Observable primordial gravitational waves from Cosmic Inflation

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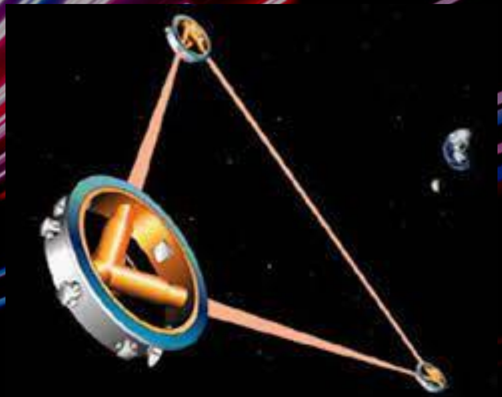
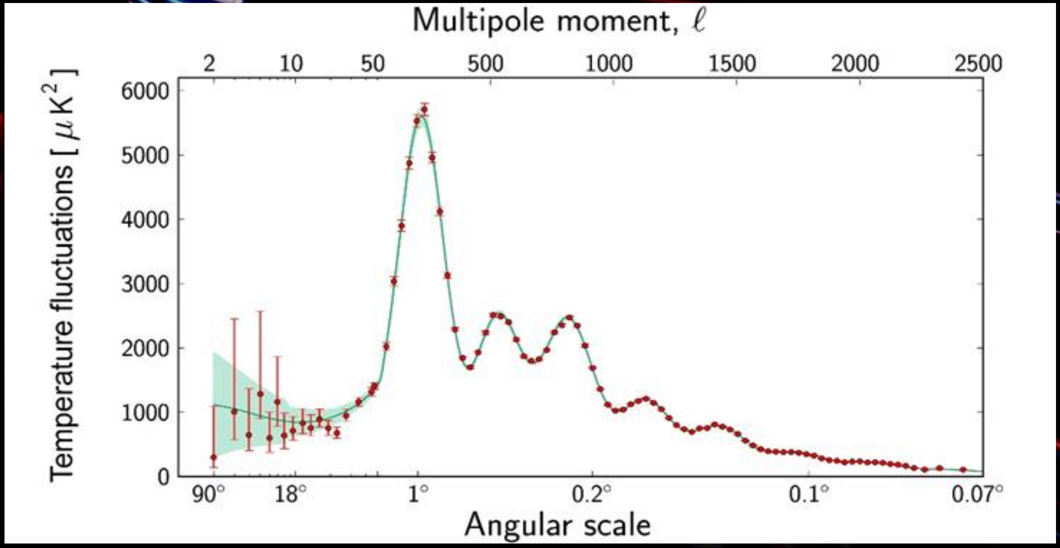
Cosmic Inflation

- The history of the Universe requires special initial conditions
- Which are arranged by cosmic inflation
- Cosmic Inflation: Period of accelerated expansion
  in the Early Universe



- Inflation produces a Universe large, flat and uniform
- Inflation also produces the primordial density perturbations (PDP) necessary for galaxies to form
- The PDP reflect themselves onto the CMB through the Sachs-Wolfe effect
- Impressive agreement with observations



- Inflation also generates
  Primordial Gravitational Waves
- This prediction of inflation will soon be tested
  - Indirectly (CMB)
  - Directly (interferometers)

Particle Production of Gravitational Waves during Cosmic Inflation

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

- **The metric perturbation is:**

  DoF: 6-1-3=2

  $+, \times$ polarisations

  - Symmetric $h_{ij} = h_{ji}$ $\quad e^s_{ij} = e^s_{ji}$
  - Traceless $h^i_i = 0$ $\quad {e^s}_i^{\ i} = 0$
  - Transverse $\nabla_i h^{ij} = 0$ $\quad k^i e^s_{ij} = 0$

- **The polarization tensor is:**

Fourier Xform: $$h_{ij}(\tau, \vec{x}) = \sqrt{16\pi G} \int \frac{d^3k}{(2\pi)^{3/2}} h_{ij}(\vec{k}, \tau) e^{i\vec{k}\cdot\vec{x}}$$

$$h_{ij}(\vec{k}, \tau) = \sum_{s=+,\times} h^s_k \, e^s_{ij}(\vec{k})$$

Action: $$S_{GW} = \frac{1}{64\pi G} \int d^4x \sqrt{-g}\, g^{\mu\nu} \partial_\mu h_{ij} \partial_\nu h^{ij}$$

EoM: $$h''_{ij} + 2\frac{a'}{a} h'_{ij} - \nabla^2 h_{ij} = 0 \Rightarrow h^{s\,\prime\prime}_k + 2\frac{a'}{a} h^{s\,\prime}_k + k^2 h^s_k = 0$$

Mukhanov-Sasaki Eqn

$$\left.\begin{aligned} h^s_k(\tau) &\equiv \sqrt{16\pi G}\, \phi^s_k(\tau) \\ v^s_k(\tau) &\equiv a(\tau)\phi^s_k(\tau) \end{aligned}\right\} \Rightarrow v^s_k = a\, h^s_k\, m_P/\sqrt{2}$$

$$m_P^{-2} = 8\pi G$$

$$\boxed{v^{s\,\prime\prime}_k + \left(k^2 - \frac{a''}{a}\right) v^s_k = 0}$$

Particle Production of Gravitational Waves during Cosmic Inflation

Quantize: $$\boxed{v^s(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left[v_k^s\, \hat{a}^s_{\vec{k}}\, e^{i\vec{k}\cdot\vec{x}} + (v_k^s)^*\, \hat{a}^{s\dagger}_{\vec{k}}\, e^{-i\vec{k}\cdot\vec{x}}\right]}$$

$$[\hat{a}^s_{\vec{k}}, \hat{a}^{r\dagger}_{\vec{q}}] = \delta^{sr}\delta^{(3)}(\vec{q}-\vec{k}) \quad \& \quad [\hat{a}^s_{\vec{k}}, \hat{a}^r_{\vec{q}}] = [\hat{a}^{s\dagger}_{\vec{k}}, \hat{a}^{r\dagger}_{\vec{q}}] = 0$$

Solution: $$\boxed{v_k^s(\tau) = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)e^{-k\tau}}$$

- **Subhorizon:** $-k\tau \to +\infty \quad \Rightarrow \quad v_k^s$
- **Superhorizon:** $-k\tau = \frac{k}{aH} \to 0 \quad \Rightarrow \quad v_k^s$

$$h_k^s = \frac{\sqrt{2}v_k^s}{a\, m_P} = \frac{iH}{m_P k^{3/2}} \Rightarrow |h_k^s|^2 = \frac{H^2}{m_P^2 k^3}$$ = c

$$\mathcal{P}_h(k) = \frac{k^3}{2\pi^2}\langle h_{ij}(k)h^{ij}(k)\rangle = \frac{k^3}{\pi^2}\sum_{s=+,\times}|h_k^s|^2 =$$

$$\left.\begin{array}{l} \mathcal{P}_\zeta(k) = \dfrac{H^2}{8\pi^2\epsilon m_P^2} \\ \epsilon \equiv -\dfrac{\dot{H}}{H^2} \end{array}\right\} \Rightarrow \boxed{r \equiv \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = 16\epsilon}$$

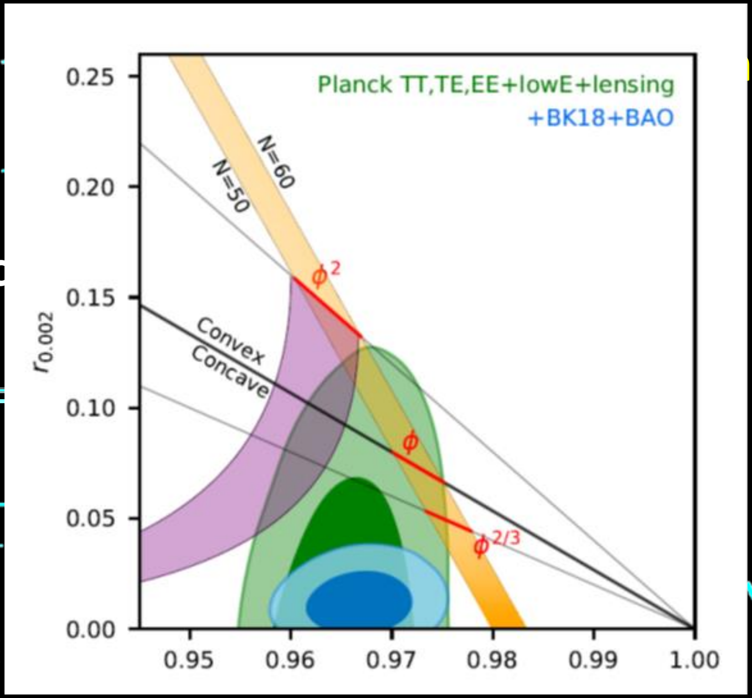
$h_k^s(\tau) \equiv \sqrt{1}$

$$0 < r < 0.036$$

$$\Rightarrow H \lesssim 10^{13}\ \text{GeV}$$

$v_k^s(\tau) \equiv$

$\sqrt{2}$



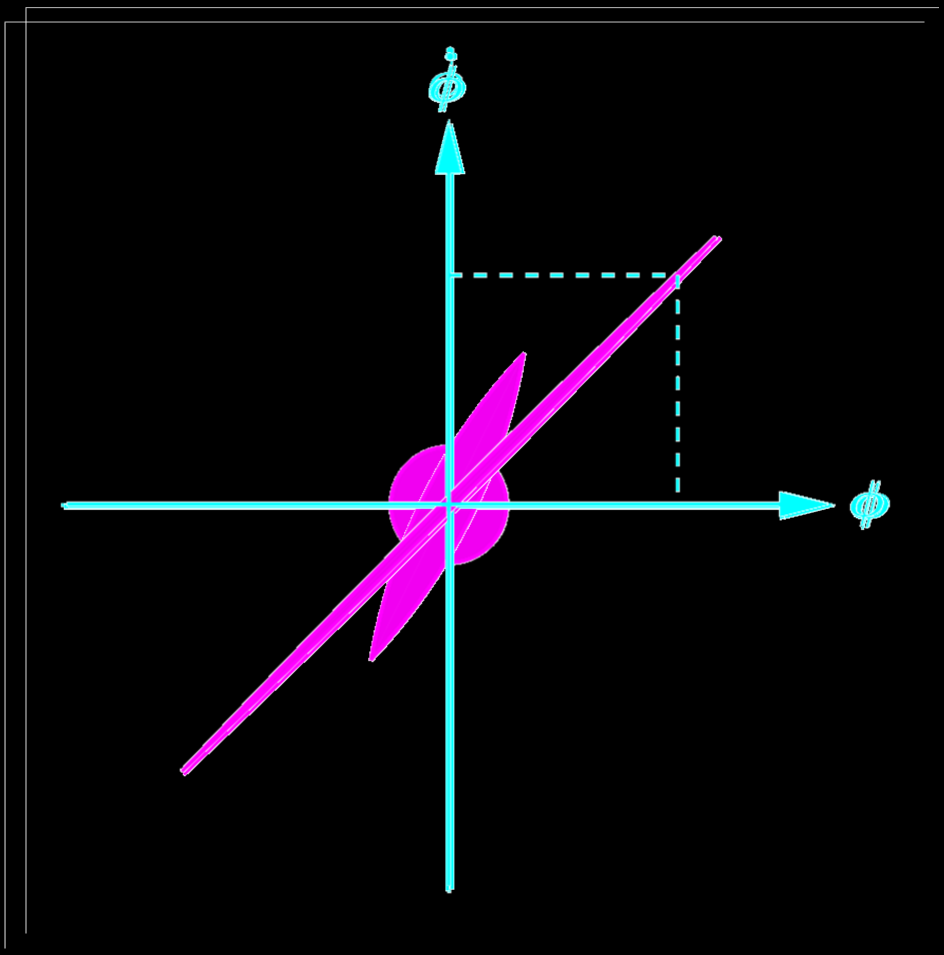
Quantum decoherence

- The quantum states become squeezed and approximate classicality

$$[\phi, \dot{\phi}] = i\hbar$$

$$\phi, \dot{\phi} \gg [\phi, \dot{\phi}] \to 0$$

- The commutator becomes negligible



Density of Gravitational Waves

Action: $$S_{\mathrm{GW}} = \frac{1}{64\pi G}\int \mathrm{d}^4x\sqrt{-g}\, g^{\mu\nu}\partial_\mu h_{ij}\partial_\nu h^{ij}$$

Energy-momentum: $$T_{\mu\nu}^{\mathrm{GW}} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{GW}}}{\delta g^{\mu\nu}} = \frac{\langle \nabla_\mu h_{ij}\nabla_\nu h^{ij}\rangle}{32\pi G}$$

$$\rho_{\mathrm{GW}} = -T_0^0 = a^{-2}T_{00} = \frac{1}{32\pi G}\frac{|h'_{ij}|^2+|\nabla h_{ij}|^2}{a^2} \quad \& \quad p_{\mathrm{GW}} = a^{-2}T_i^i$$

$$\langle\rho_{\mathrm{GW}}\rangle = \int_0^{+\infty}\frac{k^3}{2\pi^2}\sum_{s=+,\times}\frac{|h_k^{s\prime}|^2+k^2|h_k^s|^2}{a^2}\,\mathrm{d}(\ln k)$$

$$\langle p_{\mathrm{GW}}\rangle = \int_0^{+\infty}\frac{k^3}{2\pi^2}\sum_{s=+,\times}\frac{|h_k^{s\prime}|^2-\frac{1}{3}k^2|h_k^s|^2}{a^2}\,\mathrm{d}(\ln k)$$

$$h_k^s \propto e^{-ik\tau}/a \;\Rightarrow\; h_k^{s\prime} = ik\,h_k^s \;\Rightarrow\; |h_k^{s\prime}|^2 = k^2|h_k^2|^2$$

$$w_{\mathrm{GW}} = \frac{p_{\mathrm{GW}}}{\rho_{\mathrm{GW}}} = \frac{\frac{2}{3}k^2|h_k^s|^2}{2k^2|h_k^s|^2} = \frac{1}{3} \quad \Rightarrow \quad \boxed{\rho_{\mathrm{GW}} \propto a^{-4}} \qquad \rho_c = \frac{3H^2}{8\pi G}$$

GW redshift as radiation

- **Density per momentum interval:**

$$\Omega_{\mathrm{GW}}(k) \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln k} \;\Rightarrow\; \Omega_{\mathrm{GW}}(k) = \frac{k^3}{2\pi^2}\frac{8\pi G}{3H^2}\sum_{s=+,\times}\frac{|h_k^{s\prime}|^2+k^2|h_k^s|^2}{a^2}$$

Density of Gravitational Waves

Time evolution: $\Omega_{GW}(k,\tau) = \frac{k^2\Delta_h^2(k,\tau)}{12a^2H^2}$ with $\Delta_h^2(k,\tau) = T_h(k,\tau)\mathcal{P}_h(k)$

- Transfer function $T_h =$

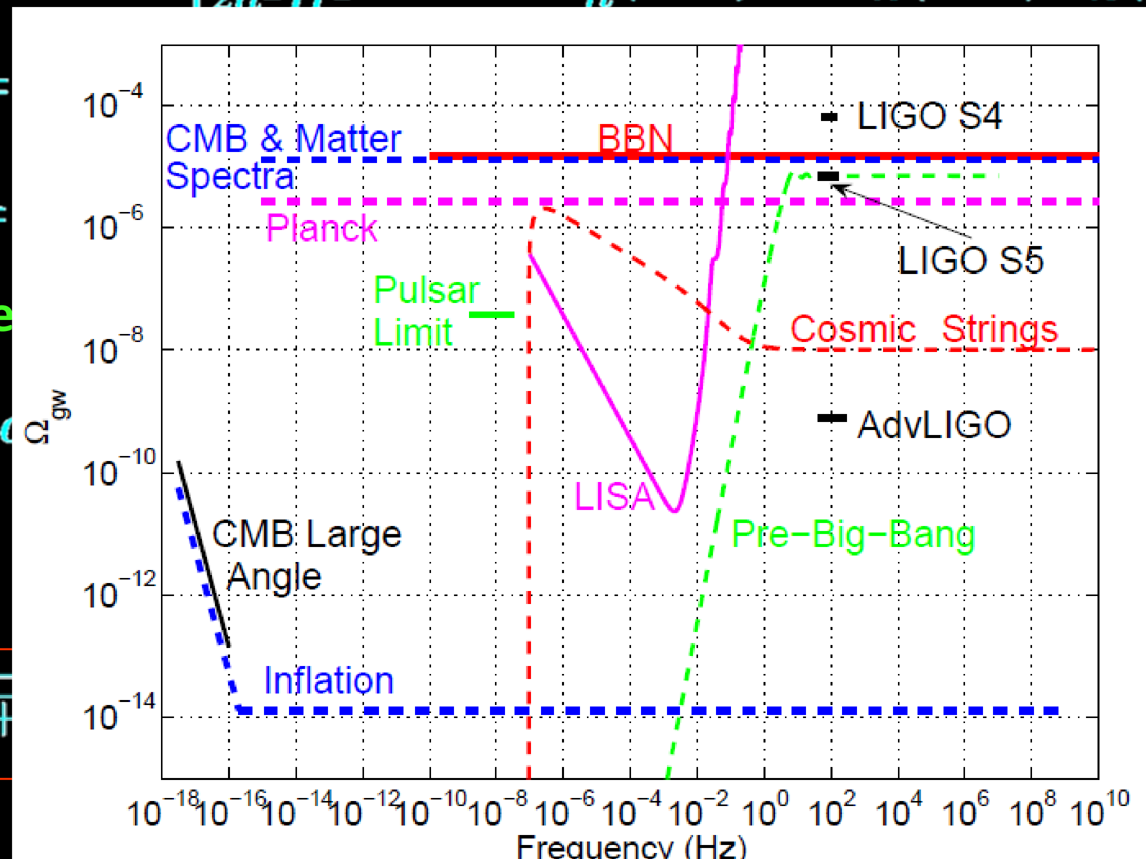
Today: $T_h^0 =$

- The transfer function depe

$a(\tau) = a_i\left[1 + \frac{1+3w}{2}\right.$

$\Omega_{GW}(k) \to \Omega_{GW}(f)$

$\Omega_{GW}(f) \propto f^{-2(\frac{1-}{1+}}$



- Radiation domination

$w = \frac{1}{3} \Rightarrow \Omega_{GW}(f) = \text{constant}$

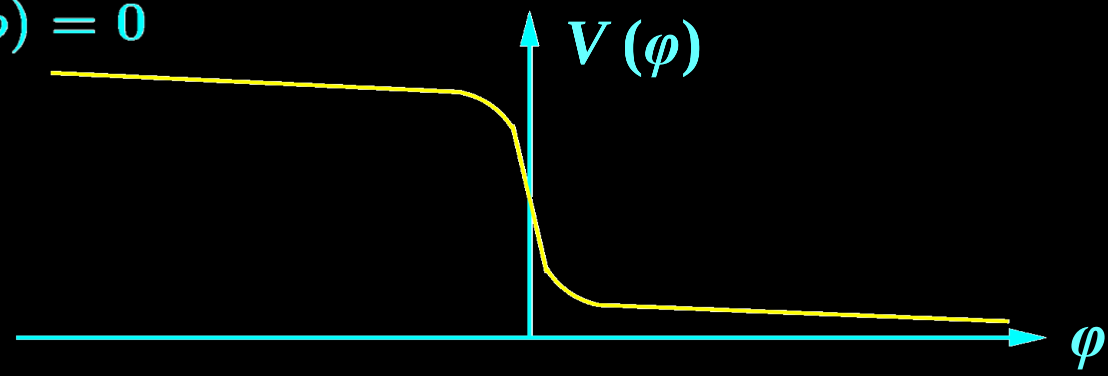
- The spectrum is flat at an unobservable level

Kination

- **Inflationary Paradigm:** The Universe inflates when dominated by the potential energy density of a scalar field (inflaton field)

EoM: $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

- Non-oscillatory inflation has a runaway inflaton scalar potential



- After inflation, the inflaton becomes dominated by the kinetic energy density

$$\ddot{\phi} + 3H\dot{\phi} \simeq 0 \;\Rightarrow\; w = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} \approx 1 \;\Rightarrow\; \Omega_{\mathrm{GW}}(f) \propto f$$

$$\boxed{\Omega_{\mathrm{GW}}(f) \propto f^{-2\left(\frac{1-3w}{1+3w}\right)}}$$

- This period is called **kination**

- The GW spectrum features a peak but at unobservable frequencies
- The GW peak cannot be extended to lower frequencies because of BBN

Kination

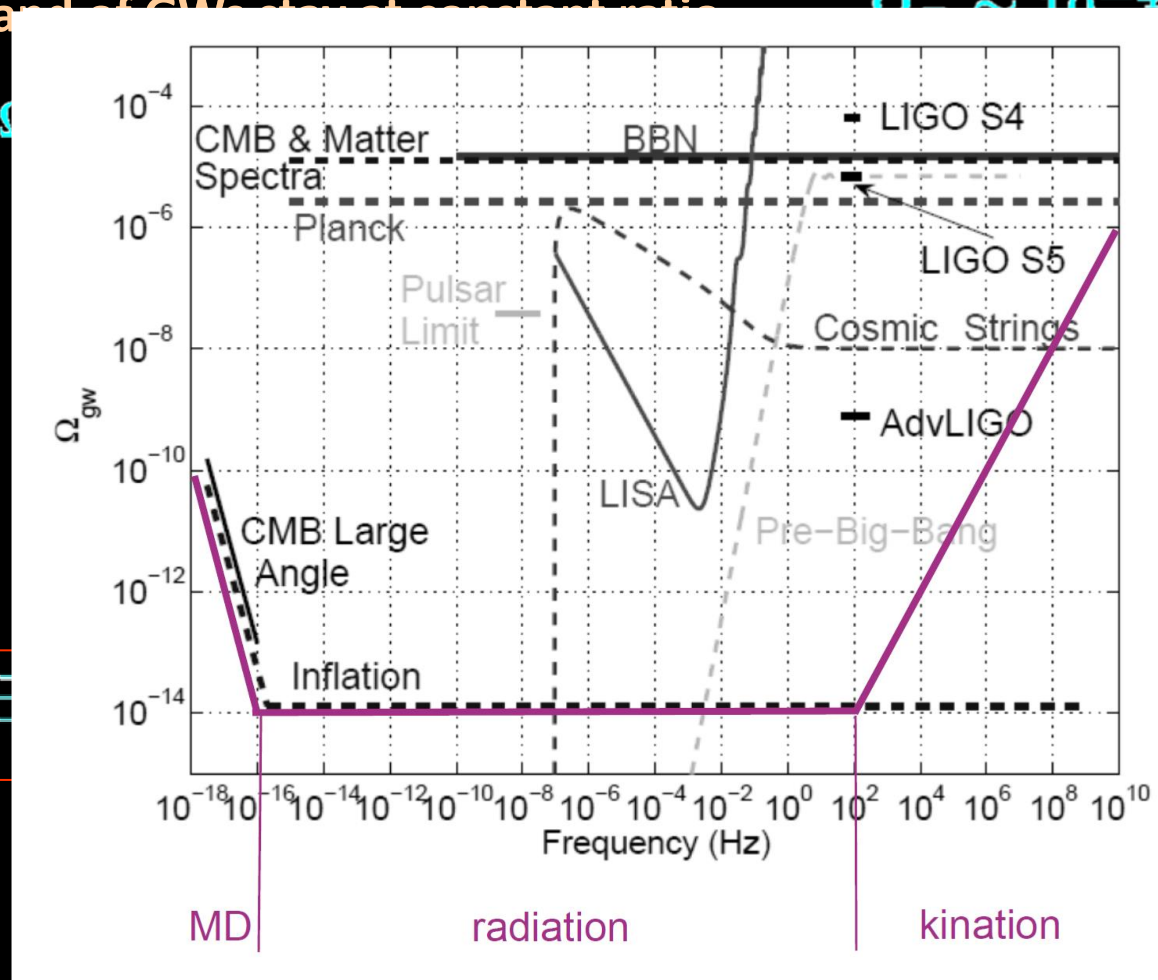
- **The GW density cannot be too large during BBN:** $\Omega^{GW}_{BBN} < 10^{-2}$
- **The density of radiation a**

$$\Omega^0_{GW} = \frac{\rho^0_{GW}}{\rho^0_c} = \frac{\rho_{GW}}{\rho_r}\Big|_0$$

$$\Rightarrow \boxed{\Omega^0_{GW} < 10^{-6}}$$

- **Kination per se cannot boost observable GWs**

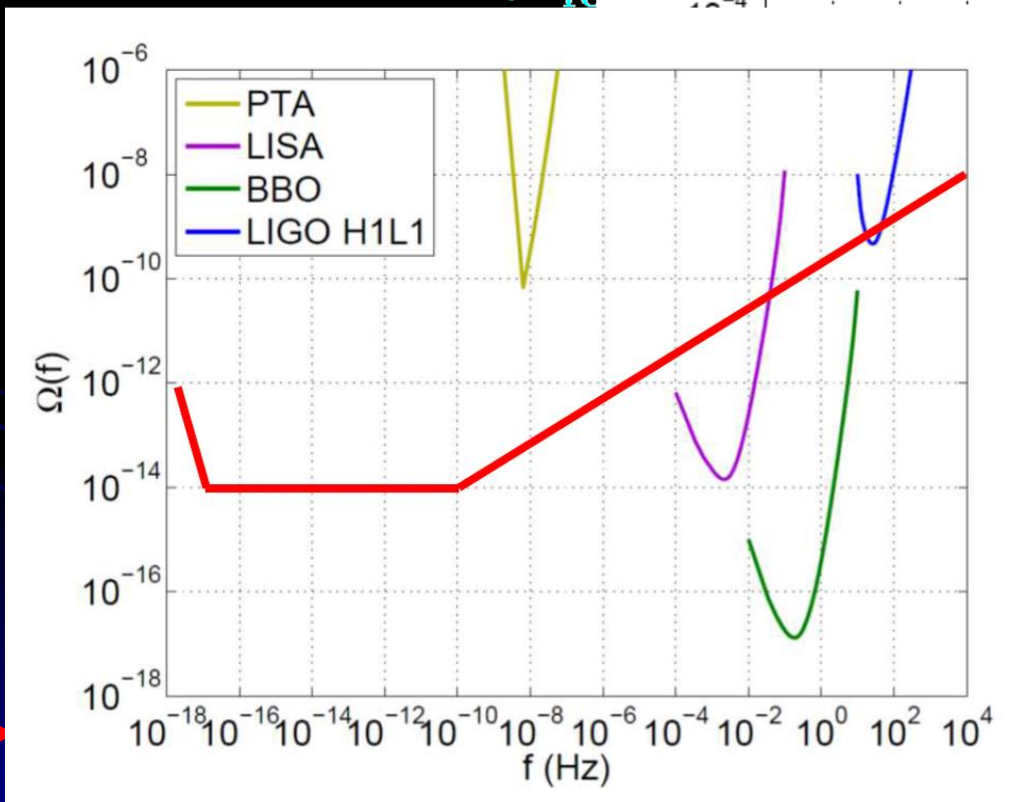
$$\Omega_{GW}(f) \propto f$$

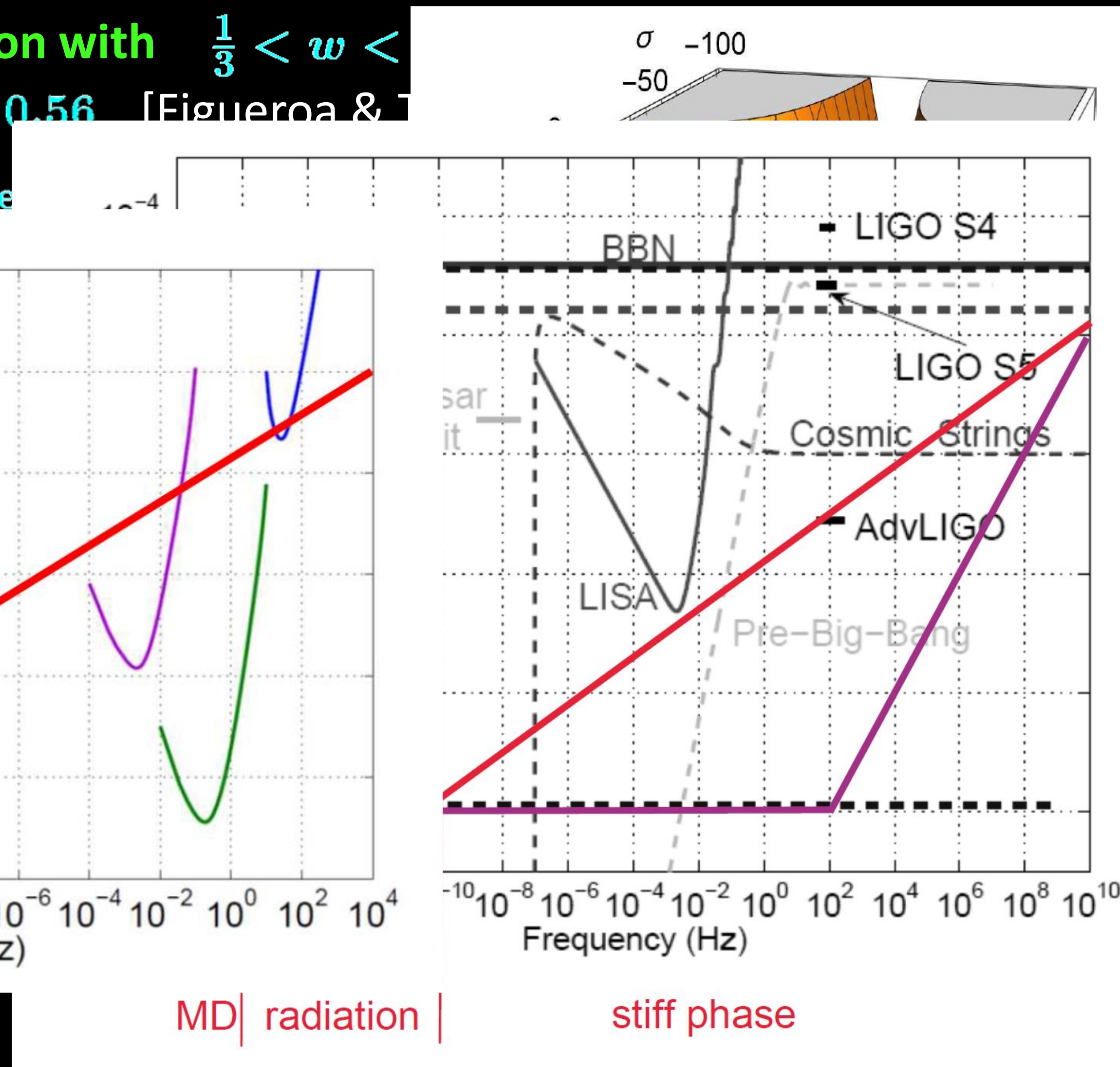


Stiff period

- A stiff period after inflation with $\frac{1}{3} < w <$
- One needs: $0.46 \lesssim w \lesssim 0.56$ [Figueroa & T

$1\ \mathrm{MeV} < T_{\mathrm{re}}$





- Attractor roll: $\phi \gg M$

Hyperkination

- **In Palatini gravity:** $\mathcal{L} = \frac{1}{2}m_{\rm P}^2 R + \frac{1}{4}\alpha R^2 + \frac{1}{2}\xi\varphi^2 R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V$

  $\mathcal{L} = \frac{1}{2}m_{\rm P}^2 R - \frac{1}{2}\partial_\mu\phi\partial^\mu$ … canonical

- **Kinetic energy density d**…

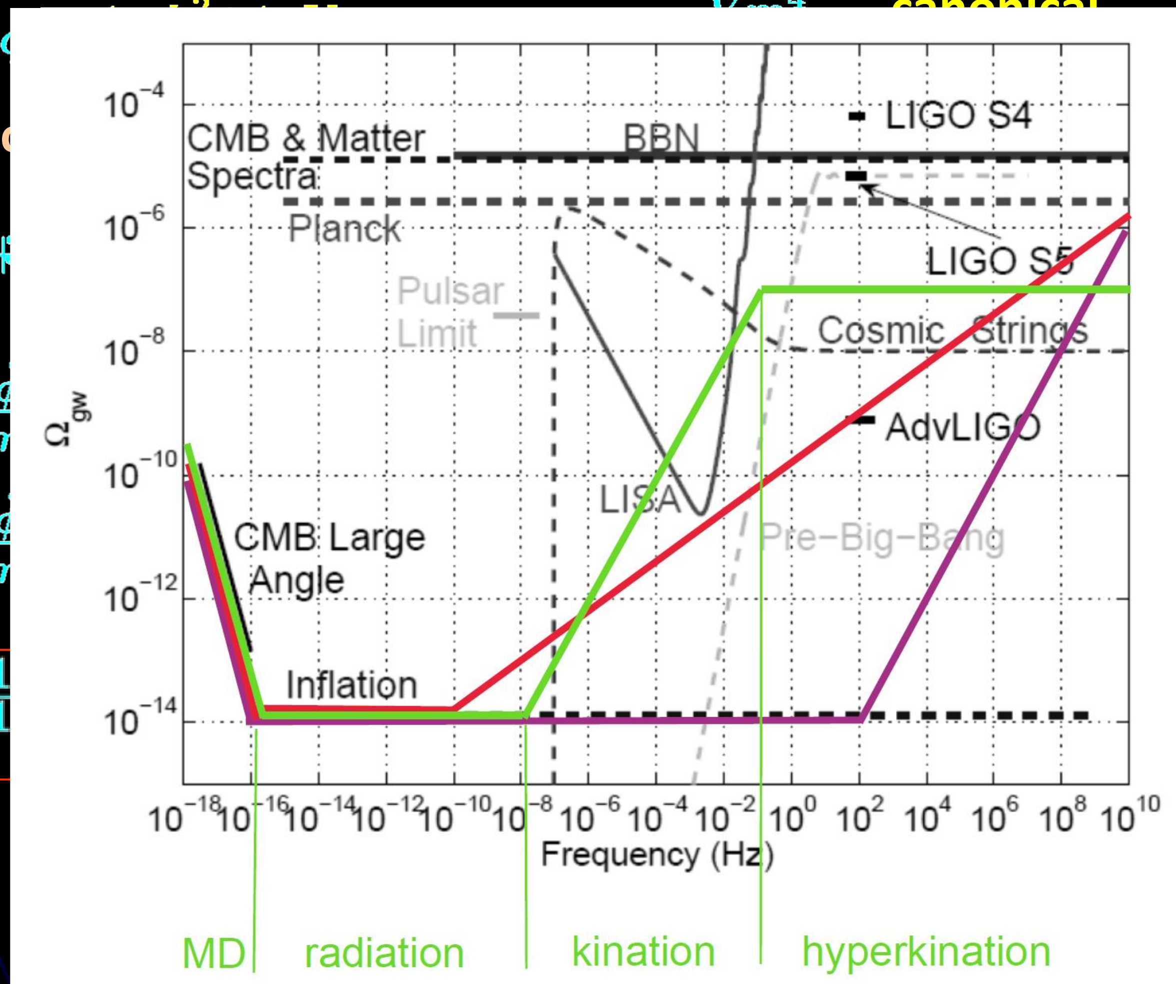
$\left(1+3\alpha\frac{\dot\phi^2}{m_{\rm P}^4}\right)$ …

$\rho_\phi \simeq \frac{1}{2}\left[\left(1+\frac{3}{2}\alpha\frac{\dot\phi^2}{m_{\rm P}^4}\right)\right.$ …

$p_\phi \simeq \frac{1}{2}\left[\left(1+\frac{1}{2}\alpha\frac{\dot\phi^2}{m_{\rm P}^4}\right)\right.$ …

$\boxed{\Omega_{\rm GW}(f) \propto f^{-2(\frac{1}{1}\cdots}}$

- **Quadratic term:** $p_\phi =$ …
- **Quartic term:** $p_\phi =$ …



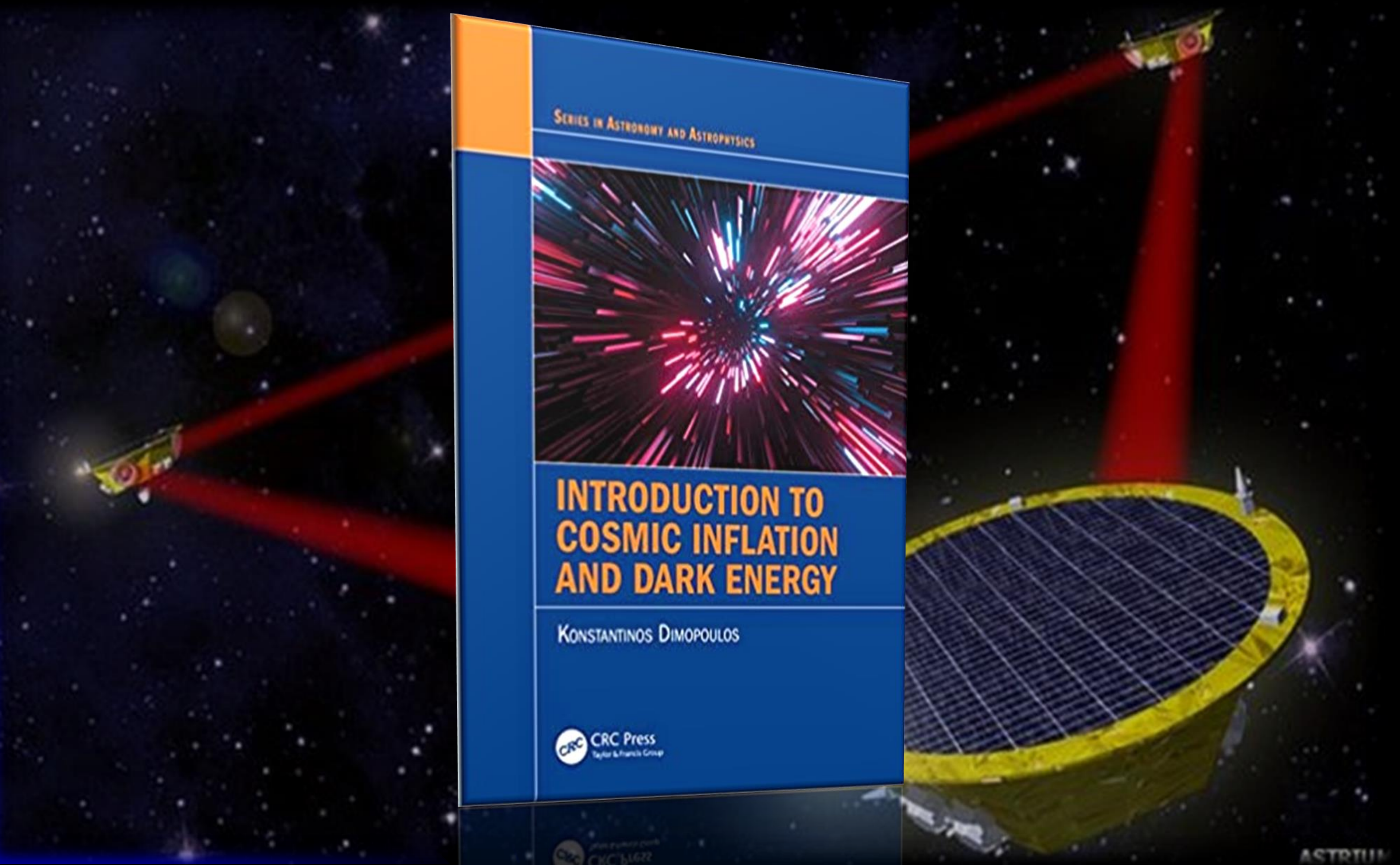
Conclusions

- Cosmic Inflation resolves the fine-tunings of Big Bang and provides seeds for structure formation. Inflation is spectacularly verified by CMB observations
- Another generic prediction of inflation is a superhorizon spectrum of Primordial Gravitational Waves generated through Particle Production.
- The form of the resulting GW spectrum depends on the post-Inflation history
- When GW modes re-enter the horizon during radiation domination they form a flat spectrum, too faint to be observable at present
- A stiff period enhances primordial GWs creating a peak in their spectrum
- N-O Inflation is followed by a period, dominated by the inflaton's kinetic energy density, called kination, but the frequencies of the peak are too high
- The GW peak can be extended to observable frequencies if the stiff period is milder than that of kination, with $w \approx 1/2$
- A model realisation considers two flat directions which intersect at an ESP and give rise to the hybrid mechanism with Planckian waterfall VEV, which is also a kinetic pole of the waterfall field (α-attractors)

Conclusions

- Another possibility to obtain a boost in primordial GWs down to observable frequencies is by considering higher order kinetic terms as with k-essence
- This is possible to realise in Palatini modified gravity
- Considering $R+R^2$ gravity an a non-minimally coupled scalar field, results in additional quartic kinetic terms
- When the quartic kinetic terms dominate, this results to hyperkination
- Hyperkination is followed by regular kination, when the kinetic terms become canonical
- The resulting truncated GW peak can be extended to observable frequencies without disturbing BBN
- Forthcoming observations of LISA, DesiGO and BBO may well detect the primordial GWs generated by inflation
- Detection of primordial GWs will not only confirm a prediction of inflation but offer tantalising evidence of quantum gravity

My book!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