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Black Hole Solutions in Chern-Simons Gravity with Axion Hair

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Chern-Simons gravity is a 4-dimensional modified theory of gravity postulated by Jackiw and Pi. [1] The action of the theory is described by the following 3 terms:

$$S = S_{EH} + S_{CS} + S_{ax} \tag{1}$$

$$S_{EH} = \int d^4x \sqrt{|g|} \frac{R}{2\kappa^2}$$

$$S_{CS} = A \int d^4x \sqrt{|g|} \, b \, R_{CS}$$

$$S_{ax} = \int d^4x \sqrt{|g|} \left[-\frac{1}{2}(\partial b)^2 - V(b) \right]$$

where A is a coupling constant with dimensions of length, κ is the inverse of the Planck mass i.e. $\kappa^2 = 8\pi G$ and b is a pseudoscalar axion field (signature convention: $-, +, +, +$).

[1]R. Jackiw and S. Y. Pi, "Chern-Simons modification of general relativity," [arXiv:gr-qc/0308071 [gr-qc]]

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The R_{CS} term, known as the **Pontryagin density**, is a topological term described by the contraction of the Riemann tensor to its dual

$$R_{CS} = \frac{1}{2} R^\mu_{\ \nu\rho\sigma} \widetilde{R}_\mu^{\ \nu\ \rho\sigma}, \tag{2}$$

where

$$\widetilde{R}_{\alpha\beta\gamma\delta} = \frac{1}{2} R_{\alpha\beta}^{\ \ \rho\sigma} \varepsilon_{\rho\sigma\gamma\delta}, \tag{3}$$

with $\varepsilon_{\rho\sigma\kappa\lambda} = \sqrt{-g(x)}\, \hat{\epsilon}_{\rho\sigma\kappa\lambda}$ the covariant Levi-Civita under the convention that the symbol $\hat{\epsilon}_{0123} = 1$, *etc*.

The case of a constant axion field, $b = constant$, reduces the theory to GR, since the Pontryagin term can be expressed as

$$\nabla_\alpha K^\alpha = \frac{1}{2} R_{CS}, \qquad K^\alpha = \varepsilon^{\alpha\beta\gamma\delta} \Gamma^\nu_{\beta\mu} (\partial_\gamma \Gamma^\mu_{\delta\nu} + \frac{2}{3} \Gamma^\mu_{\gamma\lambda} \Gamma^\lambda_{\delta\nu}) \tag{4}$$

and can thus be integrated out.

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Motivation for CS-gravity theories

- The CS term naturally arises in some low-energy string theories upon 4-dimensional compactification via the anomaly cancellation mechanism.

- The axion field is a dark matter candidate of geometric origin (torsion)[2].

- The theory has promising results in cosmological running vacuum models [3].

- Possibility of evading the no-hair theorem [4]

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[2]N. E. Mavromatos, "Geometrical origins of the universe dark sector: string-inspired torsion and anomalies as seeds for inflation and dark matter," [arXiv:2108.02152 [gr-qc]]

[3]N. E. Mavromatos and J. Solà Peracaula, "Stringy-running-vacuum-model inflation: from primordial gravitational waves and stiff axion matter to dynamical dark energy," [arXiv:2012.07971 [hep-ph]]

[4]M. J. Duncan, N. Kaloper and K. A. Olive, "Axion hair and dynamical torsion from anomalies," Nucl. Phys. B **387**, 215-235 (1992) doi:10.1016/0550-3213(92)90052-D

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In our work[5] we focused on a CS gravity theory with vanishing axion potential. The action under consideration reads:

$$S = \int d^4 x \sqrt{|g|} \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial b)^2 - A\, b\, R_{CS} \right] \tag{5}$$

$$\Downarrow$$

$$G_{\mu\nu} = \kappa^2 T^b_{\mu\nu} + 4\kappa^2 A C_{\mu\nu} \ , \tag{6}$$

$$\Box b = A\, R_{CS} \ , \tag{7}$$

where

$$T^b_{\mu\nu} = \nabla_\mu b \nabla_\nu b - \frac{1}{2} g_{\mu\nu}(\nabla b)^2 \ . \tag{8}$$

$$C_{\mu\nu} = -\frac{1}{2} \nabla^\alpha \left[(\nabla^\beta b) \widetilde{R}_{\alpha\mu\beta\nu} + (\nabla^\beta b) \widetilde{R}_{\alpha\nu\beta\mu} \right] \ . \tag{9}$$

[5]N. Chatzifotis, P. Dorlis, N. E. Mavromatos and E. Papantonopoulos, "Scalarization of Chern-Simons-Kerr black hole solutions and wormholes," [arXiv:2202.03496 [gr-qc]]

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- The Cotton tensor satisfies a covariant non-conservation law,

$$\nabla_\mu C^{\mu\nu} = -\frac{1}{4}(\nabla^\nu b)R_{CS} . \tag{10}$$

- The conservation of the matter stress tensor, $T^b_{\mu\nu}$ is violated

$$\nabla^\mu T^b_{\mu\nu} = -4A\nabla^\mu C_{\mu\nu} = A\frac{1}{4}(\nabla^\nu b)R_{CS} . \tag{11}$$

- It is the effective stress-energy tensor $\mathscr{T}_{\mu\nu} = T^b_{\mu\nu} + 4AC_{\mu\nu}$, which is conserved by virtue of the Bianchi identity.

$\implies$ **Exchange of energy between matter (the axion field b) and the gravitational anomaly**
(only for non-spherically symmetric backgrounds, e.g. *gravitational wave perturbations, rotating black holes*,etc).

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We consider a metric ansatz for a **slowly rotating black hole** (keep only the leading order in the angular momentum parameter α) in CS gravity :

$$ds^2 = -H(r)dt^2 + F(r)dr^2 - 2r^2\alpha \sin^2\theta W(r)dtd\phi + r^2 d\Omega^2 \ . \qquad (12)$$

From the axion equation of motion (7) we find that the axion field b may be written as

$$b = \alpha A u(r) P_1(\cos\theta) \ , \qquad (13)$$

where P_1 denotes the Legendre polynomial of the first order.

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- The axion field is of order $\mathcal{O}(\alpha)$
- The tt and rr component of the gravitational equations of motion are satisfied in vacuum, since the corresponding stress energy tensor components are of order $\mathcal{O}(\alpha^2)$
- This naturally means that

$$H(r) = \frac{1}{F(r)} = 1 - \frac{2M}{r} \ , \quad M \equiv G\,\mathcal{M} \ , \tag{14}$$

- Any backreaction on our spacetime will be encoded in the off-diagonal component of the metric, i.e.

$$W(r) = \frac{2M}{r^3} + w(r) \ , \tag{15}$$

where $w(r)$ describes corrections on the recovered slowly rotating Kerr spacetime (Hartle-Thorne).

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The $t\phi$ gravitational equation of motion can be straightforwardly integrated to yield that

$$u(r) = -\frac{r^5 w'}{24A^2\kappa^2 M} \ . \tag{16}$$

Plugging the result into the axion equation of motion, we extract the differential equation

$$\begin{aligned}
r^{11}(r - 2M)w''' &+ 2r^{10}(6r - 11M)w'' \\
&+ (28r^{10} - 50Mr^9 - 576A^2\kappa^2 M^2 r^4)w' + 3456A^2\kappa^2 M^3 = 0 \ .
\end{aligned} \tag{17}$$

To solve the above equation we consider a series expansion on the correction function w. In particular, in order the radial component of the axion field asymptotically to vanish $w(r)$ to be at least of order $\mathcal{O}(r^{-4})$

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We define w in a non-closed form as

$$w(r) = \sum_{n=4}^{\infty} \frac{d_n M^{n-2}}{r^n} , \tag{18}$$

Hence, the problem of finding the geometric correction $w(r)$ has been reduced to the determination of the coefficients d_n.
We extract the recursive equation

$$\boxed{d_n = \frac{2(n-5)^2(n-1)}{n(n-6)(n-3)} d_{n-1} + \frac{576 A^2 \kappa^2}{n(n-3)M^4} d_{n-6}, \text{ for } n \geq 10 .} \tag{19}$$

with the constraints:

$$d_4 = d_5 = 0 ,$$
$$-28 d_7 + 48 d_6 = 0 ,$$
$$-80 d_8 + 126 d_7 = 0 , \tag{20}$$
$$256 d_8 - 162 d_9 = -3456 \frac{A^2 \kappa^2}{M^4} .$$

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The series coefficients cannot be completely determined. However:

- Weak field limit, i.e. the case of no backreaction provides one more constraint.
- Slowly rotating Kerr metric background yields the asymptotic behaviour of the axion to be

$$u(r) = -\frac{5}{4Mr^2} - \frac{5}{2r^3} - \frac{9M}{2r^4} \ . \tag{21}$$

- Matching with the result from the series expansion, we find that

$$d_4 = d_5 = 0 \ , \ \ d_6 = -5\gamma^2 \ , \ \ d_7 = -\frac{60\gamma^2}{7} \ , \ \ d_8 = -\frac{27\gamma^2}{2} \ , \ \ d_9 = 0, \tag{22}$$

where

$$\gamma^2 = \frac{A^2\kappa^2}{M^4} \tag{23}$$

$\implies$ The correction on the metric $w(r)$ produces coefficients of even power

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Then our scalarized slowly rotating Kerr metric reads:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 \, d\,\Omega^2 - 2\,r^2\,\alpha\,\sin^2\theta\,W(r)\,dt\,d\phi \,, \tag{24}$$

where

$$W(r) = \frac{2M}{r^3} - \frac{A^2\kappa^2(189M^2 + 120Mr + 70r^2)}{14r^8} + \mathscr{O}(A^{2n}), \quad \text{with } n \geq 2 \,, \tag{25}$$

while the corresponding axion (pseudo)scalar reads:

$$b = \alpha A \cos\theta\left(-\frac{5}{4Mr^2} - \frac{5}{2r^3} - \frac{9M}{2r^4}\right) + \mathscr{O}(A^m), \quad \text{for } m = 2n+1, \ n \in \mathbb{Z}^+ \tag{26}$$

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The series of the corrections on the metric converges

1. An increasing sequence tends either to a finite limit or to ∞.

2. If the correction on the event horizon is convergent, the series is convergent $\forall r \geq r_h$.

3. The coefficients of the recursive relation are bounded, since they converge for $n \to \infty$.

4. There exists a subsequence that is bounded $\overset{\text{bounded coeff}}{\Longrightarrow}$ The sequence is bounded by induction, $|d_n| \leq \mathscr{D}$, $\forall n$

5. $\sum_{n=4}^{\infty} \frac{|d_n|}{2^{n-2}} \leq \mathscr{D} \sum_{n=4}^{\infty} \left(\frac{1}{2}\right)^{n-2} \implies \sum_{n=4}^{\infty} \frac{|d_n|}{2^{n-2}} \leq \frac{\mathscr{D}}{2} \implies$ The series converges.

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- Solution is valid for any value of the coupling .

- Theoretical framework to probe arbitrarily close to the horizon of the axionic black hole.

- The axionic distribution modifies the angular momentum of the horizon [6]

- The angular momentum of the whole spacetime is $J_{total} = \alpha M \implies$ The axionic black hole hair αA is a secondary charge of the scalarized black hole.

- The black hole spacetime violates the energy conditions in the near horizon region $\implies$ Allows for the violation of the no-hair theorem due to the Chern-Simons term.

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[6]N. Chatzifotis, P. Dorlis, N. E. Mavromatos and E. Papantonopoulos, "Axion induced angular momentum reversal in Kerr-like black holes," [arXiv:2206.11734 [gr-qc]].

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- Near the horizon, the violation takes its maximum value, while far away is negligible.
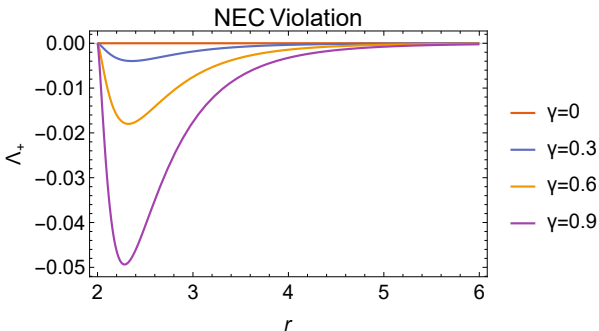
NEC Violation

- NEC violation is dependent on $\gamma \implies$ Axionic black holes of different sizes will differ on the axionic hair they support and their corresponding deformation.

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Test for possible observable effects??

- Backreaction effects are more significant for larger values of the deformation parameter γ.

- In order to investigate the effects of the axion hair on the spacetime geometry

$$\Downarrow$$

Explore the black hole exterior for larger values of γ

$$\Downarrow$$

- We consider γ as if it was an order parameter and we focus on:

1. "phase transitions" in the effective potential of the timelike/null geodesics

2. the structure of the system's total angular momentum, in connection with its parts (axion hair and event horizon)

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Geodesics in a Slowly Rotating Spacetime

- Geodesic analysis in a rotating spacetime yields that

$$\Downarrow^7$$

$$g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 = \frac{(E - V^{(+)})(E - V^{(-)})}{\tilde{\Delta}} \tag{27}$$

$$V^{(\pm)}(r,\theta) = -\frac{L_z g_{t\phi}}{g_{\phi\phi}} \pm \frac{\sqrt{\tilde{\Delta}}}{g_{\phi\phi}}\sqrt{L_z^2 - \epsilon g_{\phi\phi}} \tag{28}$$

$$\tilde{\Delta} = g_{t\phi}^2 - g_{tt}g_{\phi\phi} > 0, \quad \epsilon = -1, 0 \tag{29}$$
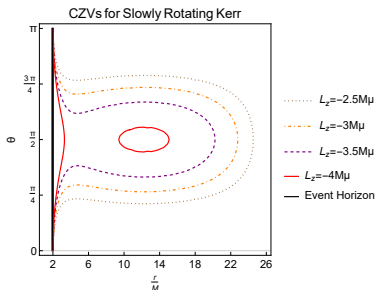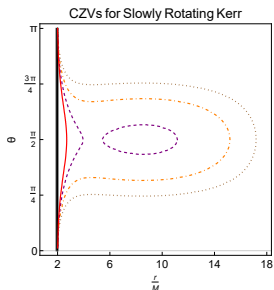
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[7]Similar approach is followed in: *D. Pugliese, H. Quevedo and R. Ruffini, "Equatorial circular motion in Kerr spacetime," Phys. Rev. D* **84**, 044030 (2011)

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$V^{(+)}$ **can be interpreted as the effective potential, for all the geodesics outside the black hole**

$$V_{eff}(r, \theta) = -\frac{L_z g_{t\phi}}{g_{\phi\phi}} + \frac{\sqrt{\tilde{\Delta}}}{g_{\phi\phi}} \sqrt{L_z^2 - \epsilon g_{\phi\phi}} \ , \ \tilde{\Delta} = g_{t\phi}^2 - g_{tt} g_{\phi\phi} > 0 \quad (30)$$

- Allowed regions are determined by $V^{(+)} \leq E$
- At the turning points $V^{(+)} = E \implies$ Curves of Zero Velocity
- Hartle-Thorne metric $g_{t\phi}(r, \theta) = -\frac{2M\alpha}{r} \sin^2(\theta)$
- Deformed Hartle-Thorne metric:
  $g_{t\phi}(r, \theta) = -\left(\frac{2M}{r} + w(r)\right) \alpha \ \sin^2(\theta)$

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Curves of Zero Velocity

- A test particle that starts its geodesic motion inside a curve of zero velocity cannot escape to infinity.
- If a test particle starts its geodesic motion inside a closed CZV, then the particle is trapped in the region signifying a stable bound orbit.
- Whether any test particle inside a CZV falls into the black hole or not depends highly on its initial conditions.

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- For the **deformed** Kerr metric:

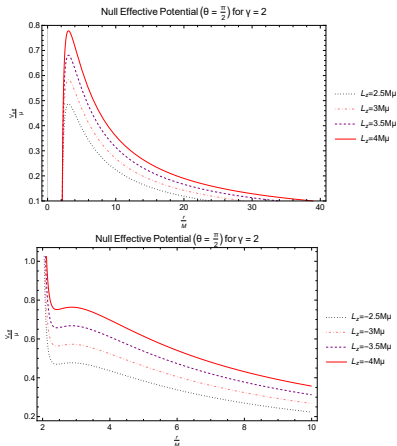$$V_{eff}(r,\theta) = V_{eff}^{K}(r,\theta) + \frac{L_z \alpha w(r)}{r^2} . \qquad (31)$$

- The first term is the effective potential of the Kerr metric, while **the second one is the extra term due to axion backreaction**

- We are interested on the behavior of the potential for increasing γ .

- **For $\gamma \sim 2$, a new minimum of the effective potential appears near the event horizon for counter-rotating geodesics**

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- This new structure can also be verified from the following CZV :

Timelike Effective Potential ($\theta = \frac{\pi}{2}$) for $\gamma = 2$

CZVs for Slowly Rotating Deformed Kerr for $\gamma = 2$
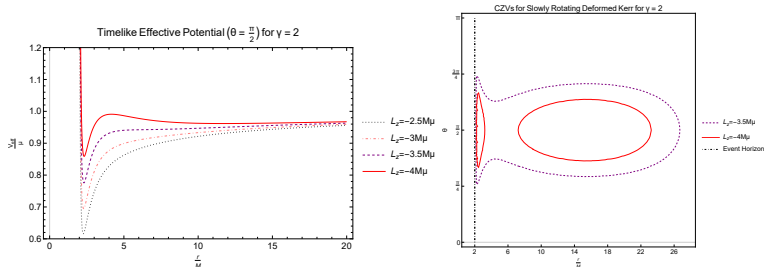
- New minimum of the effective potential $\rightarrow$ **New family of bound orbits**
- **Turning points close to the horizon appear**, preventing particles to fall into the black hole.
- **Repulsive nature of the potential close to the horizon**

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- Similar behavior for the **null effective potential**

Null Effective Potential $\left(\theta = \frac{\pi}{2}\right)$ for $\gamma = 2$

$L_z = 2.5M\mu$
$L_z = 3M\mu$
$L_z = 3.5M\mu$
$L_z = 4M\mu$

Null Effective Potential $\left(\theta = \frac{\pi}{2}\right)$ for $\gamma = 2$

$L_z = -2.5M\mu$
$L_z = -3M\mu$
$L_z = -3.5M\mu$
$L_z = -4M\mu$

- **Massless particles are scattered close to the horizon**
- Beyond the local maximum (photon sphere) there appears a local minimum $\rightarrow$ **Stable bound orbits for massless particles!**

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- **Similar repulsive nature appears in the extreme case of the Kerr metric for the counter-rotating geodesics**[8]
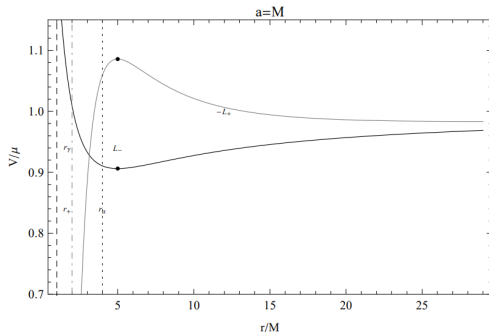
- The extreme case of the Kerr metric corresponds to a **highly rotating spacetime** and since there is no matter coupled to gravity, **corresponds to a highly rotating event horizon.**

[8]D. Pugliese, H. Quevedo and R. Ruffini, "Equatorial circular motion in Kerr spacetime," Phys. Rev. D **84**, 044030 (2011)

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- **A highly rotating event horizon seems to be responsible for this repulsive nature of the effective potential**
- This is actually true even in our case, where the whole spacetime remains slowly rotating

$$\Downarrow$$

Angular momentum reversal: *contributions coming from the event horizon, the axion matter field and their interplay, make possible such a behavior.*

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The Angular Momentum of the Axionic Black Hole

- Following the standard formalism of *Bardeen, Carter and Hawking*[9], we wish to analytically compute the angular momentum of the event horizon of our black hole solution.

- The Killing vector corresponding to polar isometry of the spacetime, $\xi = \partial_\phi$, obeys the identity:

$$\nabla_\beta \nabla^\beta \xi^\alpha = -R^\alpha{}_\beta \xi^\beta \ , \ R^\alpha{}_\beta : \text{Ricci tensor} \tag{32}$$

Integration over a hypersurface S
$$\Downarrow$$
$$\int_{\partial S} d\Sigma_{\alpha\beta} \nabla^\alpha \xi^\beta = -\int_S d\Sigma_\alpha R^\alpha{}_\beta \xi^\beta \ . \tag{33}$$

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[9] J. M. Bardeen, B. Carter S.W. Hawking, "The Four laws of black hole mechanics," Commun. math. Phys. 31, 161-170 (t973)

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S: black hole's exterior

$$\Downarrow$$

$$\partial S \rightarrow \partial S_\infty + \mathscr{H}$$

$$\Downarrow$$

$$\int_{\partial S} d\Sigma_{\alpha\beta} \nabla^\alpha \xi^\beta = \int_{\partial S_\infty} d\Sigma_{\alpha\beta} \nabla^\alpha \xi^\beta + \int_{\mathscr{H}} d\Sigma_{\alpha\beta} \nabla^\alpha \xi^\beta . \qquad (34)$$

Integral at spatial infinity $\rightarrow$ Komar's integral, i.e. total angular momentum J as measured at spatial infinity.

$$\Downarrow (33)$$

$$J = -\frac{1}{8\pi} \int_{\mathscr{H}} d\Sigma_{\alpha\beta} \nabla^\alpha \xi^\beta - \frac{1}{8\pi} \int_S d\Sigma_\alpha R^\alpha{}_\beta \xi^\beta . \qquad (35)$$

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- Expressing the Ricci tensor with respect to the energy momentum tensor from the gravitational equations of motion:

$$J = J_H + J_M \ , \tag{36}$$

$$J_H = -\frac{1}{8\pi} \int_{\mathscr{H}} d\Sigma_{\alpha\beta} \nabla^\alpha \xi^\alpha \ , \tag{37}$$
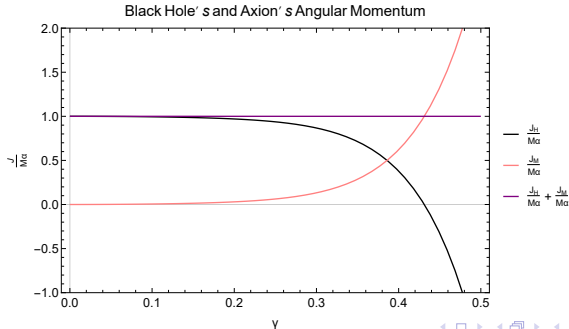
$$J_M = -\int_S d\Sigma_\alpha \left(T^\alpha_{\ \beta} - \frac{1}{2}\delta^\alpha_{\ \beta} T \right) \xi^\beta \ . \tag{38}$$
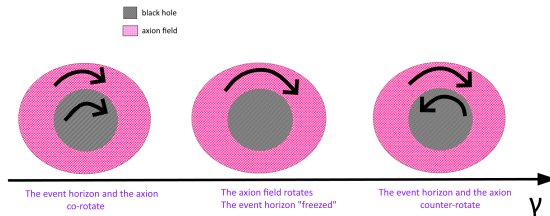
- J_H: angular momentum of event horizon
- J_M: angular momentum of axionic matter field

- No matter ($T_{\mu\nu} = 0$) $\rightarrow J = J_H$; $\Downarrow$
  highly/slowly rotating spacetime $\rightarrow$ highly/slowly rotating event horizon$\Downarrow$
  this is not true if a matter field contributes!

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$$J_H = \left[1 + \frac{2rw(r) - r^2 w'(r)}{6M}\right]_{r=2M} M\alpha \ . \tag{39}$$

$$J_M = -\left[\frac{2rw(r) - r^2 w'(r)}{6M}\right]_{r=2M} M\alpha \ . \tag{40}$$

$$J = J_H + J_M = M\alpha \tag{41}$$

Black Hole' s and Axion' s Angular Momentum

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- **The magnitude of each particular angular momentum can reach large values**, but the angular momentum of the spacetime remains constant ⇓

  highly/slowly rotating spacetime does not imply highly/slowly rotating event horizon ⇓

  "Highly rotating effects" around the black hole in a slowly rotating spacetime are possible

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- We extracted a slowly rotating black hole solution in axionic Chern-Simons gravity in all order of the effective coupling parameter $\gamma \sim A/M^2$. This coupling parameter measures the strength of the backreaction of the axion field on the spacetime geometry.
- As γ increases:
  1. new potential structure for counter-rotating geodesics with novel stable bound orbits
  2. Black hole angular momentum decreases and starts to rotate in the opposite direction with increasing magnitude. **(angular momentum reversal phenomenom $\rightarrow$ responsible for the above)**

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- We aim to extend the study to higher rotating paradigms in CS gravity.
- We also aim to consider the extension of the theory with a Gauss-Bonnet-dilaton term and test the interplay of the axion and the dilaton on the local solutions.

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Thank you for listening!