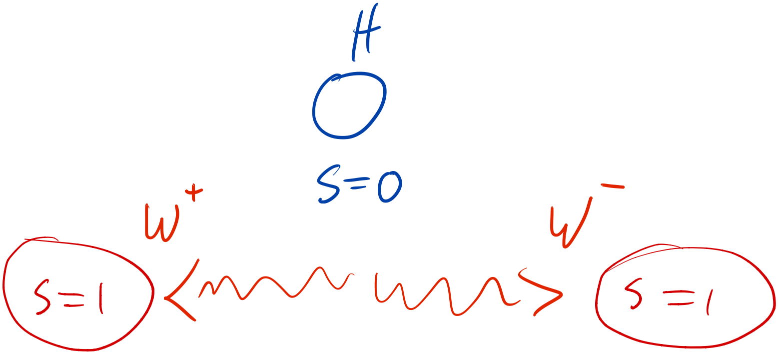
Bell Inequalities in Higgs boson decays (to vector bosons)

Alan Barr

A quantumTANGO talk

22$^{\mathrm{nd}}$ November 2022

AJB, Phys.Lett.B 825 (2022) 136866 — 2106.01377 [hep-ph]
AJB, P. Caban, J.Rembieliński — 2204.11063 [quant-ph]
R.Ashby-Pickering, AJB, A.Wierzchucka — 2209.13990 [quant-ph]
C.Altomonte, AJB, ORA-2022


H
S=0
W+
W−
S=1
S=1

Spin in the $H \to W^+W^-$ decay

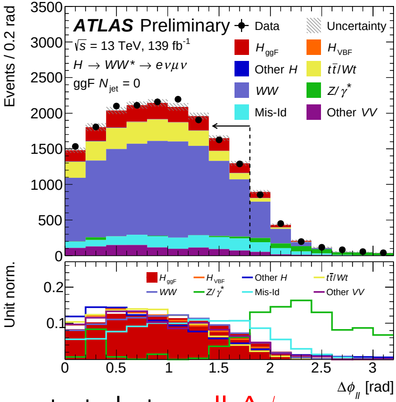
- $H \to W^+W^-$ decays produce pairs of W bosons in a singlet spin state
- In the narrow-width and non-relativistic approximations:

$$|\psi_s\rangle = \frac{1}{\sqrt{3}}\big(|+\rangle\,|-\rangle - |0\rangle\,|0\rangle + |-\rangle\,|+\rangle \big)$$

This is a Bell state of qu<u>tr</u>its

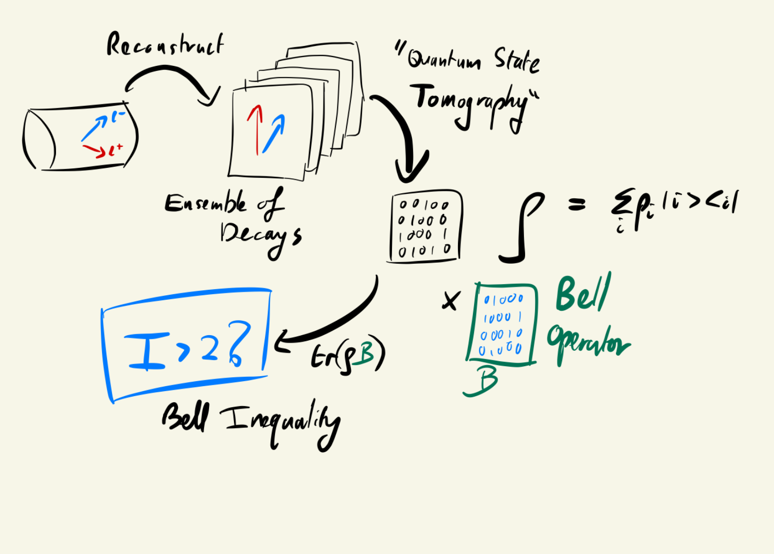
$$|\psi\rangle \in \mathcal{H}_3 \otimes \mathcal{H}_3$$

$\ell^+\ell^-$ azimuthal correlations in $H \to W^+W^-$



- Higgs signal concentrated at small $\Delta\phi_{\ell\ell}$
- Used e.g. in discovery searches

Bell tests in self-measuring quantum systems



Parameterising the density matrix

Need some parameters for ρ. We chose to use the **Gell-Mann** parameterisation:

The *WW* spin density matrix (9x9 matrix, 80 free parameters):

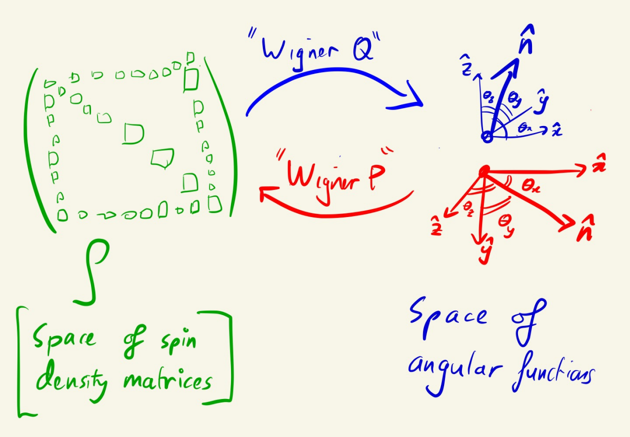
$$\rho = \frac{1}{9} I_3 \otimes I_3 + \sum_{i=1}^{8} f_i \lambda_i \otimes I_3 + \sum_{j=1}^{8} g_j I_3 \otimes \lambda_j + \sum_{i,j=1}^{8} h_{ij} \lambda_i \otimes \lambda_j,$$

- A natural extension of the Bloch sphere representation to qu<u>tr</u>its
- Perfect for tomography due to trace orthogonality

$$\mathrm{tr}(\lambda_i \lambda_j) = 2\delta_{ij}$$

- Commonly used in quantum information applications due to symmetry

Measuring the parameters



$$p(\hat{\mathbf{n}};\rho) = \frac{3}{4\pi}\left(\frac{1}{3} + \sum_{j=1}^{8} \Phi_j^Q a_j\right) \qquad a_j = \frac{1}{2}\left\langle \Phi_j^P(\hat{\mathbf{n}})\right\rangle$$

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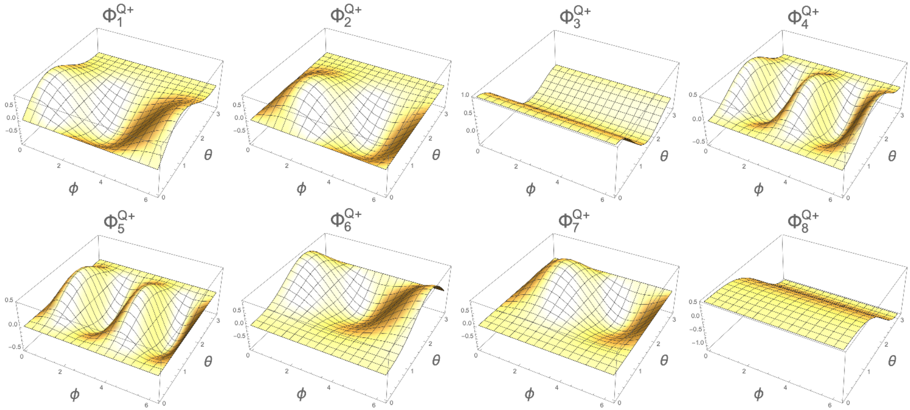

Wigner Q symbols for the eight Gell-Mann matrices

$$p(\hat{\mathbf{n}};\rho) = \frac{3}{4\pi}\left(\frac{1}{3} + \sum_{j=1}^{8} \Phi_j^Q a_j\right)$$

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Weak gauge bosons measure their own spin

SU(2) weak force is chiral: $\gamma^\mu(1-\gamma^5)$

W boson

$$W^+ \to \ell_R^+ + \nu_L$$
$$W^- \to \ell_L^- + \bar{\nu}_R$$

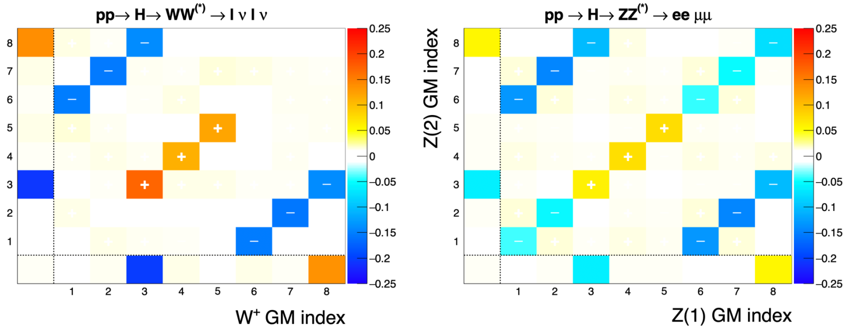
Decay of a $W^\pm$ boson is equivalent to a projective (von Neumann) quantum measurement of its spin along the axis of the emitted lepton

Z boson

Z bosons also have spin-sensitive decays

Left, right couplings determined by electroweak mixing
Equivalent to a non-projective quantum measurement

Density matrices from simulated Higgs boson decays



- High stats Madgraph simulation, parton level truth
- Tomographic reconstruction of parameters from Wigner-Weyl methods
- Almost perfect qutrit Bell states
- Can perform Bell Tests, entanglement tests, . . .

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The CGLMP inequality from data

CGLMP operator

$$\mathcal{B}^{xy}_{\rm CGLMP} = -\frac{2}{\sqrt{3}}\left(S_x \otimes S_x + S_y \otimes S_y\right) + \lambda_4 \otimes \lambda_4 + \lambda_5 \otimes \lambda_5$$

where

$$S_x = \frac{1}{\sqrt{2}}(\lambda_1 + \lambda_6) \quad \text{and} \quad S_y = \frac{1}{\sqrt{2}}(\lambda_2 + \lambda_7).$$

For *WW* systems:

$$\mathcal{I}_3 = \mathrm{tr}(\rho \mathcal{B}^{xy}_{\rm CGLMP}) = \frac{8}{\sqrt{3}}\left\langle \xi_x^+ \xi_x^- + \xi_y^+ \xi_y^- \right\rangle_{\rm av} + 25\left\langle \left((\xi_x^+)^2 - (\xi_y^+)^2\right)\left((\xi_x^-)^2 - (\xi_y^-)^2\right)\right\rangle_{\rm av} + 100\left\langle \xi_x^+ \xi_y^+ \xi_x^- \xi_y^- \right\rangle_{\rm av}$$

where $\xi_i^{\pm} \equiv \hat{\mathbf{p}}_{\ell^\pm} \cdot \hat{\mathbf{x}}_i$ is the Cartesian direction cosine of the emitted lepton in the parent W boson rest frame

Near maximal violation of CGLMP in idealised sample

$m_W^<$ [GeV]	20	30	40	50
$\mathcal{I}_3^{\mathrm{xyz}}$	2.76	2.81	2.82	2.77

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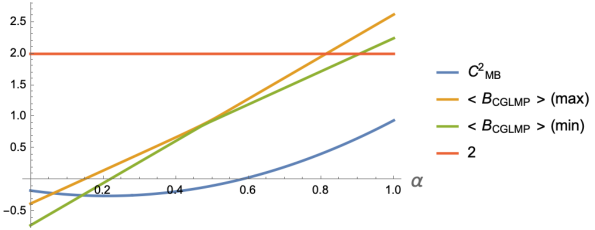
$\mathcal{I}_3 > 2 \implies$ violation

CAVEAT: In the absence of backgrounds, smearings, ...

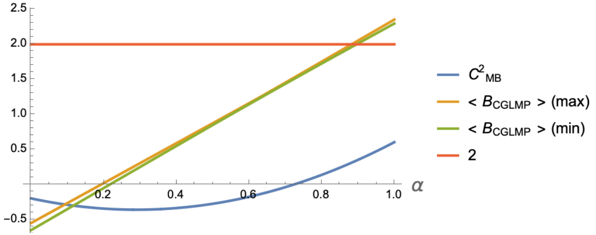
Confirmed for relativistic QFT in AJB, P. Caban, J. Rembieliński, 2204.11063

Signal purity needed?

If we perform full quantum tomographic reconstruction



$H \to WW^*$



$H \to ZZ^*$

$$\rho = \alpha \rho_{H \to VV^*} + (1 - \alpha) \rho_{pp \to VV^*}$$

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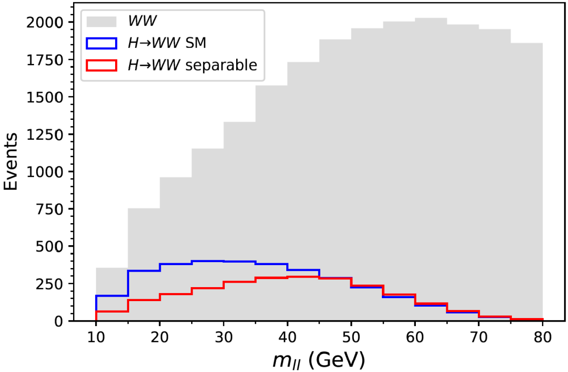
Laboratory-frame tests of quantum entanglement in $H \to WW$

Aguilar-Saavedra 2209.14033

- Parameterise ρ in terms of tensor operators
- Assume WW system is in scalar state
- Use CMS-like selection
- Look at lab-frame observables such as rapidity and azimuth differences
- Distinguish SM vs longitudinal polarisation

  "For the specific case of the dilepton invariant mass, which is a quite robust variable already measured by the ATLAS and CMS Collaborations, the expected statistical difference between the two hypotheses is of 7.1σ *with a luminosity of 138* fb^{-1}*. Therefore, the entanglement in* $H \to WW$ *could be established with the already collected Run 2 data."*



Aguilar-Saavedra 2209.14033

Testing entanglement and Bell inequalities in $H \to ZZ$

Aguilar-Saavedra, Bernal, Casas, Moreno 2209.13441

- Again parameterise in terms of Cartesian Tensor operators
- Assume scalar state of ZZ
- Optimize over unitary transformations of the operators
- Parton level simulation with no background

  "The numerical analysis shows that with a luminosity of $L = 300\mathrm{fb}^{-1}$ entanglement can be probed at $> 3\sigma$ level.

  For $L = 3\mathrm{ab}^{-1}$ (HL-LHC) entanglement can be probed beyond the 5σ level, while the sensitivity to a violation of the Bell inequalities is at the 4.5σ level."

The LHC: a laboratory for probing quantum foundations

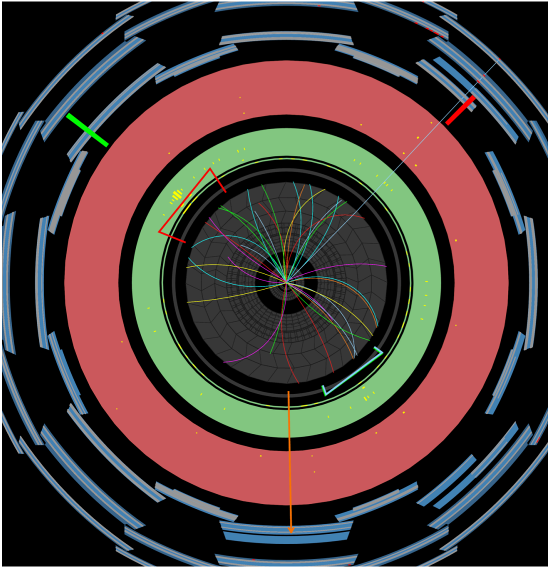
Weak bosons are wonderful quantum probes

- Quantum spin self measurement via chiral weak decays
- Expect entanglement and even Bell inequality violation
- Spin density matrix can be reconstructed from angular distributions ('tomography')

A wide-ranging quantum programme is possible @ LHC

- Local realism tests at $\sim 10^{12}$ higher energy
- Probes of quantum measurement
- Exchange symmetry and distinguishablity
- All in an unexplored region

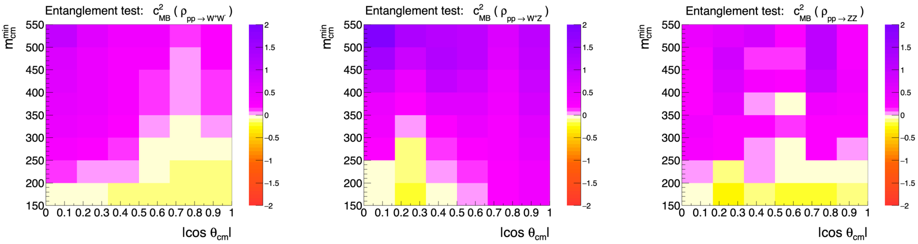
EXTRAS



Recent related works by other authors

- Gong, Parida, Tu and Venugopalan, *"Measurement of Bell-type inequalities and quantum entanglement from Λ-hyperon spin correlations at high energy colliders"*, 2107.13007
- Severi, Boschi, Degli Esposti, Maltoni and Sioli, *"Quantum tops at the LHC: from entanglement to Bell inequalities"*, 2110.10112
- Afik, de Nova and Ramón Muñoz, *"Quantum information with top quarks in QCD"*, 2203.05582
- Fabbrichesi, Floreanini and Gabrielli, *"Constraining new physics in entangled two-qubit systems: top-quark, tau-lepton and photon pairs"*, 2208.11723
- Afik, de Nova and Ramón Muñoz, *"Quantum discord and steering in top quarks at the LHC"* 2209.03969
- Aguilar-Saavedra, Bernal, Casas and Moreno, *"Testing entanglement and Bell inequalities in $H \to ZZ$"*, 2209.13441
- Aguilar-Saavedra *"Laboratory-frame tests of quantum entanglement in $H \to WW$"*, 2209.14033

Entanglement in diboson continuum?



WW, WZ, ZZ, as a function of m_{VV} and $\cos\theta$

Pink/Purple means entangled

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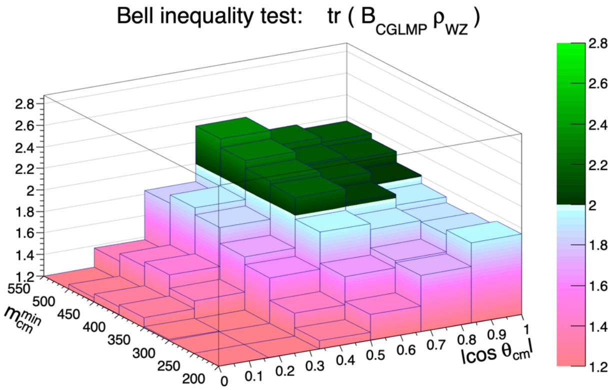
Bell violation in *WZ* continuum?



Green means Bell-inequality violating

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QFT calculations

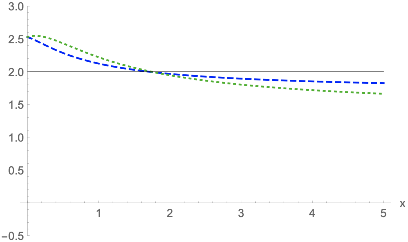


FIG. 3: Comparison of the violation of the CGLMP inequality in the state $|\xi(k, k^\pi)\rangle$ (blue, dashed line) and in the state $|\psi(k, k^\pi)\rangle$ (green, dotted line). The configuration of particles momenta and measurements directions is the following: $\mathbf{n} = (0, 0, 1)$, $\mathbf{w} = (\cos\phi_w \sin\theta_w, \sin\phi_w \sin\theta_w, \cos\theta_w)$, $\mathbf{w} \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ and $\theta_a = 2.667$, $\phi_a = 4.109$, $\theta_b = 0.924$, $\phi_b = 0.974$, $\theta_c = 2.699$, $\phi_c = 1.005$, $\theta_d = 0$, $\phi_d = 0$.

AJB, P. Caban, J. Rembieliński — 2204.11063 [quant-ph]

Experimental dependence @ LHC?

- Simulate LHC: 140/fb pp @ 13 TeV with Madgraph Monte Carlo simulation
- No backgrounds, some basic selection cuts, Gaussian smearing of each of the W boson momentum components

Expt. Assumptions	Truth	'A'	'B'	'C'
Min $p_T(\ell)$ [GeV]	0	5	20	20
Max $\lvert\eta(\ell)\rvert$	—	2.5	2.5	2.5
σ_{smear} [GeV]	0	5	5	10
$\mathcal{I}_3^{\mathrm{xyz}}$	2.62	2.40	2.75	2.16
Signif. $(\mathcal{I}_3^{\mathrm{xyz}} - 2)$	11.7σ	5.2σ	5.3σ	1.0σ

CAVEAT: Indicative only – more realistic version being investigated

In case you're curious

The CGLMP operator is[1]

$$
\mathcal{B}^{xy}_{\mathrm{CGLMP}} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\
0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\
0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

[1]after a minor tweak – see 2106.01377

CGLMP limits?

In a local realist theory

$$\mathcal{I}_3 \leq 2$$

In QM

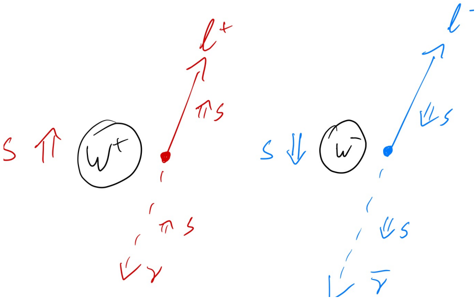
$$\mathcal{I}_3^{\mathrm{QM}} \leq 1 + \sqrt{11/3} \approx 2.9149$$

In QM for a maximally entangled state

$$\mathcal{I}_3^{\mathrm{QM,singlet}} \leq 4/(6\sqrt{3} - 9) \approx 2.8729$$

This is the tightest Bell inequality for pairs of three-outcome experiments

Getting the directions right



- ℓ^+ is emitted preferentially along spin direction (of W^+)
  ℓ^- is emitted preferentially against spin direction (of W^-)
- The $W^\pm$ spins are in different directions
- So the two leptons prefer to go in the same direction as each other

The density matrix ρ

- A fully-characterised quantum system is described by a ket $|\psi\rangle$
- Expectation values of measurement operator $\mathcal{A}$ are given by

$$\langle\psi|\,\mathcal{A}\,|\psi\rangle$$

- A more general, not-fully-characterised, quantum system is described by a **density matrix** ρ

$$\rho = \sum_i p_i\, |\psi\rangle_i \langle\psi|_i$$

  p_i is classical probability
  ρ is a non-negative hermitian operator with unit trace
- Expectation values for operator $\mathcal{A}$ for ρ are given by:

$$\langle\mathcal{A}\rangle = \mathrm{tr}(\rho\mathcal{A})$$

CHSH for spin-1

We can build a generalised CHSH Bell operator for pairs of spin-1 QM systems:

$$\mathcal{B}_{\text{CHSH}} = \hat{\mathbf{n}}_1 \cdot \mathbf{S} \otimes (\hat{\mathbf{n}}_2 - \hat{\mathbf{n}}_4) \cdot \mathbf{S} + \hat{\mathbf{n}}_3 \cdot \mathbf{S} \otimes (\hat{\mathbf{n}}_2 + \hat{\mathbf{n}}_4) \cdot \mathbf{S}$$

where now

$$S_{x,y,z} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Local realist expectations

Measurement outcomes: $\in \{+1, 0, -1\}$

The additional 0-outcome can only dilute CHSH expectation value

$\implies$ CHSH Bell inequality $|\mathcal{I}_2| \leq 2$ still must be satisfied in LR theory

Simulate $pp \to H \to WW^* \to \ell^+ \nu_\ell \ell^- \bar{\nu}_\ell$

Monte Carlo

Generate $gg \to H \to \ell^+ \nu_\ell \ell^- \bar{\nu}_\ell$ in Madgraph Monte Carlo simulation (10^6 pp events with $\sqrt{s} = 13\,\mathrm{TeV}$)

- Idealise: no detector, assume we can reconstruct $W^\pm$ rest frames
- Cut out the $e^+ e^-$ and $\mu^+ \mu^-$ events to remove $H \to ZZ^*$
- Place a lower bound $m_W^<$ on the m_W masses
- Optimise over the CHSH measurement directions
- The CHSH Bell inequality is violated iff

$$\mathcal{I}_2 > 2$$