Quantum Entanglement in Two-Qubit Systems

Marco Fabbrichesi, INFN, Italy

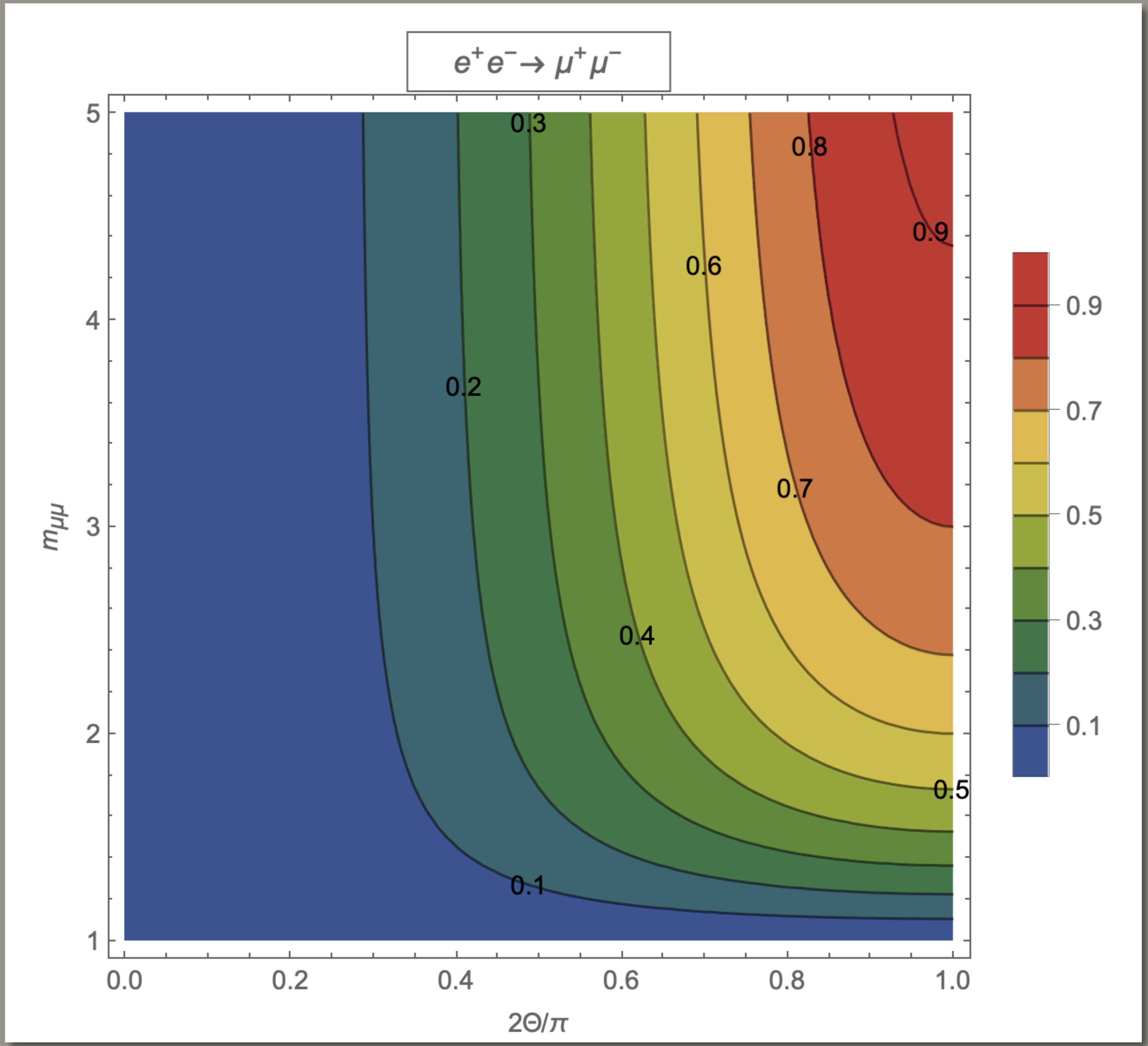
a QuantumTANGO presentation

November 22, 2022

together with
R. Floreanini, E. Gabrielli, L. Marzolo, G. Panizzo, M. Pinamonti

Y. Afik and J. R. M. de Nova,
``Entanglement and quantum tomography with top quarks at the LHC,"
Eur. Phys. J. Plus 136 (**2021**) 907 [arXiv:2003.02280 [quant-ph]]

- **there is entanglement!**
- **key insight: entanglement depends on the kinematical region**



violation of Bell inequalities makes entanglement real

measuring polarizations leads to heavy particles

Barr et al., 2106.0137
Barr et al., 2204.11063
Ashby-Pickering et al., 2209.13990
Aguilar-Saavedra et al., 2209.13441
Aguilar-Saavedra, 2209.09330

$$\underbrace{t\bar{t} \qquad \tau^+\tau^-}_{\text{qubits}}$$

MF et al, 2102.11883
Severi et al. 2110.10112
Aoude et al., 2203.05619
Aguilar-Saavedra et al., 2205.00542
MF et al, 2208.11723
Mantani, 2211.03428
Altakach et al., 2211.10513

$$\underbrace{W^+W^- \qquad W^\pm Z \qquad ZZ}_{\text{qutrits}}$$

exception: photon pairs

the importance of choosing the right observables

- entanglement witness vs. measure
- basis vs. maximization

✓ $\rho\,(\sigma_2 \otimes \sigma_2)\,\rho^*\,(\sigma_2 \otimes \sigma_2)$ (concurrence)

$$C[\rho] = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

✓ $M = C^T C$ $m_1 + m_2$ (best for Bell inequalities)

$$\rho = \frac{1}{4}\Big[1 \otimes 1 + \sum_i A_i(\sigma_i \otimes 1) + \sum_j B_j(1 \otimes \sigma_j) + \sum_{ij} C_{ij}(\sigma_i \otimes \sigma_j)\Big]$$

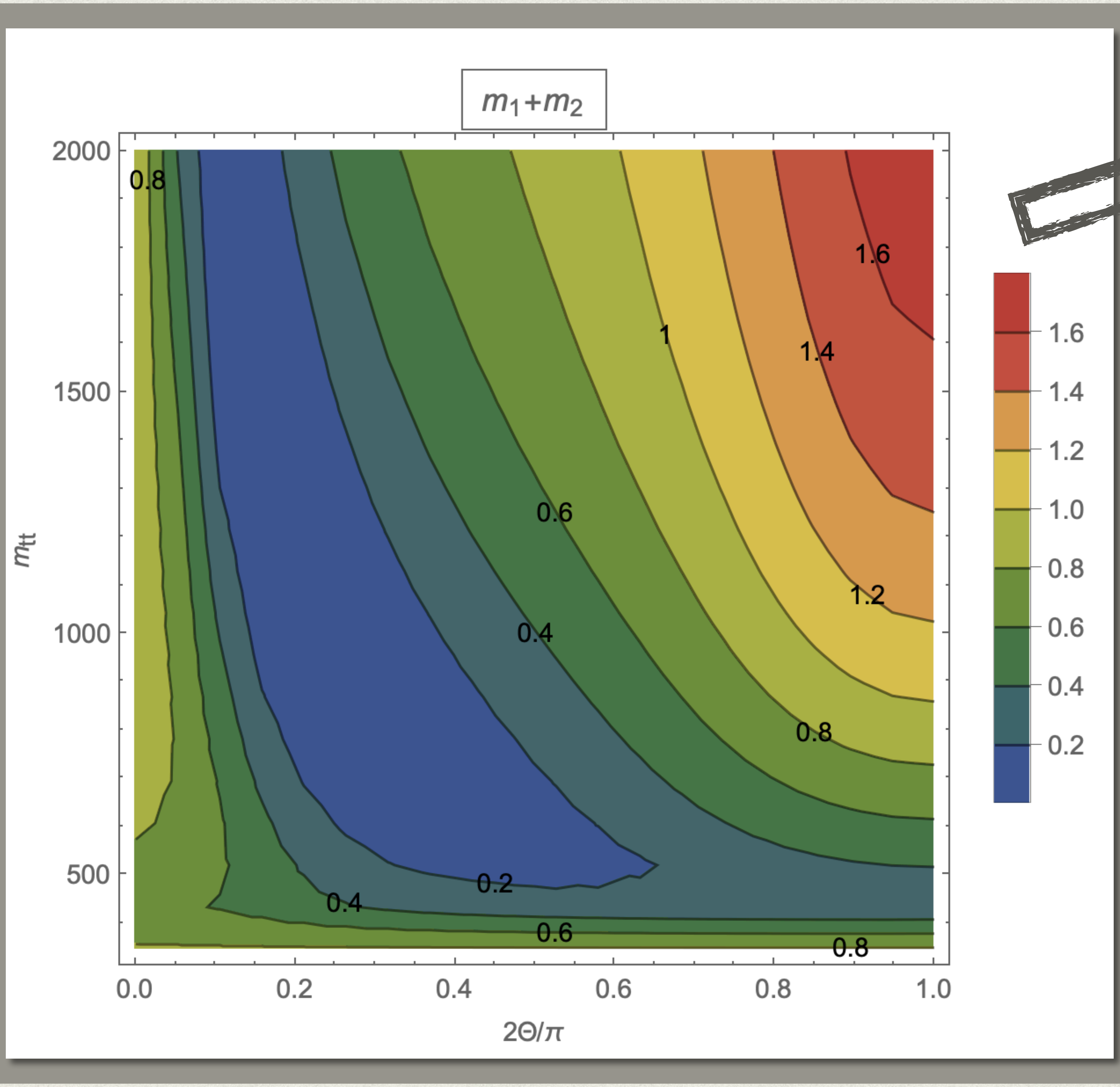
automatically in the best basis

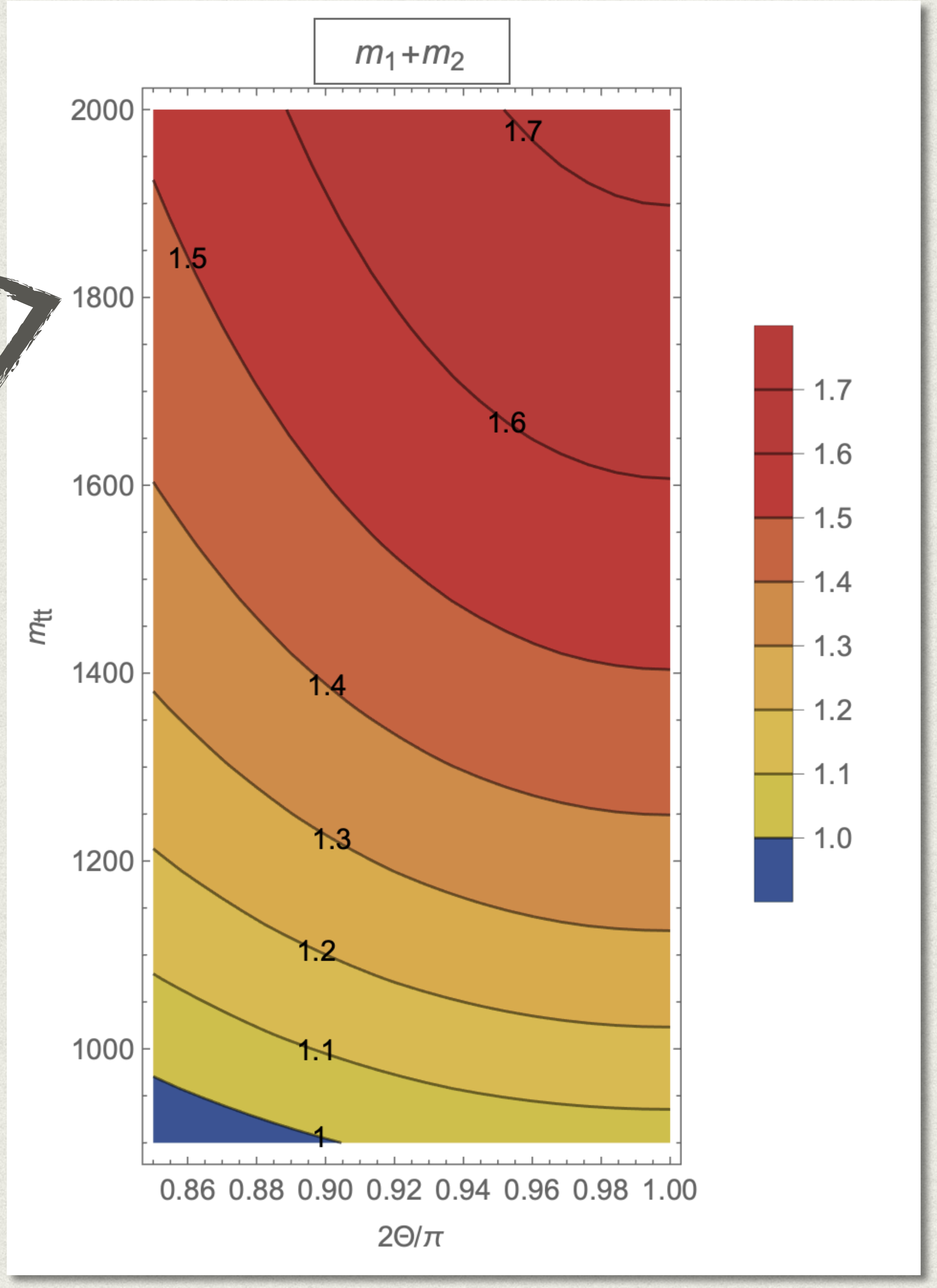
consistent estimator

F. Clauser, M.A. Horne, A. Shimony and R.A. Holt, Phys. Rev. Lett. 23 (1969) 880

R. Horodecki et al., Phys. Lett. A200 (1995) 340

the study of entanglement leads to that of Bell inequalities violation





what is the uncertainty?

spin correlations of top quarks

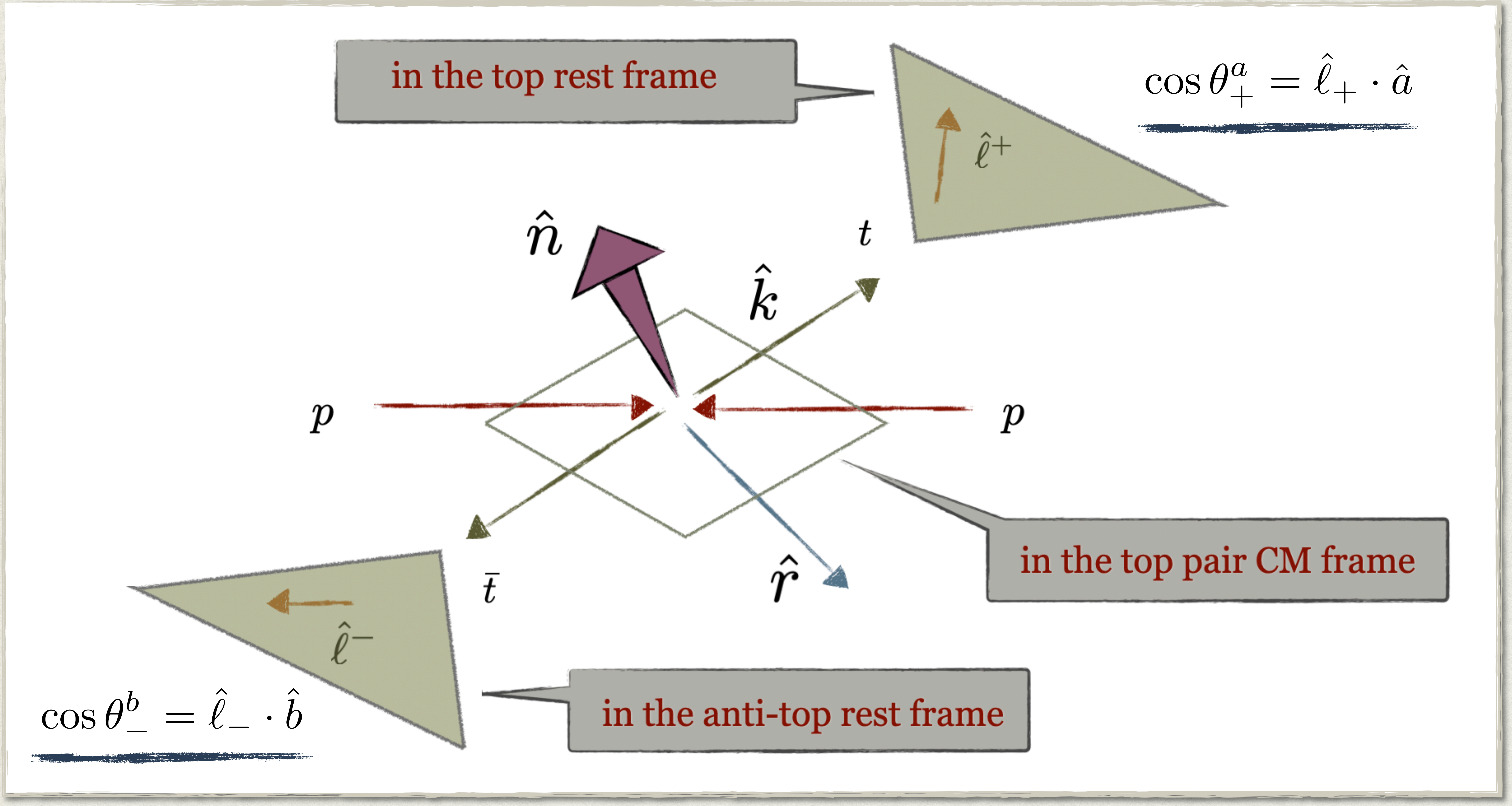
at the level of pseudo-observables: significance of 20!

at the level of observables: significance of 2 (?)

spin correlations of final leptons in the detector

Down the rabbit hole

$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$



in the top rest frame

$$\cos\theta_{+}^{a} = \hat{\ell}_{+}\cdot\hat{a}$$

in the top pair CM frame

in the anti-top rest frame

$$\cos\theta_{-}^{b} = \hat{\ell}_{-}\cdot\hat{b}$$

$$\hat{r} = \frac{1}{r}(\hat{p} - y\hat{k})$$

$$y = \hat{p}\cdot\hat{k}$$

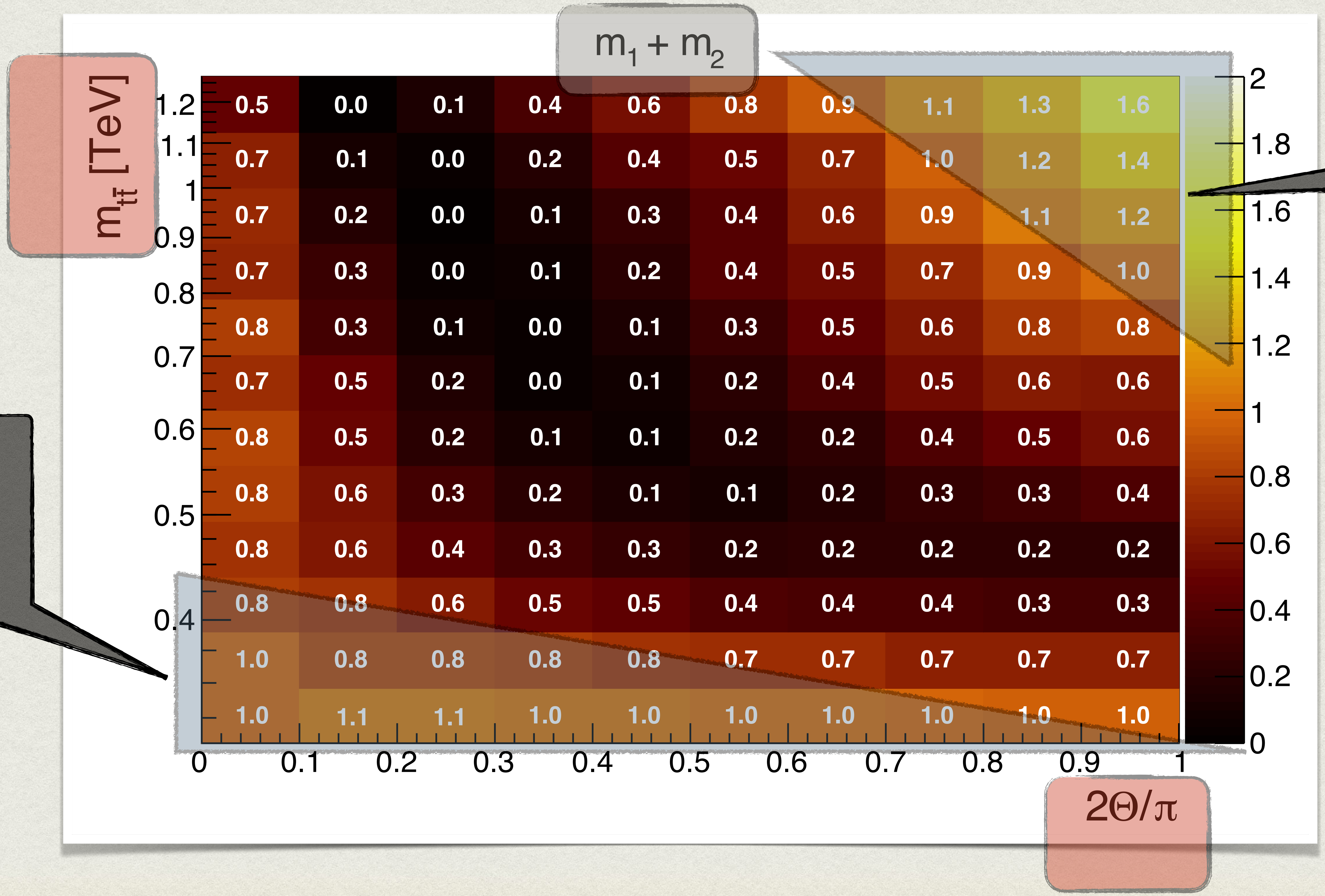
$$r = \sqrt{1-y^2}$$

$$\hat{n} = \frac{1}{r}(\hat{p}\times\hat{k})$$

W. Bernreuther, D. Heisler and Z. G. Si, JHEP 12, 026 (2015)

analysis

MF, R. Floreanini, G. Panizzo, PRL 127 (2021) 16



0.5	0.0	0.1	0.4	0.6	0.8	0.9	1.1	1.3	1.6
0.7	0.1	0.0	0.2	0.4	0.5	0.7	1.0	1.2	1.4
0.7	0.2	0.0	0.1	0.3	0.4	0.6	0.9	1.1	1.2
0.7	0.3	0.0	0.1	0.2	0.4	0.5	0.7	0.9	1.0
0.8	0.3	0.1	0.0	0.1	0.3	0.5	0.6	0.8	0.8
0.7	0.5	0.2	0.0	0.1	0.2	0.4	0.5	0.6	0.6
0.8	0.5	0.2	0.1	0.1	0.2	0.2	0.4	0.5	0.6
0.8	0.6	0.3	0.2	0.1	0.1	0.2	0.3	0.3	0.4
0.8	0.6	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2
0.8	0.8	0.6	0.5	0.5	0.4	0.4	0.4	0.3	0.3
1.0	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7
1.0	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0

both qq and gg
give top pair
max. entangled

only gg
gives top pair
max. entangled

Aside: correct for the bias

An intrinsically *differential* observable

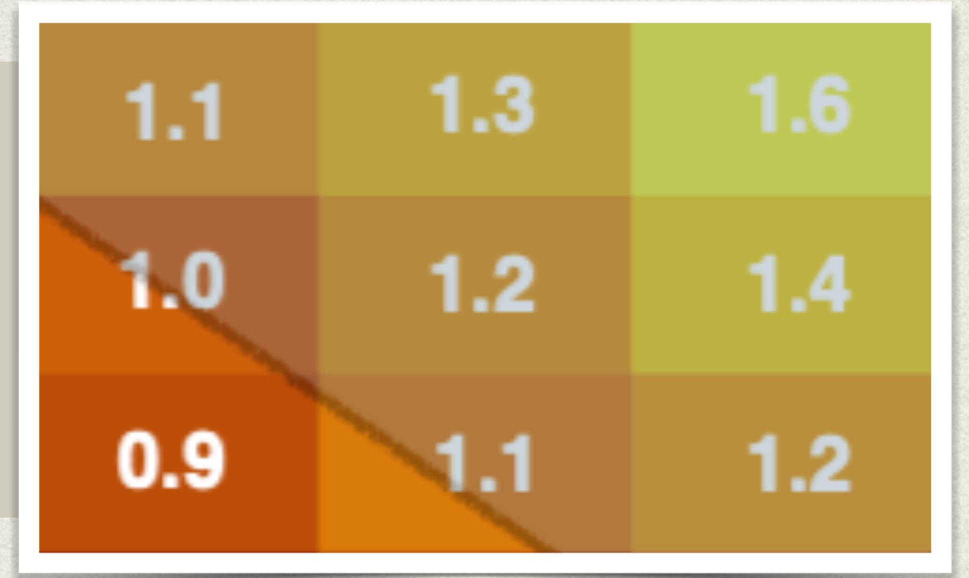
- **increasing** the **cell size** decreases the *expected* value of $m_1 + m_2$ (expected feature considering its meaning)

- In order to **increase statistical significance, statistically combine measurements** on different cells

Results

bins

$$\frac{2\,\Theta}{\pi} \gtrsim 0.7 \qquad m_{t\bar{t}} \gtrsim 0.9\ \mathrm{TeV}$$



1.1	1.3	1.6
1.0	1.2	1.4
0.9	1.1	1.2

null hypothesis: $m_1 + m_2 \leq 1$

Hypothesis test

$$\chi^2 = \sum_i \frac{(1 - m_1^i - m_2^i)^2}{s_i^2}$$

violation: 98% CL w/ Run II data (139 fb-1)
99.99% CL with Run III

systematic uncertainties (e.g. from unfolding) not included

MF, R. Floreanini, E. Gabrielli, 2208.11723

can we use entanglement to study new physics?
it would be a new and powerful tool

example: magnetic dipole moment in top-gluon interaction

$$\mathcal{L}_{\text{dipole}} = -\mu \, \frac{g_s}{2m_t} \, \bar{t} \, \sigma^{\mu\nu} \, T^a \, t \, G^a_{\mu\nu}$$

$$\underset{\nwarrow \text{sign}}{-} g_s \bar{q} \gamma^\mu T^a t G^a_\mu$$

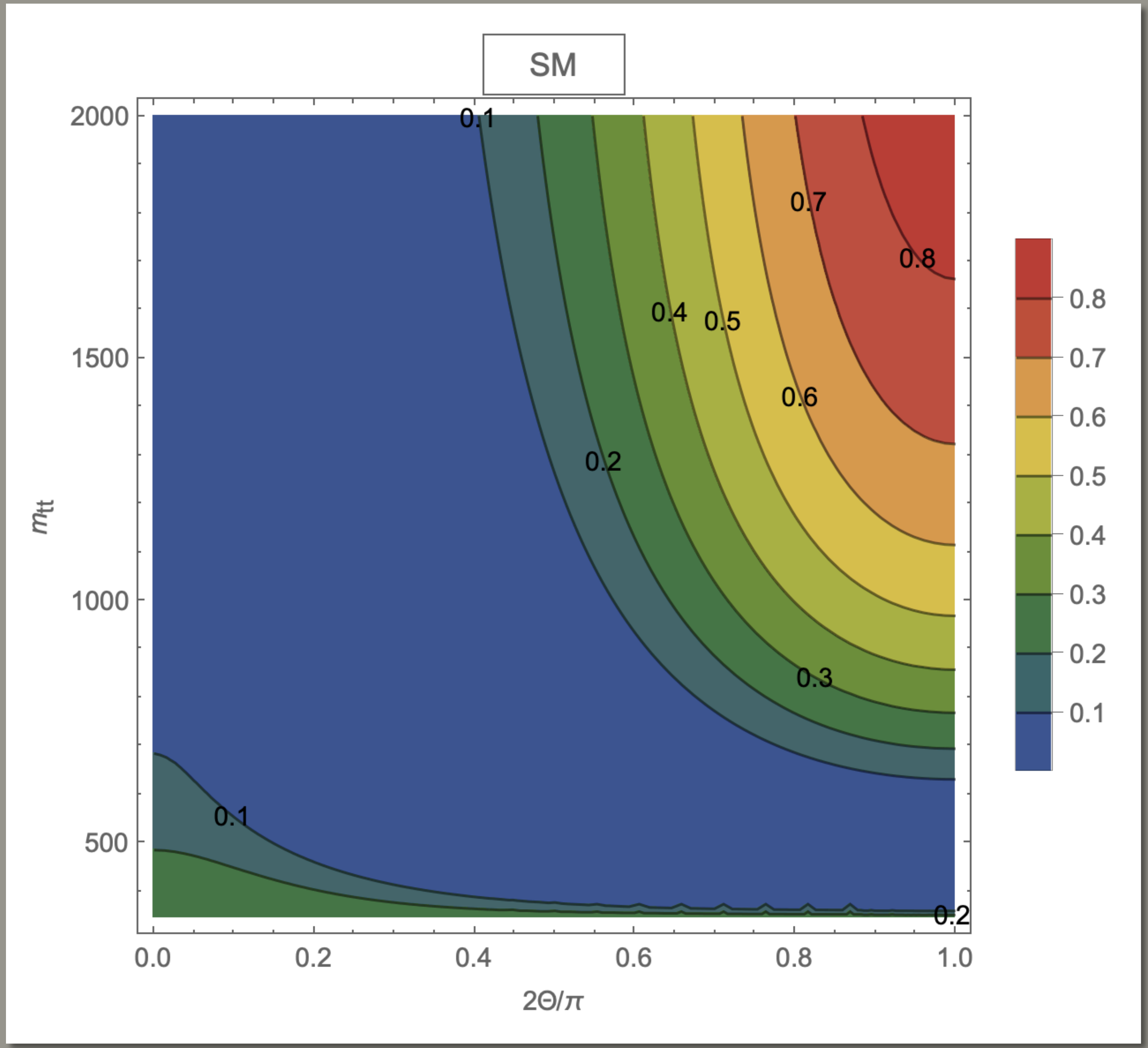
Aoude et al., 2203.05619
Severi, 2210.09330

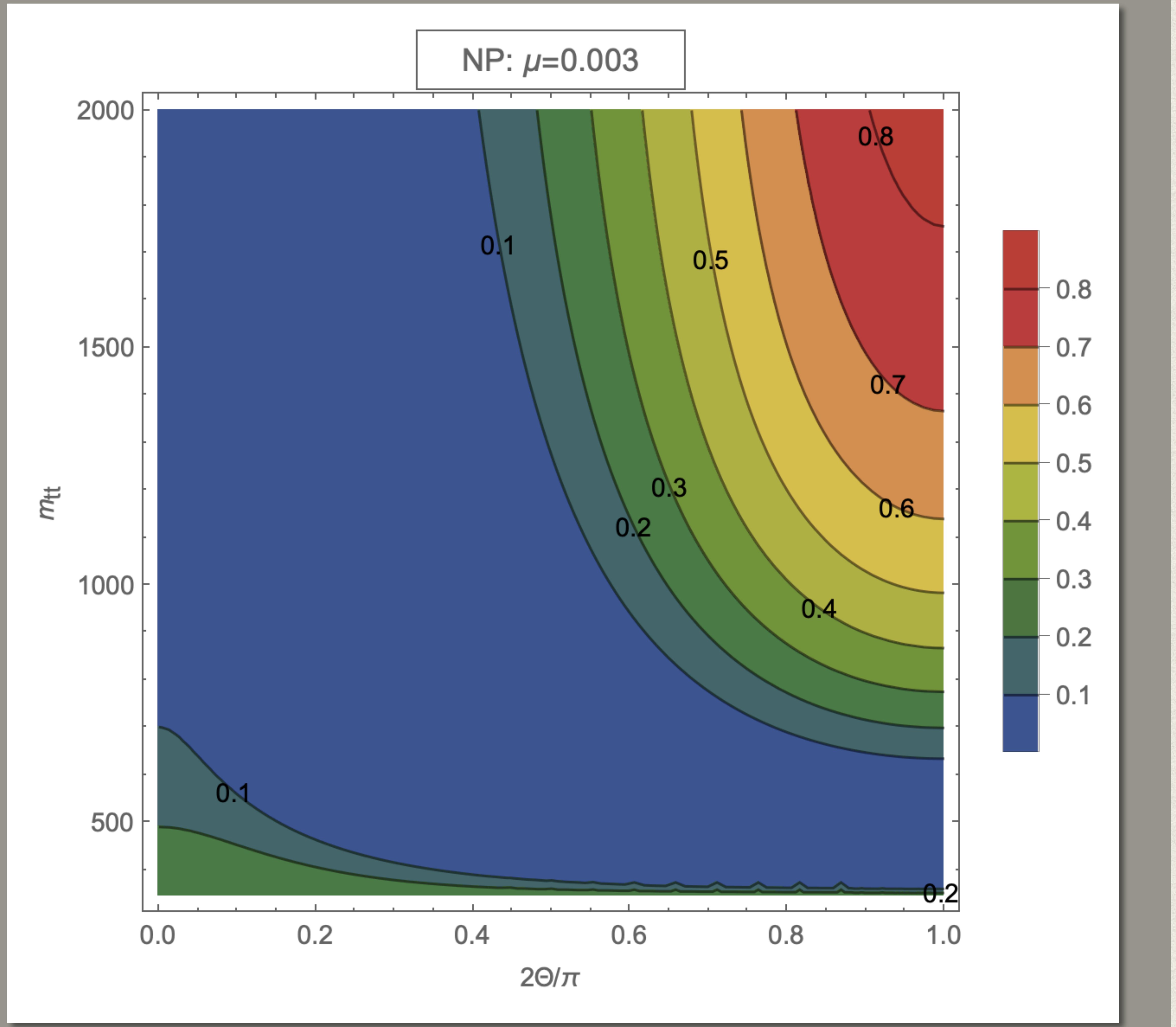
$$\rho = \frac{1}{4}\Big[1\otimes 1 + \sum_i A_i(\sigma_i\otimes 1) + \sum_j B_j(1\otimes\sigma_j) + \sum_{ij} C_{ij}(\sigma_i\otimes\sigma_j)\Big]$$

consistency conditions on density matrix

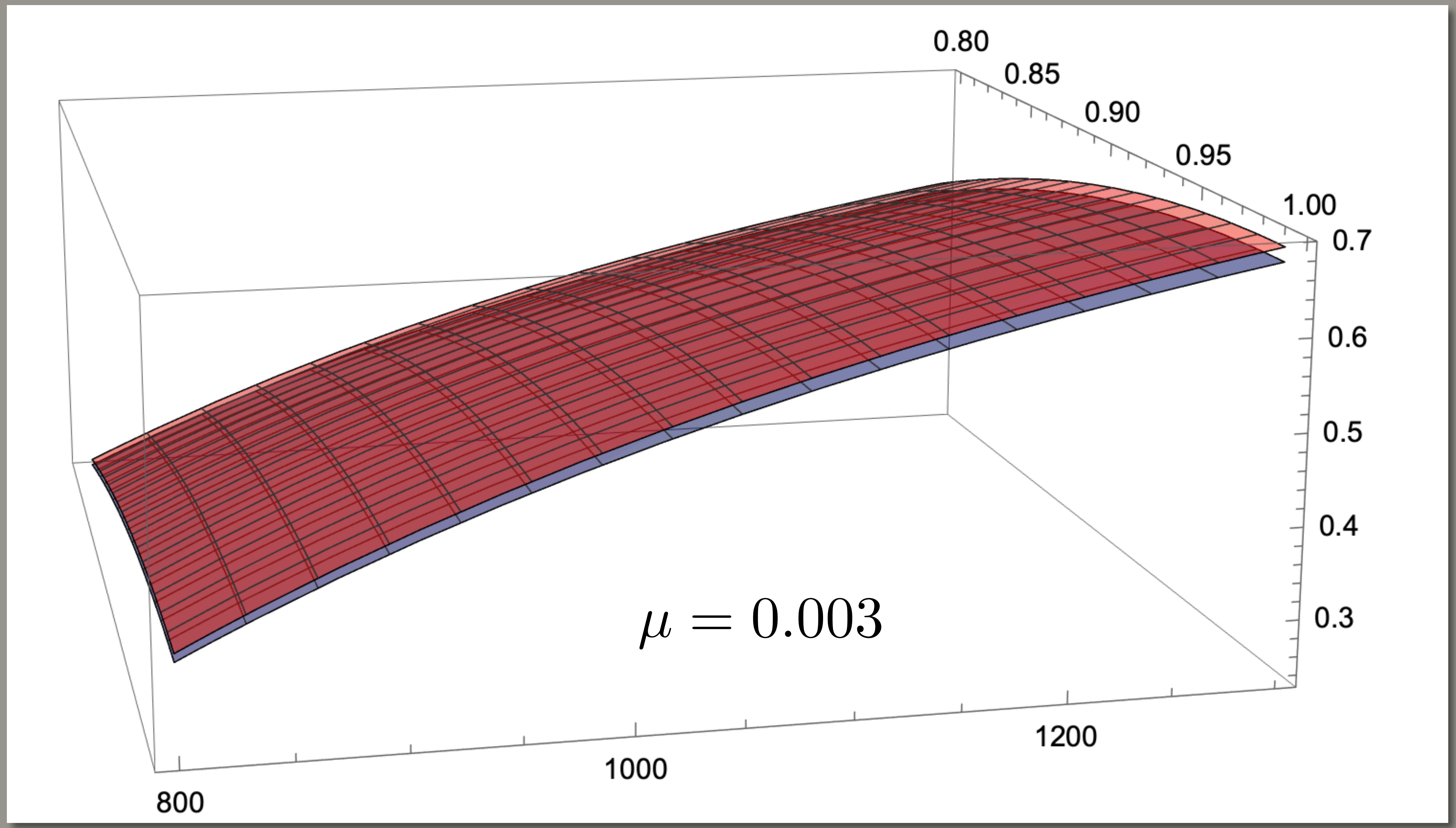
Hermitian, Tr =1, definite positive

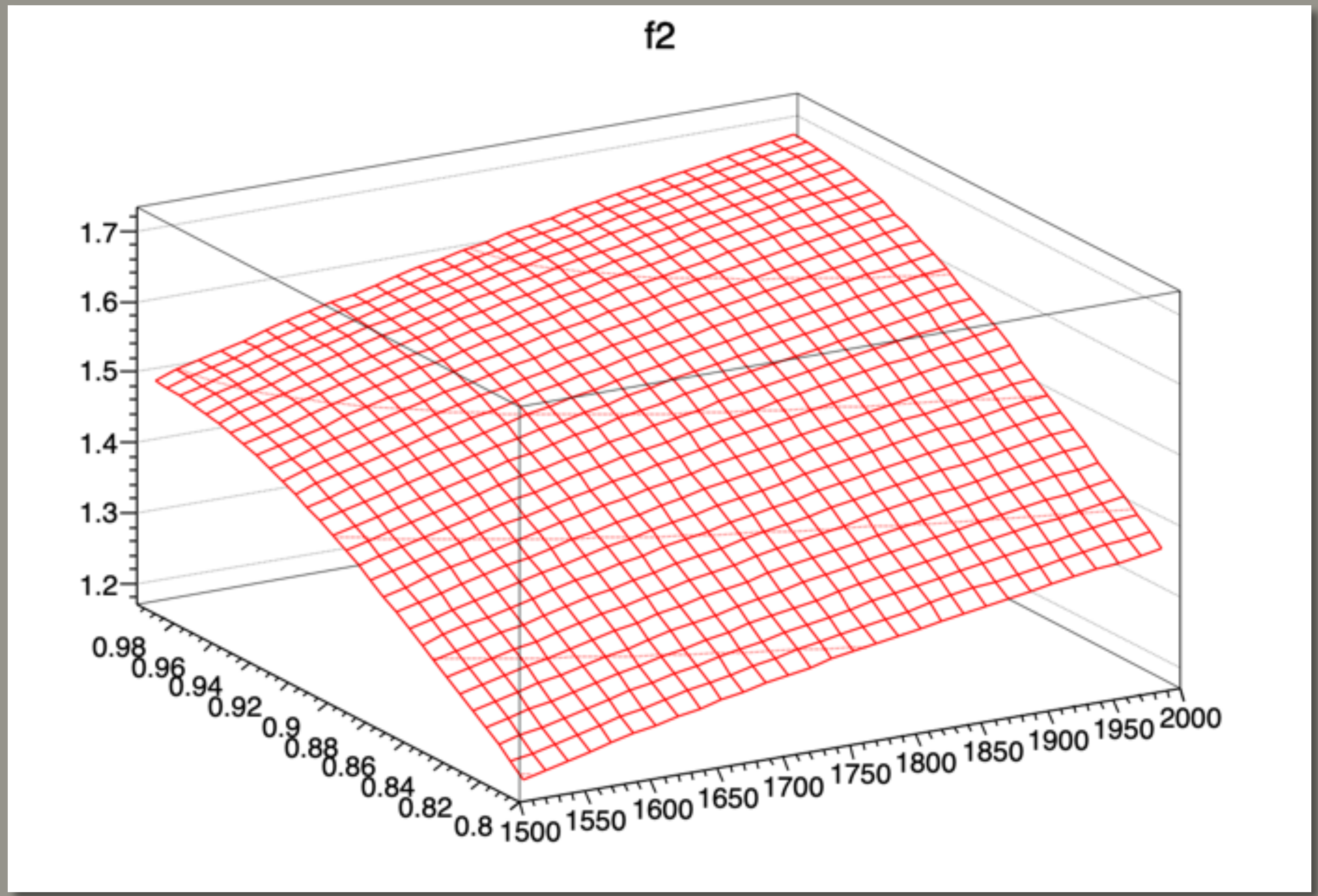
it should be tested in all numerical and analytical computations!

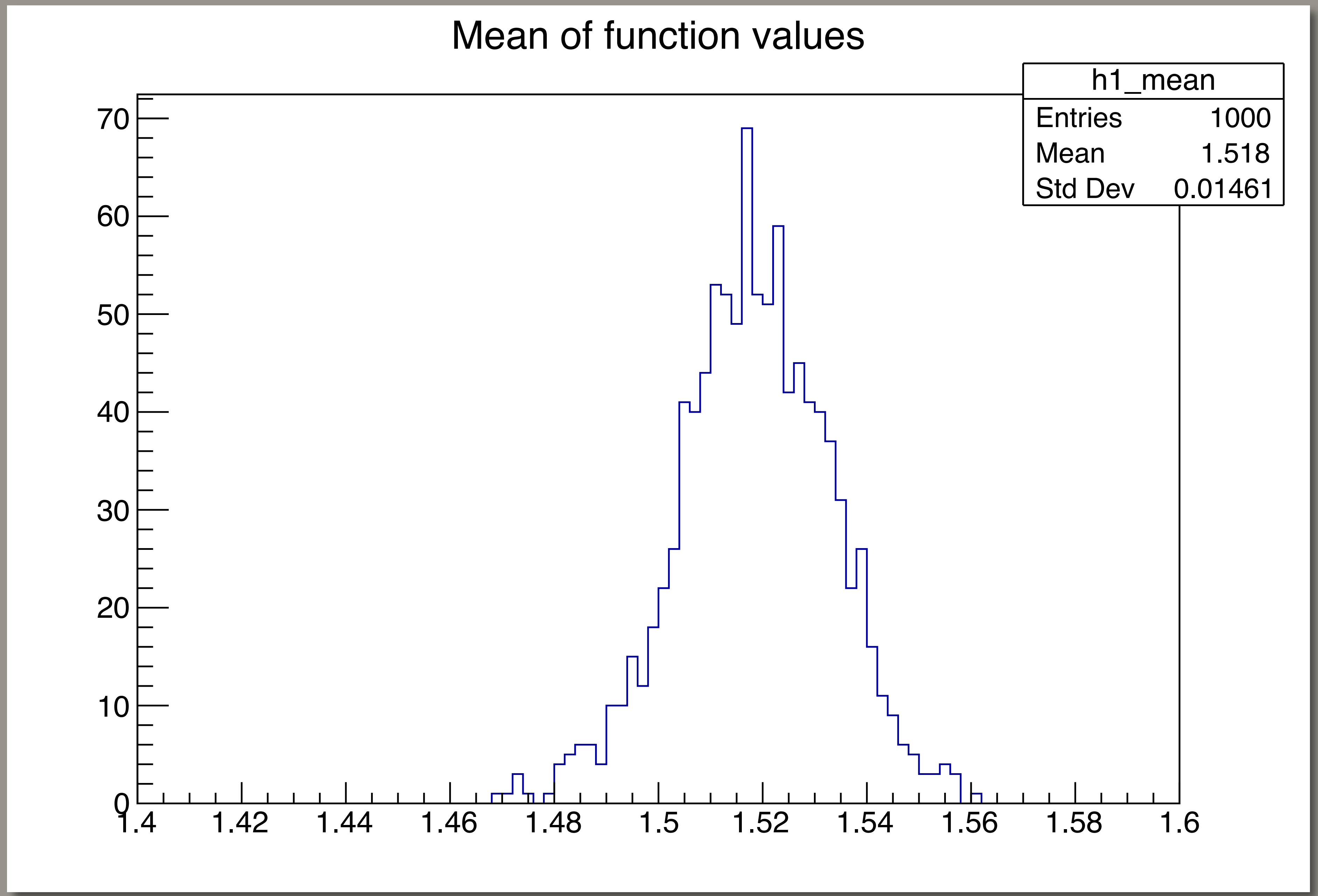

SM
2000
1500
1000
500
m_{tt}
0.0
0.2
0.4
0.6
0.8
1.0
2Θ/π
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.1
0.2
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

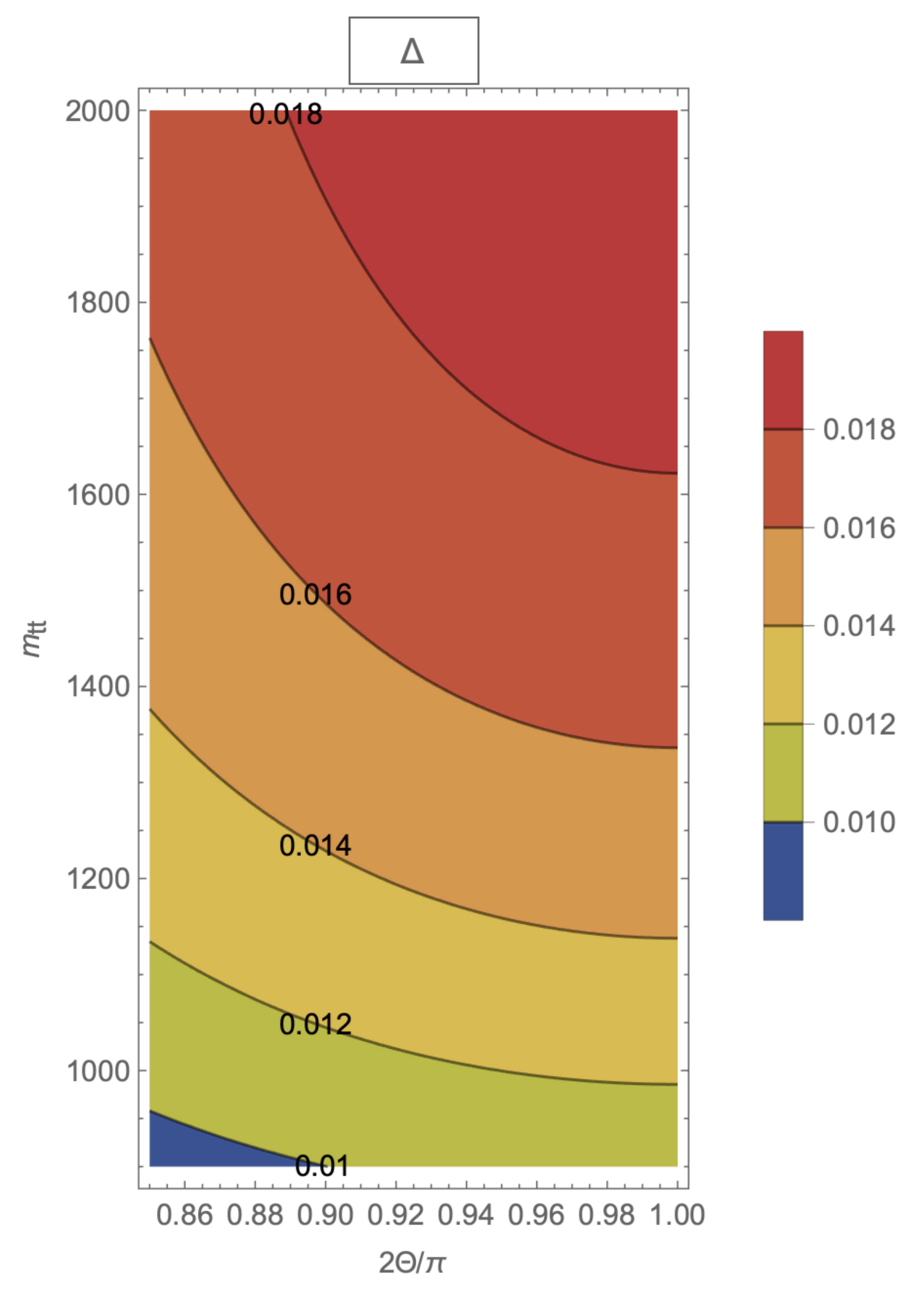

NP: μ=0.003
2000
1500
1000
500
m_{tt}
0.0
0.2
0.4
0.6
0.8
1.0
2Θ/π
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.1
0.2
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

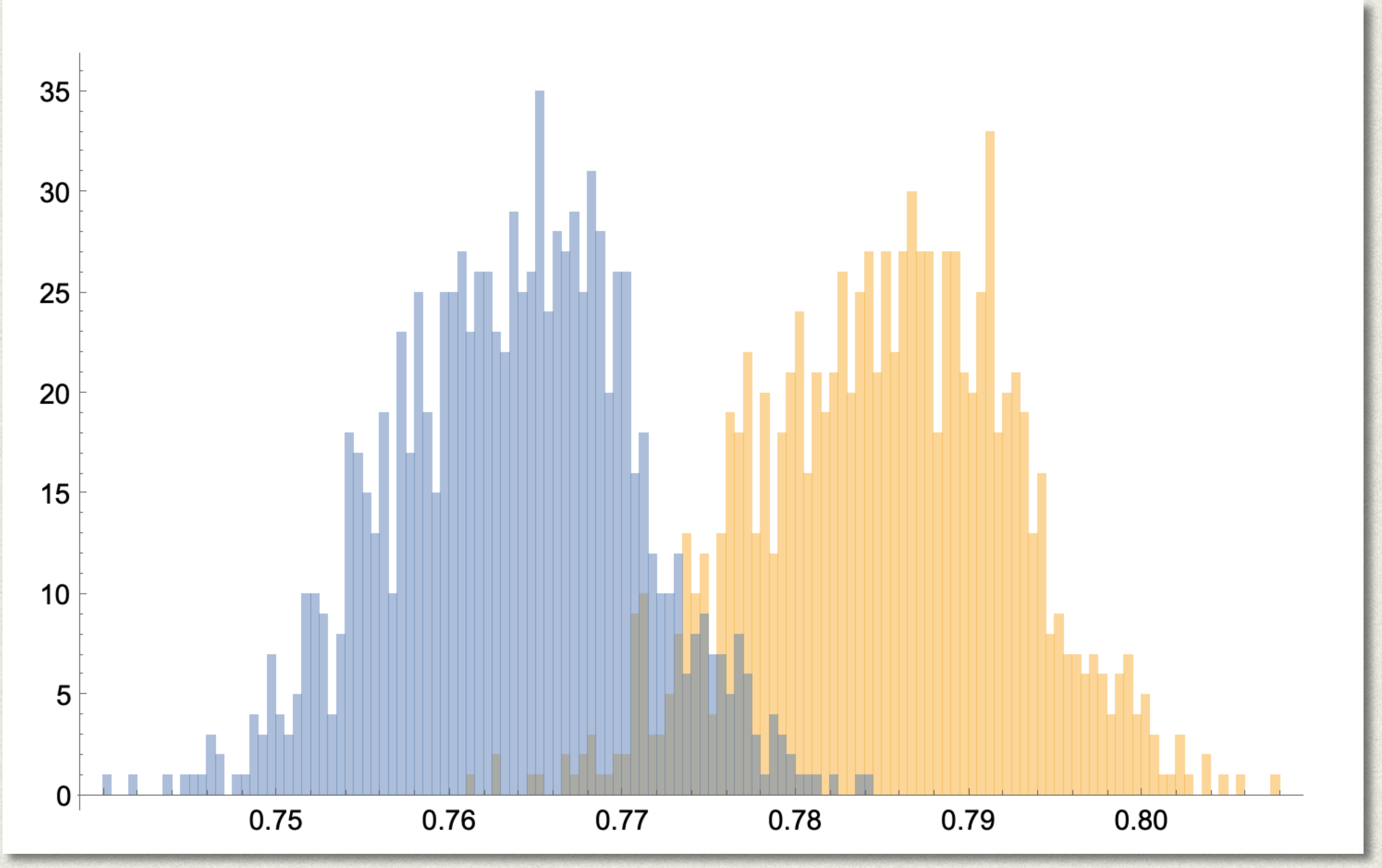
again, what is the uncertainty?




f2
1.7
1.6
1.5
1.4
1.3
1.2
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
1500
1550
1600
1650
1700
1750
1800
1850
1900
1950
2000

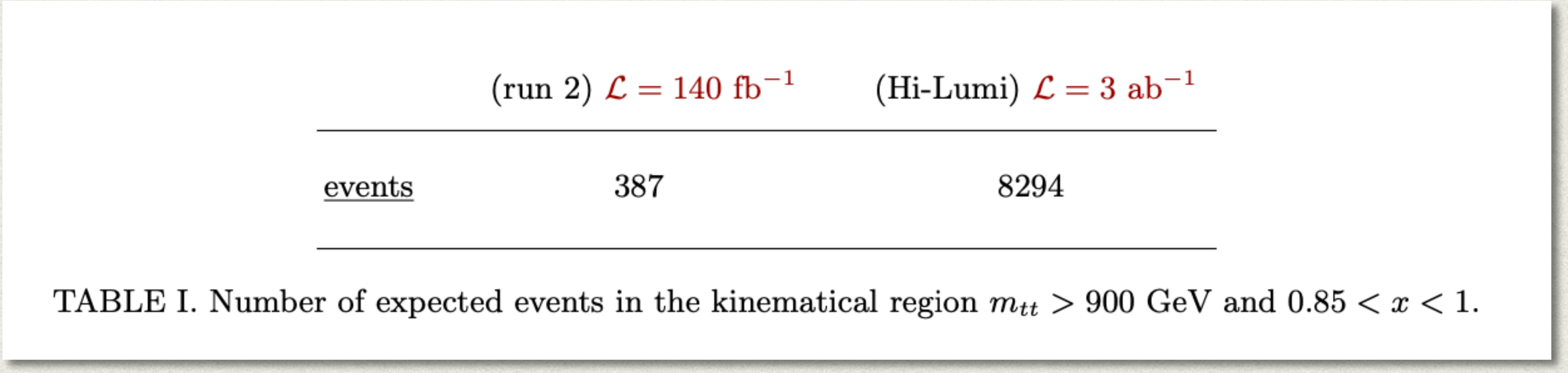

Mean of function values
h1_mean
Entries 1000
Mean 1.518
Std Dev 0.01461
70
60
50
40
30
20
10
0
1.4
1.42
1.44
1.46
1.48
1.5
1.52
1.54
1.56
1.58
1.6





$\mu = 0.003$

significance: 3



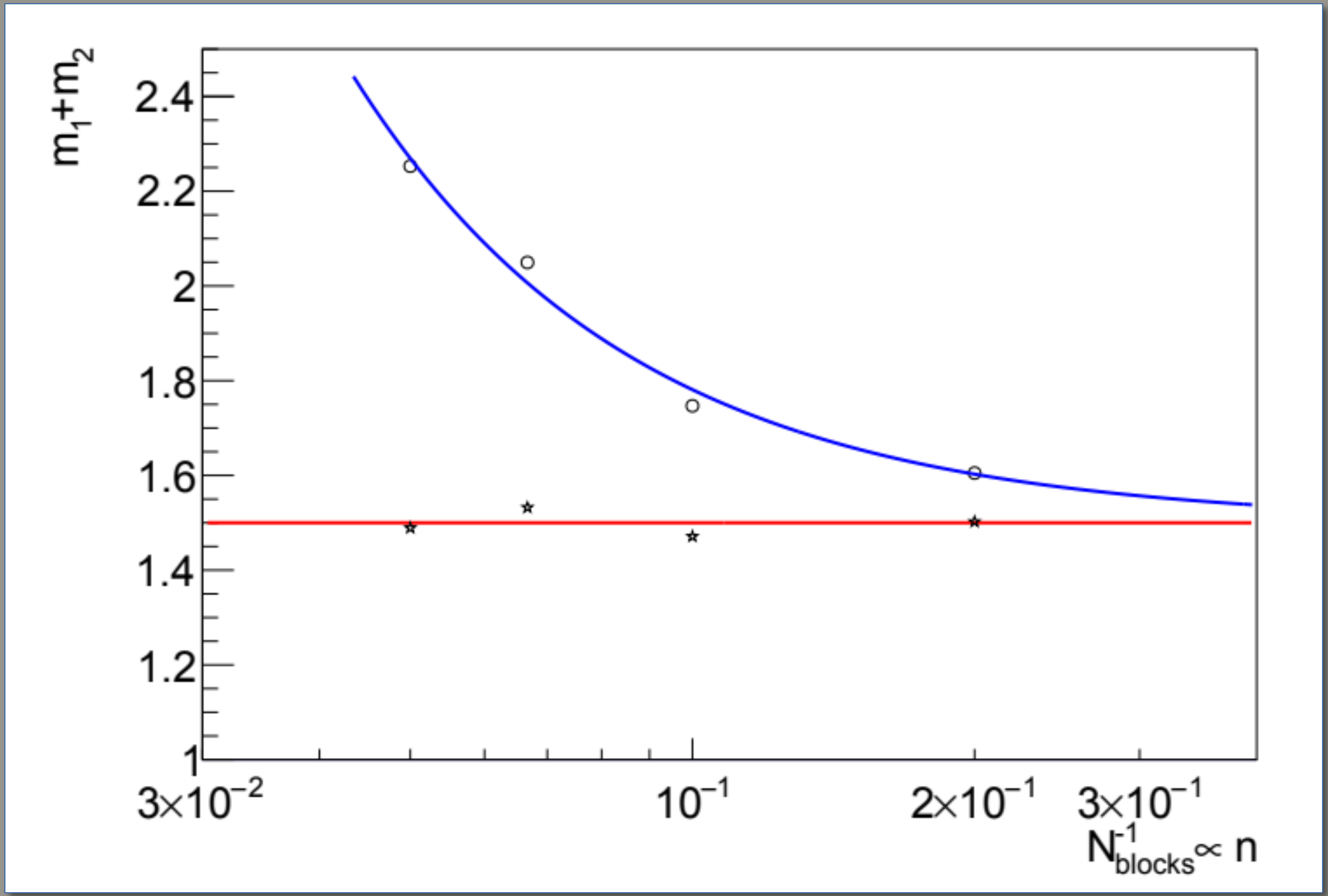
	(run 2) $\mathcal{L} = 140\ \text{fb}^{-1}$	(Hi-Lumi) $\mathcal{L} = 3\ \text{ab}^{-1}$
events	387	8294

TABLE I. Number of expected events in the kinematical region $m_{tt} > 900$ GeV and $0.85 < x < 1$.

previous limit: $\mu < 0.02$

in case someone asks

consistent estimator



$$m_{t\bar{t}} \simeq 1.1\mathrm{TeV}\ 2\Theta/\pi \simeq 0.95$$

MadGraph5 (NNPDF23)
DELPHES (fast simulation
ATLAS detector)

Event generation

exactly two opposite sign leptons (e,mu) of different flavor

- at least 2 anti-k_t jets with R=0.4
- at least 1 b-tagged jet
- $p_T > 25$ GeV $|\eta| < 2.5$ jets
- $p_T > 20$ GeV $|\eta| < 2.47$ leptons
- neutrino weighting technique (top quark momenta)

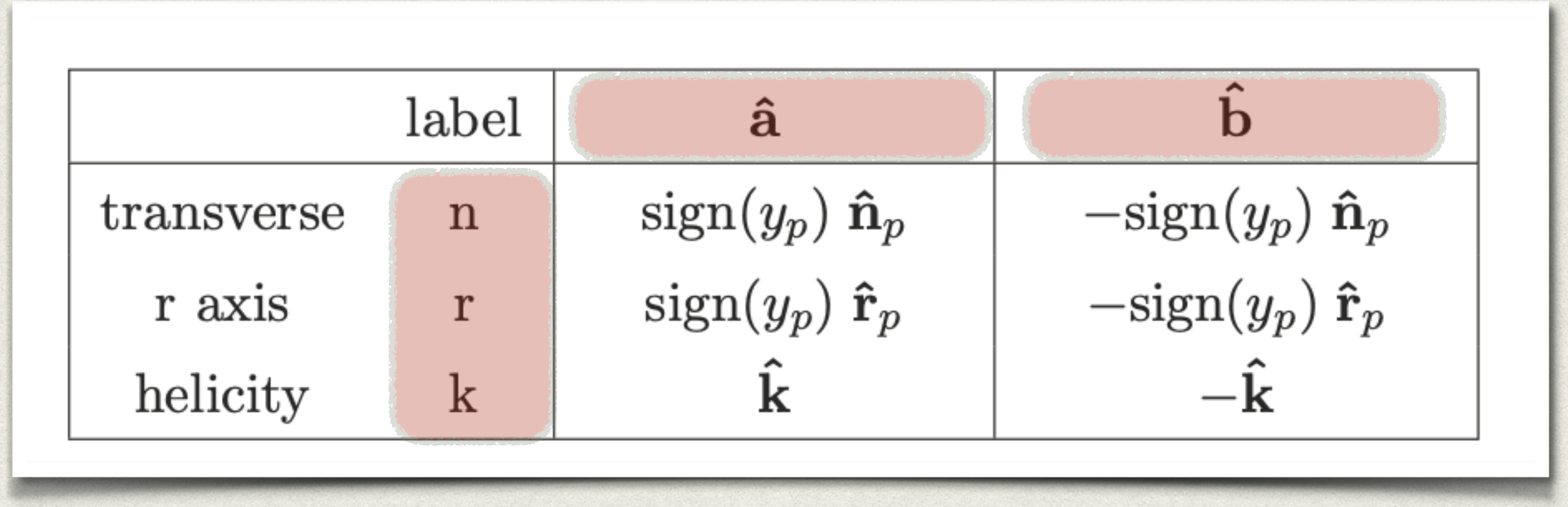
Implementing at the LHC

W. Bernreuther, D. Heisler and Z. G. Si, JHEP 12, 026 (2015)
Y. Afik and J. R. M. de Nova, Eur.Phys.J.Plus 136 (2021) 9, 907

$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

$$\xi_{ab} = \cos\theta_+^a \cos\theta_-^b$$

3 x 3 matrix



	label	$\hat{\mathbf{a}}$	$\hat{\mathbf{b}}$
transverse	n	$\text{sign}(y_p)\ \hat{\mathbf{n}}_p$	$-\text{sign}(y_p)\ \hat{\mathbf{n}}_p$
r axis	r	$\text{sign}(y_p)\ \hat{\mathbf{r}}_p$	$-\text{sign}(y_p)\ \hat{\mathbf{r}}_p$
helicity	k	$\hat{\mathbf{k}}$	$-\hat{\mathbf{k}}$

$$C_{ab}\left[\sigma(m_{t\bar{t}}, \cos\Theta)\right] = -9\ \frac{1}{\sigma}\int d\xi_{ab}\frac{d\sigma}{d\xi_{ab}}\xi_{ab}$$

diagonalization for each value
of invariant mass and scattering angle

$$m_1 + m_2 > 1$$