Antenna subtraction in colour space: automation and application to high-multiplicity processes

Matteo Marcoli

Zurich PhD Seminars 2022

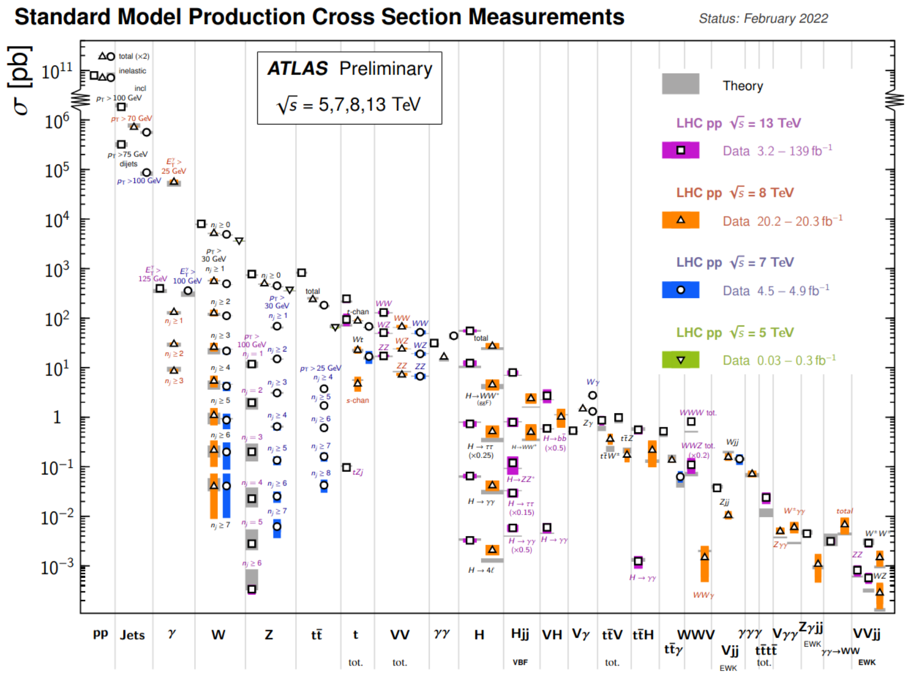
27 January 2023

Universität Zürich UZH
Swiss National Science Foundation
European Research Council
Established by the European Commission

Precision Phenomenology

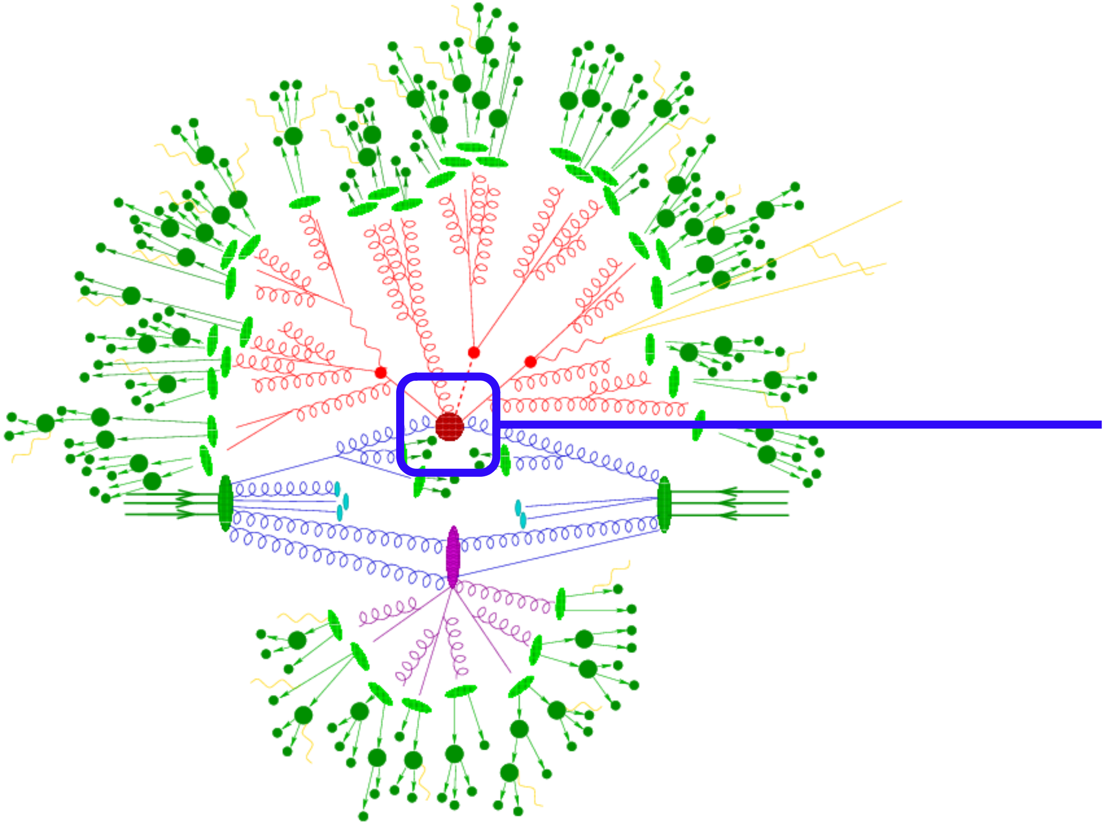
Precise theoretical predictions are crucial to **probe the Standard Model** and search for **new physics**.

- How are **precision calculations** performed?
- Can we define a **universal** approach?
- How can we address **high-multiplicity processes**?



ATL-PHYS-PUB-2022-009

Hard Scattering



Hard scattering:

- scale: ~TeV (LHC);
- QCD can be studied **perturbatively** (α_s ~ 0.1);
- Determines the **size** of the cross section and the **shape** of the distributions;

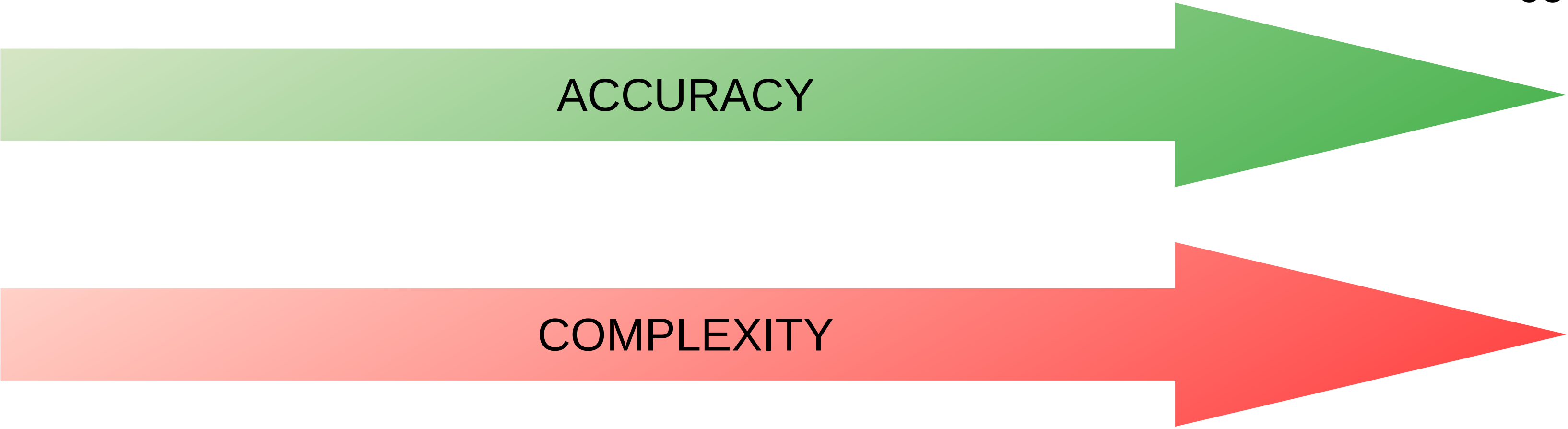
Perturbative Calculations in QCD

Theoretical predictions are computed as an **expansion** in the **small parameter α_s**:

$$\sigma = \sigma_0 + \alpha_s \, \sigma_1 + \alpha_s^2 \, \sigma_2 + \ldots$$

radiative/quantum corrections:

- Extra emissions
- Loop corrections



Perturbative Calculations in QCD

Theoretical predictions are computed as an **expansion** in the **small parameter αₛ**:

$$\sigma = \sigma_0 + \alpha_s\,\sigma_1 + \alpha_s^2\,\sigma_2 + \ldots$$

Leading Order (**LO**)

✓ Easy;

✗ Not accurate;

Next-to-LO (**NLO**)

😐 Hard;

✓ Automated;

✗ Still not accurate enough;

Next-to-NLO (**NNLO**)

☹ Harder;

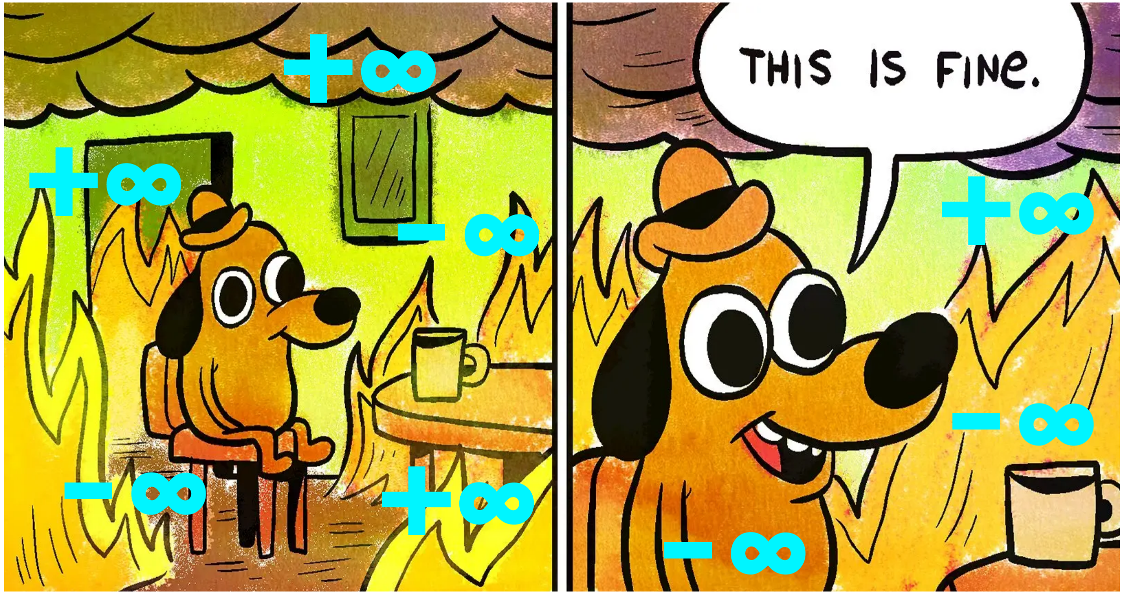
✓ Computed for many processes;

✗ Not automated;

✗ Mostly 2 → 2;

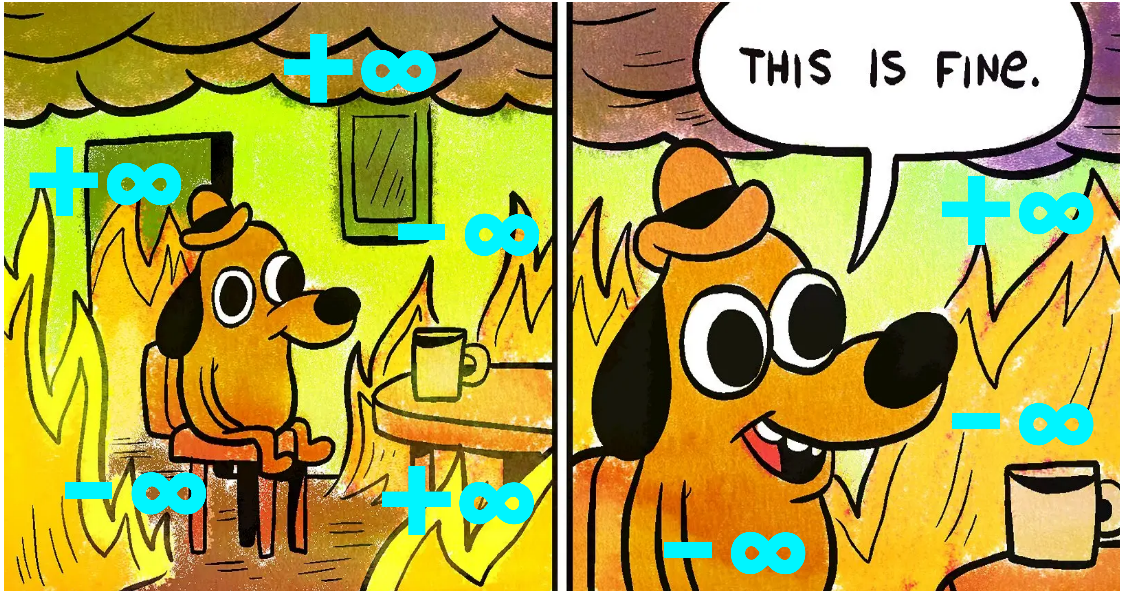
Perturbative Calculations in QCD

Problem: in perturbative calculations **infinities emerge**, in the form of **divergent integrals!**



Perturbative Calculations in QCD

Problem: in perturbative calculations **infinities emerge**, in the form of **divergent integrals!**



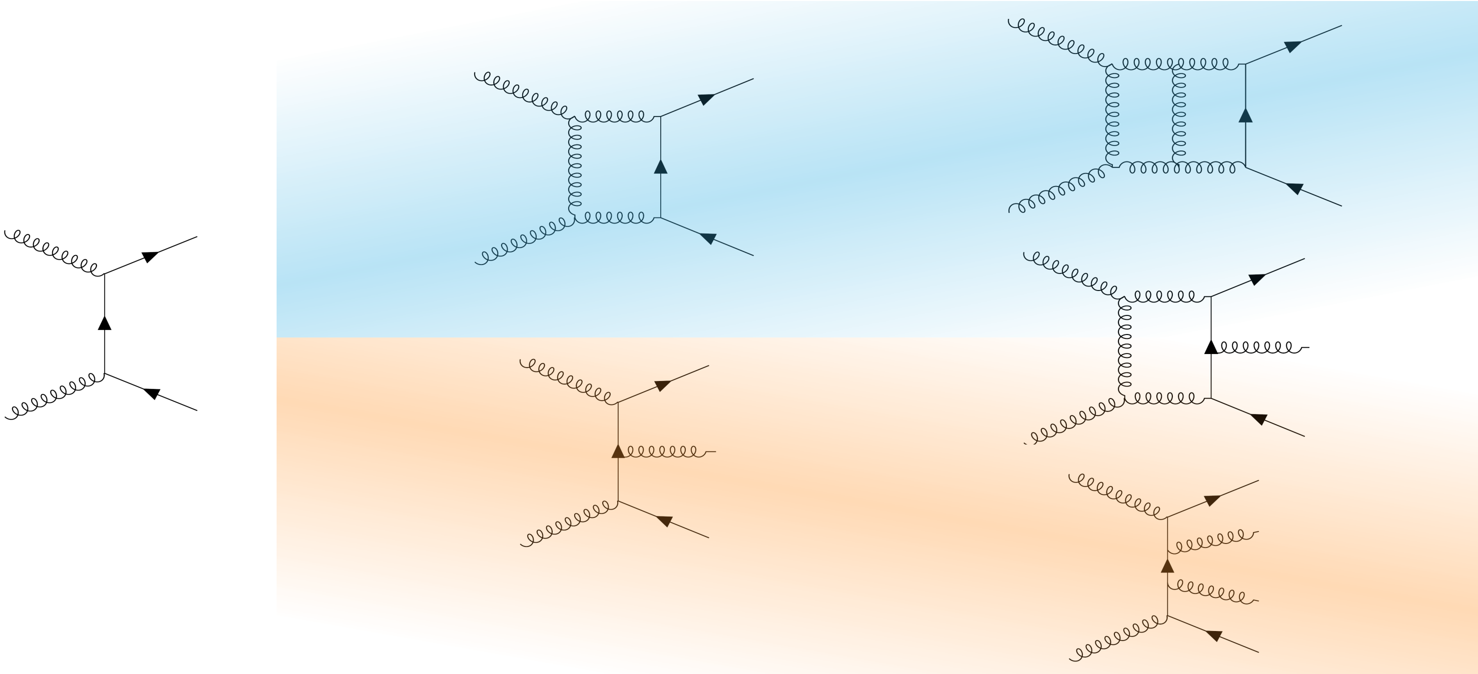
Two types of divergences:

- **Ultraviolet**: cured by **renormalization**;
- **Infrared**: **cancel in the final result** for physical observables. To achieve the cancellation is **highly non trivial!**

Perturbative Calculations in QCD

Infrared (IR) divergences arise both in **virtual** and **real** corrections.

LO **NLO** **NNLO**



VIRTUALS

REALS

Virtual corrections: singularities from loop integrals.

The sum is finite!

Real corrections: divergences after phase space integration.

Perturbative Calculations in QCD

- Infrared divergences are due to the emission of particles with small momentum (**soft limit**) or with a momentum alligned to other hard particles (**collinear limit**).
- The cross section calculation is ultimately done with **Monte Carlo numerical simulations**. Computers can't deal with **infinities**.

Strategy:

- **Regularization**: explicit **poles** appear;
- Remove infinities from virtual and real corrections **separately**;
- Numerical evaluation of **finite quantities**;

Subtraction scheme

Subtraction scheme at NLO

$$\sigma_1 = \underbrace{\int_n V}_{\text{Virtuals: \textbf{divergent}}} + \underbrace{\int_{n+1} R}_{\text{Reals: \textbf{divergent}}}$$

Sum: **finite**

Subtraction scheme at NLO

$$\sigma_1 = \underbrace{\int_n V}_{\text{Virtuals: divergent}} + \underbrace{\int_{n+1} R}_{\text{Reals: divergent}} \Bigg] \ \text{Sum: finite}$$

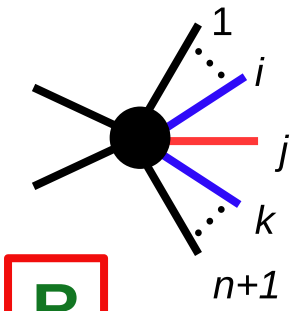
$$\sigma_1 = \underbrace{\int_n [V - T]}_{\text{Subtracted virtuals: finite}} + \underbrace{\int_{n+1} [R - S]}_{\text{Subtracted reals: finite}} , \qquad T = -\int_1 S$$

T: virtual subtraction term

S: real subtraction term

Antenna Subtraction (NLO)

Real:



R

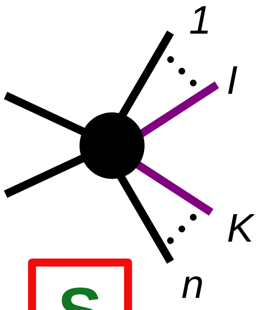
parton j soft or collinear to i or k

$$\xrightarrow{\hspace{6cm}}$$

factorization properties of QCD

3-parton tree antenna

$$\underbrace{X_3^0(i,j,k)\,\cdot}_{\textbf{Antenna function (universal)}}$$



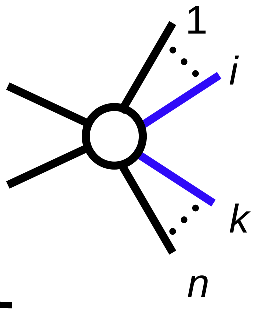
S

$\int_1$ Analytical integration over PS of the unresolved radiation

Virtual:

V

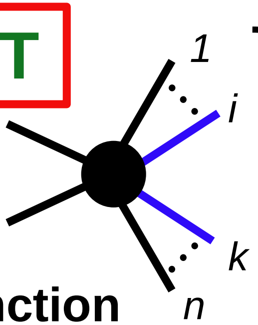
Poles $\Big[$



$\Big]$ = Poles $\Big[$

$$\underbrace{\mathcal{X}_3^0(s_{ik})\,\cdot}_{\textbf{Integrated antenna function}}$$

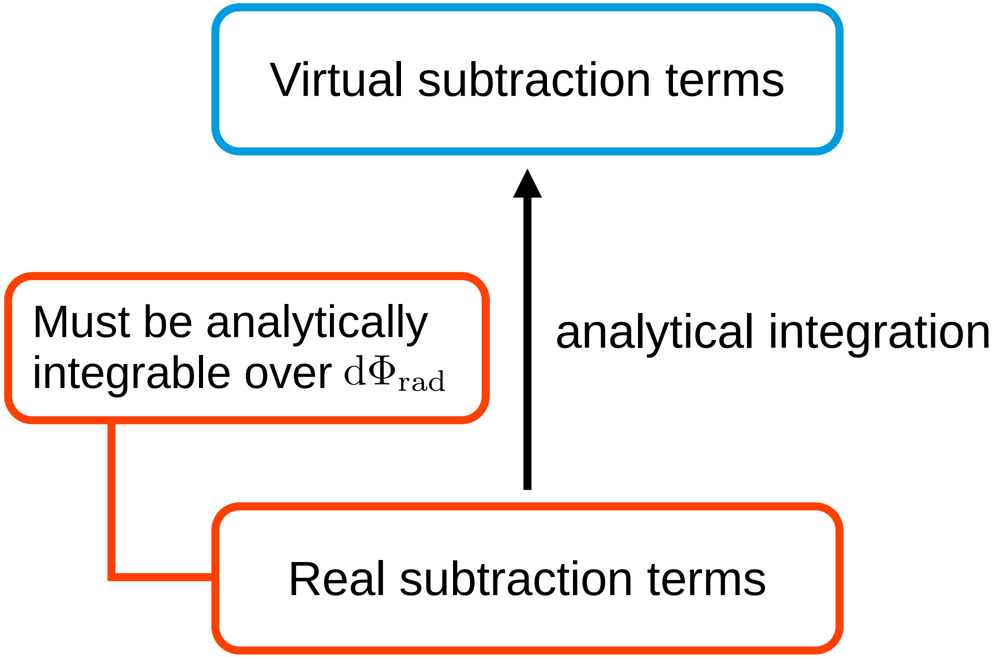
T



$\Big]$

Colourful Antenna Subtraction

Traditional approach



Virtual subtraction terms

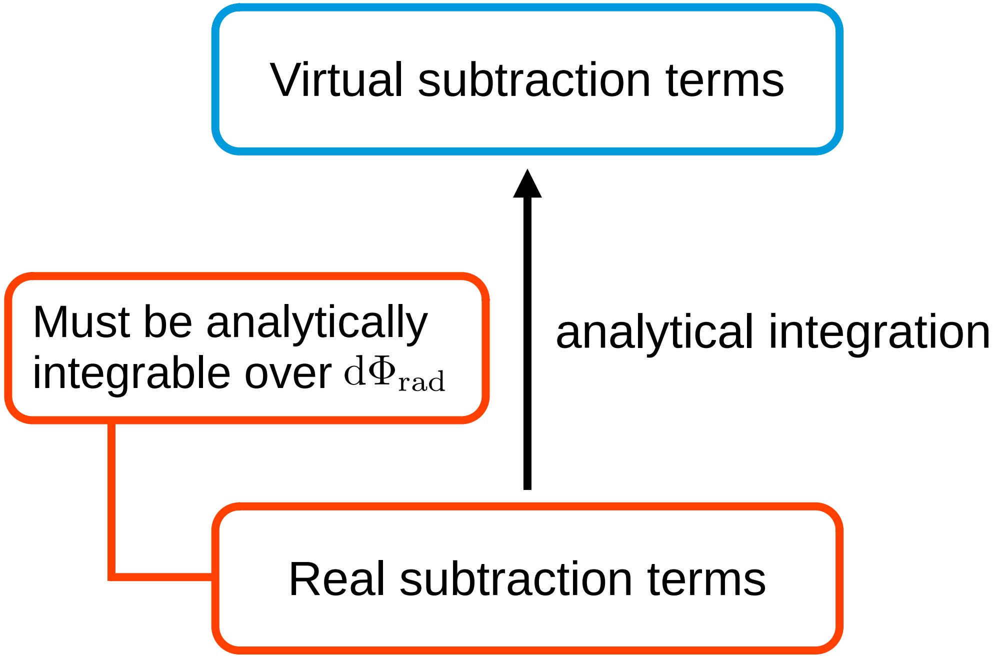
Must be analytically integrable over $\mathrm{d}\Phi_{\text{rad}}$

analytical integration

Real subtraction terms

Colourful Antenna Subtraction

Traditional approach



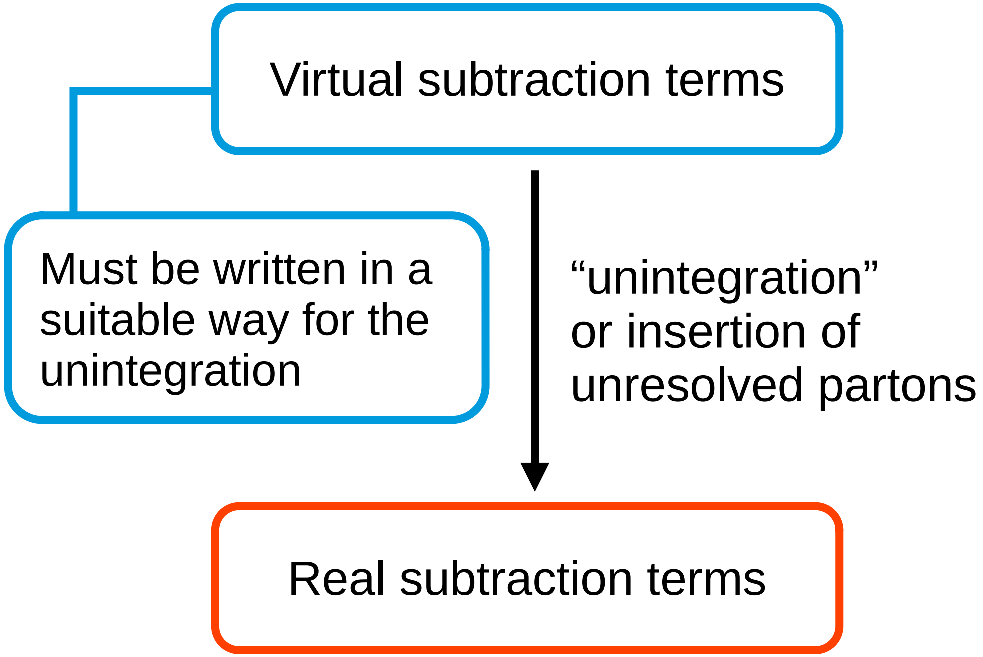
Virtual subtraction terms

analytical integration

Must be analytically integrable over $\mathrm{d}\Phi_{\mathrm{rad}}$

Real subtraction terms

New approach



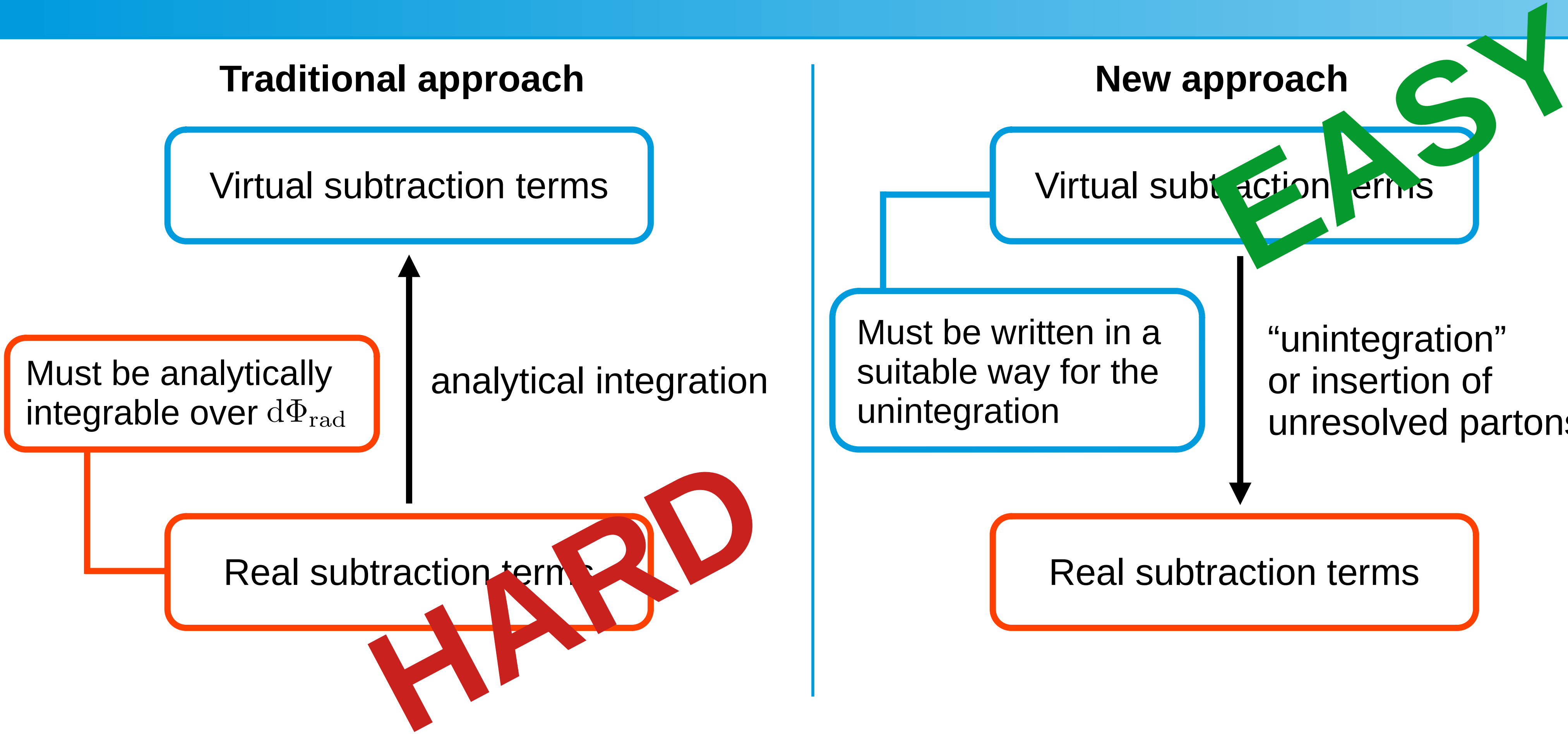
Virtual subtraction terms

Must be written in a suitable way for the unintegration

“unintegration” or insertion of unresolved partons

Real subtraction terms

Colourful Antenna Subtraction



Traditional approach

Virtual subtraction terms

analytical integration

Must be analytically integrable over $\mathrm{d}\Phi_{\mathrm{rad}}$

Real subtraction terms

HARD

New approach

Virtual subtraction terms

EASY

Must be written in a suitable way for the unintegration

“unintegration” or insertion of unresolved partons

Real subtraction terms

Colour space

The IR singularity structure of loop amplitudes in QCD is best described in **colour space.**

An n-parton ℓ-loop QCD amplitude can be written as:

$$|\mathcal{A}_n^\ell(\{p\}_n)\rangle = \sum_{i\in I^\ell} \underbrace{\boldsymbol{\mathcal{C}}_{n,i}^\ell}_{\text{\textbf{colour basis}, vectors in colour space}} \cdot \underbrace{A_{n,i}^\ell(\{p\}_n)}_{\text{colour-ordered partial amplitudes}}$$

Colour space

IR singularity structure of (renormalised) one- and two-loop amplitudes:

$$|\mathcal{A}_n^1\rangle = \boldsymbol{I}^{(1)}(\epsilon, \mu_r^2)|\mathcal{A}_n^0\rangle + \text{ finite terms}$$

$$|\mathcal{A}_n^2\rangle = \boldsymbol{I}^{(1)}(\epsilon, \mu_r^2)|\mathcal{A}_n^1\rangle + \boldsymbol{I}^{(2)}(\epsilon, \mu_r^2)|\mathcal{A}_n^0\rangle + \text{ finite terms}$$

[Catani ‘98]
[Bern, De Freitas, Dixon ‘03]
[Becher, Neubert ‘09]

$\boldsymbol{I}^{(1)}$ and $\boldsymbol{I}^{(2)}$ are **infrared insertion operators** in colour space:

$$\boldsymbol{I}^{(1)}(\epsilon, \mu_r^2) = \sum_{(i,j)} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, \mathcal{I}_{ij}^{(1)}(\epsilon, \mu_r^2)$$

$$\boldsymbol{I}^{(2)}(\epsilon, \mu_r^2) = -\frac{1}{2} \sum_{(i,j)} \sum_{(k,l)} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, (\boldsymbol{T}_k \cdot \boldsymbol{T}_l)\, \mathcal{I}_{ij}^{(1)}(\epsilon, \mu_r^2) \mathcal{I}_{kl}^{(1)}(\epsilon, \mu_r^2)$$
$$-\frac{b_0 N_c}{\epsilon} \sum_{(i,j)} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, \mathcal{I}_{ij}^{(1)}(\epsilon, \mu_r^2) + \sum_{(i,j)} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, \mathcal{I}_{ij}^{(2)}(\epsilon, \mu_r^2)$$

- Colour charge dipole structure;
- Retain full colour correlations;
- Universal;

Colourful Antenna Subtraction

We exploit this to write down the IR singularities of loop matrix elements as:

$$Poles\left\{|\mathcal{M}_n^1|^2\right\} = Poles\left\{2\mathrm{Re}\langle\mathcal{A}_n^1|\mathcal{A}_n^0\rangle\right\} = 2Poles\left\{\langle\mathcal{A}_n^0|\boldsymbol{\mathcal{J}}^{(1)}|\mathcal{A}_n^0\rangle\right\}$$

$$Poles\left\{|\mathcal{M}_n^2|^2\right\} = Poles\left\{2\mathrm{Re}\langle\mathcal{A}_n^2|\mathcal{A}_n^0\rangle + \langle\mathcal{A}_n^1|\mathcal{A}_n^1\rangle\right\} = 2Poles\left\{2\mathrm{Re}\langle\mathcal{A}_n^1|\boldsymbol{\mathcal{J}}^{(1)}|\mathcal{A}_n^0\rangle - \langle\mathcal{A}_n^0|\boldsymbol{\mathcal{J}}^{(1)}\otimes\boldsymbol{\mathcal{J}}^{(1)}|\mathcal{A}_n^0\rangle \right.$$
$$\left. -\frac{\beta_0 N_c}{\epsilon}\langle\mathcal{A}_n^0|\boldsymbol{\mathcal{J}}^{(1)}|\mathcal{A}_n^0\rangle + \langle\mathcal{A}_n^0|\boldsymbol{\mathcal{J}}^{(2)}|\mathcal{A}_n^0\rangle\right\}$$

$\boldsymbol{\mathcal{J}}^{(1)}$ and $\boldsymbol{\mathcal{J}}^{(2)}$ are analogous to $\boldsymbol{I}^{(1)}$ and $\boldsymbol{I}^{(2)}$, but are constructed using **integrated antenna functions**:

- exact extraction of virtual IR poles;
- explicit connection with real IR divergences via the **correspondence of integrated and unintegrated antenna functions**;

Colourful Antenna Subtraction

The structure of the IR divergences for real emissions is obtained from the previous expressions replacing integrated antenna functions with their unintegrated counterparts:

$$\mathcal{X}(s_{ij}) \quad \rightarrow \quad X(i,k,j)$$

Colourful Antenna Subtraction

The structure of the IR divergences for real emissions is obtained from the previous expressions replacing integrated antenna functions with their unintegrated counterparts:

Insertion of the unresolved parton *k*

$$\mathcal{X}(s_{ij}) \quad \rightarrow \quad X(i,k,j)$$

Integrated antenna function: reproduces poles of **virtual** corrections

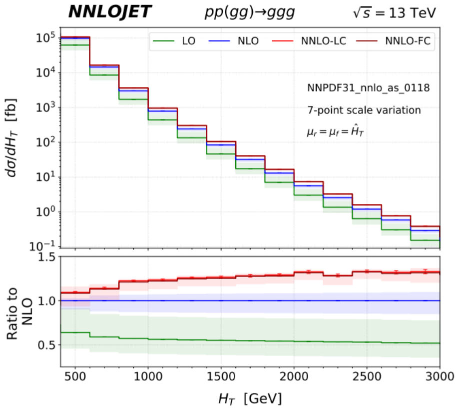
Unintegrated antenna function: reproduces divergent behaviour of **real** corrections in soft and collinear limits

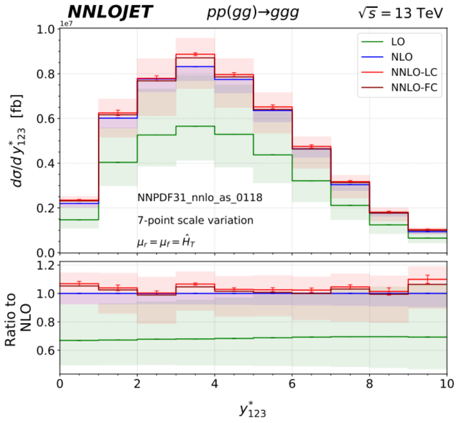
- Cancellation of IR divergences;
- Systematic generation of the subtraction infrastructure;
- Knowledge of all the antenna functions is crucial;

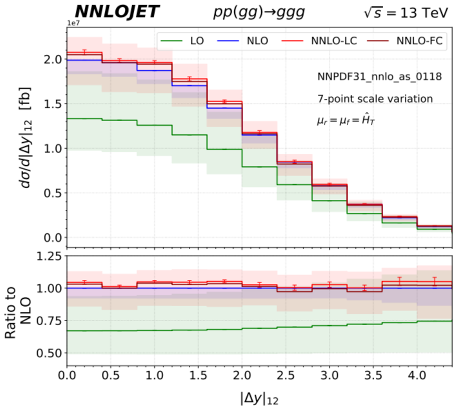
Status

- Colorful antenna subtraction: a formalism to achieve a **systematic and automatable** extraction of IR singularities at NNLO for any number of external partons;

- Successful calculation of gg→ggg at NNLO in the **gluons-only** assumption (see 2203.13531);





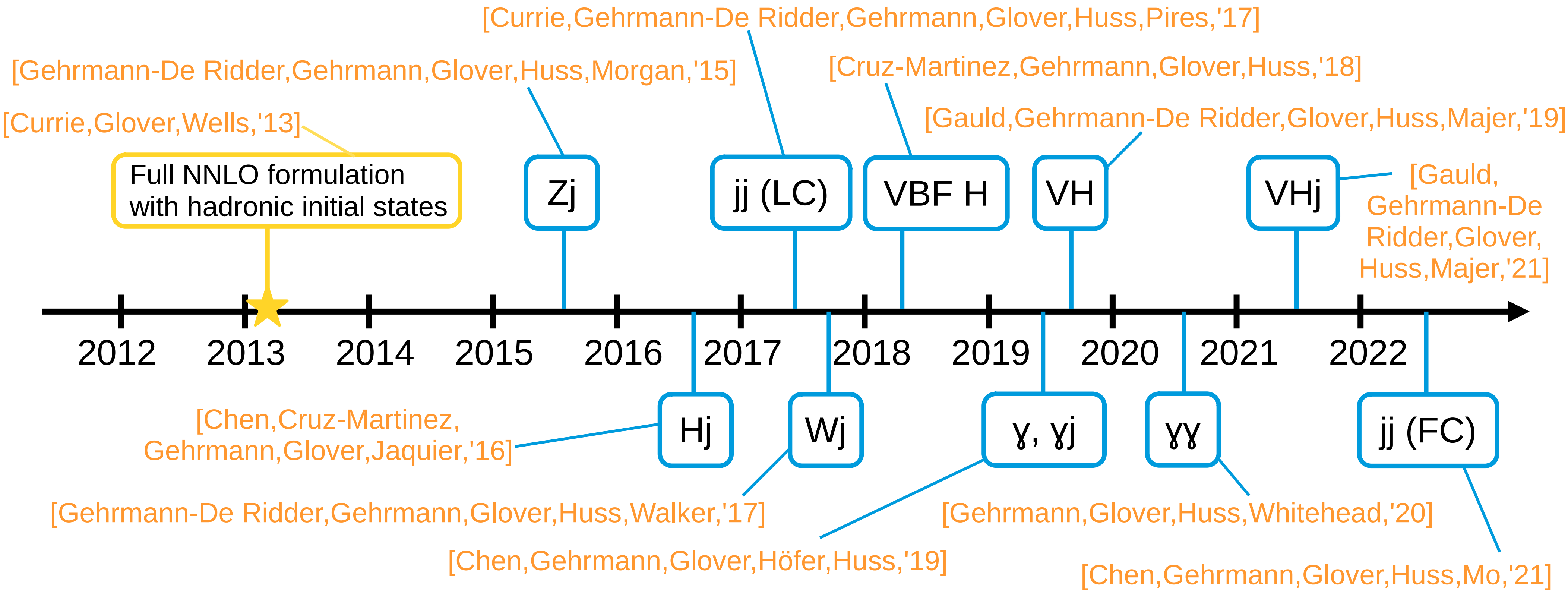


- Work in progress towards full **3-jet production at NNLO**: complete establishment of this approach;

Thanks for your attention!

Antenna Subtraction

Succesfully applied to a variety of LHC processes in the past decade with ***NNLOJET***:



[Currie,Gehrmann-De Ridder,Gehrmann,Glover,Huss,Pires,'17]

[Gehrmann-De Ridder,Gehrmann,Glover,Huss,Morgan,'15]

[Cruz-Martinez,Gehrmann,Glover,Huss,'18]

[Gauld,Gehrmann-De Ridder,Glover,Huss,Majer,'19]

[Currie,Glover,Wells,'13]

Full NNLO formulation
with hadronic initial states

Zj

jj (LC)

VBF H

VH

VHj

[Gauld,
Gehrmann-De
Ridder,Glover,
Huss,Majer,'21]

Hj

Wj

ɣ, ɣj

ɣɣ

jj (FC)

[Chen,Cruz-Martinez,
Gehrmann,Glover,Jaquier,'16]

[Gehrmann-De Ridder,Gehrmann,Glover,Huss,Walker,'17]

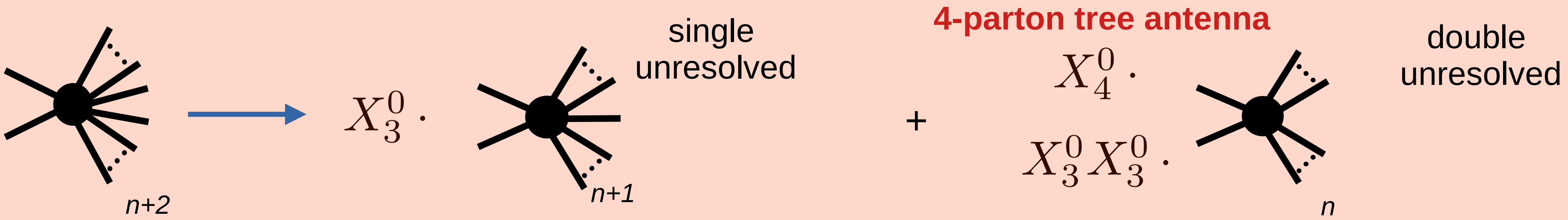
[Gehrmann,Glover,Huss,Whitehead,'20]

[Chen,Gehrmann,Glover,Höfer,Huss,'19]

[Chen,Gehrmann,Glover,Huss,Mo,'21]

Antenna Subtraction (NNLO)

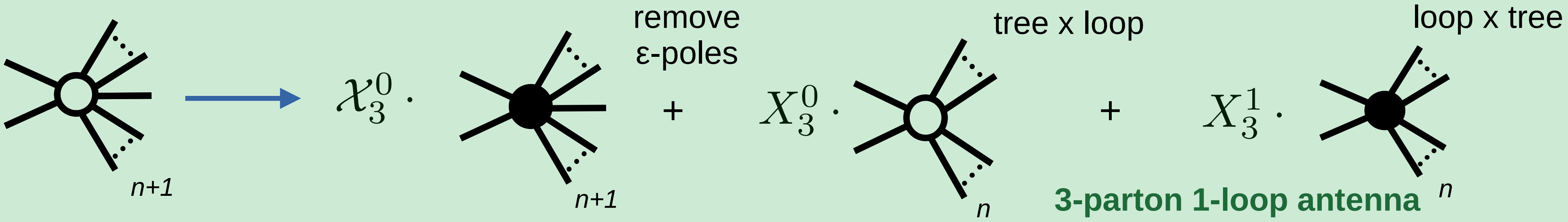
RR:



$$\xrightarrow{} X_3^0 \cdot$$ single unresolved $+$ $X_4^0 \cdot$, $X_3^0 X_3^0 \cdot$ double unresolved

4-parton tree antenna

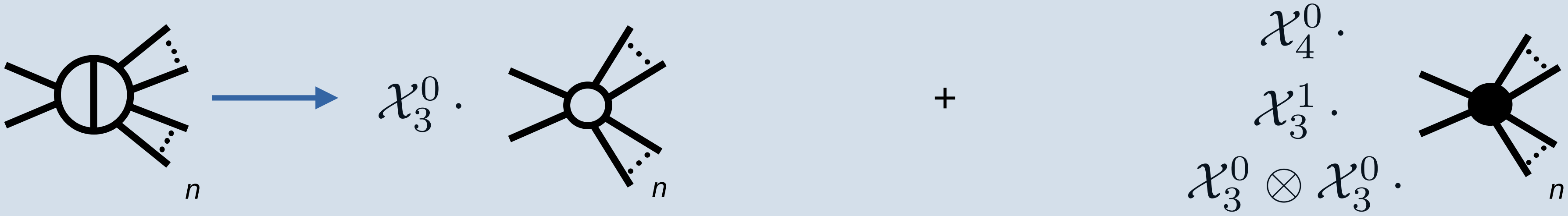
RV:



$$\xrightarrow{} \mathcal{X}_3^0 \cdot$$ remove ε-poles $+$ $X_3^0 \cdot$ tree x loop $+$ $X_3^1 \cdot$ loop x tree

3-parton 1-loop antenna

VV:



$$\xrightarrow{} \mathcal{X}_3^0 \cdot$$ $+$ $\mathcal{X}_4^0 \cdot$, $\mathcal{X}_3^1 \cdot$, $\mathcal{X}_3^0 \otimes \mathcal{X}_3^0 \cdot$

Colourful Antenna Subtraction @NNLO

- Single insertion from VV to RV;
- New terms at RV level:
    - ε-finiteness;
    - Oversubtraction;
    - Large angle soft radiation;
- Single insertion from RV to RR;
- Double insertion from VV to RR (iterated or simultaneous);