T-Model Higgs Inflation in Supergravity

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Based on:

- C.P., *J. Cosmol. Astropart. Phys.* **05**, *043 (2021)* [arXiv:2103.05534].

Outline

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Inflationary Observables and Requirements

- Inflation is Called a Period of **Exponential Expansion** of The Universe, During Which

$$\left(d\widehat{\phi}/\sqrt{2}dt\right)^2 \ll V_{\rm I}(\widehat{\phi}) \simeq \text{Cst} \rightsquigarrow R(t)=R(t_{\rm i})e^{\Delta N_e} \quad \text{with} \quad \Delta N_e \text{ the Number of } e\text{-Folding}$$

t the Cosmic Time, $R(t)$ the Scale Factor & $\widehat{\phi}=\widehat{\phi}(t)$ the **Canonically Normalized Inflaton**.

- A **Successful** Inflationary Scenario **In Principle** Requires:
  - The **Number of e-foldings**, $N_\star$, that the Scale $k_\star=0.05/{\rm Mpc}$ Underwent During HI has to be Sufficient to Resolve the Horizon and Flatness Problems of Standard Big Bang:

$$N_\star=\int_{\widehat{\phi}_{\rm f}}^{\widehat{\phi}_\star} d\widehat{\phi}\,\frac{V_{\rm I}}{V_{{\rm I},\widehat{\phi}}}\simeq(44-56) \text{ Depending on } w_{\rm rh}\simeq(-0.24-0.58), \quad \text{Where}$$

    - The **Barotropic Index** $w_{\rm rh}$ Depends on the Degree of the Polynomial in $V_{\rm I}$;
    - $\widehat{\phi}_\star$ is The Value of $\widehat{\phi}$ When $k_\star$ Crosses Outside The Inflationary Horizon;
    - $\widehat{\phi}_{\rm f}$ is the Value of $\widehat{\phi}$ at the end of HI Which Can Be Found From The Condition:

$$\max\{\epsilon(\widehat{\phi}_{\rm f}),|\eta(\widehat{\phi}_{\rm f})|\}=1, \quad \text{With} \quad \epsilon=\left(V_{{\rm I},\widehat{\phi}}/\sqrt{2}V_{\rm I}\right)^2 \quad \text{and} \quad \eta=V_{{\rm I},\widehat{\phi}\widehat{\phi}}/V.$$

  - The **Amplitude $A_{\rm s}$ of the Power Spectrum** of the Curvature Perturbations is To Be Consistent with ***Planck*** Data:

$$A_{\rm s}^{1/2}=\frac{1}{2\sqrt{3}\pi}\frac{V_{\rm I}(\widehat{\phi}_\star)^{3/2}}{|V_{{\rm I},\widehat{\phi}}(\widehat{\phi}_\star)|}=4.588\cdot10^{-5}$$

- The Models Fulfilling The Restrictions Above Can be Further Qualified by Computing the **(Scalar) Spectral Index**, $n_{\rm s}$, its Running, $\alpha_{\rm s}$, and the **Tensor-to-Scalar Ratio**, r from the Formulas:

$$n_{\rm s}=1-6\epsilon_\star+2\eta_\star, \quad \alpha_{\rm s}=2\left(4\eta_\star^2-(n_{\rm s}-1)^2\right)/3-2\xi_\star \quad \text{and} \quad r=16\epsilon_\star,$$

Where $\xi=V_{{\rm I},\widehat{\phi}}V_{{\rm I},\widehat{\phi}\widehat{\phi}\widehat{\phi}}/V_{\rm I}^2$ And The Variables With Subscript $\star$ Are Evaluated at $\widehat{\phi}=\widehat{\phi}_\star$.

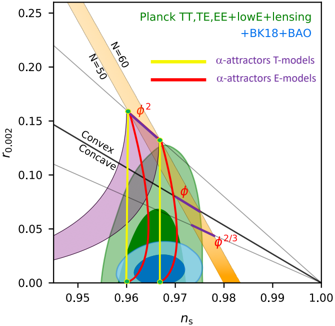
Observational Status of Higgs Inflation (HI)

- We Aspire to **Identify** ϕ With the Radial Component of a **Higgs Field**, $\Phi = \phi e^{i\theta}/\sqrt{2}$, Within **a Grand Unified Theory (GUT)**. Therefore, We Choose As Inflationary Potential The One Employed for the Realization of the **Higgs Mechanism**,

$$V_{\rm HI}(\phi) = \lambda^2(\phi^2 - M^2)^2/16 \simeq \lambda^2\phi^4/16 \quad \text{For} \quad M \ll m_{\rm P} = 1. \quad (\because \textbf{H})$$

- For $\phi = \widehat{\phi}$, the **Theoretically Derived Values** $n_{\rm s} \simeq 0.947$ and $r \simeq 0.28$ Are Not Compatible With the Observational Ones.
- The Combined Bicep2/*Keck Array* and *Planck* Results Require, for Fitted $A_{\rm s}$ and $N_\star$,

$$n_{\rm s} = 0.965 \pm 0.009 \quad \text{and} \quad r \lesssim 0.032 \quad \text{at } 95\% \text{ c.l.}$$



- On the Contrary, **Observationally Friendly** Are Models Called **α-Attractors** Which Employ Chaotic Potentials and so can be Activated with $V_{\rm HI}$ in Eq. (H).
- These Are Based on the Specific Relation Established Between **the Initial**, ϕ, and the **Canonically Normalized** Inflaton $\widehat{\phi}$ and can be Classified into **E-Model Inflation** (**EMI**) (or α-Starobinsky model) and **T-Model Inflation** (**TMI**) And I.e.

$$\phi = \begin{cases} 1 - \mathrm{Exp}\left(-\sqrt{2/N}\widehat{\phi}\right) & \text{For EMI,} \\ \tanh\left(\widehat{\phi}/\sqrt{2N}\right) & \text{For TMI,} \end{cases} \quad \text{With} \quad N > 0.$$

- Such Relations Between ϕ and $\widehat{\phi}$ Can be Achieved in The Presence Of A **Pole** In The Inflaton **Kinetic Term**.

T-Model HI From A Kinetic Pole of Second Order

- **TMI** Is "Taylor Made" for HI Since It Arises from a **Kinetic Pole of Order Two** Which Includes the **GUT-Invariant** Quantity $|\Phi|^2 := \Phi^\dagger\Phi$. In Particular, the Lagrangian $\mathcal{L}$ of $\phi = \phi(t)$ Reads

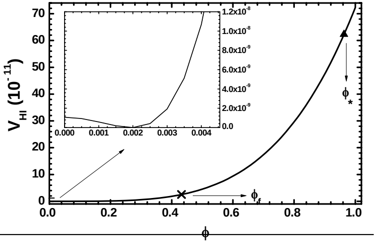
$$\mathcal{L} = \sqrt{-\mathfrak{g}}\left(N_2\dot\phi^2/2f_2^2 - V_{\rm HI}(\phi)\right) \text{ with } {}^{\cdot} = d/dt,\;\; f_2 = 1-\phi^2 \text{ and } N_2 > 0.$$

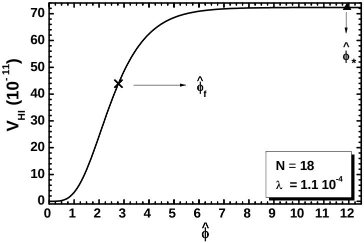
Also, $\mathfrak{g}$ is the Determinant of the Space-time Metric $g_{\mu\nu}$.

- If We **Extract $\widehat\phi$**, Via the Relation $N_2\dot\phi^2/2f_2^2 = (d\widehat\phi/\sqrt{2}dt)^2$, We Obtain[1]

$$\frac{d\widehat\phi}{d\phi} = J = \frac{\sqrt{N_2}}{f_2} \;\;\Rightarrow\;\; \phi = \tanh\frac{\widehat\phi}{\sqrt{N_2}}. \text{ Therefore } V_{\rm HI}(\widehat\phi) \simeq \frac{\lambda^2}{16}\tanh^4\frac{\widehat\phi}{\sqrt{N_2}}.$$

$V_{\rm HI}$ Expressed as a Function of $\widehat\phi$ Develops a **Plateau** for $\widehat\phi > 1$ Which Renders it Convenient for the Realization of a Observationally Viable HI.





[1] *R. Kallosh and A. Linde (2013); J. Ellis, D.V. Nanopoulos and K.A. Olive (2013).*

Inflation Analysis

• The **Parameters ϵ & η** (Expressed as a Function of ϕ) Decrease With **the Pole Function** $f_2 = 1-\phi^2$

$$\epsilon \simeq 16{f_2}^2/N_2\phi^2 \quad \text{and} \quad \eta \simeq 8f_2(3-5\phi^2)/N_2\phi^2$$

And Can be Kept Below Unity As $\phi \to 1$, Assuring Thereby an Inflationary Period.

• The **Number of e-Foldings** Turns out to be Inverse Proportional of $f_{2\star} = 1-\phi_\star^2$,

$$N_\star \simeq N_2\phi_\star^2/4f_{2\star} \;\Rightarrow\; \phi_\star = \sqrt{4N_\star}/\sqrt{4N_\star+N_2} \sim 1 \gg \phi_{\rm f},$$

And so $N_\star$ Can be Adequately Large For $\phi_\star \to 1$.

• The Proximity of $\phi_\star$ to 1 Signals a Just **Mild Tuning** In the Initial Conditions Since

$$0.01 \lesssim \Delta_\star \lesssim 0.04 \quad \text{for} \quad 0.1 \lesssim N_2 \lesssim 55 \quad \text{Where} \quad \Delta_\star = (1-\phi_\star)$$

• The Normalization of $A_{\rm s}$ Provides the **Value of λ**, i.e.,

$$A_{\rm s}^{1/2} \simeq \frac{\sqrt{2}\lambda N_\star}{\sqrt{3N_2}\pi} = 4.588\cdot 10^{-5} \;\Rightarrow\; \lambda \simeq 2\sqrt{3N_2A_{\rm s}}\pi/N_\star \;\Rightarrow\; \underline{\lambda \sim 10^{-5}} \quad \text{For} \quad N_\star \simeq 55 \;\;\&\;\; N_2 = 1.$$

• For the Remaining Inflationary Observables We obtain

$$n_{\rm s} \simeq 1-2/N_\star \simeq 0.965, \quad \alpha_{\rm s} \simeq -2/N_\star^2 = 9.5\cdot 10^{-4} \quad \text{and} \quad r \simeq 2N_2/N_\star^2 \leq 0.032 \quad\Rightarrow\quad N_2 \lesssim 55,$$

Consistently With the Data, **Provided we Pose an Upper Bound on N_2**.

• The Effective Theory Describing HI Remains **Valid Up to a "Ultraviolet" Cutoff Threshold**, $\Lambda_{\rm UV} \sim m_{\rm P}$, Assuring the Stability of the Inflationary Solutions,

$$\text{(a)}\;\; V_{\rm I}(\phi_*)^{1/4} \leq \Lambda_{\rm UV} \quad \text{for} \quad \text{(b)}\;\; \phi \leq \Lambda_{\rm UV}\,.$$

• The Natural Framework for the Analysis Of a GUT Is **Supersymmetry (SUSY)** – and its Topical Extension, **Supergravity (SUGRA)** – Where the **Gauge Hierarchy** Problem Can Be Naturally Arranged.

SUGRA Scalar Potential

• The General **Lagrangian For The Scalar Fields** z^α Plus Gravity In Four Dimensional, $\mathcal{N}=1$ SUGRA is:

$$\mathcal{L} = \sqrt{-\mathfrak{g}}\left(K_{\alpha\bar\beta}g^{\mu\nu}D_\mu z^\alpha D_\nu z^{*\bar\beta} - V_{\rm SUGRA}\right) \quad \text{where} \quad K_{\alpha\bar\beta} := \partial^2_{z^\alpha z^{*\bar\beta}}K > 0, \quad K^{\bar\beta\alpha}K_{\alpha\bar\gamma} = \delta^{\bar\beta}_{\bar\gamma};$$

$$V_{\rm SUGRA} = V_{\rm F} + V_{\rm D} \quad \text{With} \quad \begin{cases} V_{\rm D} = g^2{\rm D}_a^2/2, \quad {\rm D}_a = z_\alpha\left(T_a\right)^\alpha_\beta K_{,z^\beta} \\ V_{\rm F} = e^K\left(K^{\alpha\bar\beta}D_\alpha W D^*_{\bar\beta}W^* - 3|W|^2\right) \end{cases} \text{and} \quad \begin{cases} D_\mu z^\alpha = \partial_\mu z^\alpha + igA^a_\mu T^a_{\alpha\beta}z^\beta, \\ D_\alpha = \partial_\alpha + \partial_\alpha K. \end{cases}$$

A^a_μ is The Vector Gauge Fields, g is the Gauge Coupling and T_a are the Generators of the Gauge Transformations Of z^α.

• The Kinetic Mixing is Controlled by The **Kähler Potential** K Which Affects **Also** $V_{\rm F}$. This Depends on an Holomorphic Function of the Superfields Called **Superpotential** W Too.

• Therefore, Possible Appearance of f_2 in $K_{\alpha\bar\beta}$ Is Expected to Impact on $V_{\rm F}$ too, In Contrast to Non-SUSY Case, Making More Difficult the Realization of TMI in SUGRA. We Propose Below **Two Ways Out** of this Difficulty.

• We Concentrate on **HI Driven by $V_{\rm F}$** Which Requires $V_{\rm D}=0$ During HI.

Introduction of the Stabilizer Field

• In General, the Realization of Chaotic Inflation in SUGRA Can be Facilitated, If We Introduce A Gauge-Singlet Superfield $z^1=S$ Called **Stabilizer or Goldstino**[2]. Its Introduction is Necessary For the Following Reasons:

- It can be **Stabilized** at $S=0$ Without Invoking Higher Order Terms, if we Select[3]:
  $K_2 = N_S\ln\left(1+|S|^2/N_S\right) \;\Rightarrow\; K_2^{SS^*}=1$ With $0<N_S<6$ Which Parameterizes the Compact Manifold $SU(2)/U(1)$.
- It Assures the **Boundedness of $V_{\rm F}$:** If We set $S=0$ During HI, the Terms $K_{,z^\alpha}W$, $\alpha\neq1$, and $-3|W|^2$ Vanish. The 2nd one May Render $V_{\rm F}$ Unbounded From Below.
- It Generates the **non-SUSY Potential** From the Term $|W_{,S}|^2$ for $S=0$. E.g., For $W=\lambda S\Phi^{\mathbf{n}/2}$ We Obtain

$$\langle V_{\rm F}\rangle_{\rm I} = \langle e^K K^{SS^*}|W_{,S}|^2\rangle_{\rm I} \in V_{\rm non-SUSY} = \lambda^2\phi^n \quad \text{with} \quad \phi = {\rm Re}(\Phi) \quad \text{the (Initial) Inflaton.}$$

[2] R. Kallosh, A. Linde and T. Rube (2011). [3] C.P. and N. Toumbas (2016).

Selection of K & W

- We Employ a Pair of Chiral Superfields $z^2 = \Phi$ & $z^3 = \bar{\Phi}$, With **Charged Oppositely** Under a Gauge Symmetry, E.g., $U(1)_{B-L}$ and a **Superpotential**

$$W = S\left(\lambda_2\bar{\Phi}\Phi/2 - M^2/4 + \lambda_4(\bar{\Phi}\Phi)^2\right)$$

Determined **Uniquely** From a $U(1)_{B-L}$ & a Global $U(1)_R$ Under Which $R(W) = R(S)$.

Charge Assignments

Superfields:	S	Φ	$\bar{\Phi}$
$U(1)_R$	1	0	0
$U(1)_{B-L}$	0	2	-2

W Leads to a GUT **Phase Transition** in SUSY Vacuum $\langle S\rangle = 0, |\langle\Phi\rangle| = |\langle\bar{\Phi}\rangle| \sim M/\sqrt{2\lambda_2}$.

- The **Introduction of f_2** in the Kinetic Terms is Achieved, if We Adopt One of the Following Kähler

$$K_{21} = -N\ln\left(1 - |\Phi|^2 - |\bar{\Phi}|^2\right) \quad \text{Or} \quad \widetilde{K}_{21} = -N\ln\frac{\left(1 - |\Phi|^2 - |\bar{\Phi}|^2\right)}{(1-2\bar{\Phi}\Phi)^{1/2}(1-2\bar{\Phi}^*\Phi^*)^{1/2}},$$

Which Share the Same **Kähler Metric**, $K_{\alpha\bar{\beta}}$ Parameterizing the Kähler Manifold $SU(2,1)/(SU(2)\times U(1))$.

- For Both K, the D Term Due to $U(1)_{B-L}$ is $\mathrm{D}_{BL} = N\left(|\Phi|^2 - |\bar{\Phi}|^2\right)/\left(1 - |\Phi|^2 - |\bar{\Phi}|^2\right) \rightsquigarrow V_{\rm D} = 0$ If $|\Phi| = |\bar{\Phi}|$

I.e., the D Term is Eliminated During HI, If we Choose as Inflaton the **Common Radial Part** of Φ & $\bar{\Phi}$.

- For $K = K_{21}$ We Obtain $\langle e^K\rangle_{\rm I} = 1/f_2^N$ And So, A Pole Appears in $\langle V_{\rm F}\rangle_{\rm I}$. However, This **Pole Can be Eliminated** in $\langle V_{\rm F}\rangle_{\rm I}$, If we set $N = 2$ & $\lambda_2 \simeq -\lambda_4 = \lambda$ Resulting to $W \simeq \lambda S\Phi\bar{\Phi}(1-\bar{\Phi}\Phi)$ – for $M \ll 1$.
- For $K = \widetilde{K}_{21}$ We Obtain $\langle e^K\rangle_{\rm I} = 1$ and so, **no Pole Appears** in $\langle V_{\rm F}\rangle_{\rm I}$.
- In all, We End up With the **Following Models**:
    - δ T-Model **(δTM)** With $K = K_{221} = K_2 + K_{21}$, $N = 2$ and $\lambda_4 = -\lambda_2(1+\delta_\lambda)$ in W with $\delta_\lambda = \mathcal{O}(10^{-5})$; The Results Deviate With Those Obtained in non-SUSY Regime.
    - T-Model 4 & 8 **(TM4 & TM8)** With $K = \widetilde{K}_{221} = K_2 + \widetilde{K}_{21}$ with Hierarchy $\lambda_2 \gg \lambda_4$ and $\lambda_2 \ll \lambda_4$ Respectively; N Remains a Free Parameter as in the non-SUSY Regime.

Inflationary Potential

- If We Use The **Parameterizations**: $\Phi = \phi e^{i\theta}\cos\theta_\Phi$ and $\bar\Phi = \phi e^{i\bar\theta}\sin\theta_\Phi$ with $0 \leq \theta_\Phi \leq \pi/2$ and $S = (s + i\bar s)/\sqrt{2}$

We Select as **Inflationary Path** The **D-Flat Direction** Is $\langle\theta\rangle_{\rm I} = \langle\bar\theta\rangle_{\rm I} = 0,\ \langle\theta_\Phi\rangle_{\rm I} = \pi/4$ and $\langle S\rangle_{\rm I} = 0$ $(:\mathbf{I})$

- The only Surviving term of $V_{\rm F}$ Along the Path in Eq. (P) is (With $r_{ij} = -\lambda_i/\lambda_j$ with $i, j = 2, 4$)

$$V_{\rm HI} = \langle e^K K^{SS^*}|W_{,S}|^2\rangle_{\rm I} = \frac{\lambda^2}{16}\begin{cases}\left(\phi^2 - r_{42}\phi^4 - M_2^2\right)^2/f_2^N & \text{for } \delta\text{TM},\\ \left(\phi^2 - r_{42}\phi^4 - M_2^2\right)^2 & \text{for TM4},\\ \left(\phi^4 - r_{24}\phi^2 - M_4^2\right)^2 & \text{for TM8},\end{cases}\quad \text{where } \lambda = \begin{cases}\lambda_2 \text{ and } M_2 = \frac{M}{\sqrt{\lambda_2}} & \text{for } \delta\text{TM and TM4},\\ \lambda_4 \text{ and } M_4 = \frac{M}{\sqrt{\lambda_4}} & \text{for TM8}.\end{cases}$$

- In All Three Cases, T-Model HI Can Be Realized Since the **Convenient Relation** $\phi - \widehat\phi$ can Be Achieved.
- To Verify This, We Compute $K_{\alpha\bar\beta}$ Along Eq. (I), Which takes the Form

$$\left(\langle K_{\alpha\bar\beta}\rangle_{\rm I}\right) = \left(\langle M_{\Phi\bar\Phi}\rangle_{\rm I}, \langle K_{SS^*}\rangle_{\rm I}\right) \text{ with } \langle M_{\Phi\bar\Phi}\rangle_{\rm I} = \frac{\kappa\phi^2}{2}\begin{pmatrix}2/\phi^2 - 1 & 1\\ 1 & 2/\phi^2 - 1\end{pmatrix},\ \kappa = \frac{N}{f_2^2}\ \&\ \langle K_{SS^*}\rangle_{\rm I} = 1.$$

- Upon Diagonalisation of $\langle M_{\Phi\bar\Phi}\rangle_{\rm I}$ the **Canonically Normalized Fields**, Are

$$\frac{d\widehat\phi}{d\phi} = J = \frac{\sqrt{2N}}{f_2} \ \Rightarrow\ \phi = \tanh\frac{\widehat\phi}{\sqrt{2N}},\ \widehat\theta_+ = \sqrt{\kappa}\phi\theta_+,\ \widehat\theta_- = \sqrt{\kappa f_2}\phi\theta_-\ \&\ \widehat\theta_\Phi = \sqrt{2\kappa f_2}\phi\left(\theta_\Phi - \pi/4\right).$$

- We Check The **Stability** of the Trajectory in Eq. (I) w.r.t the Fluctuations of the Various Fields. I.e.,

$$\left\langle\frac{\partial V_{\rm SUGRA}}{\partial\widehat z^\alpha}\right\rangle_{\rm I} = 0\ \&\ \widehat m^2_{z^\alpha} > 0,\ \text{Where } \widehat m^2_{z^\alpha} = \text{Egv}\left[\widehat M^2_{\alpha\beta}\right] \text{ With } \widehat M^2_{\alpha\beta} = \left\langle\frac{\partial^2 V_{\rm SUGRA}}{\partial\widehat z^\alpha\partial\widehat z^\beta}\right\rangle_{\rm I}\ \&\ z^\alpha = \theta_\pm, \theta_\Phi, s, \bar s.$$

Here Egv Are the Eigenvalues of $\widehat M^2_{\alpha\beta}$ & the Subscript I Denotes Computation along Eq. (I).

Stability of The Inflationary Direction

Scalar **Mass-Squared Spectrum** for $K = K_{221}$ and $\widetilde{K}_{221}$ Along The Inflationary Trajectory

Fields	Eigen-states	Masses Squared		
			$K = K_{221}$	$K = \widetilde{K}_{221}$
2 real scalars	$\widehat{\theta}_+$	$m^2_{\widehat{\theta}+}$	$3H_{\rm I}^2$	
	$\widehat{\theta}_\Phi$	$\widehat{m}^2_{\theta_\Phi}$	$M^2_{BL} + 6H_{\rm I}^2(1 + 4/N - 2/N\phi^2 - 2\phi^2/N)$	
1 complex scalar	$s, \bar{s}$	$\widehat{m}^2_s$	$6H_{\rm I}^2(1/N_S - 8(1-\phi^2)/N + N\phi^2/2 + 2(1-2\phi^2) + 8\phi^2/N)$	$6H_{\rm I}^2(1/N_S - 4/N + 2/N\phi^2 + 2\phi^2/N)$
1 gauge boson	A_{BL}	M^2_{BL}	$2Ng^2\phi^2/f_2^2$	
4 Weyl spinors	$\widehat{\psi}_\pm$	$\widehat{m}^2_{\psi\pm}$	$12f_2^2H_{\rm I}^2/N^2\phi^2$	
	$\lambda_{BL}, \widehat{\psi}_{\Phi-}$	M^2_{BL}	$2Ng^2\phi^2/f_2^2$	

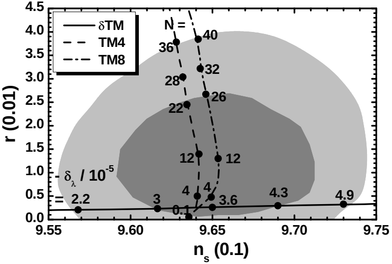
- We can Obtain $\forall\,\alpha$, $\widehat{m}^2_{\chi^\alpha} > 0$. Especially $\widehat{m}^2_s > 0 \;\Leftrightarrow\; N_S < 6$.
- We can Obtain $\forall\,\alpha$, $\widehat{m}^2_{\chi^\alpha} > H_{\rm I}^2$ and So No other Inflationary Perturbations Besides that of ϕ Contribute to $A_{\rm s}$;
- $M_{BL} \neq 0$ Signals the Fact that That $U(1)_{B-L}$ Is Broken and so, **no Topological Defects** are Produced.
- M_2 & M_4 Can be Determined **Demanding** the GUT Scale $M_{\rm GUT} \simeq 2/2.4\times10^{-2}$ In the Context of **Minimal SUSY Standard Model (MSSM)** Coincides With $\langle M_{BL}\rangle$, E.g.,

$$\langle M_{BL}\rangle = \sqrt{2N}gM_2/\langle f_2\rangle = M_{\rm GUT} \;\Rightarrow\; M_2 \simeq M_{\rm GUT}/g\sqrt{2N} \ll m_{\rm P} \;\text{ with }\; g \simeq 0.7 \text{ (GUT Coupling Constant)}.$$

- The One-Loop **Radiative Corrections** à la Coleman-Weinberg to $V_{\rm I}$ Can Be Kept Under Control Once the Renormalization Scale is Conveniently Selected.

Testing Against The Inflationary Data

- Enforcing $N_\star \simeq 44-56$ and $\sqrt{A_s} = 4.588 \cdot 10^{-5}$, we Obtain the Allowed Curves for Our Models In the $n_s - r$ Plane
- The **Free Parameters** of δTM, TM4, TM8 Are $\delta_\lambda = r_{42} - 1,\ (N, r_{42})$ And (N, r_{24}) Respectively.
- We set $r_{42} = 0.01$ for TM4 and $r_{24} = 10^{-6}$ for TM8.



Model:	δTM	TM4	TM8
δ_λ / r_{42} / r_{24} N	$-3.6 \cdot 10^{-5}$ 2	0.01 12	10^{-6} 12
$\phi_\star/0.1$ $\Delta_\star(\%)$ $\phi_f/0.1$	9.9555 0.445 5.9	9.75 2.5 3.9	9.877 1.23 6.5
w_{rh} $N_\star$	0.33 55.2	0.266 56.4	0.58 58
$\lambda/10^{-5}$	3.6	8.6	8.5
$n_s/0.1$ $r/10^{-2}$	9.65 0.26	9.64 1.4	9.65 1.3

- In **δTM** We Have **$N = 2$** & All Allowed n_s Are Possible with $r < 0.01$, $\delta_\lambda \sim 10^{-5}$ & $\Delta_\star \sim 10^{-3}$.
- For **TM4 & TM8** n_s Turns out to be Close to Its Observationally Favorable Value, r Increase With N and $\Delta_\star$ **Covering** all the Allowed Values, I.e.,

$$0.963 \lesssim n_s \lesssim 0.965, \quad 0.1 \lesssim N \lesssim 40, \quad 0.45 \gtrsim \Delta_\star/10^{-2} \gtrsim 13.6 \ \ \& \ \ 0.0025 \lesssim r \lesssim 0.032\,.$$

The Upper Bound on $r \lesssim 0.032$ Implies an **Upper Bound $N \lesssim 40$**.

- For **TM8** An Additional Tuning is Required Since $r_{24} \sim 10^{-6}$.
- **TM4** Can be Qualified As the Most **Natural** one Regarding the Choice of the Parameters.

Inflaton Mass

- For **TM4** The **Mass of the Inflaton** at the SUSY Vacuum is

$$\widehat{m}_{\delta\phi} = \langle V_{\mathrm{HI},\widehat{\phi\phi}}\rangle^{1/2} \simeq (4.4-25)\cdot 10^{10}\ \mathrm{GeV} \quad \text{For} \quad 1 \lesssim N \lesssim 36.$$

I.e., $\widehat{m}_{\delta\phi}$ Crucially Depends on the Imposed GUT Constraint and Lies at the Intermediate Energy Scale.

Embedding Of the Model

- T Model HI Can Be Embedded in a $B-L$ Extension of MSSM **Promoting** to Gauge the Pre-Existing Global $U(1)_{B-L}$. The Terms of the **Total Superpotential** Which Control the Coexistence of the Inflationary and the MSSM Sectors Are

$$\Delta W = \lambda_\mu S H_u H_d \ + \ \lambda_{ij\nu}\bar{\Phi}N_i^c N_j^c$$

Where H_u and H_d are the Electroweak Higgs Superfields & N_i^c the ith Generation **Right-handed Neutrino** with $i=1,..,3$.

Charge Assignments

Superfields:	N_i^c	H_u	H_d
$U(1)_R$	1	0	0
$U(1)_{B-L}$	1	0	0

- The Terms Above Allow for The Perturbative **Inflaton Decay** Into:
  - **A Pair of** (N_j^c) With Majorana Masses M_{jN^c} Through The Following Decay Width

$$\widehat{\Gamma}_{\delta\phi\to N_i^c} = \frac{g_{iN^c}^2}{16\pi}\widehat{m}_{\delta\phi}\left(1-\frac{4M_{iN^c}^2}{\widehat{m}_{\delta\phi}^2}\right)^{3/2} \quad \text{With} \quad g_{iN^c} = \lambda_{iN^c}/\langle J\rangle \quad \text{Arising from} \quad \mathcal{L}_{\widehat{\delta\phi}\to N_i^c} = g_{iN^c}\,\widehat{\delta\phi}\left(N_i^c N_i^c \ + \mathrm{h.c}\right).$$

  - **H_u and H_d** Through The Following Decay Width

$$\widehat{\Gamma}_{\delta\phi\to H} = \frac{2}{8\pi}g_H^2\widehat{m}_{\delta\phi} \quad \text{with} \quad g_H = \frac{\lambda_\mu}{\sqrt{2}} \quad \text{Arising from} \quad \mathcal{L}_{\widehat{\delta\phi}\to H_u H_d} = -g_H\widehat{m}_{\delta\phi}\widehat{\delta\phi}\left(H_u^* H_d^* + \mathrm{h.c}\right).$$

- The Reheating Temperature, T_{rh}, is given by

$$T_{\mathrm{rh}} = \left(72/5\pi^2 g_*\right)^{1/4}\widehat{\Gamma}_{\delta\phi}^{1/2} m_{\mathrm{P}}^{1/2} \quad \text{with} \quad \widehat{\Gamma}_{\delta\phi} = \widehat{\Gamma}_{\delta\phi\to N_i^c} + \widehat{\Gamma}_{\delta\phi\to H}, \quad \text{with} \quad g_* \simeq 228.75\,.$$

Leptogenesis and $\widetilde{G}$ Abundance

- The **Out-Of-Equilibrium Decay** of N_i^c can Generate an L Asymmetry Which Can Be Converted to the **B Yield**:

$$Y_B = -0.35\; 2\;\frac{5}{4}\;\frac{T_{\rm rh}}{\widehat{m}_{\delta\phi}}\;\frac{\widehat{\Gamma}_{\delta\phi\to N_i^c}}{\widehat{\Gamma}_{\delta\phi}}\,\varepsilon_i \quad \text{Where } \varepsilon_i = \sum_{j\neq i}\frac{\mathsf{Im}\left[(m_{\rm D}^\dagger m_{\rm D})^2_{ij}\right]}{8\pi\langle H_u\rangle^2 (m_{\rm D}^\dagger m_{\rm D})_{ii}}\Big(F_{\rm S}\left(x_{ij}, y_i, y_j\right) + F_{\rm V}(x_{ij})\Big).$$

Here $x_{ij} := M_{jN^c}/M_{iN^c}$ and $y_i := \Gamma_{iN^c}/M_{iN^c} = (m_{\rm D}^\dagger m_{\rm D})_{ii}/8\pi\langle H_u\rangle^2$ and $\widehat{m}_{\delta\phi} < 2M_{iN^c}$ For Some i with $i = 1, 2, 3$.
Also $F_{\rm V}$ and $F_{\rm S}$ Represent, Respectively, The Contributions From **Vertex And Self-Energy** Diagrams.

- $m_{i{\rm D}}$ are the Dirac Masses Which May Be Diagonalized In the **Weak (primed) Basis**

$$U^\dagger m_{\rm D} U^{c\dagger} = d_{\rm D} = \mathsf{diag}\left(m_{1{\rm D}}, m_{2{\rm D}}, m_{3{\rm D}}\right) \quad \text{Where } L' = LU \quad \text{and} \quad N^{c\prime} = U^c N^c.$$

And Are Related to M_{iN^c} via the **Type I Seesaw** Formula

$$m_\nu = -m_{\rm D}\, d_{N^c}^{-1}\, m_{\rm D}^{\mathsf{T}}, \quad \text{Where } d_{N^c} = \mathsf{diag}\left(M_{1N^c}, M_{2N^c}, M_{3N^c}\right) \text{ with } M_{1N^c} \leq M_{2N^c} \leq M_{3N^c} \text{ Real and Positive.}$$

- Replacing $m_{\rm D}$ in the See-Saw Formula We Extract The Mass Matrix of Light Neutrinos In The Weak Basis

$$\bar{m}_\nu = U^\dagger m_\nu U^* = -d_{\rm D} U^c d_{N^c}^{-1} U^{c\mathsf{T}} d_{\rm D},$$

Which Can Be Diagonalized by the Unitary **PMNS Matrix** U_ν Parameterized As Follows:

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} e^{-i\varphi_1/2} & & \\ & e^{-i\varphi_2/2} & \\ & & 1 \end{pmatrix},$$

with $c_{ij} := \cos\theta_{ij}$, $s_{ij} := \sin\theta_{ij}$, δ the CP-Violating Dirac Phase and φ_1 and φ_2 the two CP-violating Majorana Phases.

- The **Thermally Produced $\widetilde{G}$ Yield** At The Onset of **Big-Bang Nucleosynthesis (BBN)** Is Estimated To Be:

$$Y_{\widetilde{G}} \simeq 1.9\cdot 10^{-22}\, T_{\rm rh}/{\rm GeV}.$$

Post-Inflationary Requirements

The **Baryogenesis** Scenario via **non-Thermal Leptogenesis (nTL)** Can be Characterized as Successful If:

(i) We Obtain the **Observationally Required B Yield** Which is $Y_B = (8.697 \pm 0.054) \cdot 10^{-11}$ at 95% c.l.

(ii) Constraints on M_{iN^c} Are Satisfied. We have To Avoid Any Erasure Of The Produced Y_L; Ensure That The ϕ Decay To N_i^c Is Kinematically Allowed; and M_{iN^c} are Theoretically Acceptable, We Have To Impose The Constraints:

$$(a)\ M_{1N^c} \gtrsim 10T_{\rm rh},\ \ (b)\ \widehat{m}_{\delta\phi} \geq 2M_{1N^c}\ \text{ and }\ (c)\ M_{iN^c} \lesssim 7.1M\ \Leftrightarrow\ \lambda_{iN^c} \lesssim 3.5.$$

(iii) $\widetilde{G}$ Constraint Is Under Control. Assuming Unstable $\widetilde{G}$, We Impose an Upper Bound[4] on $Y_{\widetilde{G}}$ In Order to Avoid Problems With the BBN:

$$Y_{3/2} \lesssim \begin{cases} 10^{-14} \\ 10^{-13} \end{cases} \Rightarrow\ T_{\rm rh} \lesssim \begin{cases} 5.3 \cdot 10^{7}\ {\rm GeV} \\ 5.3 \cdot 10^{8}\ {\rm GeV} \end{cases}\ \text{ for } \widetilde{G} \text{ Mass }\ m_{3/2} \simeq \begin{cases} 0.69\ {\rm TeV}, \\ 10.6\ {\rm TeV}. \end{cases}$$

(iv) It is In Agreement With the **Light Neutrino Data**.

Parameter	Best Fit Value (2021)	
	Normal	Inverted
	Hierarchy	
$\Delta m^2_{21}/10^{-3}{\rm eV}^2$	7.5	
$\Delta m^2_{31}/10^{-3}{\rm eV}^2$	2.55	2.45
$\sin^2\theta_{12}/0.1$	3.18	
$\sin^2\theta_{13}/0.01$	2.2	2.225
$\sin^2\theta_{23}/0.1$	5.74	5.78
δ/π	1.08	1.58

- The Masses, $m_{i\nu}$, of ν_i Are Calculated as Follows:

$$m_{2\nu} = \sqrt{m^2_{1\nu} + \Delta m^2_{21}}\ \text{ and}$$

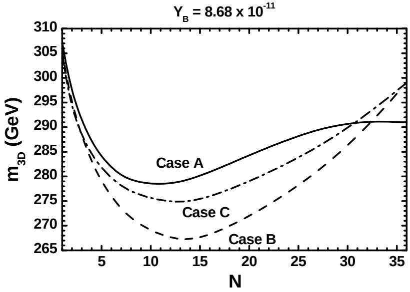
$$\begin{cases} m_{3\nu} = \sqrt{m^2_{1\nu} + \Delta m^2_{31}}, & \text{for Normally Ordered (NO) } m_\nu\text{'s} \\ \text{or} & \\ m_{1\nu} = \sqrt{m^2_{3\nu} + \left|\Delta m^2_{31}\right|}, & \text{for Invertedly Ordered (IO) } m_\nu\text{'s} \end{cases}$$

- $\sum_i m_{i\nu} \leq 0.12\ [0.15]\ {\rm eV}$ at 95% c.l. For NO [IO] m_ν's.

[4] *M. Kawasaki, K. Kohri, and T. Moroi (2005); J.R. Ellis, K.A. Olive, and E. Vangioni (2005).*

Combining Inflationary and Post-Inflationary Requirements

- Enforcing the Post-Inflationary Constraints, We Can Obtain **Predictions for m_{iD}'s or M_{iN^c}** Employing as **Input Parameters** $m_{r\nu}, \varphi_1$ and φ_2, (Where $m_{r\nu}$ is A **Reference Scale** for the Neutrino Masses).
- **All** the Requirements can be Met Along the Lines Presented in the $N - m_{3\rm D}$ Plane for $\mu \simeq 4$ TeV.



CASES : Hierarchy :	A NO	B NO	C IO
$m_{r\nu}$ / eV	0.02	0.01	0.007
$\Sigma_i m_{i\nu}$ / eV	0.096	0.075	0.11
m_{1D} / GeV	0.4	0.1	1.1
m_{2D} / GeV	1	0.58	0.6
φ_1	$\pi / 1.5$	$\pi / 4$	$-3\pi/4$
φ_2	0	π	$5\pi/4$
M_{1N^c} / 10^{10} GeV	1.3	0.16	1.94
M_{2N^c} / 10^{10} GeV	2.5	1.97	2.4
M_{3N^c} / 10^{15} GeV	1.8 - 2	1.1 - 1.45	3.9 - 4.8

- We take $m_{r\nu} = m_{1\nu}$ for NO m_ν's and $m_{r\nu} = m_{3\nu}$ for IO m_ν's.
- **The Inflaton Decays into the Lightest and Next-to-Lightest of N_i^c** Since $2M_{iN^c} > \widehat{m}_{\delta\phi}$ for $i = 3$.
- Y_B and $Y_{\widetilde{G}}$ can be **Reconciled With Data** for $m_{3/2}$ Even Lighter than 10 TeV, Since We Obtain

$$3.2 \lesssim Y_{\widetilde{G}}/10^{-15} \lesssim 44.3 \quad \text{With} \quad 1.6 \lesssim T_{\rm rh}/10^7\text{GeV} \lesssim 23.3 \quad \text{for} \quad 0.89 \lesssim \lambda_\mu/10^{-6} \lesssim 5.2.$$

- Successful nTL Requires M_{iN^c} and $m_{3\rm D}$ in the Ranges $(10^9 - 10^{15})$ GeV and $(270 - 310)$ GeV Respectively.

Generation of the μ-Term of MSSM Applying the Mechanism of G. Dvali, G. Lazarides and Q. Shafi (1999)

- The Origin of the μ Term Can be Explained, **If We Combine** the Terms $W_{\rm HI}+W_\mu=\lambda S\left(\bar\Phi\Phi/2-M^2/4\right)+\lambda_\mu S H_u H_d$.
- The **Soft SUSY Breaking Terms** Corresponding to $W_{\rm HI}+W_\mu$ Are Included In

$$V_{\rm soft}=\left(\lambda A_\lambda S\,\bar\Phi\Phi/2+\lambda_\mu A_\mu S\,H_uH_d-{\rm a}_S\,S\,\lambda M^2/4+{\rm h.c.}\right)+m^2_{\tilde\alpha}\left|z^{\tilde\alpha}\right|^2 \quad \text{with } z^{\tilde\alpha}=\Phi,\bar\Phi,S,H_u,H_d$$

where $m_\alpha, A_\lambda, A_\mu$ and ${\rm a}_S$ are Soft SUSY Breaking Mass Parameters Of the Order of Gravitino Mass $m_{3/2}$.

- **Minimizing** $V_{\rm tot}=V_{\rm SUSY}+V_{\rm soft}$ w.r.t Phases and **Substituting** in $V_{\rm soft}$ $\langle\Phi\rangle=\langle\bar\Phi\rangle\simeq M/\sqrt{2}$ we get

$$\langle V_{\rm tot}(S)\rangle=\lambda^2M^2S^2/4N-\lambda{\rm a}_\mu m_{3/2}M^2S\,, \quad \text{where } m_S\ll M \text{ and } (|A_\lambda|+|{\rm a}_S|)=2{\rm a}_\mu m_{3/2}.$$

Minimizing Finally $\langle V_{\rm tot}(S)\rangle$ w.r.t S We Obtain a **non-Vanishing $\langle S\rangle$** as Follows:

$$\langle S\rangle\simeq 2N{\rm a}_\mu m_{3/2}/\lambda\simeq NN_\star{\rm a}_\mu m_{3/2}/2\pi\sqrt{6NA_{\rm s}}$$ Due to $\lambda-A_{\rm s}$ Relation – see Page 5.

- Therefore, the **Generated μ Parameter** From W_μ is $\mu=\lambda_\mu\langle S\rangle$ **is Of the Order $m_{3/2}$ if $\lambda_\mu\sim10^{-6}$** Since $N_\star/\sqrt{A_{\rm s}}\sim10^6$.
- This λ_μ Value is Welcome Since **Stability of the H_u-H_d System** Requires $\lambda_\mu\leq\lambda(1+N_S)\phi_{\rm f}^2/4N_S\sim10^{-5}$.
- The Allowed λ_μ Values Render Our Models Compatible With The **Best-Fit Points** in the CMSSM[5] Setting, E.g.,

$$m_0=m_{3/2} \quad \text{and} \quad |A_\lambda|=|{\rm a}_S|=|A_0|$$

CMSSM Region ($m_h\simeq125$ GeV & $\Omega_\chi h^2\lesssim0.12$)	$\lvert A_0\rvert$ (TeV)	m_0 (TeV)	$\lvert\mu\rvert$ (TeV)	${\rm a}_\mu$	λ_μ (10^{-6}) $N=1$	λ_μ (10^{-6}) $N=36$
(I) A/H Funnel	9.9244	9.136	1.409	1.086	**1.81**	**3.5**
(II) $\tilde\tau_1-\chi$ Coannihilation	1.2271	1.476	2.62	0.831	***14.48***	**5**
(III) $\tilde t_1-\chi$ Coannihilation	9.965	4.269	4.073	2.33	**5.2**	**1**
(IV) $\tilde\chi_1^\pm-\chi$ Coannihilation	9.2061	9.000	0.983	1.023	**1.35**	**0.2**

[5] *P. Athron et al. [GAMBIT Collaboration] (2018) – It is obtained* $m_{\tilde g}\geq2.9$ TeV, $m_{\tilde\chi^\pm}\geq1.1$ TeV & $m_{\tilde t_1}\geq3.6$ TeV *(Besides Region III) so, Regions I, II, IV Are Still Alive. On the Other hand, The muon $g-2$ Anomaly is not Interpreted in These Regions.*

Summary

- We Proposed New **Implementations** of T-Model **Inflation** in SUGRA Using as Inflaton a Higgs Field.
- We Employ W Consistent with The **GUT and an R Symmetries** and two K's Which Parameterize the Kähler Manifold $SU(2,1)/(SU(2)\times U(1))$.
- We Analyzed Three (δTM, TM4 & TM8) **Cosmologically Successful** Inflationary Models, From Which one (TM4) Can Be Qualified as the **Most Natural** One. It Predicts $n_s \sim 0.965$, and r **Increasing** with the Coefficient N of $\widetilde{K}_{21}$.
- We Proposed a Post-Inflationary Completion for TM4 Which Offers a Nice Solution to **the μ Problem of MSSM** and Allows for **Baryogenesis** via non-TL With $M_{i\nu^c}$ in the Range (10^9-10^{15}) GeV.
- It Remains the Introduction of a Consistent **Soft SUSY Breaking Sector** to Achieve a Fully Self-Contained Theory.

Thank You!

The Kähler Manifold Corresponding to K_{21} and $\widetilde{K}_{21}$

- $K = K_{21}$ and $\widetilde{K}_{21}$ Parameterize the Kähler Manifold $SU(2,1)/(SU(2)\times U(1))$.
- **To Show it**, We Extract the **Line Element and the Scalar Curvature** In the Moduli Space, Which are

$$ds^2_{21} = K_{\alpha\bar\beta}dz^\alpha dz^{*\bar\beta} = N\left(\frac{|d\Phi|^2+|d\bar\Phi|^2}{1-|\Phi|^2-|\Phi|^2}+\frac{|\Phi^* d\Phi+\bar\Phi^* d\bar\Phi|^2}{\left(1-|\Phi|^2-|\bar\Phi|^2\right)^2}\right) \quad \text{kai} \quad \mathcal{R}_{21} = -\frac{6}{N}.$$

- The Action of $SU(2,1)/(SU(2)\times U(1))$ to Φ & $\bar\Phi$ May be Found if an Element of $U\in SU(2,1)$, Which Fulfils the Relations

$$U^\dagger \eta_{21} U = \eta_{21} \quad \text{and} \quad \det U = 1 \quad \text{With} \quad \eta_{21} = \mathrm{diag}\,(1,1,-1)\,,$$

is Parameterized by $a,b,d,f\in\mathbb{C},\gamma\in\mathbb{R}_+,\vartheta\in\mathbb{R}$ as Follows $U=\mathcal{U}\mathcal{P}$ With

$$\underset{\in SU(2,1)/(SU(2)\times U(1))}{\mathcal{U} = \begin{pmatrix} 1/N_a & 0 & a \\ N_a b a^* & N_a\gamma & b \\ N_a\gamma a^* & N_a b^* & \gamma\end{pmatrix}} \quad \& \quad \underset{\in SU(2)\times U(1)}{\mathcal{P} = e^{i\vartheta}\begin{pmatrix} d & f & 0 \\ -f^* & d^* & 0 \\ 0 & 0 & e^{-3i\vartheta}\end{pmatrix}}, \quad \text{Where} \quad \begin{cases} N_a = 1/\sqrt{1+|a|^2} \\ |a|^2+|b|^2-\gamma^2 = -1 \\ |d|^2+|f|^2 = 1. \end{cases}$$

- Acting With the Line Parameters of $\mathcal{U}^\dagger$ to Φ & $\bar\Phi$, We Define the **Isometréc Transformations**

$$\Phi \to \frac{(1/N_a)\Phi + N_a b^* a\bar\Phi + N_a a\gamma}{a^*\Phi + b^*\bar\Phi+\gamma} \quad \text{and} \quad \bar\Phi \to \frac{N_a\gamma\bar\Phi + N_a b}{a^*\Phi + b^*\bar\Phi+\gamma} \quad \text{with} \quad (B-L)(a,b,\gamma) = (1,-1,0)$$

Which **Let Invariant ds^2_{21}** and Justifies the Symmetry of K_{21} and $\widetilde{K}_{21}$.

- **Counting of Free Parameters** of Each Space Shows That the Demonstration is Self-Consistent.

Space:	$SU(2,1)$	$SU(2,1)/(SU(2)\times U(1))$	$SU(2)\times U(1)$
Parameters:	a,b,d,f,γ,ϑ	a,b,γ	d,f,ϑ
Constraints:	2	1	1
Dimension:	$10-2=8$	$5-1=4$	$5-1=4$