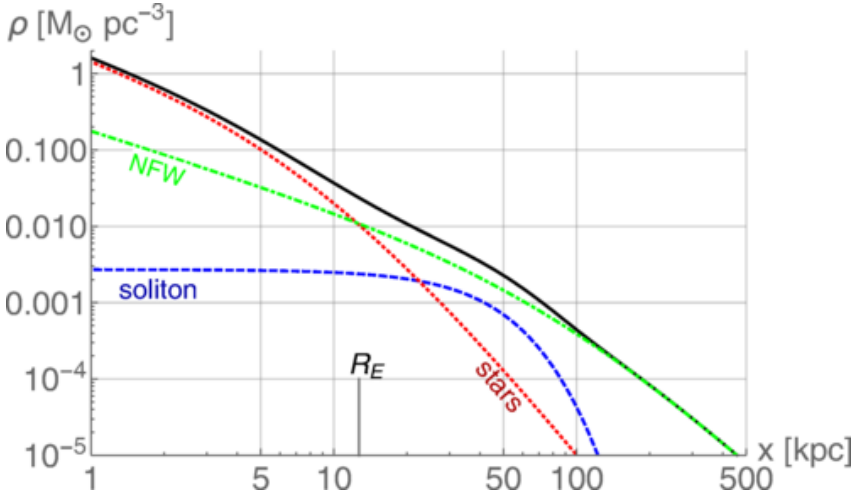


from V. Bonvin et al (2016)

Effects of Large Scale Structure



External convergences κ are unobservable, they can be removed from modeling (not completely true in multiple source systems!)



Blum, Teodori 2021

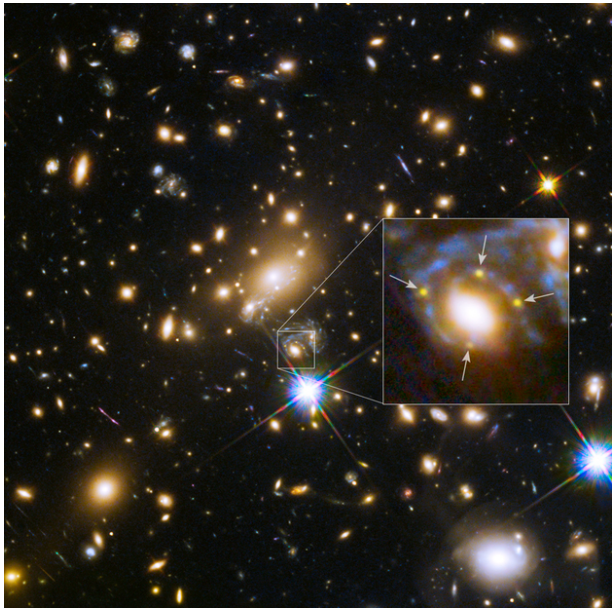
- Stellar kinematics can constrain mass in the galaxy. Bias:

$$\frac{H_0^{\mathrm{inferred}}}{H_0^{\mathrm{true}}} = 1 + \kappa^{\mathrm{L}}$$

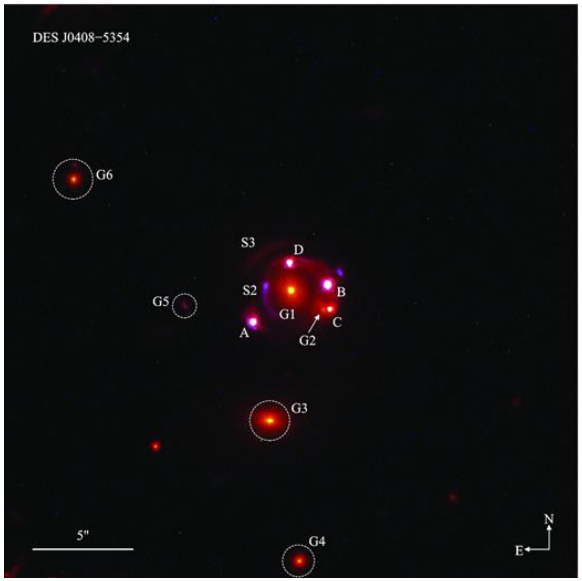
- Large unaccounted cores have similar effects to weak lensing κ^{S} (internal mass sheet degeneracy), but it can have larger effects, $1 - \lambda = \kappa_{\mathrm{c1}} \implies \kappa_{\mathrm{c1}} \simeq \frac{\delta H_0}{H_0}$

Multi-source systems

Cluster MACS J1149.5+2223. Credit: NASA



Shajib et al 2019



- Observable (C_i involves ratio of angular diameter distances):

$$\delta\kappa^{\rm obs} = \delta\kappa^{\rm S}_{1i} - \delta\kappa^{\rm LS}_{1i} + \kappa_{\rm c1}\,(1 - C_i)$$

- Differential external convergences are measurable! Help characterizing the density field on the line of sight, or spot an unaccounted feature of the lens model.

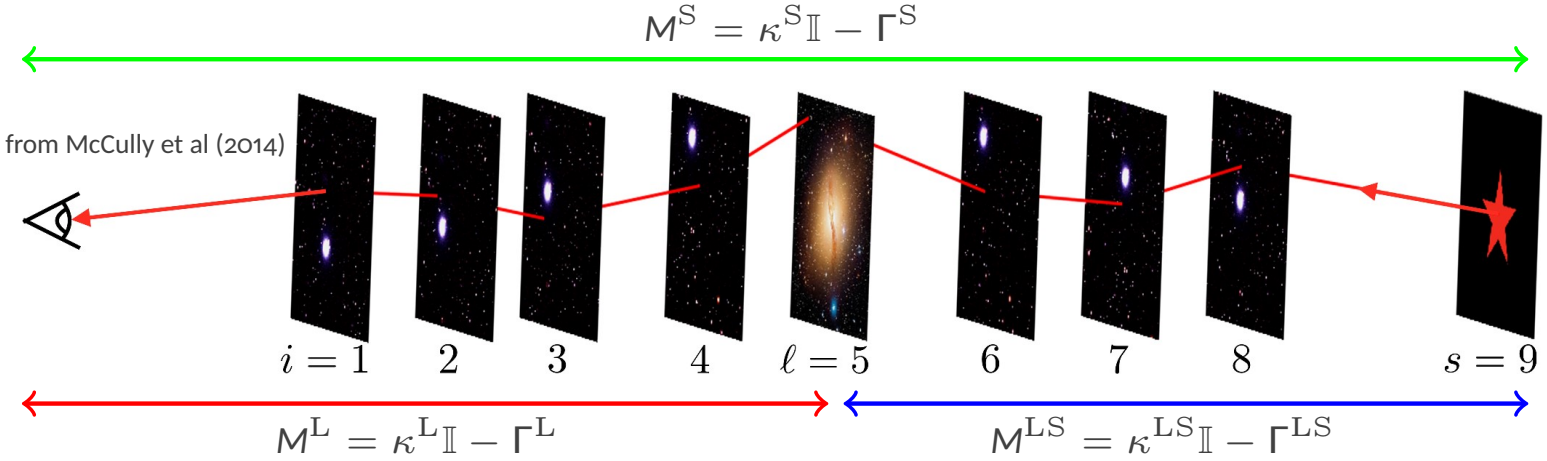
Summary

Can we measure H_0 with Gravitational Lensing?

- Challenge: modeling degeneracies; important to point out all the shortcomings to lensing collaborations, in other to achieve an *accurate* H_0 measurement.
- Promising! New data arriving, including precise stellar kinematics, new lensed systems etc.
- Effects from observer-lens and lens-source sightlines *must* be taken into account for a non-biased inference of time delays and non-biased stellar kinematics constraint.
- With multi-source systems, you might be able to spot a (large) core component, and/or better characterize the density field on the line of sight.
- With an H_0 prior, we can measure galactic features like large cores, difficult to spot otherwise, and we can have a better characterization of the weak lensing field

The multilens equation

The tidal approximation



Lens equation in tidal approximation

$$\vec{\beta} = (\mathbb{I} - M^{\rm S})\vec{\theta} - (\mathbb{I} - M^{\rm LS})\vec{\alpha}((\mathbb{I} - M^{\rm L})\vec{\theta})$$

Degeneracy ("revised" MSD)

$$\begin{aligned} 1 - M^{\rm R} &\longmapsto \lambda_{\rm R}(1 - M^{\rm R}), \\ \vec{\beta} &\longmapsto \lambda_{\rm S}\vec{\beta}, \\ \vec{\alpha}(\vec{\theta}) &\longmapsto \lambda_{\rm S}\,\lambda_{\rm LS}^{-1}\,\vec{\alpha}(\lambda_{\rm L}^{-1}\,\vec{\theta}), \\ \Psi(\vec{\theta}) &\longmapsto \lambda_{\rm S}\,\lambda_{\rm LS}^{-1}\,\lambda_{\rm L}\,\Psi(\lambda_{\rm L}^{-1}\,\vec{\theta}). \end{aligned}$$

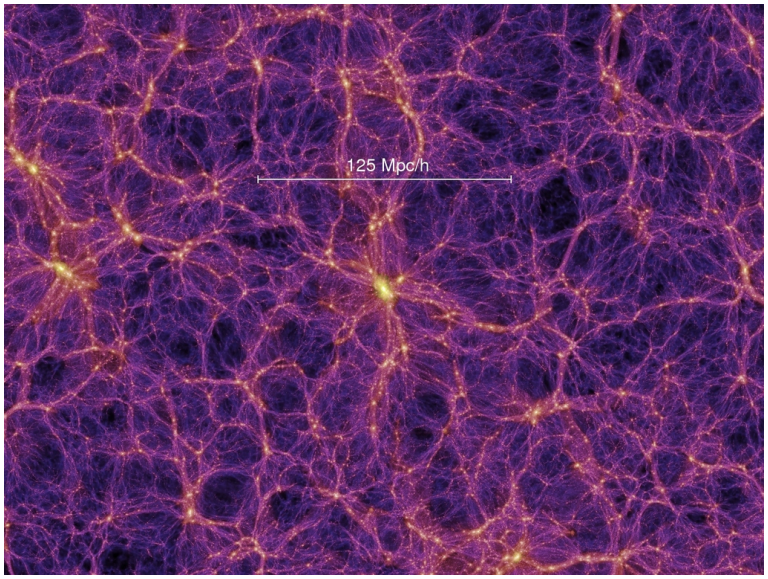
Choose

$$\lambda_{\rm S} = \frac{1}{1-\kappa^{\rm S}}, \quad \lambda_{\rm LS} = \frac{1}{1-\kappa^{\rm LS}}, \quad \lambda_{\rm L} = \frac{1}{1-\kappa^{\rm L}},$$

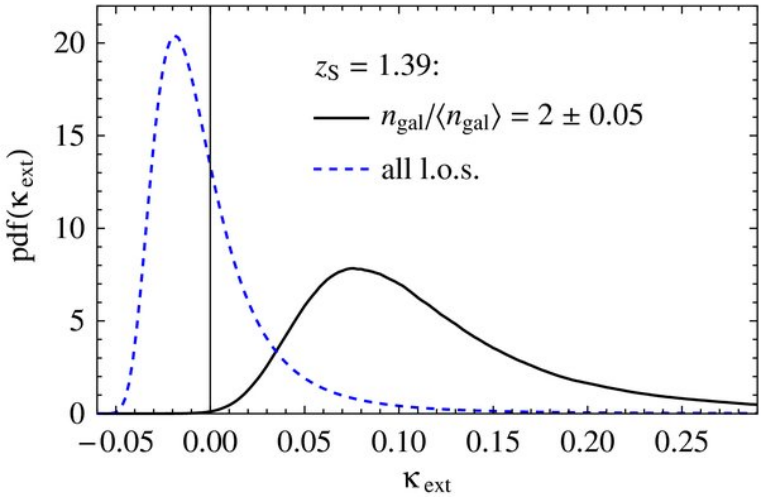
external convergence removed from the modeling

Interpreting the Mass Sheet Degeneracy

Springer et al 2005



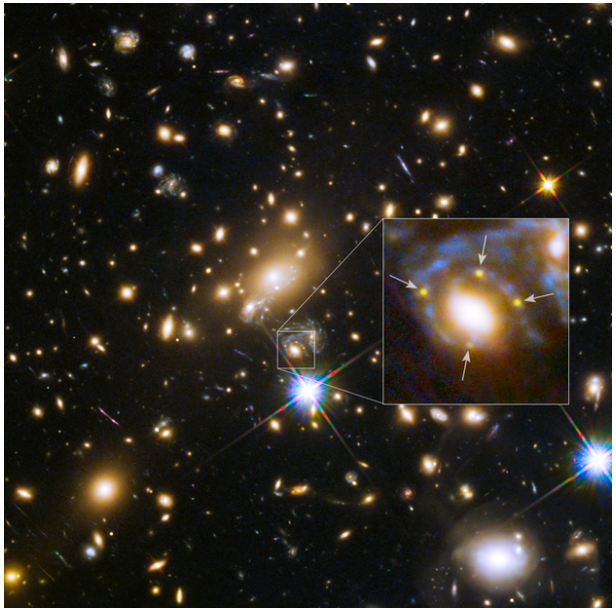
Suyu et al 2010



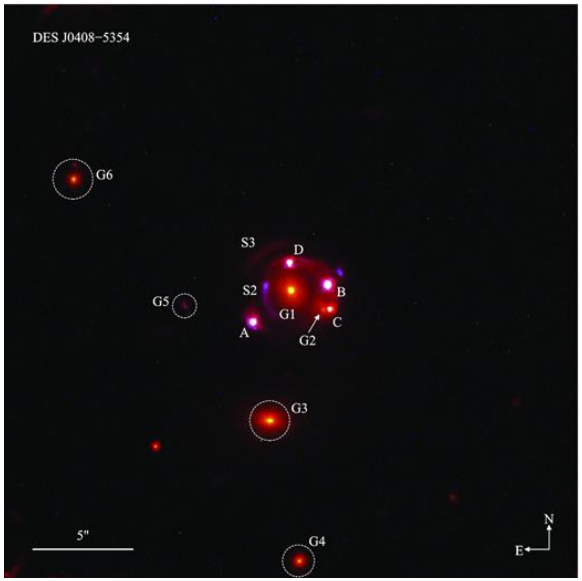
- Change λ_S: changing one's mind about the true κ^S
- $$\kappa^R \longmapsto \lambda_R \kappa^R + (1 - \lambda_R)$$
  $$\Gamma^R \longmapsto \lambda_R \Gamma^R$$
- Time delays do change!
  $$\Delta\tau \to \lambda_S \lambda_{LS}^{-1} \lambda_L \Delta\tau$$
  $$H_0 \to \lambda_S \lambda_{LS}^{-1} \lambda_L H_0$$
- Estimate κ^S via ray-tracing through Millennium Simulation and characterization of the lens field
- Degeneracy is limited by priors on weak lensing quantities and constraints on mass of lens galaxy (stellar kinematics)

Multi-source systems and MSD

Cluster MACS J1149.5+2223. Credit: NASA



Shajib et al 2019



- New lens equation (ignoring lens-lens coupling)

$$\vec{\beta}_i = (\mathbb{I} - M_i^{\mathrm{S}})\vec{\theta} - (\mathbb{I} - M_i^{\mathrm{LS}})C_i\vec{\alpha}_1((\mathbb{I} - M^{\mathrm{L}})\vec{\theta})$$

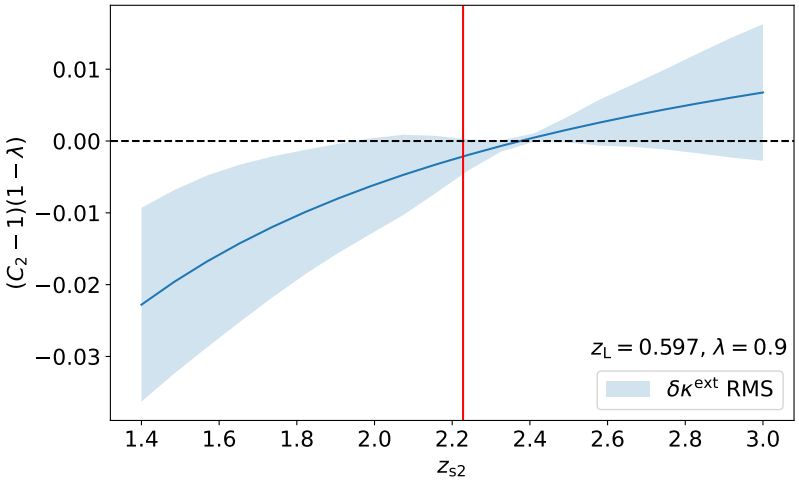
$$C_i := \frac{D_{\mathrm{S}_1} D_{\mathrm{LS}_i}}{D_{\mathrm{LS}_1} D_{\mathrm{S}_i}}\ , \quad \left(\hat{\alpha}\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}} =: \vec{\alpha}\right)$$

- MSD untouched! But I cannot rescale away all external convergences $\implies$ differential external convergences
- Observable:

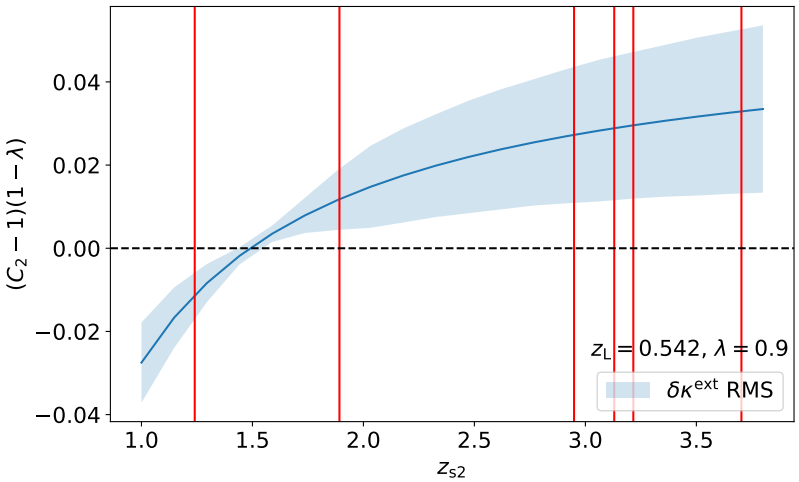
$$C_i \frac{\left|\vec{\alpha}_1^{\mathrm{model}}(\vec{\theta})\right|}{\left|\vec{\alpha}_2^{\mathrm{model}}(\vec{\theta})\right|} \approx \left(1 + \delta\kappa_{12}^{\mathrm{S}} - \delta\kappa_{12}^{\mathrm{LS}}\right)$$

External convergence reinterpretation?

DESJ0408-5354



Cluster MACS J1149.5+2223



- With $\delta\kappa^{\rm ext} := \delta\kappa^{\rm S}_{21} - \delta\kappa^{\rm LS}_{21}$,

  $\delta\kappa^{\rm ext} \to \delta\kappa^{{\rm ext},\lambda} = \delta\kappa^{\rm ext} + \kappa_{\rm c1}\,(C_2 - 1)$

- "Good systems" we have now cannot say much; clusters have still a lot of modeling uncertainties.
- With future surveys, many more multisource systems $\implies$ we could spot the core!