Entanglement in SMEFT: top pair production

arXiv:2203.05619

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In collaboration with:
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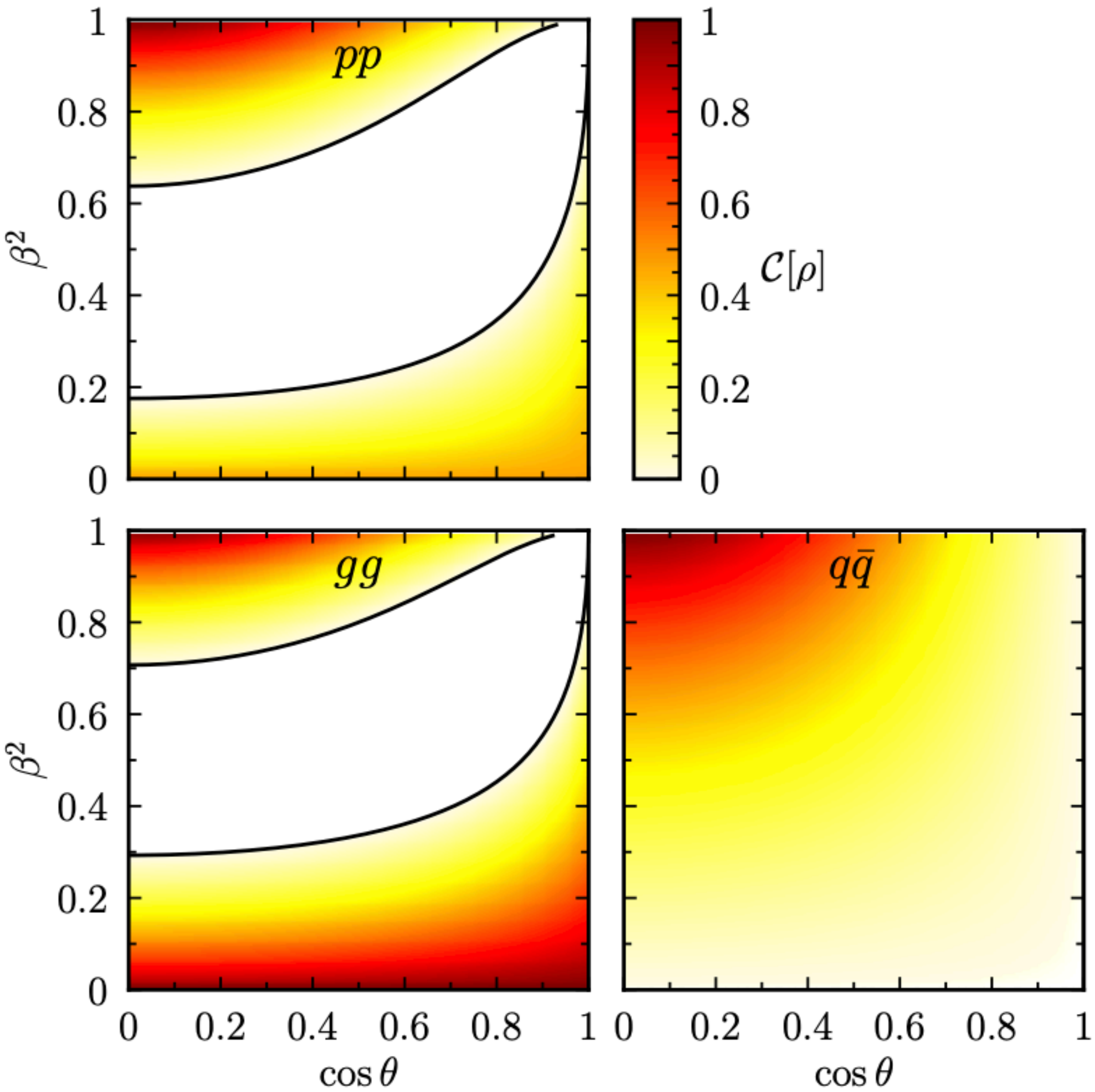
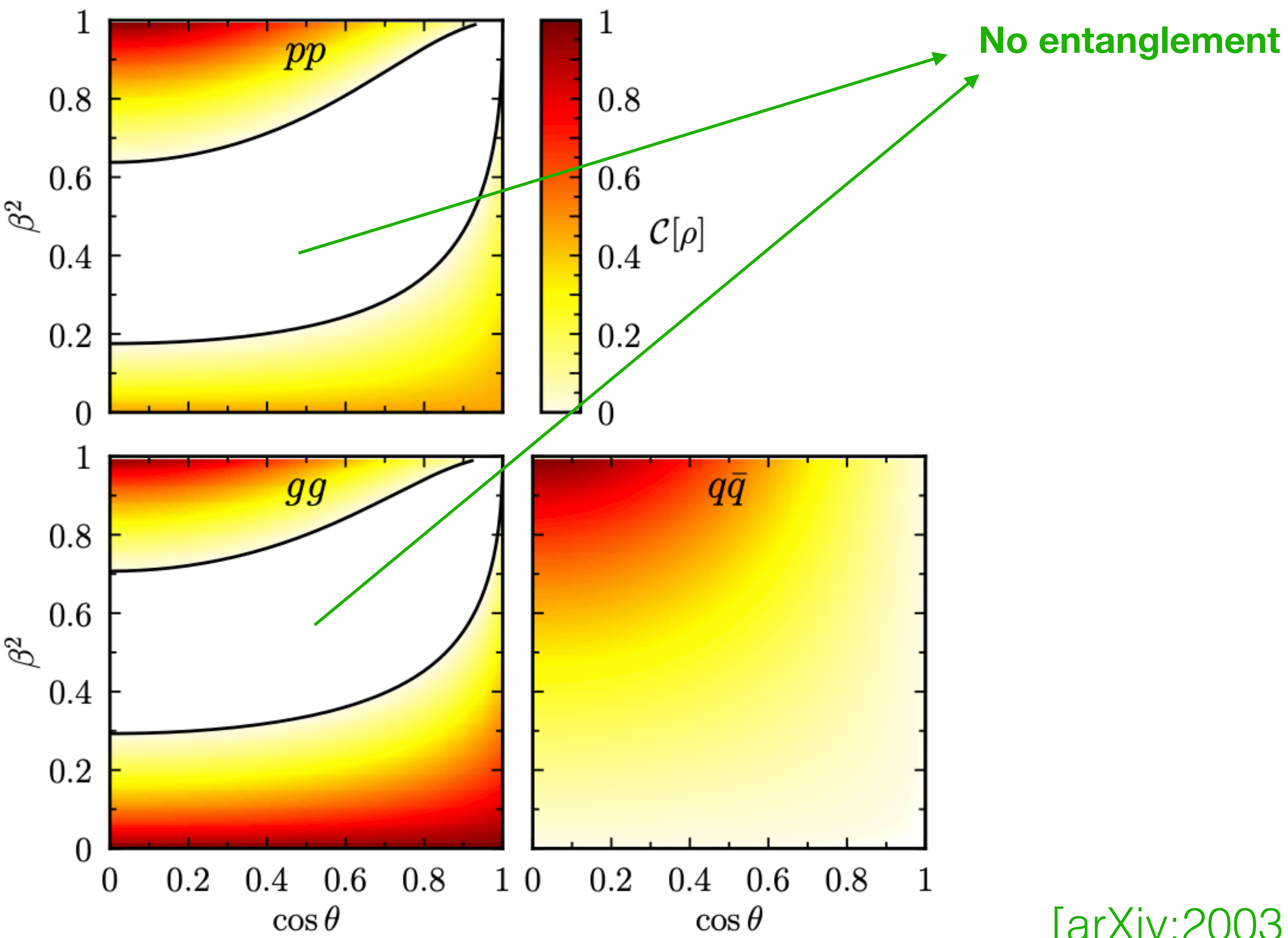
PBSP



[arXiv:2003.02280]



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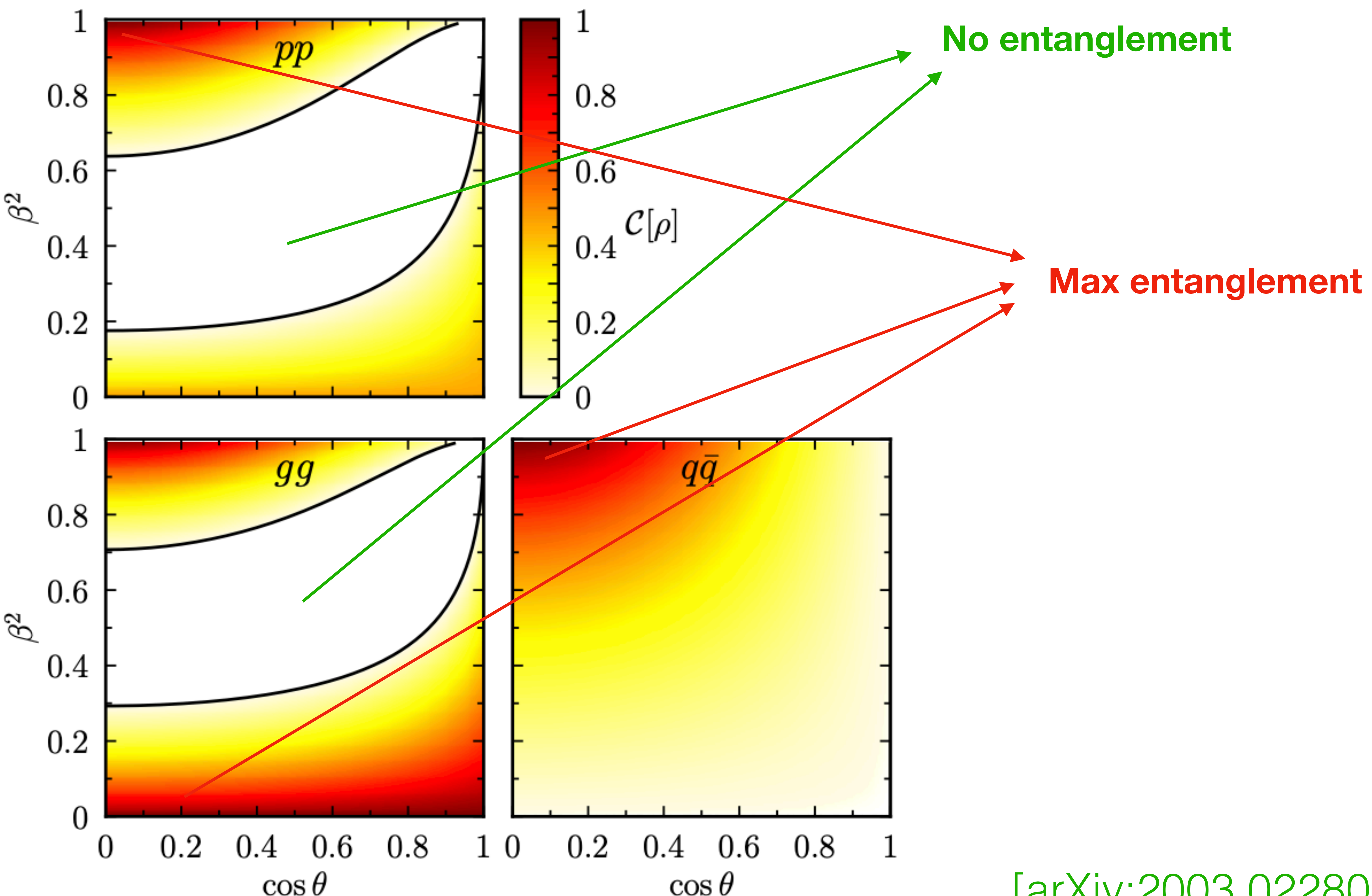

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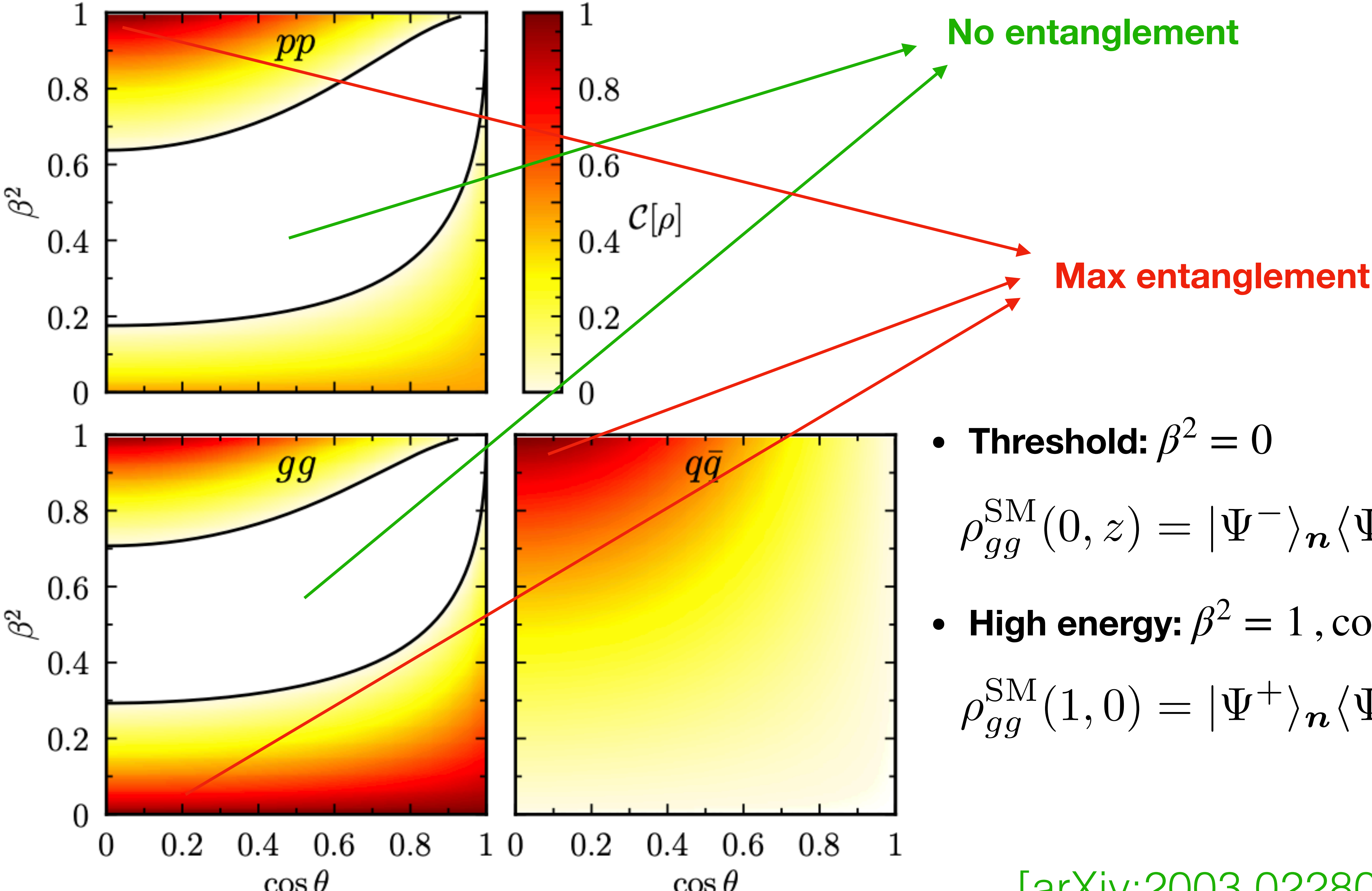

- **Threshold:** $\beta^2 = 0$

$$\rho_{gg}^{\rm SM}(0,z) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}}$$

- **High energy:** $\beta^2 = 1\,, \cos\theta = 0$

$$\rho_{gg}^{\rm SM}(1,0) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}}$$

[arXiv:2003.02280]

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$$\mathcal{O}_G = g_s f^{ABC} G_\nu^{A,\mu} G_\rho^{B,\nu} G_\mu^{C,\rho}$$

$$\mathcal{O}_{\varphi G} = \left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) G_A^{\mu\nu} G_{\mu\nu}^A$$

$$\mathcal{O}_{tG} = g_s(\bar{Q}\sigma^{\mu\nu}T^A\, t)\tilde{\varphi}G_{\mu\nu}^A + \text{h.c.}$$

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4-Fermion operators

$$\mathcal{O}_{Qq}^{(8,1)}, \mathcal{O}_{Qq}^{(8,3)}, \mathcal{O}_{tu}^{(8)}, \mathcal{O}_{td}^{(8)}, \mathcal{O}_{Qu}^{(8)}, \mathcal{O}_{Qd}^{(8)}, \mathcal{O}_{tq}^{(8)}$$

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What are the effects of NP on the entanglement regions?

Is NP affecting the quantum state?

$$R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2} \mathcal{M}_{\alpha_1\beta_1}$$

$$\mathcal{M}_{\alpha\beta} = \mathcal{M}^{\mathrm{SM}}_{\alpha\beta} + \frac{1}{\Lambda^2}\mathcal{M}^{(\mathrm{d6})}_{\alpha\beta} \quad \Longrightarrow \quad \rho = \frac{R^{\mathrm{SM}} + R^{\mathrm{EFT}}}{\mathrm{tr}(R^{\mathrm{SM}}) + \mathrm{tr}(R^{\mathrm{EFT}})}$$

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At $\mathcal{O}(1/\Lambda^2)$

$$\tilde{A}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{g_s^2 v m_t(9\beta^2 z^2+7)}{12\sqrt{2}}c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2-(1-\beta^2)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right],$$

$$\tilde{C}^{gg,(1)}_{nn} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{-7g_s^2 v m_t}{12\sqrt{2}}c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2-(1-\beta^2)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right],$$

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$$\Delta = -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$

$\Delta_1 \equiv \Delta - \Delta_0$ Δ computed up to $\mathcal{O}(1/\Lambda^2)$

$\Delta_2 \equiv \Delta - \Delta_1 - \Delta_0$ Δ computed up to $\mathcal{O}(1/\Lambda^4)$

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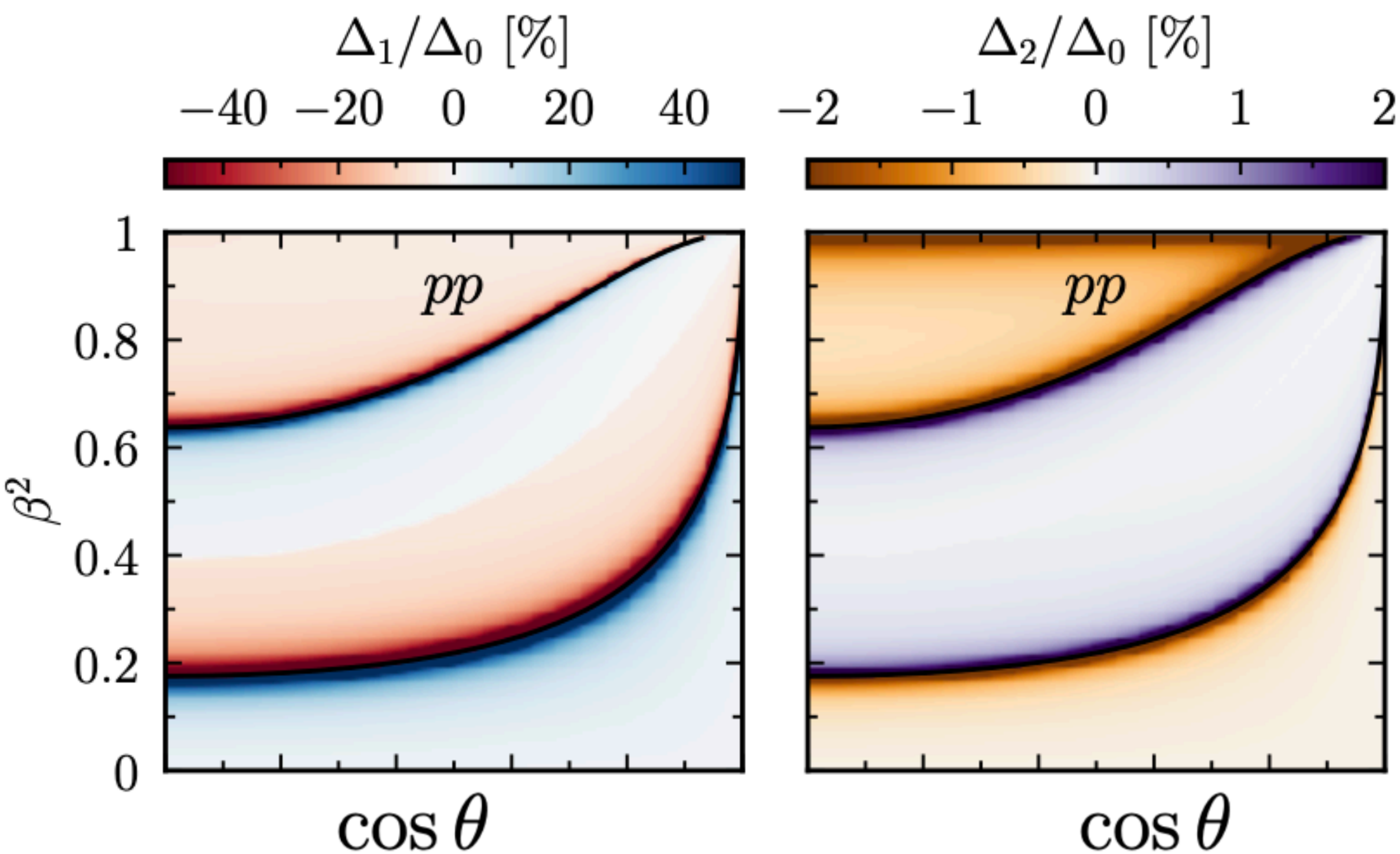

$$c_{tG}/\Lambda^2 = 0.1\,\mathrm{TeV}^{-2}$$

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega \, R(\hat{s}, \boldsymbol{k}) \,,$$

→

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$$C[\rho] = \max(\delta/2, 0)$$

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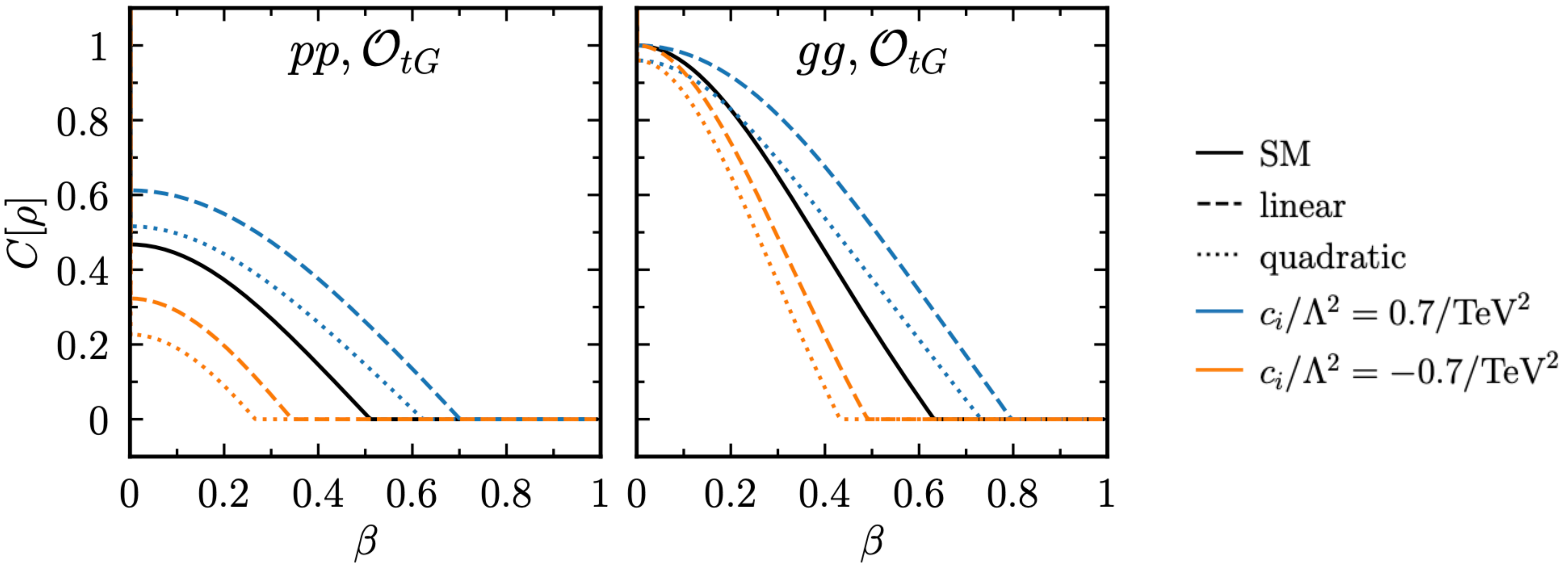

gg-induced

$$\rho_{gg}^{\mathrm{EFT}}(0,z) = p_{gg}|\Psi^+\rangle_{\boldsymbol{p}}\langle\Psi^+|_{\boldsymbol{p}} + (1-p_{gg})|\Psi^-\rangle_{\boldsymbol{p}}\langle\Psi^-|_{\boldsymbol{p}}$$

$$p_{gg} = \frac{72}{7\Lambda^4} m_t^2 (3\sqrt{2} m_t\, c_G + v\, c_{tG})^2$$

Only quadratic effects!

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$$p_{q\bar{q}} = \frac{1}{2} - 4\frac{c_{VA}^{(8),u}}{\Lambda^2} + \frac{8m_t^4}{\Lambda^4}\left(\frac{v\sqrt{2}}{m_t} c_{VA}^{(8),u} c_{tG} - 9c_{VA}^{(1),u} c_{VV}^{(1),u} + 2c_{VA}^{(8),u} c_{VV}^{(8),u}\right)$$

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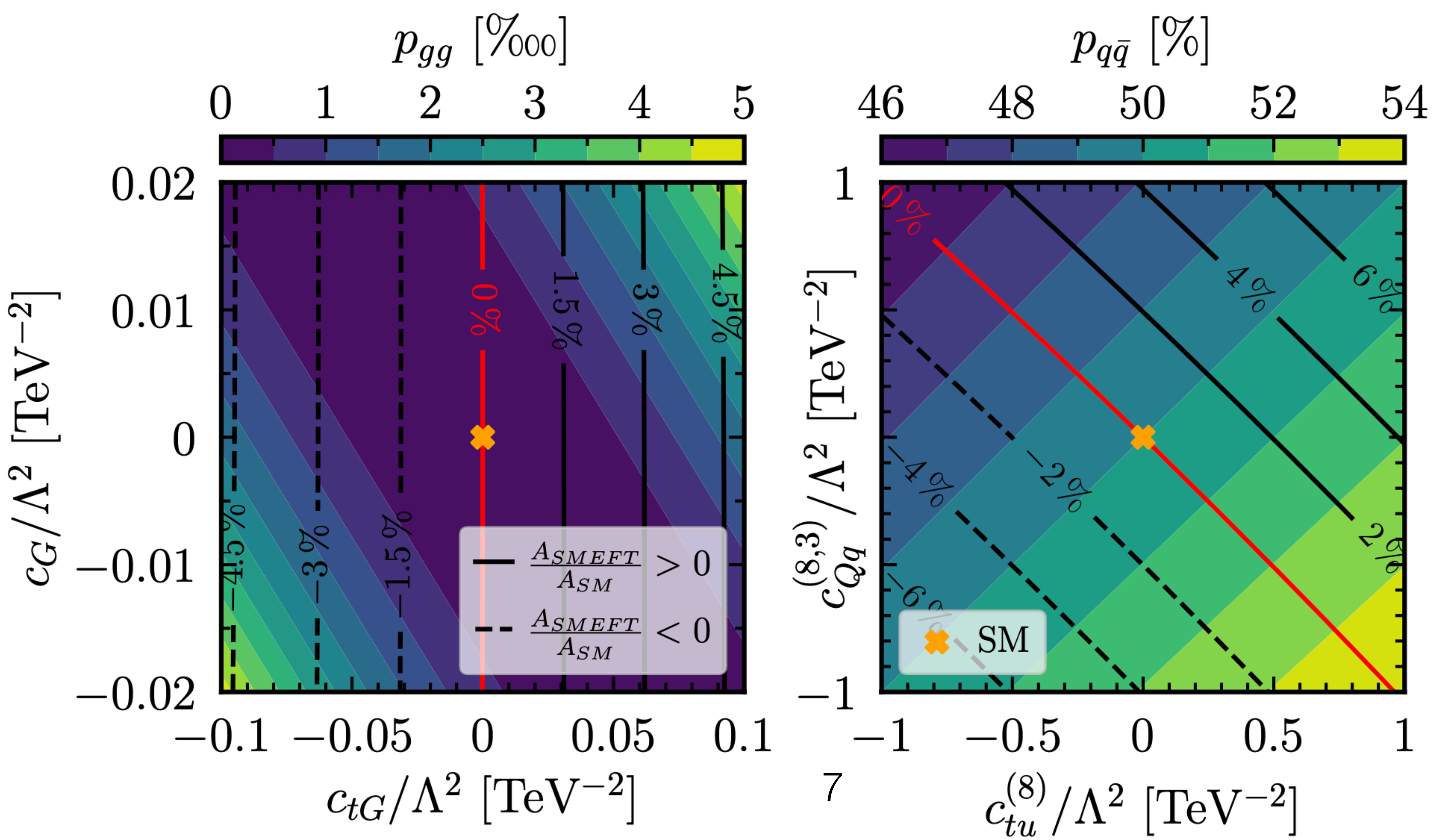
$$p_{q\bar q} = \frac{1}{2} - 4\frac{c_{VA}^{(8),u}}{\Lambda^2} + \frac{8m_t^4}{\Lambda^4}\left(\frac{v\sqrt{2}}{m_t} c_{VA}^{(8),u} c_{tG} - 9 c_{VA}^{(1),u} c_{VV}^{(1),u} + 2 c_{VA}^{(8),u} c_{VV}^{(8),u}\right)$$



- Possibility to exploit quantum observables as entanglement proposed.
- Measurement of top pair entanglement would be highest energy evidence ever.
- In the SM, top pairs are maximally entangled at threshold and very high energy.
- SMEFT effects induce presence of different quantum states, decreasing entanglement at threshold and modifying the overall pattern.
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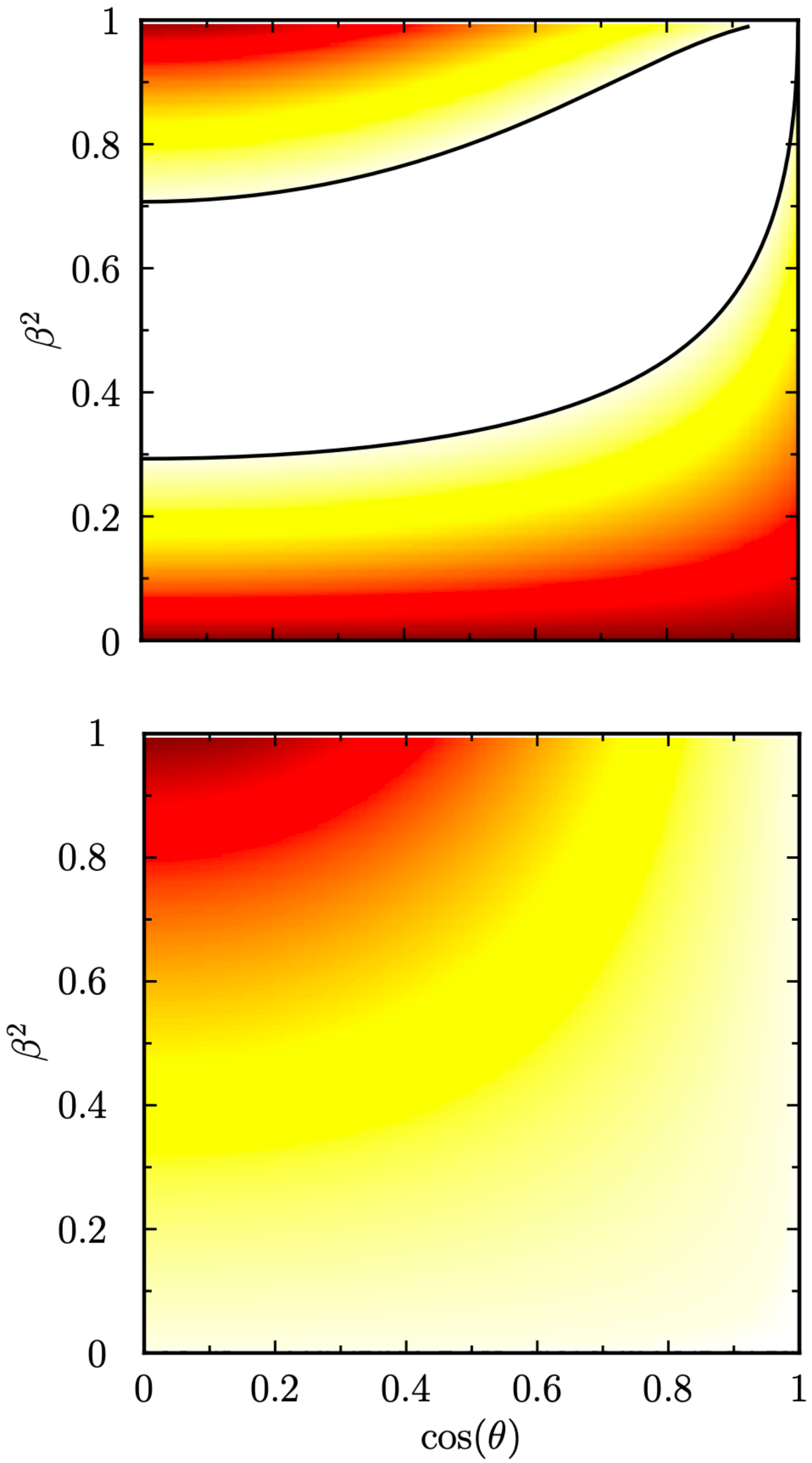
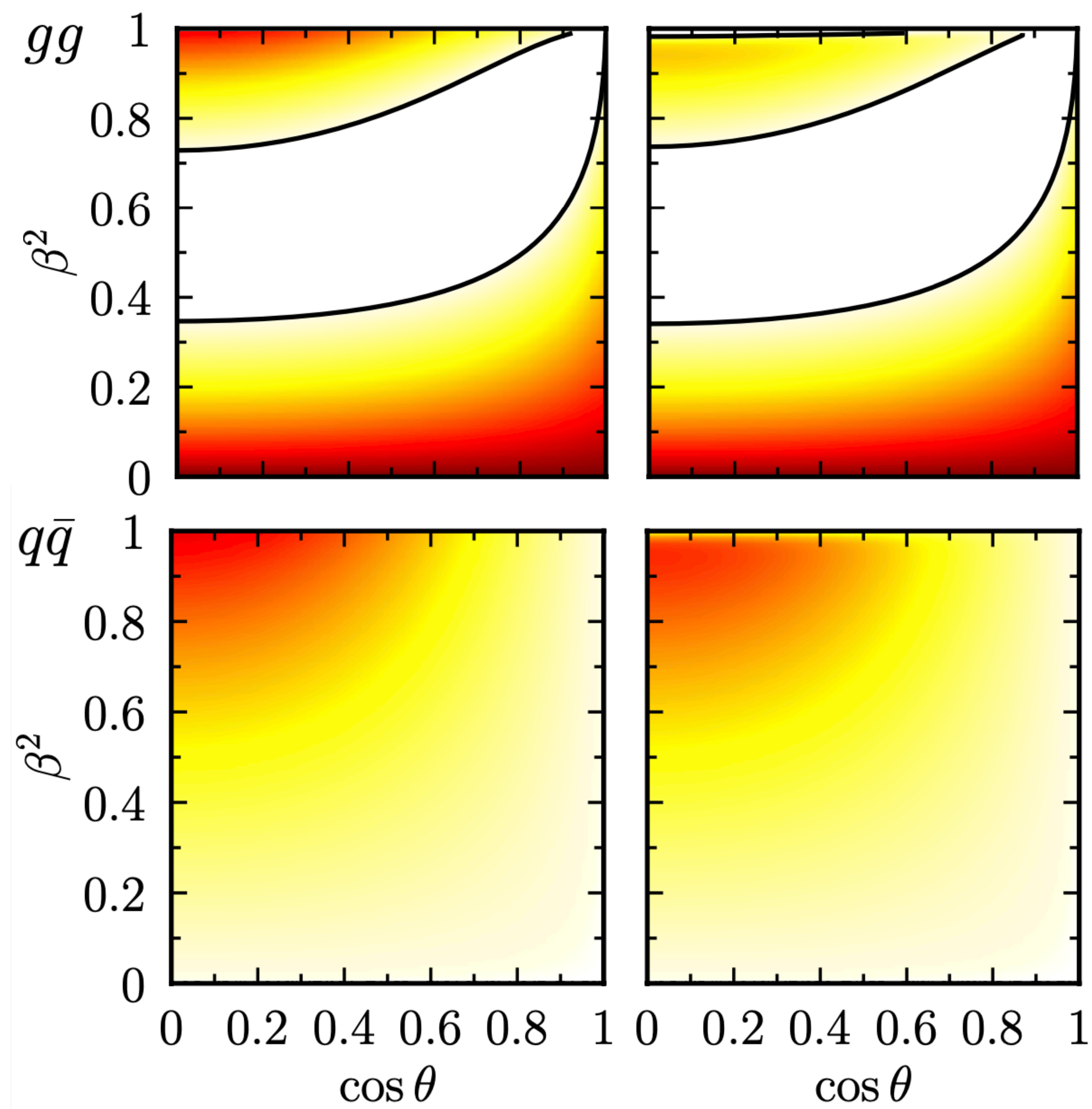
Backup

$$\mathcal{O}_{tG} = g_s(\bar{Q}\sigma^{\mu\nu}T^A\, t)\tilde{\varphi}G^A_{\mu\nu} + \text{h.c.}$$

SM



Linear Quad



Quantum Information → Unveil the inner behaviour of quantum mechanics.

Entanglement is a pure quantum phenomenon.
Measurement at high energies undertaken in recent years.

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Top pairs ideal probe: spin correlations preserved after decay

[arXiv:2003.02280]
[arXiv:2110.10112]
[arXiv:2102.11883]
[arXiv:2203.05582]
[arXiv:2205.00542]

The fundamental object is the spin correlation matrix

$$R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2} \mathcal{M}_{\alpha_1\beta_1}$$

At LO in QCD

$$I = gg, q\bar{q}$$

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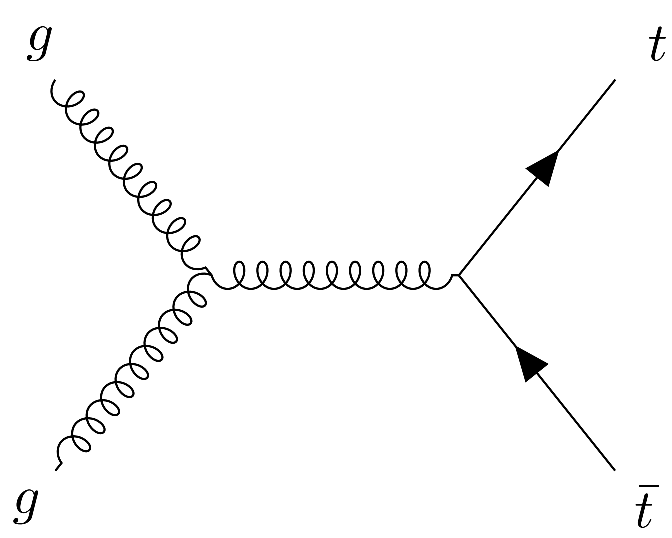
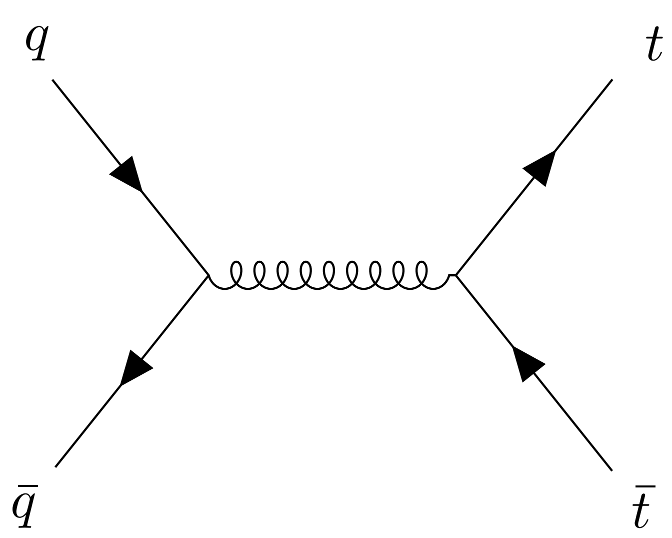


We collide protons



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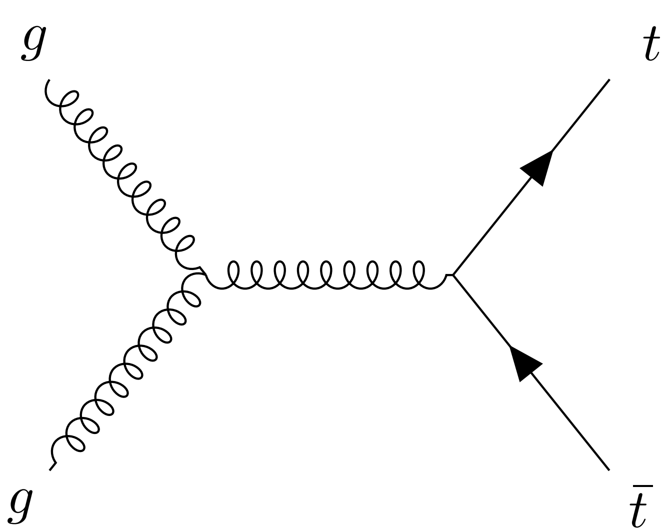
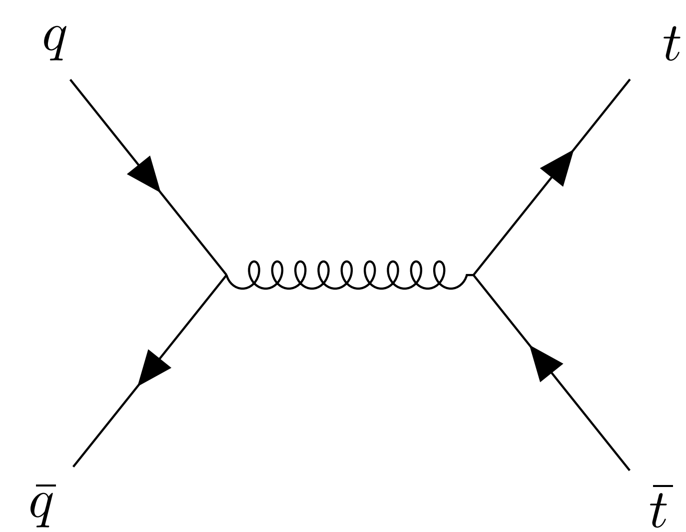


We collide protons

$$R(\hat{s},\boldsymbol{k}) = \sum_I L^I(\hat{s}) R^I(\hat{s},\boldsymbol{k})$$



Full correlation matrix is mixed state, weighted by parton luminosity

The R matrix can be decomposed in the spin space

$$R = \tilde{A}\,\mathbb{1}_2 \otimes \mathbb{1}_2 + \tilde{B}_i^+ \sigma^i \otimes \mathbb{1}_2 + \tilde{B}_i^- \mathbb{1}_2 \otimes \sigma^i + \tilde{C}_{ij}\, \sigma^i \otimes \sigma^j$$

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Cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\mathrm{d}\hat{s}} = \frac{\alpha_s^2\beta}{\hat{s}^2}\tilde{A}\left(\hat{s}, \boldsymbol{k}\right)$$

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Degree of top and anti-top polarisation (zero if interactions P-invariant)

Spin correlations

If normalised, we define the density matrix of the system

$$\rho = \frac{\mathbb{1}_2 \otimes \mathbb{1}_2 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_i^- \mathbb{1}_2 \otimes \sigma^i + C_{ij}\, \sigma^i \otimes \sigma^j}{4}.$$

Given a bipartite system, with Hilbert space $\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$

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Maximally entangled states

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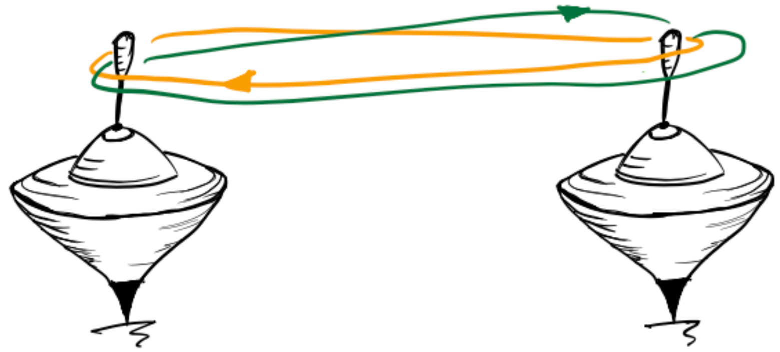

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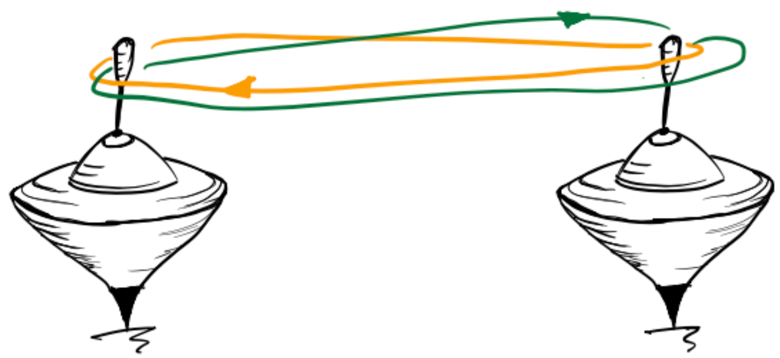

In the case of a statistical ensemble (mixed state)

$$\rho = \sum_k p_k \rho_k$$

entangled if $\rho_k \neq \rho_1 \otimes \rho_2$

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Operative definition of entanglement: **Peres-Horodecki criterion**

$$\Delta = -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$ **entangled**

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$$\Delta = -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$ **entangled**

We can then define the concurrence

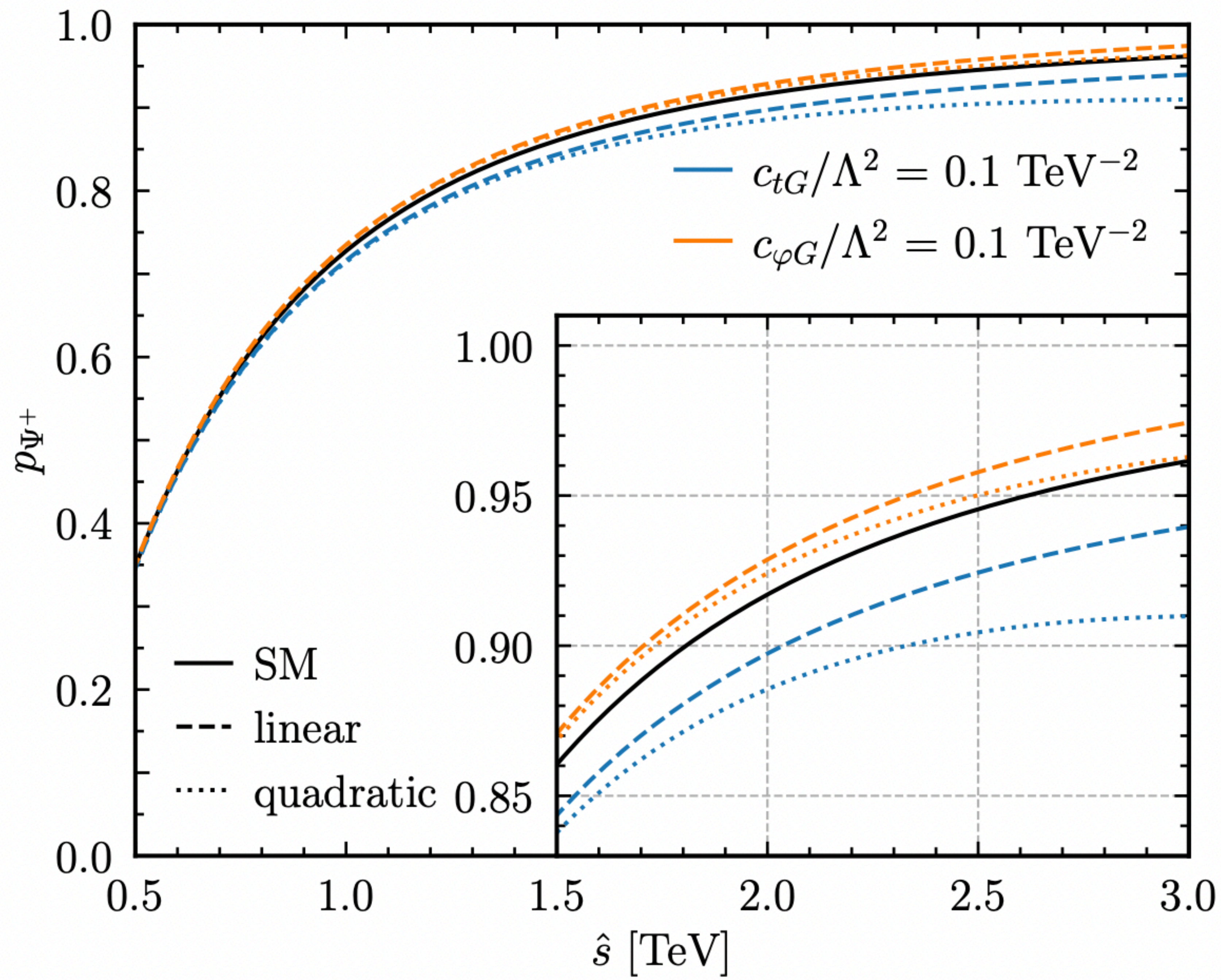
$$C[\rho] = \max(\Delta/2, 0)$$

$$C[\rho] = 1$$

Max entanglement

$$p_{\Psi^+} = \langle \Psi^+|_{\boldsymbol{n}}\, \rho\, |\Psi^+\rangle_{\boldsymbol{n}}$$

Probability triplet state



LO coefficients - gg channel

$$\tilde{A}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{g_s^2 v m_t(9\beta^2 z^2+7)}{12\sqrt{2}}c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2-(1-\beta^2)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right],$$

$$\tilde{C}_{nn}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{-7g_s^2 v m_t}{12\sqrt{2}}c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2-(1-\beta^2)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right],$$

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$$\tilde{C}_{rr}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{g_s^2 v m_t\left(-9\beta^4\left(z-z^3\right)^2-7\beta^2\left(z^4-z^2+1\right)+7\right)}{12\sqrt{2}\left(\beta^2 z^2-1\right)}c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2-\left(1-\beta^2\right)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right],$$

$$\tilde{C}_{rk}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{g_s^2 v m_t \beta^2 z\left(1-z^2\right)\left(9\beta^2+\left(\beta^2-2\right)z^2\left(9\beta^2\left(z^2-1\right)+7\right)-2\right)}{24\sqrt{2}\sqrt{\left(\beta^2-1\right)\left(z^2-1\right)}\left(\beta^2 z^2-1\right)}c_{tG} + \frac{9g_s^2\beta^2 m_t^2 z}{8}\sqrt{\frac{1-z^2}{1-\beta^2}}c_G\right].$$

LO coefficients - qq channel

$$\tilde{A}^{q\bar{q},(1)} = \frac{4g_s^2 m_t^2}{9\Lambda^2(1-\beta^2)}\left[\sqrt{2}g_s^2\frac{v}{m_t}(1-\beta^2)c_{tG} + \left(2-(1-z^2)\beta^2\right)c_{VV}^{(8),u} + 2z\beta c_{AA}^{(8),u}\right],$$

$$\tilde{C}_{nn}^{q\bar{q},(1)} = -\frac{g_s^2 m_t^2}{\Lambda^2}\frac{4\beta^2(1-z^2)}{9(1-\beta^2)}c_{VV}^{(8),u},$$

$$\tilde{C}_{kk}^{q\bar{q},(1)} = \frac{2g_s^2 m_t^2}{9\Lambda^2(1-\beta^2)}\left[2\sqrt{2}g_s^2\frac{v}{m_t}(1-\beta^2)z^2 c_{tG} + \left(2+\beta^2-(2-\beta^2)(1-2z^2)\right)c_{VV}^{(8),u} + 4\beta z c_{AA}^{(8),u}\right]$$

$$\tilde{C}_{rr}^{q\bar{q},(1)} = \frac{4g_s^2 m_t^2(1-z^2)}{9\Lambda^2(1-\beta^2)}\left[\sqrt{2}g_s^2\frac{v}{m_t}(1-\beta^2)c_{tG} + (2-\beta^2)c_{VV}^{(8),u}\right],$$

$$\tilde{C}_{rk}^{q\bar{q},(1)} = -\frac{2g_s^2 m_t^2}{9\Lambda^2}\sqrt{\frac{1-z^2}{1-\beta^2}}\left[\sqrt{2}g_s^2\frac{v}{m_t}(2-\beta^2)z c_{tG} + 4z c_{VV}^{(8),u} + 2\beta c_{AA}^{(8),u}\right],$$

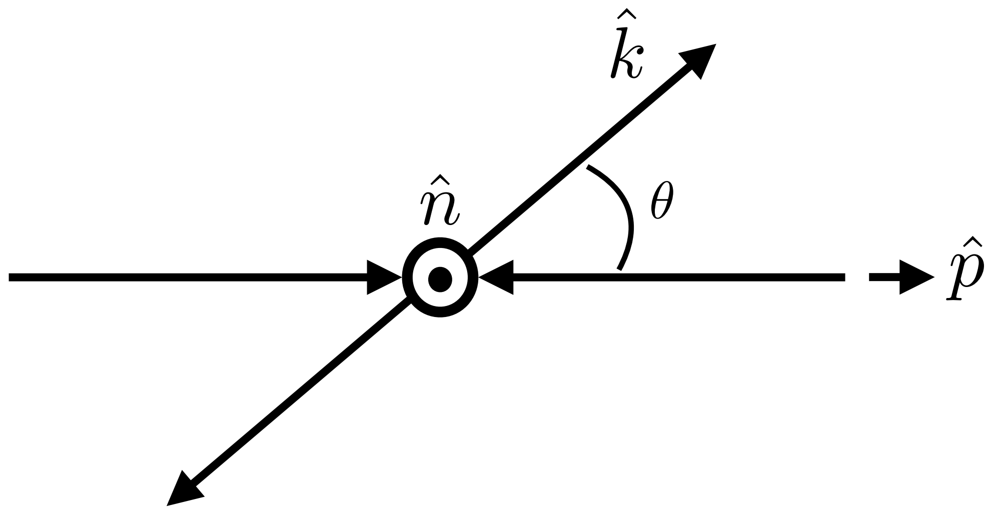
$$B_k^{\pm,q\bar{q},(1)} = 4g_s^2\frac{m_t^2}{9\Lambda^2}\frac{1}{1-\beta^2}\left(\beta(z^2+1)c_{AV}^{(8),u} + 2z c_{VA}^{(8),u}\right),$$

$$B_r^{\pm,q\bar{q},(1)} = -4g_s^2\frac{m_t^2}{9\Lambda^2}\sqrt{\frac{1-z^2}{1-\beta^2}}\left(\beta z c_{AV}^{(8),u} + 2c_{VA}^{(8),u}\right).$$

$$c_{VV}^{(8),u} = (c_{Qq}^{(8,1)} + c_{Qq}^{(8,3)} + c_{tu}^{(8)} + c_{tq}^{(8)} + c_{Qu}^{(8)})/4, \qquad c_{AA}^{(8),u} = (c_{Qq}^{(8,1)} + c_{Qq}^{(8,3)} + c_{tu}^{(8)} - c_{tq}^{(8)} - c_{Qu}^{(8)})/4,$$

$$c_{AV}^{(8),u} = (-c_{Qq}^{(8,1)} - c_{Qq}^{(8,3)} + c_{tu}^{(8)} + c_{tq}^{(8)} - c_{Qu}^{(8)})/4, \qquad c_{VA}^{(8),u} = (-c_{Qq}^{(8,1)} - c_{Qq}^{(8,3)} + c_{tu}^{(8)} - c_{tq}^{(8)} + c_{Qu}^{(8)})/4,$$

$$\{\boldsymbol{k}, \boldsymbol{n}, \boldsymbol{r}\} : \ \boldsymbol{r} = \frac{(\boldsymbol{p} - z\boldsymbol{k})}{\sqrt{1-z^2}}, \quad \boldsymbol{n} = \boldsymbol{k} \times \boldsymbol{r},$$



To expand in this basis, e.g.

$$C_{nn} = \mathrm{tr}[C_{ij}\, \boldsymbol{n} \otimes \boldsymbol{n}\,]$$

Phase-space parametrized by:

$$\beta^2 = (1 - 4m_t^2/\hat{s})$$

$$\cos\theta$$