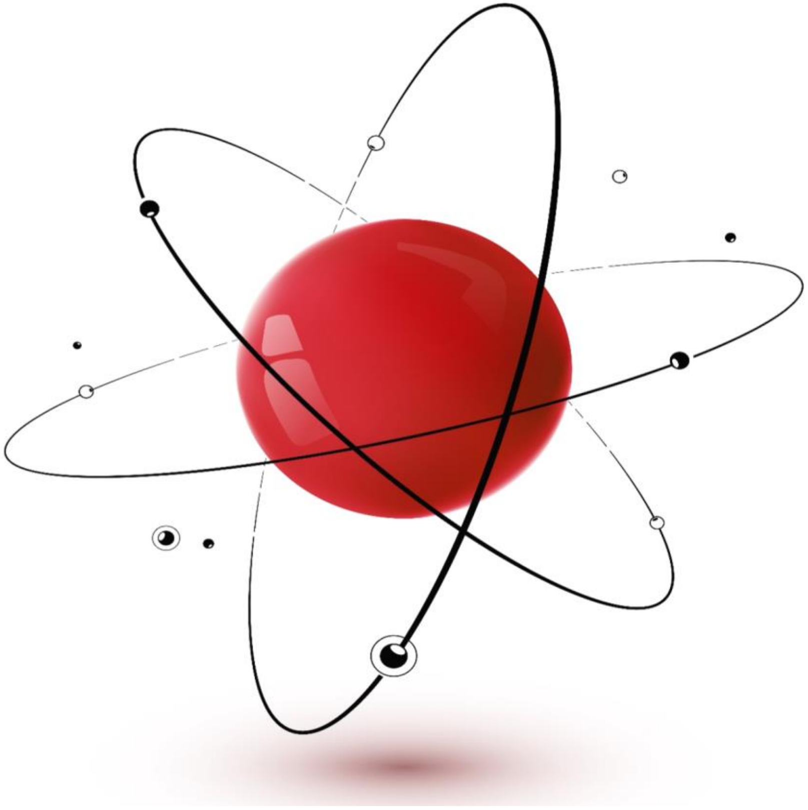
Low-energy Nucleon Structure at the Precision Frontier

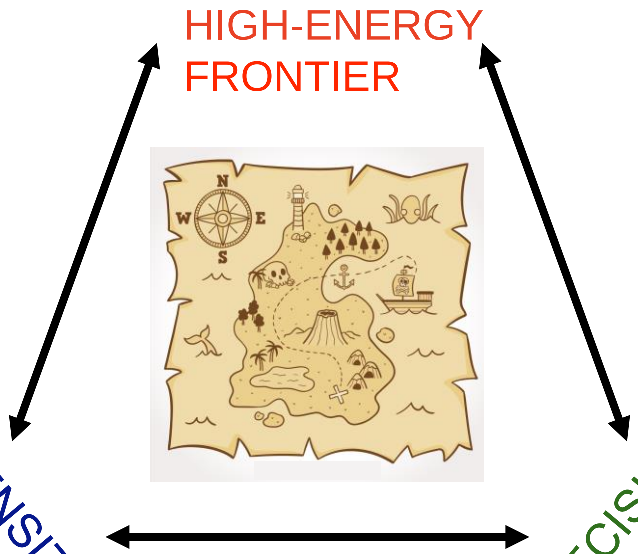


Vladyslava Sharkovska (PSI&UZH)

NEW PHYSICS SEARCHES

Lab searches for New Physics proceed along 3 frontiers:

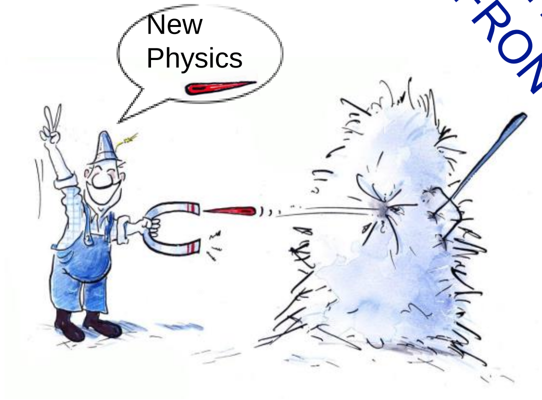
SM prediction vs.
experimental observation

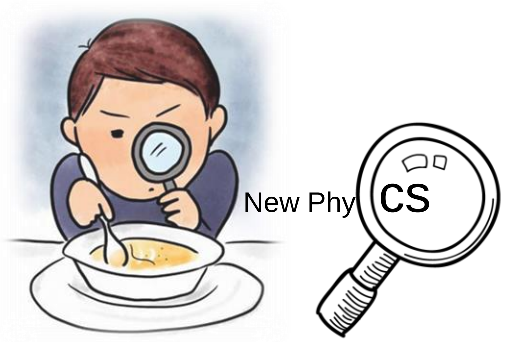


HIGH-ENERGY FRONTIER

INTENSITY FRONTIER

PRECISION FRONTIER

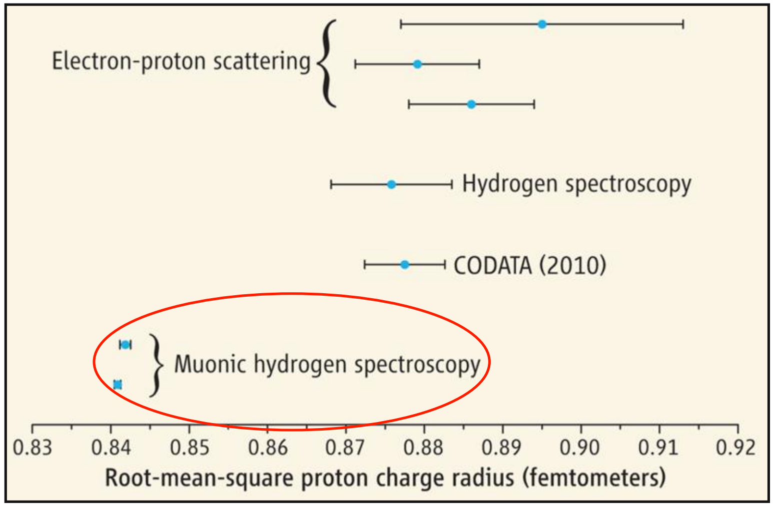




PRECISION FRONTIER

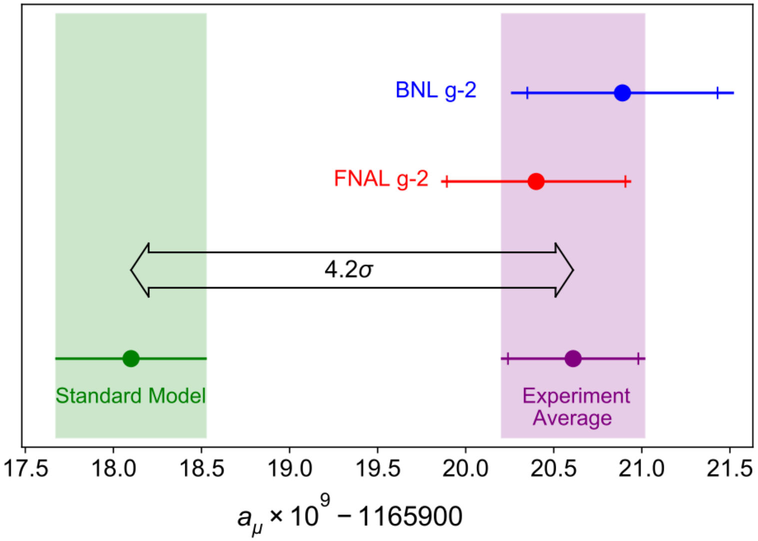
Low-energy observables measured to high precision provide stringent tests of the Standard Model (SM) of particle physics

Proton Radius



@PSI

Muon g-2



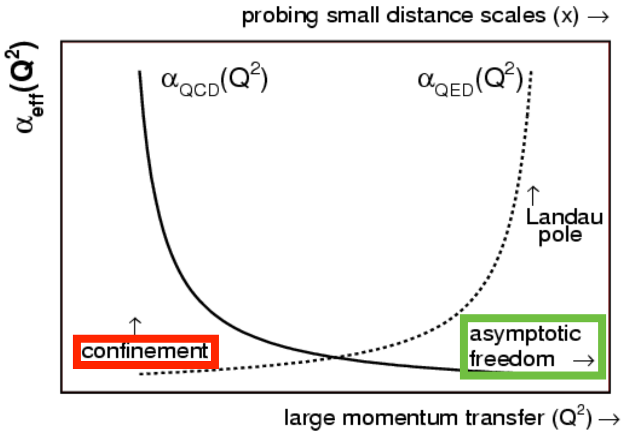
@Fermilab

HADRONIC CORRECTIONS

Hadrons (*e.g.*, the proton) are often involved in “puzzles” — disagreements between SM predictions and experiments.

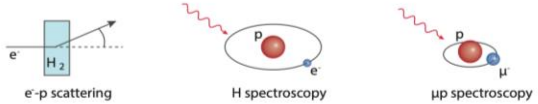
- Often limit precision of SM predictions

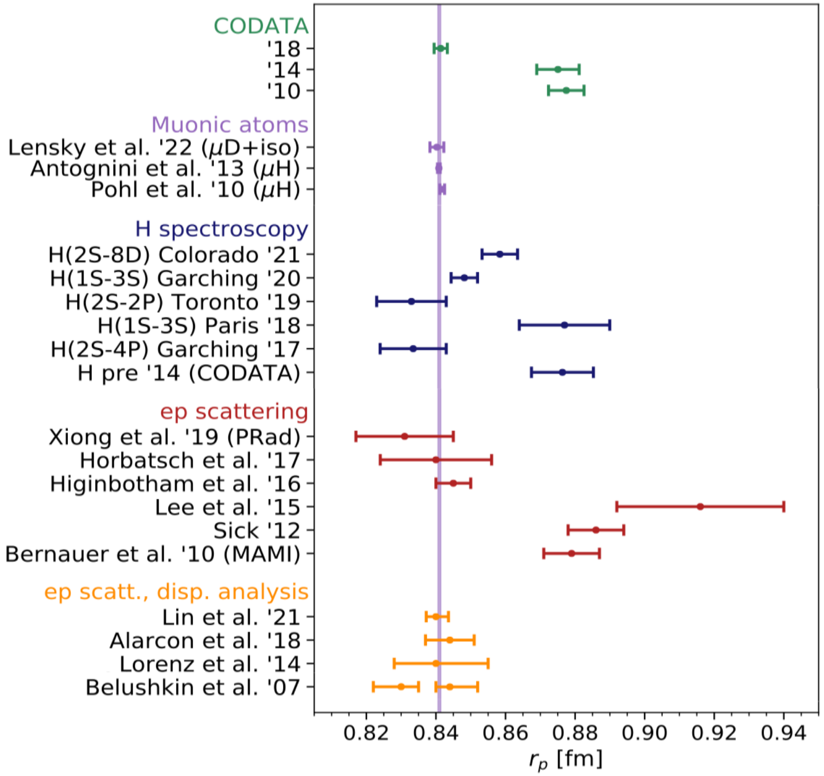
- QCD is non-perturbative at low energies



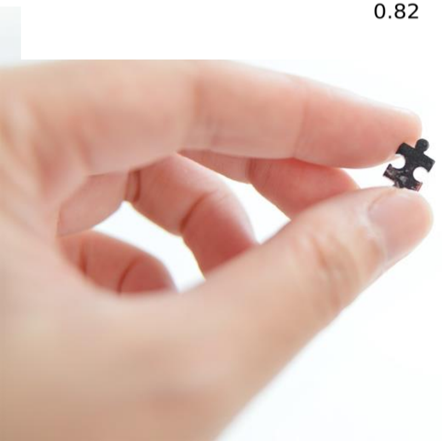
- Hadronic contributions are challenging to calculate:
  - Low-energy effective field theories, e.g., chiral perturbation theory (ChPT)
  - Lattice QCD
  - Dispersion relations — basis for data-driven evaluations

STATUS OF THE PROTON RADIUS PUZZLE





- μH Lamb shift measurements allowed for r_p extraction with unprecedented precision but gave a smaller value
- e-p scattering improved (**PRad** gives now the smaller value)
- H Lamb shift was remeasured and now agrees with the smaller value
- Still open issues: H(2S-8D), H(1S-3S) transitions
  (systematic effects?)



Considered resolved, but future efforts will still be necessary to deliver a clear verdict

FROM PUZZLE TO PRECISION

- Several experimental activities ongoing and proposed:
  - 1S hyperfine splitting in μH (ppm accuracy) and μHe
  - Improved measurement of Lamb shift in μH, μD and μHe$^+$ possible ($\times$ 5)
  - Medium- and high-Z muonic atoms

- **Theory improvements** needed!

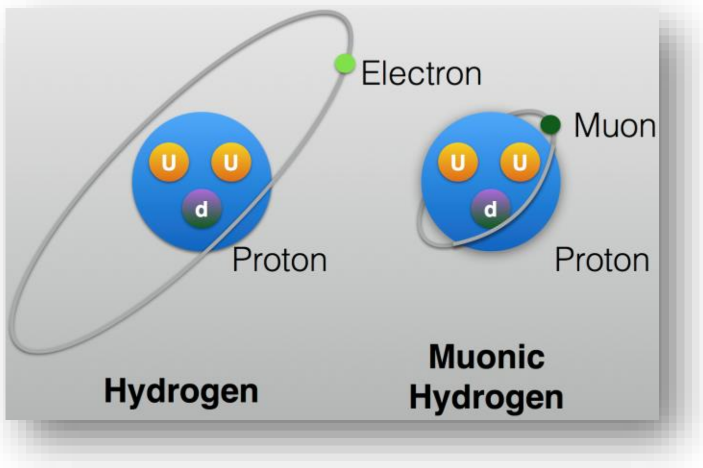
$$r_p = 0.84087(12)_{\mathrm{sys}}(23)_{\mathrm{stat}}(29)_{\mathrm{theory}} \quad \mathrm{fm}$$

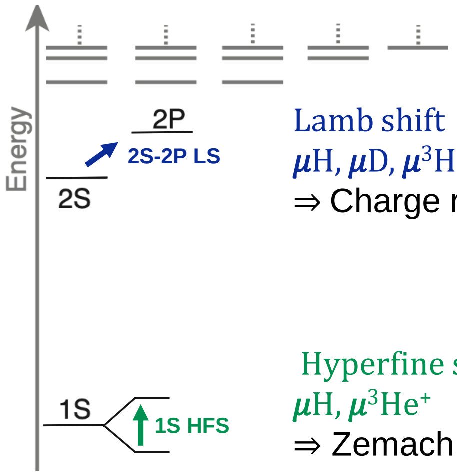
$$r_d = 2.12562(5)_{\mathrm{sys}}(12)_{\mathrm{stat}}(77)_{\mathrm{theory}} \quad \mathrm{fm}$$

$$r_\alpha = 1.67824(2)_{\mathrm{sys}}(13)_{\mathrm{stat}}(82)_{\mathrm{theory}} \ \mathrm{fm}$$

- Charge radius extractions from Lamb shift in muonic atoms:
  - μH: present accuracy comparable with experimental precision
  - μD, μ^3He$^+$ and μ^4He$^+$: present accuracy factor 5-10 worse than experimental precision

WHY MUONIC ATOMS?





Lamb shift
μH, μD, μ^3He$^+$, μ^4He$^+$
⇒ Charge radii

Hyperfine splitting
μH, μ^3He$^+$
⇒ Zemach radii, magnetic properties

- Lamb shift:

$$\Delta E_{nl}(\text{LO+NLO}) = \delta_{l0}\frac{2\pi Z\alpha}{3}\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R_{E(2)}^3\right]$$

Fermi energy:

$$E_F(nS) = \frac{8}{3}\frac{Z\alpha}{a^3}\frac{1+\kappa}{mM}\frac{1}{n^3}$$

NLO becomes appreciable in μH

- Hyperfine splitting (HFS):

Bohr radius:

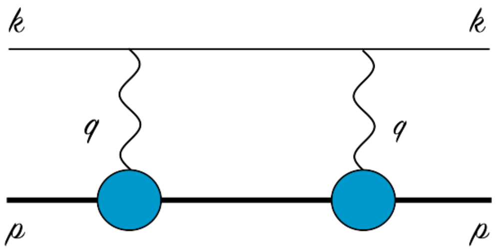
$$a = 1/(Z\alpha m_r)$$

$$\Delta E_{nS}(\text{LO} + \text{NLO}) = E_F(nS)\,[1 - 2\,Z\alpha m_r\,R_Z]$$

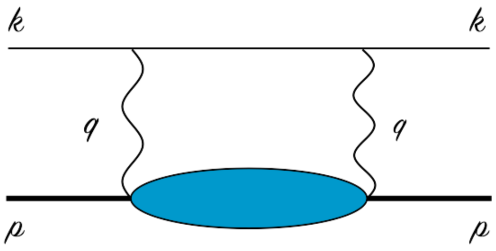
STRUCTURE EFFECTS THROUGH TPE

- Proton-structure effects at subleading orders arise through **multi-photon processes**

forward two-photon exchange (TPE)



Elastic
finite-size recoil,
3rd Zemach moment (Lamb shift),
Zemach radius (HFS)



Non-Born
polarizability contribution

- "Blob" corresponds to doubly-virtual Compton scattering:

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\boxed{T_1(\nu,Q^2)} + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right)\boxed{T_2(\nu,Q^2)}$$
$$-\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha\boxed{S_1(\nu,Q^2)} - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right)\boxed{S_2(\nu,Q^2)}$$

- Proton structure functions: $\boxed{f_1(x,Q^2),\ f_2(x,Q^2)}, \boxed{g_1(x,Q^2),\ g_2(x,Q^2)}$

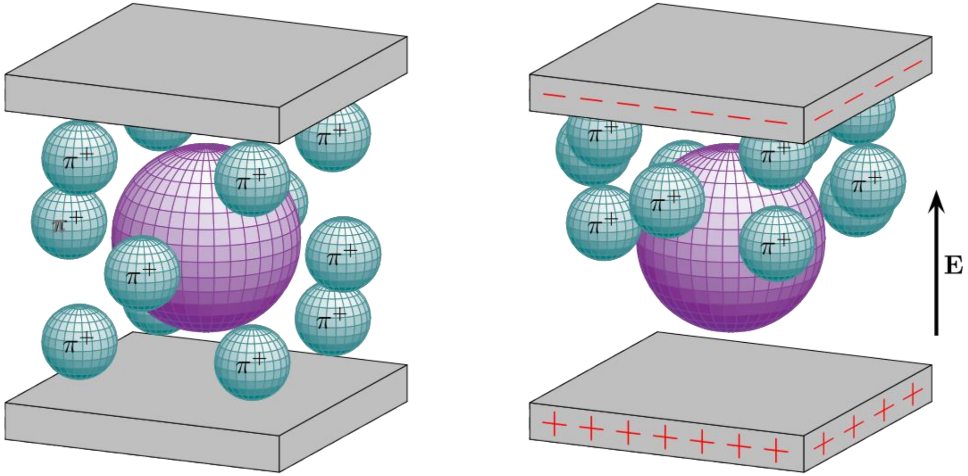
Lamb shift HFS

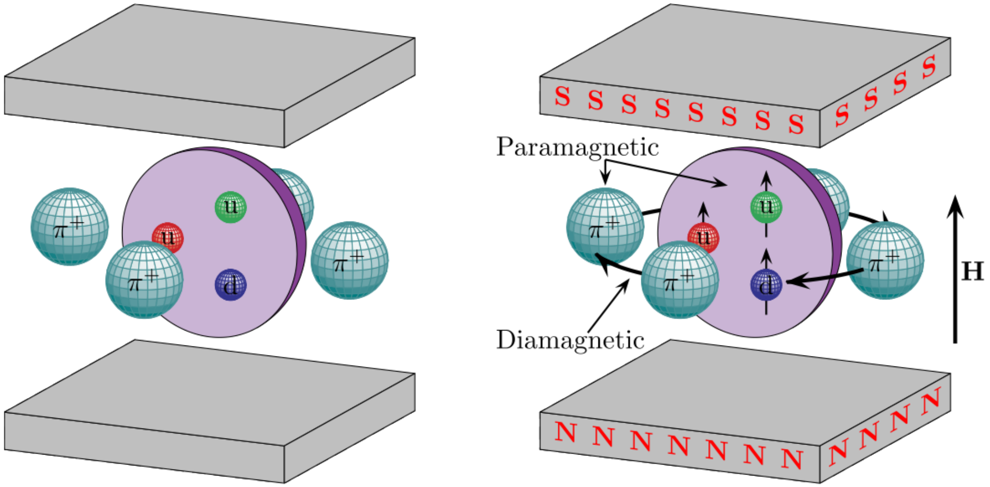
POLARIZABILITIES

Electric dipole polarizability:

$$\vec{P} = \alpha_{E1}\,\vec{E}$$

induced electric dipole polarization





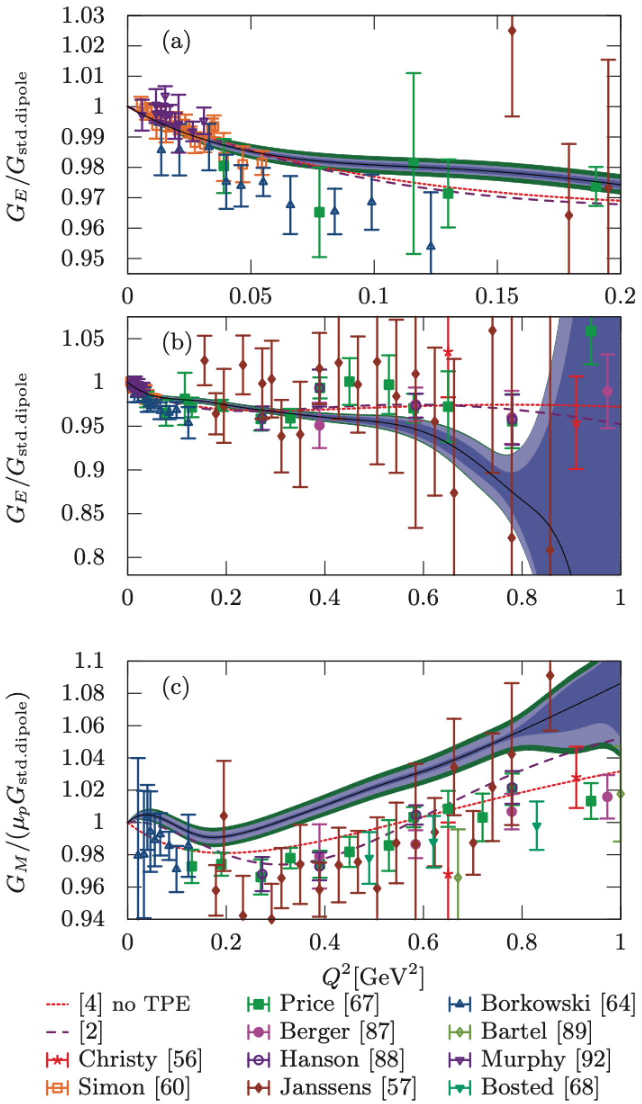
Magnetic dipole polarizability:

$$\vec{P} = \beta_{M1}\vec{H}$$

polarization induced by magnetic field

diamagnetic: $\beta_{M1} < 0$
paramagnetic: $\beta_{M1} > 0$

PROTON FORM FACTORS



J. C. Bernauer *et al.*, Phys. Rev. C **90**, 015206 (2014)

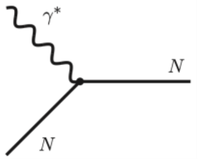
- Form factors (FF): Fourier transforms of charge and magnetization distributions:

$$\rho(r) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} G(\boldsymbol{q}^2) e^{-i\boldsymbol{q}\boldsymbol{r}}$$

- Root-mean-square (rms) charge radius:

$$R_E = \sqrt{\langle r^2\rangle_E}$$

$$\langle r^2\rangle_E \equiv \int \mathrm{d}\boldsymbol{r}\, r^2\, \rho_E(\boldsymbol{r}) = -6\frac{\mathrm{d}}{\mathrm{d}Q^2} G_E(Q^2)\Big|_{Q^2=0}$$



- Extraction of the proton charge radius from e-p scattering requires extrapolation of FF data to zero momentum transfer

HYPERFINE SPLITTING IN μH

$$\Delta E_{\mathrm{HFS}}(nS) = \left[1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}\right] E_F(nS)$$

with $\Delta_{\mathrm{structure}} = \boxed{\Delta_Z} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$

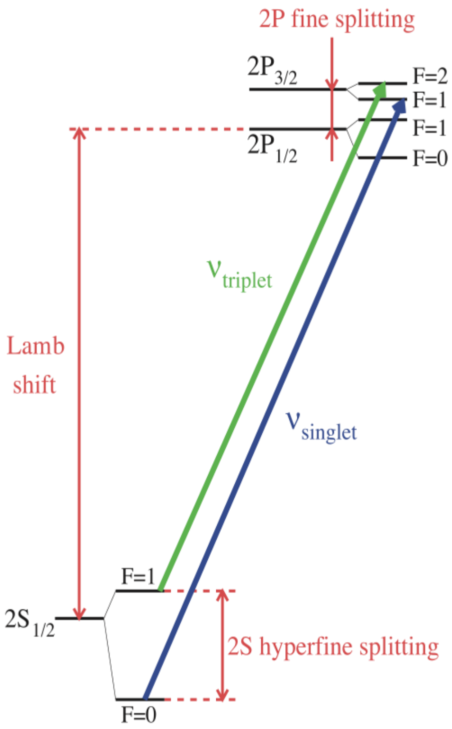
$\downarrow$

Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1\right] \equiv -2Z\alpha m_r R_Z$$

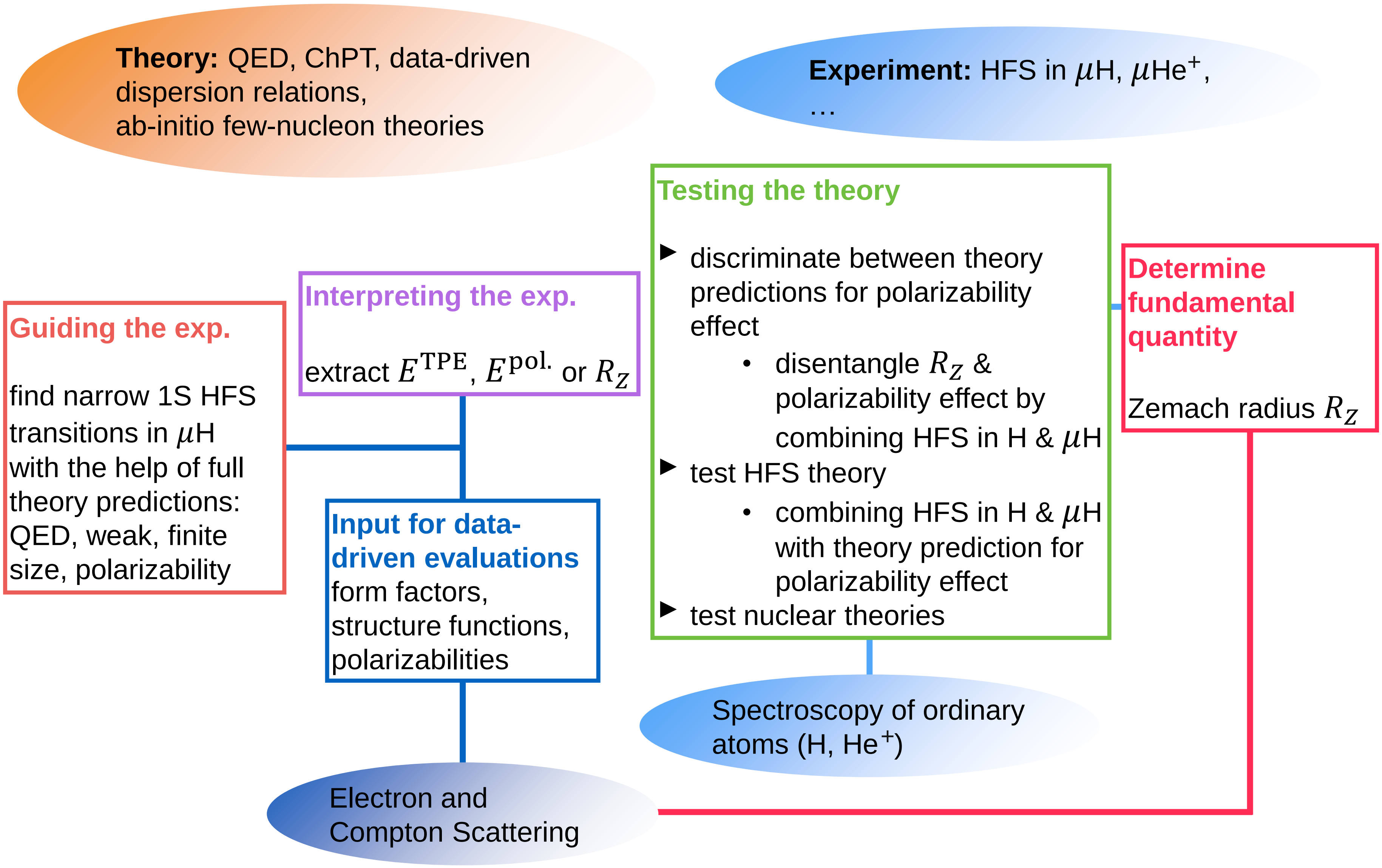
experimental value: $R_Z = 1.082(37)\ \mathrm{fm}$

A. Antognini, et al., Science **339** (2013) 417–420



- Measurements of the μH ground-state HFS planned by the CREMA, FAMU and J-PARC / Riken-RAL collaborations !
- Zemach radius can help to pin down the magnetic properties of the proton

HYPERFINE SPLITTING



Theory: QED, ChPT, data-driven dispersion relations, ab-initio few-nucleon theories

Experiment: HFS in μH, μHe$^+$, …

Guiding the exp.

find narrow 1S HFS transitions in μH with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

extract E^{TPE}, $E^{\text{pol.}}$ or R_Z

Input for data-driven evaluations

form factors, structure functions, polarizabilities

Testing the theory

- discriminate between theory predictions for polarizability effect
  - disentangle R_Z & polarizability effect by combining HFS in H & μH
- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Determine fundamental quantity

Zemach radius R_Z

Spectroscopy of ordinary atoms (H, He$^+$)

Electron and Compton Scattering

HYPERFINE SPLITTING

Theory: QED, ChPT, data-driven dispersion relations, ab-initio few-nucleon theories

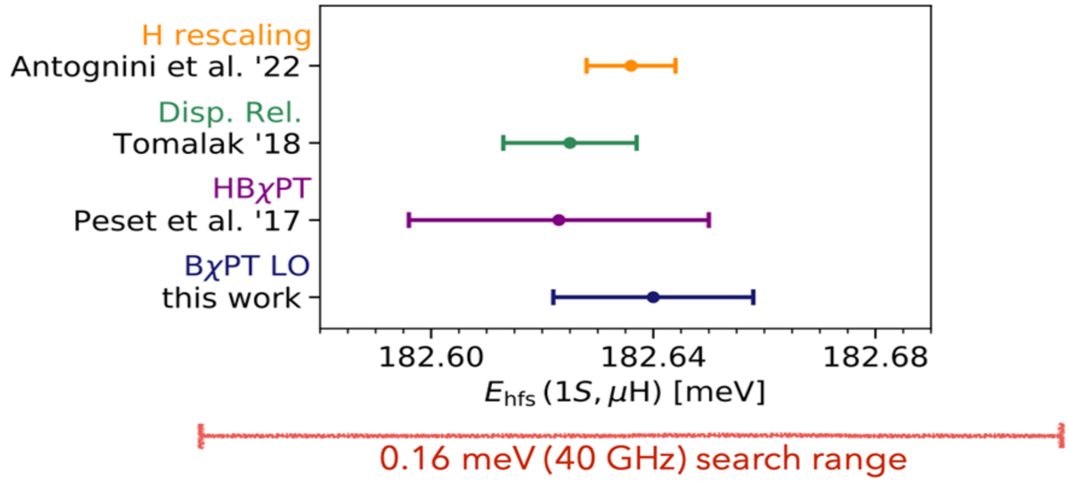
Experiment: HFS in μH, μHe$^+$, …

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

In

ex



A. Antognini, F. Hagelstein, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022)

polarizabilities

Spectroscopy of ordinary atoms (H, He$^+$)

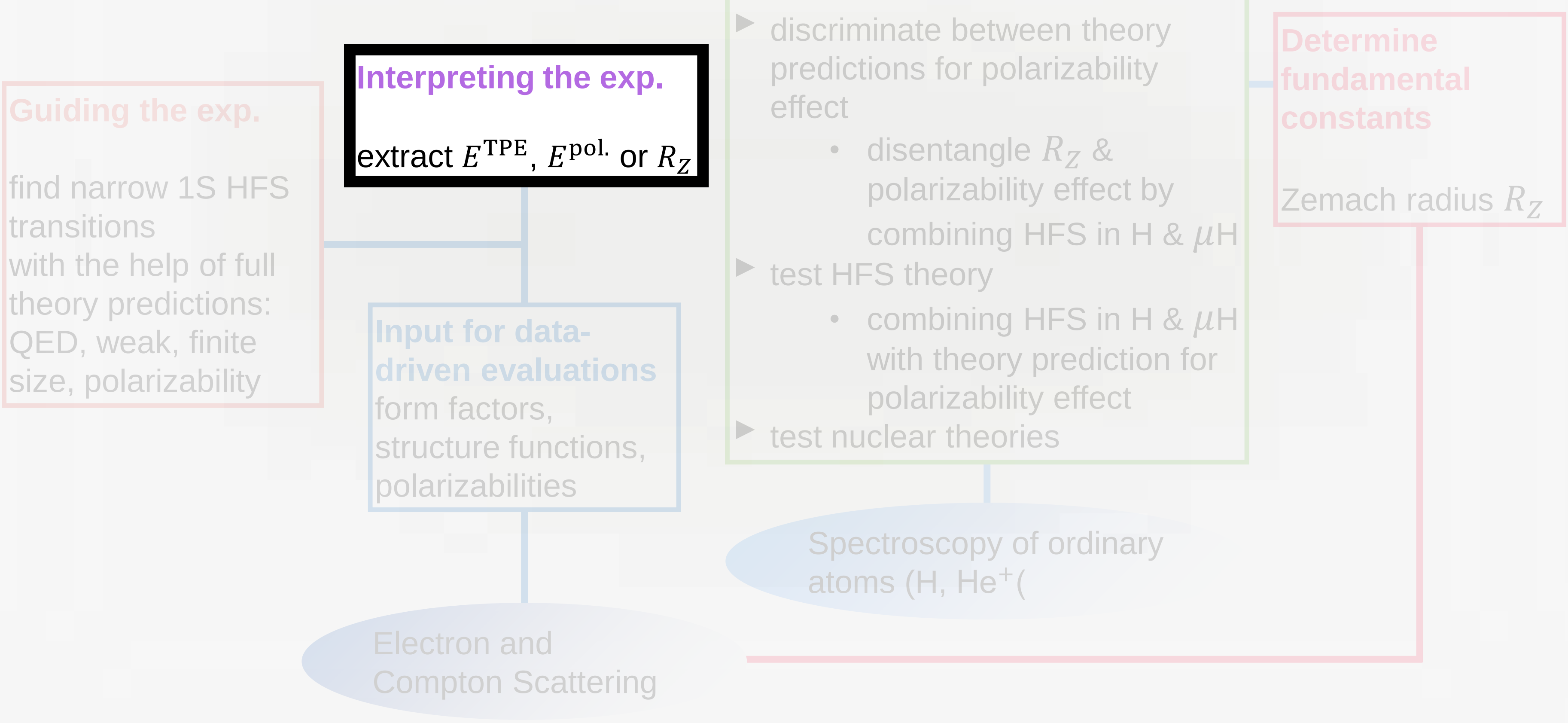
Electron and Compton Scattering

HYPERFINE SPLITTING

The hyperfine splitting of μH (theory update):

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022)

$$E_{1S\text{-hfs}} = \Big[\underbrace{182.443}_{E_\mathrm{F}}\ \underbrace{+1.350(7)}_{\text{QED+weak}}\ \underbrace{+0.004}_{\text{hVP}}\ \underbrace{-1.30653(17)\left(\frac{r_{Zp}}{\text{fm}}\right) + E_\mathrm{F}\left(1.01656(4)\,\Delta_{\text{recoil}} + 1.00402\,\Delta_{\text{pol}}\right)}_{2\gamma\text{ incl. radiative corr.}}\Big]\,\text{meV}$$



Interpreting the exp.

extract E^{TPE}, $E^{\text{pol.}}$ or R_Z

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Input for data-driven evaluations

form factors, structure functions, polarizabilities

Electron and Compton Scattering

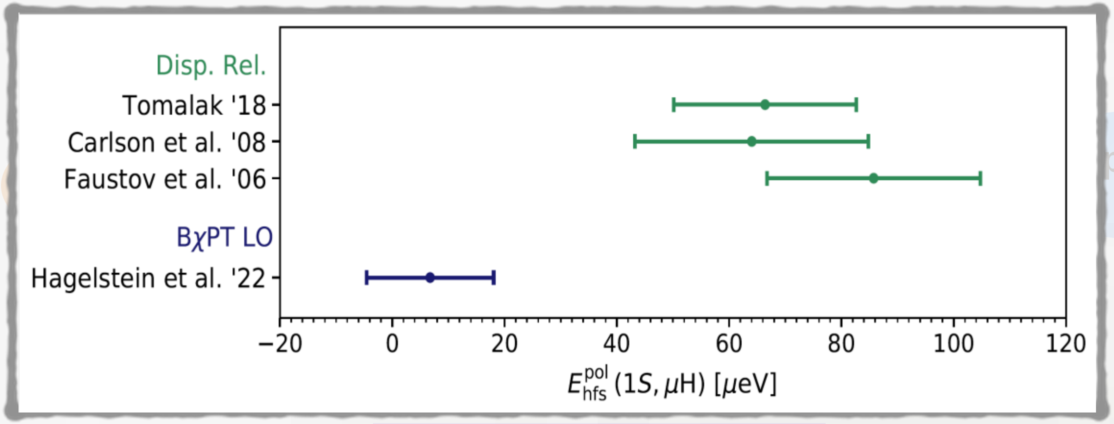
- discriminate between theory predictions for polarizability effect
  - disentangle R_Z & polarizability effect by combining HFS in H & μH
- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Determine fundamental constants

Zemach radius R_Z

Spectroscopy of ordinary atoms (H, He$^+$(

HYPERFINE SPLITTING



periment: HFS in μH, μHe$^+$,

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

extract E^{TPE}, $E^{\text{pol.}}$ or R_Z

Testing the theory

- **discriminate between theory predictions for polarizability effect**
  - disentangle R_Z & polarizability effect by combining HFS in H & μH
- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Determine fundamental constants

Zemach radius R_Z

Input for data-driven evaluations

form factors, structure functions, polarizabilities

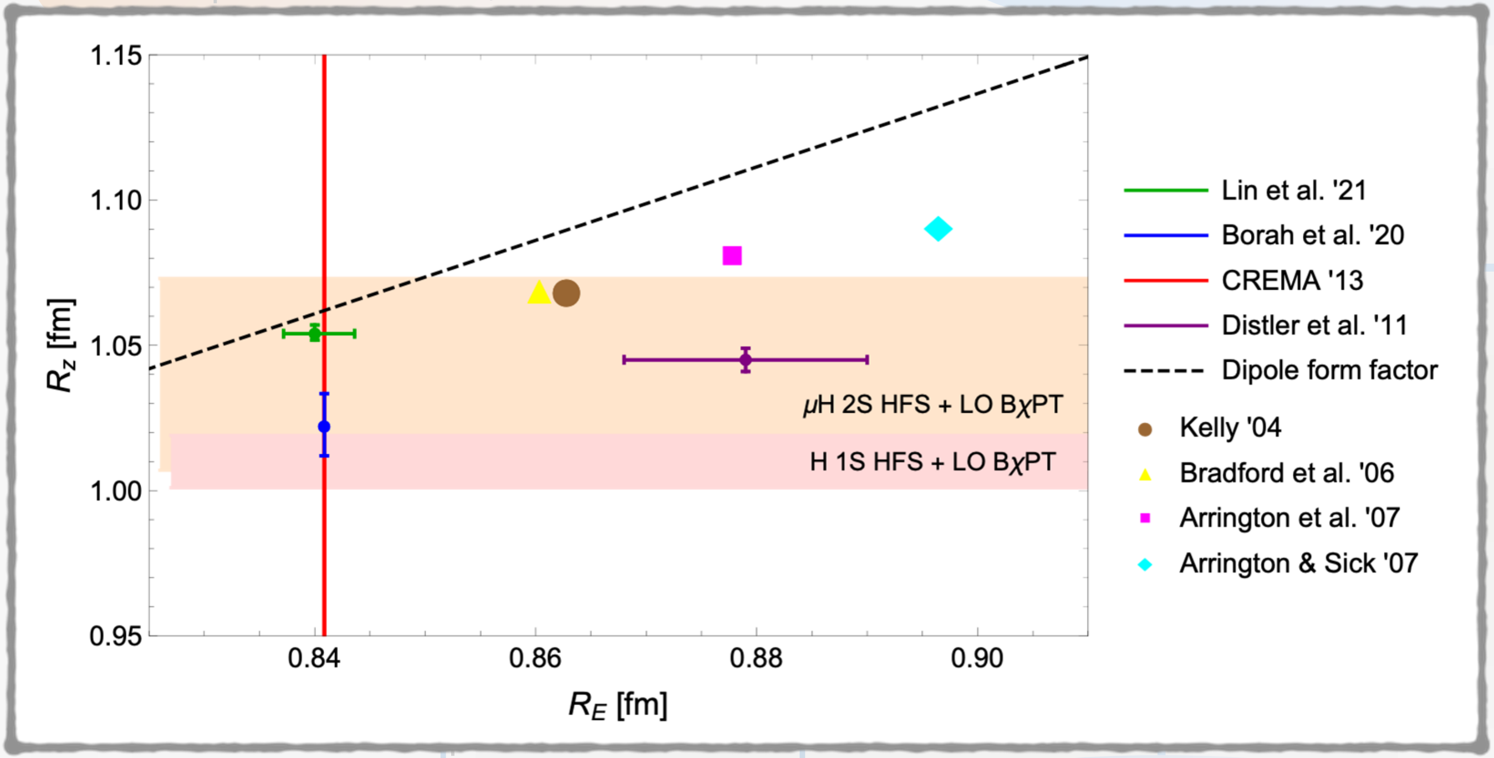
Spectroscopy of ordinary atoms (H, He$^+$)

Electron and Compton Scattering

HYPERFINE SPLITTING

Theory: QED, ChPT, data-driven dispersion relations,

Experiment: HFS in μH, μHe$^+$,



Determine fundamental constants

Zemach radius R_Z

Spectroscopy of ordinary atoms (H, He$^+$)

Electron and Compton Scattering

CONCLUSIONS!

During my PhD the main interest is low-energy nucleon structure at the precision frontier

- Experimental precision presents a challenge for theory
- Further progress in studying the proton structure is important for matching the precision, but also as input for heavier nuclei
- Precise theoretical prediction for μH 1S HFS is needed to guide the upcoming experiment
- Future plans: improve the (data-driven) dispersive prediction for Δ_{pol} contribution to HFS in (μ)H based on the global analysis of nuclear structure functions

Thank you for having me here!