

Anomalous and axial Z' contributions to the $g - 2$ of the muon

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Intro / motivation

- Standard Model (SM) is the **most accurate theory even made**.
- However, we know that it is **not the final theory**. It is effective.
- In this talk, we will extend the SM with **an additional gauge boson** (often called Z').
- However, we are going to explore an **“exotic” version** of the Z' : **an anomalous Z'** .

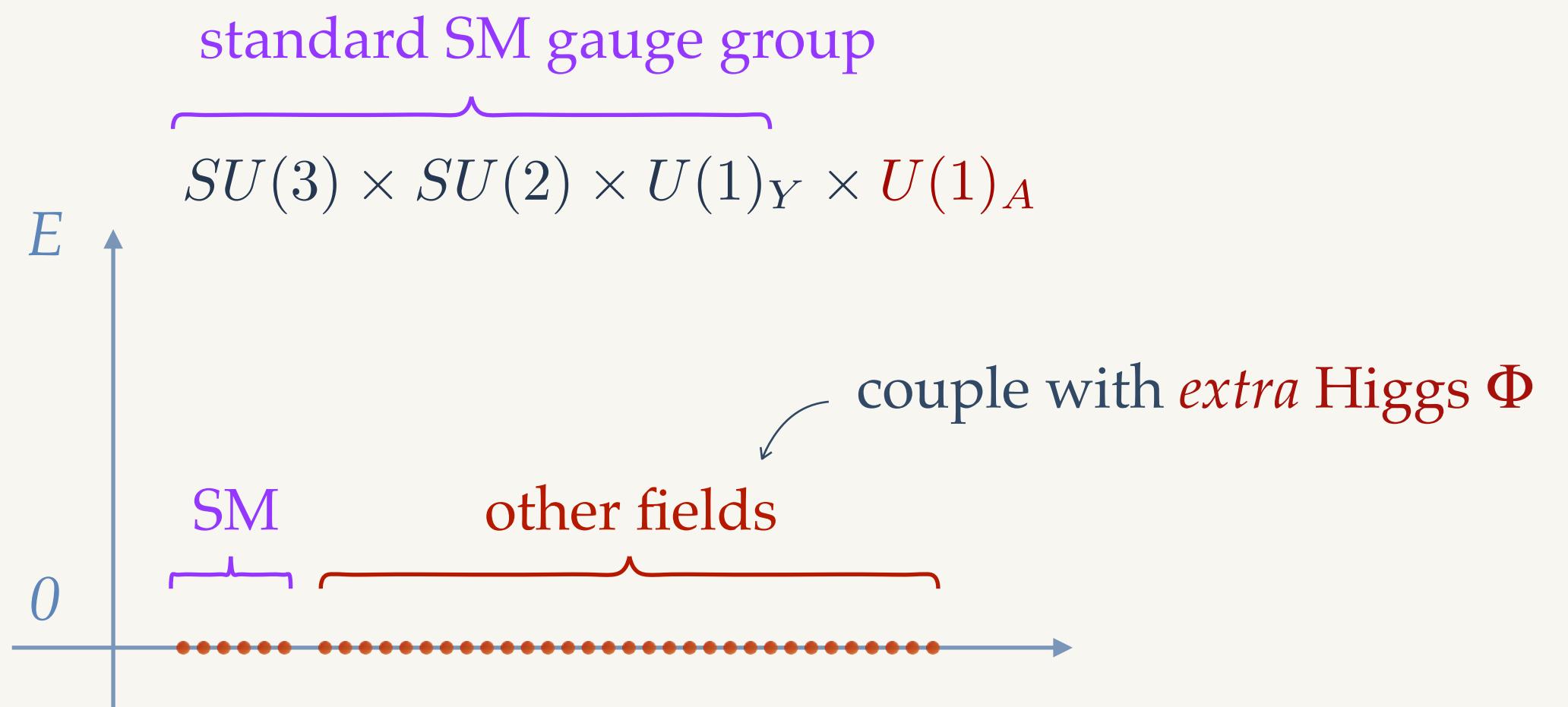
Anomalies

Anomalies

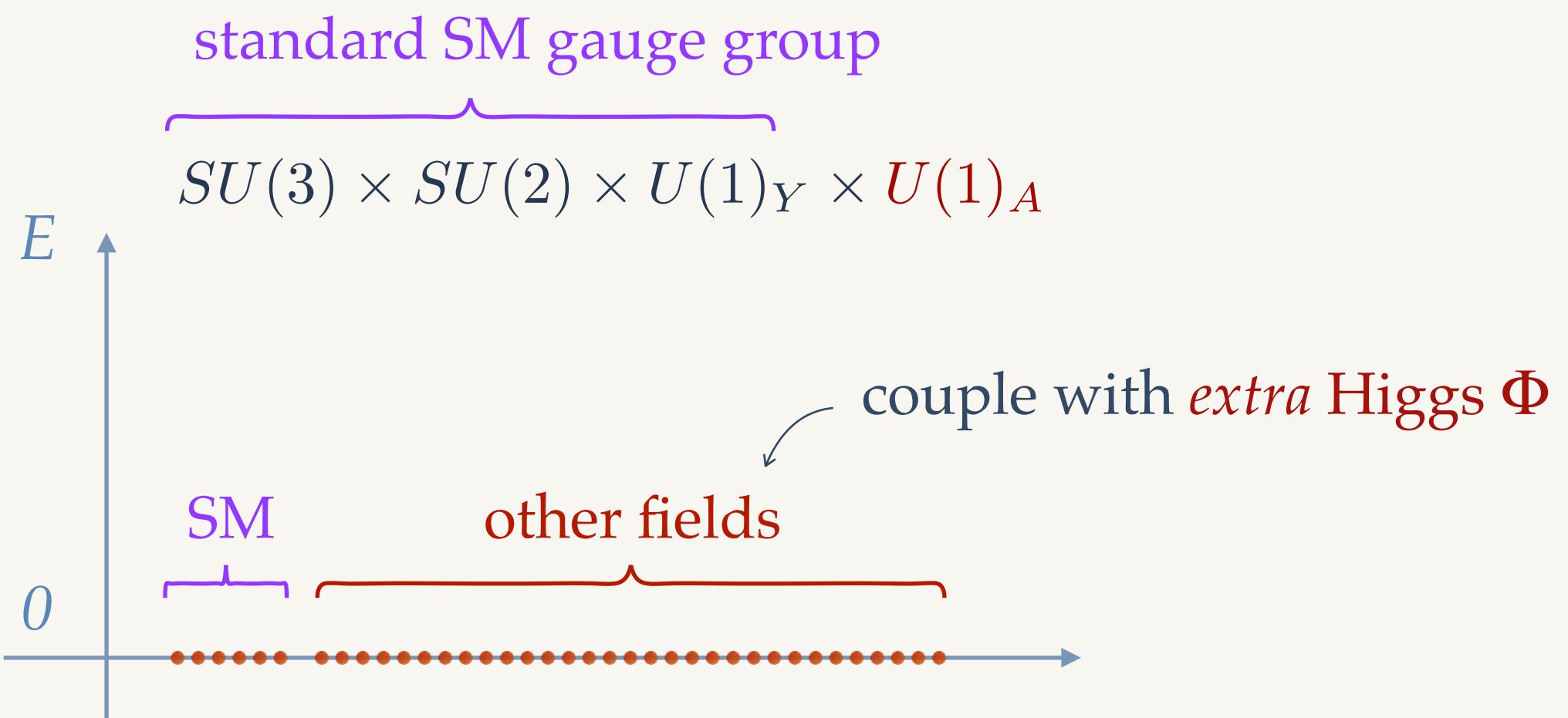
- Anomalies
 - Violations of symmetries via quantum corrections (loop-effects).
 - They split into: violations of
 - global symmetries: acceptable (for example the $\pi \rightarrow \gamma\gamma$).
 - gauge symmetries: unacceptable. Have to be omitted!
 - Gauge anomalies are forbidden in Fundamental Theories, but can appear in effective theories!
 - Such effective theories can come from string theory, GUTs, theories with a Higgs mech at a higher scale.

Standard Model as an anomalous EFT

Standard Model as an anomalous EFT



Standard Model as an anomalous EFT



SM

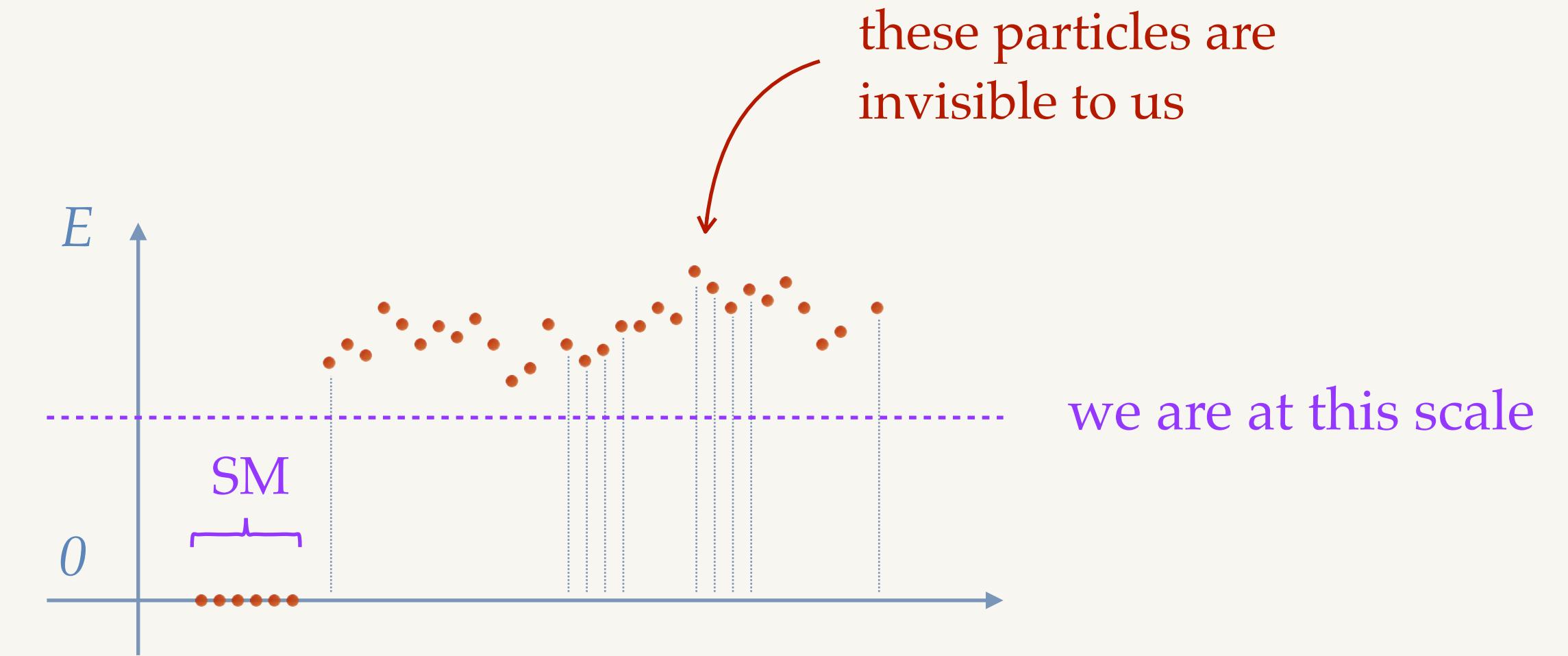
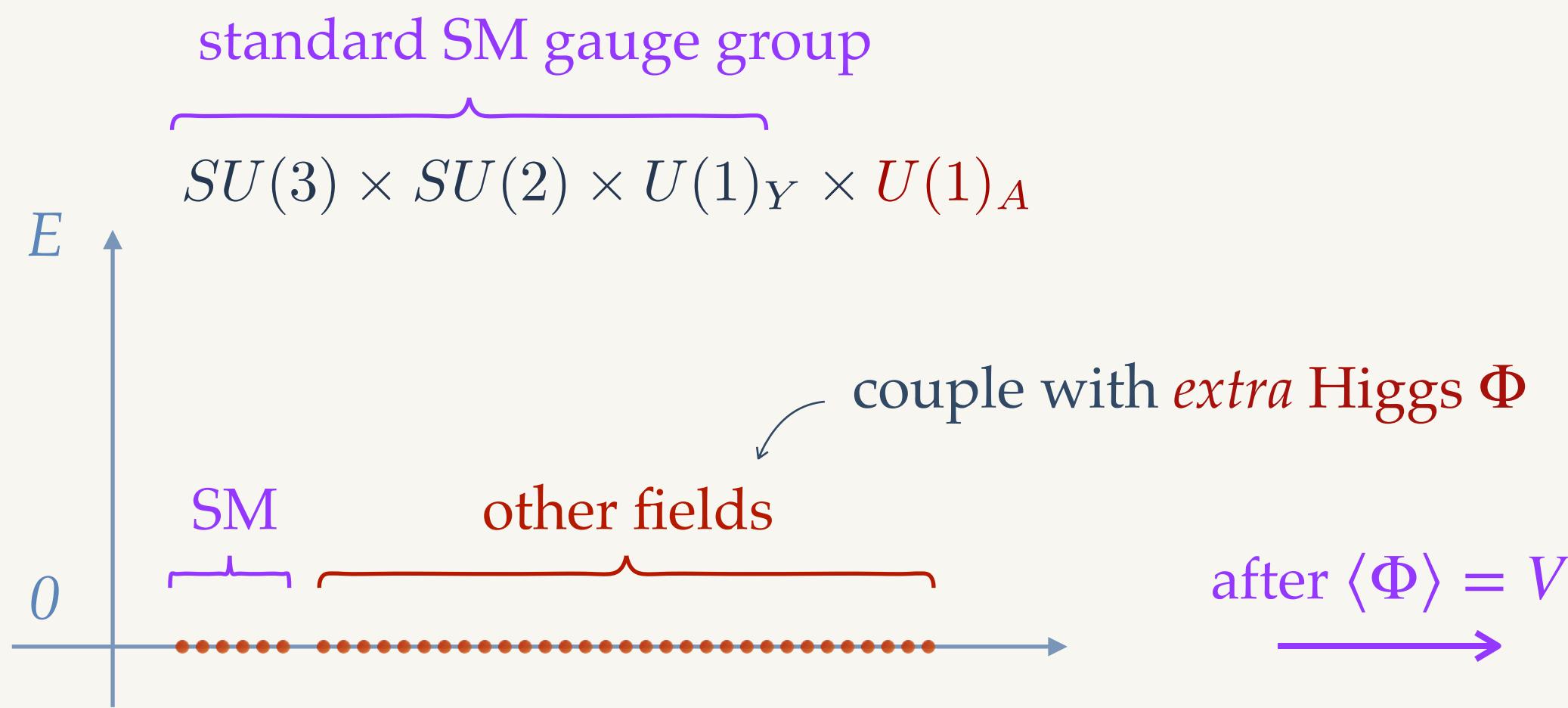
other fields

$= 0$ anomaly free

A

$-A$

Standard Model as an anomalous EFT



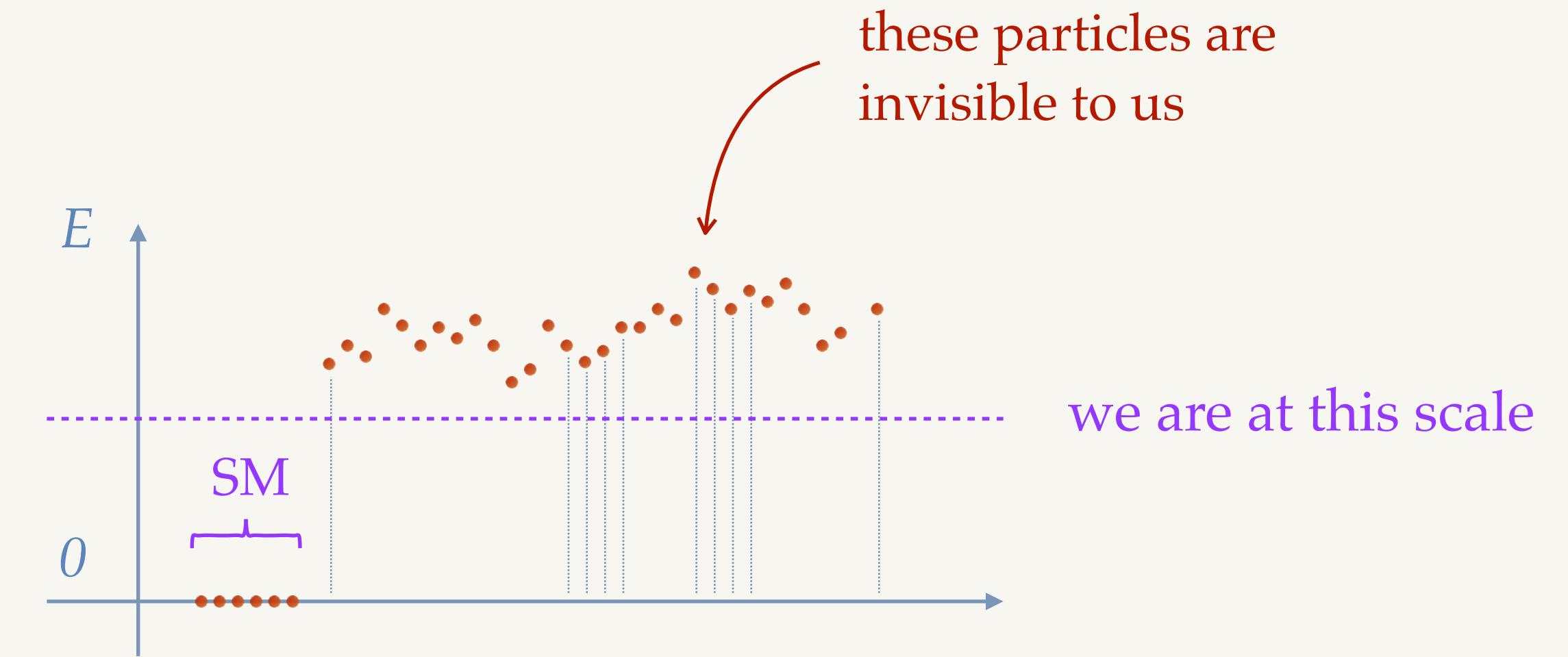
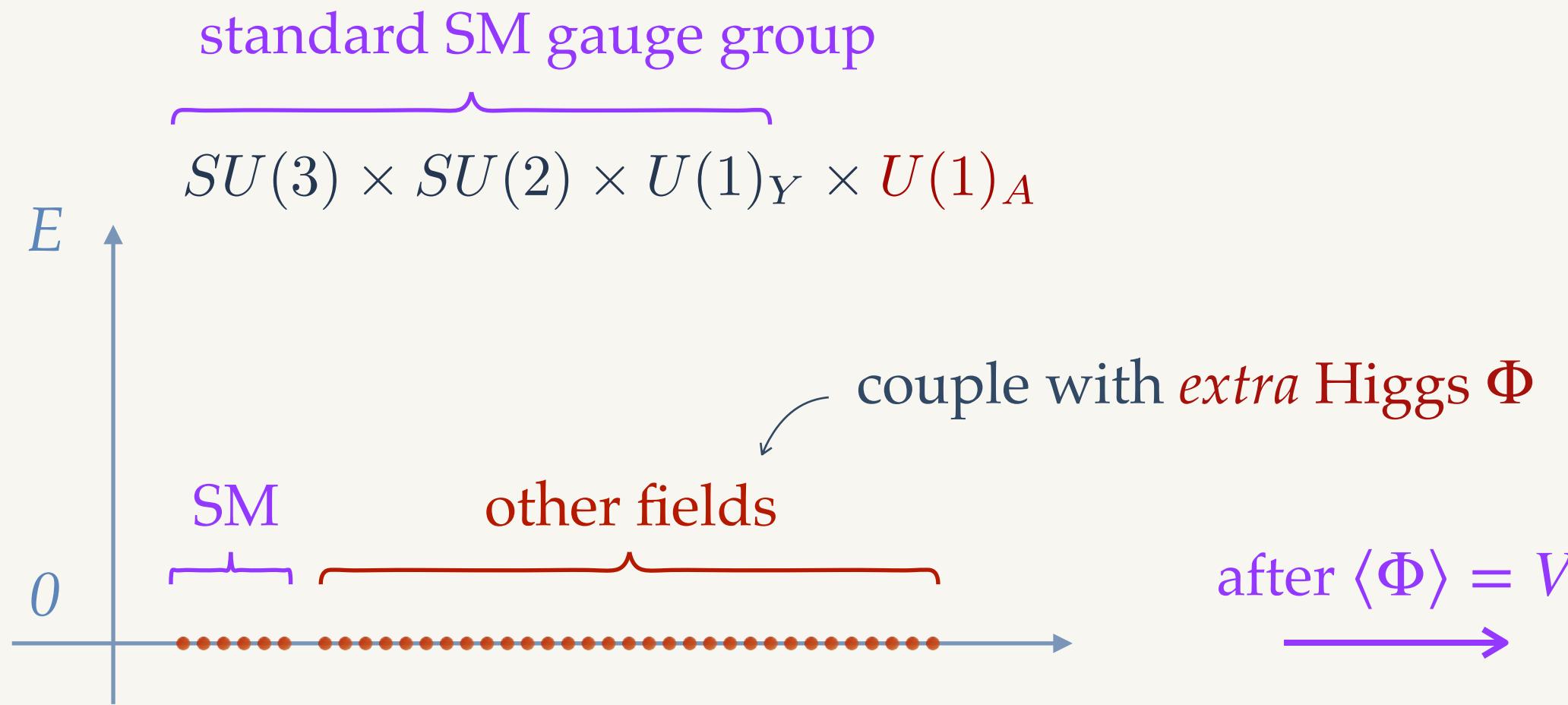
SM other fields

$\text{---} + \text{---} + \text{---} + \text{---} = 0$ anomaly free

\mathcal{A} $-\mathcal{A}$

This equation shows the cancellation of anomalies. It consists of two sets of four Feynman diagrams each, representing the SM and other fields respectively. The first set of diagrams is enclosed in a purple bracket and labeled \mathcal{A} . The second set is enclosed in a red bracket and labeled $-\mathcal{A}$. The sum of all diagrams is zero, indicating that the theory is anomaly free.

Standard Model as an anomalous EFT



SM other fields

$\text{---} + \text{---} + \text{---} + \text{---} = 0$ anomaly free

\mathcal{A} $-\mathcal{A}$

This diagram shows a sum of four Feynman-like diagrams involving the SM and other fields. The result is zero, indicating the model is anomaly free.

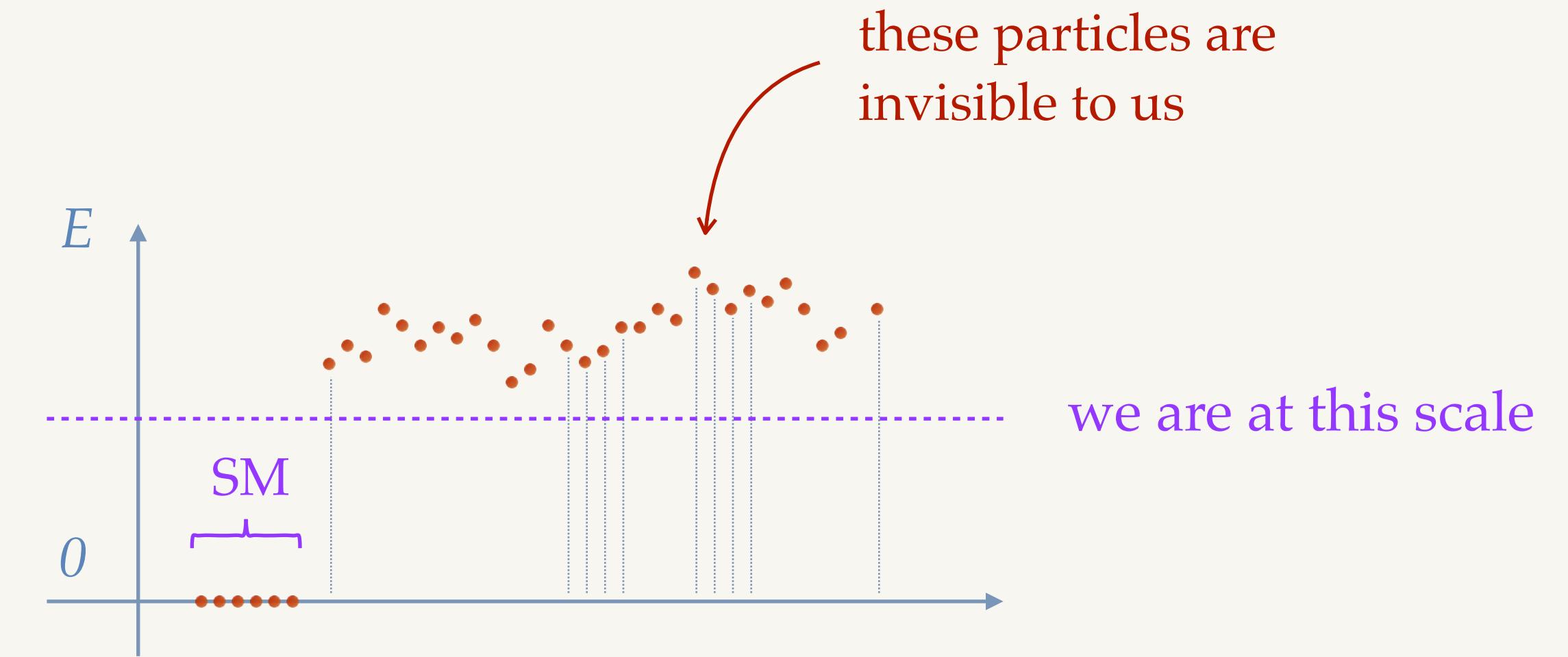
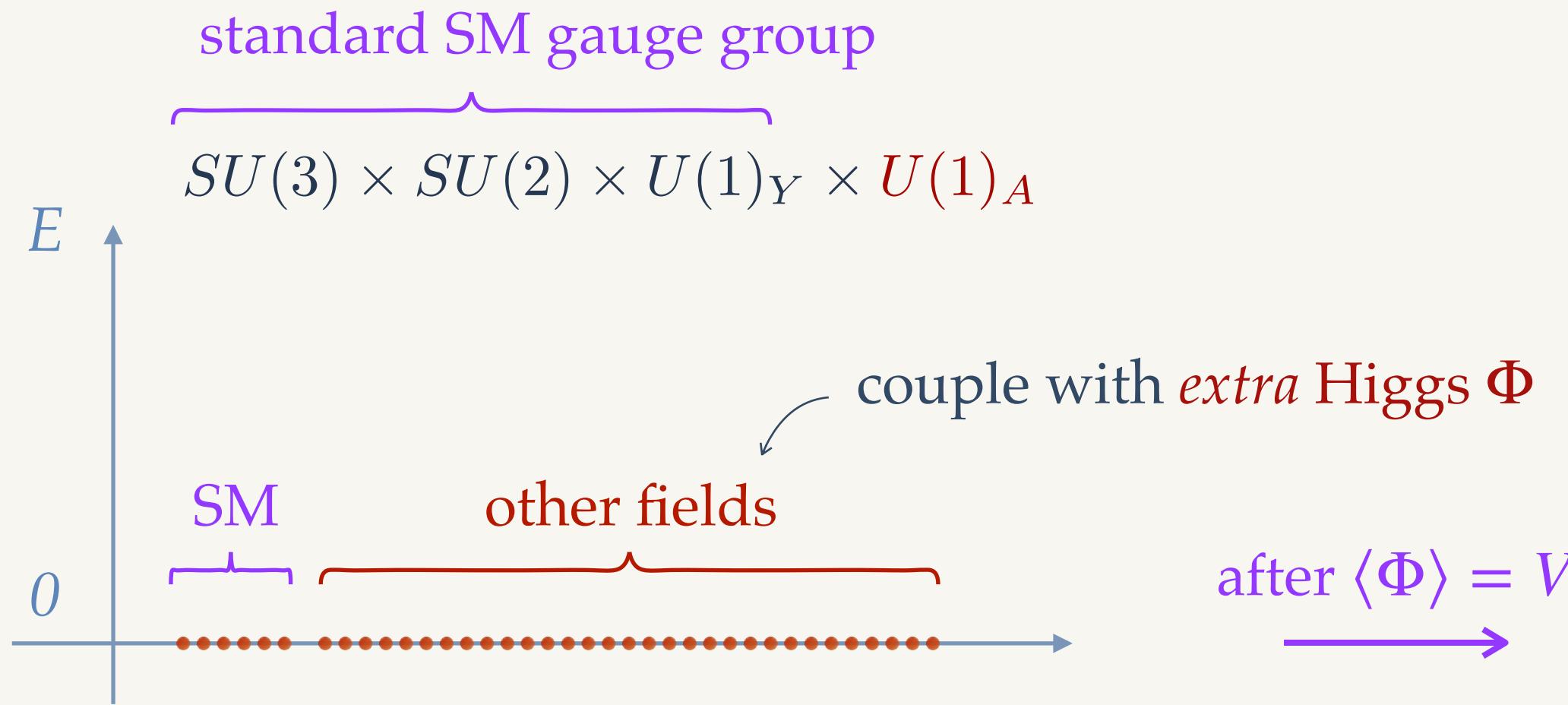
SM other fields

$\text{---} + \text{---} + \text{---} + \text{---} \neq 0$ (superficially) anomalous?!

\mathcal{A} $-\mathcal{A}$

This diagram shows a sum of four Feynman-like diagrams involving the SM and other fields, resulting in a non-zero value, indicating the model is (superficially) anomalous.

Standard Model as an anomalous EFT



$$\begin{array}{c} \text{SM} \quad \text{other fields} \\ \overbrace{\quad\quad\quad}^{\text{wavy lines}} + \overbrace{\quad\quad\quad}^{\text{wavy lines}} + \overbrace{\quad\quad\quad}^{\text{wavy lines}} + \overbrace{\quad\quad\quad}^{\text{wavy lines}} = 0 \quad \text{anomaly free} \\ \overbrace{\quad\quad\quad}^{\text{wavy lines}} \quad \overbrace{\quad\quad\quad}^{\text{wavy lines}} \\ \downarrow \quad \quad \quad \downarrow \\ \mathcal{A} \quad -\mathcal{A} \end{array}$$

This diagram shows the cancellation of anomalies. It consists of two columns of four Feynman-like diagrams each. The left column is labeled "SM" and the right column is labeled "other fields". The first two rows of diagrams have wavy lines, while the last two rows have solid lines. The first two rows are grouped by a purple bracket and the last two by a red bracket. A plus sign is placed between the two groups. The result is set equal to zero, indicating that the model is anomaly free. Arrows point from the bottom of each column to the labels \mathcal{A} and $-\mathcal{A}$.

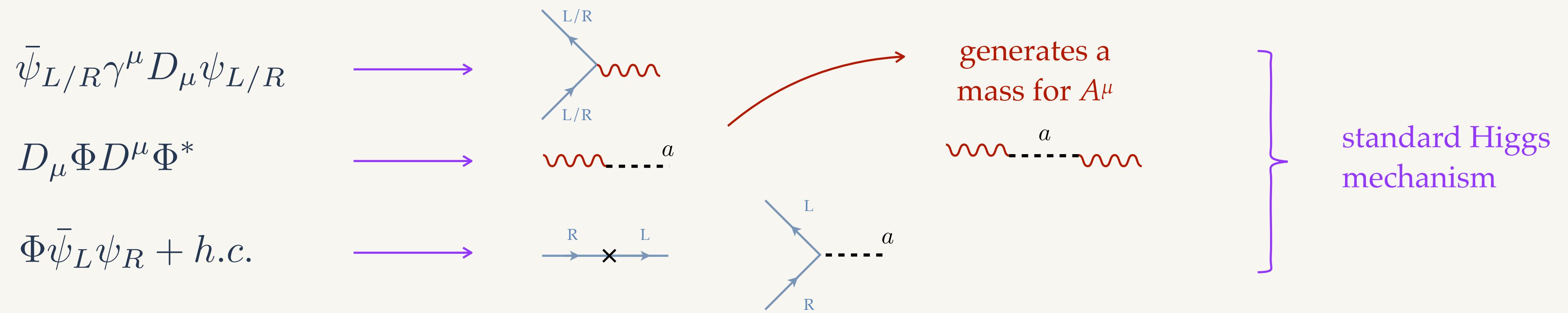
$$\begin{array}{c} \text{SM} \quad \text{other fields} \\ \overbrace{\quad\quad\quad}^{\text{wavy lines}} + \overbrace{\quad\quad\quad}^{\text{wavy lines}} + \overbrace{\quad\quad\quad}^{\text{wavy lines}} + \overbrace{\quad\quad\quad}^{\text{wavy lines}} \neq 0 \quad (\text{superficially} \text{ anomalous?}) \\ \overbrace{\quad\quad\quad}^{\text{wavy lines}} \quad \overbrace{\quad\quad\quad}^{\text{wavy lines}} \\ \downarrow \quad \quad \quad \downarrow \\ \mathcal{A} \quad -\mathcal{A} \end{array}$$

This diagram shows the cancellation of anomalies after a Higgs mechanism. It has the same structure as the previous diagram, but the wavy lines in the "other fields" column are now colored red. The result is not zero, indicated by the inequality symbol $\neq 0$. Arrows point from the bottom of each column to the labels \mathcal{A} and $-\mathcal{A}$.

- * After a Higgs mechanism the model is **anomalous?!**

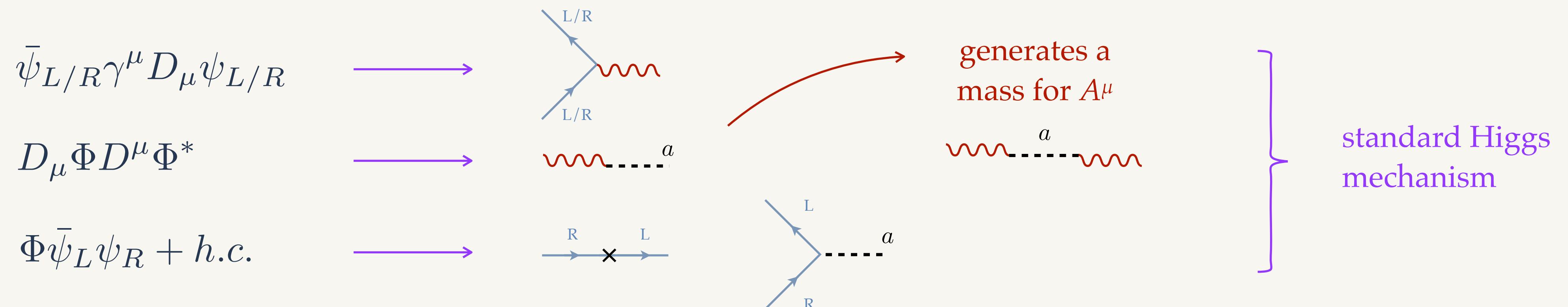
Higgs mechanism & effective terms

- After the **Higgs mechanism** (in a higher scale), the Higgs field $\Phi = (v + r)e^{ia/v}$ gives

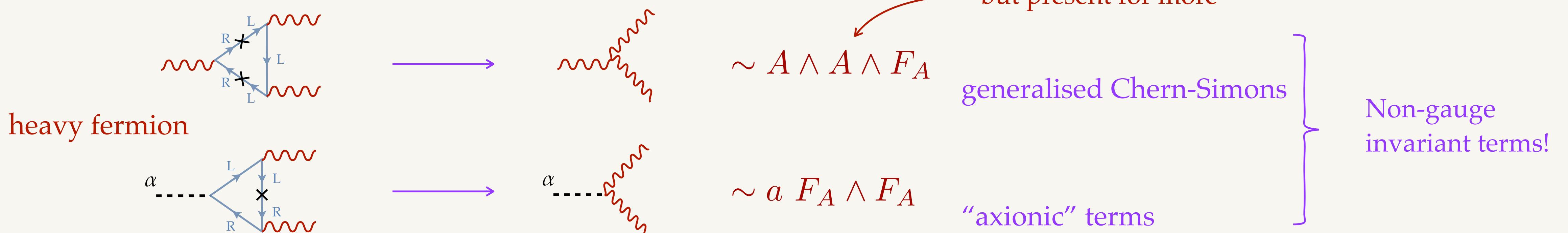


Higgs mechanism & effective terms

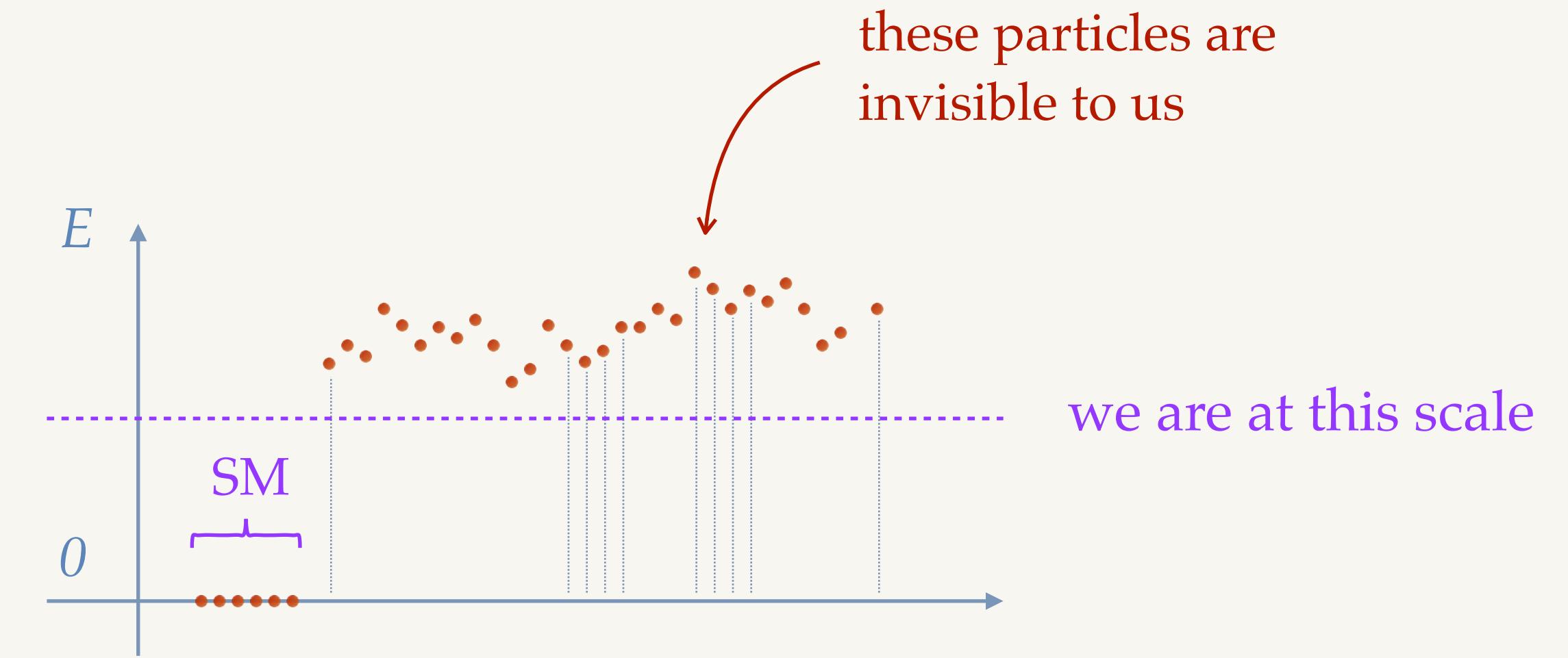
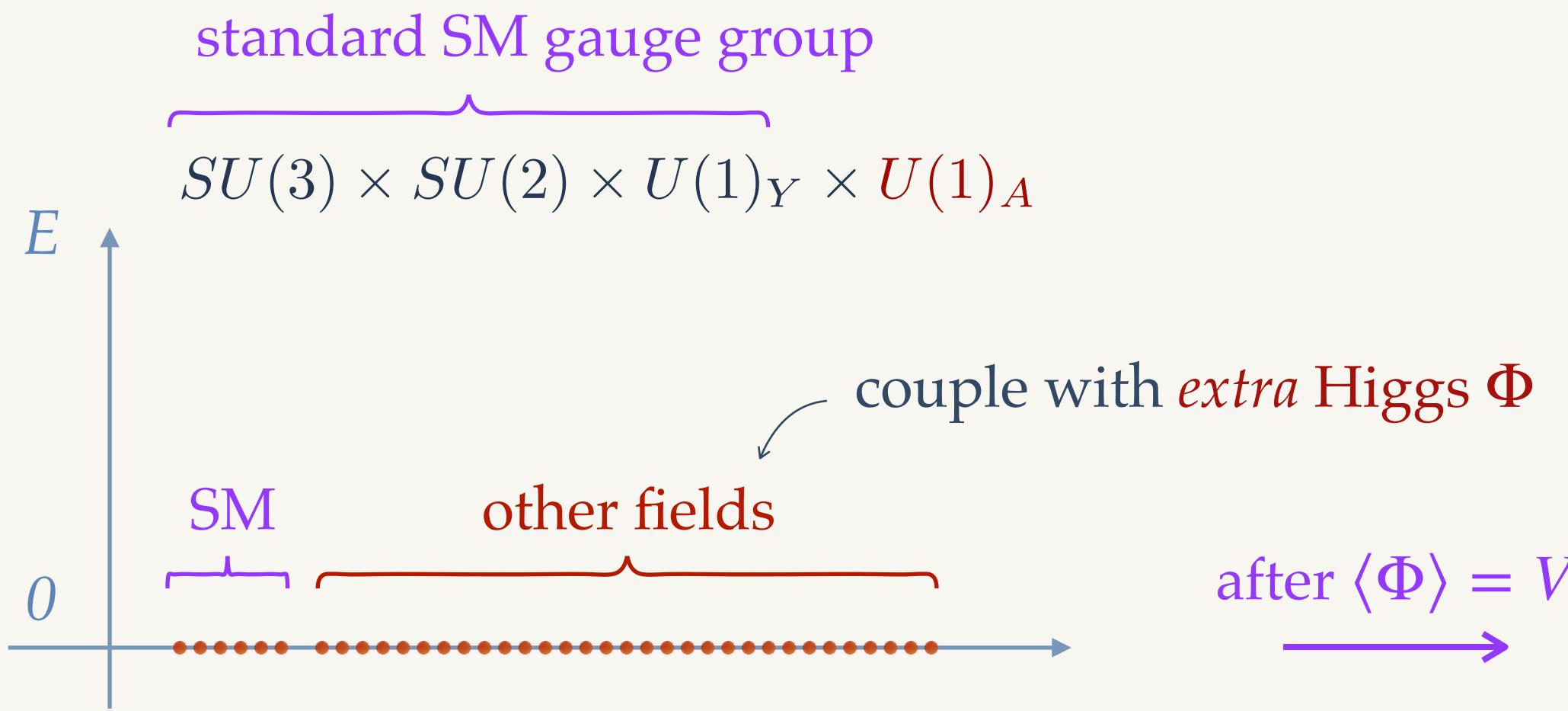
- After the **Higgs mechanism** (in a higher scale), the Higgs field $\Phi = (v + r)e^{ia/v}$ gives



- Out of which we have diagrams that generate **effective terms**



Standard Model as an anomalous EFT (back to our puzzle)



SM other fields

$\text{---} + \text{---} + \text{---} + \text{---} = 0$ anomaly free

--- ---

\mathcal{A} $-\mathcal{A}$

This diagram shows a cancellation of anomalies. It consists of two parts: "SM" and "other fields", each containing two Feynman-like diagrams. The first part has a purple bracket under it, and the second part has a red bracket under it. The two parts are separated by a plus sign. The entire expression equals zero, indicating it is "anomaly free". Arrows point from the SM and other fields parts to the labels \mathcal{A} and $-\mathcal{A}$ respectively.

SM other fields

$\text{---} + \text{---} + \text{---} + \text{---} \neq 0$ (superficially) anomalous?!

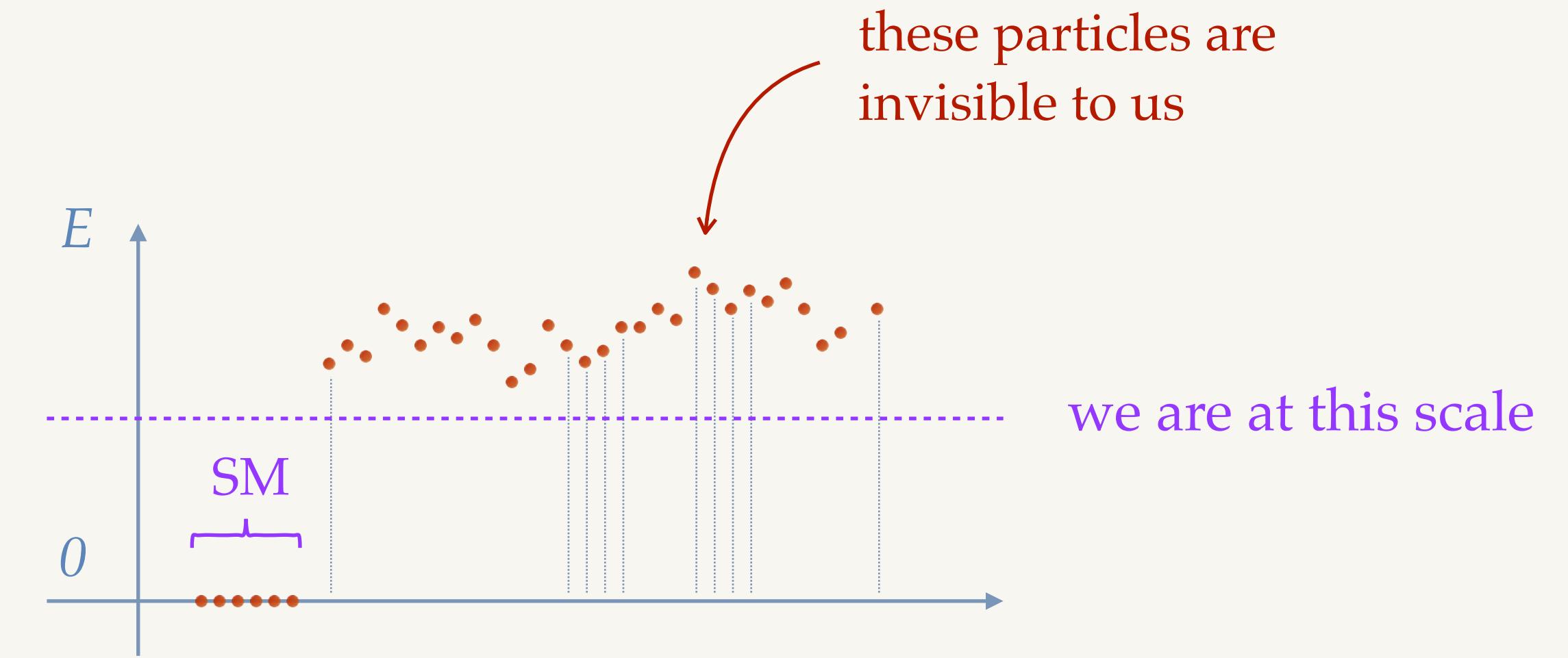
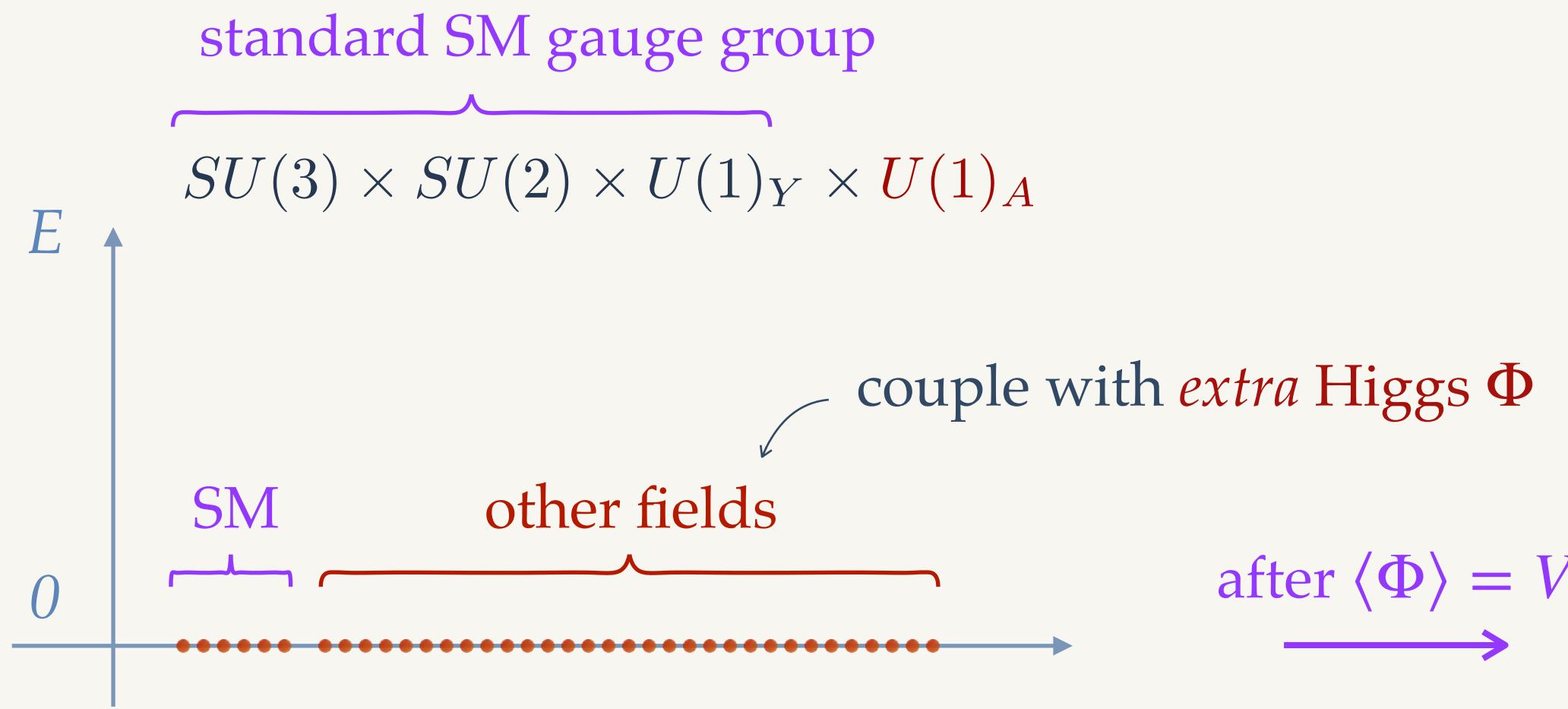
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\mathcal{A} $-\mathcal{A}$

This diagram shows a non-cancellation of anomalies. It consists of two parts: "SM" and "other fields", each containing two Feynman-like diagrams. The first part has a purple bracket under it, and the second part has a red bracket under it. The two parts are separated by a plus sign. The entire expression does not equal zero, indicated by the symbol $\neq 0$. Arrows point from the SM and other fields parts to the labels \mathcal{A} and $-\mathcal{A}$ respectively.

- * After a Higgs mechanism the model is **anomalous?!**

Standard Model as an anomalous EFT (back to our puzzle)



$$\begin{array}{c} \text{SM} \quad \text{other fields} \\ \overbrace{\quad\quad\quad}^{\text{}} + \overbrace{\quad\quad\quad}^{\text{}} + \overbrace{\quad\quad\quad}^{\text{}} + \overbrace{\quad\quad\quad}^{\text{}} = 0 \quad \text{anomaly free} \\ \overbrace{\quad\quad\quad}^{\text{}} \quad \overbrace{\quad\quad\quad}^{\text{}} \\ \downarrow \quad \quad \quad \downarrow \\ A \quad -A \end{array}$$

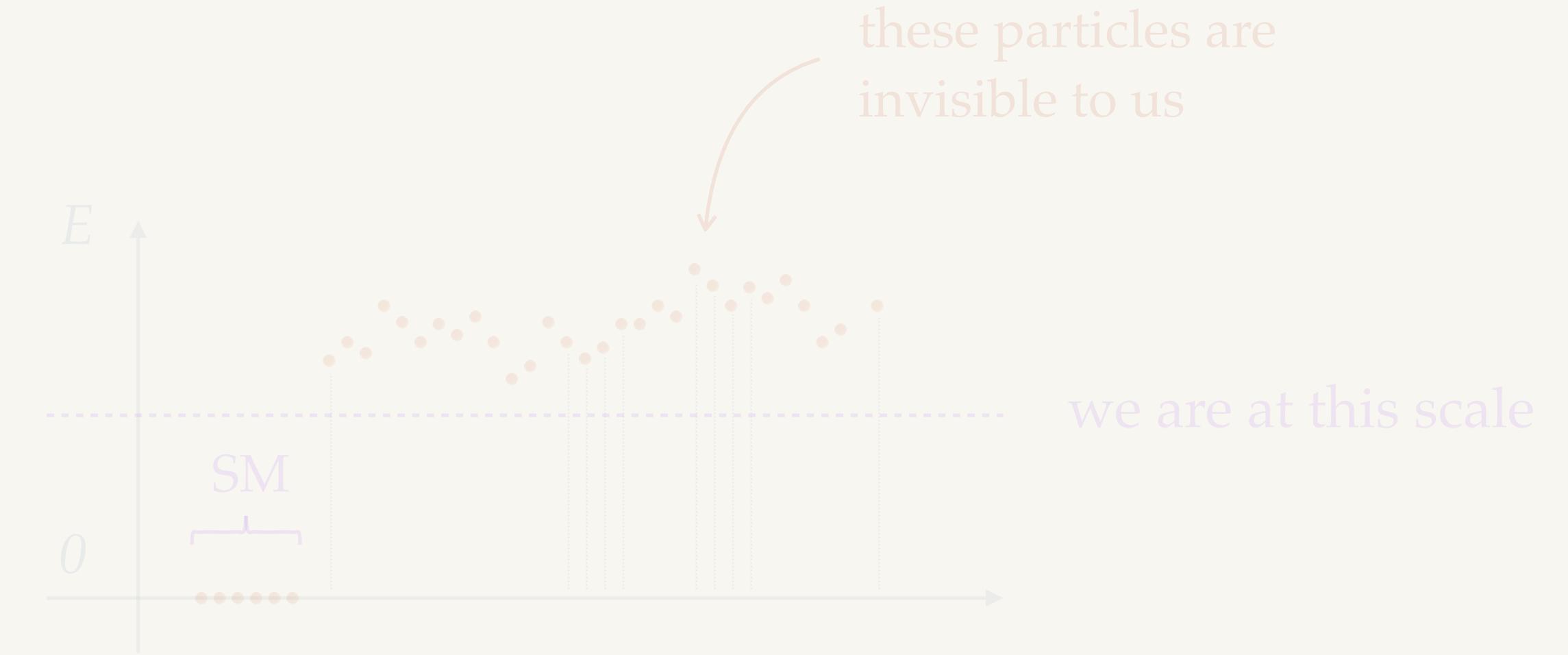
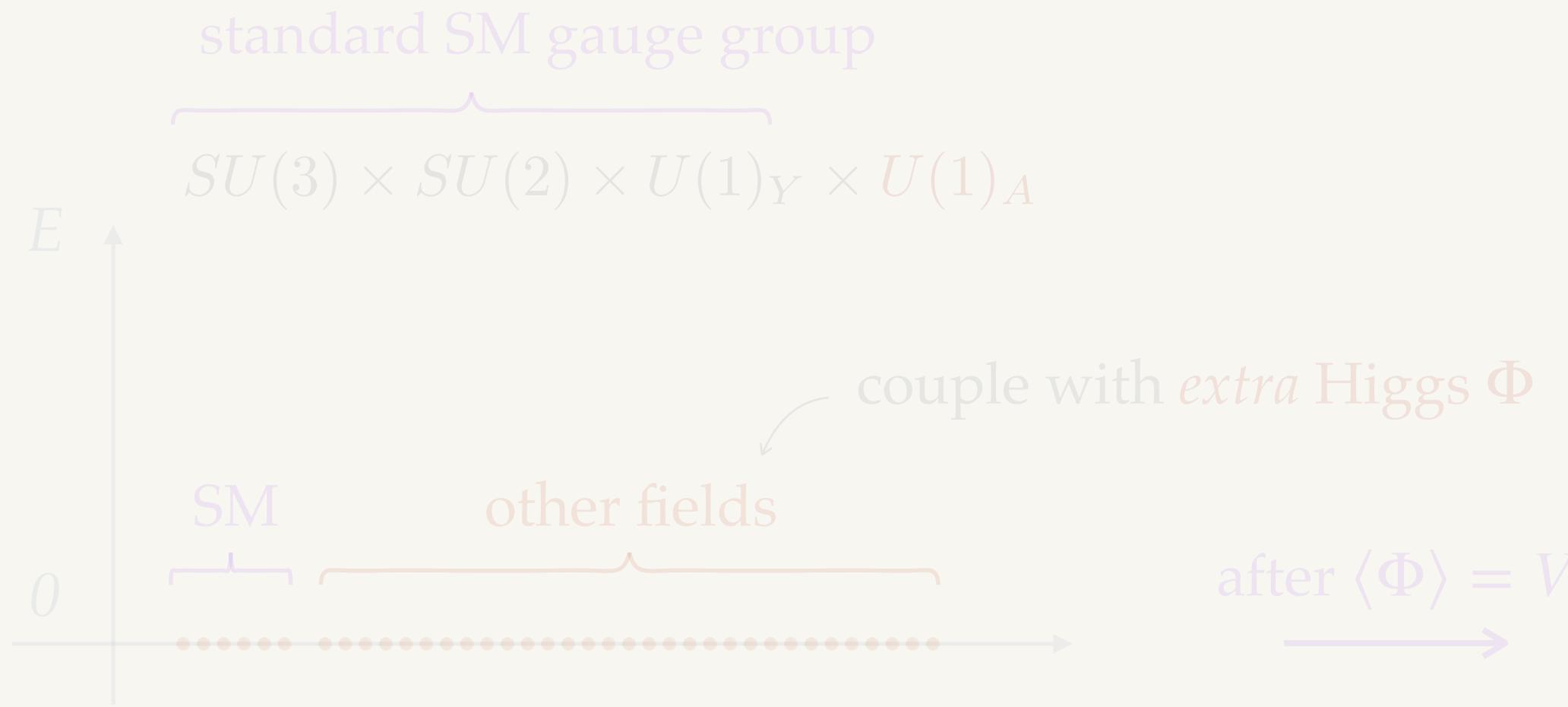
Diagram illustrating the cancellation of anomalies. The sum of four diagrams involving the SM and other fields is zero, resulting in an anomaly-free theory. The diagrams are labeled A and $-A$.

$$\partial_\mu \begin{array}{c} \text{SM} \quad \text{other fields} \\ \overbrace{\quad\quad\quad}^{\text{}} + \overbrace{\quad\quad\quad}^{\text{}} + \overbrace{\quad\quad\quad}^{\text{}} + \overbrace{\quad\quad\quad}^{\text{}} = 0 \quad \text{anomaly free} \\ \overbrace{\quad\quad\quad}^{\text{}} \quad \overbrace{\quad\quad\quad}^{\text{}} \\ \downarrow \quad \quad \quad \downarrow \\ A \quad -A \end{array}$$

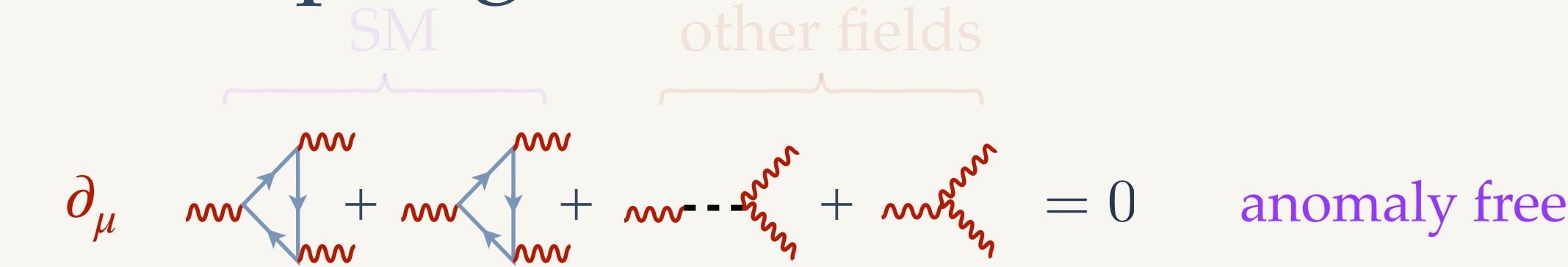
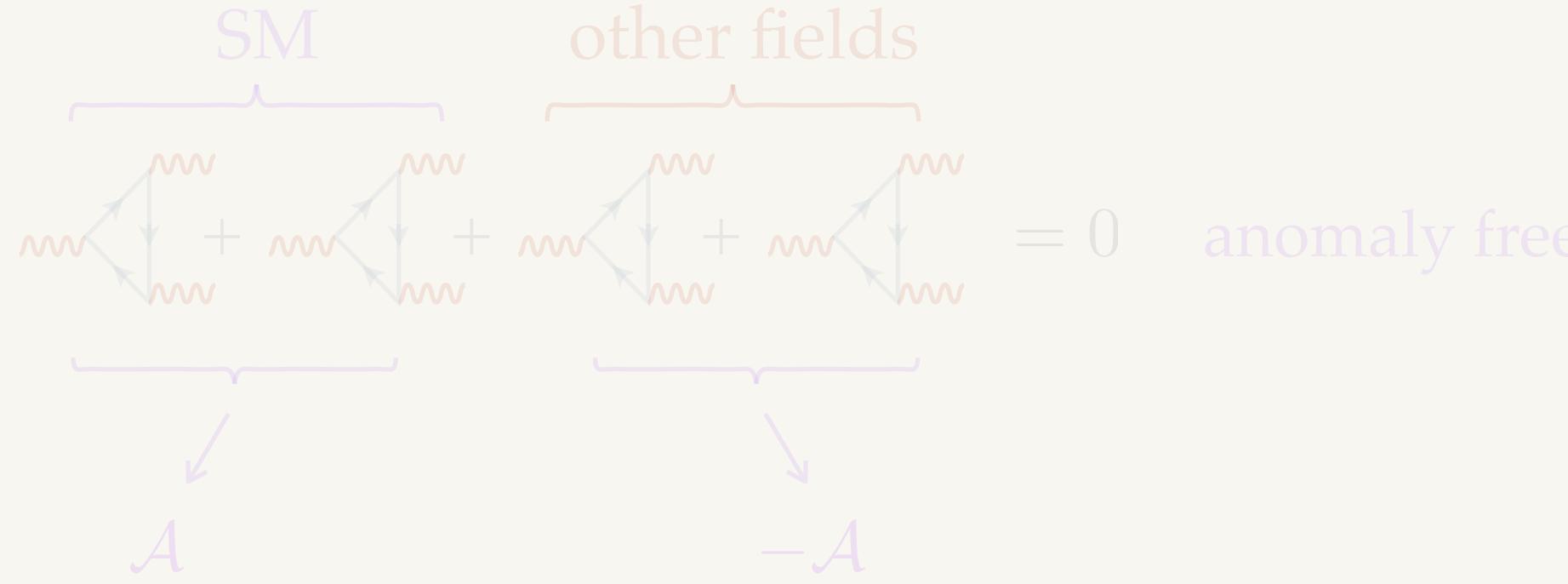
Diagram illustrating the cancellation of anomalies after a Higgs mechanism. The sum of four diagrams involving the SM and other fields is zero, resulting in an anomaly-free theory. The diagrams are labeled A and $-A$.

- * After a Higgs mechanism the model is **anomalous**?!?

Standard Model as an anomalous EFT (back to our puzzle)



The anomalous coupling

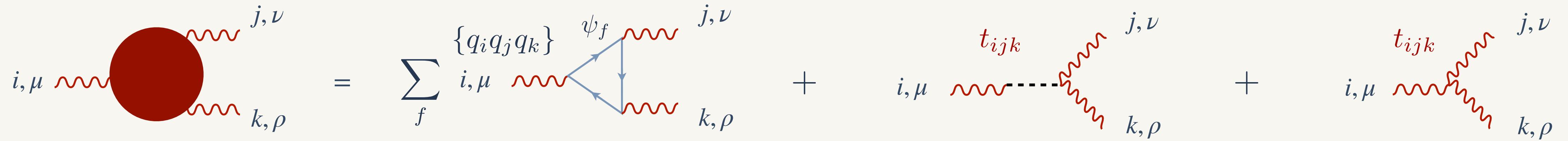


superficially

- * After a Higgs mechanism the model is **anomalous**?!

The anomalous 3-point coupling

these terms effective: they are coming from the contribution of all “heavy” fermions



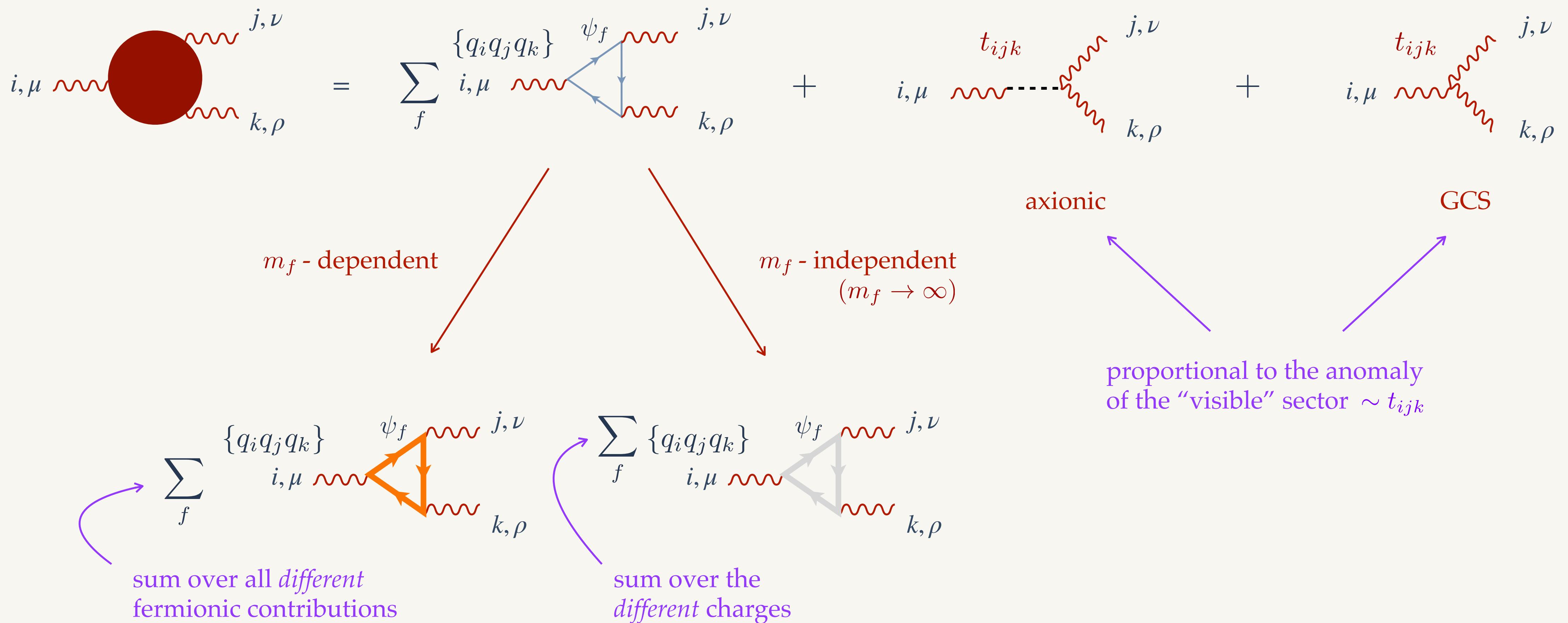
- Anomaly cancellation conditions: the Ward Identities vanish.

$$k_1^\nu \times \quad k_2^\rho \times \quad k_3^\mu \times \quad i, \mu \quad \overrightarrow{k_3} \quad \text{red circle with } \begin{matrix} \leftarrow \\ k_1 \end{matrix} \quad j, \nu \quad \begin{matrix} \leftarrow \\ k_2 \end{matrix} \quad k, \rho = 0$$

proportional to the anomaly of the “visible” sector $\sim t_{ijk}$

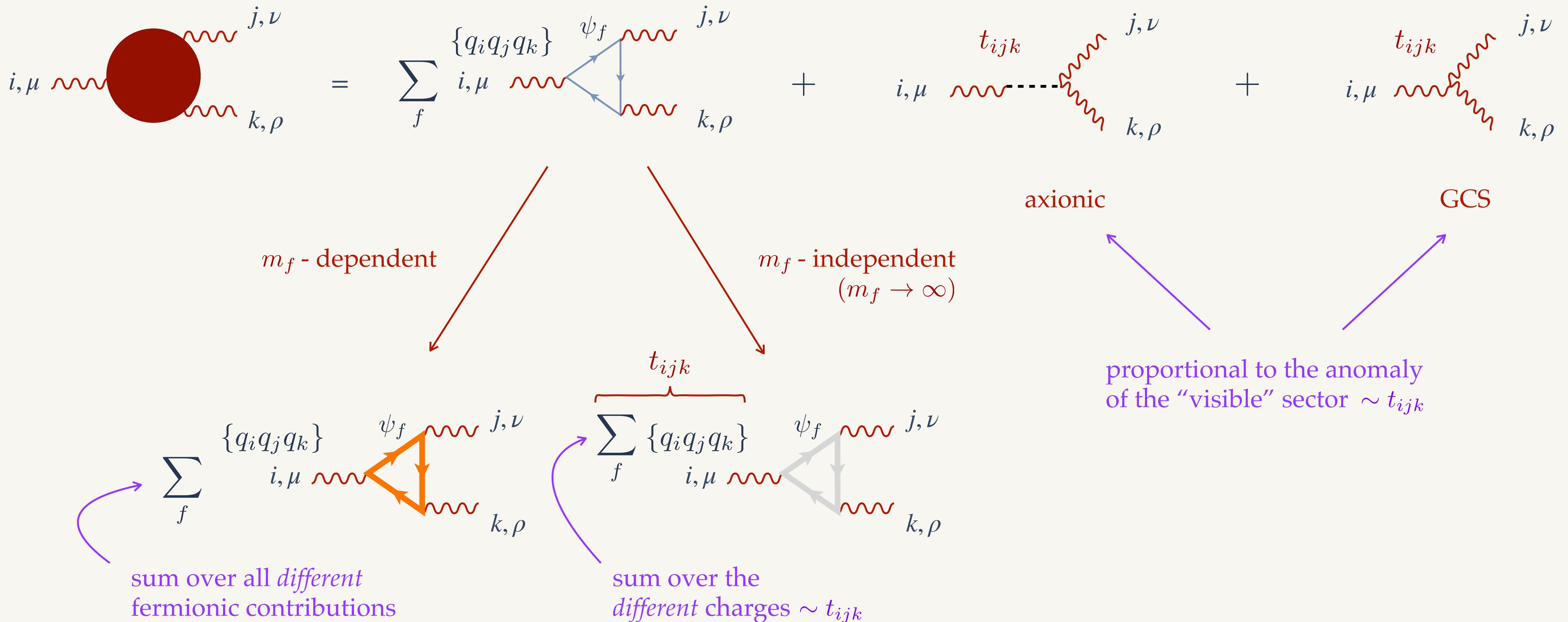
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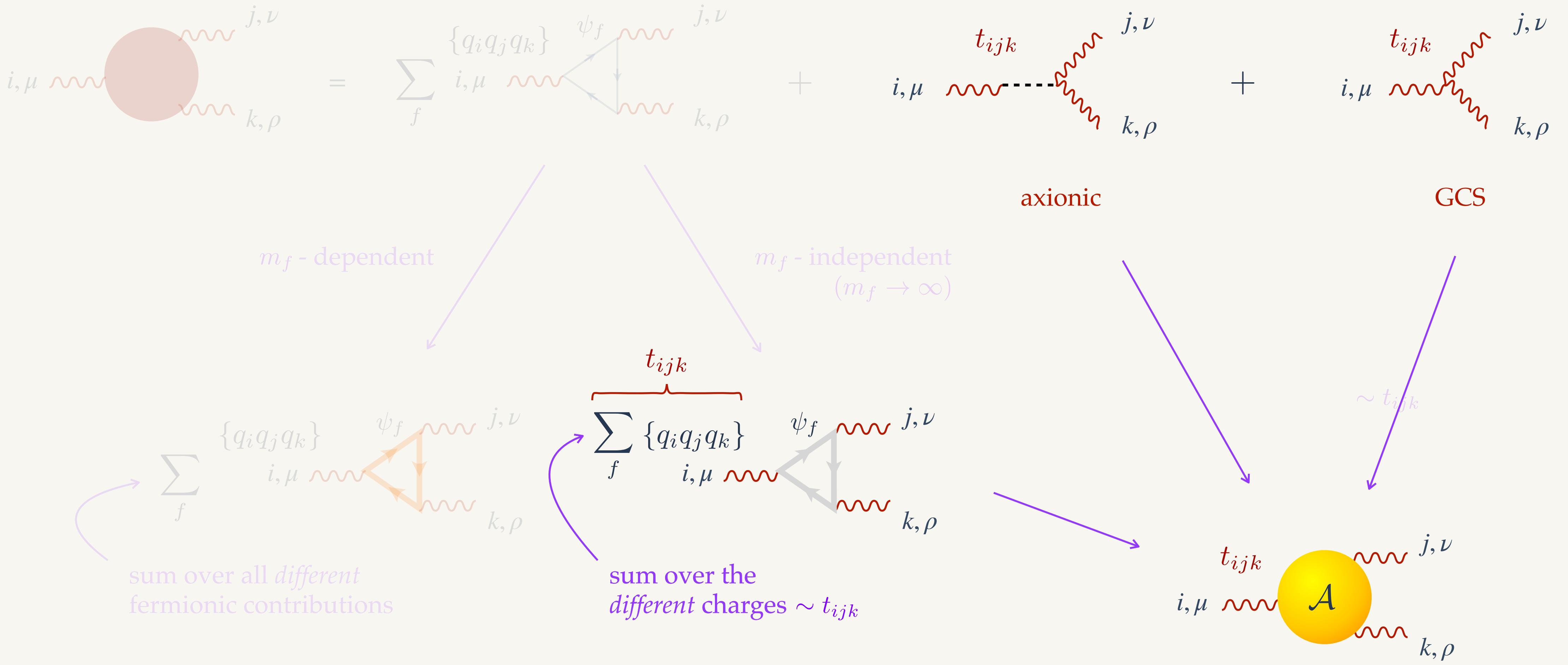
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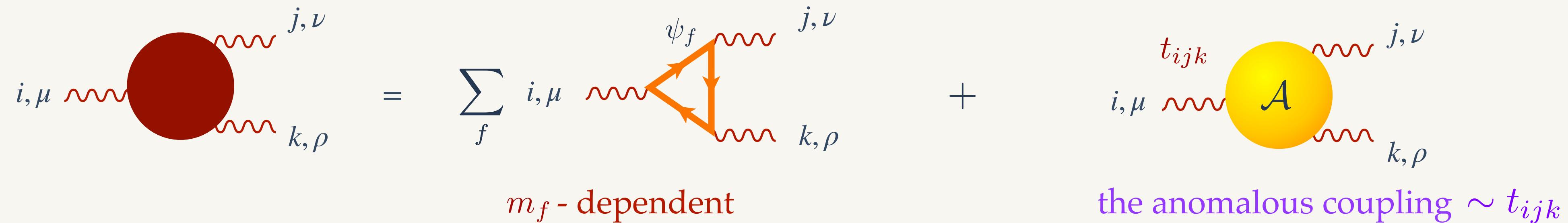
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the anomalous coupling $\sim t_{ijk}$

The anomalous 3-point coupling



The anomalous 3-point coupling

$$\begin{array}{c}
 \text{Diagram 1: } i, \mu \text{ (red wavy line)} \rightarrow \text{Red Circle} \xrightarrow{j, \nu} k, \rho \\
 \text{Diagram 2: } i, \mu \text{ (red wavy line)} \xrightarrow{\sum_f} \psi_f \text{ (orange triangle)} \xrightarrow{j, \nu} k, \rho \\
 \text{Diagram 3: } i, \mu \text{ (red wavy line)} \xrightarrow{t_{ijk}} \text{Yellow Circle} \xrightarrow{j, \nu} k, \rho
 \end{array}$$

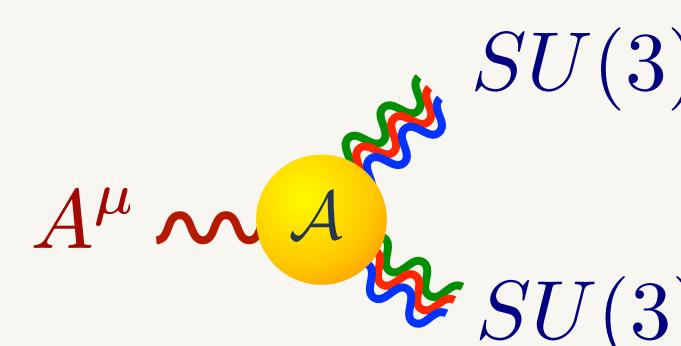
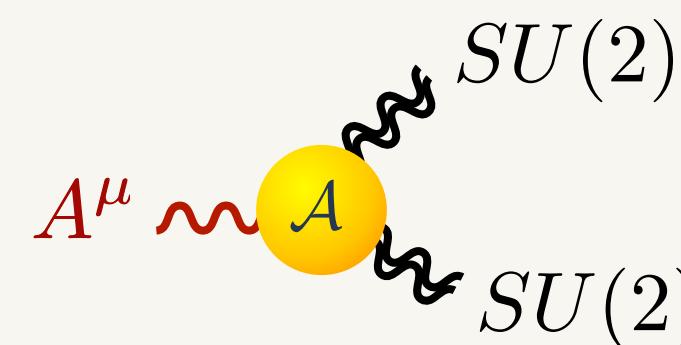
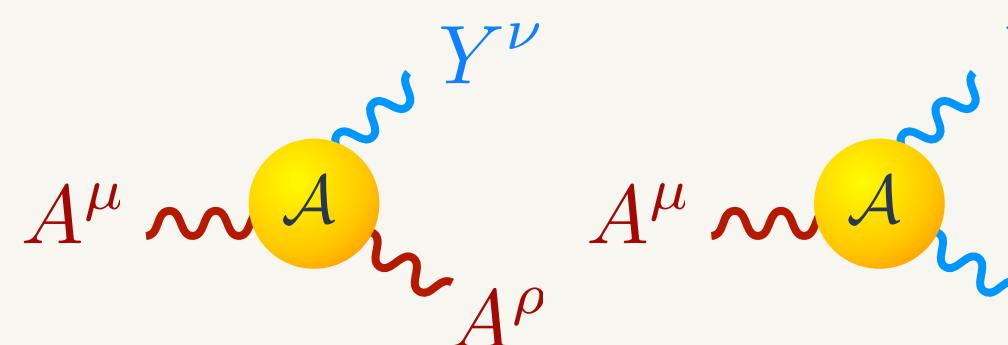
- always present
(anomalous/non-anomalous models)
 - depends on the mass m_f
The heavier the fermion the smaller
the contribution
 - depends on individual charges of the
fermion in the loop
 - drops in anomaly free models
 - does not depend on any mass
 - depends on the full anomaly

Standard Model as an anomalous EFT

Predictions

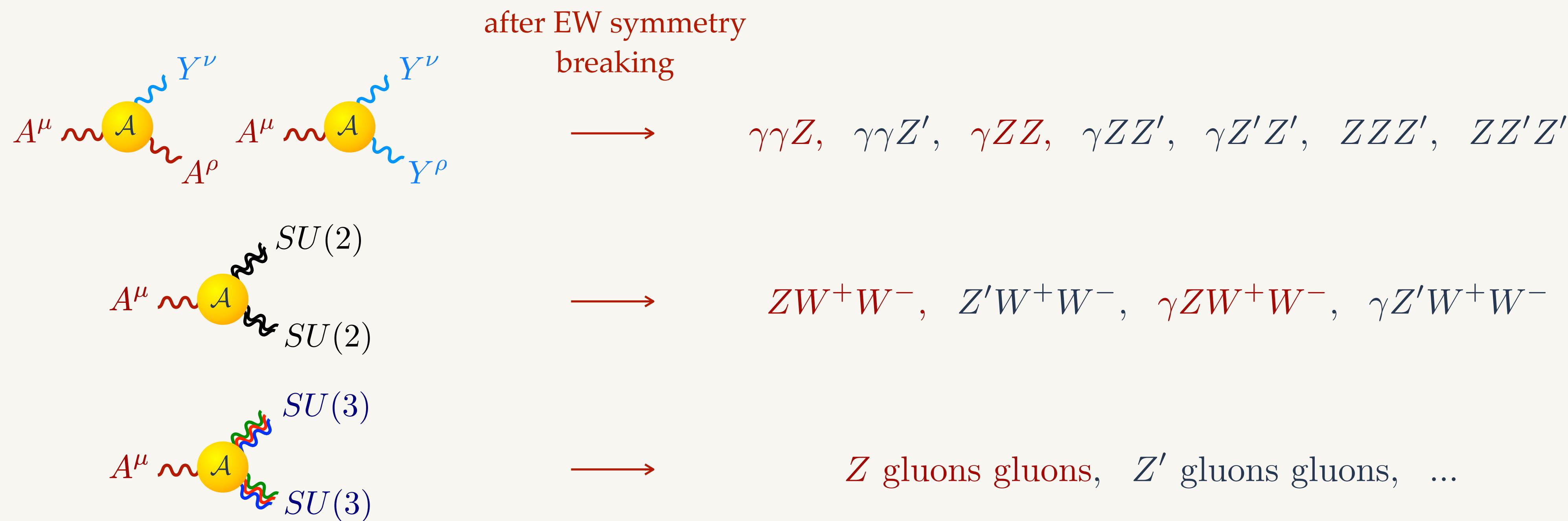
Standard Model as an anomalous EFT - Predictions

- This anomalous extension of the SM **predicts**
 - an **(anomalous) Z'** , (A^μ becomes Z' after EW symmetry breaking),
 - new anomaly related couplings (not gauge-invariant) that **cure** the model.



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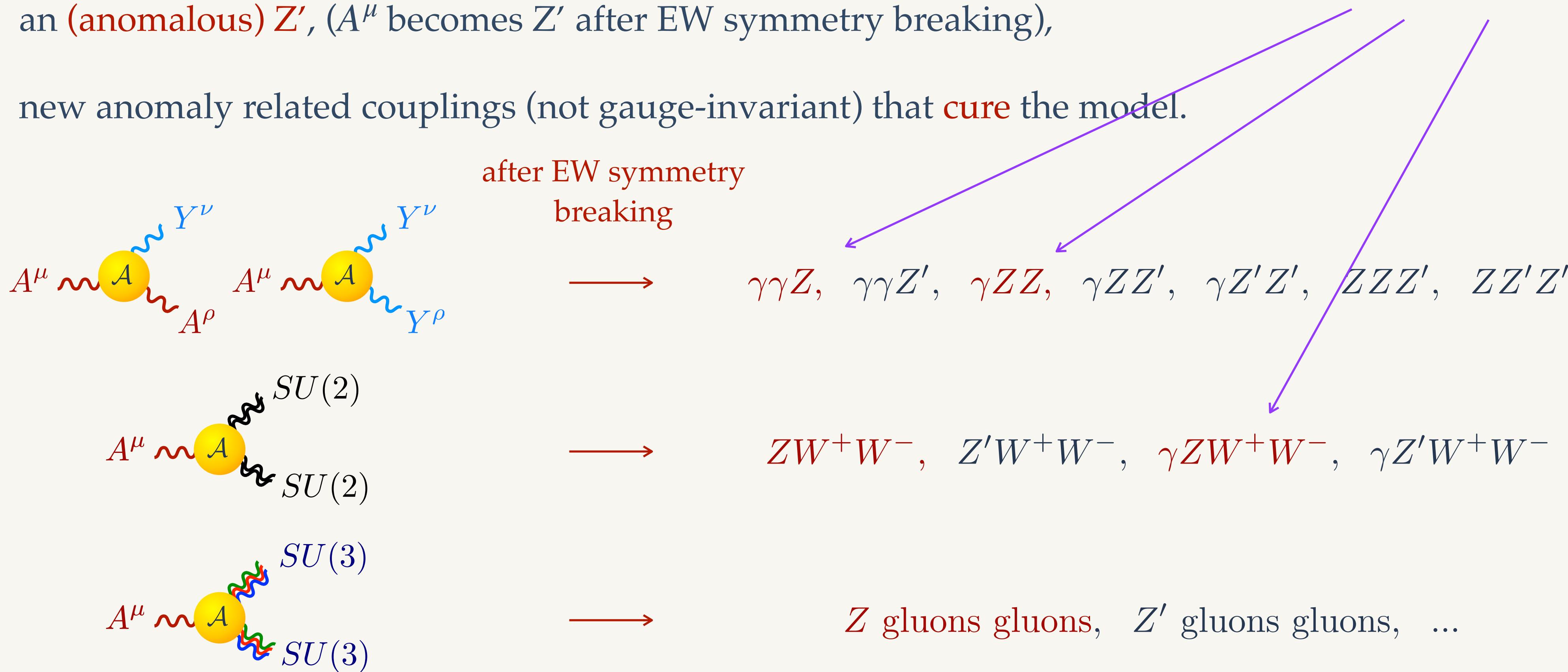
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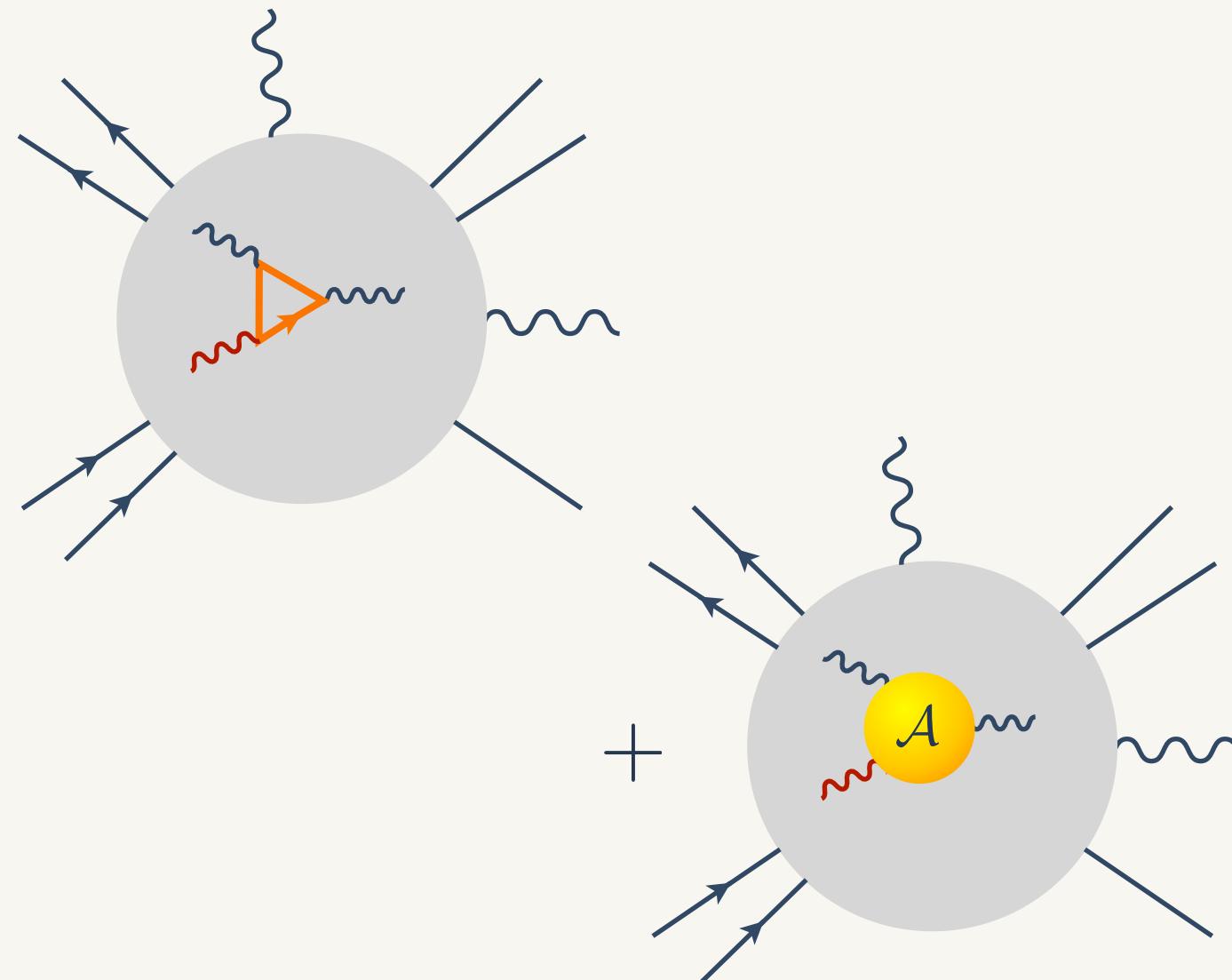
- * new anomaly related couplings (not gauge-invariant) that **cure** the model.

these couplings can
be tested today!



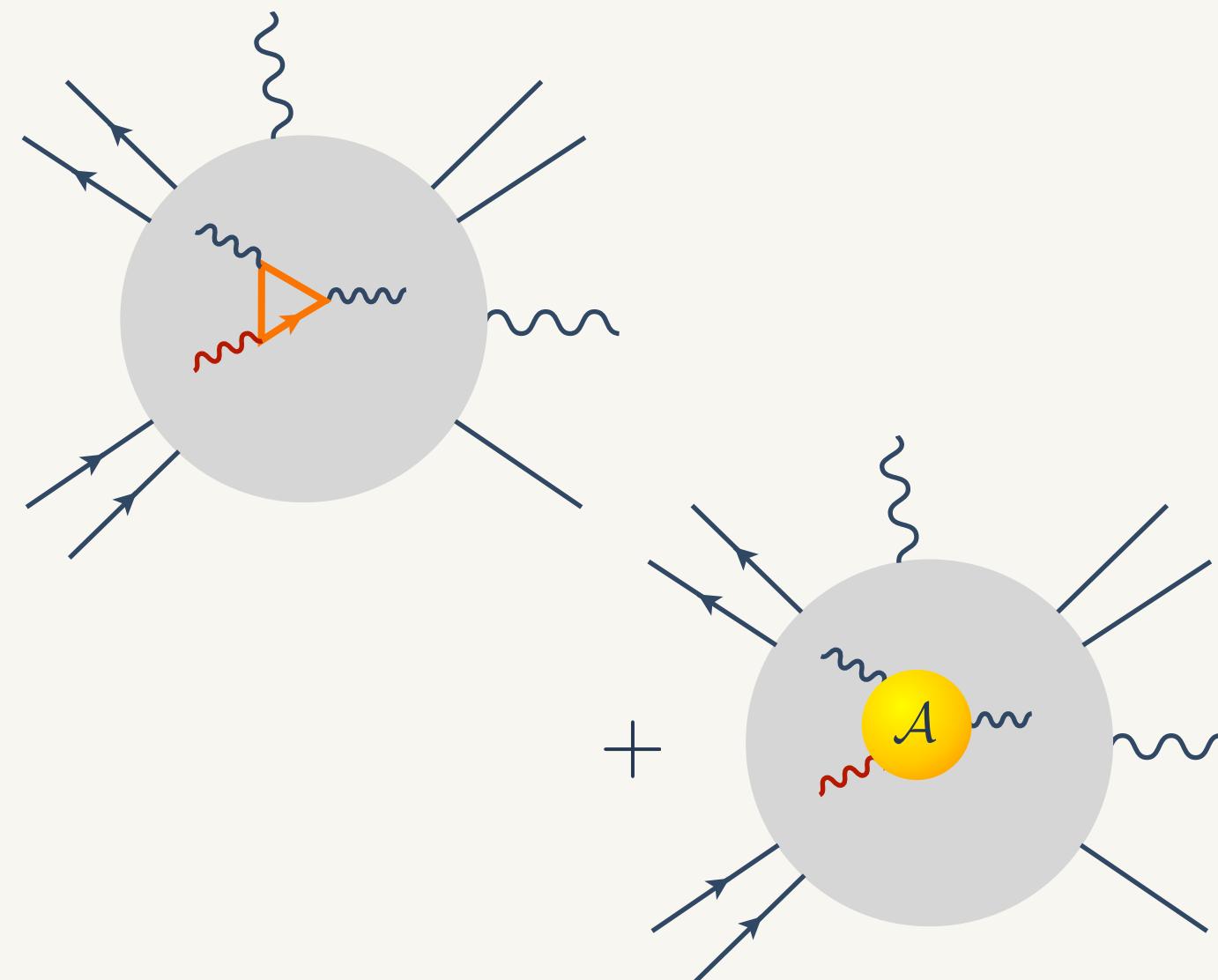
Standard Model as an anomalous EFT - Predictions

- The new couplings have the **same “structure”** like the standard fermionic triangle diagram.
- **New diagrams** appear where the triangle sub-diagram is replaced by the anomalous coupling.

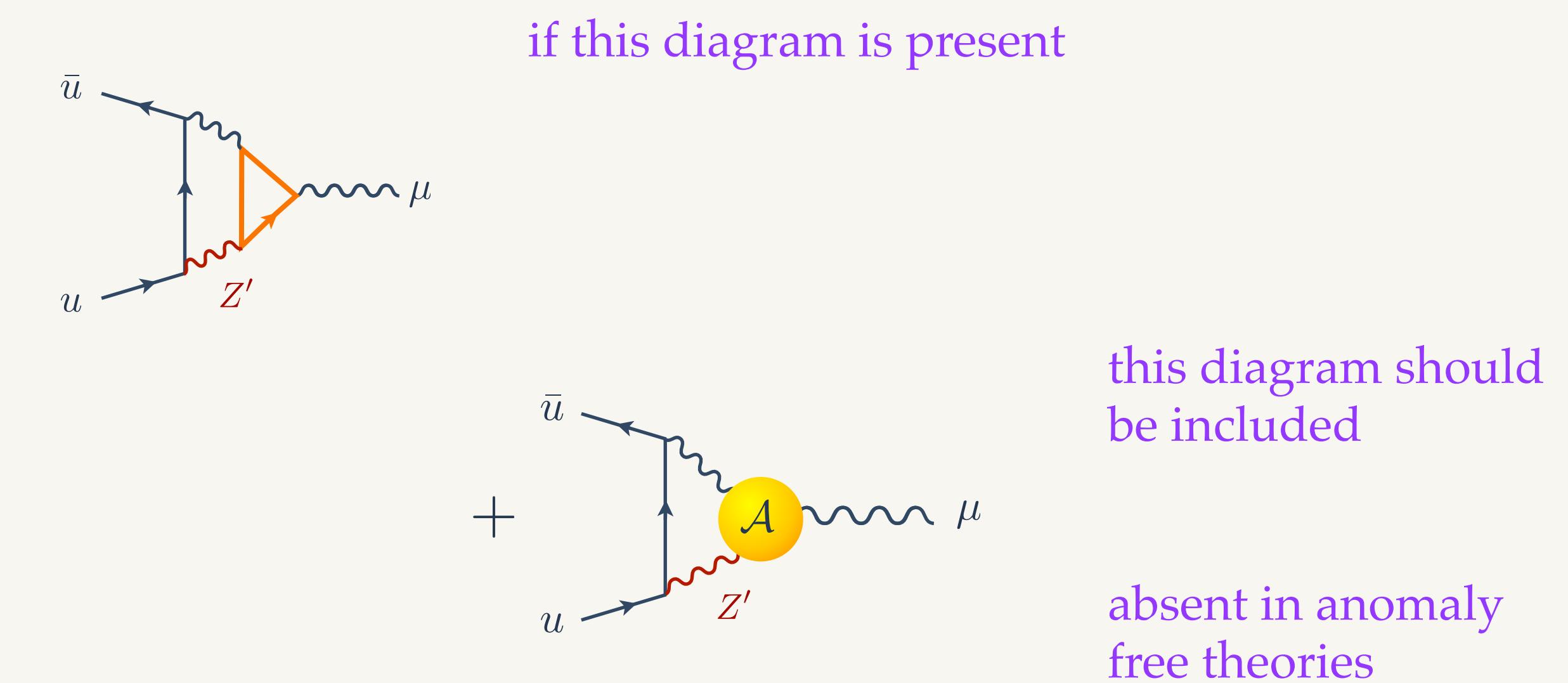


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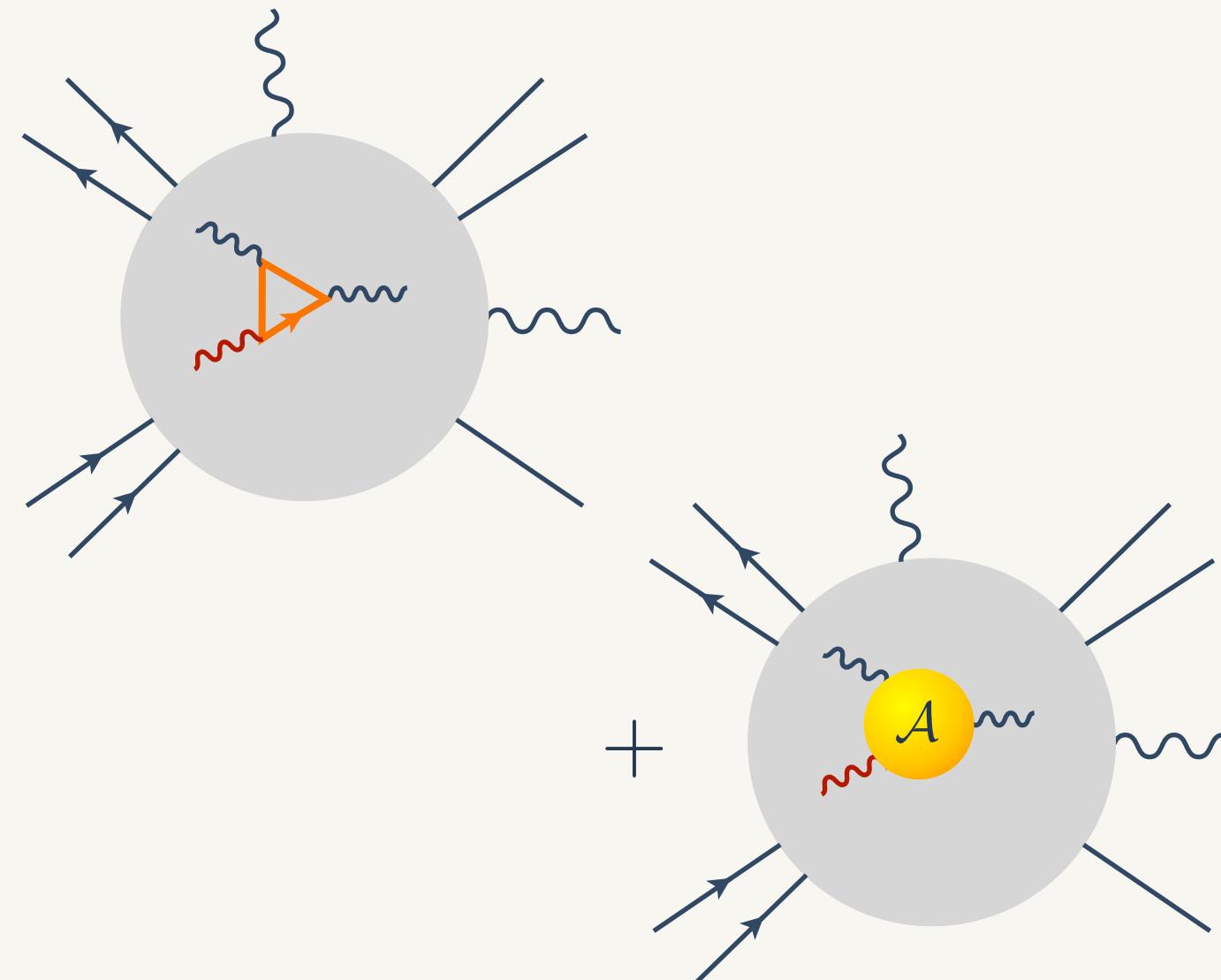


An example:

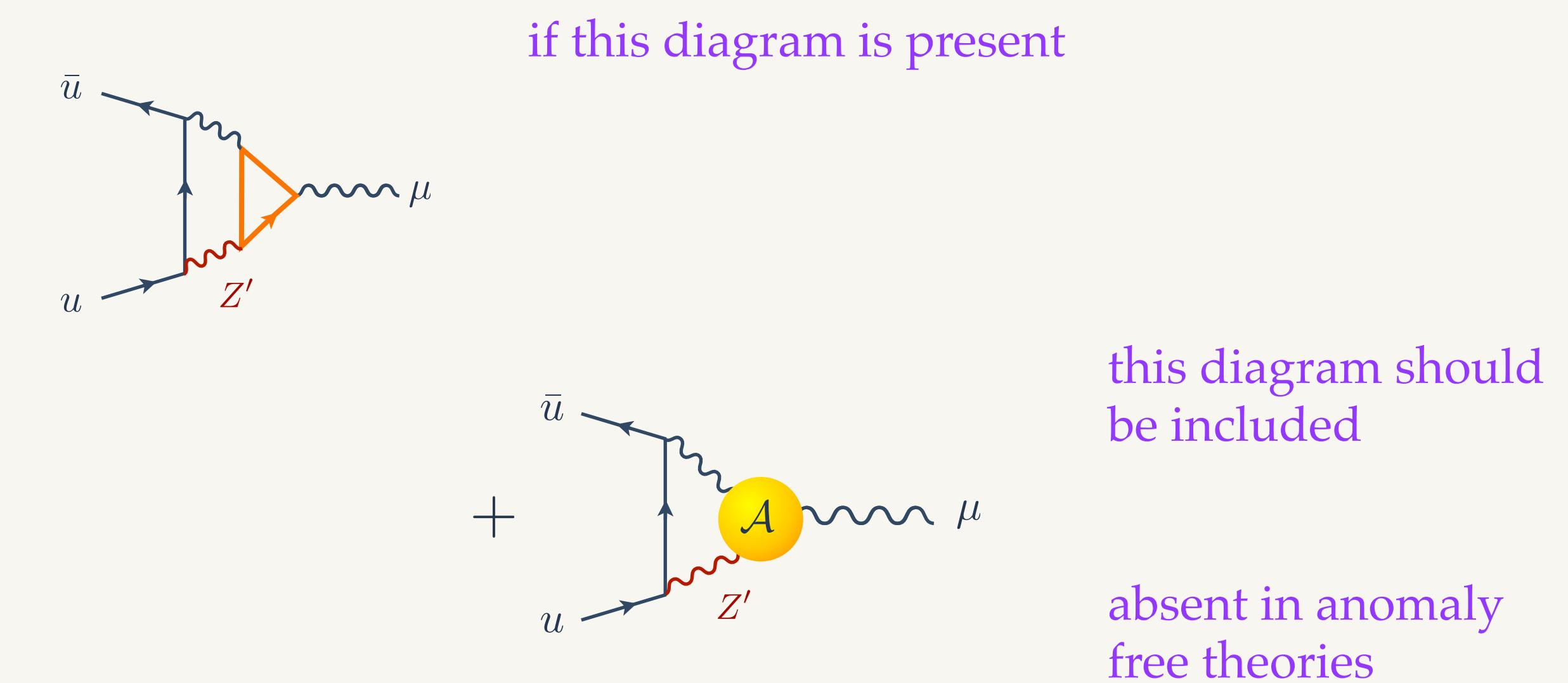


Standard Model as an anomalous EFT - Predictions

- The new couplings have the **same “structure”** like the standard fermionic triangle diagram.
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An example:



- **How significant** are the new (anomaly related) couplings?

How significant these terms are? How big is the anomaly?

- All these new terms are proportional to the anomaly.
 - How big is the anomaly?
 - How significant these new three-point couplings can be?
- Assuming that all 24 fermions of the SM have the same couplings, $q_{V/A}^f = 2$ it is

$$t_{ijk} = \sum_f \left(q_{A,i}^f q_{V,j}^f q_{V,k}^f + q_{V,i}^f q_{A,j}^f q_{V,k}^f + q_{V,i}^f q_{V,j}^f q_{A,k}^f + q_{A,i}^f q_{A,j}^f q_{A,k}^f \right)$$
$$24 \times (8 + 8 + 8 + 8) = 768$$

- The anomaly can be huge, making the anomalous coupling the leading at 1-loop.

Applications to the $g - 2$ of the muon

The $g - 2$ of the muon

- * Theoretical evaluation of the $g - 2$ of a fermion.

$$(-ie)\bar{u}(p') \left(\gamma^\mu F_1(q) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\ell} F_2(q) + \gamma^5 \frac{\sigma^{\mu\nu}q_\nu}{2m_\ell} F_3(q) + \gamma^5 (q^2\gamma^\mu - \gamma_\nu q^\nu q^\mu) F_4(q) \right) u(p)$$

(g - 2)/2 of the fermion

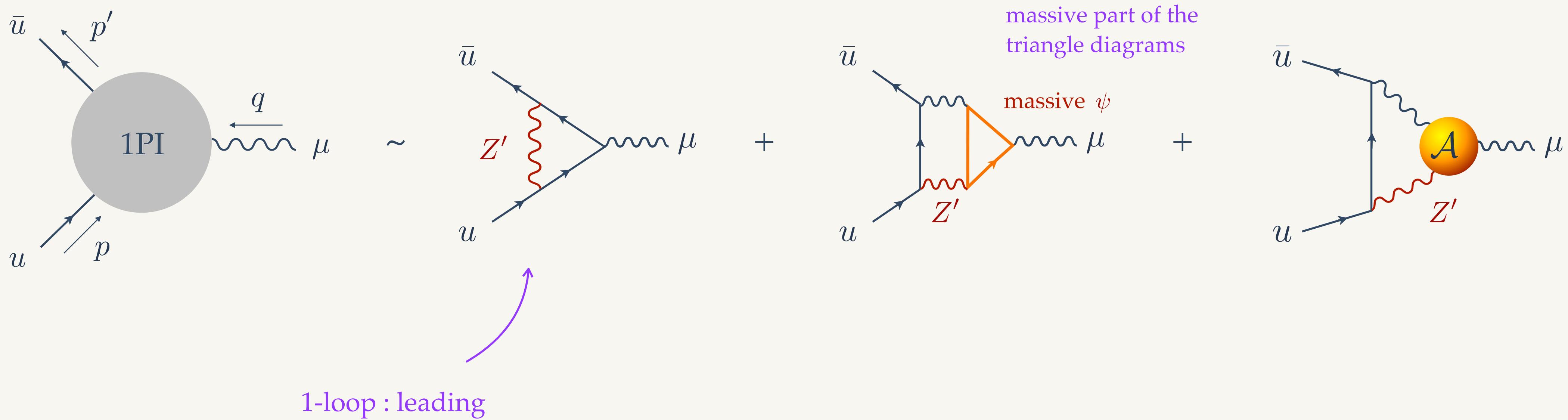
all possible diagrams

Schwinger

- * In “all possible diagrams”, propagate *all* possible fields in a model.
- * Several models have been built, to explain the discrepancy (extra Higgs, Z' , ...).
- * Our goal is to evaluate the contribution of *an anomalous Z'* to the $g - 2$ of the muon.

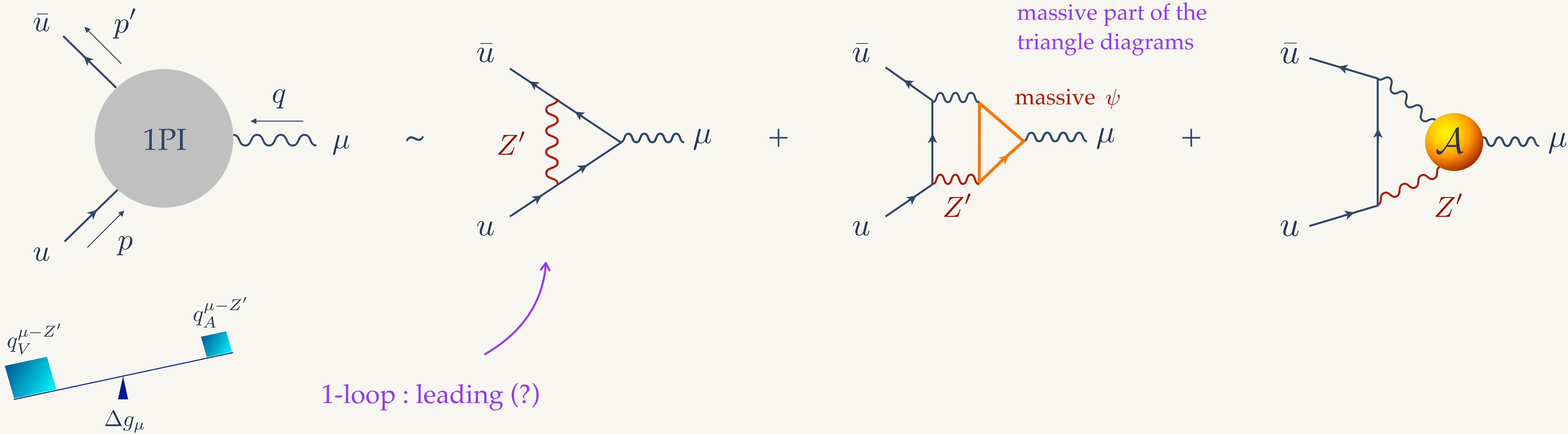
The $g - 2$ of the muon

- * We focus on the extra diagrams (from the presence of an anomalous Z')



The $g - 2$ of the muon

- We focus on the extra diagrams (from the presence of an anomalous Z')



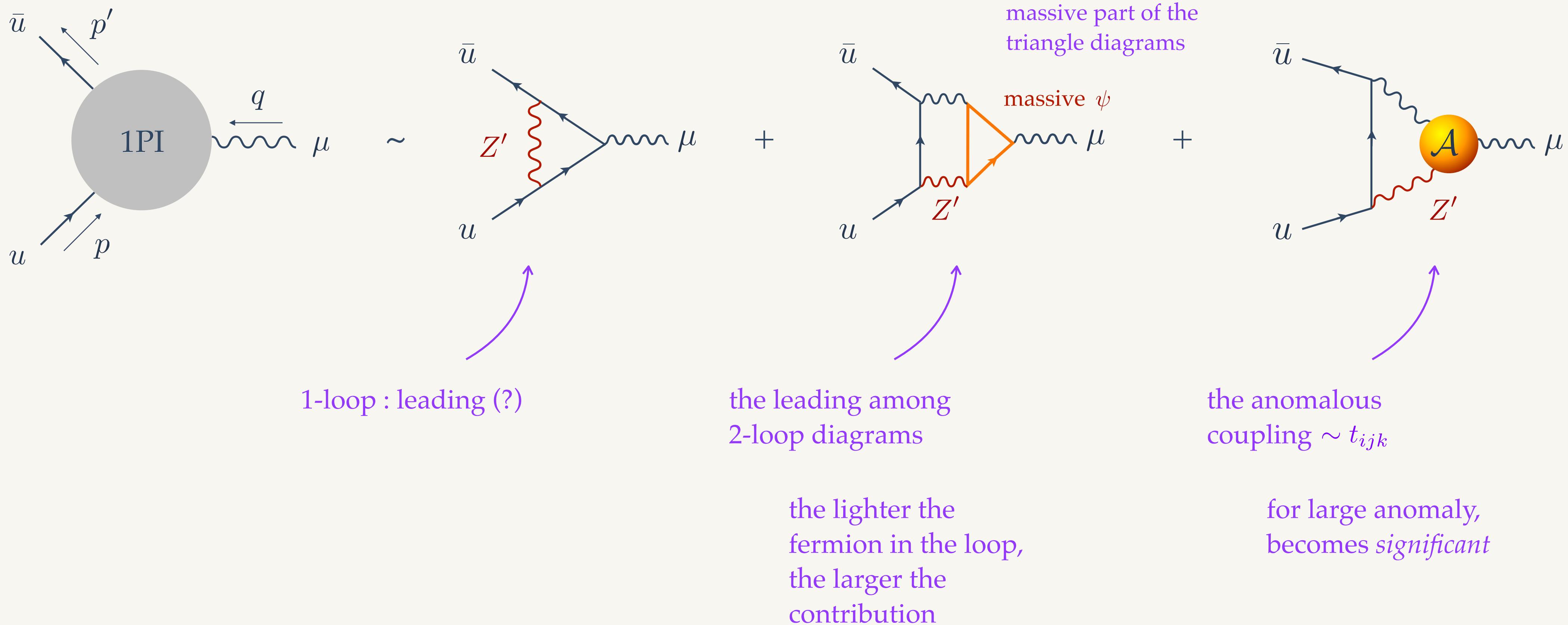
$$\Delta g_\mu/2 = \frac{1}{4\pi^2} \frac{m_\mu^2}{m_{Z'}^2} g_{Z'}^2 \left((q_V^{\mu-Z'})^2 \mathcal{F}_V \left[\frac{m_\mu^2}{m_{Z'}^2} \right] - (q_A^{\mu-Z'})^2 \mathcal{F}_A \left[\frac{m_\mu^2}{m_{Z'}^2} \right] \right)$$

$$\mathcal{F}_V \left[\frac{m_\mu^2}{m_{Z'}^2} \right] = \int_0^1 dx \frac{x^2(1-x)}{1-x+x^2m_\mu^2/m_{Z'}^2}$$

$$\mathcal{F}_A \left[\frac{m_\mu^2}{m_{Z'}^2} \right] = \int_0^1 dx \frac{x(1-x)(4-x) + 2x^3m_\mu^2/m_{Z'}^2}{1-x+x^2m_\mu^2/m_{Z'}^2}$$

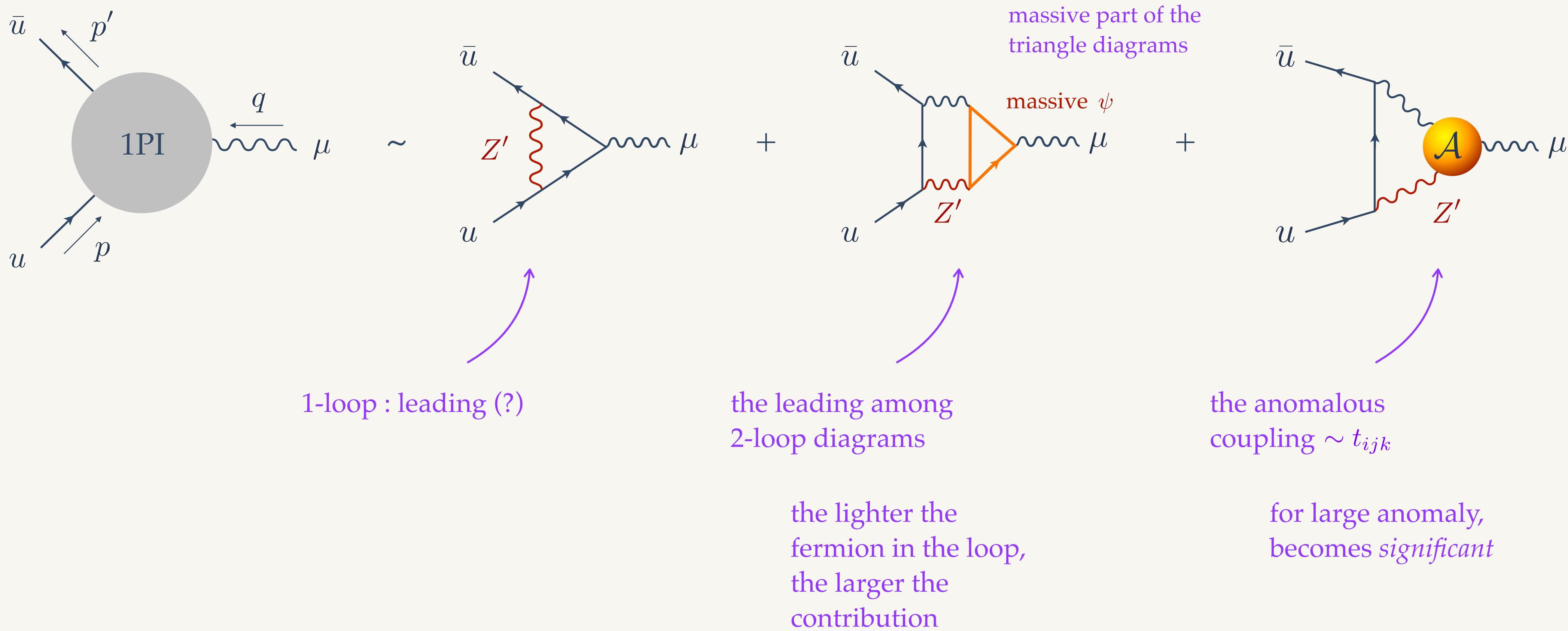
The $g - 2$ of the muon

- * We focus on **the extra diagrams** (from the presence of an anomalous Z')



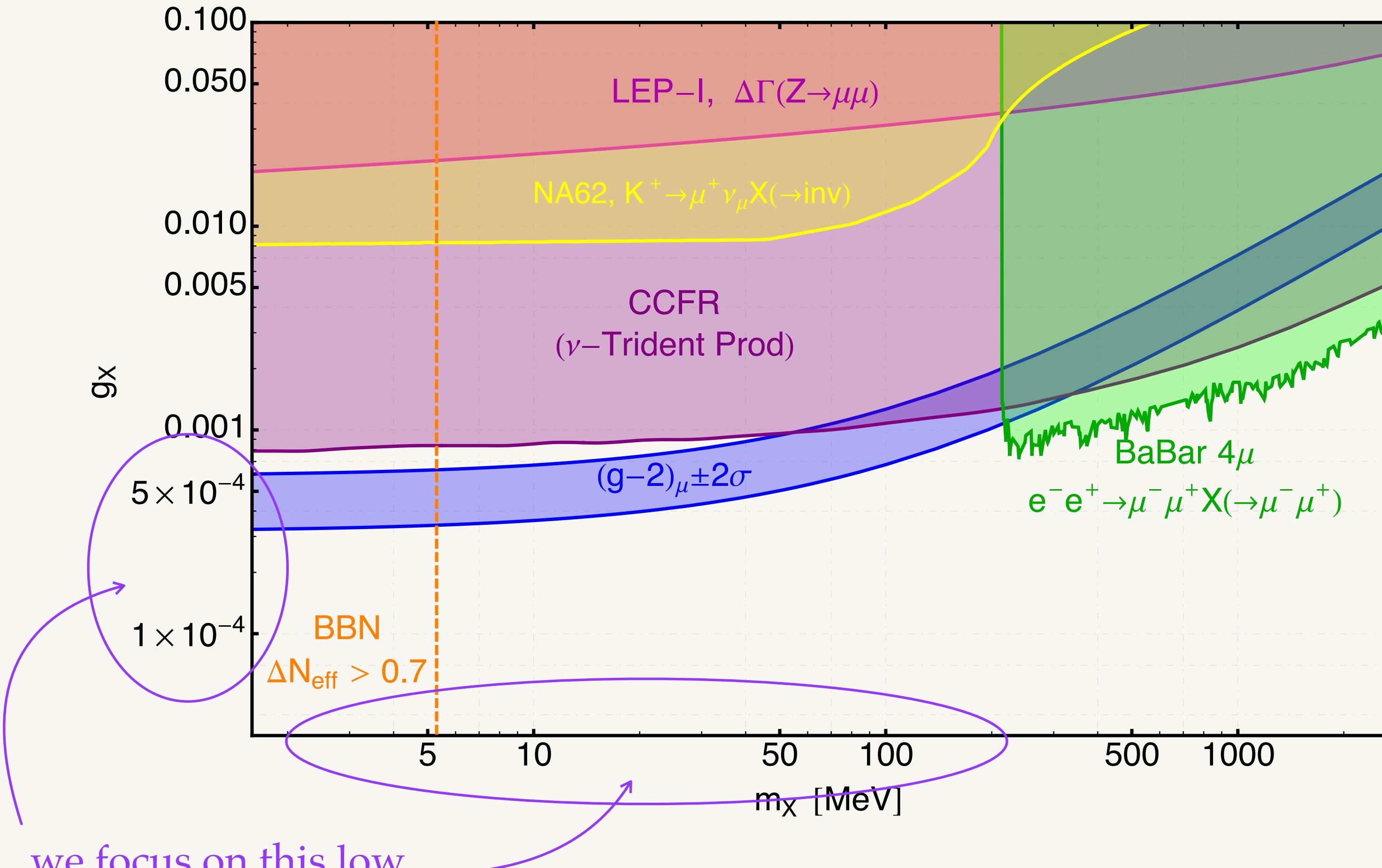
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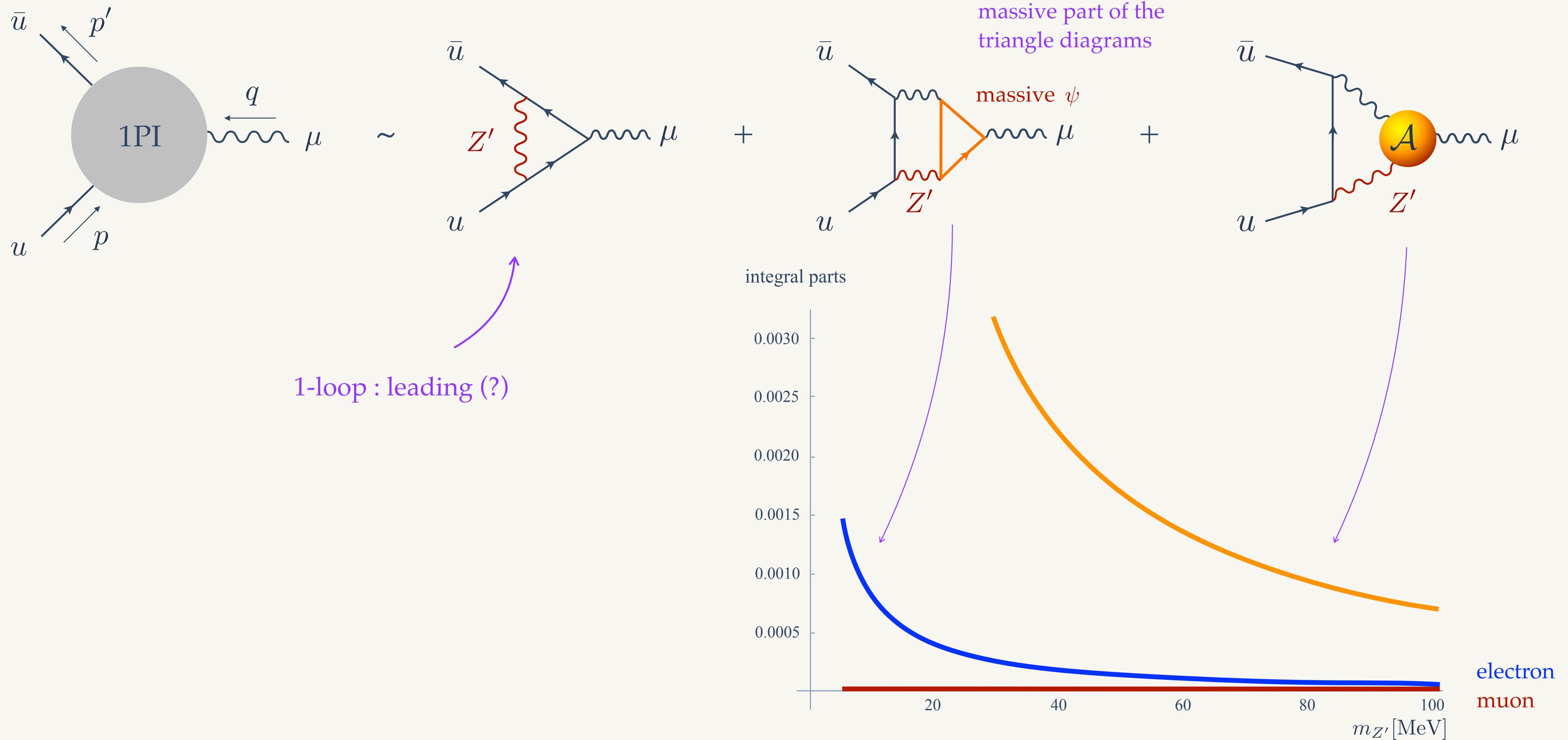
We can compare we need to some numbers

Mass and couplings



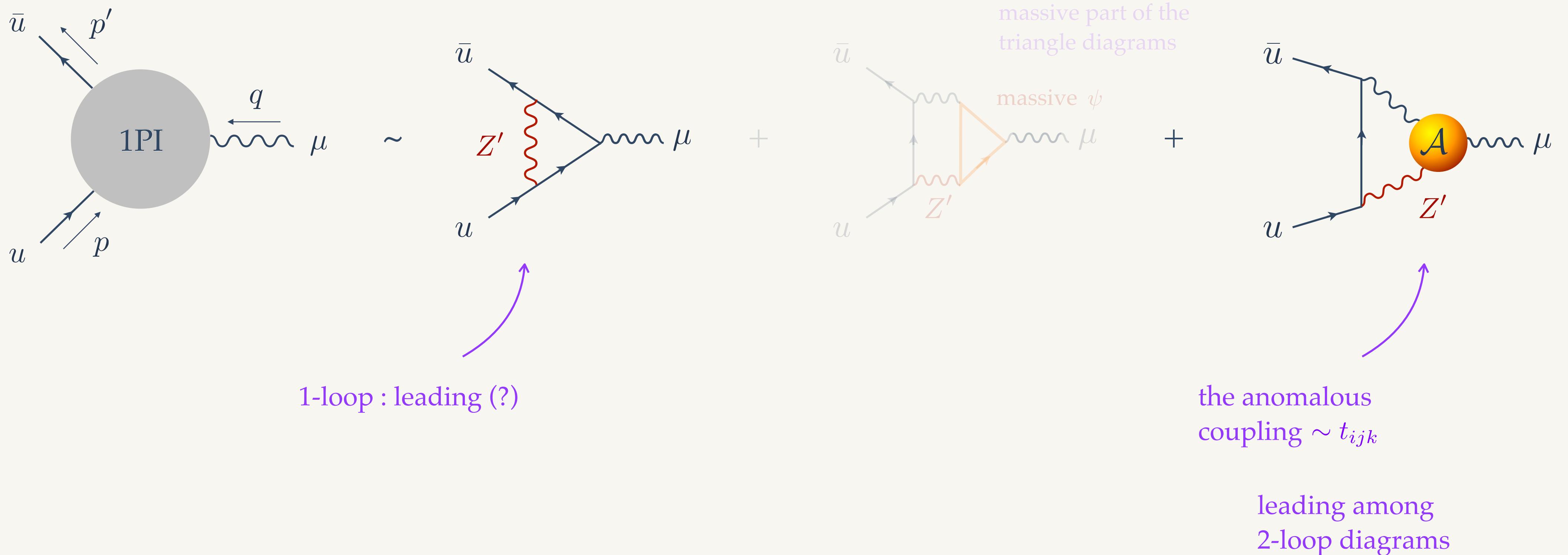
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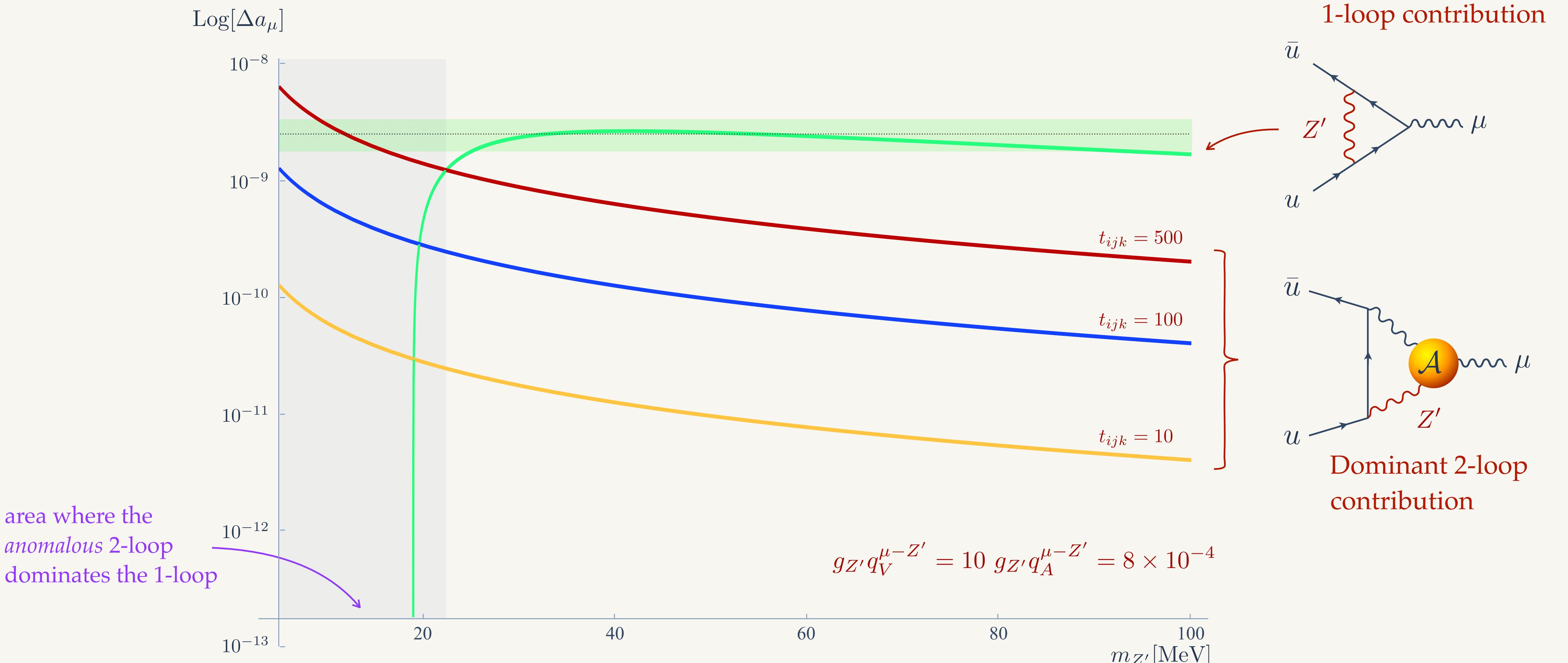


The $g - 2$ of the muon

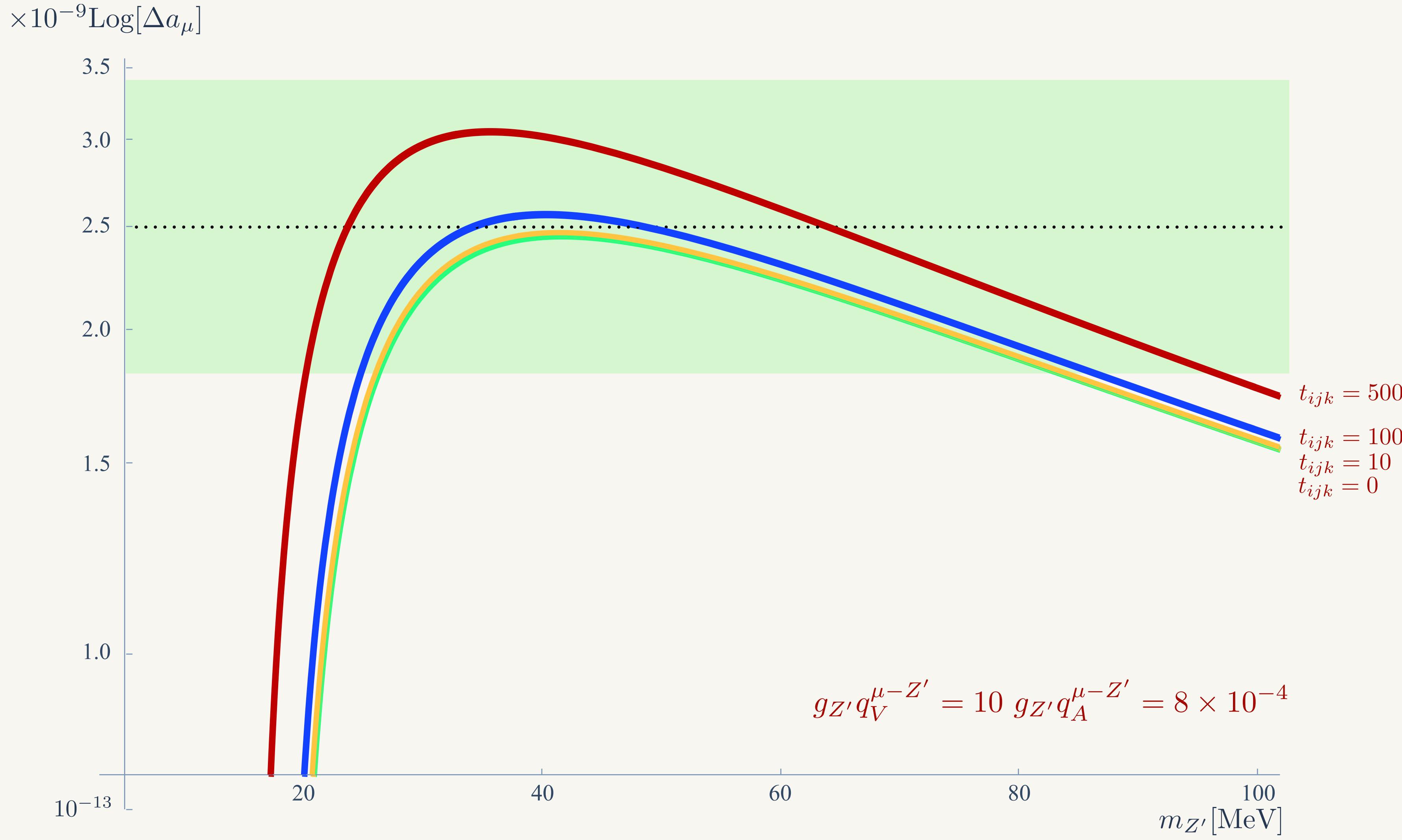
- * We focus on **the extra diagrams** (from the presence of an anomalous Z')



The $g - 2$ of the muon



The $g - 2$ of the muon



Conclusions and future plans

Conclusions and future plans

- * I argued that
 - * fundamental theories (**anomaly free** in the UV) can appear to be **anomalous** in the IR.
 - * Effective 3-point couplings (axionic & GCS) **cure** the “anomalies”.
 - * These new couplings **depend on the anomaly**.

Conclusions and future plans

- * I argued that
 - * Z' gauge bosons are more “likely” to be (superficially) **anomalous** than not.
 - * The anomalous behaviour appears even in couplings that **involve only SM fields**.
 - * The **anomaly can be huge**, these new terms can be significant (leading at 2-loops).

Conclusions and future plans

- * Theoretical directions
 - * Generalise the anomalous cancellation mechanism in **effective theories in higher dims (6D)**.
Anastasopoulos Benakli Goodsell
 - * Study the cutoff scale of the effective actions in comparison with the **weak gravity conjecture**.
Anastasopoulos Benakli Goodsell
- * Phenomenological directions
 - * Analyse **anomalous Z' models** in view of the **new anomalous couplings**, especially the ones containing only SM gauge fields $\gamma\gamma Z$, γZZ , ZW^+W^- , γZW^+W^- , Z gluons gluons ...
Anastasopoulos Dedes Kiritis
 - * Study effective models with the **features described before** (total anomaly cancellation but not individually in each sector).
Anastasopoulos Antoniadis Rondeau
 - * Apply the new anomalous couplings to **Dark Matter**.
Anastasopoulos Cléry Dudas Mambrini

Thank you