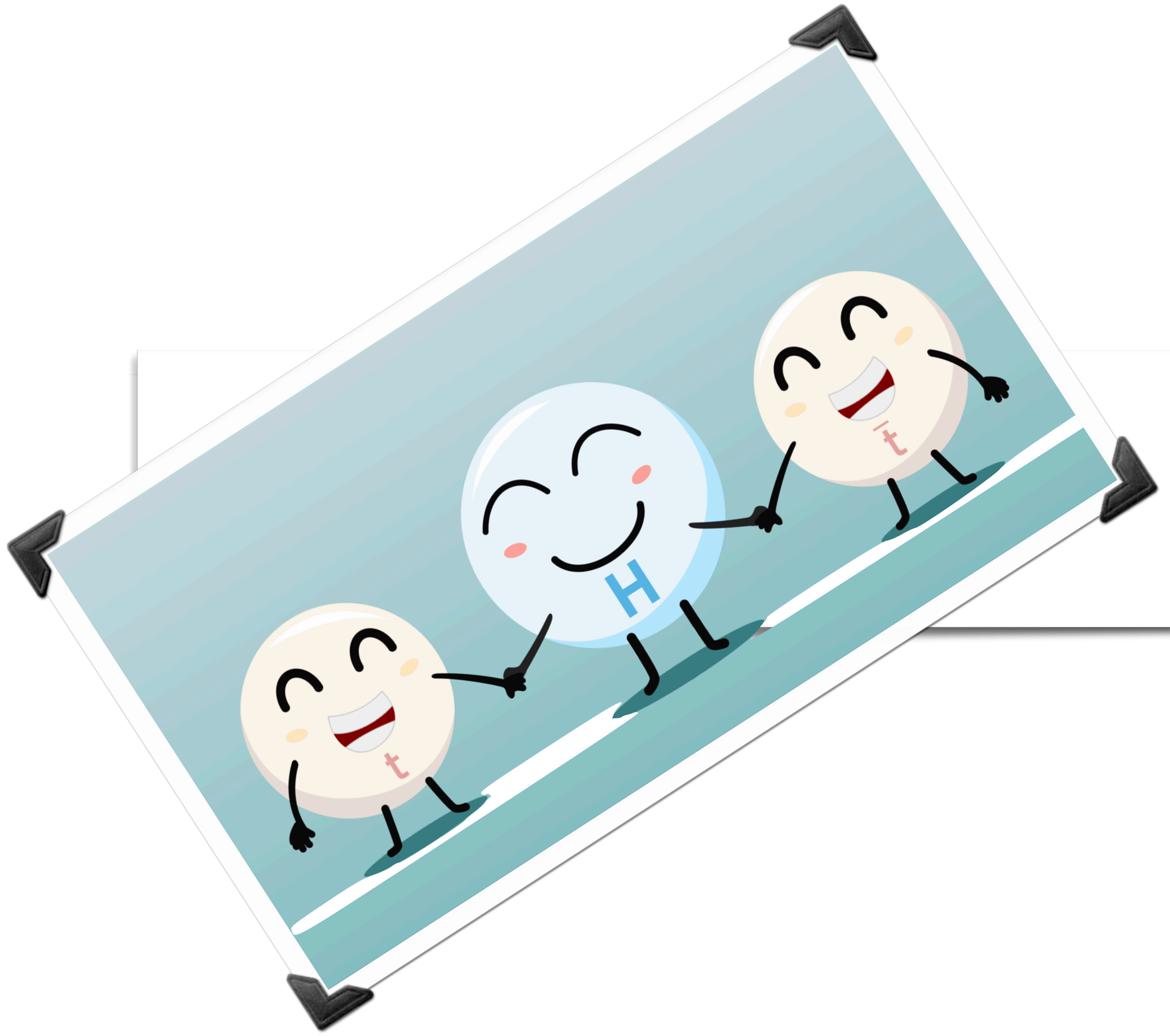


$t\bar{t}H$ production at NNLO

Chiara Savoini

University of Zurich

(based on the paper **2210.07846**, in collaboration with *S.Catani, S.Devoto, M.Grazzini, S.Kallweit, J.Mazzitelli*)

Zürich PhD Seminars 2022 – January 26^{th}- 27^{th} 2023

Universität Zürich UZH

Outline

- *Introduction*
- *Bottleneck of two-loop amplitudes:* soft Higgs boson approximation
- *The computation:* q_T - slicing formalism
- *Numerical results*
- *Conclusions*

Introduction: the Higgs boson

Motivations :

- the study of the Higgs boson is **one of the priorities in the LHC experimental program**, after its discovery in 2012
- it is responsible for giving mass to the SM particles (both bosons and fermions)
- the Higgs boson couplings to SM particles are proportional to their masses:

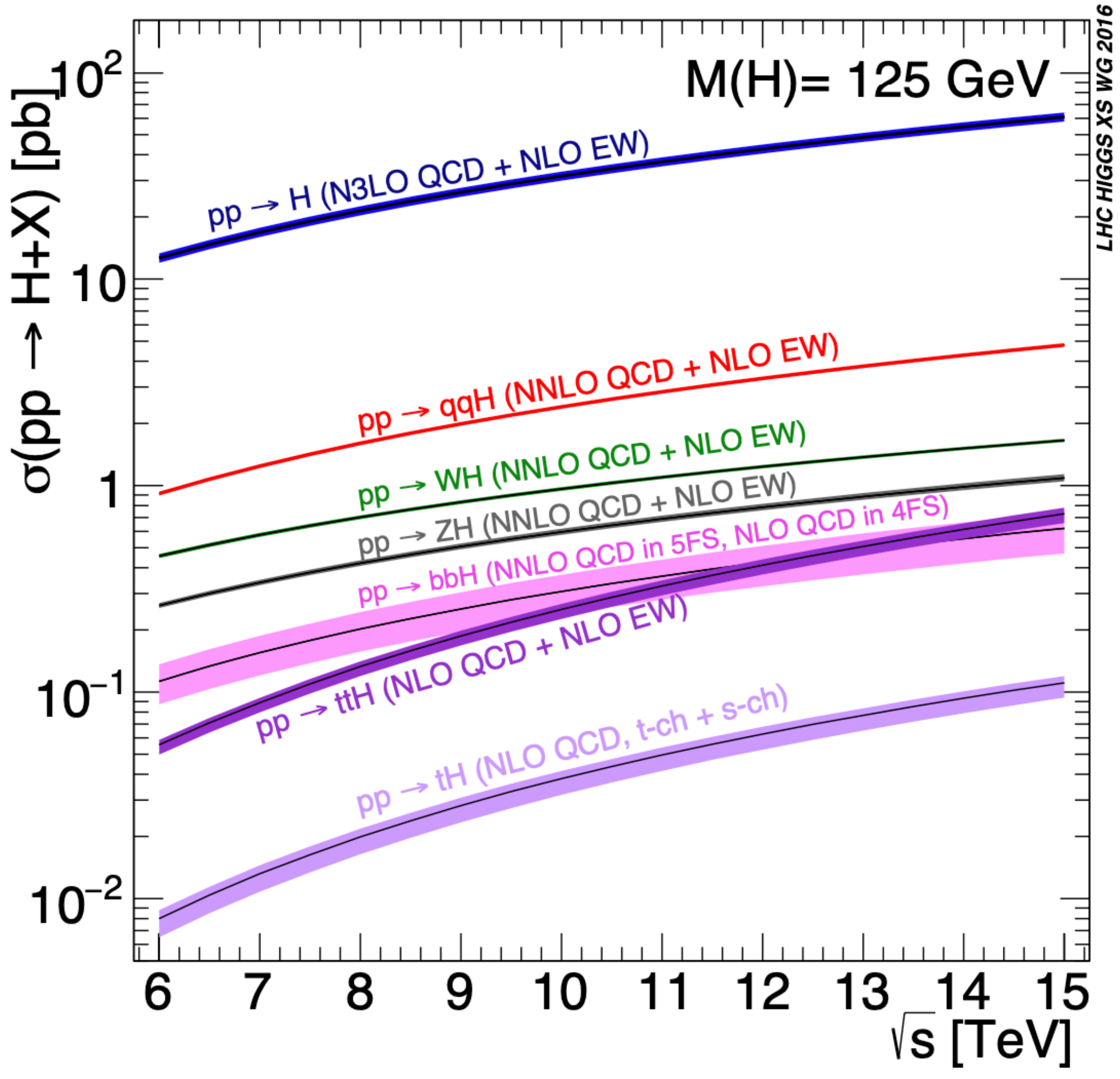
  the larger is the mass, the stronger is the coupling!

- there are **4 main production modes** in proton-proton collisions:

  gluon fusion (87%), *vector boson fusion* (7%), *Higgs strahlung* (4%)

  Higgs production in association with one or two top quarks (~1%)

- the dominant mechanism is gluon fusion via a top-quark quantum loop



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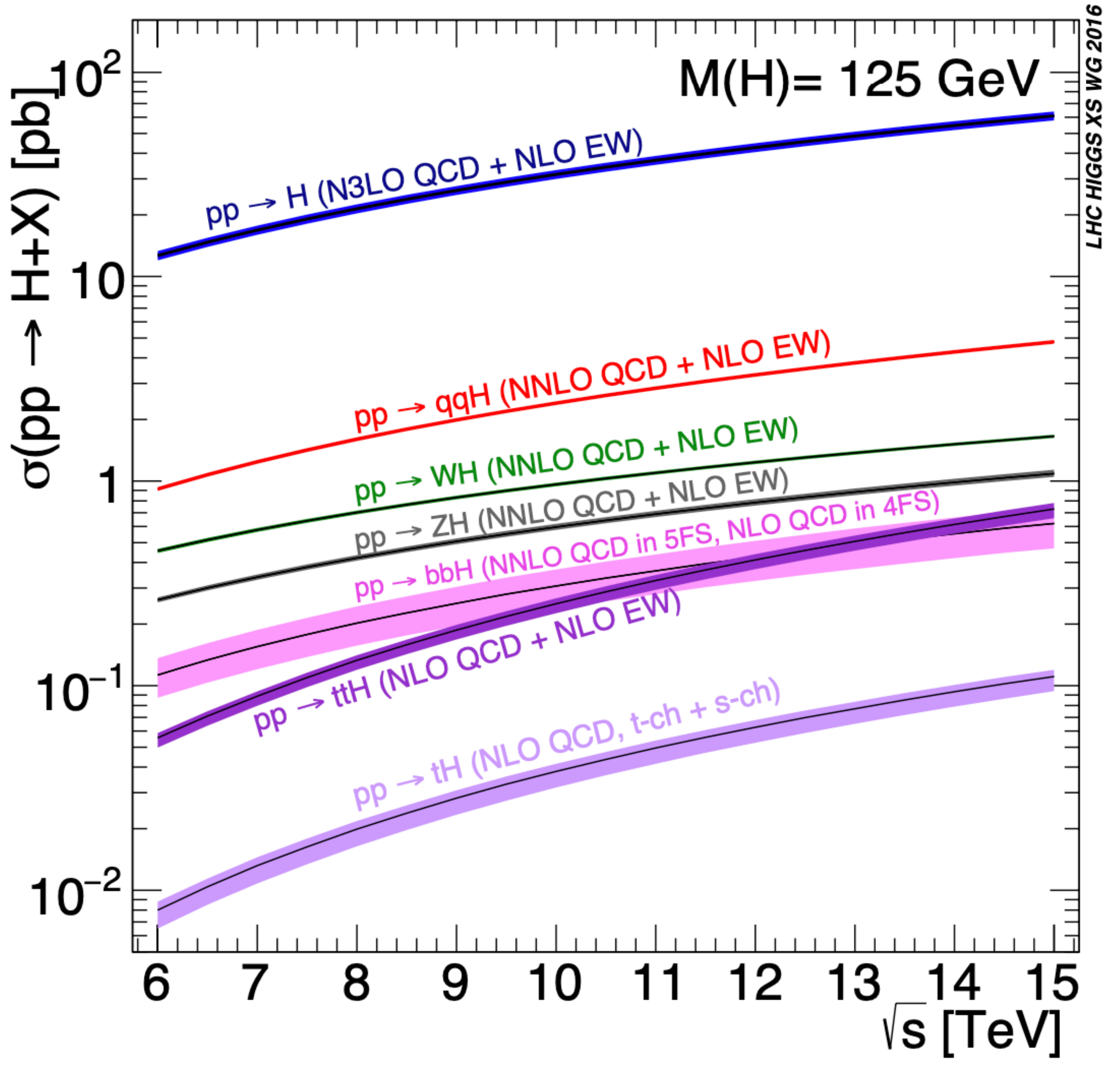
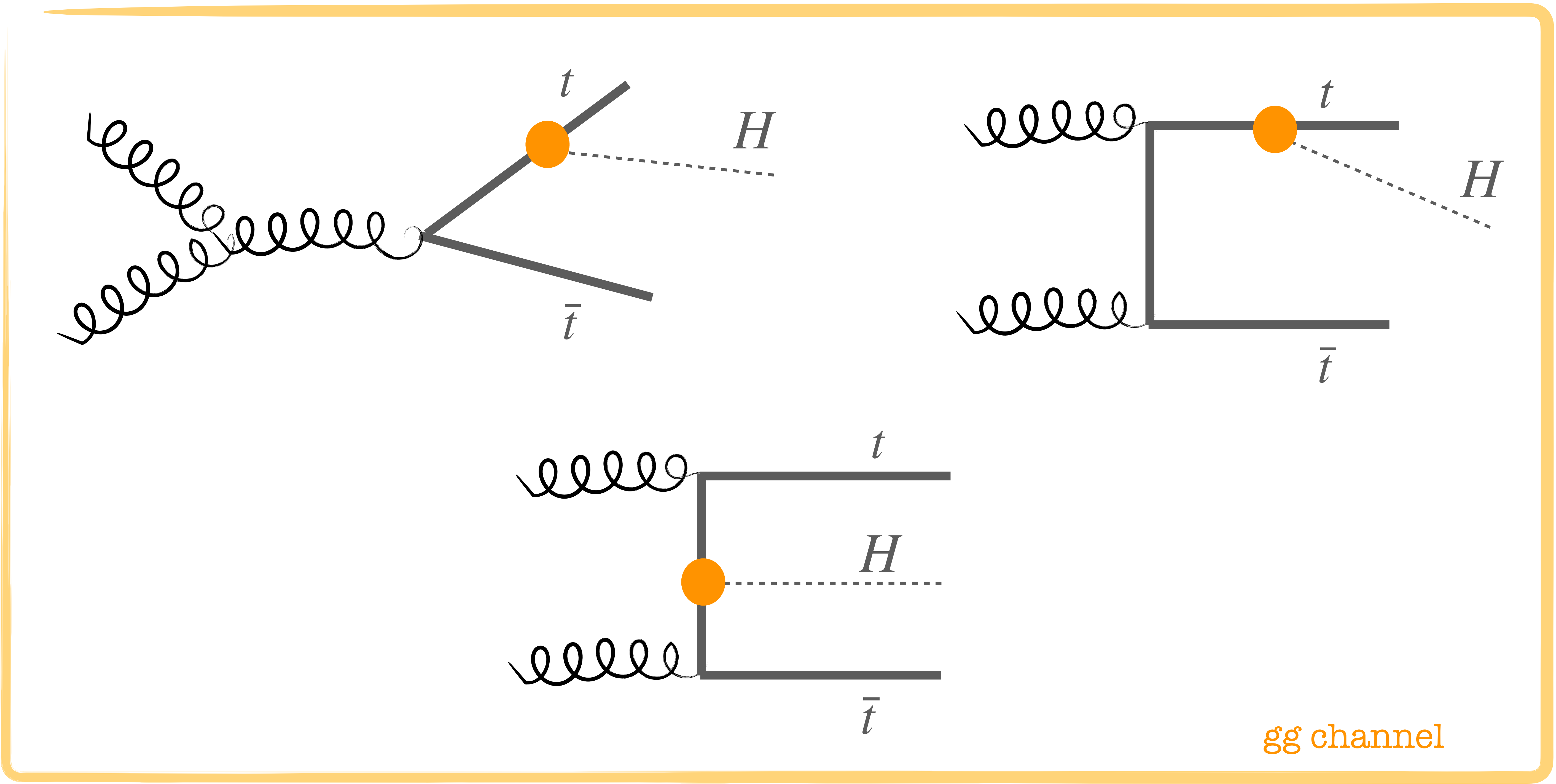
we will focus on $t\bar{t}H$ production mode, even if it has a smaller cross section compared to other production mechanisms

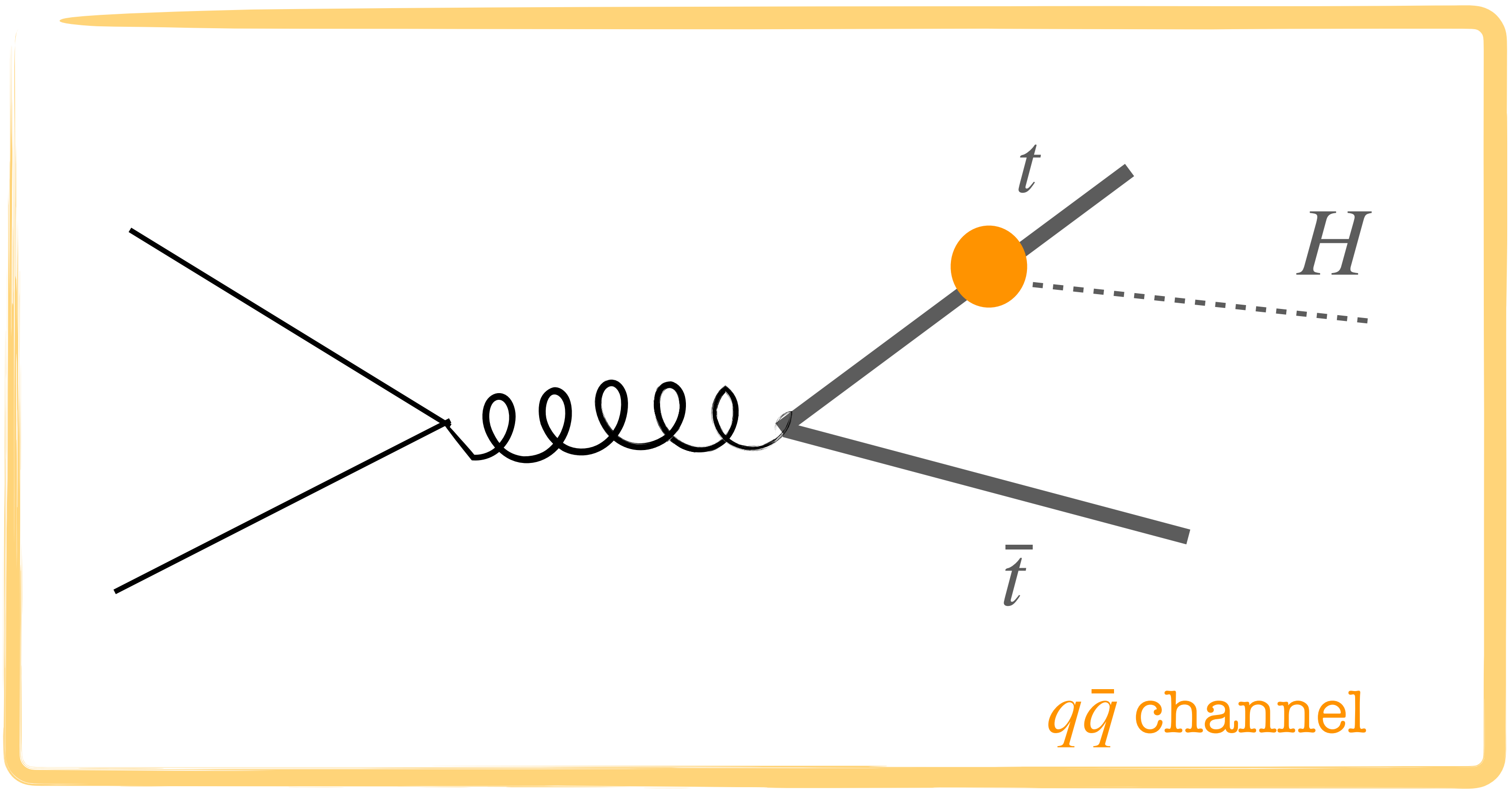


Introduction: $t\bar{t}H$ production

Motivations :

- the Higgs boson couplings to SM particles are proportional to their masses: **special role played by the top quark**!
- the top quarks are not evanescent quantum fluctuations as in the gluon fusion, they are produced as short-lived real particles and detected together with the Higgs
- the production mode $pp \to t\bar{t}H$ is relevant for a direct measurement of the **top-quark Yukawa coupling**





Introduction: $t\bar{t}H$ production

Motivations :

[CERN Yellow Report (2019)]

- the current **experimental accuracy** is $\mathcal{O}(20\%)$ but it is expected to go down to $\mathcal{O}(2\%)$ at the end of HL-LHC
- the extraction of the $t\bar{t}H$ signal is, at the moment, limited by the theoretical uncertainties in the modelling of the backgrounds, mainly $t\bar{t}b\bar{b}$ and $t\bar{t}W + jets$
- from the theoretical point of view:
  - ☑ **NLO QCD** corrections (*on-shell top quarks*) [Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas] [Reina, Dawson, Wackeroth, Jackson, Orr]
  - ☑ **NLO EW** corrections (*on-shell top quarks*) [Frixione, Hirschi, Pagani, Shao, Zaro]
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  - ☑ **NLO QCD + EW** corrections (*off-shell top quarks*) [Denner, Lang, Pellen, Uccirati (2017)]
  - ☑ current predictions based on: **NLO QCD + EW** corrections (*on-shell top quarks*), including **NNLL** soft-gluon resummation [Broggio et al.] [Kulesza et al.]
- the current predictions are affected by an uncertainty of $\mathcal{O}(10\%)$

[LHC cross section WG (2016)]

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  - ☑ first step completed by the evaluation of **NNLO QCD** contributions for the **off-diagonal** partonic channels [Catani, Fabre, Grazzini, Kallweit (2021)]

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  - ☑ **complete NNLO QCD** with approximated two-loop amplitudes in this talk!

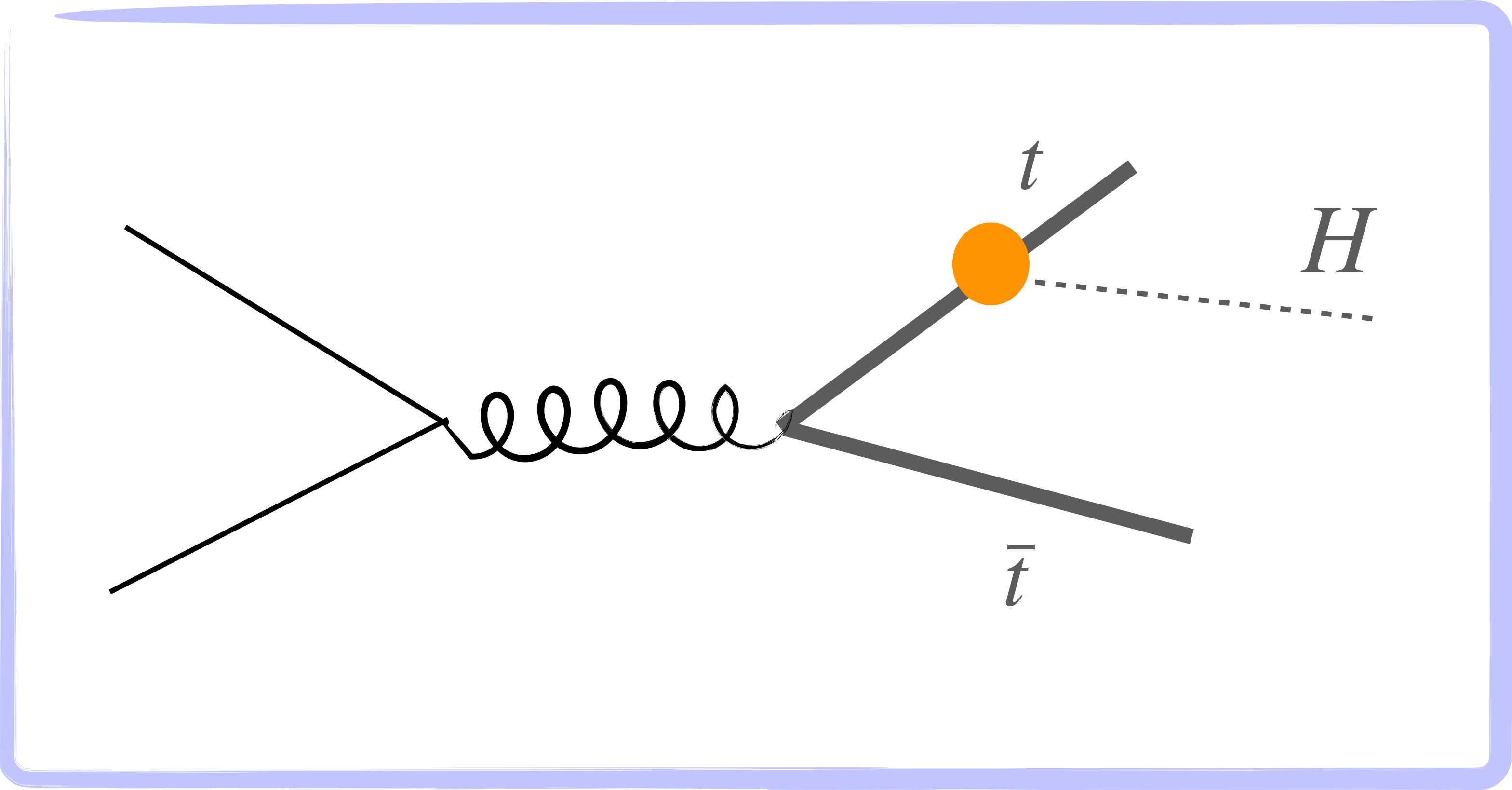
to match the expected experimental accuracy, the inclusion of **NNLO corrections is mandatory**!

Introduction: importance of radiative corrections

- we perturbatively expand the $t\bar{t}H$ partonic cross section, in the strong coupling,

$$d\hat{\sigma} = d\hat{\sigma}^{(0)} + \frac{\alpha_s(\mu_R)}{2\pi} d\hat{\sigma}^{(1)} + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 d\hat{\sigma}^{(2)} + \mathcal{O}(\alpha_s^3)$$

σ_{LO}



Born contribution

$\sim \mathcal{O}(50\% - 30\%)$ precision

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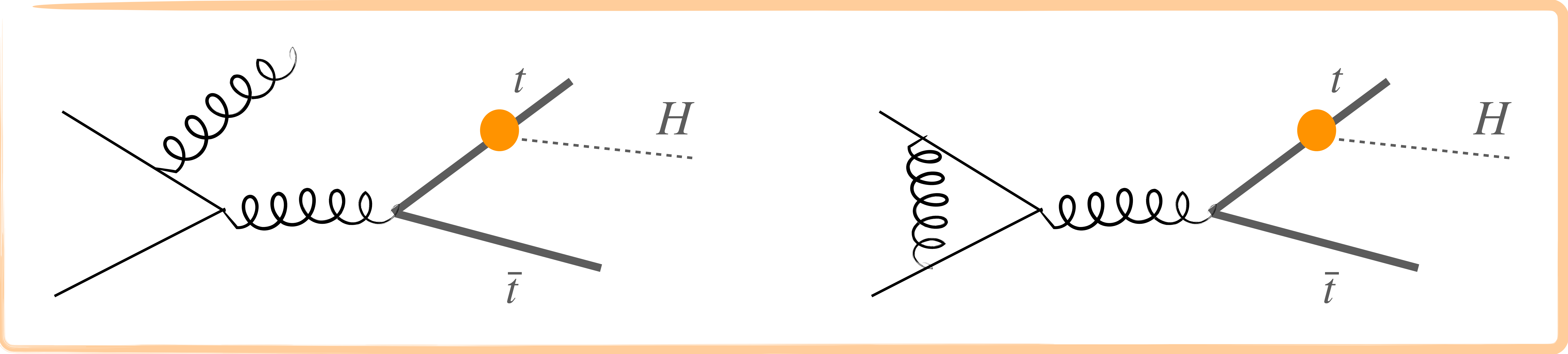

real contribution

virtual contribution

$\sim \mathcal{O}(10\%)$ precision

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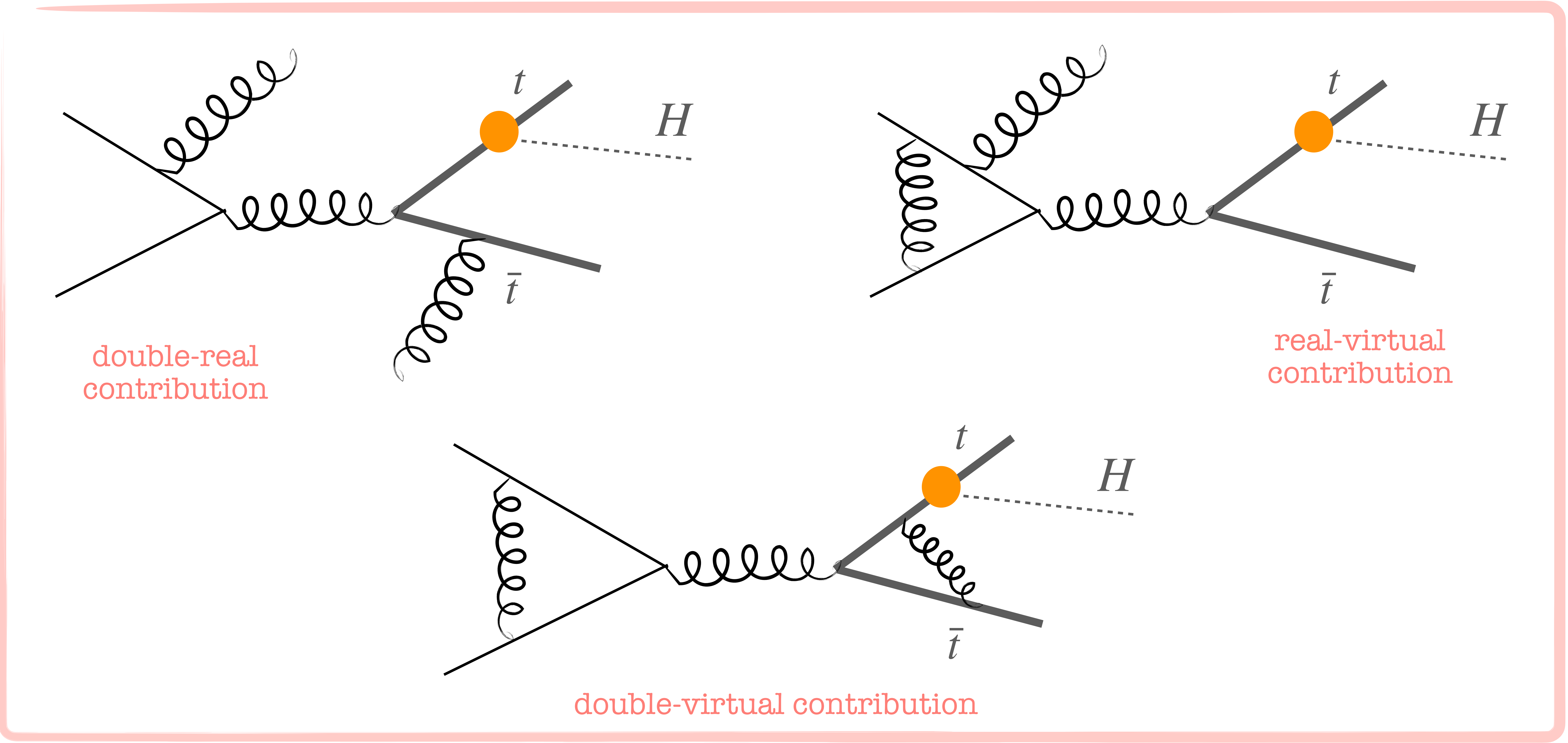
$\Delta\sigma_{NNLO}$



$\sim \mathcal{O}(2\%)$ precision

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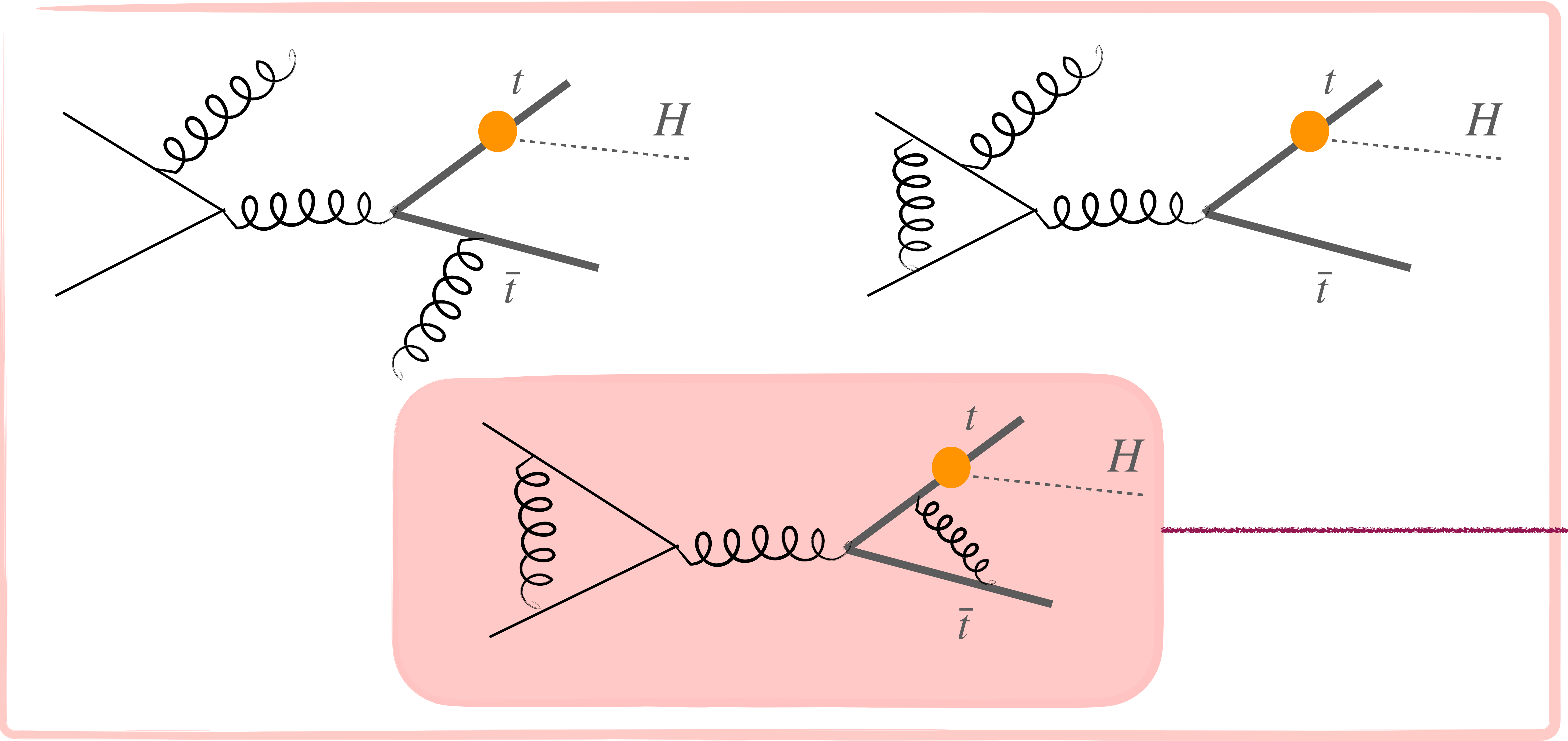
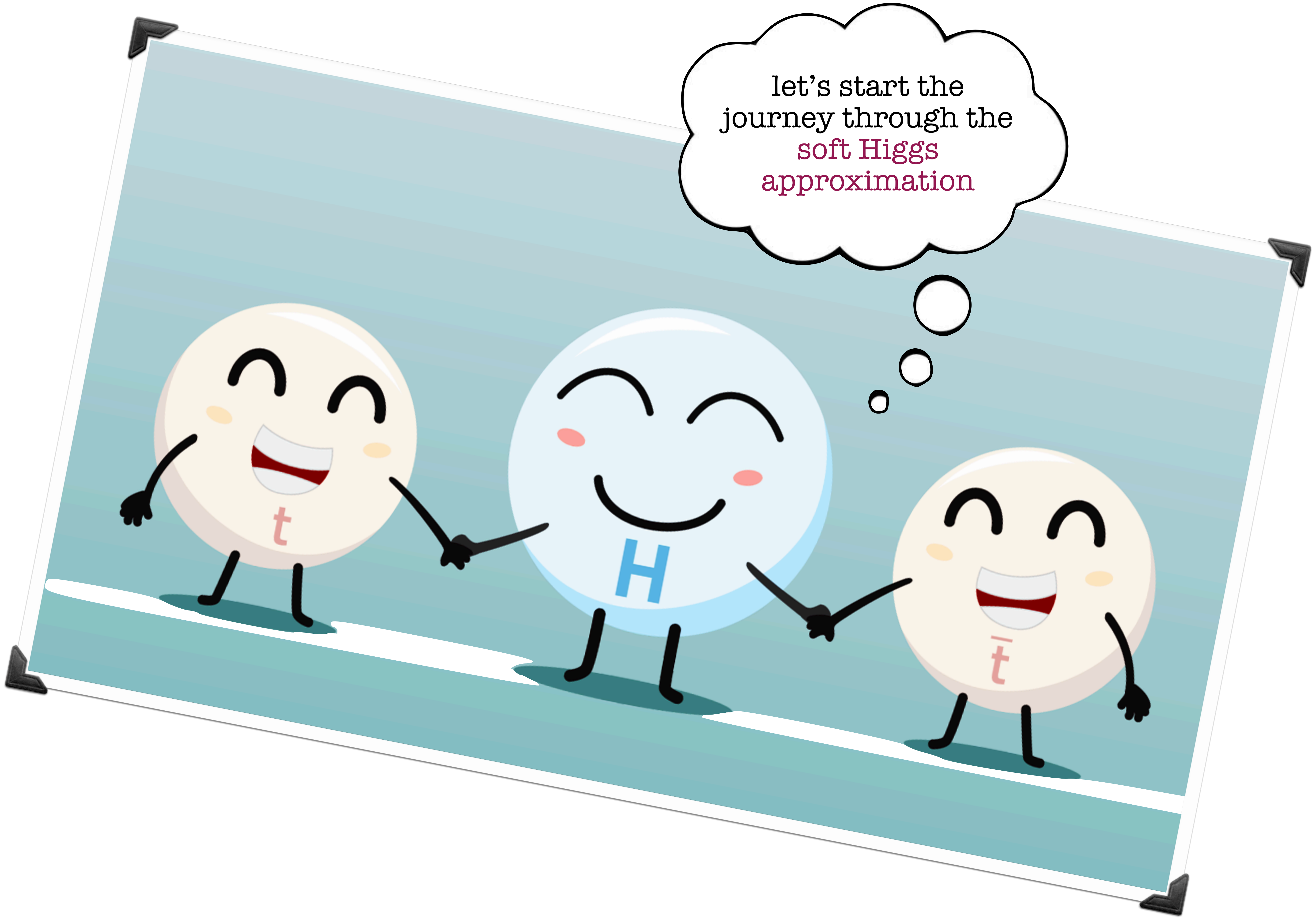
higher accuracy implies larger complexity in the calculations!



$\sim \mathcal{O}(2\%)$ precision

main bottleneck...

$2 \to 3$ process with several masses and scales involved


let's start the journey through the soft Higgs approximation
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Soft Higgs boson approximation

bottleneck: the two-loop amplitudes are at the frontier of the current techniques

solution: development of a soft Higgs boson approximation

- the main idea is to find an analogous formula to the well known factorisation in the case of **soft gluons**

$$\lim_{k\to 0} \mathcal{M}^{bare}(\{p_i\},k) = J(k)\,\mathcal{M}^{bare}(\{p_i\})$$

see e.g. [Catani, Grazzini (2000)]

$$J(k) = g_s \mu^{\epsilon}(J^{(0)}(k) + g_s^2 J^{(1)}(k) + \ldots)$$

purely non abelian

- for a **soft scalar Higgs** radiated off a heavy quark i, we have that

soft insertion rules, only external legs matter!

$$\lim_{k\to 0} \mathcal{M}^{bare}(\{p_i\},k) = J^{(0)}(k)\,\mathcal{M}^{bare}(\{p_i\})$$

bare mass of the heavy quark

$$J^{(0)}(k) = \sum_i \frac{m_{i,0}}{v}\frac{m_{i,0}}{p_i \cdot k}$$

- the naïve factorisation formula does not hold at the level of renormalised amplitudes!

Soft Higgs boson approximation

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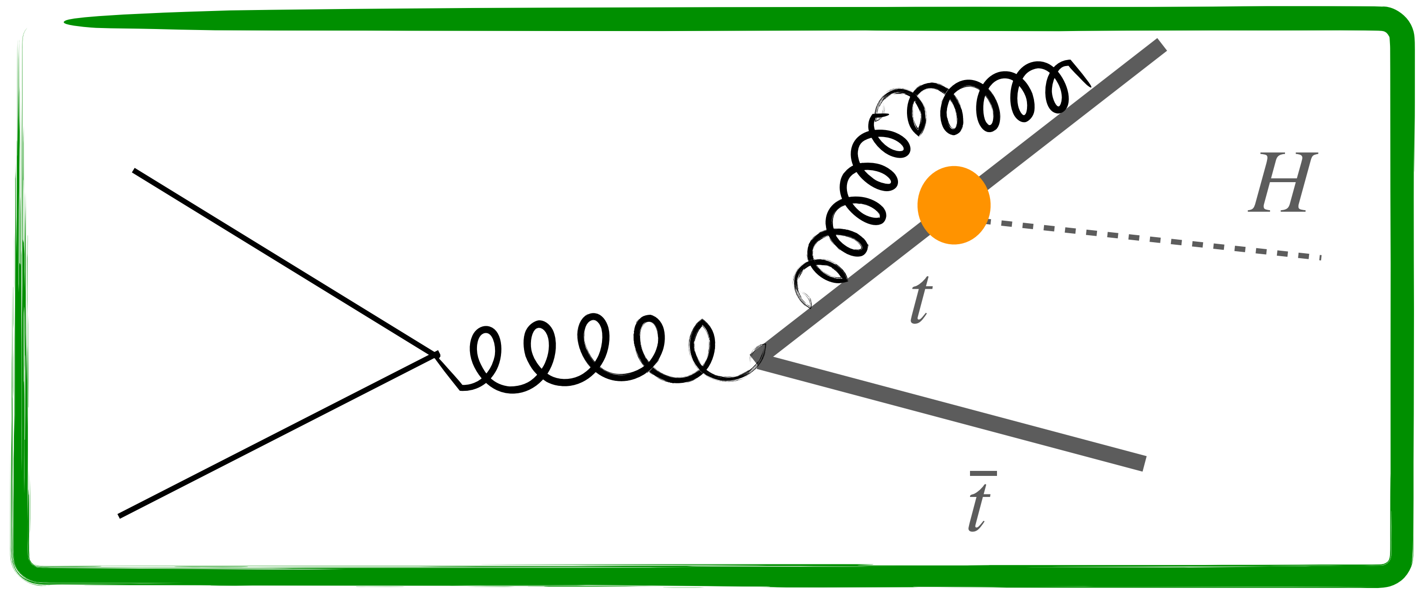
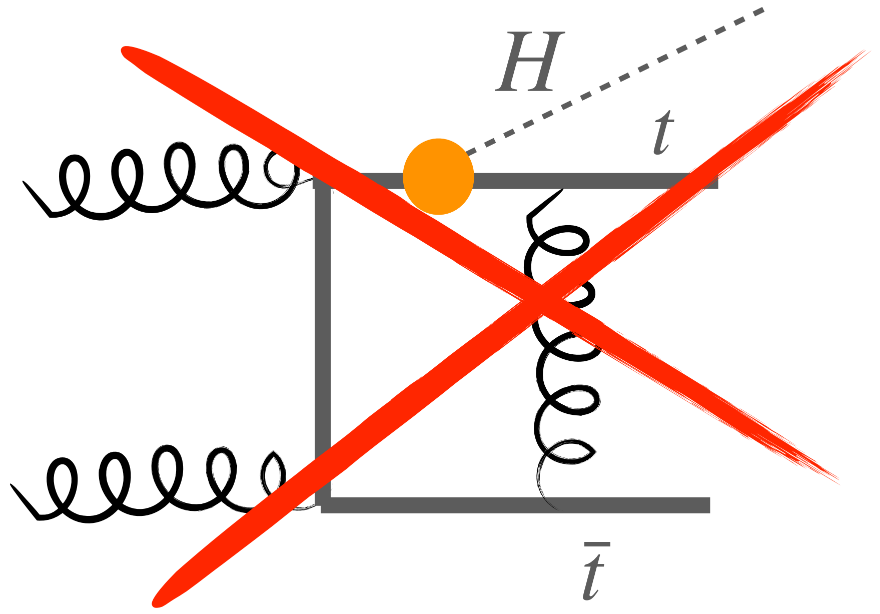
solution: development of a soft Higgs boson approximation

- there are diagrams that are not captured by the naïve factorisation formula, but they give an **additional contribution** in the soft Higgs limit





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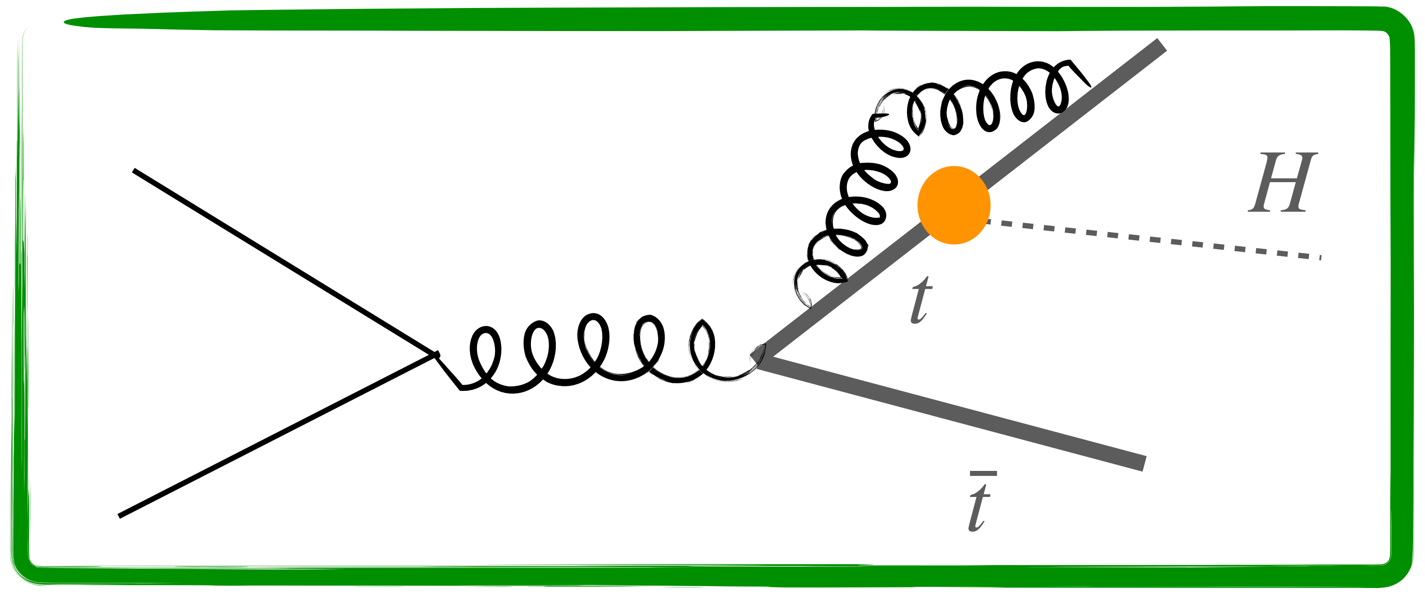

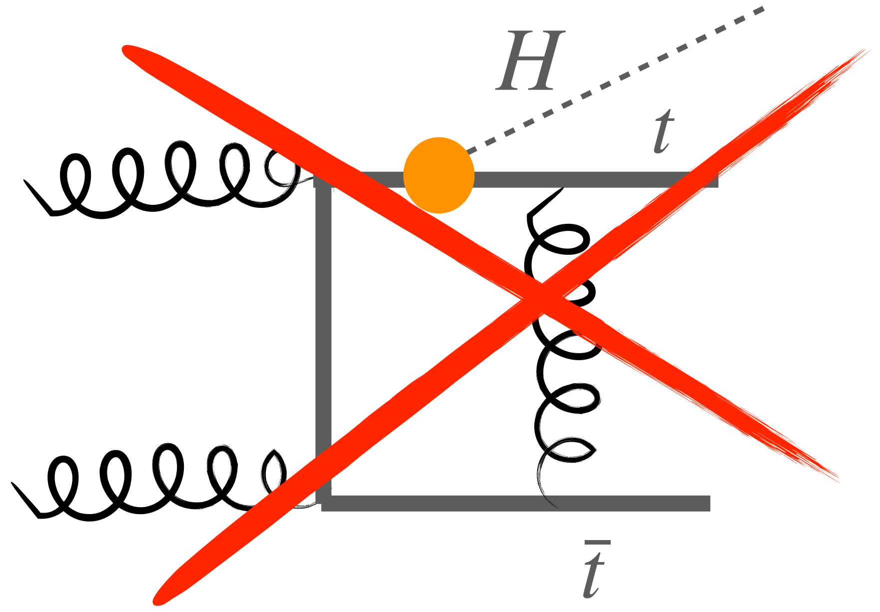

- the **renormalisation** of the heavy-quark mass and wave function induces a modification of the Higgs coupling to the heavy quark

$$\lim_{k\to 0} \mathscr{M}(\{p_i\},k) = F(\alpha_s(\mu_R); m/\mu_R)\, J^{(0)}(k)\, \mathscr{M}(\{p_i\})$$

renormalised mass of the heavy quark

$$J^{(0)}(k) = \sum_i \frac{m}{v} \frac{m}{p_i \cdot k}$$

we assume that all heavy quarks involved in the process have the same mass

overall normalisation, finite, gauge-independent and perturbatively computable

Soft Higgs boson approximation

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solution: development of a soft Higgs boson approximation

- **master formula** in the soft Higgs limit ($k \to 0$, $m_H \ll m_t$)

$$\lim_{k\to 0} \mathcal{M}(\{p_i\}, k) = F(\alpha_s(\mu_R); m/\mu_R)\, J^{(0)}(k) \mathcal{M}(\{p_i\})$$

soft limit of the scalar form factor for the heavy quark [Bernreuther et al. (2005)] [Blümlein et al. (2017)]

$$F(\alpha_s(\mu_R); m/\mu_R) = 1 + \frac{\alpha_s(\mu_R)}{2\pi}(-3C_F) + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 \left(\frac{33}{4}C_F^2 - \frac{185}{12}C_F C_A + \frac{13}{6}C_F(n_L+1) - 6C_F\beta_0 \ln\frac{\mu_R^2}{m^2}\right) + \mathcal{O}(\alpha_s^3)$$

- the form factor can also be derived by using Higgs **low-energy theorems** (LETs) [Kniehl, Spira (1995)]

$$\lim_{k\to 0} \mathcal{M}^{bare}_{Q\to QH}(p,k) = \frac{1}{v}\frac{\partial}{\partial \log m_0} \mathcal{M}^{bare}_{Q\to Q}(p)\Big|_{p^2=m^2}$$

heavy-quark self-energy

[Broadhurst, Grafe, Gray, Schilcher (1990)]
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valid also at the level of finite remainders (after subtracting the IR ϵ poles)

- how did we test it? …in the strict soft Higgs limit ($m_H = 0.5 GeV$, $E_H < 1 GeV$)

☑ $t\bar{t}H$: up to 1loop against OpenLoops

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less than per mille difference, pointwise, at the amplitude level

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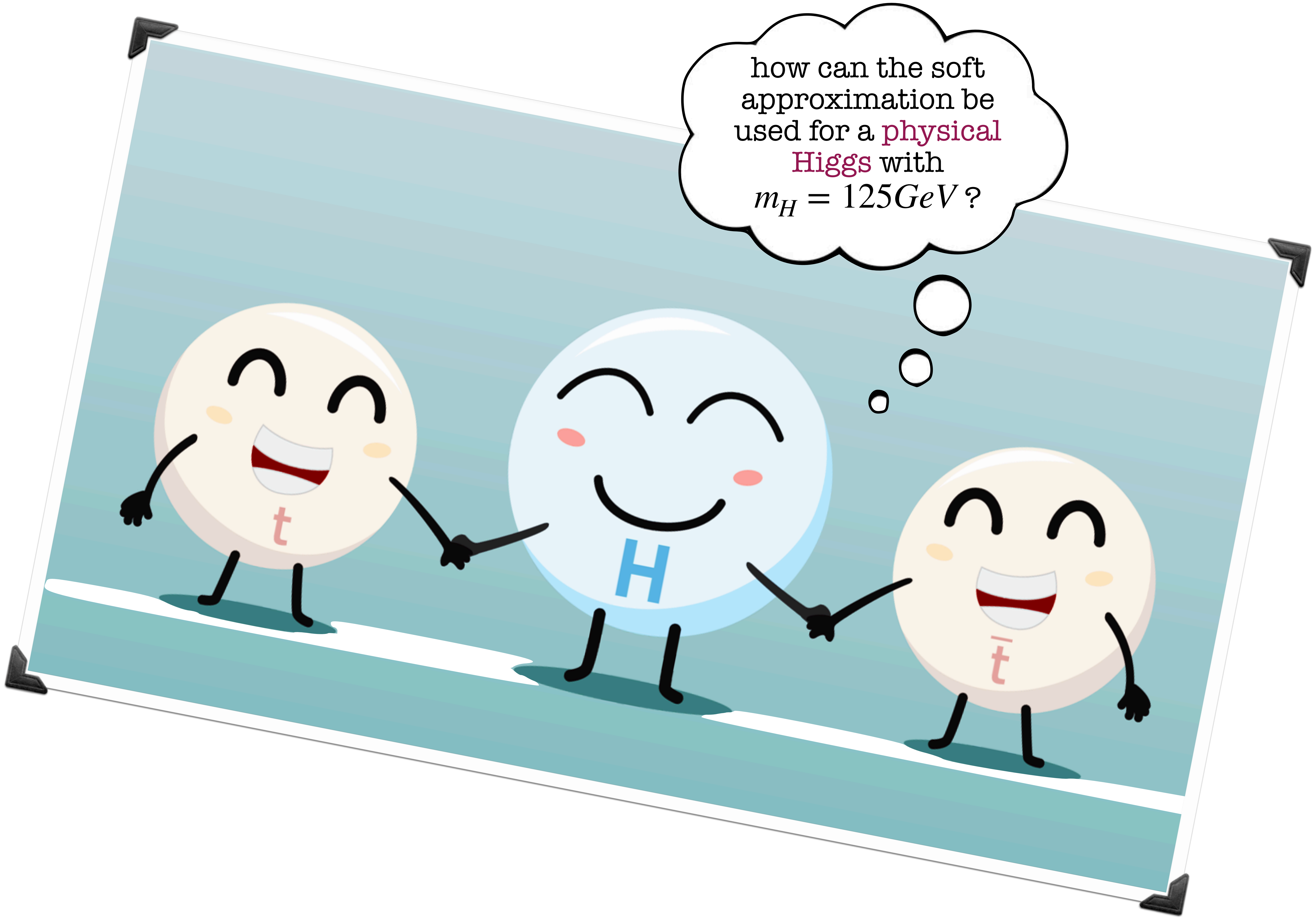
- can it be used to complete the NNLO calculation?

  - ☑ absolutely yes!!


how can the soft approximation be used for a physical Higgs with $m_H = 125GeV$?
t
H
$\bar{t}$

The computation: a divergent world!

- as soon as all the building blocks (i.e. renormalised amplitudes) are available, there is still a problem to face: the **appearance of infrared (IR) divergences** associated to soft and/or collinear limits
- these singularities arise both in virtual and real contributions

virtual → explicit exposed at the integrand level, before performing the phase space integral

real → manifest themselves only after integration over the radiation phase space

- the cancellation of the IR divergences is guaranteed by *KLN and factorisation theorems* for sufficiently inclusive **physical observables**
- the cancellation could be achieved **analytically** by working in $d = 4 - 2\epsilon$ space-time dimensions
- but the multidimensional integrals become **soon intractable** analytically for generic observables with phase space cuts

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1. develop a **SUBTRACTION** or **SLICING** formalism
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3. exploit **Monte Carlo methods** for the numerical integration of the finite quantities

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see Matteo's talk! ← SUBTRACTION

SLICING → method exploit in this talk!

The computation: slicing method in a nutshell

main idea: use a **resolution variable** to disentangle the fully unresolved from the single resolved regions

- we consider an NNLO computation (i.e. at most two additional emissions wrt a Born configuration)
- a *good resolution variable* X is defined as an infrared-safe (non-negative) observable such that:
  - for $X > 0$, at most one parton can become soft and/or collinear (NLO-type singularities)
  - for $X = 0$, double-unresolved limits occur (genuine NNLO-type singularities)
- we introduce a cut X_{cut} on the phase space, which acts as a small but finite **resolution cut-off**

$$\Delta\sigma_{NNLO} = \int_n d\sigma_{VV} + \int_{n+1} d\sigma_{RV}\Theta(X_{cut} - X) + \int_{n+2} d\sigma_{RR}\Theta(X_{cut} - X) + \int_{n+1} d\sigma_{RV}\Theta(X - X_{cut}) + \int_{n+2} d\sigma_{RR}\Theta(X - X_{cut})$$

we exploit the **factorisation properties** of the real matrix elements in the singular limits to analytically extract the poles that cancel the explicit ones from $d\sigma_{VV}$

we can apply an **NLO subtraction scheme** to reorganise the contributions and define finite integrands that can be numerically integrated

The computation: q_T -slicing

- in our slicing method we exploit the transverse momentum q_T as resolution variable
- q_T -slicing was **initially formulated for colour singlet processes** [Catani, Grazzini (2007)] and successfully applied for the calculation of NNLO QCD corrections see e.g. [Grazzini, Kallweit, Wiesemann (2018)]
- the formalism was extended to the case of **heavy-quark production** [Bonciani, Catani, Grazzini, Sargsyan, Torre (2015)]
- and successfully employed to calculate NNLO QCD corrections for $t\bar{t}$ [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan (2019)] and $b\bar{b}$ [Catani, Devoto, Grazzini, Mazzitelli (2021)] production
- the role of the **heavy quark mass** is crucial: q_T cannot regularise final-state collinear singularities
- the extension of the formalism to **heavy-quark production in association of a colourless system** does not pose any additional conceptual complication but …

not trivial ingredient:

two-loop soft function for arbitrary kinematics

[Catani, Devoto, Grazzini, Mazzitelli (in preparation)]

The computation: q_T-slicing

- we perturbatively expand the $t\bar{t}H$ partonic cross section, in the strong coupling, and we consider the contribution of order α_s^n $(n = 1,2)$

- the **master formula** is $$d\hat{\sigma}^{(n)} = \mathcal{H}^{(n)} \otimes d\hat{\sigma}_{LO} + [d\hat{\sigma}^{(n)}_{real} - d\hat{\sigma}^{(n)}_{ctrm}]_{q_t/Q > r_{cut}}$$

q_T and Q are the transverse momentum and invariant mass of the $t\bar{t}H$ system

- **hard-collinear coefficient** living at $q_T = 0$
- in order to expose the *irreducible* virtual contribution, we introduce the following decomposition

$$\mathcal{H}^{(n)} = H^{(n)} \delta(1 - z_1)\delta(1 - z_2) + \delta\mathcal{H}^{(n)}(z_1, z_2)$$

UV renormalised and IR subtracted amplitudes at scale μ_{IR} (overall normalisation $(4\pi)^{\epsilon} e^{-\gamma_E \epsilon}$)

where $$H^{(1)} = \left. \frac{2\Re(\mathcal{M}^{(1)}_{fin}(\mu_{IR}, \mu_R)\mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2} \right|_{\mu_R = Q}$$ and $$H^{(2)} = \left. \frac{2\Re(\mathcal{M}^{(2)}_{fin}(\mu_{IR}, \mu_R)\mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2} \right|_{\mu_R = Q}$$

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- in order to expose the *irreducible* virtual contribution, we introduce the following decomposition

$$\mathcal{H}^{(n)} = H^{(n)}\delta(1-z_1)\delta(1-z_2) + \delta\mathcal{H}^{(n)}(z_1, z_2)$$

where $$H^{(1)} = \left.\frac{2\Re(\mathcal{M}^{(1)}_{fin}(\mu_{IR}, \mu_R)\mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2}\right|_{\mu_R=Q}$$ and $$H^{(2)} = \left.\frac{2\Re(\mathcal{M}^{(2)}_{fin}(\mu_{IR}, \mu_R)\mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2}\right|_{\mu_R=Q}$$ only missing ingredient

- for $n = 2$, $H^{(2)}$ contains the genuine **two-loop virtual contribution** while $\delta\mathcal{H}^{(2)}$ includes the one-loop squared plus finite remainders to restore the unitarity

The computation: our prescription

Strategy :

- we want to apply the soft approximation in the **physical Higgs** region ($m_H = 125\,\text{GeV}$)
- construct a **mapping** that allows to project a $t\bar{t}H$ event $\{p_i\}_{i=1,\ldots,4}$ onto a $t\bar{t}$ one $\{q_i\}_{i=1,\ldots,4}$

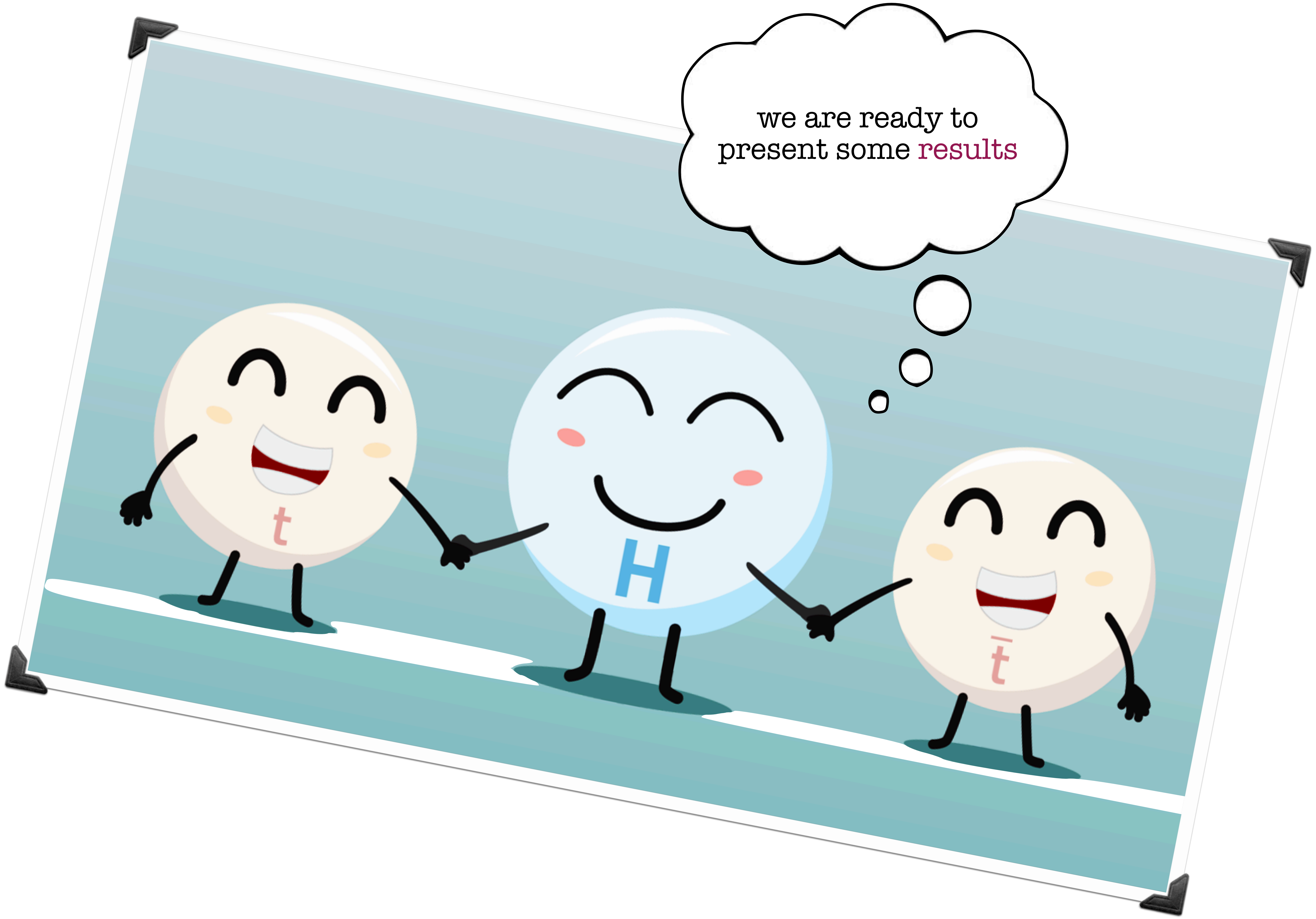
q_T recoil prescription

we apply the formula at the level of the finite remainders

$$\mathcal{M}_{t\bar{t}H}(\{p_i\}, p_H) \to F(\alpha_s(\mu_R); m/\mu_R)\, J^{(0)}(p_H)\mathcal{M}_{t\bar{t}}(\{q_i\})$$

$\mu_{IR} = \mu_R = Q_{t\bar{t}H}$ $\qquad$ $\mu_{IR} = \mu_R = Q_{t\bar{t}}$

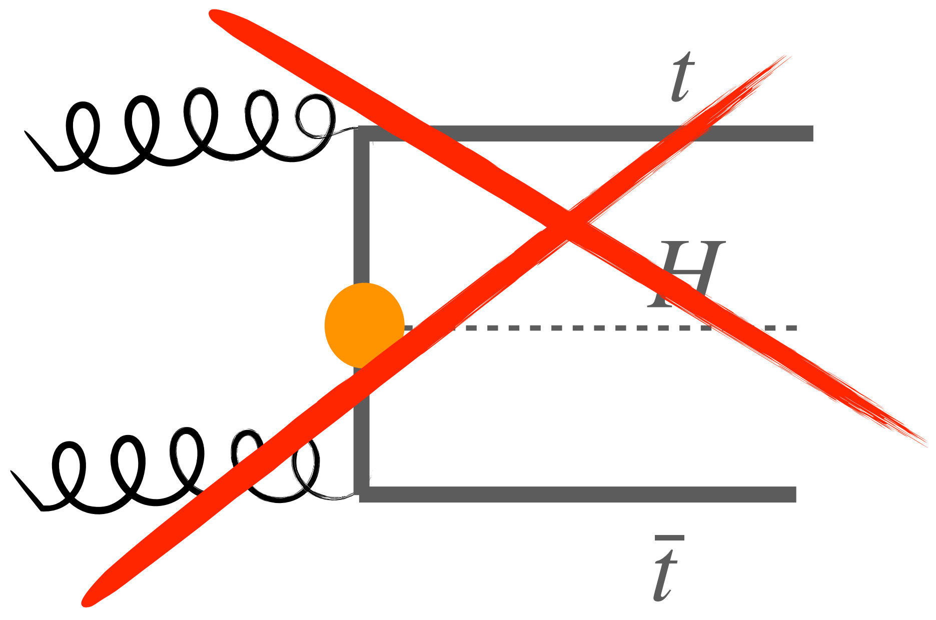
- the required tree-level and one-loop amplitudes are evaluated with OpenLoops
- the two-loop $t\bar{t}$ amplitudes are those provided by [Bärnreuther, Czakon, Fiedler (2013)]
- we test the **quality** of the approximation at Born and one-loop level
- @NNLO, **all the ingredients** are treated **exactly** except the $H^{(2)}$ contribution, on which we apply the same prescription tested at one-loop


we are ready to
present some results

Numerical results: LO benchmark

setup: NNLO NNPDF31, $m_H = 125 GeV$, $m_t = 173.3 GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

- the soft Higgs approximation gives the right order of magnitude of the exact LO result but it **overestimates** it by
  - $q\bar{q}$: factor **1.11 (1.06)** larger at $\sqrt{s} = 13\,(100)\,TeV$
  - gg : factor **2.3 (2)** larger at $\sqrt{s} = 13\,(100)\,TeV$
- for $q\bar{q}$ the approximation is expected to work better, for the absence of t-channel diagrams



not captured by the soft approximation since they are finite (not singular) in the soft Higgs limit

- **do not worry!** in our computation we need to approximate $H^{(1)}$ and $H^{(2)}$

$$H^{(n)}\big|_{\rm soft} = \left.\frac{2\Re(\mathcal{M}^{(n)}_{fin}(Q_{t\bar{t}},\mu_R)\mathcal{M}^{(0)*})_{\rm soft}}{|\mathcal{M}^{(0)}|^2_{\rm soft}}\right|_{\mu_R = Q_{t\bar{t}}}$$

effective reweighting

Numerical results: NLO benchmark

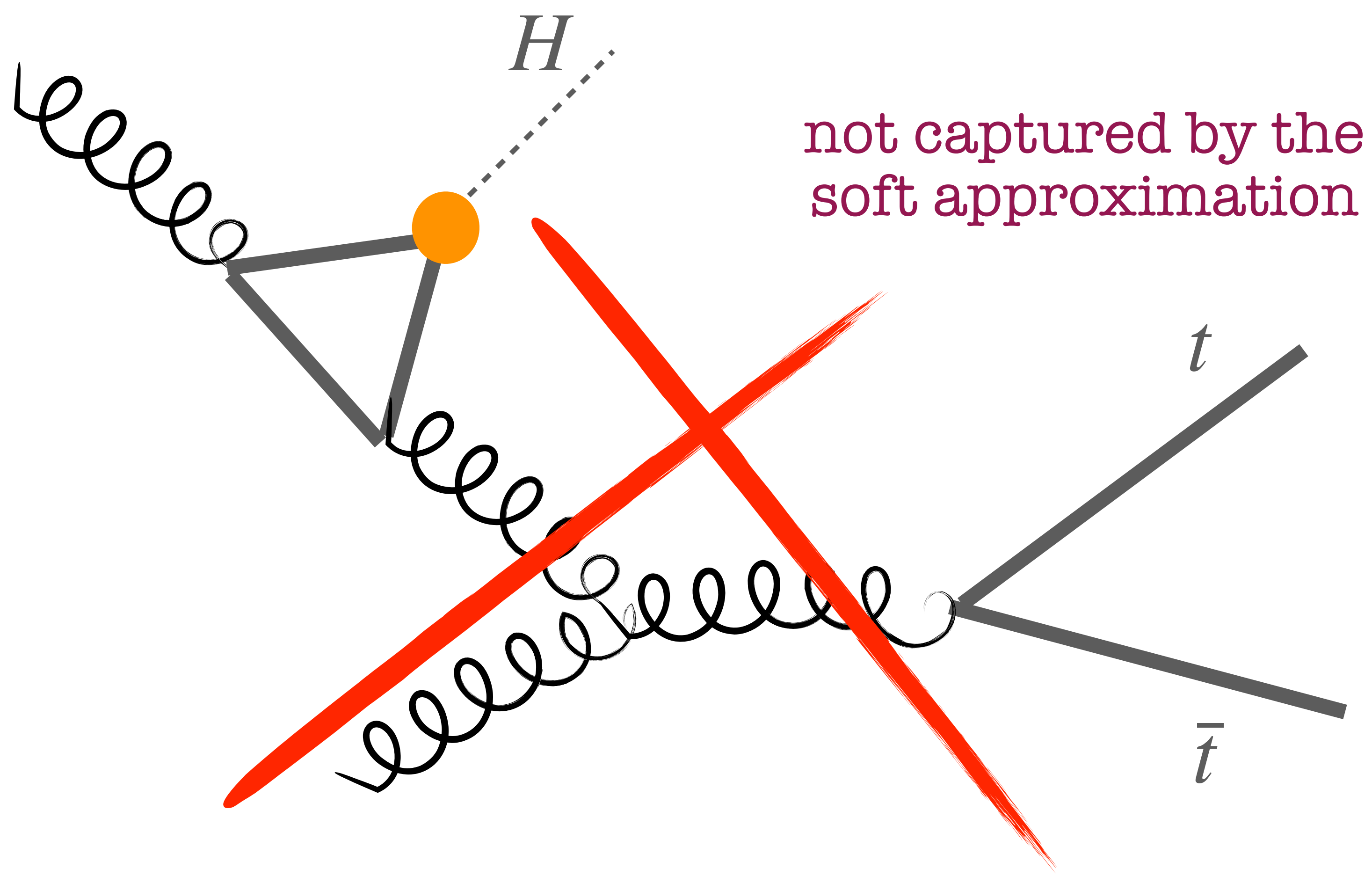
setup: NNLO NNPDF31, $m_H = 125GeV$, $m_t = 173.3GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

- the soft Higgs approximation works better wrt LO (mainly due to the reweighting):
  - $q\bar{q}$: **5%** of difference at $\sqrt{s} = 13\,(100)\,TeV$
  - gg : **30%** of difference at $\sqrt{s} = 13\,(100)\,TeV$

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\text{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\text{NLO,H}}\vert_{\text{soft}}$	61.98	7.413	5612	206.0

- in both channels, there are diagrams with virtual top quarks radiating a Higgs boson

  but… in $q\bar{q}$ there are no diagrams like



not captured by the soft approximation

the observed deviation can be used to estimate the uncertainty at NNLO

the quality of the final result will depend on the size of the contribution we approximate

Numerical results: uncertainties?

setup: NNLO NNPDF31, $m_H = 125GeV$, $m_t = 173.3GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

- @NNLO, the hard contribution is about **1%** of the LO cross section in gg and **2-3%** in $q\bar{q}$

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
$\sigma_{\rm LO}$	261.58	129.47	23055	2323.7
$\Delta\sigma_{\rm NLO,H}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\rm NLO,H}\|_{\rm soft}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\rm NNLO,H}\|_{\rm soft}$	$-2.980(3)$	$2.622(0)$	$-239.4(4)$	$65.45(1)$

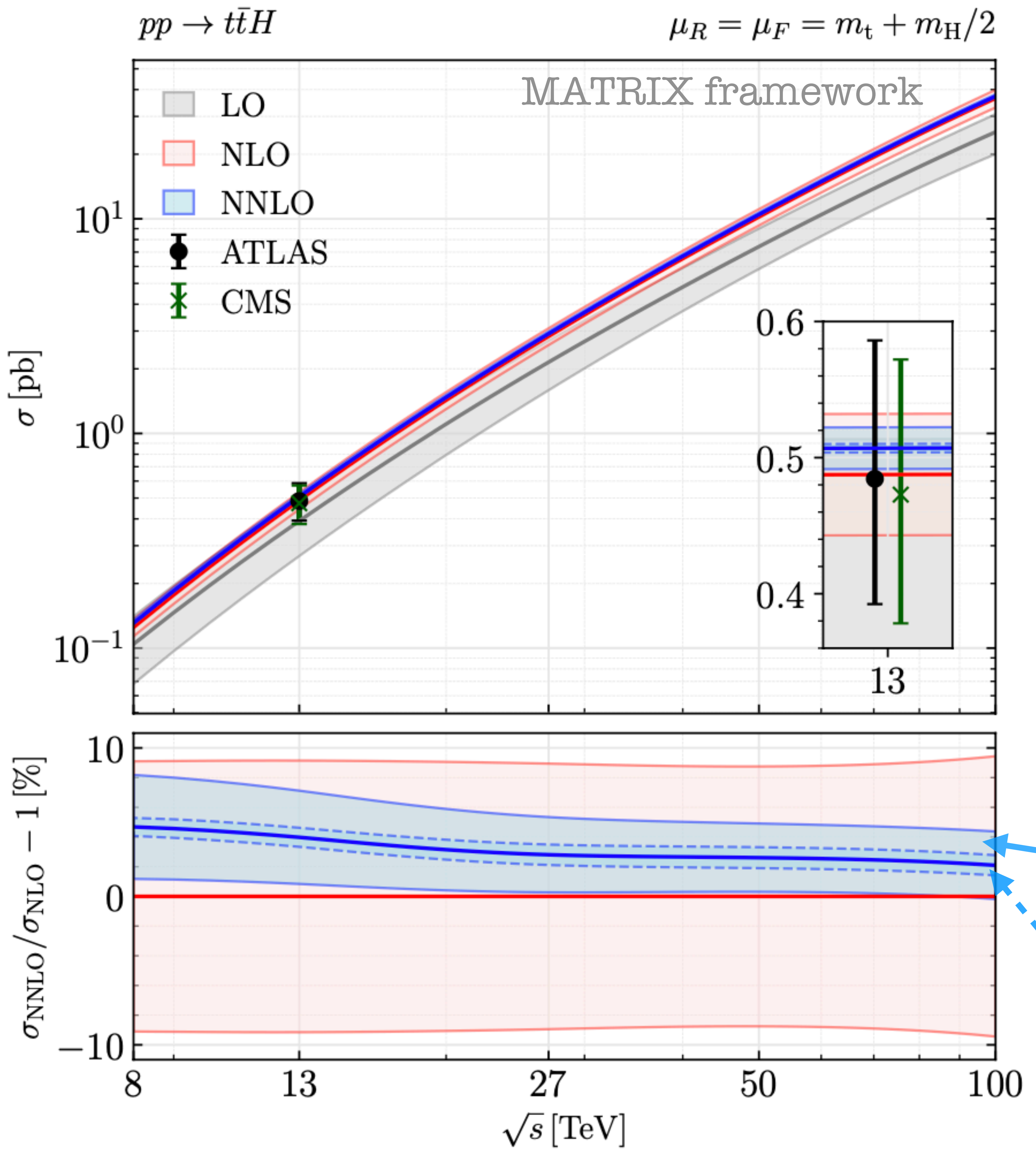
- how do we estimate the uncertainties?
  - ☑ test different recoil prescriptions
  - ☑ apply the soft factorisation formula at different subtraction scales $\mu_{IR} = Q_{t\bar{t}}/2$ and $\mu_{IR} = 2Q_{t\bar{t}}$
  - ☑ a conservative uncertainty cannot be smaller than the NLO discrepancy
  - ☑ multiply the NLO uncertainties for gg and $q\bar{q}$ by a **tolerance factor 3**
  - ☑ combine the gg and $q\bar{q}$ **linearly**

FINAL UNCERTAINTY:

$\pm 0.6\,\%$ on σ_{NNLO}, $\pm 15\,\%$ on $\Delta\sigma_{NNLO}$

Numerical results: inclusive cross section

setup: NNLO NNPDF31, $m_H = 125GeV$, $m_t = 173.3GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$



σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910^{+31.3\%}_{-22.2\%}$	$25.38^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875^{+5.6\%}_{-9.1\%}$	$36.43^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)^{+0.1\%}_{-2.2\%}$

- @NLO: **+25 (+44)%** at $\sqrt{s} = 13\,(100)\,TeV$
- @NNLO: **+4 (+2)%** at $\sqrt{s} = 13\,(100)\,TeV$
- significant reduction of the perturbative uncertainties

symmetrised 7-point scale variation

systematic + soft-approximation

Conclusions

- the current and expected precision of LHC data requires **NNLO QCD predictions**
- the actual frontier is represented by NNLO corrections for $2 \to 3$ processes with **several massive external legs**
- the **associated production of a Higgs boson with a top-quark pair** ($t\bar{t}H$) belongs to this category and it is crucial for the measurement of the top-Yukawa coupling

- the IR divergencies are regularised within the q_T **-slicing** framework

two-loop soft function for arbitrary kinematics

- the only missing ingredient is represented by the **two-loop amplitudes**

soft Higgs boson approximation

- our formula will provide a strong check of future computations of the exact two-loop amplitudes

- this is the **first (almost) exact** computation, at this perturbative order, for a $2 \to 3$ process with massive coloured particles
- the quantitative impact of the genuine two-loop contribution, in our computation, is relatively small (~1% on σ_{NNLO})
- significant reduction of the perturbative uncertainties

Conclusions

- the current and expected precision of LHC data requires **NNLO QCD predictions**
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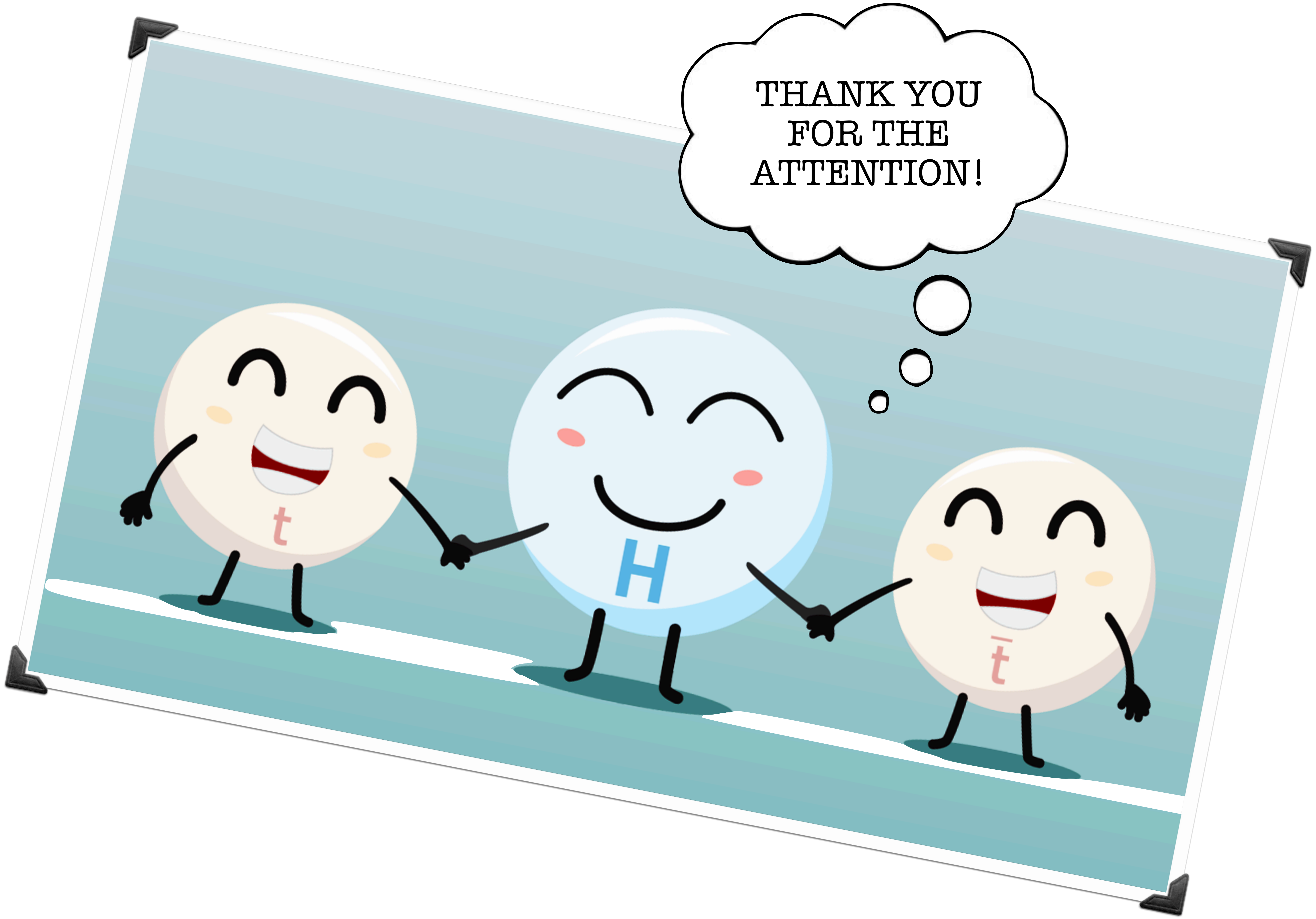
- significant reduction of the perturbative uncertainties

our prediction + NLO EW corrections will provide the most advanced perturbative prediction to date! STAY TUNED !!


THANK YOU
FOR THE
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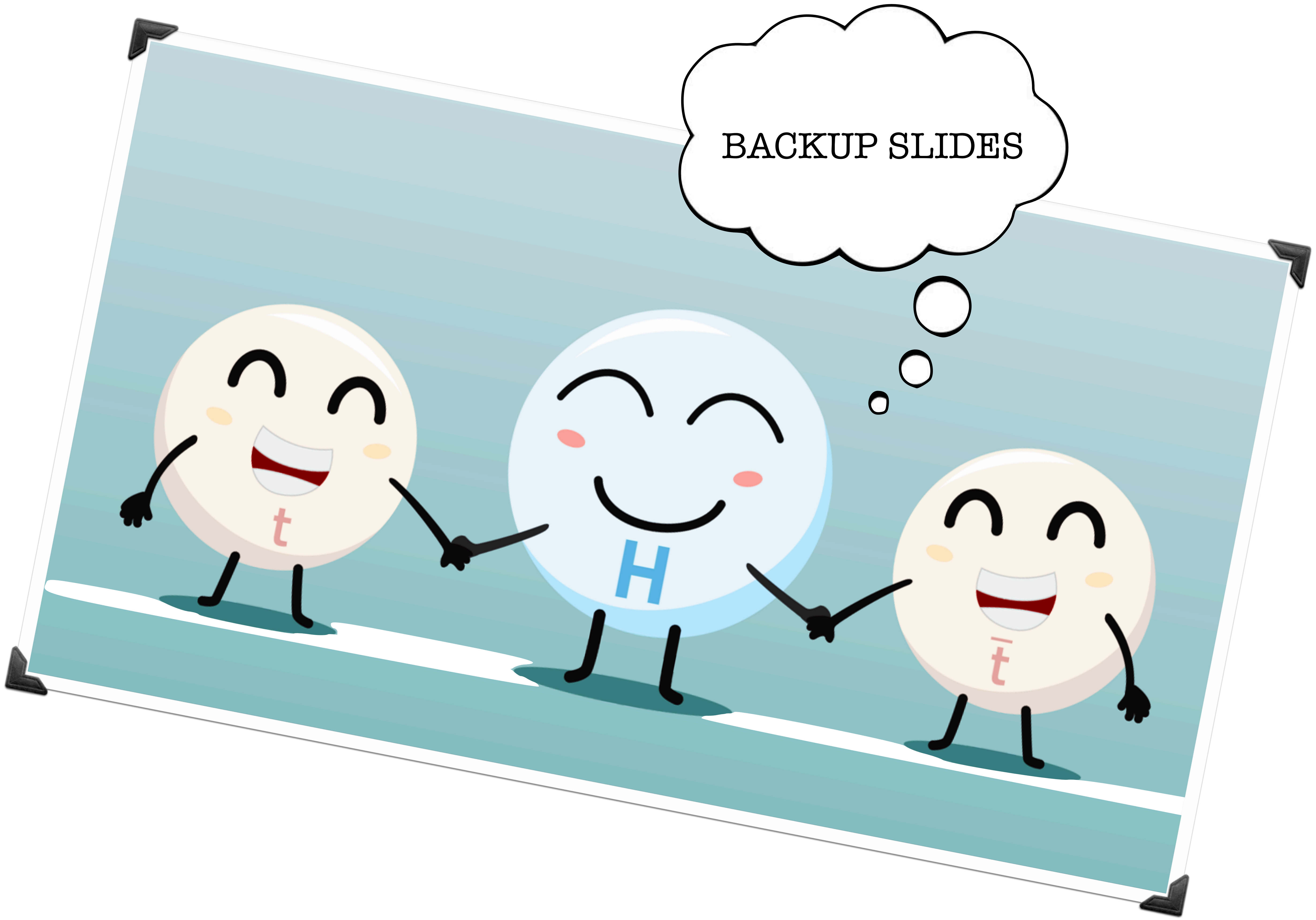
BACKUP SLIDES
t
H
$\bar{t}$

Differences wrt other approximations

- in our approximation we formally consider the limit in which the **Higgs boson** is **purely soft** $(p_H \to 0, m_H \ll m_t)$
- in [Dawson, Reina (1997)], [Brancaccio et al. (2021)] the main idea is to treat the Higgs boson as a **parton radiating off of a top quark**. Both approaches are based on a **collinear factorisation**.
  - in [Dawson, Reina (1997)] they consider the limit $m_H \ll m_t \ll \sqrt{s}$ and they introduce a function expressing the probability to extract a massless Higgs boson from a top quark (not full mass dependence + soft gluon approximation)
  - in [Brancaccio et al. (2021)] they compute the perturbative fragmentation functions (PFFs) $D_{t\to H}$ and $D_{g\to H}$ at NLO (full mass dependence)
  - this is an attempt towards an NNLO computation for $t\bar{t}H$ in the high $p_{T,H}$ region

- another difference is that we apply the soft approximation only the finite part of the two-loop amplitudes

Soft approximation: more details

- the form factor can also be derived by using Higgs **low-energy theorems** (LETs) [Kniehl, Spira (1995)]

$$\lim_{k\to 0} \mathscr{M}^{bare}_{Q\to QH}(p,k) = \frac{1}{v}\frac{\partial}{\partial \log m_0} \mathscr{M}^{bare}_{Q\to Q}(p)\Big|_{p^2=m^2}$$

heavy-quark self-energy

In the soft limit, the Higgs boson is not a dynamical d.o.f.

Its effect is to shift the mass of the heavy quark:

$$m_0 \to m_0\left(1+\frac{H}{v}\right)$$

$$\mathscr{M}^{bare}_{Q\to Q}(p) = \bar{Q}_0 \left\{ m_0[-1+\Sigma_S(p)] + \not{p}\,\Sigma_V(p) \right\} Q_0$$

[Broadhurst, Grafe, Gray, Schilcher (1990)]
[Broadhurst, Gray, Schilcher (1991)]

$$\Sigma_S(p) = -\sum_{n=1}^{+\infty}\left[\frac{g_0^2}{(4\pi)^{D/2}(p^2)^{\epsilon}}\right]^n \left(A_n(m_0^2/p^2) - B_n(m_0^2/p^2)\right)$$

$$\Sigma_V(p) = -\sum_{n=1}^{+\infty}\left[\frac{g_0^2}{(4\pi)^{D/2}(p^2)^{\epsilon}}\right]^n B_n(m_0^2/p^2)$$

- renormalisation of the quark mass and wave function $m_0\bar{Q}_0 Q_0 = m\bar{Q}QZ_m Z_2$
- $\overline{MS}$ renormalisation of the strong coupling + decoupling of the heavy quark

Soft approximation: scale variation

- in order to test our prescription, we **vary the subtraction scale** μ at which we apply the soft factorisation formula
- the **renormalisation scale** μ_R is kept **fixed** at $Q_{t\bar{t}H}$ in the $t\bar{t}H$ amplitudes and at $Q_{t\bar{t}}$ in the $t\bar{t}$ ones
- the running terms are added exactly

$gg : {}^{+164\%}_{-25\%}$ at 13TeV (similar pattern ${}^{+142\%}_{-20\%}$ at 100TeV)

gg channel @13TeV

approximation		$\sigma^{\text{VT only H1}}_{\text{NLO QCD}}$ [fb]		
		$\mu = Q/2$	$\mu = Q$	$\mu = 2Q$
exact		123.12 ± 0.04	88.61 ± 0.02	4.568 ± 0.013
		$\mu = Q_{proj}/2$	$\mu = Q_{proj}$	$\mu = 2Q_{proj}$
	$Q_{t\bar{t}}$	100.73 ± 0.03	61.98 ± 0.02	-26.308 ± 0.015
		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	66.24 ± 0.04	61.98 ± 0.02	57.76 ± 0.03

approximation		$\sigma^{\text{VT2 only H2 M2M0}}_{\text{NNLO QCD}}$ [fb]		
		$\mu = Q_{proj}/2$	$\mu = Q_{proj}$	$\mu = 2Q_{proj}$
	$Q_{t\bar{t}}$	13.114 ± 0.007	-2.977 ± 0.002	-29.03 ± 0.02
		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	1.882 ± 0.005	-2.977 ± 0.002	-3.715 ± 0.005
$\mathbf{F_2(Q)}$		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	0.378 ± 0.005	-4.487 ± 0.003	-5.222 ± 0.005

$Q = Q_{t\bar{t}H}$ $\quad$ $Q_{proj} = Q_{t\bar{t}}$

$$\left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \left(H^{(n)}|_{\mu=\mu_R=Q}\right)|\mathcal{M}^{(0)}|^2 \to \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \left(H^{(n)}_{soft}|_{\mu=\xi Q_{proj};\mu_R=Q_{proj}} + (\mu : \xi Q \to Q)\right)|\mathcal{M}^{(0)}|^2$$

where $n = 1{,}2$ and $\xi = \left\{\frac{1}{2}, 1, 2\right\}$

exact running terms

Soft approximation: scale variation

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- the **renormalisation scale** μ_R is kept **fixed** at $Q_{t\bar{t}H}$ in the $t\bar{t}H$ amplitudes and at $Q_{t\bar{t}}$ in the $t\bar{t}$ ones
- the running terms are added exactly

$q\bar{q}$: $^{+4\%}_{-0\%}$ at 13TeV (similar pattern $^{+3\%}_{-0\%}$ at 100TeV)

$q\bar{q}$ channel @13TeV

approximation		$\sigma^{\text{VT only H1}}_{\text{NLO QCD}}$ [fb]		
		$\mu = Q/2$	$\mu = Q$	$\mu = 2Q$
exact		18.048 ± 0.006	7.825 ± 0.005	-13.32 ± 0.01
		$\mu = Q_{proj}/2$	$\mu = Q_{proj}$	$\mu = 2Q_{proj}$
	$Q_{t\bar{t}}$	18.380 ± 0.006	7.413 ± 0.005	-14.47 ± 0.01
		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	8.156 ± 0.007	7.413 ± 0.005	6.671 ± 0.008

approximation		$\sigma^{\text{VT2 only H2 M2M0}}_{\text{NNLO QCD}}$ [fb]		
		$\mu = Q_{proj}/2$	$\mu = Q_{proj}$	$\mu = 2Q_{proj}$
	$Q_{t\bar{t}}$	2.7703 ± 0.0014	2.607 ± 0.001	4.193 ± 0.002
		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	2.6956 ± 0.0014	2.607 ± 0.001	2.7099 ± 0.0015
$\mathbf{F_2(Q)}$		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	1.8432 ± 0.0008	1.7550 ± 0.0007	1.8565 ± 0.0006

$Q = Q_{t\bar{t}H}$ $\qquad$ $Q_{proj} = Q_{t\bar{t}}$

$$\left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \left(H^{(n)}|_{\mu=\mu_R=Q}\right)|\mathcal{M}^{(0)}|^2 \to \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \left(H^{(n)}_{soft}|_{\mu=\xi Q_{proj};\mu_R=Q_{proj}} + (\mu : \xi Q \to Q)\right)|\mathcal{M}^{(0)}|^2$$

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