Topological Defects & Gravitational Waves

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[redacted] SO(10)

Usually broken via one or more intermediate steps to the SM

- $G = SO(10)/\text{Spin}(10)$
- $H = SU(3)_c \times U(1)_{e.m.}$
- $\Pi_2(G/H) \cong \Pi_1(H) \Rightarrow$ Monopoles
- $\Pi_1(G/H) \cong \Pi_0(H) = \mathbb{Z}_2 \Rightarrow$ Cosmic Strings (provided $G \to H$ breaking uses only tensor representations)
- $\mathbb{Z}_2 \subset \mathbb{Z}_4$ (center of $SO(10)$)
  [T. Kibble, G. Lazarides, Q.S., PLB, 1982]
- Intermediate scale monopoles and cosmic strings may survive inflation.
- Recent work suggests that this Z_2 symmetry can yield plausible cold dark matter candidates.
  [Mario Kadastik, Kristjan Kannike, and Martti Raidal Phys. Rev. D 81 (2010), 015002; Yann Mambrini, Natsumi Nagata, Keith A. Olive, Jeremi Quevillon, and Jiaming Zheng Phys.Rev. D91 (2015) no.9, 095010 ; Sofiane M. Boucenna, Martin B. Krauss, Enrico Nardi Phys.Lett. B755 (2016) 168-17]

[redacted]

SO(10) Breaking Chains

$$
\mathrm{SO(10)}\left\{\begin{array}{llll}
\xrightarrow{1} & 5\ 1_{\rm V} & \left\{\begin{array}{llll}
\xrightarrow{2\ (2)} & 5\ (Z_2) & \xrightarrow{1} & \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm Z}\ 1_{\rm V} & \xrightarrow{2\ (2)} & \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{1,2\ (1,2)} & \mathrm{G_{SM}}\ (Z_2) & &
\end{array}\right. & \\
\xrightarrow{1} & 5_{\rm F}\ 1_{\rm V} & \xrightarrow{2'\ (2)} & \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{0\ (2)} & 5\ (Z_2) & \xrightarrow{1} & \mathrm{G_{SM}}\ (Z_2)
\end{array}\right.
$$

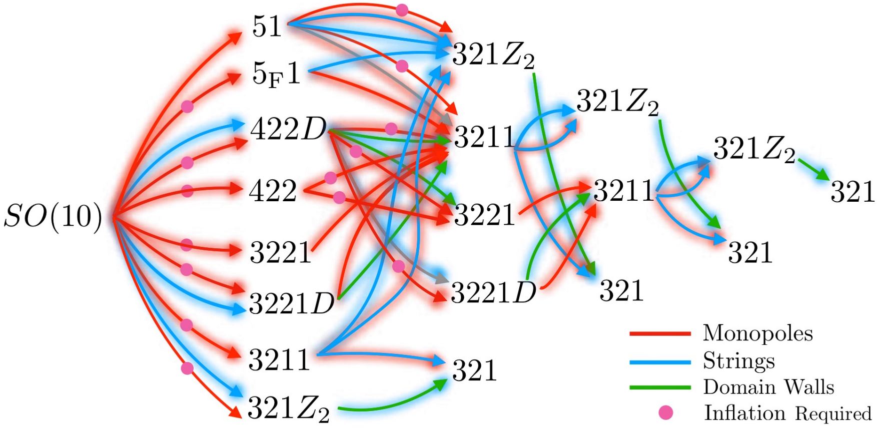
$$
\mathrm{SO(10)}\left\{\begin{array}{llll}
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 2_{\rm R} & \longrightarrow & \\
\xrightarrow{1,2} & 4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ Z_2^{\rm C} & \longrightarrow & \\
\xrightarrow{1,2} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ Z_2^{\rm C} & \longrightarrow & \cdots\\
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \longrightarrow & \cdots\\
\xrightarrow{1,2} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L}\ Z_2^{\rm C} & \longrightarrow & \cdots\\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \longrightarrow & \cdots\\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)} & \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{1\ (1,2)} & \mathrm{G_{SM}}\ (Z_2) & &
\end{array}\right.
$$

$$
4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\left\{\begin{array}{lll}
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \left\{\begin{array}{llll}
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)} & \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{2'\ (2)} & \mathrm{G_{SM}}\ (Z_2) & &
\end{array}\right.\\
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \left\{\begin{array}{llll}
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)} & \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{2'\ (2)} & \mathrm{G_{SM}}\ (Z_2) & &
\end{array}\right.\\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)}\ \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{1\ (1,2)} & \mathrm{G_{SM}}\ (Z_2) &
\end{array}\right.
$$

$$
4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ Z_2^{\rm C}\left\{\begin{array}{llll}
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L}\ Z_2^{\rm C} & \left\{\begin{array}{llll}
\xrightarrow{3} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \longrightarrow & \cdots\\
\xrightarrow{1,3} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)} & \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{2',3\ (2,3)} & \mathrm{G_{SM}}\ (Z_2) & &
\end{array}\right. & \\
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ Z_2^{\rm C} & \left\{\begin{array}{llll}
\xrightarrow{3} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \longrightarrow & \cdots\\
\xrightarrow{1,3} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)} & \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{3\ (2,3)} & \mathrm{G_{SM}}\ (Z_2) & &
\end{array}\right. & \\
\xrightarrow{3} & 4_{\rm C}\ 2_{\rm L}\ 2_{\rm R} & \longrightarrow & \\
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \longrightarrow & \cdots\\
\xrightarrow{1,3} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \longrightarrow & \cdots\\
\xrightarrow{1,3} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)} & \mathrm{G_{SM}}\ (Z_2)\\
\xrightarrow{1,3\ (1,2,3)} & \mathrm{G_{SM}}\ (Z_2). & &
\end{array}\right.
$$

R. Jeannerot, J. Rocher, and M. Sakellariadou, Phys. Rev. D 68, 103514 (2003)

SO(10) Breaking Chains



D. I. Dunsky, A. Ghoshal, H. Murayama, Y. Sakakihara, and G. White, Phys. Rev. D 106, 075030 (2022)

Topological defects in GUTs

GUT $\to \mathcal{G}_{\mathrm{I}} \to \mathcal{G}_{\mathrm{II}} \to$ SM	Topological defects		
	GUT $\to \mathcal{G}_{\mathrm{I}}$	$\mathcal{G}_I \to \mathcal{G}_{II}$	$\mathcal{G}_{II} \to$ SM
$E(6) \to \mathcal{G}_{3_L 3_R 3_C D} \to \mathcal{G}_{2_L 2_R 3_C 1_{LR} D} \to$ SM	Unstable $\mathbb{Z}_2$-strings $+\mathbb{Z}_3$-monopoles	Stable monopoles	Domain walls + embedded strings
$E(6) \to \mathcal{G}_{3_L 3_R 3_C} \to \mathcal{G}_{2_L 2_R 3_C 1_{LR} \not{D}} \to$ SM	$\mathbb{Z}_3$-monopoles	Stable monopoles	Embedded strings
$E(6) \to \mathcal{G}_{2_L 2_R 4_C 1_X D} \to \mathcal{G}_{2_L 1_X 4_C} \to$ SM	Unstable $\mathbb{Z}_2$-strings + stable monopoles + unstable $\mathbb{Z}_2$-monopoles	Domain walls	Embedded strings
$E(6) \to \mathcal{G}_{2_L 2_R 4_C 1_X \not{D}} \to \mathcal{G}_{2_L 1_X 4_C} \to$ SM	Stable monopoles + unstable $\mathbb{Z}_2$-monopoles	No defects	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 2_R 3_C 1_{B-L} D} \to$ SM	$\mathbb{Z}_2$-strings (stable upto M_{II}) $+\mathbb{Z}_2$-monopoles	Stable monopoles	Domain walls + embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 2_R 3_C 1_{B-L} \not{D}} \to$ SM	Unstable $\mathbb{Z}_2$-strings + $\mathbb{Z}_2$-monopoles	Domain walls + stable monopoles	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C \not{D}} \to \mathcal{G}_{2_L 2_R 3_C 1_{B-L} \not{D}} \to$ SM	$\mathbb{Z}_2$-monopoles	Stable monopoles	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 1_R 4_C} \to$ SM	Unstable $\mathbb{Z}_2$-strings + $\mathbb{Z}_2$-monopoles	Domain walls + stable monopoles	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C \not{D}} \to \mathcal{G}_{2_L 1_R 4_C} \to$ SM	$\mathbb{Z}_2$-monopoles	Stable monopoles	Embedded strings

J. Chakrabortty, **RM**, S. F. King, PRD **99** (2019) 095008

Composite Topological Structures in $\mathbf{SO(10)}$

$SO(10)$ breaking via:

- $SU(5) \times U(1)_\chi$,
- $SU(4)_c \times SU(2)_L \times SU(2)_R$,
- and $SU(5) \times U(1)_X$ (flipped $SU(5)$).

We find composite topological structures that include:

- a network of $\mathbb{Z}$ strings which develop monopoles and turn into necklaces with the structure of $\mathbb{Z}_2$ strings,
- dumbbells connecting two different types of monopoles, or monopoles and antimonpoles,
- starfish-like configurations,
- polypole configurations, and
- walls bounded by a necklace.

Dumbbell configuration in $\mathbf{SO(10)}$ breaking via $\mathbf{SU(5)} \times \mathbf{U(1)}_\chi$

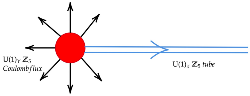


Figure 1. $SO(10)$ monopole carrying $U(1)_Y\ \mathbb{Z}_5$ Coulomb flux and $U(1)_\chi\ \mathbb{Z}_5$ magnetic flux tube.

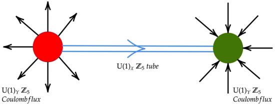


Figure 2. Dumbbell consisting of an $SO(10)$ monopole (red)-antimonopole (green) pair connected by a $U(1)_\chi\ \mathbb{Z}_5$ flux tube.

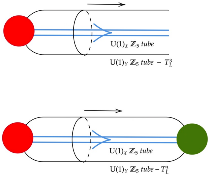


Figure 3. $SO(10)$ monopole and dumbbell configurations after the EW breaking.

Starfish configuration in $\mathbf{SO(10)}$ breaking via $\mathbf{SU(5)} \times \mathbf{U(1)}_\chi$

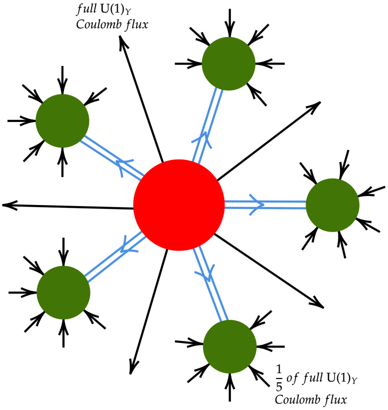


Figure: Starfish-like configuration with a central multimonopole connected to five $SO(10)$ antimonopoles by $U(1)\chi$ $\mathbb{Z}_5$ tubes.

Necklace configuration in $\mathbf{SO(10)}$ breaking via $\mathbf{SU(5)} \times \mathbf{U(1)}_\chi$

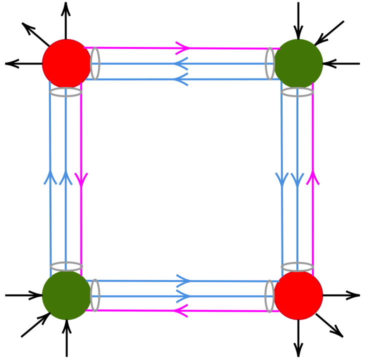
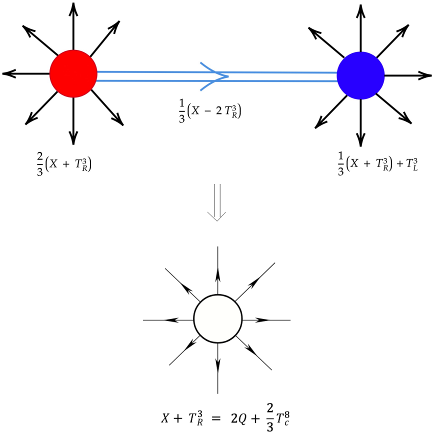


Figure: Necklace of monopoles (red) and antimonopoles (green) connected by $U(1)_\chi$ $\mathbb{Z}_{10}$ tubes, which carry half the flux of the $\mathbb{Z}_5$ tubes and correspond to a rotation by $2\pi/10$ along $U(1)_\chi$. These tubes can be thought of as hybrid structures consisting of a $\mathbb{Z}_2$ tube (magenta) and two $\mathbb{Z}_5$ anti-tubes (blue)

'Schwinger' Monopole

$$SU(4)_c \times SU(2)_L \times SU(2)_R \to SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R$$
$$\to SU(3)_c \times SU(2)_L \times U(1)_Y \qquad (1)$$



Necklace configuration in $\mathbf{SO(10)}$ breaking via $4-2-2$

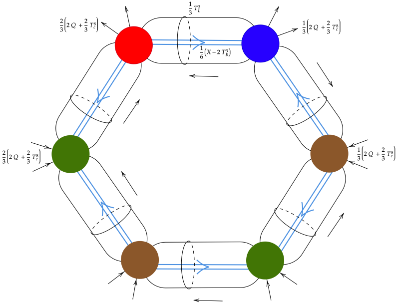


Figure: The antimonopole of the blue monopole is the brown monopole, and the antimonopole of the red monopole is the green monopole. This necklace is a realization of the $\mathbb{Z}_2$ string.

Polypole configuration in $\mathbf{SO(10)}$ breaking via $4-2-2$

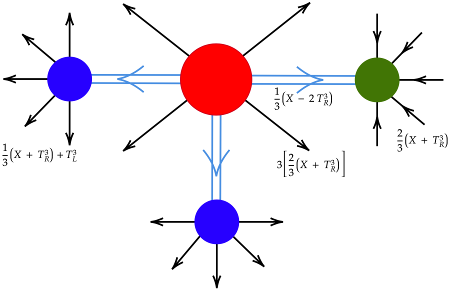


Figure: A polypole configuration with the three tubes from the red trimonopole after the breaking of $U(1)_{BL} \times U(1)_R$ terminating on one green (anti-red) and two blue monopoles.

Necklace configuration in $\mathbf{SO(10)}$ breaking via Flipped $\mathbf{SU(5)}$

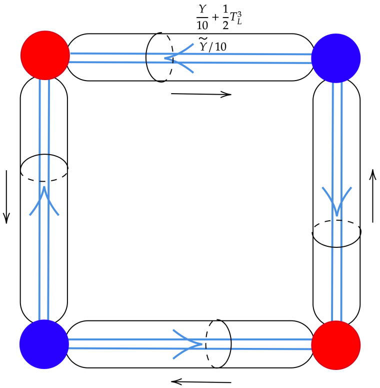


Figure: Necklace corresponding to $\mathbb{Z}_2$ strings in the flipped $SU(5)$ model.

Walls Bounded by Strings

- Consider the breaking chain

$$\begin{array}{ccc} SO(10) & \xrightarrow{\underset{\sim}{54}} & SU(4)_c \times SU(2)_L \times SU(2)_R \\ & & \Big\downarrow \underset{\sim}{126} \\ & & SU(3)_c \times SU(2)_L \times U(1)_Y \,. \end{array}$$

- The first step leaves unbroken the discrete symmetry ‘C’ (also known as ‘D’) that interchanges left and right, and conjugates the representations.
- The $\underset{\sim}{126}$ vev breaks ‘C’ which produces domain walls
- Thus we end up with walls bounded by strings.
  Similar structures also arise in axion models.

Walls bounded by a Necklace in $\mathbf{SO(10)}$ breaking via $\mathbf{SU(5)\times U(1)_\chi}$

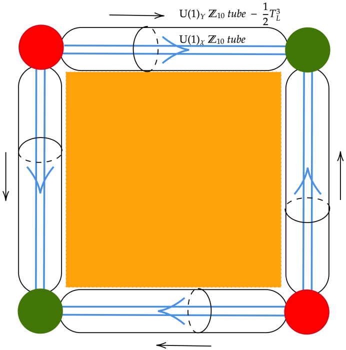


Figure: Emergence of a $\mathbb{Z}_2$ domain wall (orange) from each segment of the necklace due to the breaking of the $\mathbb{Z}_2$ subgroup of $U(1)_\chi$ by the VEV of a ν^c-type Higgs field and its conjugate, with the necklace ultimately becoming the boundary of the $\mathbb{Z}_2$ wall.

HQVs in the PdB phase

KIBBLE-LAZARIDES-SHAFI (KLS) WALL or WALL BOUNDED BY STRINGS

Composite defect suggested in the context of phase transitions in the early Universe:

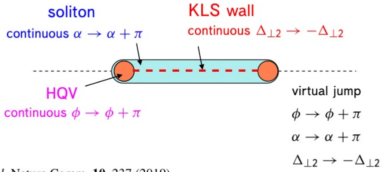
Phase 1 (Normal) $\Rightarrow$	Phase 2 (Polar) $\Rightarrow$	Phase 3 (PdB)
	Strings (HQVs) become topologically stable	Strings (HQVs) are not protected by topology — walls emerge

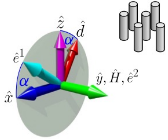
Kibble *et al*, PRD **26**, 435 (1982)

Polar-distorted B phase:

$$A_{\mu i} = e^{i\phi}(\Delta_\parallel \hat{d}_\mu \hat{z}_i + \Delta_{\perp 1} \hat{e}^1_\mu \hat{x}_i + \Delta_{\perp 2} \hat{e}^2_\mu \hat{y}_i)$$

$$|\Delta_{\perp 1}| = |\Delta_{\perp 2}| = q|\Delta_\parallel|, \quad q < 1$$





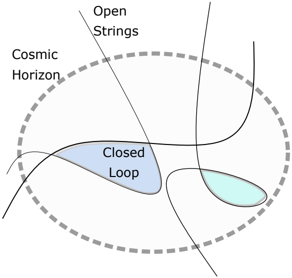
$$\hat{\mathbf{d}} = \hat{\mathbf{x}} \cos\alpha - \hat{\mathbf{z}} \sin\alpha$$
$$\hat{\mathbf{e}}^1 = \hat{\mathbf{z}} \cos\alpha + \hat{\mathbf{x}} \sin\alpha$$
$$\hat{\mathbf{e}}^2 = \hat{\mathbf{y}} \parallel \mathbf{H}$$

Mäkinen *et al*, Nature Comm. **10**, 237 (2019)

Cosmic Strings from SO(10)

Cosmic Strings arise during symmetry breaking of $G \to H$ if $\pi_1(G/H)$ is non-trivial. Consider

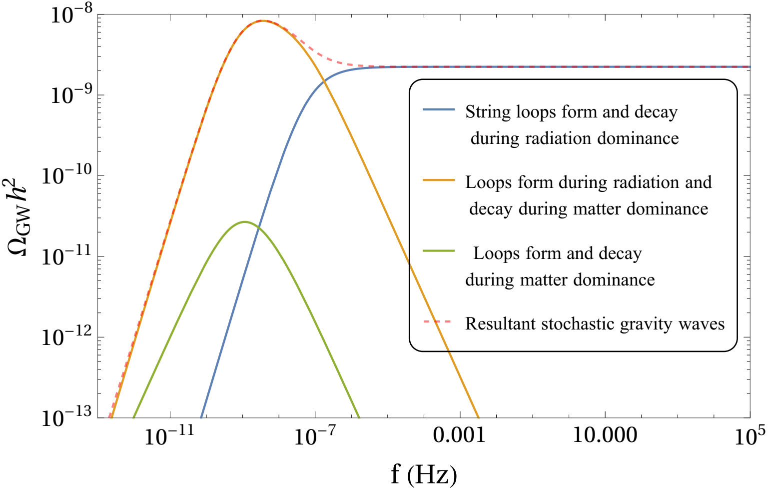
$SO(10) \xrightarrow{M_{GUT}} SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{M_I} SM \times Z_2$ Mass per unit length of string is $\mu \sim M_I^2$, with $M_I \ll M_P$. The strength of string gravity is determined by the dimensionless parameter $G\mu \ll 1$.



Stochastic Gravitational Waves from Strings

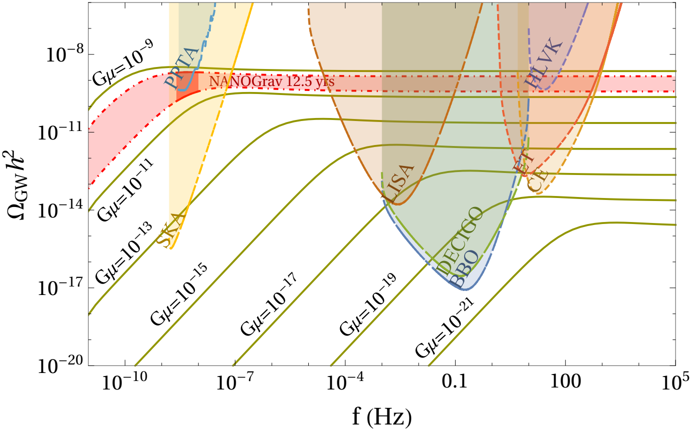
- Unresolved GWs bursts from string loops at different cosmic era produces the stochastic background.
- Loops that are formed and decay during radiation produce a plateau in the spectrum in the high frequency regime.
- Loops that are produced during radiation dominance but decay during matter dominance generate a sharply peaked spectrum at lower frequencies.
- Loops that are produced and decay during matter domination also generate a sharply peaked spectrum which, however, is overshadowed by the previous case.

Stochastic Gravitational Wave Background



Sousa, Avelino, Guedes, PRD 101 (2020) 10, 103508

GWs without Inflation and Observational Prospects



- Stringent constraint from PPTA: $G\mu \lesssim 10^{-11}$.
- Provisional GWs signal in NANOGrav: $G\mu \sim 10^{-10}$.

Evolution of Strings in Inflationary Cosmology

- The mean inter-string distance at cosmic time t (temp $= T$):

$$\boxed{d_{\rm str} = \; p\,\xi(\phi_I) \;\; \exp(N_{\rm str}) \;\; \left(\frac{t_r}{\tau}\right)^{\frac{2}{3}} \;\; \frac{T_r}{T}}$$

Inter-string seperation at production $\xi = \min\left(H^{-1}, m_{\rm eff}^{-1}\right)$

Expansion during Inflation

Expansion during Inflaton oscillations

Expansion after reheating

- The string network re-enters the post-inflationary horizon at cosmic time t_F if

$$\boxed{d_{\rm str}(t_F) = d_{\rm hor}(t_F)}$$

$$\text{with} \;\; d_{\rm hor}(t_F) = \begin{cases} 2t_F & \text{(radiation dominance)} \\ 3t_F & \text{(matter domination).} \end{cases}$$

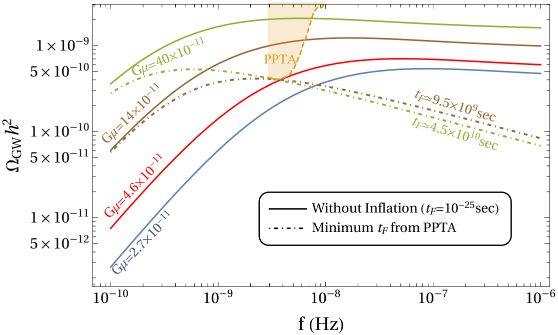
Chakrabortty, Lazarides, Maji, Shafi JHEP 02 (2021) 114

String Loops and Gravitational Waves

- After horizon re-entry the strings inter-commute and form loops at any subsequent time t_i.
- Loops of initial length $l_i = \alpha t_i$ decay via emission of gravity waves.
- The redshifted frequency of a normal mode k, emitted at time $\tilde{t}$, as observed today, is given by

$$f = \frac{a(\tilde{t})}{a(t_0)} \frac{2k}{\alpha t_i - \Gamma G\mu(\tilde{t} - t_i)}, \quad \text{with} \quad k = 1, 2, 3, \ldots$$

Inflation, GWs and PPTA bound



- Partially inflated strings re-enter horizon at time t_F in post-inflationary universe and decay via emission of GWs.
- Modified GWs spectra from ‘diluted’ strings can satisfy the PPTA bound.

Lazarides, Maji, Shafi, PRD 104 (2021) 9, 095004

String Loops and Gravitational Waves

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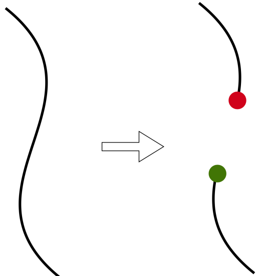
Metastable Strings

- The metastable string network decays via the Schwinger production of monopole-antimonopole pairs with a rate per string unit length of

$$\Gamma_d = \frac{\mu_{cs}}{2\pi} exp(-\pi\kappa_{cs}), \quad \kappa_{cs} = \frac{m^2}{\mu_{cs}}$$

where $m \sim M_G$ is the monopole mass and κ_{cs} quantifies the metastability of cosmic strings network with $\sqrt{\kappa_{cs}} \sim 10$ being the stability limit as the lifetime of cosmic strings becomes larger than the age of the Universe.



Quasistable Strings

- The strings are not topologically stable and connect monopoles and anti-monopoles.
- However, their lifetime of decay via quantum mechanical tunneling is larger than the age of the Universe, and the monopoles are partially inflated.
- Therefore, the strings make random walks with steps of the order of the horizon and form a network of stable strings before the horizon reentry of the monopoles.
- We call these quasi-stable strings as they form a stable network until the horizon reentry of monopoles.

Quasi-stable Cosmic Strings

- Example:

$$\begin{aligned} SO(10) &\xrightarrow{M_{\rm GUT}} SU(4)_c \times SU(2)_L \times SU(2)_R \\ &\xrightarrow{M_I} SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \\ &\xrightarrow{M_{II}} SU(3)_c \times SU(2)_L \times U(1)_Y . \end{aligned}$$

- Strings formed at M_{II} connect monopole-antimonopole ($M\bar{M}$) pairs formed at M_I.

- Strings are topologically unstable: $\Gamma_d = \frac{\mu}{2\pi} \exp\left(-\pi m_M^2/\mu\right)$ with $\mu \sim \pi M_{II}^2$ and $m_M \sim 10 M_I$.

- Strings are practically stable if $(m_M^2/\mu)^{1/2} \gtrsim 8.7$.

Lazarides, Maji, Shafi JCAP 08 (2022) 042

Gravitational Waves from Quasi-stable Strings

- Intermediate scale magnetic monopoles, created prior to the cosmic strings, experience partial inflation.

- The strings reenter the horizon (t_F) earlier than the $M\overline{M}$ pairs (t_M), form random walks with step of the order of the horizon, and inter-commute generating loops which decay into gravitational waves.

- As monopoles reenter the horizon we obtain monopole-antimonopole pairs connected by string segments which also decay into gravitational waves.

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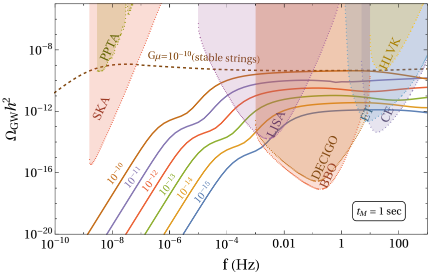
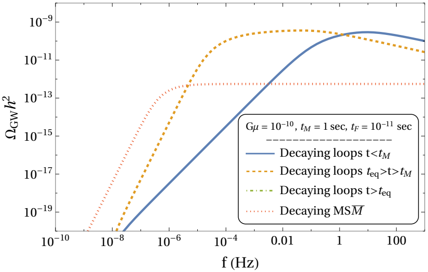
Gravitational Waves from Quasi-stable Strings

- Long string loops and segments are absent.
- Gravitational wave spectrum in the low frequency region is reduced.



GWs spectra.



Different components.

Lazarides, Maji, Shafi JCAP 08 (2022) 042

Thank You