HEP 2023, Ioannina, April 6, 2023

Quantumn Instabilities of de Sitter and Minkowski space-times

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Bibliography

Published work :

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Introduction

• Quantum effects in the presence of dynamical gravity have been investigated for over 5 decades.

• For most of this period the interest was academic.

• However, since about twenty years, cosmology has become quantitative and the issue of the influence of quantum effects resurfaced.

• In many cases in cosmology, quantum effects are not expected to play an important role.

• There are however, two contexts were they are of crucial importance:

♠ Primordial inflation, where the source of perturbations is a quantum effect, and where quantum corrections are expected to increase with time.

♠ The cosmological constant problem, a puzzle between QFT quantum effects and the measured cosmological constant today.

• In most of cosmology, gravity is treated as semiclassical while matter as quantum. The reason is that the scales at which gravitational quantum effects are expected to be non-negligible are large.

• In string theory, the same coupling constant controls both effects. But large or small volumes and other moduli can create a hierarchy between the two.

• In standard models of holography, indeed gravity and matter have similar strengths.

de Sitter space

• de Sitter space is a constant positive curvature manifold, which is the idealization of an accelerating universe.

• It is believed that the space-time during primordial inflation, as well as today, is close to (part of) de Sitter space.

• QFT on de Sitter space was considered since a long time ago.

• Perturbative quantization of QFT on non-trivial metrics is text-book material.

Birrel+Davis

• However, de Sitter space stands apart from other maximally symmetric spaces when it comes to QFT.

• It was argued by many, that quantum effects destabilize de Sitter space.

• It is a well-known fact that in classical GR, both Minkowski space and de Sitter space are non-linearly stable.

Christodoulou+Kleinerman, Anderson

• However, in the presence of quantum effects, instabilities can appear.

Horowitz+Wald,Suen,Jordan, Radjbar-Daemi, Matsui,....

• In the case of de Sitter space, the situation is more serious.

• The two-point function of a massless scalar field in de Sitter space must break de Sitter invariance due to the presence of a zero mode.

Mazur+Mottola, Allen

• Most probably, the resolution of this, may be similar with what happens in two flat dimensions,

Hollands

• In a scalar QFT in de Sitter space, scalar correlators diverge at long times.

Antoniadis+Iliopoulos+Tomaras, Antoniadis+Mottola, Tsamis+Woodard

• A stochastic formalism was developed to resum large time logs and restore the validity of perturbation theory, at least for some observables.

Starobinsky, Tsamis+Woodard, Gorbenko+Senatore

• However, a systematic approach to compute quantum effects for massless fields is lacking.

• In the context of inflation it was argued that the accumulation of long-wavelength fluctuations can have important backreaction effects on the background.

Mukhanov+Abramo+Brandenberger, Abramo+Woodard

• Several authors argued that quantum effects would destabilize the de Sitter background in the presence of gravitational dynamics, especially due to the quantum effects of massless particles like the graviton or massless scalars.

Mottola, Tsamis+Woodard,Miao+Tsamis+Woodard,Polyakov,Dvali

• It should be however remembered that quantum-triggered instabilities may be useful as they provide an exit from inflation.

• This was the case of the initial Starobinsky model, in which inflation was triggered by the conformal anomaly. However, the scalar mode was unstable but its instability time was too long to make the model observationally significant.

• Starobinsky replaced the previous model with an R^2 driven inflation in which the scalar mode has the correct instability time to trigger an exit.

Starobinsky, Vilenkin

The goals

• So far the problems mentioned, were studied using free or perturbative QFTs.

• We are interested to know what happens when QFTs are strongly coupled.

• The (solvable) strongly coupled QFTs we know are holographic QFTs.

• Holographic QFTs on de Sitter have been studied but not in connection to the problems above.

• Our strategy is to use holography and study the back-reaction of four-dimensional quantum holographic matter on the four-dimensional de Sitter geometry.

• There are two aspects of this study:

♠ The back-reaction on the exact de Sitter geometry.

♠ The back-reaction on small modifications of de Sitter.

• In both cases four-dimensional gravity is both classical and dynamical.

The theoretical setup

• We consider a holographic QFT_4 coupled to 4d classical gravity.

• The classical gravitational theory will contain all couplings necessary for renormalization.

• The holographic sector describing the strongly coupled (holographic) QFT_4, has a dual that lives in a higher-dimensional space-time (the bulk) with metric $\mathcal{G}_{ab}$.

• The 4d gravity sector's dynamical variable is a metric $g_{\mu\nu}$. This metric couples to the QFT_4 and therefore plays the role of a boundary condition for $\mathcal{G}_{ab}$.

• From the QFT point of view, $g_{\mu\nu}$ is the background metric and therefore it is the source of the QFT$_4$ stress tensor.

$$S = S_{grav}[g_{\omega\sigma}] + S_{bulk}[\mathcal{G}_{ab}, \ldots] \quad , \quad \langle T_{\mu\nu}\rangle = \frac{1}{\sqrt{g}}\frac{\delta S_{bulk}}{\delta g^{\mu\nu}}$$

The general form of S_{grav} is

$$S_{grav} = -\bar{M}_P^2 \int d^4x\sqrt{g}\left[-2\bar{\Lambda} + R - \frac{\bar{\alpha}}{384\pi M_P^2}R^2 - \right.$$

$$\left. -\frac{\bar{\beta}}{64\pi M_P^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)\right] + \mathcal{O}(R^3)$$

• These are bare coefficients.

• S_{bulk} has UV-divergent terms $\sim \Lambda^4, \Lambda^2, \log\frac{\Lambda^2}{\mu^2}$ that can be absorbed in the bare coefficients to give rise to renormalized coefficients that are finite after removing the cutoff.

$$\bar{M}_P \to M_P \quad , \quad \bar{\Lambda} \to \Lambda \quad , \quad \bar{\alpha} \to \alpha \quad , \quad \bar{\beta} \to \beta$$

• The presence of log divergences introduces a new mass parameter in the theory, μ, related to the conformal anomaly, that is scheme dependent.

• It can be shown, that it appears always in the combination

$$\beta_{eff} = \beta - \frac{N^2}{\pi}\log\left(4\mu^2 G N^2\right) \quad , \quad M_P^2 = \frac{1}{16\pi G}$$

• The higher curvature terms can be neglected if the cutoff of the gravitational theory is the "species" cutoff

$$\Lambda_{species} = \frac{M_P}{N}$$

Higher Curvature Gravity

• Quadratic-R gravity has been considered since the 70's as a candidate for a quantum theory of gravity.

Stelle

$$S_{grav} = -\int d^4x\sqrt{g}\left[M_P^2\,(-2\Lambda + R) - \frac{\alpha}{384\pi}R^2 - \right.$$

$$\left. -\frac{\beta}{64\pi}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)\right] + \mathcal{O}(R^3)$$

• It is a renormalizable theory, and has greatly improved UV behavior compared to standard gauge theories, as part of the graviton propagator comes from the R^2 terms..

• The theory has two extra elementary particles in its spectrum:

♠ A scalar originating in the R^2 term with propagator

$$\mathcal{F}^{-1}_{scalar}(k) = \frac{16\pi G}{3}\frac{1}{k^2 - \frac{4}{\alpha G}}$$

• It is a tachyon when $\alpha < 0$.

♠ A massive graviton with propagator

$$\mathcal{F}^{-1}_{\text{flat}} = 32\pi G\left\{\frac{1}{k^2} - \frac{1}{k^2 + \frac{4}{\beta G}}\right\}.$$

• It is always a ghost, and for $\beta < 0$ a tachyon.

• If however, the mass is above the cutoff, (the Planck scale), ie. $\beta \gg 1$ the effective theory is acceptable.

♠ Tachyons and ghosts are generic in higher-curvature gravity.

♠ The quantum effects of QFTs coupled to gravity, generate such higher-curvature terms, and therefore generate instabilities.

de Sitter solutions

• In the absence of QFT corrections, the gravitational action has a constant curvature solution determined by the (renormalized) cosmological constant:

$$\bar{R} = 4\Lambda$$

• The quantum corrections, however, will alter the curvature of de Sitter space.

• In this case as we seek constant curvature solution, we can replace all Ricci and Riemann tensors with their scalar R analogues.

• We consider a (holographic) QFT_4 with relevant scale m.

• The effective action in this case becomes an $f(R)$ gravity

$$S = \int d^4x\sqrt{g}\ f(R) \quad , \quad f(R) = f_{grav}(R) + f_{QFT}(R)$$

with

$$f_{grav}(R) = M_P^2(2\Lambda - R) + \alpha_{eff}R^2$$

$$f_{QFT} = \tilde{a}_{UV}\left[\frac{R^2}{96}\log\frac{R}{m^2} + m^4\left(Z\left(\frac{R}{m^2}\right) - Z(0)\right) - m^2RZ'(0)\right]$$

where

$$\tilde{a}_{UV} = (M_{bulk}\ell_{UV})^3$$

- m is the relevant scale of the QFT_4 and $Z(x)$ is computed from the data of the holographic flow and contains non-analytic contributions.

- f_{QFT} is the trace of $\langle T_{\mu\nu}\rangle$ of the QFT_4.

- Constant curvature solutions must satisfy

$$2R - R\frac{\partial f}{\partial R} = 0$$

- For the special case $m = 0$ this equation becomes

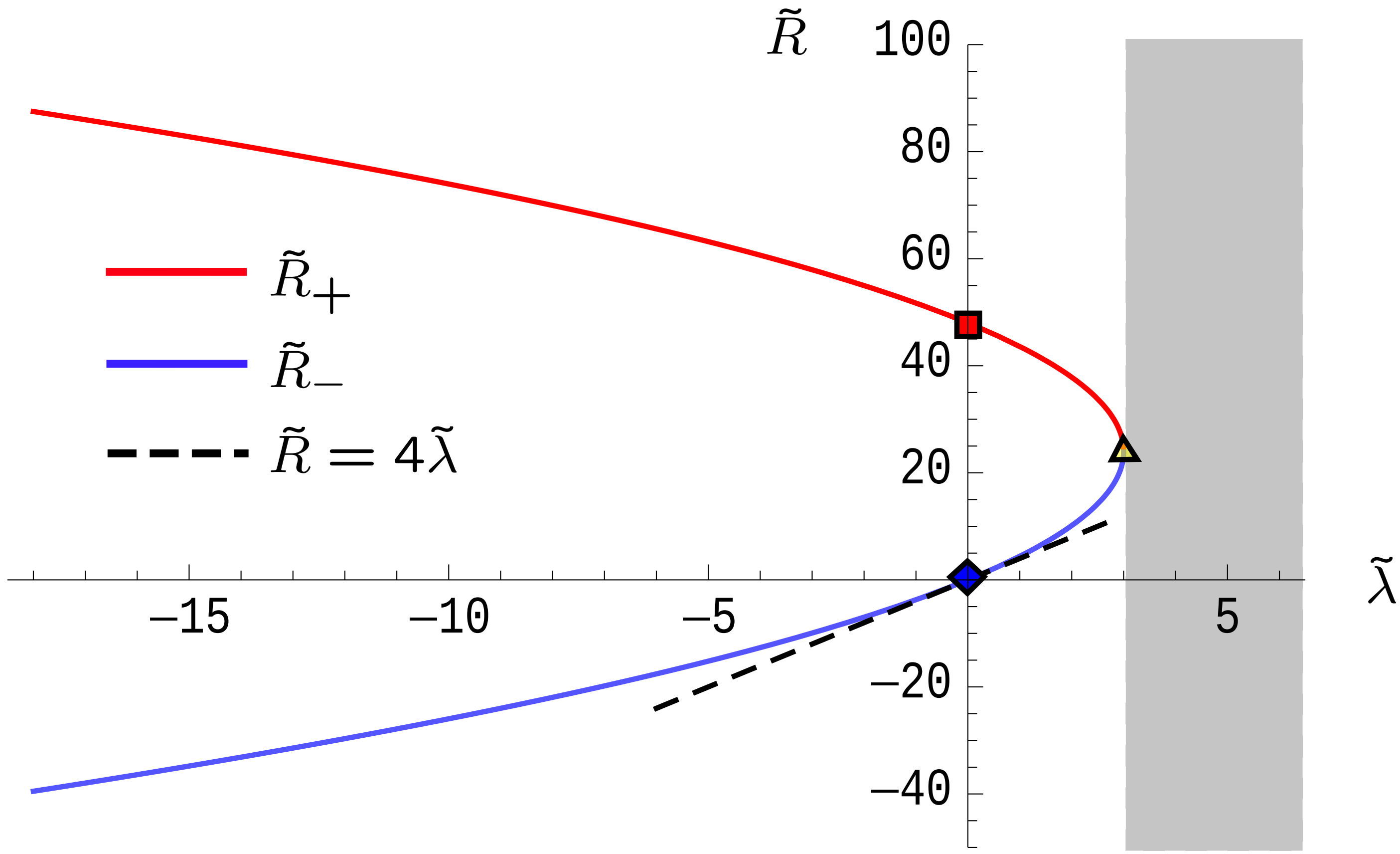
Starobinsky

$$M_P^2(R - 4\Lambda) - \frac{a}{96}R^2 = 0$$

and is valid for any CFT_4.

- There are two solutions

$$\frac{R_\pm}{M_P^2} = \frac{48}{a}\left[1 \pm \sqrt{1 - \frac{\ }{6}\frac{\Lambda}{M_P^2}}\ \right]$$



There are two branches of solutions $\tilde{R}_+$ (shown in red) and $\tilde{R}_-$ (shown in blue). The solution $\tilde{R} = 4\tilde{\lambda}$ in absence of the CFT is also shown for comparison (black, dashed). No solutions with a backreacted CFT exist in the shaded grey region. We highlighted the point where the two branches meet as well as the two solutions for $\tilde{\lambda} = 0$ by markers, to help with later comparison with the results from non-CFTs.

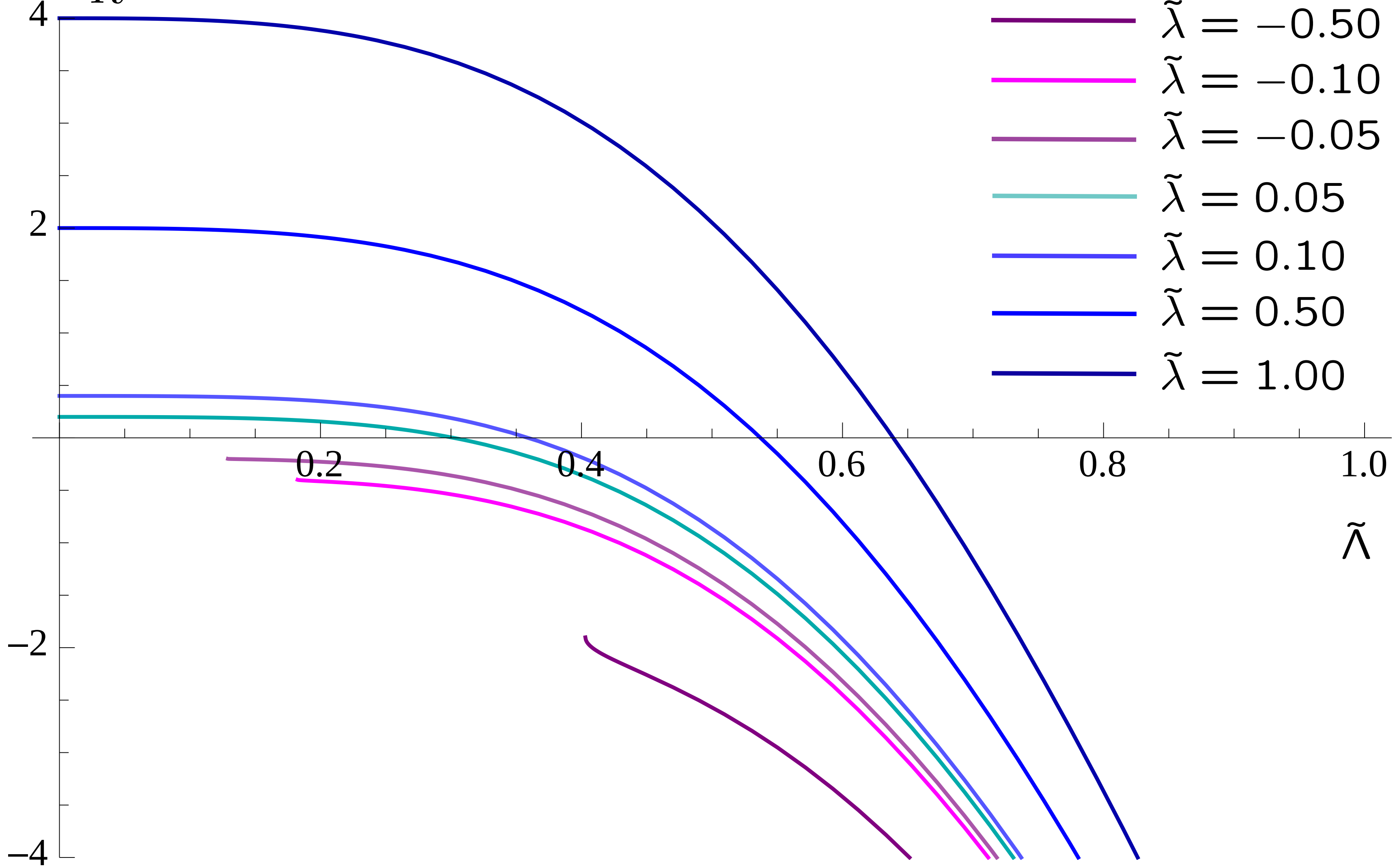
• For QFT_4 the curve interpolates (smoothly) between CFT_{UV} and CFT_{IR}.

• If we keep the cutoff Λ_{CFT} of the CFT large but finite we obtain instead

$$\bar{M}^2(R-4\bar{\Lambda})+12a{\Lambda_{CFT}}^4\sqrt{1+\frac{R}{12\Lambda_{CFT}^2}}=0\,.$$

• An important corollary is that the effective correction to the bare cosmological constant, coming from a holographic theory is always negative (and large).

• In perturbative theories it is either positive or negative.



$\tilde{R}$ vs. $\tilde{\Lambda}$ for a backreacted CFT for various values of $\tilde{\lambda}$. Note that for $\tilde{\lambda} < 0$ there is a lower bound on $\tilde{L}$ for a solution to exist.

Fluctuations

- To study the stability of our de Sitter solutions, we perturb around them

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \quad , \quad \delta g_{\mu\nu} = \psi \bar{g}_{\mu\nu} + h_{\mu\nu}$$

- We also assume that

$$QFT_4 \quad \rightarrow \quad CFT_4$$

- ψ is the (6th) scalar mode that couples to $\langle T^{\mu}{}_{\mu} \rangle_{CFT_4} = 0$

- Therefore its effective equations come only from the R^2 terms in the gravity action.

- The part of the action, quadratic in δg is

$$S^{(2)}_{eff} = \int d^4x \frac{1}{2} \delta g_{\mu\nu} \; O^{\mu\nu,\rho\sigma}_{grav} \; \delta g_{\rho\sigma} - \frac{1}{2} \int d^4x \int d^4y \, \delta g(x)_{\mu\nu} \, \langle T^{\mu\nu}(x) T^{\rho\sigma}(y) \rangle_{CFT} \, \delta g(y)_{\rho$$

• O_{grav} is the local kinetic operator of the quadratic term in the 4d gravity action S_{grav}

• $\langle T^{\mu\nu} T^{\rho\sigma} \rangle_{CFT}$ is the two-point function of the stress tensor. Its form is universal for a CFT.

• The stress tensor two-point function contains both local and non-local contributions.

• The local contributions simply renormalize the coefficients of the local terms which are already present in O_{grav} .

• The non-local contributions are genuine new effects of the CFT which cannot be found in a local gravity theory.

• This non-analyticity in p^2 (or $\Box$) exists because we are integrating out massless degrees of freedom.

• The full propagator, ($\mathcal{F}^{-1}$), is

$$\mathcal{F}^{\mu\nu\rho\sigma} \equiv O_{grav}^{\mu\nu,\rho\sigma} - \langle T^{\mu\nu} T^{\rho\sigma} \rangle_{CFT}$$

and the spectrum of the gravitational propagating modes are the solutions of the integro-differential equation

$$\mathcal{F}^{\mu\nu\rho\sigma} \cdot \delta g_{\rho\sigma} = 0.$$

- This equation can be recast into two separate *scalar* spectral equations for the scalar and tensor modes.

- This is done by decomposing the modes in eigenfunctions of the d'Alembert operator ∇^2 of the background boundary metric $\bar{g}_{\mu\nu}$:

$$\left(\nabla^2 - s\frac{\bar{R}}{12}\right)\delta\Phi(x) = \begin{cases} -H^2\left(\nu^2 - \frac{9}{4}\right)\delta\Phi(x) & dS \\ -k^2\delta\Phi(x) & \text{Minkowski} \\ \chi^2\left(\nu^2 - \frac{9}{4}\right)\delta\Phi(x) & AdS \end{cases}$$

- The values of ν^2 (or k^2 in the flat case) are determined by the spectral equation

$$\mathcal{F}(\nu) = 0$$

• The existence of tachyonic modes is important for the cosmological dynamics.

♠ . A scalar tachyonic mode may be a bonus, if it triggers the exit from a de Sitter phase.

But can also be a problem if its instability time is too short or too long.

♠ A spin-2 tachyonic mode is always disastrous for cosmology: once it is triggered it starts deforming the homogeneous metric by creating sheer and in the end completely destroys homogeneity.

♠ From what we know from cosmology, this is not good.

Tensor modes in de Sitter

• For tensor modes, the non-local contribution from the CFT stress-tensor correlator gives rise to non-polynomial expressions for the inverse propagators:

$$\mathcal{F}_{\text{tensor,dS}}(\nu) = \frac{GN^2H^2}{64\pi^2}\left(\nu^2 - \frac{9}{4}\right)\left\{1 - \frac{2\pi}{GN^2H^2} + \frac{2\pi\alpha}{N^2} + \right.$$

$$\left. -\frac{1}{2}\left(\nu^2 - \frac{1}{4}\right)\left[\log\left(GN^2H^2\right) - \frac{1}{2} + 2\mathcal{H}\left(\nu - \frac{1}{2}\right) + \frac{\pi\beta_{eff}}{N^2}\right]\right\}.$$

where $\mathcal{H}$ is the harmonic number function defined as

$$\mathcal{H}(z) = \frac{\Gamma'(z+1)}{\Gamma(z+1)} + \gamma_E.$$

• This expression with $\alpha = 0$ was already obtained by Chesler and Loeb recently.

• The logarithmic term is due to the conformal anomaly.

♠ Absence of tachyons $\Rightarrow Re(\nu) \leq \frac{3}{2}$.

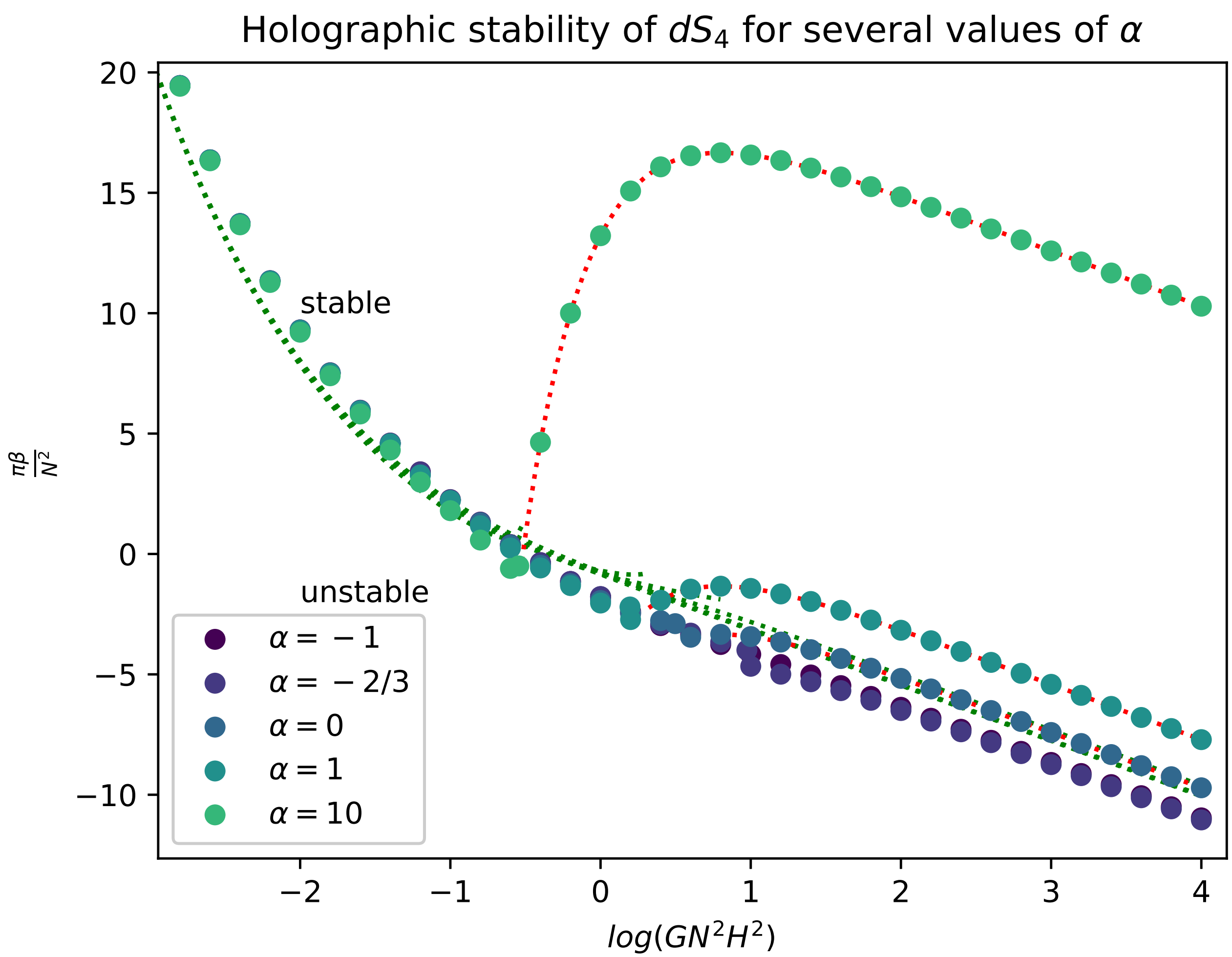
♠ Absence of ghosts $\Rightarrow Res\mathcal{F}^{-1}(\nu) < 0$.

• In de Sitter, the presence of tachyonic tensor modes depends on the curvature H and N in the combination GH^2N^2, and on the parameters α and β_{eff}.

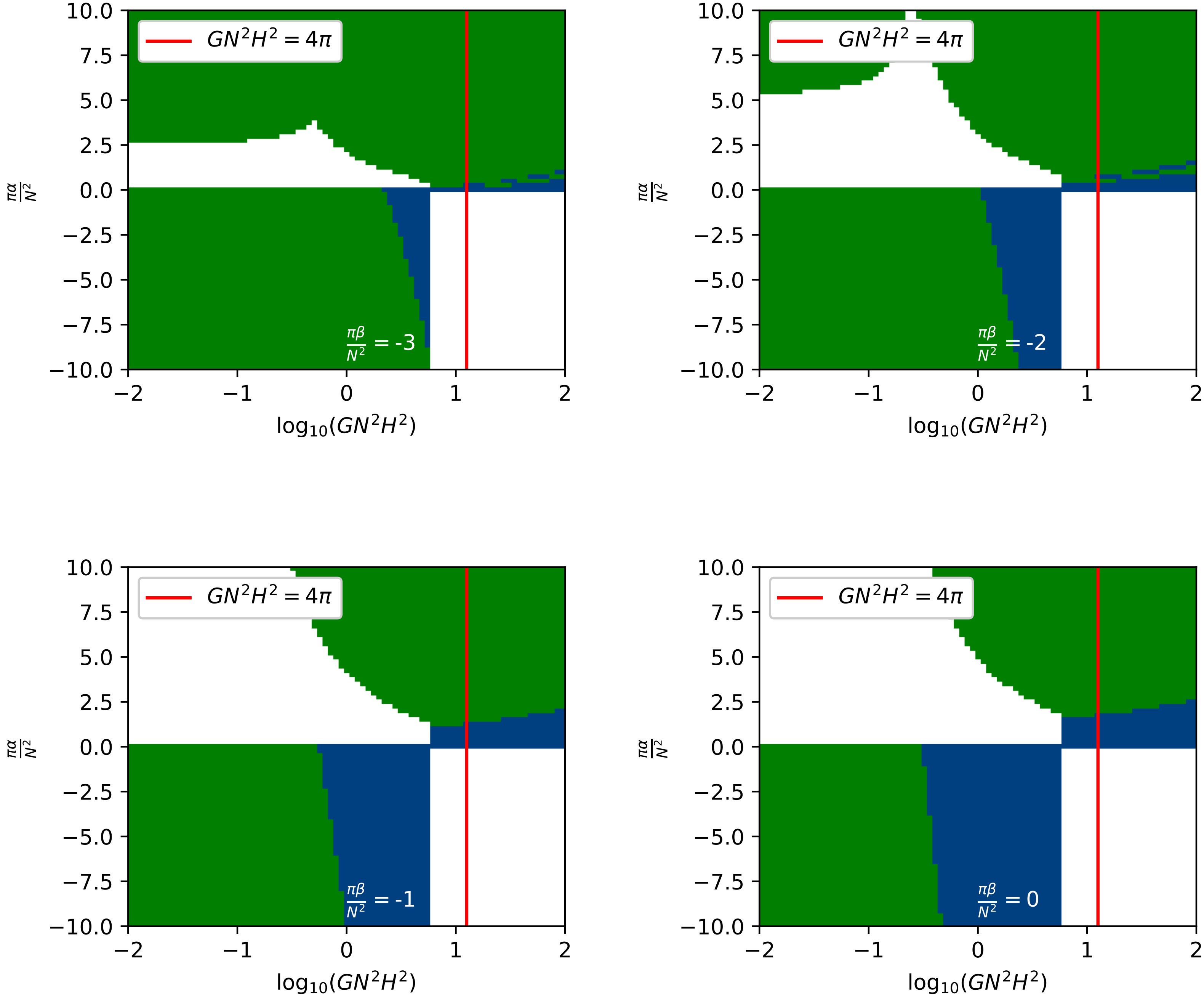
• Roughly, tachyon-stability corresponds either to large values of β_{eff}/N^2 and/or to values of H larger than the species scale $(GN^2)^{-1/2}$ (but still sub-planckian).

• If we set $\alpha = \beta_{eff} = 0$, there is a critical value for GN^2H^2, below which de Sitter space is tachyon-unstable.

$$\text{dS Instability} \quad : \quad GN^2H^2 = \frac{H^2}{\Lambda^2_{species}} \leq 0.32$$



Doted lines with large dots, are the boundaries between the stable and unstable regions and have been computed numerically. Above each curve, we are in a stable regime ($\mathrm{Re}\nu \geq -3/2$)while below, there is a tachyonic instability ($\mathrm{Re}\nu \leq -3/2$).



Green:tensor faster. White: scalar faster. Blue: no tachyons.

• There are always tensor ghosts (tachyonic or not).

• There are regimes however where all ghosts are heavy (at least in units of the "species" cut-off $(GN^2)^{-1/2}$.

• This typically occurs for small or negative β_{eff}/N^2

• When $\alpha = \beta_{eff} = 0$, the mass of the ghost is always larger, but comparable to the species scale for any curvature.

Conclusions

• We have investigated classical 4d gravity coupled to a (quantum) holographic CFT_4.

• Our quadratic fluctuation results are valid for ANY CFT_4.

• Quantum effects affect the existence of maximal symmetry solutions (like dS_4), and in some cases, (sufficiently large and positive cosmological constant), such solutions do not exist.

• Quantum effects affect also importantly the stability properties of such maximal symmetry solutions (like dS_4).

• The quantum effects of both free (massive) theories as well as CFT_4, are well-studied (in the past) on the scalar sector, that is mostly unstable but never contains a ghost.

• The quantum effects of a CFT_4 on the spectrum of spin-2 fluctuations are also revealing: they make most dS_4 spaces unstable for curvatures below the effective cutoff.

• Sometimes this instability is dominant compared to the scalar instability.

• Similar results with variations exist for Minkowski space and AdS space.

• In particular, Minkowski space is *always* unstable.

Outlook

- Quantum-driven instability results of this type are *not* new.

- However, our analysis of all possible cases, indicates that their scope is (almost) all-encompassing.

- Certainly, they must have several implications for cosmology, but these implications need a more careful analysis.

- It is not known what is the end-point of ghost instabilities in gravity theories.

- There are two (naive) possibilities:

♠ Either such instabilities are generic, independent of theory (this rimes with cosmology and "πάντα ρεί").

♠ Or semiclassical gravity and QFT have a correlated origin that *fine-tunes* these instabilities away.

• This second case could be realizable in string theory:

• However, beyond susy ground-states, even in the string theory case, the fate of flat space is unknown.

• This could be correlated with de Sitter swampland conjectures.

THANK YOU!

The scalar mode in dS

• The inverse propagator is a polynomial in ν^2 (or k^2) because the only propagating scalar mode has a purely local boundary dynamics.

• **The inverse propagator in dS$_4$**

$$\mathcal{F}_{scalar}(\nu) = -\frac{H^2}{64\pi G}\left[12\alpha G H^2 - 12 + \frac{6GN^2H^2}{\pi}\right]\left\{\frac{4}{\alpha G H^2} - \frac{2N^2}{\alpha} + \left(\nu^2 - \frac{9}{4}\right)\right\}.$$

♠ Absence of tachyons $\Rightarrow Re(\nu) \leq \frac{3}{2}$.

♠ Absence of ghosts $\Rightarrow Res\mathcal{F}^{-1}(\nu) < 0$.

• The scalar is a tachyon if

$$\alpha\left(\frac{GN^2H^2}{2\pi} - 1\right) < 0.$$

• The scalar is a ghost if

$$\left(\frac{\pi\alpha}{N^2} + \frac{1}{2}\right)\frac{GN^2H^2}{\pi} > 1.$$

• The (recurrent) expression

$$GN^2H^2 \simeq \frac{H^2}{\Lambda^2_{species}}$$

is the dimensionless curvature in units of the species cutoff.

The scalar mode in Minkowski

• The "physical" scalar inverse propagator is

Starobinsky, Vilelnkin

$$\mathcal{F}_{scalar}(k) = -\frac{3}{16\pi G}\left(k^2 - \frac{4}{\alpha G}\right)$$

♠ Absence of tachyons $\Rightarrow$ $k^2 < 0$.

♠ Absence of ghosts $\Rightarrow$ $Res\mathcal{F}^{-1}(k_*^2) < 0$.

• The scalar mode is tachyonic if $\alpha > 0$.

Starobinsky, Vilenkin

• It is never a ghost.

The scalar mode in AdS

- **The inverse propagator in AdS$_4$**

$$\mathcal{F}_{scalar}(\nu) = \frac{\chi^2}{64\pi G}\left[12\alpha G\chi^2 + 12 + \frac{6GN^2\chi^2}{\pi}\right]\left\{\frac{4}{\alpha G\chi^2} + \frac{2N^2}{\pi\alpha} + \left(\nu^2 - \frac{9}{4}\right)\right\}.$$

♠ Absence of tachyons ⇒ $Re(\nu) \neq 0$.

♠ Absence of ghosts ⇒ $Res\mathcal{F}^{-1}(\nu) > 0$.

- scalar tachyon-stability requires:

$$\frac{9}{4} - \frac{4}{\alpha G\chi^2}\left(1 + \frac{GN^2\chi^2}{2\pi}\right) \geq 0.$$

- The scalar is a ghost if

$$\left(\frac{\pi\alpha}{N^2} + \frac{1}{2}\right)\frac{GN^2\chi^2}{12\pi} < -1.$$

Tensor modes in Minkowski space

In Minkowski space the tensor inverse propagator is :

$$\mathcal{F}_{\text{tensor,Mink}}(k) = \frac{N^2}{64\pi^2}k^2\left\{-\frac{2\pi}{GN^2} + k^2\left[\frac{1}{4} - \gamma_E - \frac{1}{2}\log\left(GN^2k^2\right) - \frac{1}{2}\frac{\pi\beta_{eff}}{N^2}\right]\right\}$$

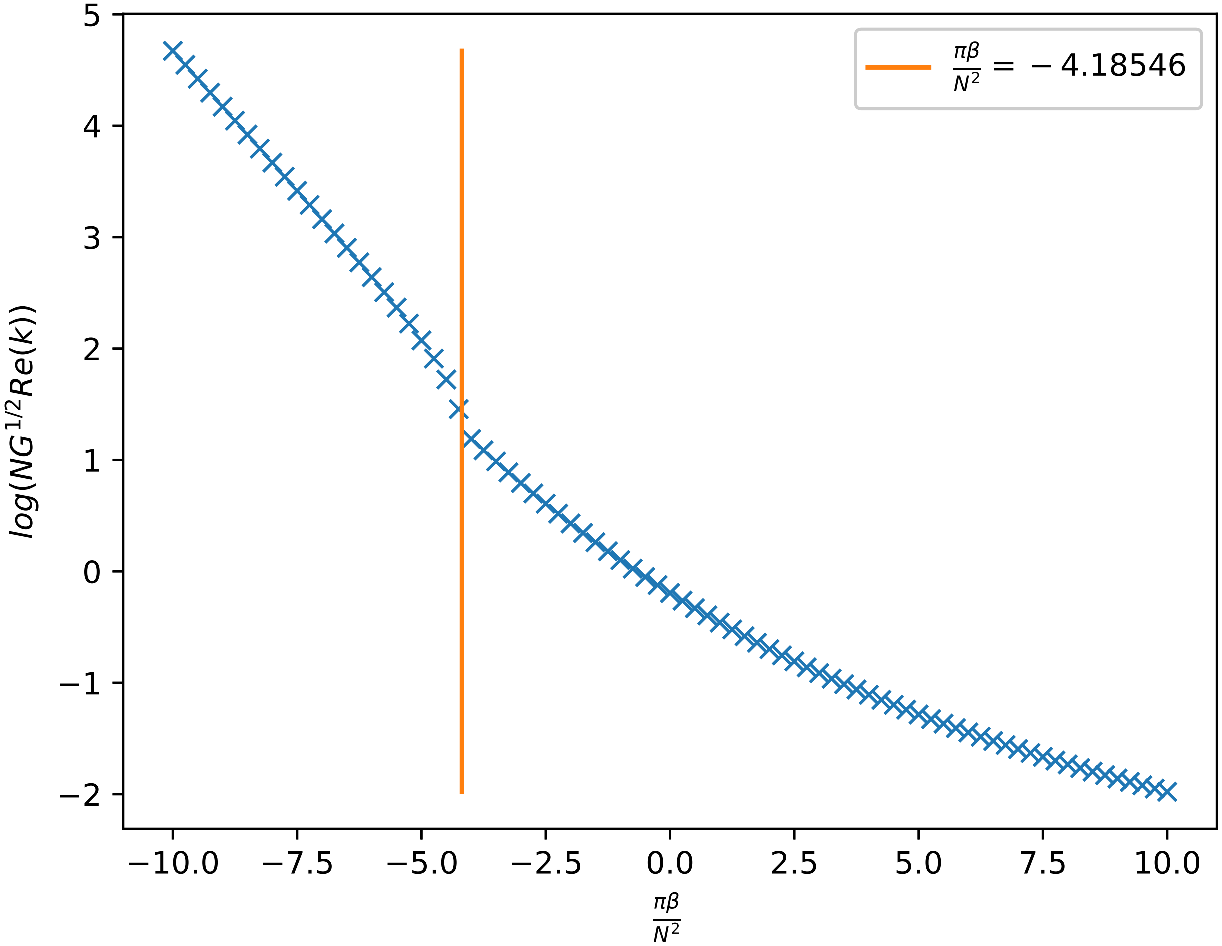
- It is independent of α because the background curvature vanishes.
- The logarithmic contribution k^2 is due to the conformal anomaly.
- This is the only non-trivial , beyond 4d-gravity contribution, in Minkowski.

♠ Absence of tachyons $\Rightarrow k^2 < 0$.

♠ Absence of ghosts $\Rightarrow Res\mathcal{F}^{-1}(k_*^2) < 0$.

• The non-trivial roots of $\mathcal{F}$ are the solutions of a transcendental equation of the type $X \log X = a$,

• *Minkowski space* *always contains two tachyon-unstable spin-2 modes* for any value of $\tilde{\alpha}$ and $\tilde{\beta}_{\text{eff}}$



tachyonic instability in units of GN^2 given as a function of $\tilde{\beta}_{\text{eff}}$ for flat slicing. The red line is the value of $\tilde{\beta}_{\text{eff}}$ where two tachyons merge and move off the real axis when $\tilde{\beta}_{\text{eff}}$ is increased.

• The theory becomes eventually tachyon-stable only in the extreme limit

$$\frac{\beta_{eff}}{N^2} \to +\infty.$$

• In this limit however one always finds also a light ghost (both with respect to the Planck scale $G^{-1/2}$ and with respect to the "species" scale $(GN^2)^{-1/2}$).

• All in all, the masses of the unstable tensor modes are above the species cut-off for $O(1)$ values of β_{eff} (this includes the special case $\alpha = \beta_{\text{eff}} = 0$), while Minkowski space is unstable within EFT iff $|\tilde{\beta}_{\text{eff}}| \gg 1$ and independently of $\tilde{\alpha}$.

Tensor modes in AdS space

• In this case, in the holographic theory, there are two connected boundaries, corresponding to two a priori independent copies of the CFT$_4$.

• The standard interpretation is in terms of two copies of a CFT$_4$ on half $\mathbb{R}^4$, interacting via a common interface.

• This is conformally related to two CFT's on AdS_4, with transparent boundary conditions at their common boundary.

• There is some freedom in how to couple 4d gravity to the CFT$_4$ on AdS$_4$.

♠ Only one of the two CFT$_4$s is coupled to dynamical gravity, and the metric on the second boundary is frozen.

$$\mathcal{F}^{-}_{\mathrm{tensor,AdS}}(\nu) = \frac{N^2\chi^2}{64\pi^2}\left(\nu^2 - \frac{9}{4}\right)\left\{1 + \frac{2\pi}{N^2}\left(\frac{1}{G\chi^2} + \alpha\right) + \right.$$

$$-\frac{1}{2}(\nu^2 - 1/4)\left[\frac{\pi\beta_{eff}}{N^2} + \log\left(GN^2\chi^2\right) - \frac{1}{2} + \right.$$

$$\left.\left. + \mathcal{H}\left(-\frac{1}{2} - \nu\right) + \mathcal{H}\left(-\frac{1}{2} + \nu\right)\right]\right\}.$$

• Again the harmonic sum function $\mathcal{H}(\nu)$ controls the complexity of graviton poles.

• Symmetric boundary conditions:

• In this case there is effectively a single boundary and there is again a single dynamical gravity theory coupled to a single 4d CFT on AdS.

• Therefore the interpretation is in terms of a single CFT_4 coupled to a dynamical near-AdS_4 metric.

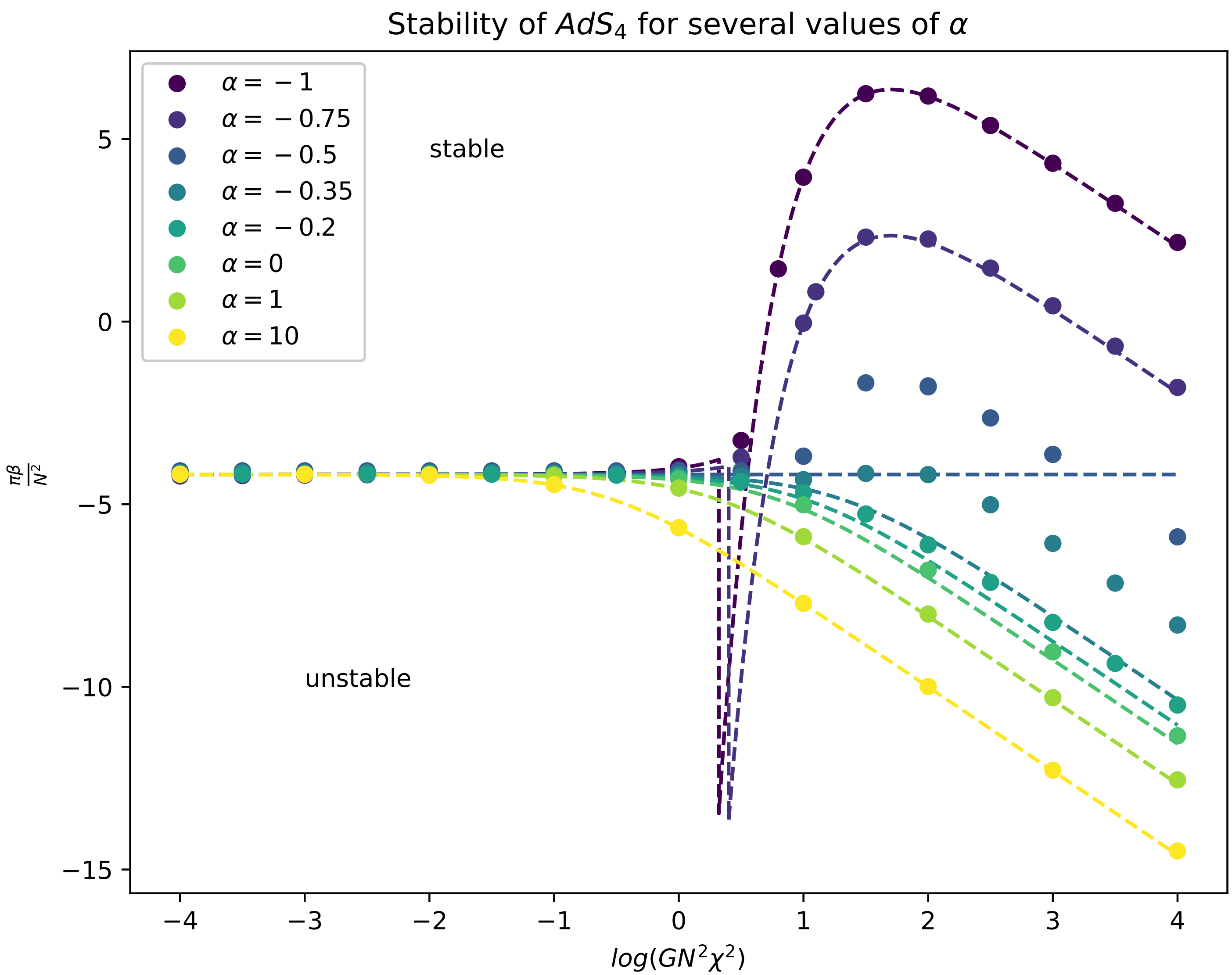
$$\mathcal{F}^{sym}_{\text{tensor,AdS}}(\nu) = \frac{N^2\chi^2}{64\pi^2}\left(\nu^2 - \frac{9}{4}\right)\left\{1 + \frac{2\pi}{N^2}\left(\frac{1}{G\chi^2} + \alpha\right) + \right.$$

$$-\frac{1}{2}(\nu^2 - 1/4)\left[\frac{\pi\beta_{eff}}{N^2} + \log\left(GN^2\chi^2\right) - \frac{1}{2}\right.$$

$$\left.\left. + \mathcal{H}(\nu - \frac{1}{2}) + \mathcal{H}(-\nu - \frac{1}{2}) - \frac{\pi}{\cos \pi\nu}\right]\right\}.$$

♠ Absence of tachyons $\Rightarrow$ $Re(\nu) \neq 0$.

♠ Absence of ghosts $\Rightarrow$ $Res\mathcal{F}^{-1}(\nu) > 0$.



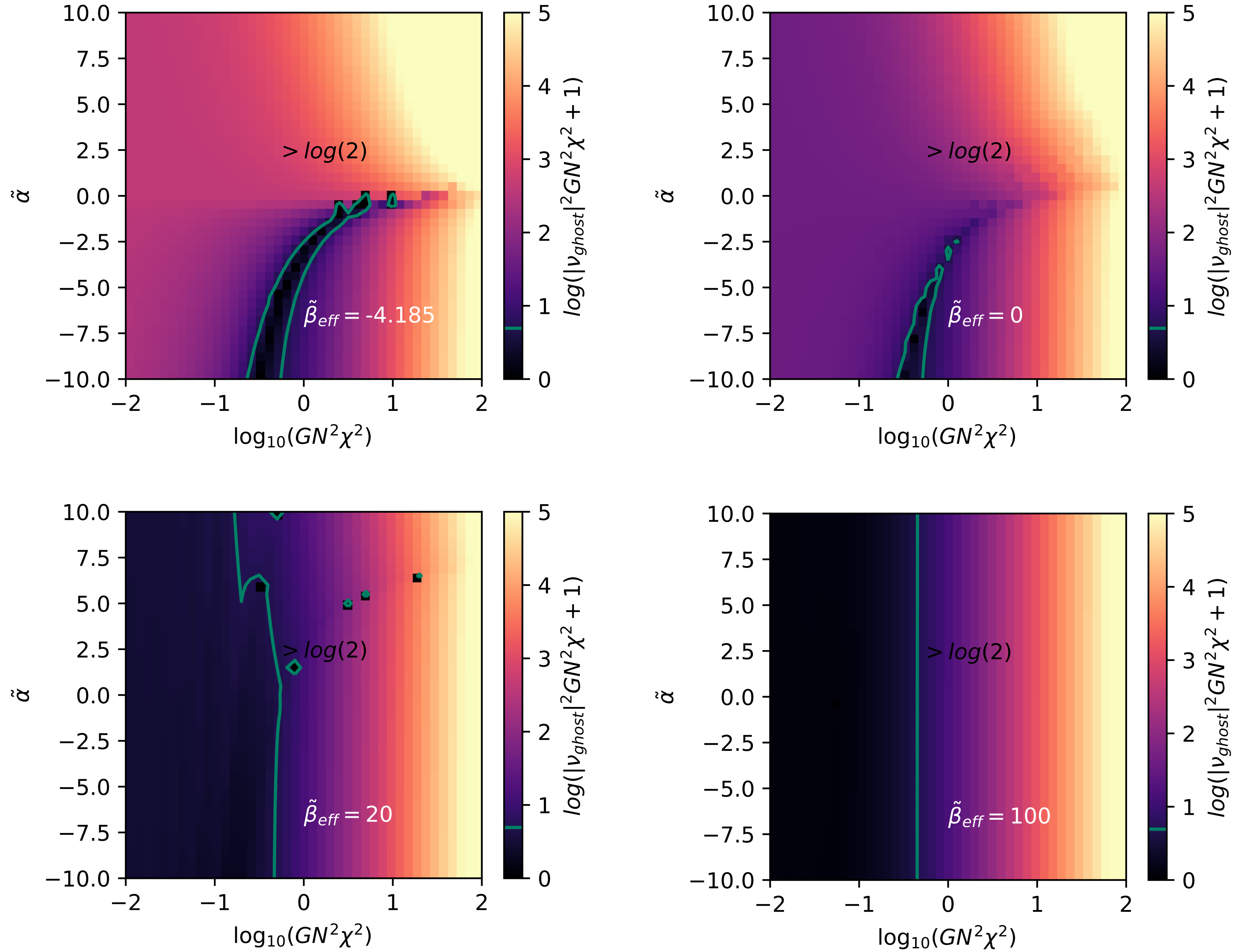
Dashed lines are the analytical predictions obtained from the large $|\nu|$ approximation. Dots are numerical results.

- The tensor modes can be tachyonic or not depending on the parameters.

- We find that the tachyon instability disappears as α/N^2 becomes large.

- If $GN^2\chi^2$ is not too large, it disappears also if β_{eff}/N^2 become large.

- In the $\alpha = \beta_{eff} = 0$ case, AdS space-time is tachyon-stable for any curvature.



This figure displays the mass of the ghost tensor pole in AdS, in units of the species scale

The holographic RG Flow at constant curvature

$$C\left(\tfrac{R}{m^2}\right) = \frac{4-\Delta}{4\,\tilde{a}_{\rm uv}}\,\langle\mathcal{O}\rangle\, m^{-\Delta} = \mathcal{G}\left(\tfrac{R}{m^2}\right) - \tfrac{1}{2}\tfrac{R}{m^2}\,\mathcal{G}'\left(\tfrac{R}{m^2}\right),$$

i.e. $C(Rm^{-2})$ is proportional to the vev in units of the (relevant) coupling constant m.

$$\langle T_\mu^{\rm ren,\mu}\rangle = -\frac{\tilde{a}_{\rm uv}}{48}R^2 - (\Delta-4)\overline{\langle\mathcal{O}\rangle}\, m^{4-\Delta}.$$

$$\overline{C}\left(\tfrac{R}{m^2}\right) = \frac{4-\Delta}{4\,\tilde{a}_{\rm uv}}\,\overline{\langle\mathcal{O}\rangle}\, m^{-\Delta} = \mathcal{G}\left(\tfrac{R}{m^2}\right) - \tfrac{1}{2}\tfrac{R}{m^2}\,\mathcal{G}'\left(\tfrac{R}{m^2}\right) - \mathcal{G}\left(0\right) - \tfrac{1}{2}\tfrac{R}{m^2}\mathcal{G}'\left(0\right),$$

$$= C\left(\tfrac{R}{m^2}\right) - C\left(0\right) - \tfrac{R}{m^2}\,C'\left(0\right).$$

• For $\left|\frac{R}{m^2}\right| \to 0$ and $\left|\frac{R}{m^2}\right| \to \infty$ we can derive analytic expressions for $\overline{C}\left(\frac{R}{m^2}\right)$.

$$\lim_{R\to\infty} \overline{C}\left(\frac{R}{m^2}\right) = \mathcal{O}\Big(\left(\frac{R}{m^2}\right)^{\Delta_{UV}-2}\Big),$$

$$\lim_{R\to 0} \overline{C}\left(\frac{R}{m^2}\right) = \frac{1}{192}\left(1 - \frac{\tilde{a}_{\text{ir}}}{\tilde{a}_{\text{uv}}}\right)\frac{R^2}{m^4} + \mathcal{O}\Big(\frac{R^3}{m^6}\Big) + \mathcal{O}\Big(\left(\frac{R}{m^2}\right)^{\Delta_-^{\text{IR}}-2}\Big).$$

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