

Detecting Stochastic Gravitational Wave Backgrounds

with future space-based observatories

Nikolaos Karnesis

Aristotle University of Thessaloniki

karnesis@auth.gr

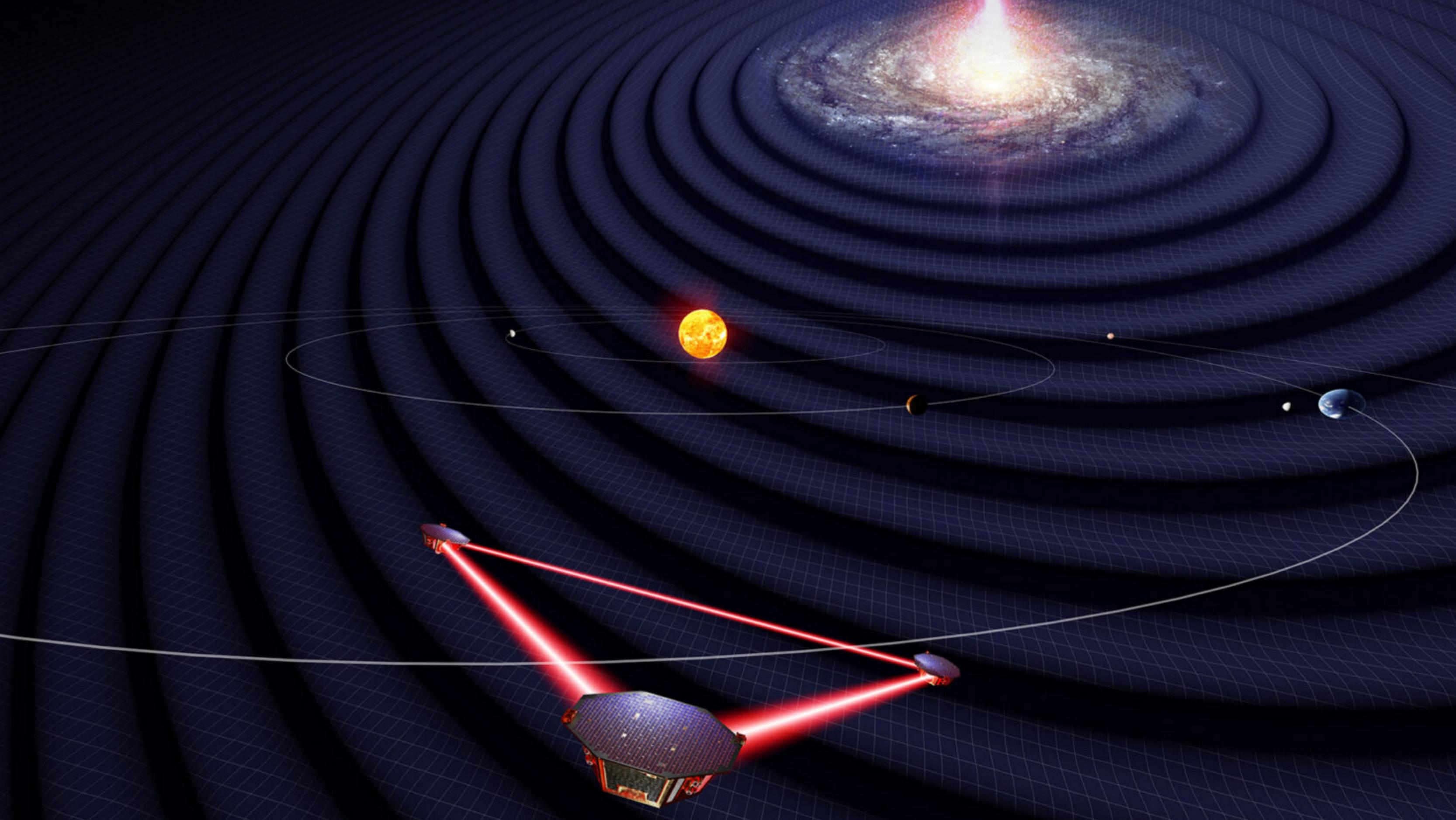
HEP2023 - University of Ioannina

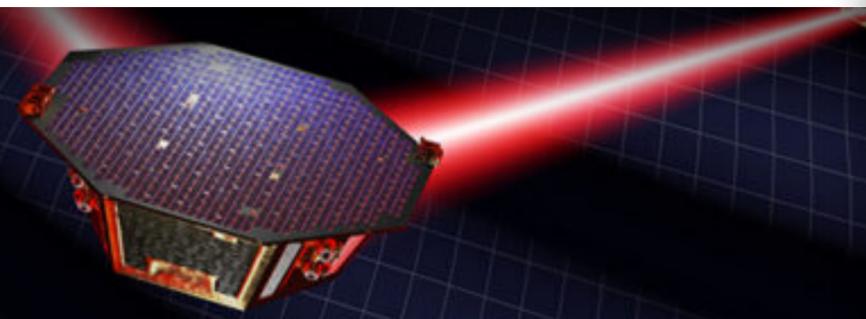
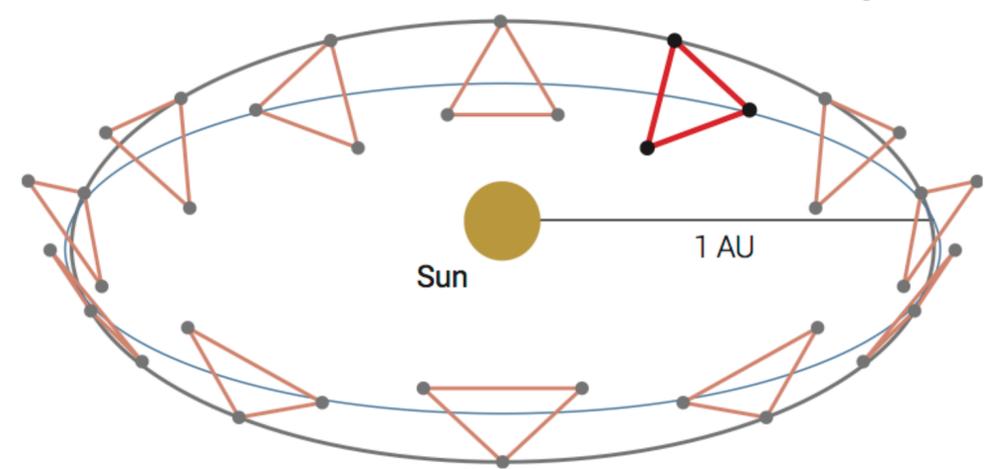
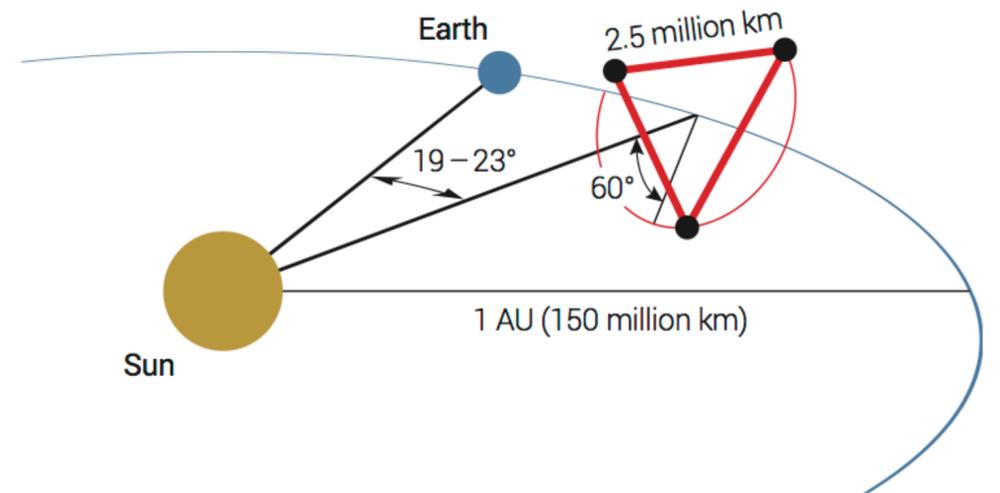
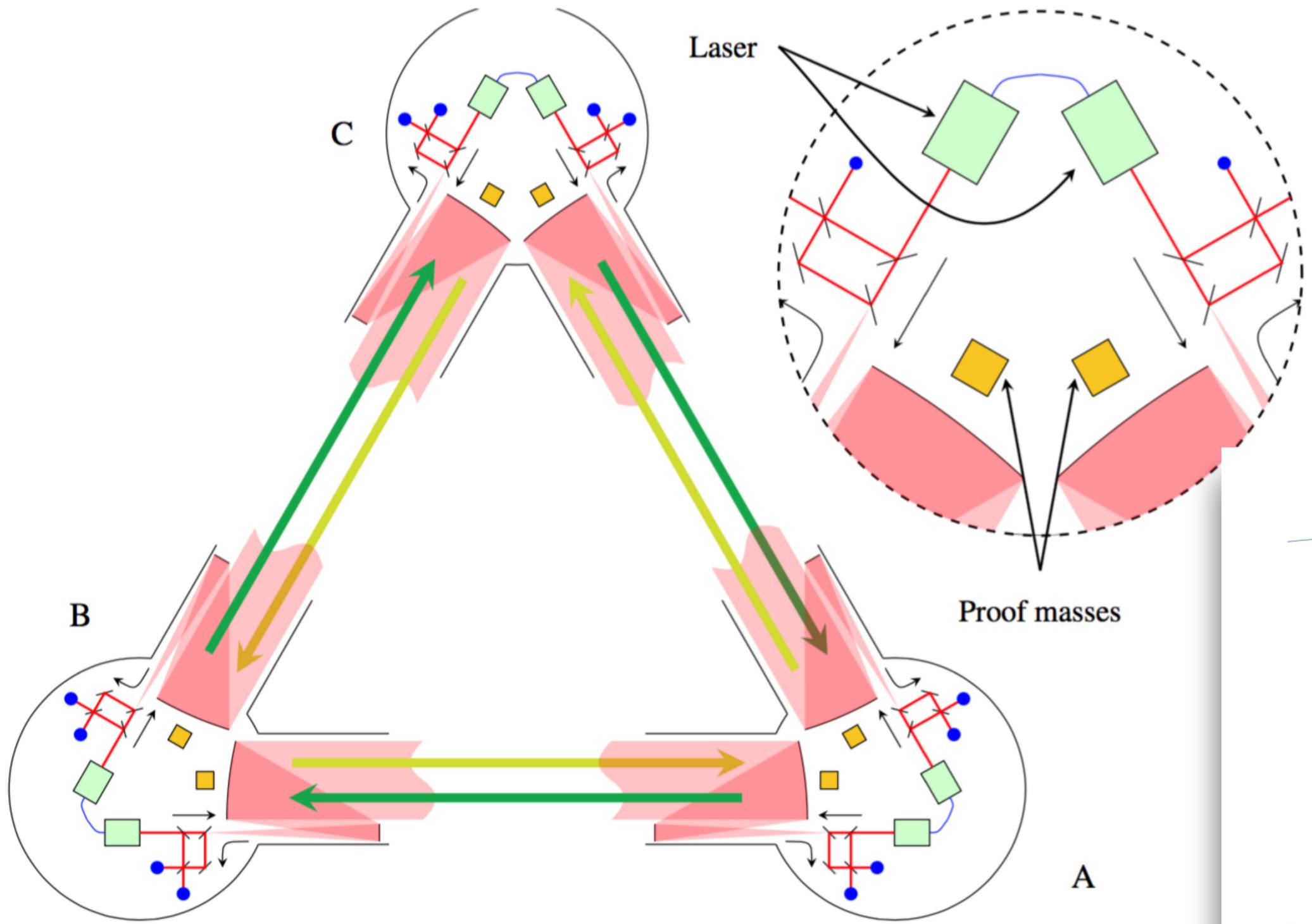
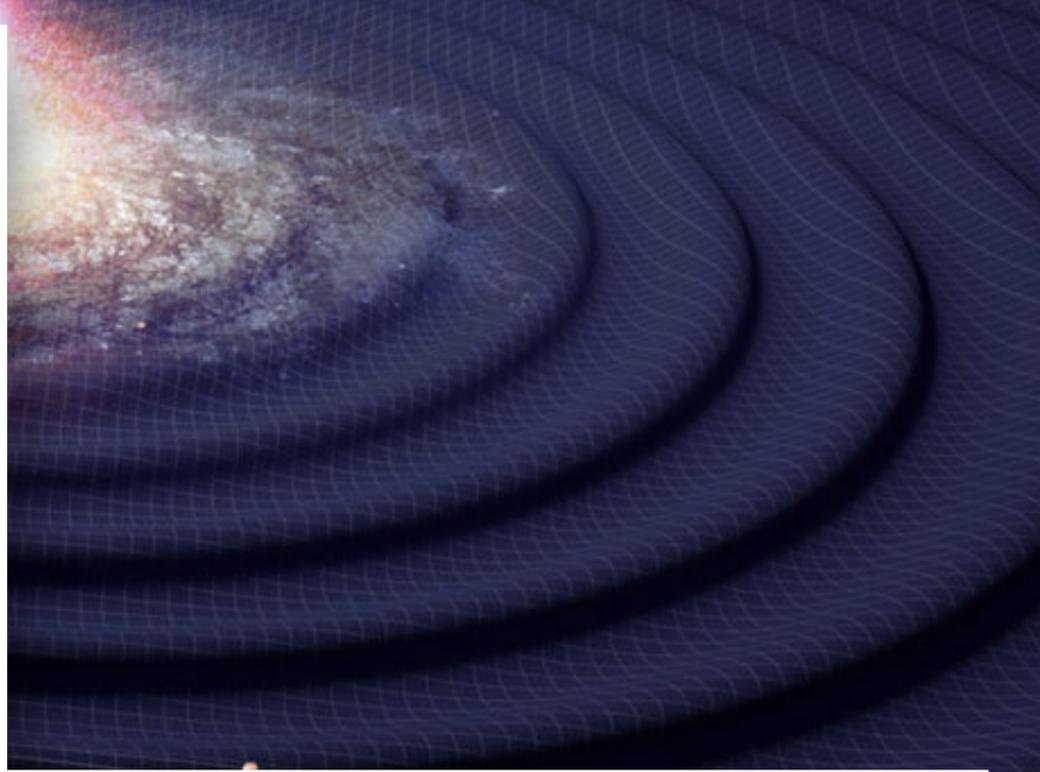
05/04/2023



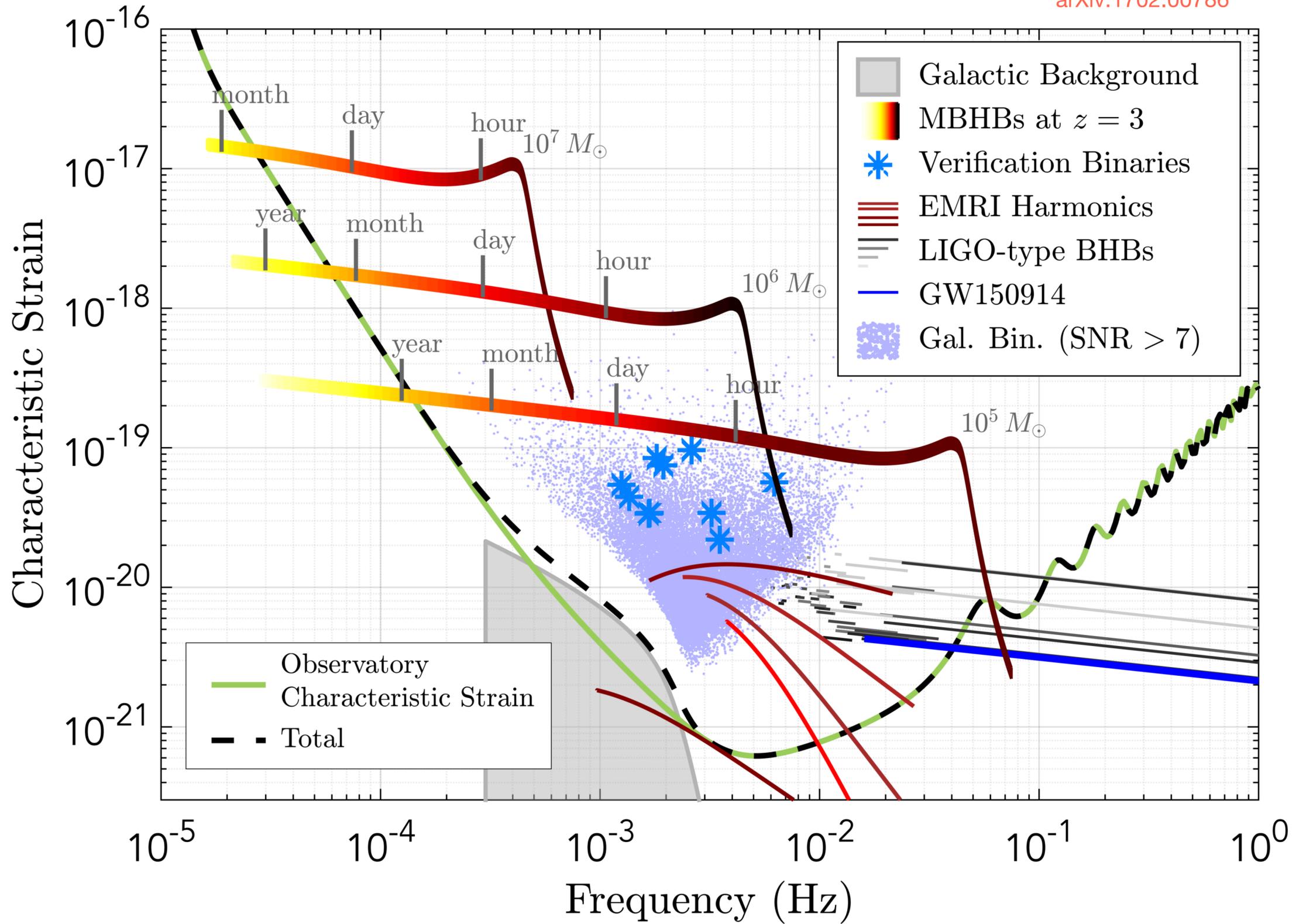
□ **Gravitational Waves Stochastic signals**

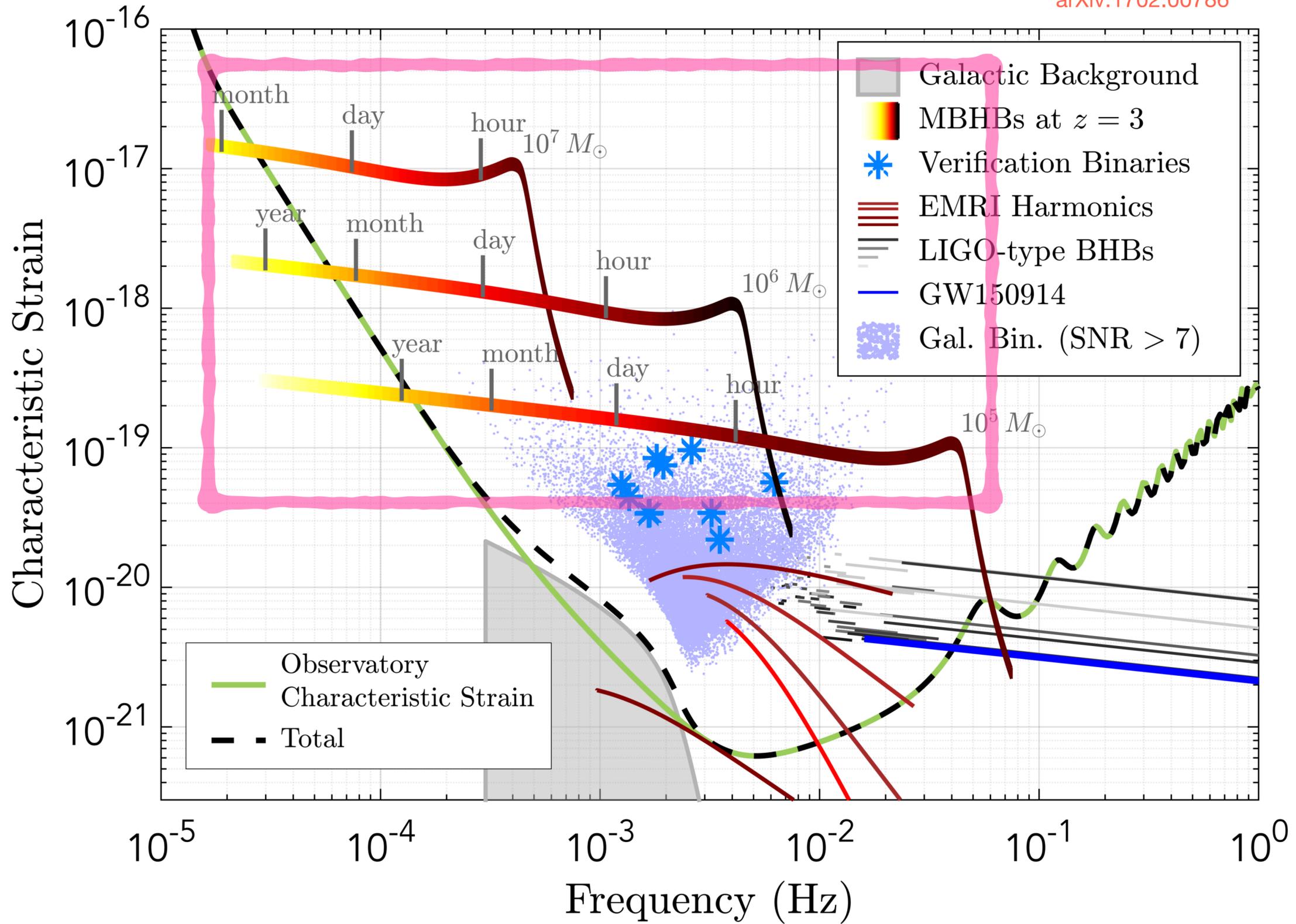
□ **What are the challenges of detection?**

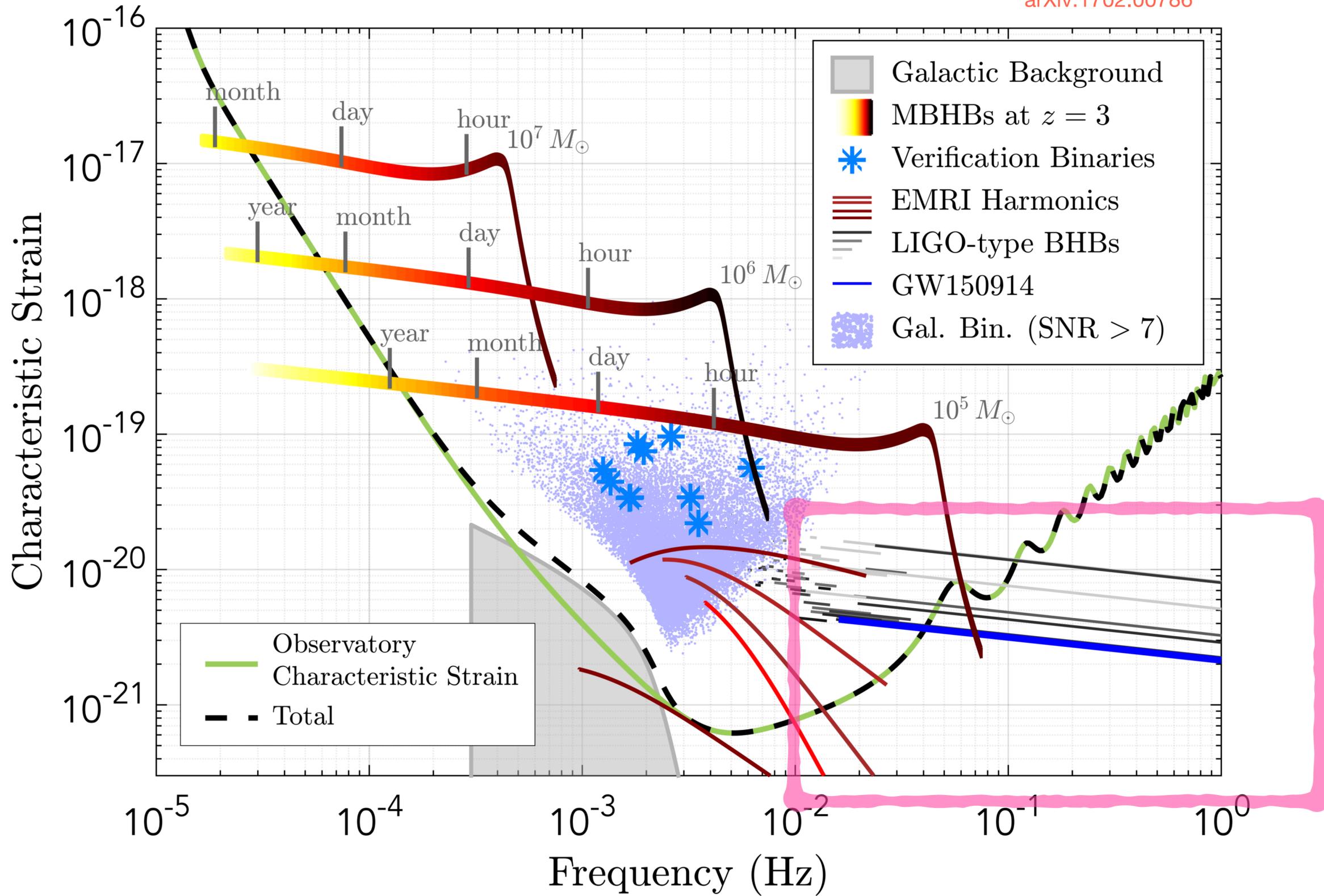


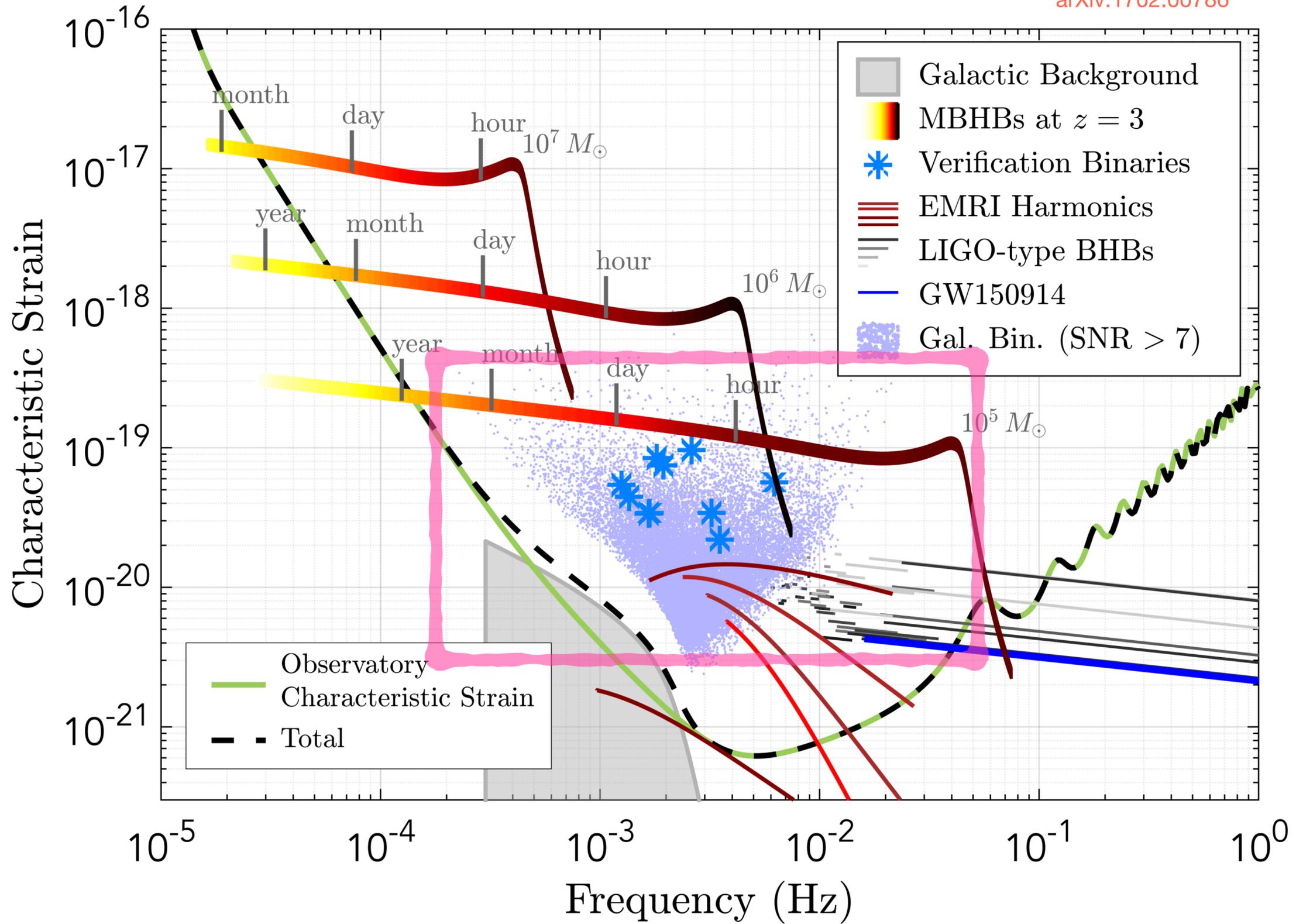


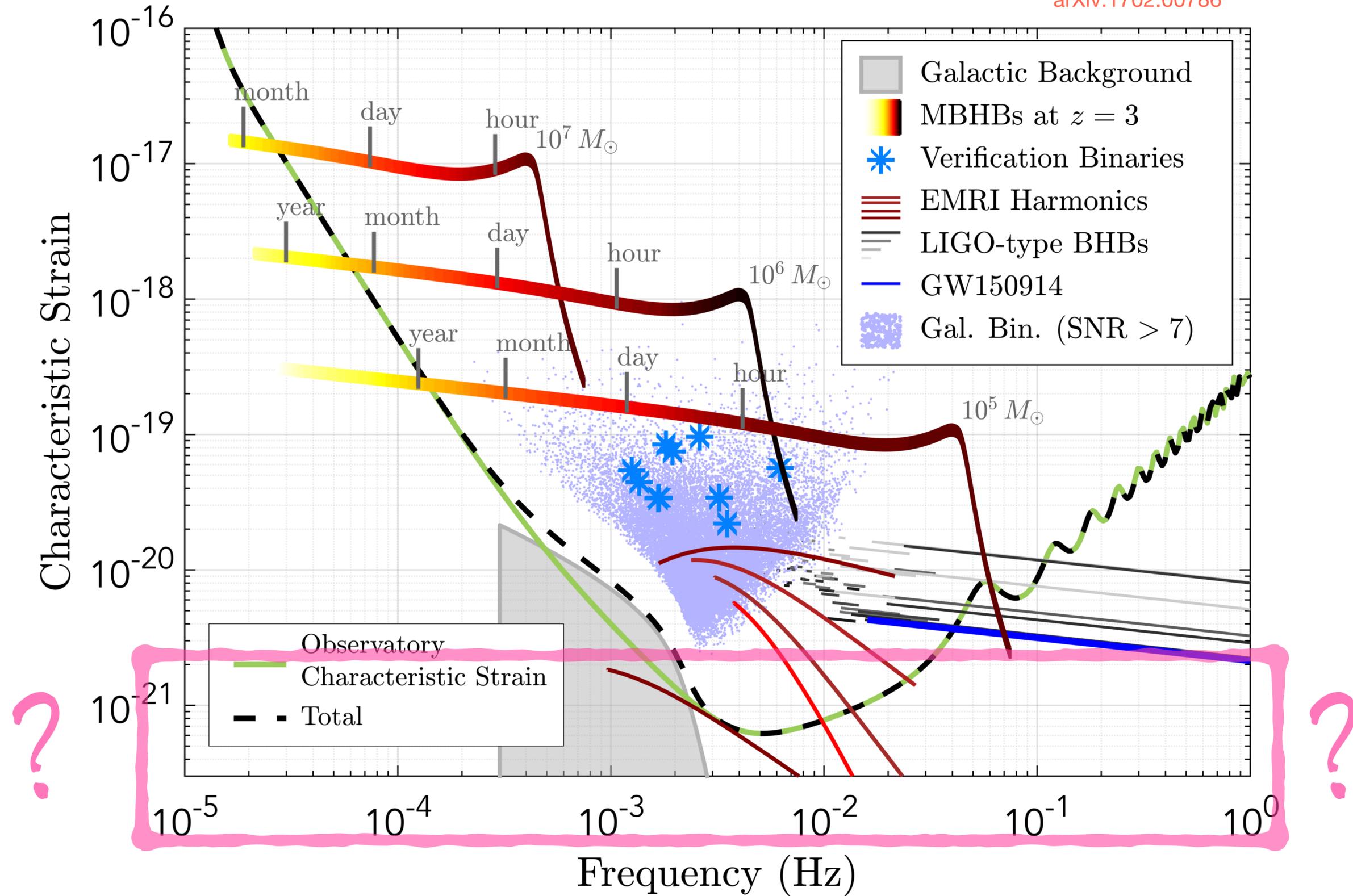
❑ **Science-rich data sets!**





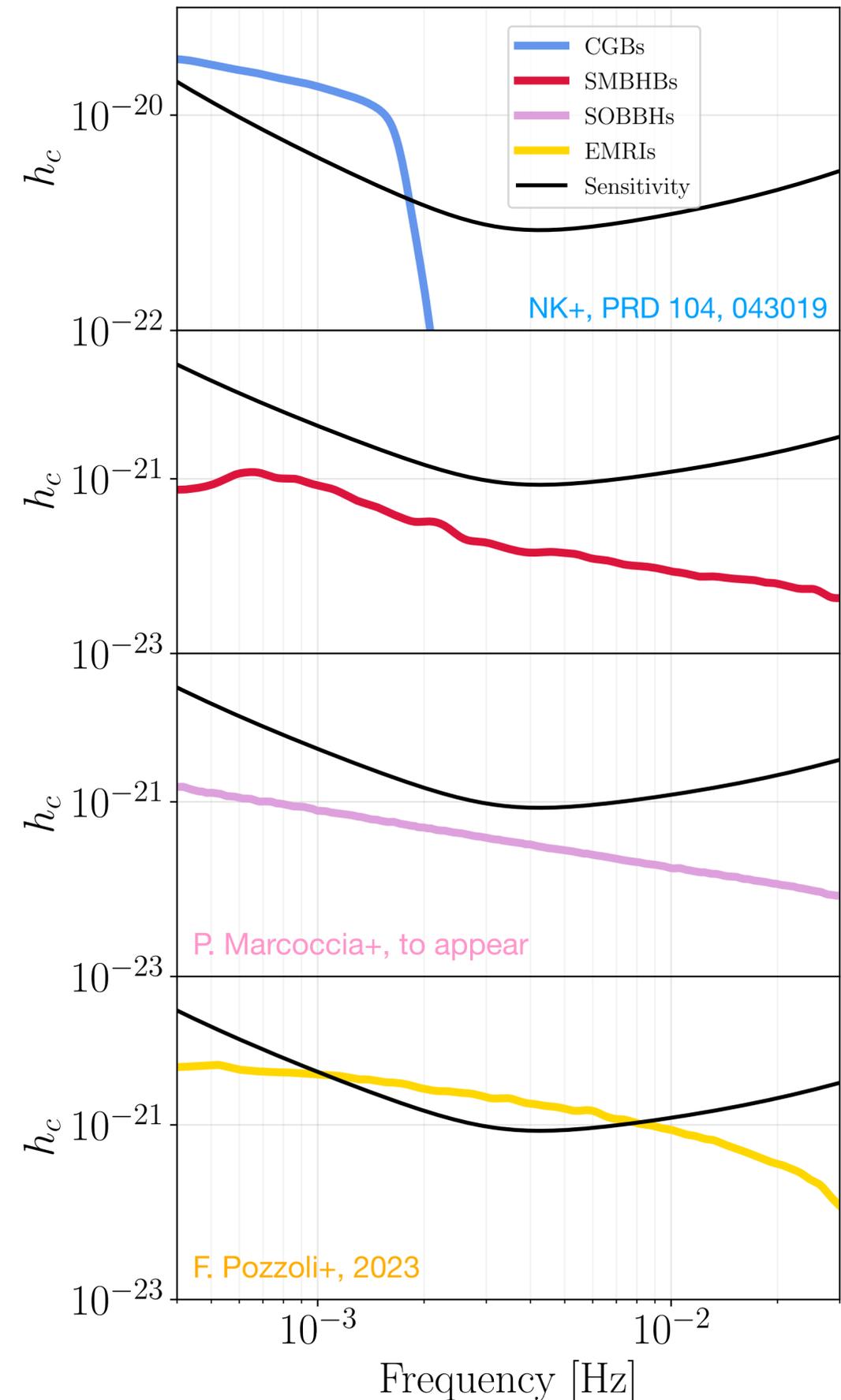






Astrophysical stochastic signals.

- UCBs [blue]: Cyclo-stationary, Anisotropic, change with time, spectral shape unveils properties of the Galaxy.
- SMBHBs [red]: Existence to be proven, dependent on the population model.
- SOBHBs [pink]: Stationary, isotropic. Extrapolated from ground-based measurements. Shape to unveil properties of their population.
- EMRIs [yellow]: Non-Stationary, isotropic, very uncertain predictions.

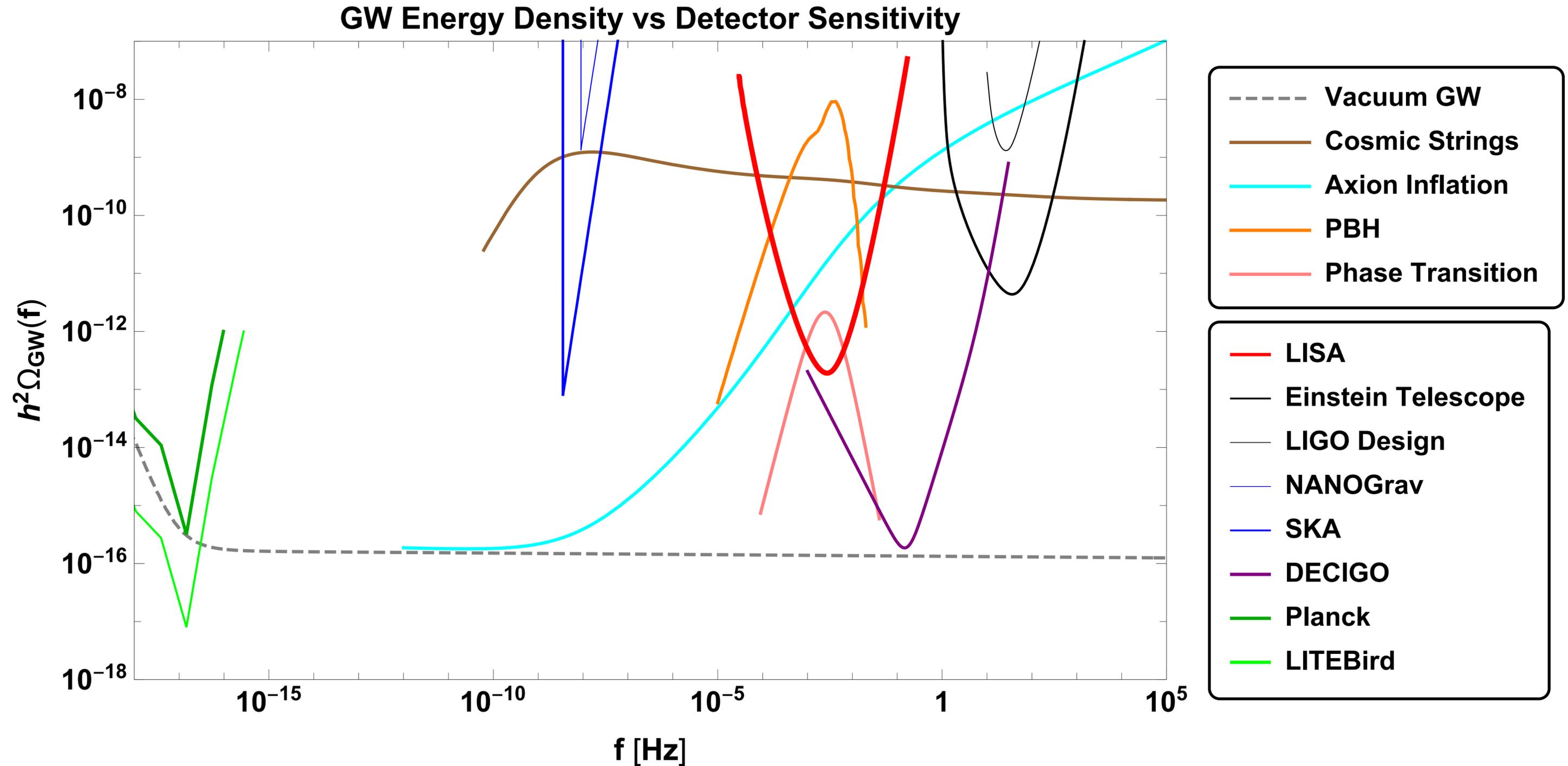


□ **Cosmological Stochastic signals**

Cosmological stochastic signals.

Cosmology with the Laser Interferometer Space Antenna
[\[2204.05434\]](#)

Cosmological stochastic signals.

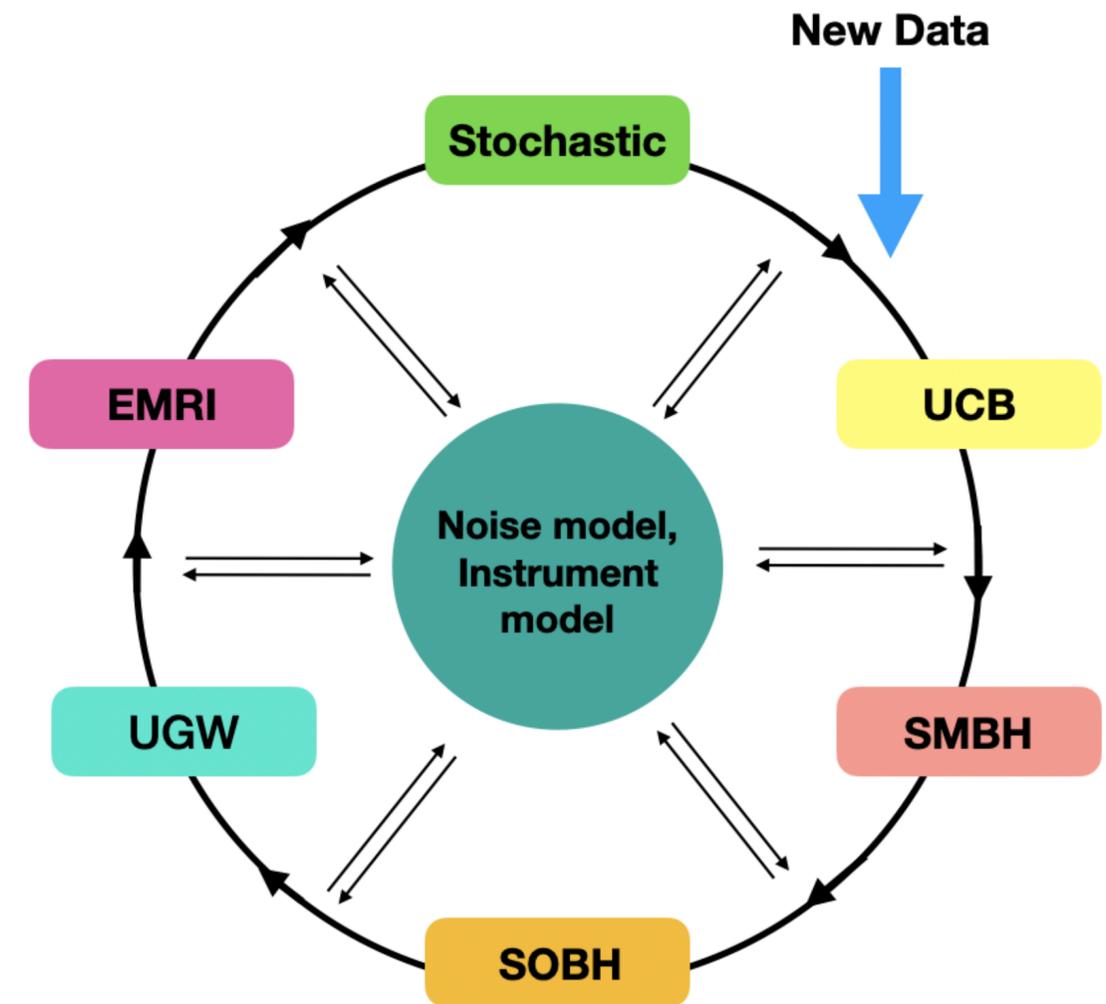


❑ How to reach that point of searching for noisy signals?

LISA Global Fit

Searching for different types of sources simultaneously

- ▷ Computational reasons: sequential fits are inefficient.
- ▷ Grid searches are impossible.
- ▷ Correlations between sources become important for that many signals.
- ▷ Imperfect source subtraction yields imperfect residuals.
- ▷ Uncertainties propagation
- ▷ **Not fixed dimensions!**



[arXiv:2004.08464](https://arxiv.org/abs/2004.08464)

[arXiv:2301.03673](https://arxiv.org/abs/2301.03673)

❑ **Extract stochastic signals
from the data.**

LISA Data Analysis

Bayesian Framework

- Define a likelihood function.

$$\pi(y|\vec{\theta}) = C \times e^{-\frac{1}{2}(y - h(\vec{\theta})|y - h(\vec{\theta}))} = C \times e^{-\chi^2/2}$$

- Define priors
- Form posterior
- Sample

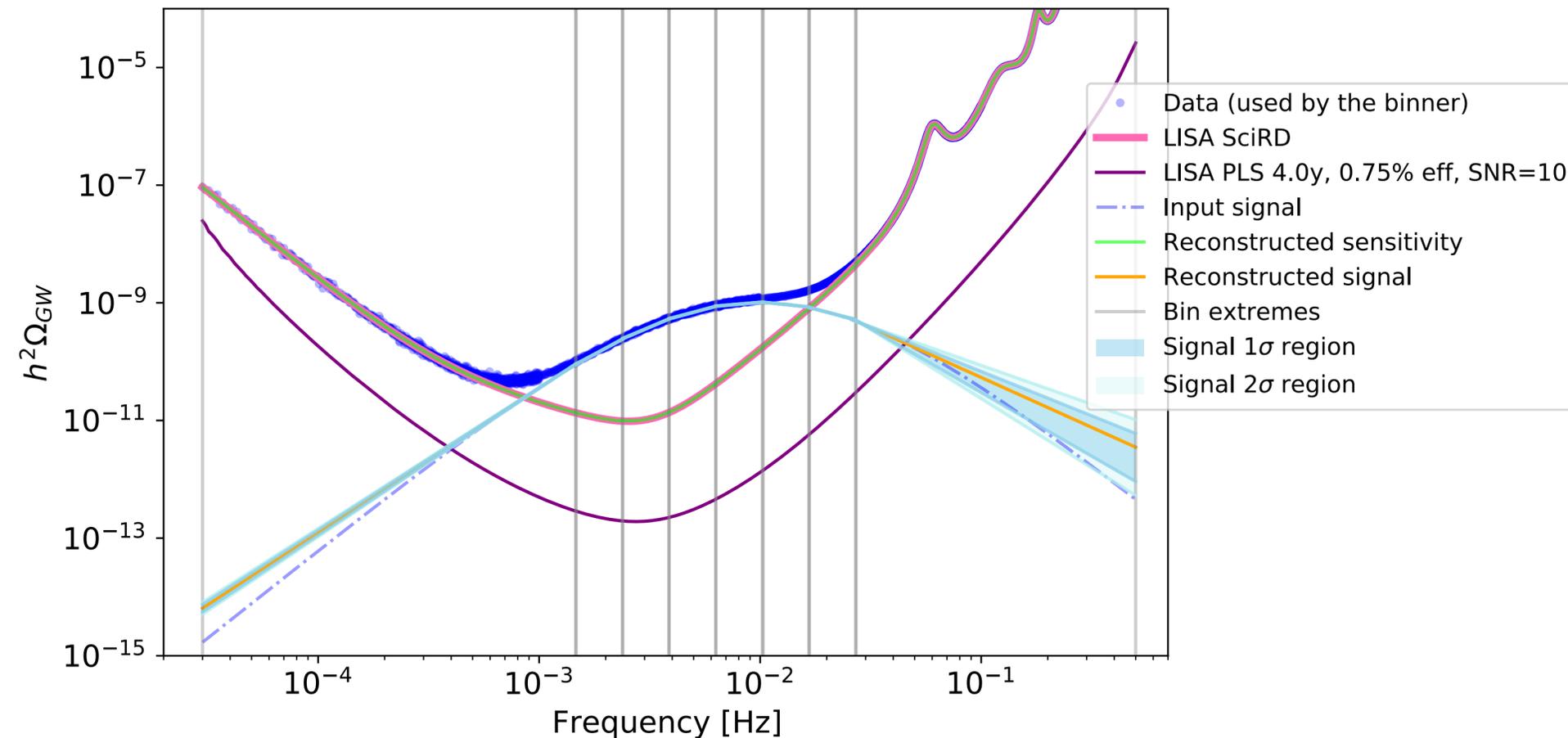
$$\pi(\vec{\theta}|y) \propto \pi(y|\vec{\theta})p(\vec{\theta})$$

$$(a|b) = 2 \int_0^\infty df \left[\tilde{a}^*(f)\tilde{b}(f) + \tilde{a}(f)\tilde{b}^*(f) \right] / \tilde{S}_n(f)$$

LISA Data Analysis

Assuming no spectral shape for the signal - Previews efforts

- Ordered by the Cosmology WG.
- Divide the data into bins.
- Fit power-laws at those bins.
- Fit also an analytic model of the noise.
- Join bins if power-law models are similar (e.g. using the AIC)



❑ But the instrumental noise knowledge is crucial

LISA Data Analysis

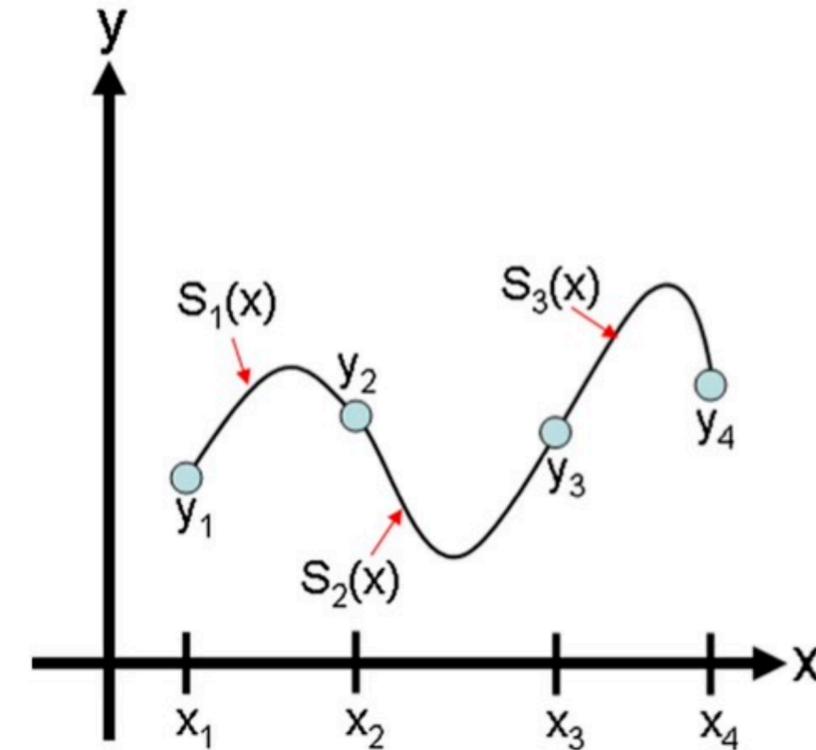
Assuming no spectral shape for the noise

- Usually, we adopt a model of the X,Y,Z or A, E, T TDI channels for the noise.
- This allows us to fit a few noise parameters together with the signal.
- More parameters for more complexity (unequal arms, unequal noise PSDs)
- In this work we adopt a shape-agnostic model, based on interpolating cubic B-spline functions.

LISA Data Analysis

Assuming no spectral shape for the noise

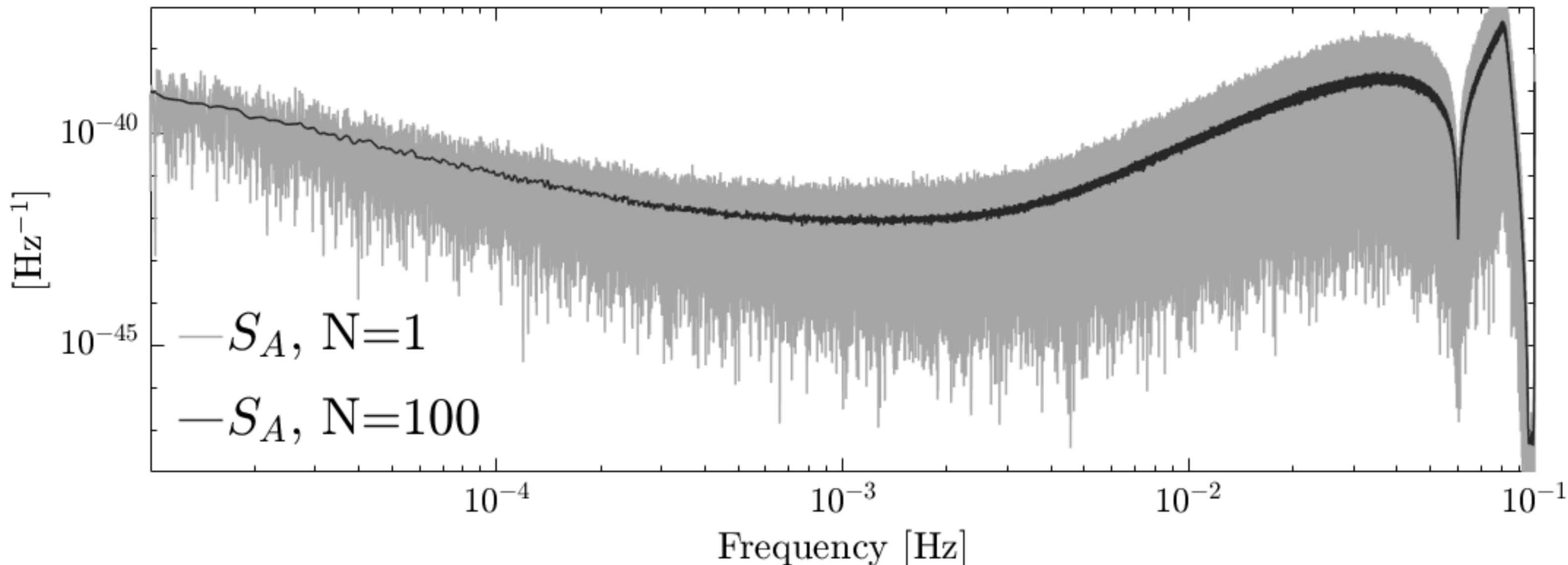
- Usually, we adopt a model of the X,Y,Z or A, E, T TDI channels for the noise.
- This allows us to fit a few noise parameters together with the signal.
- More parameters for more complexity (unequal arms, unequal noise PSDs)
- In this work we adopt a shape-agnostic model, based on interpolating cubic B-spline functions.



$$\log S_n(f) = \sum_{i=1}^{Q+1} a_i B_{i,3}(\xi, f)$$

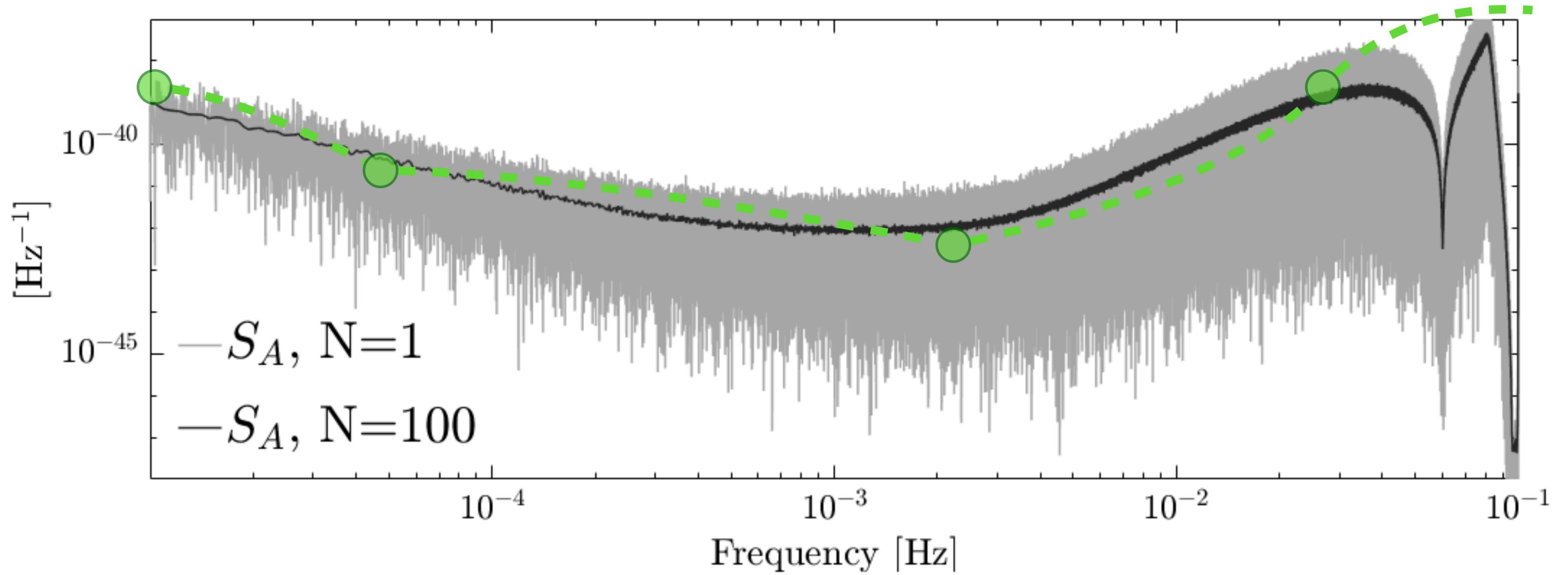
LISA Data Analysis

Assuming no spectral shape for the noise



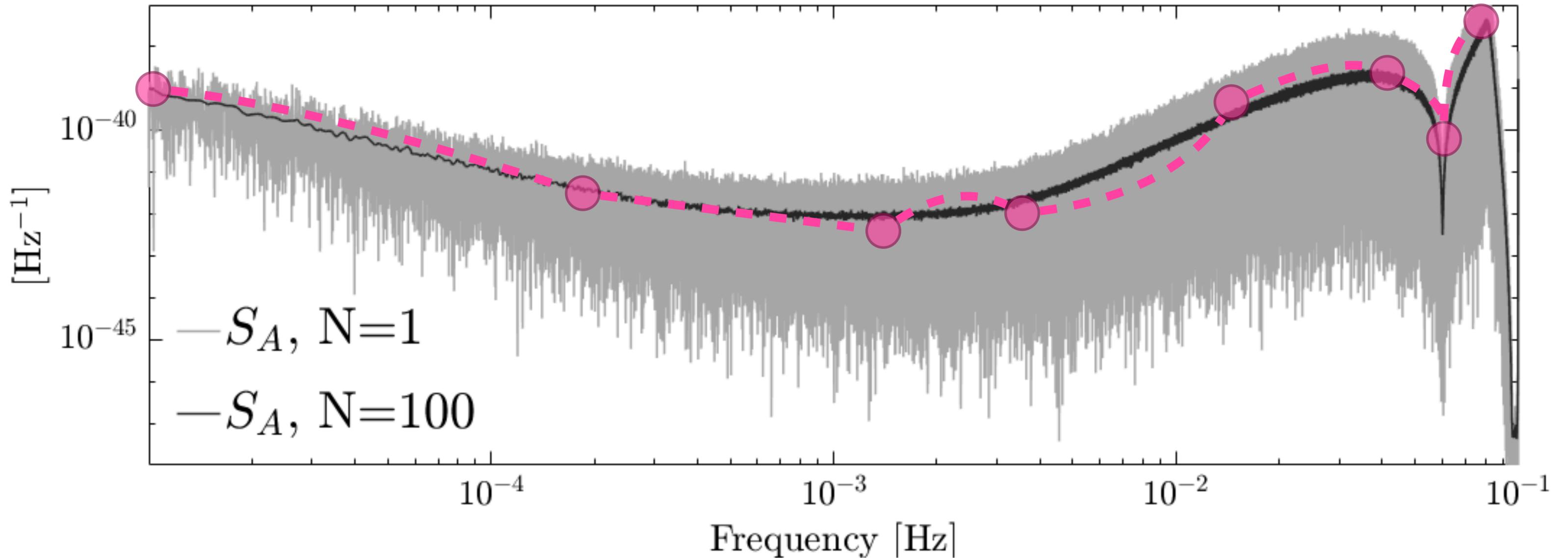
LISA Data Analysis

Assuming no spectral shape for the noise



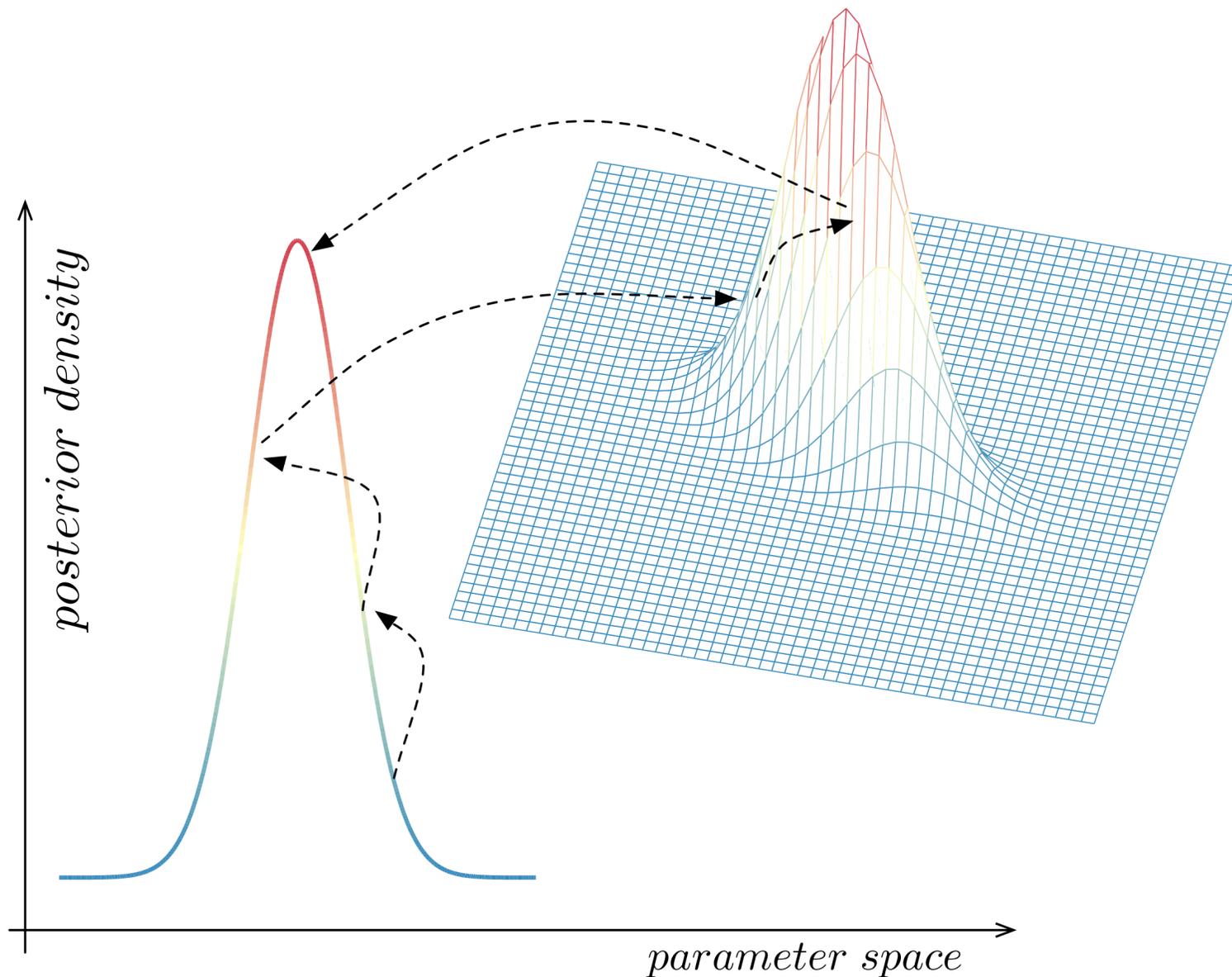
LISA Data Analysis

Assuming no spectral shape for the noise



Trans-dimensional sampling:

Assume a model with a changing dimensionality...

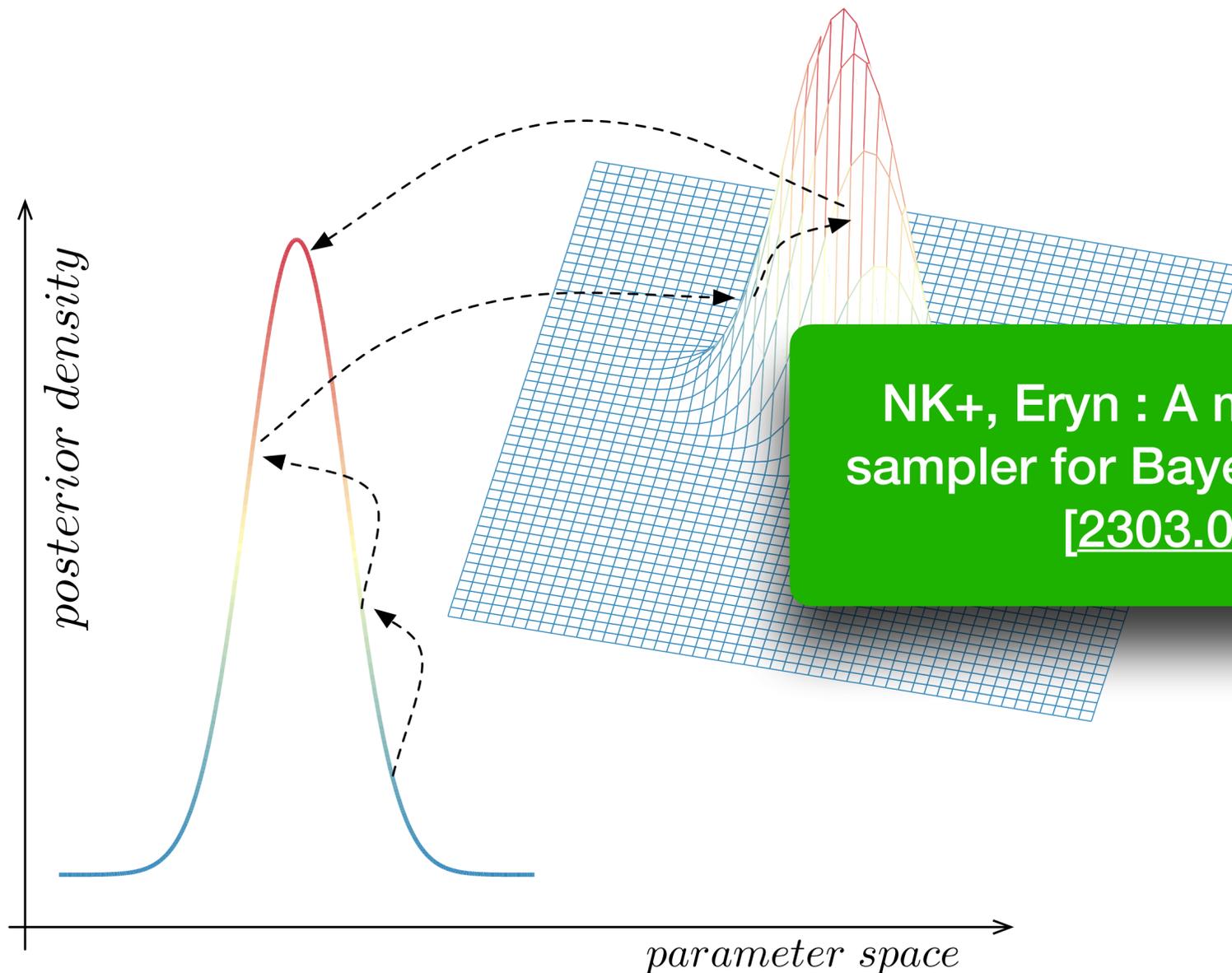


- Same procedure, now generalized for k -order of model. It is organized in two steps.
- Before all, we begin with θ_k for model k .
- 1. In-Model Step: The usual MH step, for model k .
- 2. Outer-Model Step:
 - Propose new θ_m for model m from a given proposal distribution q .
 - Essentially propose the “birth” or “death” of dimensions at each iteration.
 - Accept, or reject with a probability:

$$\alpha = \min \left[1, \frac{p(y|\vec{\theta}_k)p(\vec{\theta}_k)q(\{k, \vec{\theta}_k\}, \{m, \theta_m\})}{p(y, \vec{\theta}_m)p(\vec{\theta}_m)q(\{m, \vec{\theta}_m\}, \{k, \theta_k\})} \right]$$

Trans-dimensional sampling:

Assume a model with a changing dimensionality...



- Same procedure, now generalized for k -order of model. It is organized in two steps.

- Before all, we begin with θ_k for model k .

1. In-Model Step: The usual MH step, for model k .

2. Out-Model Step:

Propose θ_m for model m from a given proposal q .

Essentially propose the “birth” or “death” of dimensions at each iteration.

- Accept, or reject with a probability:

$$\alpha = \min \left[1, \frac{p(y|\vec{\theta}_k)p(\vec{\theta}_k)q(\{k, \vec{\theta}_k\}, \{m, \theta_m\})}{p(y, \vec{\theta}_m)p(\vec{\theta}_m)q(\{m, \vec{\theta}_m\}, \{k, \theta_k\})} \right]$$

We continue by defining a likelihood function

- We want to analyse the data into chunks, in order to make it computationally lighter.

- In that case, we get the Wishart distribution, which is written as

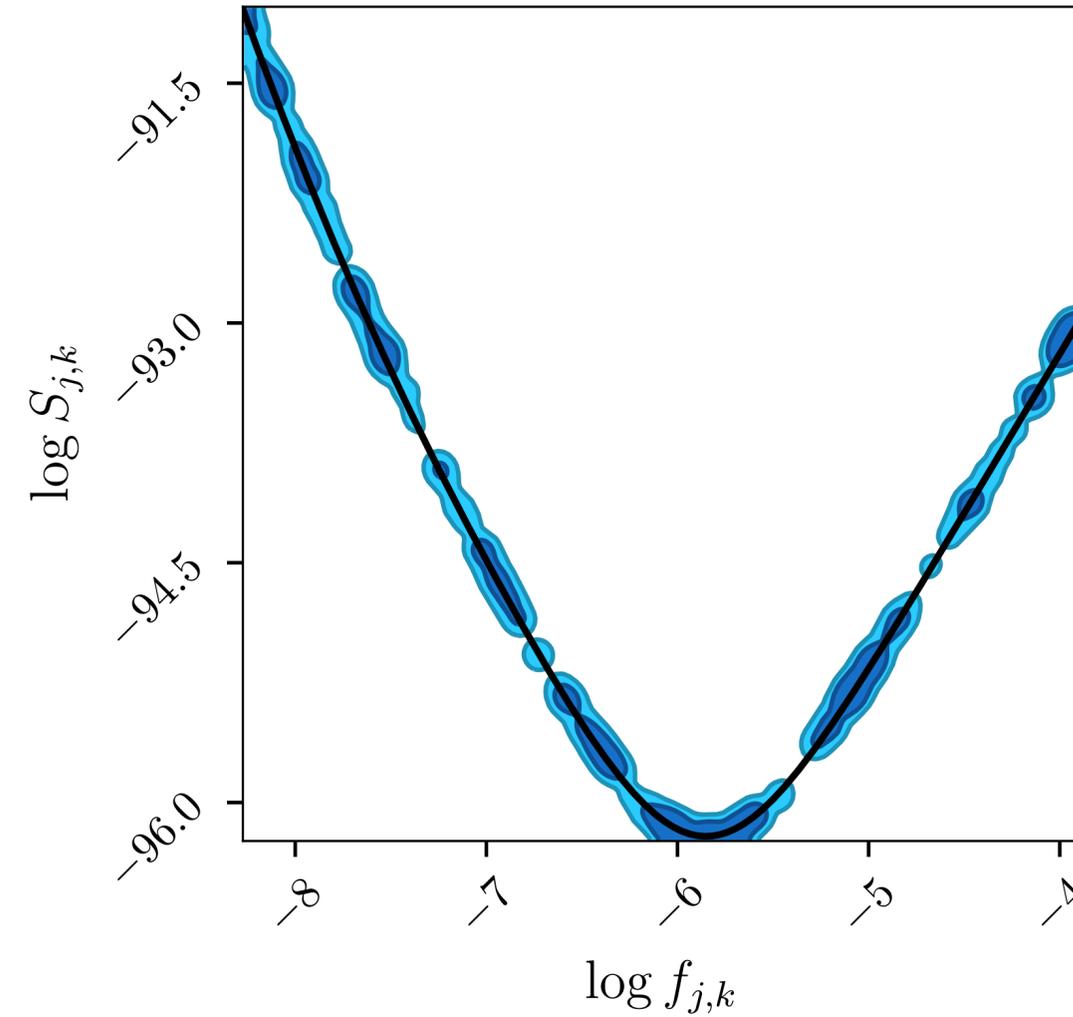
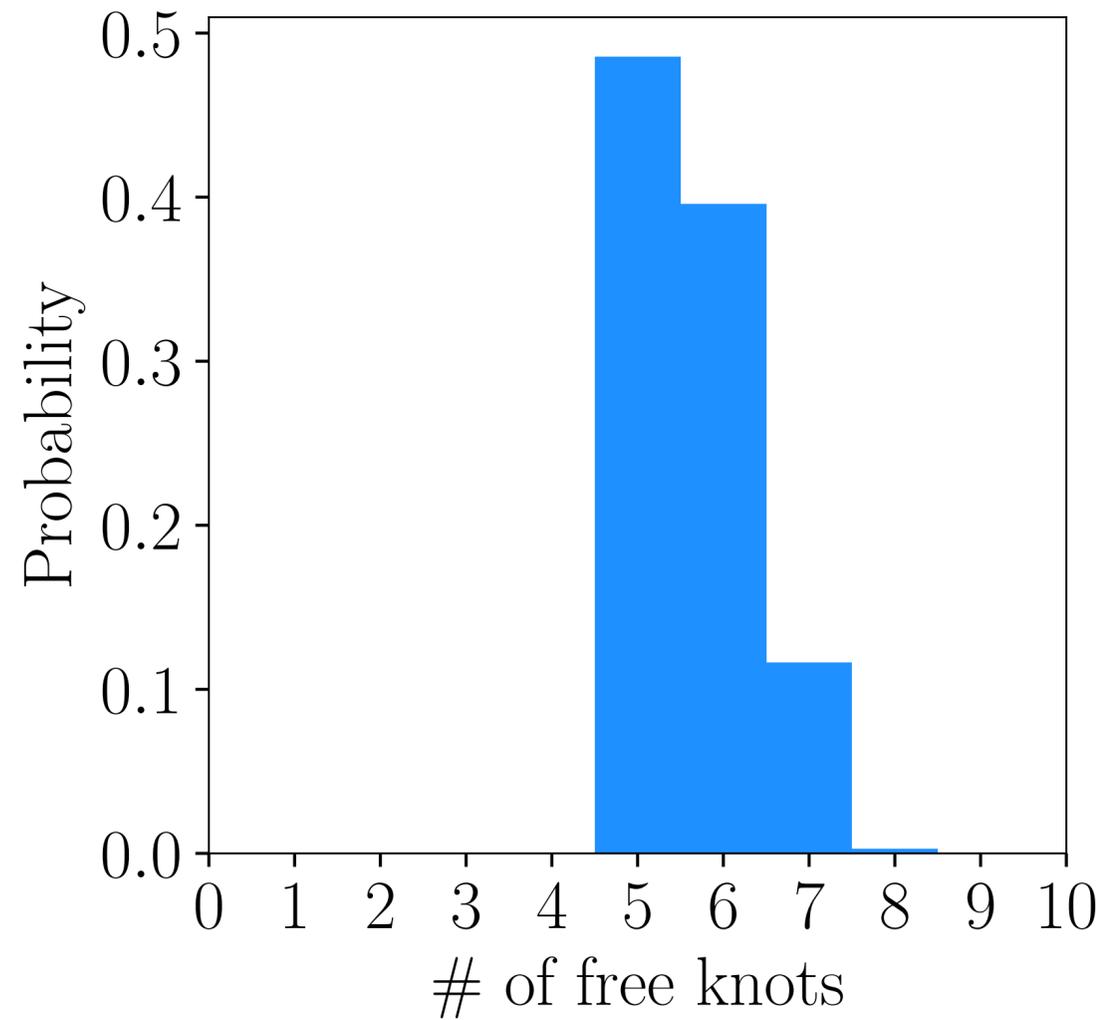
$$p(\mathbf{Y}(f)|\theta) = \frac{|\mathbf{Y}(f)|^{\nu-3} \exp\{-\text{tr}(\mathbf{C}_d^{-1}\mathbf{Y}(f))\}}{|\mathbf{C}_d(f)|^\nu \cdot \mathcal{C}\tilde{\Gamma}_3(\nu)}$$

- With its logarithm

$$\log p(\mathbf{Y}(f)|\theta) = -\text{tr}(\mathbf{C}_d^{-1}\mathbf{Y}(f)) - \nu(f) \log |\mathbf{C}_d(f)|$$

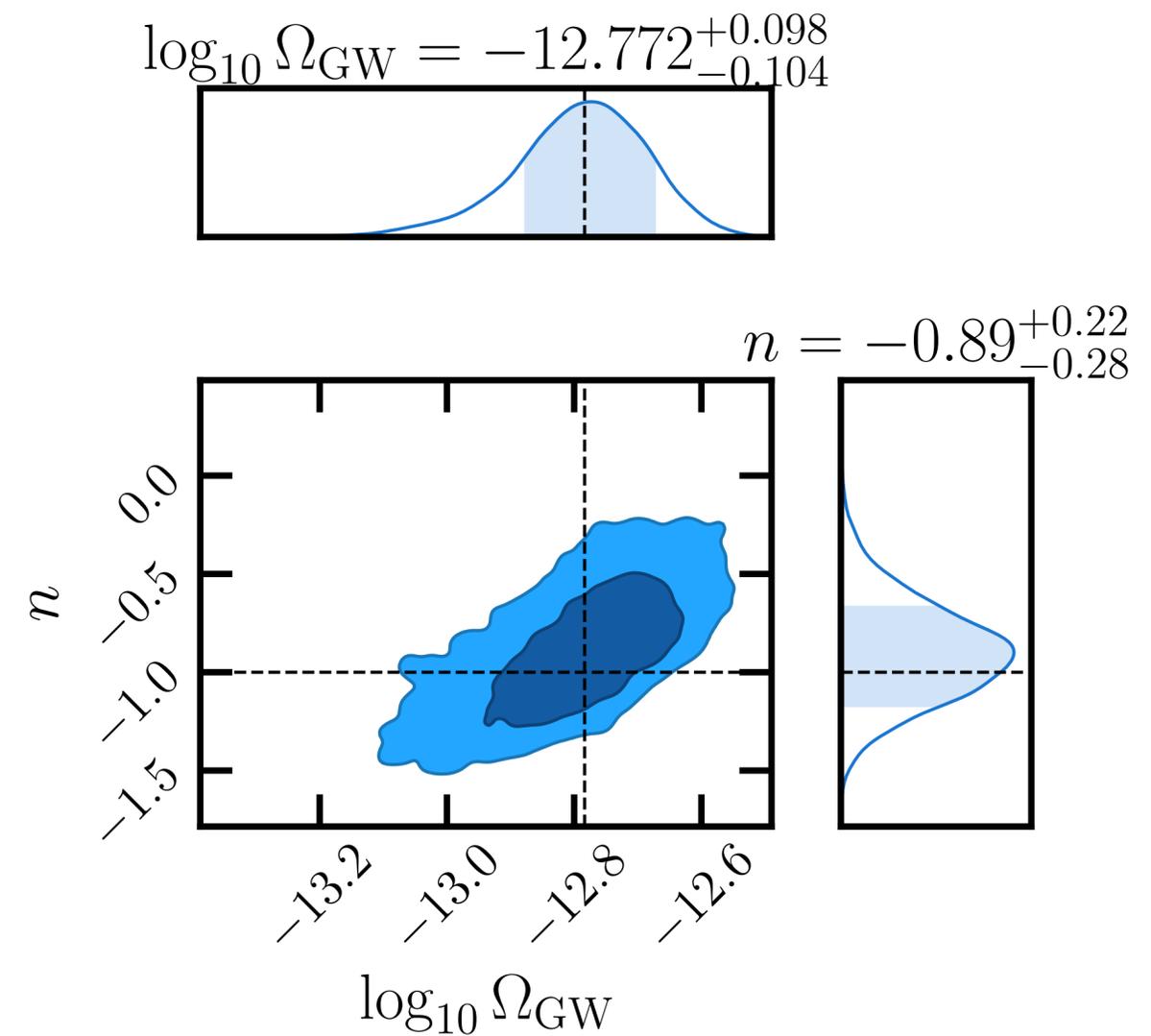
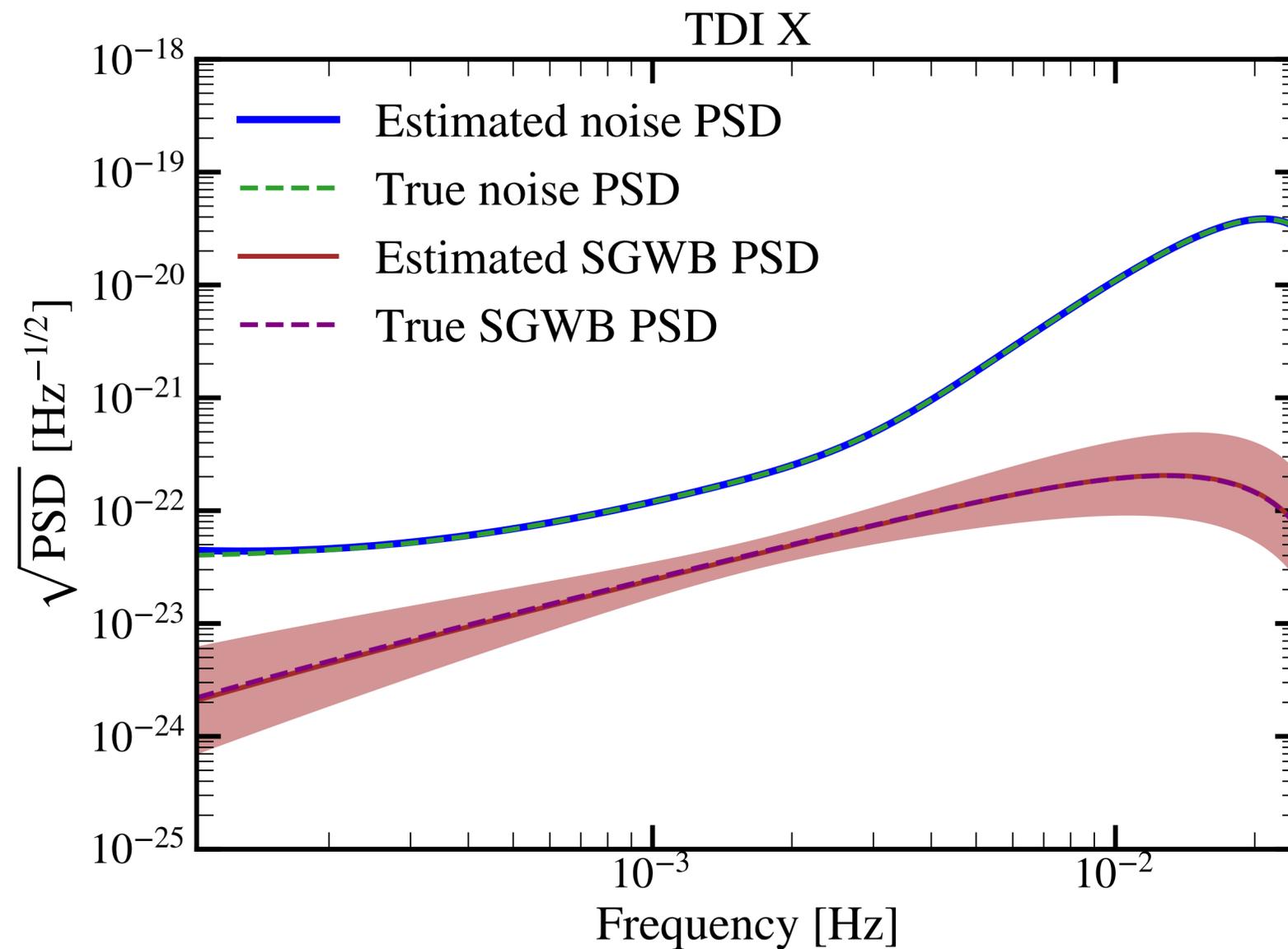
LISA Data Analysis

Assuming no spectral shape for the noise



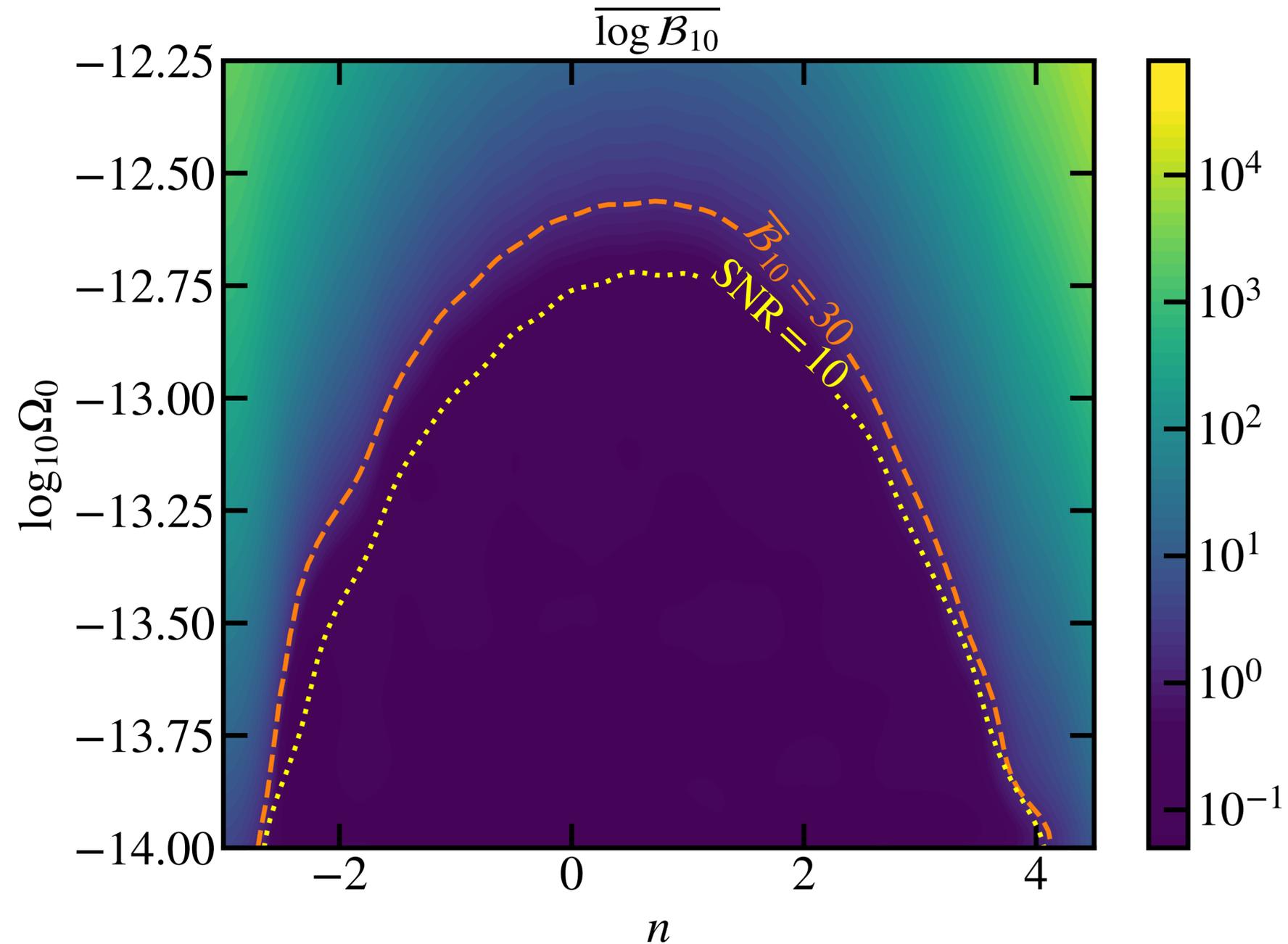
LISA Data Analysis

Parameter estimation



LISA Data Analysis

Assuming detectability of power-law signals



- ❑ Searching for stochastic GW signals (astro+cosmo) is going to be very *very* challenging.
- ❑ The Global fit is a no trivial procedure, but we are getting there.
- ❑ Combine different techniques/methodologies.
- ❑ LISA is very promising at the mHz range!

Έξτρα Ματέριαλ