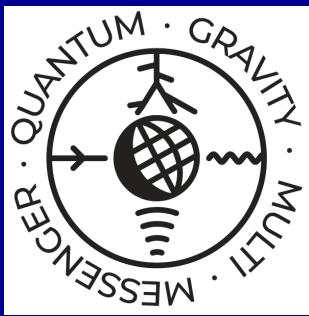


# ***Geometric origin of the Dark Sector & matter-antimatter asymmetry of the Universe***

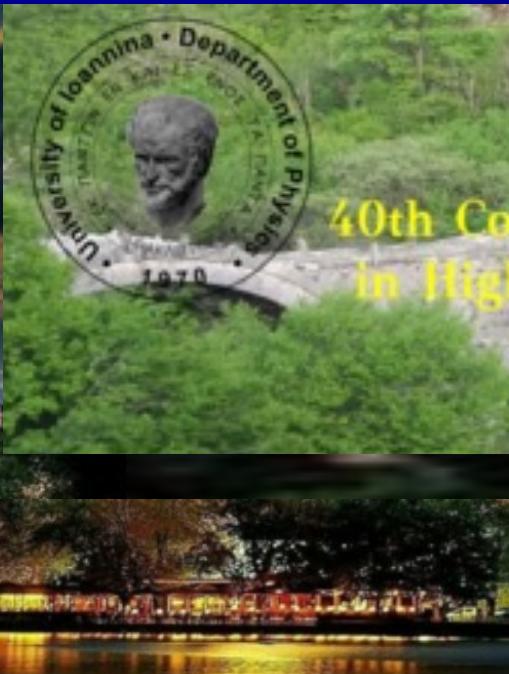
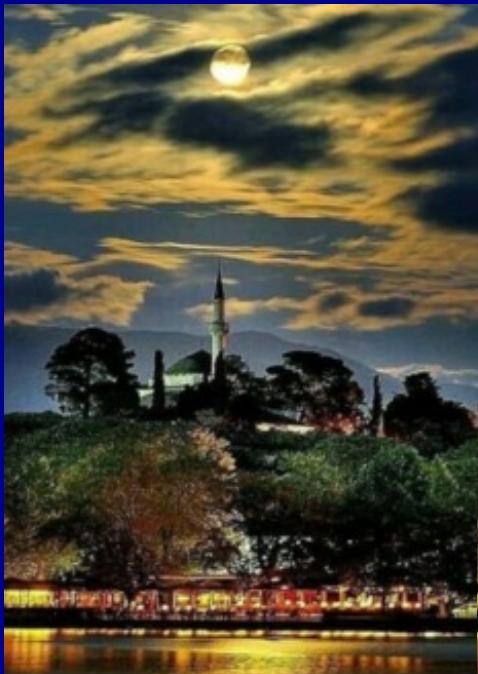


**Nick E. Mavromatos**

Natl. Tech. U. Athens,  
Physics Dept., Athens, Greece



**CA18108 - Quantum gravity  
phenomenology in the multi-  
messenger approach**



# **0. Outline**

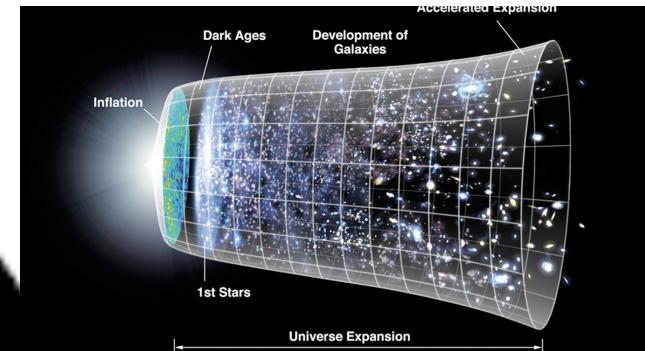
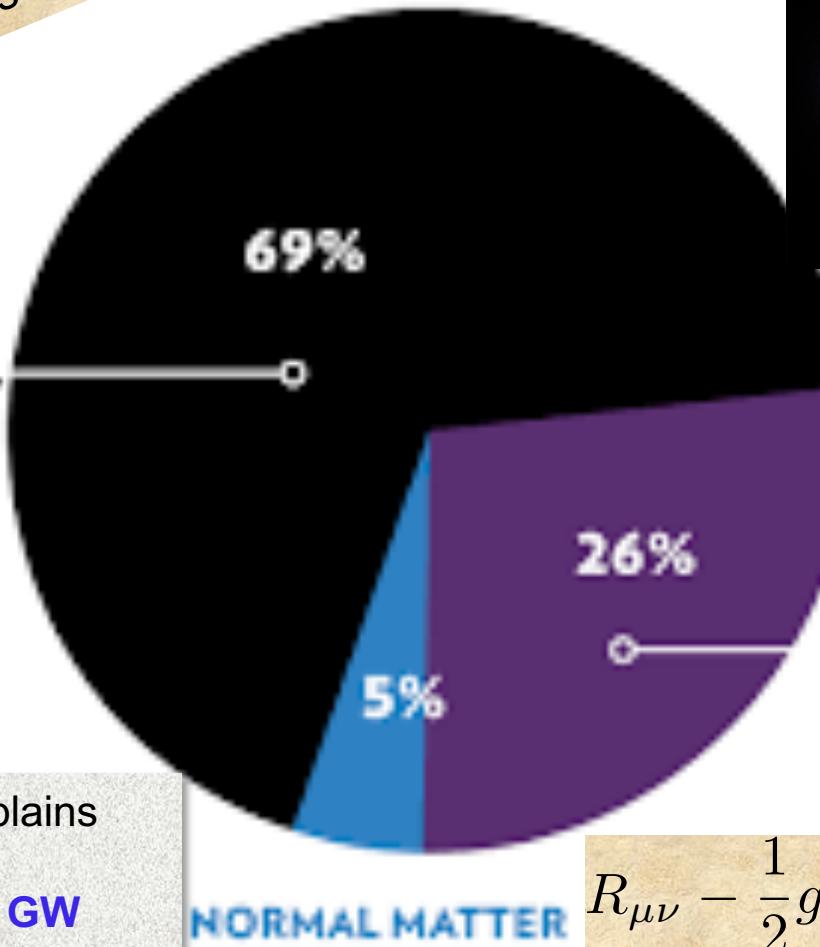
- 1. Motivation**
- 2. Gravitational origin of (axion) dark matter (DM)**
- 3. String-Inspired Gravitational Theory with Torsion & Grav. Anomalies, axions and torsion**
- 4. Primordial Gravitational Waves (GW) induced Condensates of Anomalies,**
- 5. Spontaneous Lorentz and CPT-Violation by axion backgrounds & Running Vacuum Model-type inflation without external inflatons**
- 6. Post Inflationary eras & cosmic evolution of the stringy RVM:  
Spontaneous Lorentz & CPT-Violation by axion backgrounds & Leptogenesis  
in radiation era → Baryogenesis → geometric origin of Matter-antimatter asymmetry**
- 7. Conclusions & Outlook (modern-era cosmological tensions and stringy RVM;  
enhanced primordial black hole production during inflation,  
modified GW profiles and DM)**

# **1. Motivation**

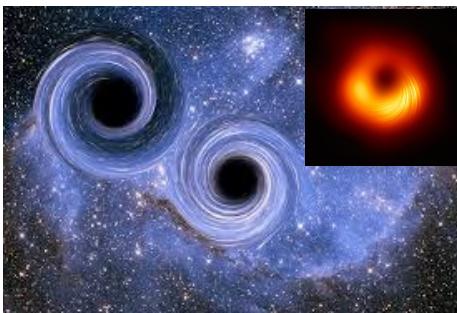
# Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

Simplest model based  
on  $\Lambda$ CDM works OK  
for large scales

## ENERGY DISTRIBUTION OF THE UNIVERSE



+ SnIa, BaO, Lensing



Also Einstein's GR explains  
sufficiently well  
Black-Hole Mergers + GW  
(since 2015 LIGO),  
Black-Hole 'photographs' (EHT),...

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} R - g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}$$

$T_{\mu\nu} \ni$  Cold Dark Matter

# Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

Simulations  
3 data

Simulations

But....

Need to go  
Beyond....

What still we do not know/did not observe:

Nature of Dark Energy

Nature of Dark matter

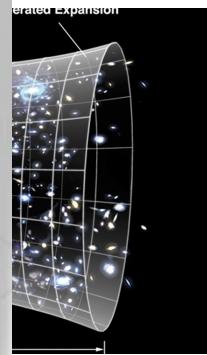
Primordial Gravitational Waves

(through detection of B-mode polarisation

in CMB from very early Universe)

Microscopic models of Inflation

(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)



Lensing

$$8\pi G T_{\mu\nu}$$



Also I  
suffice  
Black  
(since  
Black

# Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

But....

Need to go  
Beyond....

What still we

Nature of  
Nature of

Primordial

(through detection of B-mode  
polarisation

in CMB from very early Universe)

Microscopic models of Inflation

(Is it due to fundamental inflatons or  
dynamical e.g. Starobinsky type? ....)



$\Lambda$ CDM appears  
to be in tension with  
local measurements of  
present-era  $H_0$   
& also galaxy-growth  
data ?

$$8\pi G T_{\mu\nu}$$

10,000,000,001

MATTER

10,000,000,000

ANTI-MATTER



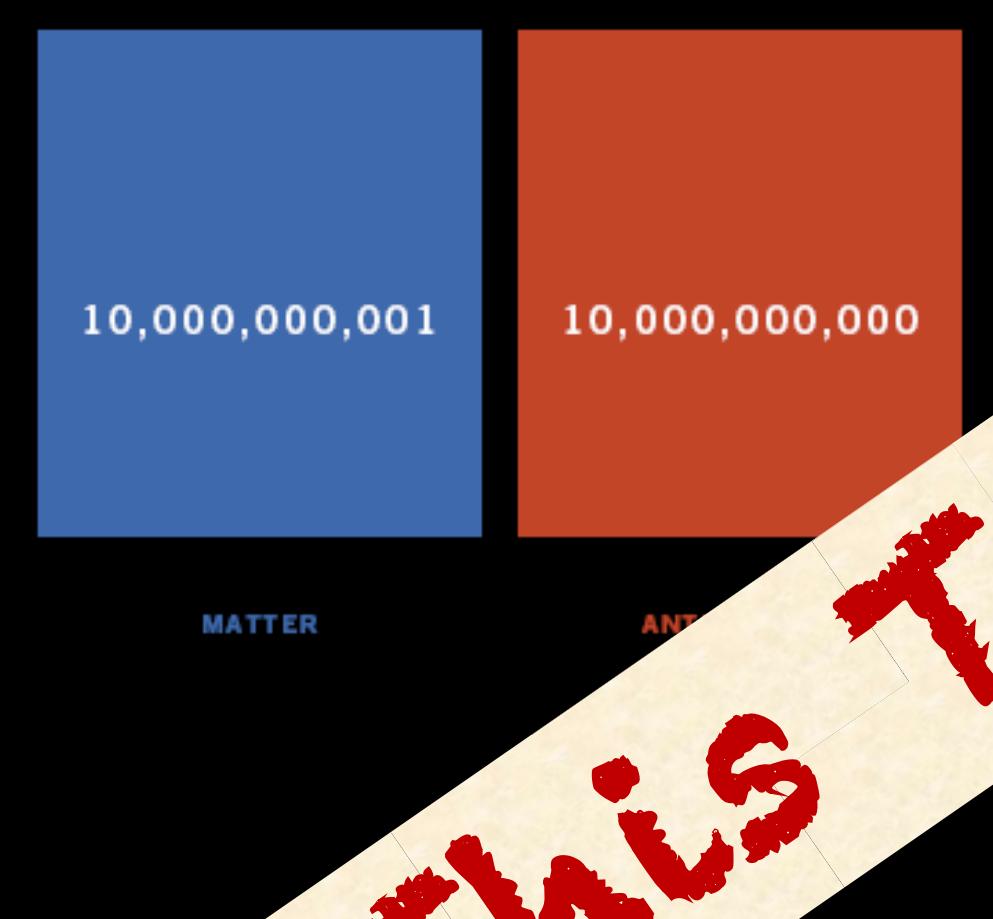
Microscopic  
understanding of  
**Matter/Antimatter  
asymmetry** in the  
Universe?

## The Baryon Asymmetry

$$\frac{n_B - \bar{n}_B}{n_B + \bar{n}_B} \sim \frac{n_B - \bar{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

*s = entropy density  
of Universe*



10,000,000,001

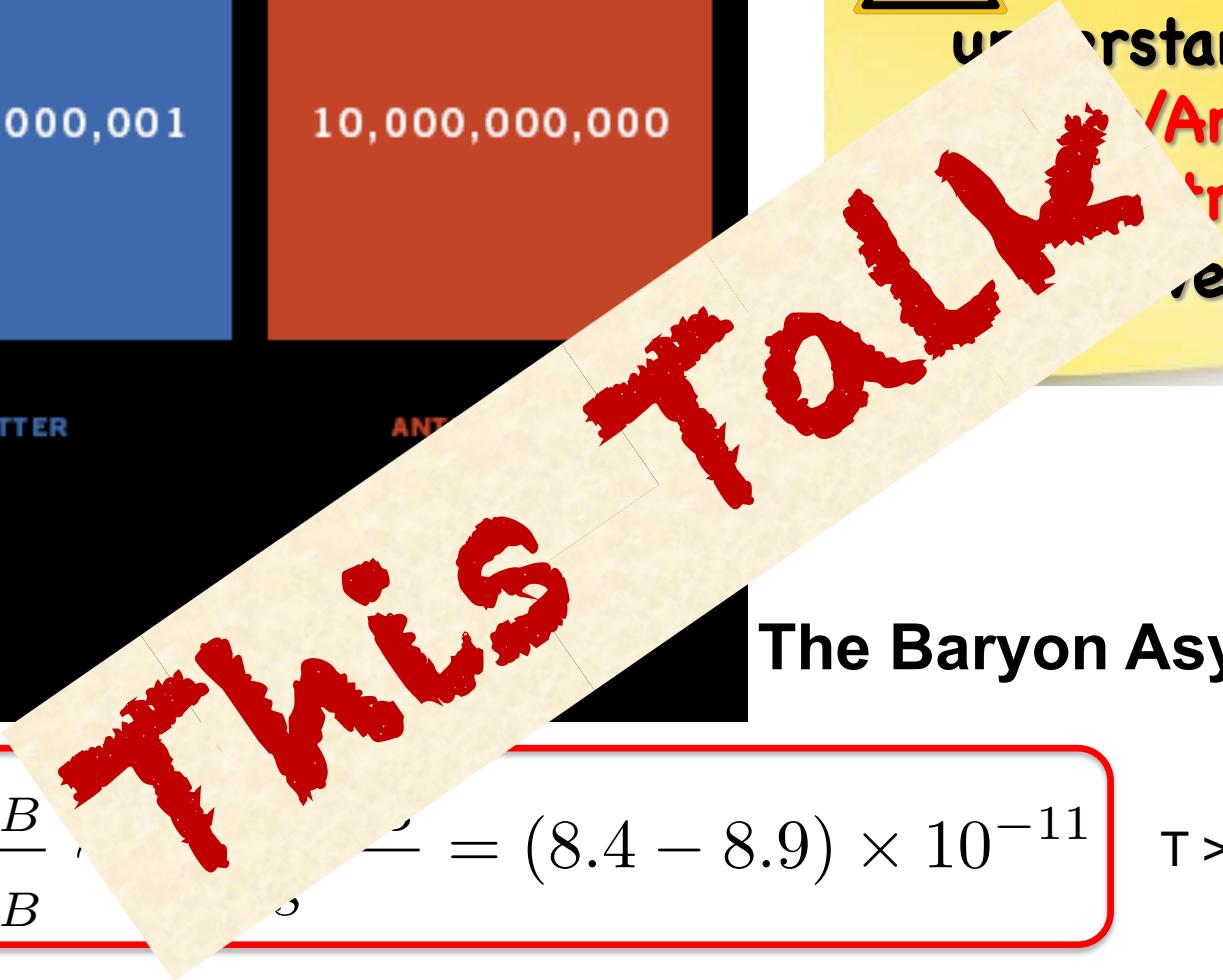
10,000,000,000

MATTER

ANTIMATTER



Microscopic  
understanding of  
Matter /Antimatter  
in the  
Universe?



THIS TALK

## The Baryon Asymmetry

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T > 1 GeV

$s$  = entropy density  
of Universe

# Attempts at Explanation of Baryon Asymmetry – Sakharov 's Conditions

**Baryon number violation**

**C-violation**

**and CP violation**



**Departure from thermodynamic equilibrium (non-stationary system)**

$CP |particle\rangle = |anti-particle\rangle$

Need new physics beyond the SM →  
new sources of CP violation?



# Attempts at Explanation of Baryon Asymmetry – Sakharov 's Conditions

**Baryon number violation**

**C-violation**

**and CP violation**



**Departure from thermodynamic equilibrium (non-stationary system)**

$CP |particle\rangle = |anti-particle\rangle$

Need new physics beyond the SM →  
new sources of CP violation?

What if CPTV geometries  
in the early Universe ?



I will argue that:

observed matter-antimatter asymmetry

can be linked with

Microscopic string-inspired models of Cosmology with ANOMALIES,  
primordial gravitational waves and induced spontaneous  
(through gravitational anomaly condensates) Lorentz + CPT Violation

+

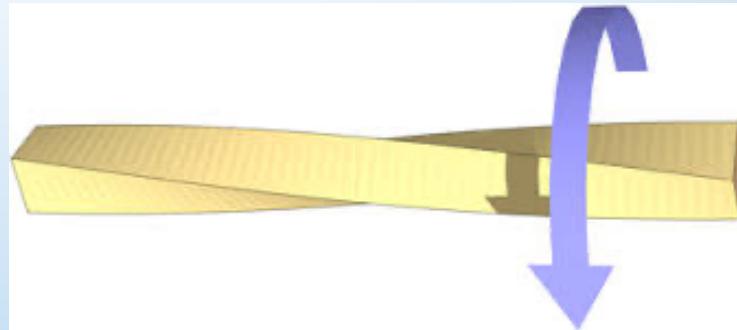
geometric torsion interpretation of axion Dark matter

## **2. Geometrical origin of axion Dark matter**

# A Geometric Origin of (axion) Dark Matter?



# A Geometric Origin of (axion) Dark Matter?



Torsion in spacetime?

The graphic features portraits of Albert Einstein and Trifunovic Stjepan. It includes the text "Einstein-Cartan" and two boxes: one labeled "only curvature" and another labeled "curvature and torsion". Below the Einstein portrait is the text "or teleparallel gravity (only torsion)".

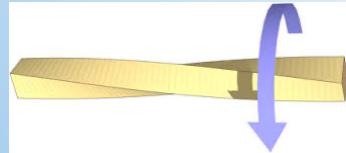
Einstein-Cartan

only curvature

curvature and torsion

or teleparallel gravity (only torsion)

## Example of Einstein-Cartan theory : QED with Torsion



$$T^a = \mathbf{d}e^a + \bar{\omega}_b^a \wedge e^b$$

$$\bar{R}_b^a = \mathbf{d}\bar{\omega}_b^a + \bar{\omega}_c^a \wedge \bar{\omega}_b^c$$

Contorted  
Spin connection

$$g_{\mu\nu} = e_\mu^a \eta_{ab} e_\nu^b$$

vielbein

Torsion 2-form

Generalised curvature 2-form

$$\bar{\omega}_b^a = \omega_b^a + K_b^a$$

contorsion

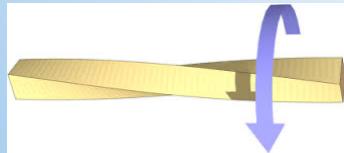
$$\bar{\mathbf{D}}e^a = T^a,$$

Metricity postulate Breaks down if torsion present

$$\bar{\nabla}_\rho g_{\mu\nu} \neq 0$$

$$\nabla_\rho g_{\mu\nu} = 0 \text{ (torsion free)}$$

## Example of Einstein-Cartan theory : QED with Torsion



$$T^a = \mathbf{d}e^a + \bar{\omega}_b^a \wedge e^b$$

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vielbein

Torsion 2-form

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contorsion

$$\bar{\mathbf{D}}e^a = T^a,$$

$$\bar{\mathbf{D}}T^a = \bar{R}_b^a \wedge e^b$$

$$\bar{\mathbf{D}}\bar{R}_b^a = 0.$$

Metricity postulate Breaks down if torsion present

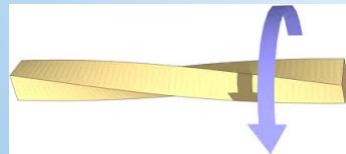
$$T_{\mu\nu}^a = e_\lambda^a (\Gamma_{\nu\mu}^\lambda - \Gamma_{\mu\nu}^\lambda) = -2e_\lambda^a \Gamma_{[\mu\nu]}^\lambda$$

$$T_{bc}^a = -2K_{[bc]}^a, \quad K_{abc} = \frac{1}{2}(T_{cab} - T_{abc} - T_{bca}).$$

$$\bar{R}_b^a = R_b^a + \mathbf{D}K_b^a + K_c^a \wedge K_b^c$$

Torsion-free

## Example of Einstein-Cartan theory : QED with Torsion



$$\begin{aligned}
 S_G &= \frac{1}{2\kappa^2} \int \bar{R}_{ab} \wedge *(\mathbf{e}^a \wedge \mathbf{e}^b) \\
 &\equiv \frac{1}{2\kappa^2} \int (\mathbf{R}_{ab} + \mathbf{D}\mathbf{K}_{ac} + \mathbf{K}_{ac} \wedge \mathbf{K}_b^c) \wedge *(\mathbf{e}^a \wedge \mathbf{e}^b) \\
 \text{* = Hodge dual} \quad * \left( \underbrace{\mathbf{e}^a \wedge \dots \wedge \mathbf{e}^b}_p \right) &= \frac{1}{(4-p)!} \epsilon^{a \dots b} {}_{c \dots d} \underbrace{\mathbf{e}^c \wedge \dots \wedge \mathbf{e}^d}_{4-p}
 \end{aligned}$$

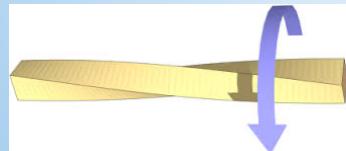
$$\int (\mathbf{D}\mathbf{K}_{ab} \wedge *(\mathbf{e}^a \wedge \mathbf{e}^b)) = \int \mathbf{d}(\mathbf{K}_{ab} \wedge *(\mathbf{e}^a \wedge \mathbf{e}^b)) = \int_{\partial\mathcal{M}} (\mathbf{K}_{ab} \wedge *(\mathbf{e}^a \wedge \mathbf{e}^b)) = 0$$

Stokes

$$S_G = \frac{1}{2\kappa^2} \int \sqrt{-g} (\mathcal{R} + \Delta) d^4x$$

$$\Delta = K^\lambda_{\mu\nu} K^{\nu\mu}{}_\lambda - K^{\mu\nu}{}_\nu K_{\mu\lambda}{}^\lambda = T^\nu{}_{\nu\mu} T^\lambda{}_\lambda{}^\mu - \frac{1}{2} T^\mu{}_{\nu\lambda} T^{\nu\lambda}{}_\mu + \frac{1}{4} T_{\mu\nu\lambda} T^{\mu\nu\lambda}$$

# Example of Einstein-Cartan theory : QED with Torsion



**fermions**

$$\bar{\mathbf{D}}\psi = \mathbf{d}\psi - \frac{i}{4}\bar{\omega}_{ab}\sigma^{ab}\psi,$$

$$\bar{\omega}_b^a = \omega_b^a + K_b^a$$

**contorsion**

$$S_\psi = \frac{i}{2} \int (\bar{\psi} \gamma^\mu \bar{\mathcal{D}}_\mu \psi - (\bar{\mathcal{D}}_\mu \bar{\psi}) \gamma^\mu \psi) \sqrt{-g} \text{ d}^4x$$

$$\bar{\mathcal{D}}_\mu = \bar{\mathbf{D}}_\mu - ieA_\mu$$

$$S_\psi = \frac{i}{2} \int (\bar{\psi} \gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \gamma^\mu \psi) \sqrt{-g} \text{ d}^4x$$

$$+ e \int A_\mu \bar{\psi} \gamma^\mu \psi \sqrt{-g} \text{ d}^4x + \frac{1}{8} \int \bar{\psi} \{\gamma^c, \sigma^{ab}\} \psi K_{ab;c} \sqrt{-g} \text{ d}^4x$$

$$\{\gamma^c, \sigma^{ab}\} = 2\epsilon^{abc}{}_d \gamma^d \gamma^5$$



# Example of Einstein-Cartan theory : QED with Torsion



**fermions**

$$\bar{\mathbf{D}}\psi = \mathbf{d}\psi - \frac{i}{4}\bar{\omega}_{ab}\sigma^{ab}\psi,$$

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$$\begin{aligned} \mathbf{T} &= (1/3!) T_{abc} \mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathbf{e}^c \\ \mathbf{S} &= * \mathbf{T} \end{aligned}$$

$$S_d = (1/3!) \epsilon^{abc}{}_d T_{abc}$$

$$\{\gamma^c, \sigma^{ab}\} = 2 \epsilon^{abc}{}_d \gamma^d \gamma^5$$



## Example of Einstein-Cartan theory : QED with Torsion



**fermions**

$$\bar{\mathbf{D}}\psi = \mathbf{d}\psi - \frac{i}{4}\bar{\omega}_{ab}\sigma^{ab}\psi,$$

$$\bar{\omega}_b^a = \omega_b^a + K_b^a$$

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$$S_\psi = \frac{i}{2} \int \left( \bar{\psi} \gamma^\mu \bar{\mathcal{D}}_\mu \psi - (\bar{\mathcal{D}}_\mu \bar{\psi}) \gamma^\mu \psi \right) \sqrt{-g} \, d^4x$$

$$\bar{\mathcal{D}}_\mu = \bar{\mathbf{D}}_\mu - ieA_\mu$$

$$\mathbf{T} = (1/3!)T_{abc}e^a \wedge e^b \wedge e^c$$

$$\mathbf{S} = * \mathbf{T}$$

$$S_d = (1/3!) \epsilon^{abc}{}_d T_{abc}$$

$$S_\psi \ni -\frac{3}{4} \int S_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi \sqrt{-g} \, d^4x = -\frac{3}{4} \int S \wedge * j^5$$

$$J^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi \quad \text{Axial current}$$

# Torsion & Axion-like d.o.f.

Fermion (Dirac) equation

$$i\gamma^\mu \mathcal{D}_\mu \psi - \frac{3}{4} S_\mu \gamma^\mu \gamma^5 \psi = 0$$

$$\mathcal{D}_\mu = D_\mu - ieA_\mu$$

Duncan, Kaloper,  
Olive

Nucl. Phys. B 387,  
215( 1992)

Einstein equations

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

$$T_{\mu\nu} = T_{\mu\nu}^A + T_{\mu\nu}^\psi + T_{\mu\nu}^S$$

Stress-energy  
tensor

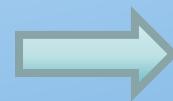
$$T_{\mu\nu}^A = F_{\mu\lambda}F_\nu^\lambda - \frac{1}{4}g_{\mu\nu}F_{\lambda\rho}F^{\lambda\rho},$$

$$T_{\mu\nu}^S = -\frac{3}{2\kappa^2}(S_\mu S_\nu - \frac{1}{2}g_{\mu\nu}S_\lambda S^\lambda)$$

$$T_{\mu\nu}^\psi = -\frac{i}{2}\left(\bar{\psi}\gamma_{(\mu}\mathcal{D}_{\nu)}\psi - (\mathcal{D}_{(\mu}\bar{\psi})\gamma_{\nu)}\psi\right) + \frac{3}{4}S_{(\mu}\bar{\psi}\gamma_{\nu)}\gamma^5\psi.$$

Classical torsion equation of motion

$$S = \frac{1}{2}\kappa^2 j^5$$



$$\mathbf{d} * \mathbf{S} = 0$$

If  $J^5$  conserved

# Torsion & Axion-like d.o.f.

Quantum chiral anomalies  $\rightarrow \mathbf{d} * \mathbf{J}^5 \neq 0$

Add counterterms (order byn order in perturbation theory)  
to ensure  $\mathbf{d} * \mathbf{S} = 0$  & thus conservation of torsion charge  $Q_S = \int * S$

Path integral over torsion d.o.f.

$$\int \mathcal{D}S \delta(\mathbf{d} * S) \exp\left(i \int \left[ \frac{3}{4\kappa^2} S \wedge * S - \frac{3}{4} S \wedge * j^5 \right] \right)$$

Lagrange  
Multiplier  $\Phi$   
(pseudoscalar)



$$\int \mathcal{D}S \mathcal{D}\Phi \exp\left(i \int \left[ \frac{3}{4\kappa^2} S \wedge * S - \frac{3}{4} S \wedge * j^5 + \Phi \mathbf{d} * S \right] \right)$$

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Integrate out torsion  $S$   
(non-propagating field)

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Multiplier  $\Phi$   
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$$\Phi = (3/2\kappa^2)^{1/2} \phi$$

$$\int \mathcal{D}S \mathcal{D}\Phi \exp\left(i \int \left[ \frac{3}{4\kappa^2} S \wedge * S - \frac{3}{4} S \wedge * j^5 + \Phi \mathbf{d} * S \right] \right)$$

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Integrate out torsion  $S$   
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$$\Phi = (3/2\kappa^2)^{1/2} \phi$$

Lagrange  
Multiplier  $\Phi$   
(pseudoscalar)

$$\int \mathcal{D}\phi \exp\left(i \int \left[ -\frac{1}{2} \mathbf{d}\phi \wedge * \mathbf{d}\phi - \frac{1}{f_\phi} \mathbf{d}\phi \wedge * j^5 - \frac{1}{2f_\phi^2} j^5 \wedge * j^5 \right] \right)$$

Axion coupling  
parameter  $f_\phi = (3\kappa^2/8)^{-1/2}$



# Torsion & Axion-like d.o.f.

$$\int \mathcal{D}\phi \exp\left(i \int \left[ -\frac{1}{2} \mathbf{d}\phi \wedge * \mathbf{d}\phi - \frac{1}{f_\phi} \mathbf{d}\phi \wedge * j^5 - \frac{1}{2f_\phi^2} j^5 \wedge * j^5 \right] \right)$$

$f_\phi = (3\kappa^2/8)^{-1/2}$

Partially integrate

# Torsion & Axion-like d.o.f.

$$\int \mathcal{D}\phi \exp \left( i \int \left[ -\frac{1}{2} \mathbf{d}\phi \wedge * \mathbf{d}\phi - \frac{1}{f_\phi} \mathbf{d}\phi \wedge * j^5 - \frac{1}{2f_\phi^2} j^5 \wedge * j^5 \right] \right)$$

Partially integrate

Axion coupling

$$f_\phi = (3\kappa^2/8)^{-1/2}$$



$$\mathbf{d} * j^5 = -\frac{e^2}{4\pi^2} F \wedge F - \frac{1}{96\pi^2} \text{tr}(\bar{R} \wedge \bar{R}) \equiv G(A, \bar{\omega})$$

$$\nabla \cdot j^5 = \frac{e^2}{8\pi^2} F^{\mu\nu} * F_{\mu\nu} - \frac{1}{192\pi^2} \bar{R}^{\alpha\beta\mu\nu} * \bar{R}_{\alpha\beta\mu\nu}$$

Can add counterterms so that  
only **torsion-free spin connection**  
 **$\omega$**  appears in the **Anomaly**

$$\int \mathcal{D}\phi \exp \left( i \int \left[ -\frac{1}{2} \mathbf{d}\phi \wedge * \mathbf{d}\phi + \frac{1}{f_\phi} \phi \mathbf{G}(A, \omega) - \frac{1}{2f_\phi^2} j^5 \wedge * j^5 \right] \right)$$

**Repulsive four-fermion**  
Characteristic of  
**Einstein-Cartan** theories

# Torsion & Axion-like d.o.f.

$$\int \mathcal{D}\phi \exp \left( i \int \left[ -\frac{1}{2} \mathbf{d}\phi \wedge * \mathbf{d}\phi - \frac{1}{f_\phi} \mathbf{d}\phi \wedge * j^5 - \frac{1}{2f_\phi^2} j^5 \wedge * j^5 \right] \right)$$

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Non-Abelian

Partially integrate



Can add counterterm  $\hat{\text{hat}}$   
only **torsion-free spin connection**  
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$$\int \mathcal{D}\phi \exp \left( i \int \left[ -\frac{1}{2} \mathbf{d}\phi \wedge * \mathbf{d}\phi + \frac{1}{f_\phi} \phi G(A, \omega) - \frac{1}{2f_\phi^2} j^5 \wedge * j^5 \right] \right)$$

Non-Abelian Gauge group Instantons can lead to potential

$$V(\phi) = \Lambda_{\text{inst}}^4 \left( 1 - \cos(\phi/f_b) \right)$$

→ massive ( $m_b = \Lambda_{\text{inst}}^2/f_b$ ) torsion-induced axion  
GEOMETRIC ORIGIN OF AXION DM?



### **3. String-Inspired Gravitational Theory with **Torsion & Grav. Anomalies, axions and torsion****

# String-inspired gravitational theories with torsion and anomalies

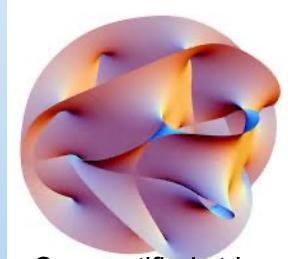
NEM,  
+ Basilakos, Solà,  
Sarkar,

## Massless gravitational (bosonic) string multiplet:

$g_{\mu\nu} = g_{\nu\mu}$ , spin = 2 (graviton)

$\Phi$ , spin = 0 (dilaton),

$B_{\mu\nu} = -B_{\nu\mu}$ , spin = 1 (Kalb – Ramond (KR) field)



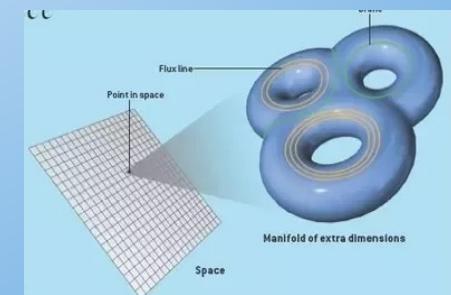
Compactified strings

Gauge symmetry in closed string sector  $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \theta_\nu - \partial_\nu \theta_\mu$

## Symmetry of string $\sigma$ -model vertex operators

$$\int_{\Sigma^{(2)}} d^2\sigma B_{\mu\nu} \epsilon^{AB} \partial_A X^\mu \partial_B X^\nu, \quad A, B = 1, 2$$

world  
sheet



Gross and Sloan, Metsaev and Tseytlin

# String-inspired gravitational theories with torsion and anomalies

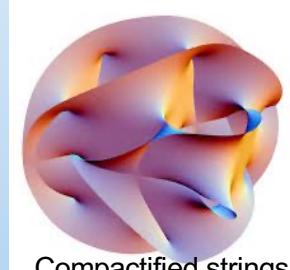
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$\Phi$ , spin = 0 (dilaton),

$B_{\mu\nu} = -B_{\nu\mu}$ , spin = 1 (Kalb – Ramond (KR) field)

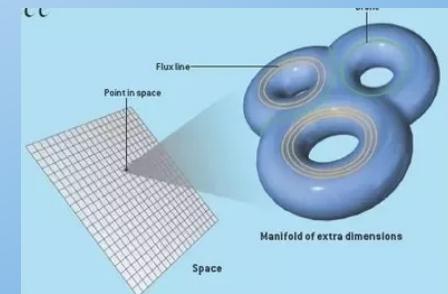
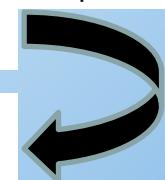


Compactified strings

Gauge symmetry in closed string sector  $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \theta_\nu - \partial_\nu \theta_\mu$

Effective target-spacetime gravitational action depends on the field strength :

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$$



# String-inspired gravitational theories with torsion and anomalies

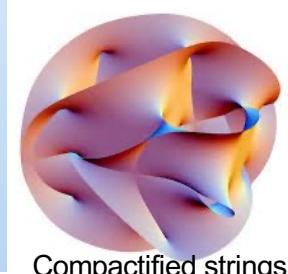
NEM,  
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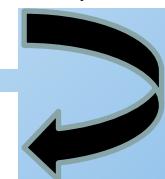
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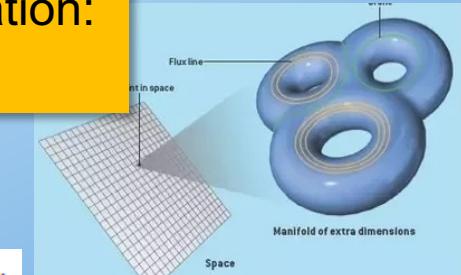
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$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa) (\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$$

Chern-Simons terms  
Gravitational                  gauge

$$\Omega_{3L} = \omega^a{}_c \wedge d\omega^c{}_a + \frac{2}{3} \omega^a{}_c \wedge \omega^c{}_d \wedge \omega^d{}_a$$

$$\Omega_{3Y} = A \wedge dA + A \wedge A \wedge A$$



$$\alpha' = \text{Regge slope} = M_s^{-2}$$

$$\kappa^2 = 8\pi G = 4d \text{ grav. constant}$$

# String-inspired gravitational theories with torsion and anomalies

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+ Basilakos, Solà,  
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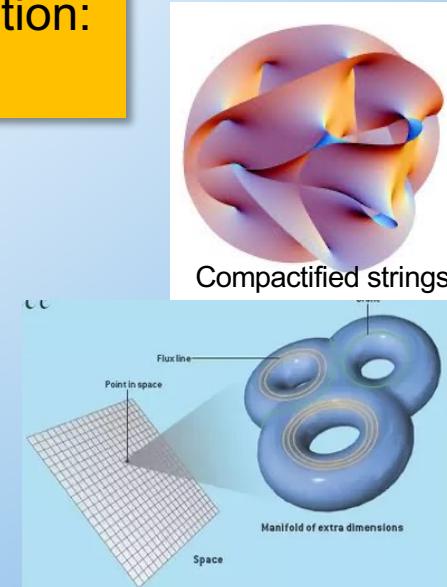
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String effective action (lowest order in Regge slope)

$$S_B = - \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} R + \frac{1}{6} \mathcal{H}_{\lambda\mu\nu} \mathcal{H}^{\lambda\mu\nu} + \dots \right).$$



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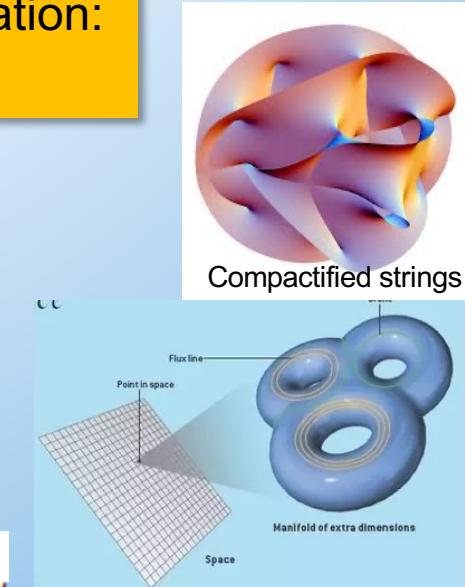
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Totally antisymmetric  
torsion

$$\overline{R}(\overline{\Gamma})$$

$$\overline{\Gamma}_{\mu\nu}^\rho = \Gamma_{\mu\nu}^\rho + \frac{\kappa}{\sqrt{3}} \mathcal{H}_{\mu\nu}^\rho \neq \overline{\Gamma}_{\nu\mu}^\rho$$



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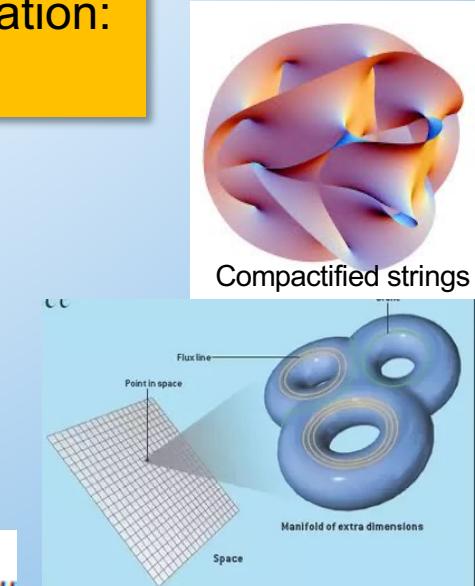
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Torsion → axion-like d.o.f. (as in **CONTORTED QED**)

String-model independent axion

Svrcek-Witten

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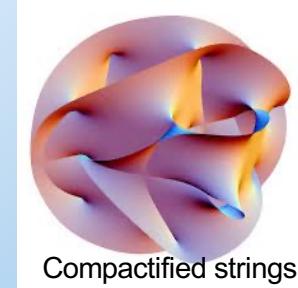
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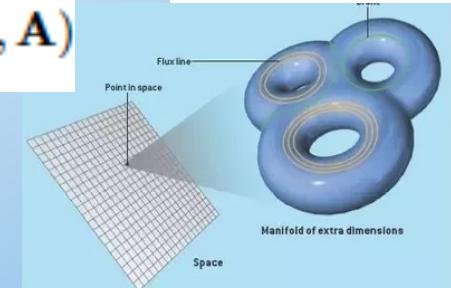
## Bianchi identity constraint

$$\varepsilon_{abc}^{\mu} \mathcal{H}^{abc}_{;\mu} = \frac{\alpha'}{32\kappa} \sqrt{-g} \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \equiv \sqrt{-g} \mathcal{G}(\omega, \mathbf{A})$$

Implementation via axion-like Lagrange multiplier field  $b(x)$



Compactified strings



$$\begin{aligned} & \Pi_x \delta \left( \varepsilon^{\mu\nu\rho\sigma} \mathcal{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A}) \right) \Rightarrow \\ & \int \mathcal{D}b \exp \left[ i \int d^4x \sqrt{-g} \frac{1}{\sqrt{3}} b(x) \left( \varepsilon^{\mu\nu\rho\sigma} \mathcal{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A}) \right) \right] \\ & = \int \mathcal{D}b \exp \left[ -i \int d^4x \sqrt{-g} \left( \partial^\mu b(x) \frac{1}{\sqrt{3}} \epsilon_{\mu\nu\rho\sigma} \mathcal{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}} \mathcal{G}(\omega, \mathbf{A}) \right) \right] \end{aligned}$$

# String-inspired gravitational theories with torsion and anomalies

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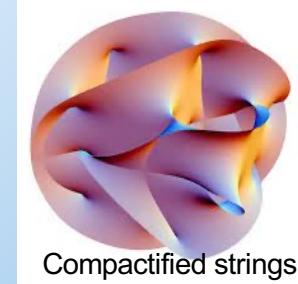
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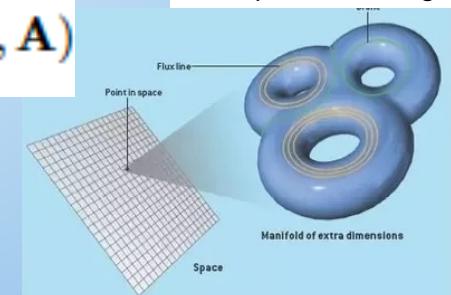
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Compactified strings

Implementation via axion-like Lagrange multiplier field  $b(x)$   
Integration of non-propagating  $H$  field



$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right) + \dots \right].$$

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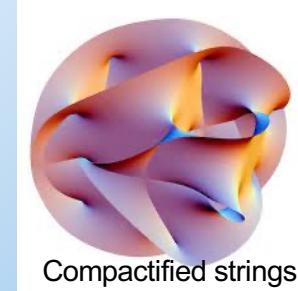
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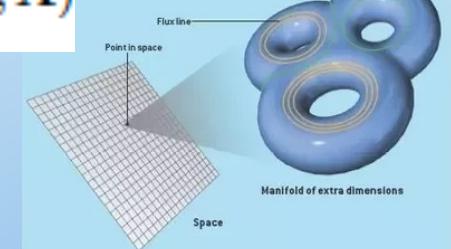
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Massive axions through  
Non-Abelian gauge group  
Instantons



Compactified strings



$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right) + \dots \right],$$

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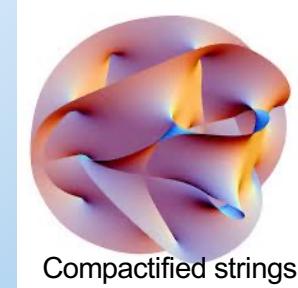
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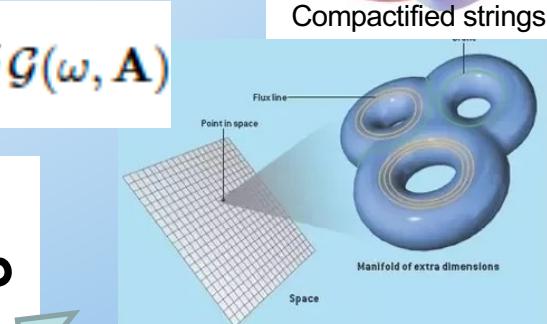
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Compactified strings

## GEOMETRIC ORIGIN OF AXION DM

Massive axions through  
Non-Abelian gauge group  
Instantons



$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right) + \dots \right],$$



Geometric origin of stringy axion DM

## Inclusion of Fermions

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \frac{\sqrt{2}\alpha'}{96\kappa\sqrt{3}} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right) + \dots \right]$$

$$+ S_{\text{Dirac}}^{\text{Free}} + \int d^4x \sqrt{-g} \left( \mathcal{F}_\mu + \frac{\kappa}{2} \sqrt{\frac{3}{2}} \partial_\mu b \right) J^{5\mu} - \frac{3\kappa^2}{16} \int d^4x \sqrt{-g} J_\mu^5 J^{5\mu} + \dots \Big] + \dots$$

or Majorana

$$J^{5\mu} = \bar{\psi}_j \gamma^\mu \gamma^5 \psi_j$$

Axial Current  
All fermion species

torsion

cf. classically in 4 dim:

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

## Inclusion of Fermions

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \frac{\sqrt{2}\alpha'}{96\kappa\sqrt{3}} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right) + \dots \right]$$

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Axial Current  
All fermion species

KR-axion anomalous  
CP-Violating interaction

cf. classically in 4 dim:

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

torsion

## Inclusion of Fermions

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**Axial Current**  
**All fermion species**

4-fermion contact interaction  
 characteristic of  
 (integrating out) torsion

cf. classically in 4 dim:  
 (duality relationship)

**torsion**

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

## Inclusion of Fermions

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Majorana

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$\mathcal{F}^d = \epsilon^{abcd} e_{b\lambda} \partial_a e_c^\lambda$ , vielbeins

Vanishes for Friedmann-Lemaître-Roberston-Walker backgrounds

torsion

cf. classically in 4 dim:  
(duality relationship)

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

## Inclusion of Fermions

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Axial Current

Kalb-Ramond (KR) or string-model independent ("gravitational") axion

torsion

cf. classically in 4 dim:  
(duality relationship)

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

# The Model

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \frac{\sqrt{2}\alpha'}{96\kappa\sqrt{3}} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right) + \dots \right]$$

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# The Model

Anomaly terms

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \frac{e\alpha'}{96\kappa\sqrt{3}} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right) + \dots \right]$$

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All fermion species

# The Model

Anomaly terms

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \frac{e\alpha'}{96\kappa\sqrt{3}} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right) + \dots \right]$$
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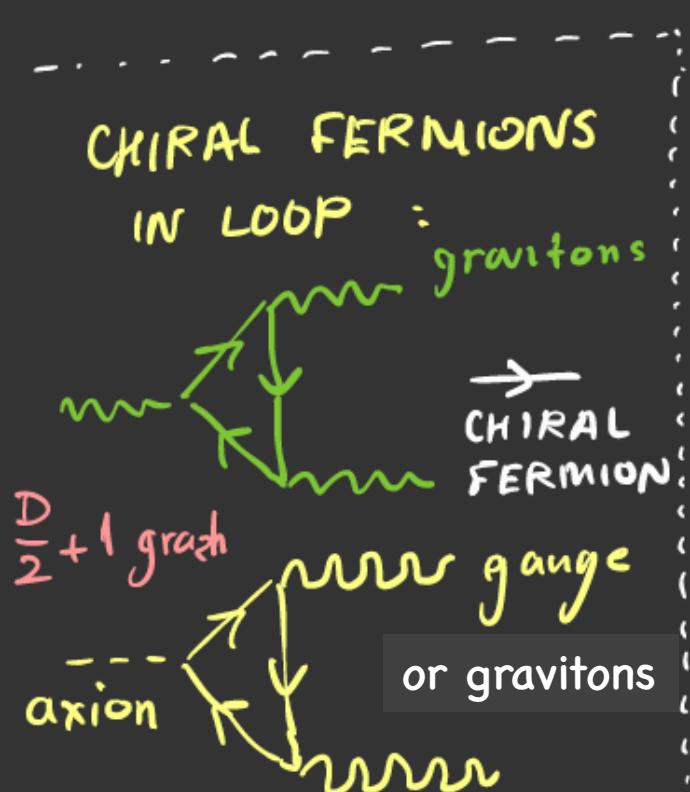
or Majorana

All fermion species



Non-trivial if chiral anomalies affect the conservation of axial current

NB: Anomalies:  
(CHIRAL)



Classically conserved current  
AXIAL FERMION CURRENT  $J^{\mu 5}$   
CEASES to be conserved @ a  
quantum level

$$V_F J^{\mu 5} \propto g R_{\mu\nu\rho} \tilde{R}^{\rho\nu\sigma} - F_{\mu\nu} \tilde{F}^{\nu\sigma}$$

$c_i \in IR$

$$J^{\mu 5} = \bar{\psi}_j \gamma^\mu \gamma^5 \psi_j, j=1 \dots N_{\text{SPECIES}}$$

chiral  
fermion

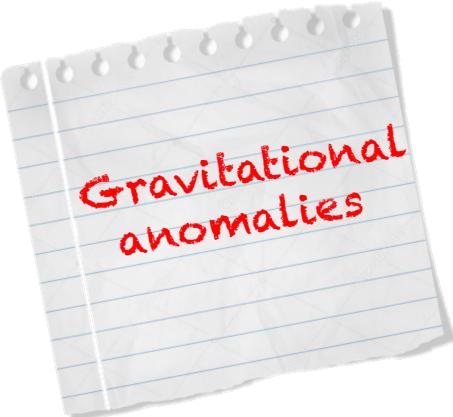
$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma},$$

$$\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} R^{\alpha\beta} \rho\sigma$$

$$\gamma^5 \psi_j = \mp \psi_j$$

(LEFT OR  
RIGHT  
HANDED)

# Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x \sqrt{-g} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

Spoils conservation  
of stress tensor  
(diffeomorphism  
invariance affected  
in quantum theory)

Topological,  
does NOT  
contribute to  
stress tensor

$$\delta \left[ \int d^4x \sqrt{-g} b R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \right] = 4 \int d^4x \sqrt{-g} \mathcal{C}^{\mu\nu} \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g} \mathcal{C}_{\mu\nu} \delta g^{\mu\nu}$$

## Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2} \left[ v_\sigma \left( \varepsilon^{\sigma\mu\alpha\beta} R^\nu_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^\mu_{\beta;\alpha} \right) + v_{\sigma\tau} \left( \tilde{R}^{\tau\mu\sigma\nu} + \tilde{R}^{\tau\nu\sigma\mu} \right) \right] = -\frac{1}{2} \left[ \left( v_\sigma \tilde{R}^{\lambda\mu\sigma\nu} \right)_{;\lambda} + (\mu \leftrightarrow \nu) \right]$$

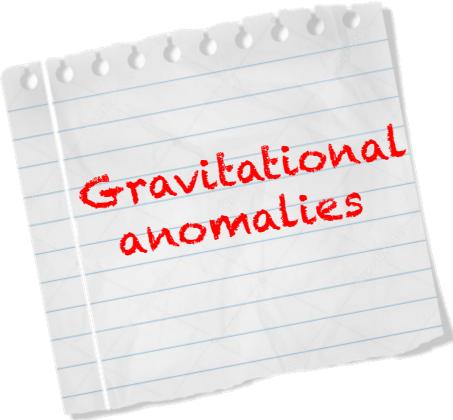
$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless

$$g_{\mu\nu} \mathcal{C}^{\mu\nu} = 0$$

Jackiw, Pi (2003)

# Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x \sqrt{-g} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

Spoils conservation  
of stress tensor  
(diffeomorphism  
invariance affected  
in quantum theory)

Topological,  
does NOT  
contribute to  
stress tensor

$$\delta \left[ \int d^4x \sqrt{-g} b R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \right] = 4 \int d^4x \sqrt{-g} \mathcal{C}^{\mu\nu} \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g} \mathcal{C}_{\mu\nu} \delta g^{\mu\nu}$$

Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2} \left[ v_\sigma \left( \varepsilon^{\sigma\mu\alpha\beta} R^\nu_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^\mu_{\beta;\alpha} \right) + v_{\sigma\tau} \left( \tilde{R}^{\tau\mu\sigma\nu} + \tilde{R}^{\tau\nu\sigma\mu} \right) \right] = -\frac{1}{2} \left[ \left( v_\sigma \tilde{R}^{\lambda\mu\sigma\nu} \right)_{;\lambda} + (\mu \leftrightarrow \nu) \right]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless

$$g_{\mu\nu} \mathcal{C}^{\mu\nu} = 0$$

 not necessarily  
positive  
contributions  
to vacuum energy



# Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - C^{\mu\nu} = \kappa^2 T_{\text{matter}}^{\mu\nu}$$

$$C^{\mu\nu}_{;\mu} = -\frac{1}{8} v^\nu R^{\alpha\beta\gamma\delta} \tilde{R}_{\alpha\beta\gamma\delta}$$

$$v_\sigma \equiv \partial_\sigma b$$



$$\kappa^2 T_{\text{matter}}^{\mu\nu} ; \mu = - C^{\mu\nu} ; \mu \neq 0$$

Diffeomorphism  
invariance breaking by  
gravitational anomalies?

# Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - C^{\mu\nu} = \kappa^2 T_{\text{matter}}^{\mu\nu}$$

$$C^{\mu\nu}_{;\mu} = -\frac{1}{8} v^\nu R^{\alpha\beta\gamma\delta} \tilde{R}_{\alpha\beta\gamma\delta}$$

$$v_\sigma \equiv \partial_\sigma b$$



$$\kappa^2 T_{\text{matter}}^{\mu\nu}_{;\mu} + C^{\mu\nu}_{;\mu} = 0$$

No problem  
with diffeo



Conserved Modified  
stress-energy  
tensor

## **4. Primordial Gravitational Waves, Anomaly condensates**

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ )

Basilakos, NEM,  
Solà (2019-20)

$$\begin{aligned} S_B^{\text{eff}} &= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} + \dots \right] \\ &= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \partial_\mu b(x) \mathcal{K}^\mu + \dots \right], \end{aligned}$$

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ )

Basilakos, NEM,  
Solà (2019-20)

**NB:**

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) R_{\mu\nu\rho\sigma} \cancel{R^{\mu\nu\rho\sigma}} + \dots \right]$$

$$= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \partial_\mu b(x) \mathcal{K}^\mu + \dots \right],$$

absent before  
formation of GW

No potential for KR axion before generation of GW

→ stiff-matter, equation of state  $w=+1$   
 → stiff-axion-matter dominance  
 during very early (pre-inflationary)  
 Universe

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ )

Basilakos, NEM,  
Solà (2019-20)

**NB:**

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) R_{\mu\nu\rho\sigma} \cancel{R^{\mu\nu\rho\sigma}} + \dots \right]$$

$$= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \partial_\mu b(x) \mathcal{K}^\mu + \dots \right],$$

absent before  
formation of GW

No potential for KR axion before generation of GW  
 → stiff-matter, equation of state  $w=+1$   
 → stiff-axion-matter dominance  
 during very early (pre-inflationary)  
 Universe

c.f. Zeldovich  
 but for baryons  
 in his model;  
 cf. also Chavanis

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ )

Basilakos, NEM,  
Solà (2019-20)

$$\begin{aligned} S_B^{\text{eff}} &= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} + \dots \right] \\ &= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \partial_\mu b(x) \mathcal{K}^\mu + \dots \right], \end{aligned}$$

**Primordial Gravitational Waves  
Potential Origins in pre-inflationary era?**

NEM, Solà  
EPJ-ST  
(2020)

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ , $\psi_\mu$ )

Basilakos, NEM,  
Solà (2019-20)

$$\begin{aligned} S_B^{\text{eff}} &= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} + \dots \right] \\ &= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \partial_\mu b(x) \mathcal{K}^\mu + \dots \right], \end{aligned}$$

## Primordial Gravitational Waves Potential Origins in pre-inflationary era?

Collapse/collisions of Domain walls formed in theories with (approximate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino  $\psi_\mu$  or gaugino)

NEM,Solà  
EPJ-ST  
(2020)

## Other Potential origins of

$$\langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle$$

Populations of rotating primordial black holes (pBH), which are sourced by axions, and also GW generated by merging of such pBH

See: Chatzifotis, Dorlis  
Talks 5 April 2023

Hence, we have several sources of primordial GW

NB: Stringy RVM may lead to Enhanced production of pBH during RVM inflation

NEM, Spanos, Stamou  
PRD106 (2022), 063532

# **5. Spontaneous Lorentz & CPT Violation by axion backgrounds and Inflation without inflatons**

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ )

Basilakos, NEM,  
Solà (2019-20)

Non-trivial if  
GW present

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} + \dots \right]$$

$$= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \partial_\mu b(x) \mathcal{K}^\mu + \dots \right],$$

**Primordial Gravitational Waves,  
&**

**De Sitter space times &  
Spontaneous Lorentz & CPT Violation**

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ )

Basilakos, NEM,  
Solà (2019-20)

Gravitational  
Chern-Simons (gCS)

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} + \dots \right]$$

$$= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \partial_\mu b(x) \mathcal{K}^\mu + \dots \right],$$

**Primordial Gravitational Waves →**  
**Condensate < ... >** of Gravitational Anomalies

$$g\mathcal{CS} = \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \int d^4x \sqrt{-g} \left( \langle b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle + :b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma}: \right)$$

quantum ordered

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ )

Basilakos, NEM,  
Solà (2019-20)

$$\begin{aligned}
 S_B^{\text{eff}} &= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} + \dots \right] \\
 &= \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \partial_\mu b(x) \mathcal{K}^\mu + \dots \right], \\
 &\quad + \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \int d^4x \sqrt{-g} \langle b(x) R_{\mu\mu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle
 \end{aligned}$$

Gravitational  
Chern-Simons (gCS)

**Condensate**  $\langle \dots \rangle$  of  
Gravitational Anomalies

Cosmological-  
Constant-like

$$g\mathcal{CS} = \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \int d^4x \sqrt{-g} \left( \langle b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle + :b(x) R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma}: \right)$$

quantum ordered

Effective action contains **CP violating axion-like coupling**

$$\sqrt{-g} \mathcal{K}^\mu(\omega)_{;\mu}$$



$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \frac{\sqrt{2} \alpha'}{96 \kappa \sqrt{3}} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \right)' + \dots \right]$$

- (i) Assume de Sitter era, first, to discuss anomaly condensate in the presence of GW perturbation
- (ii) deduce Running Vacuum Model (RVM) vacuum behaviour and
- (iii) Inflation is obtained self consistently from RVM evolution

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu \left( \sqrt{-g} \mathcal{K}^\mu(\omega) \right)$$

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \frac{\sqrt{2}\alpha'}{96\kappa\sqrt{3}} b(x) \left( R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \right) + \dots \right]$$

$$ds^2 = dt^2 - a^2(t) \left[ (1 - h_+(t, z)) dx^2 + (1 + h_+(t, z)) dy^2 + 2h_\times(t, z) dx dy + dz^2 \right]$$

Average over inflationary space time in the presence of  
**primordial Gravitational waves**

$$n_\star \equiv \frac{N(t)}{\sqrt{-g}} \quad \text{Proper density of sources}$$

$$b(x)=b(t)$$

Alexander, Peskin,  
Sheikh -Jabbari

$\mu$  = UV k-momentum Cut-off

$$\frac{d}{dt} \left( \sqrt{-g} \mathcal{K}^0(t) \right) = \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle = \frac{16}{a^4} \kappa^2 n^* \int^\mu \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3} k^4 \Theta + O(\Theta^3)$$

Homogeneity & Isotropy

$$\Theta = \sqrt{\frac{2}{3}} \frac{\kappa^3}{12} H \dot{b} \ll 1$$

**$H \approx \text{const.}$   
(inflation)**

$$\kappa = M_{\text{Pl}}^{-1},$$

$$\dot{b} \equiv db/dt$$

$$a(t) \sim e^{Ht}$$

## Solutions (backgrounds) to the Eqs of Motion

$$\alpha' = M_s^{-2}$$

$$\partial_\alpha \left[ \sqrt{-g} \left( \partial^\alpha \bar{b} - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \mathcal{K}^\alpha(t) \right) \right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \mathcal{K}^0$$

$n_\star \equiv \frac{N(t)}{\sqrt{-g}}$  Proper density of sources



$$\frac{d}{dt} \left( \sqrt{-g} \mathcal{K}^0(t) \right) = \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle = \frac{16}{a^4} \kappa^2 n^* \int \frac{\mu}{(2\pi)^3} \frac{d^3 k}{2k^3} H^2 k^4 \Theta + O(\Theta^3)$$

$$\Theta = \sqrt{\frac{2}{3}} \frac{\kappa^3}{12} H \dot{b} \propto \mathcal{K}^0$$

time evolution of Anomaly

$\mu$  = UV k-momentum Cut-off

$$\mathcal{K}^0(t) \simeq \mathcal{K}_{\text{begin}}^0(0) \exp \left[ -3Ht \left( 1 - 3 \times 10^{-4} n_\star \left( \frac{H}{M_{\text{Pl}}} \right)^2 \left( \frac{\mu}{M_s} \right)^4 \right) \right]$$

$$n_\star^{1/4} \frac{\mu}{M_s} \sim 7.6 \times \left( \frac{M_{\text{Pl}}}{H} \right)^{1/2} \quad \rightarrow \quad \mathcal{K}^0 = \text{const.} \quad \mu / M_s = 1$$

Planck Data

$$H/M_{\text{Pl}} < 10^{-4}$$



$$n_\star \gtrsim 3.3 \times 10^{13}$$

## Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha \left[ \sqrt{-g} \left( \partial^\alpha \bar{b} - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \mathcal{K}^\alpha(t) \right) \right] = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}} = \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \mathcal{K}^0 \sim \text{constant}}$$

↓

$$\dot{\bar{b}} \propto \epsilon^{ijk} H_{ijk} = \text{constant}$$

$$\frac{d}{dt} \left( \sqrt{-g} \mathcal{K}^0(t) \right) = \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle = \frac{16}{a^4} \kappa^2 n^* \int \frac{\mu}{(2\pi)^3} \frac{d^3 k}{2k^3} \frac{H^2}{k^4} \Theta + O(\Theta^3)$$

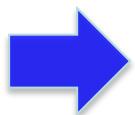
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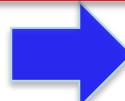


$$\mathcal{K}^0 = \text{const.}$$

**Spontaneous LV (+ CPTV) solution** !

**Planck Data**

$$H/M_{\text{Pl}} < 10^{-4}$$



$$n_\star \gtrsim 3.3 \times 10^{13}$$

$$\mu / M_s = 1$$

## Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha \left[ \sqrt{-g} \left( \partial^\alpha \bar{b} - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \mathcal{K}^\alpha(t) \right) \right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \mathcal{K}^0 \sim \text{constant}$$

Using **slow-roll assumption**  $b$

$$\varepsilon = \frac{1}{2} \frac{1}{(H M_{\text{Pl}})^2} \dot{\bar{b}}^2 \sim 10^{-2} \quad \text{Planck Data}$$



$$\dot{\bar{b}} \sim \sqrt{2\varepsilon} M_{\text{Pl}} H \sim 0.14 M_{\text{Pl}} H$$

$$H = H_{\text{infl}} \simeq \text{const.}$$

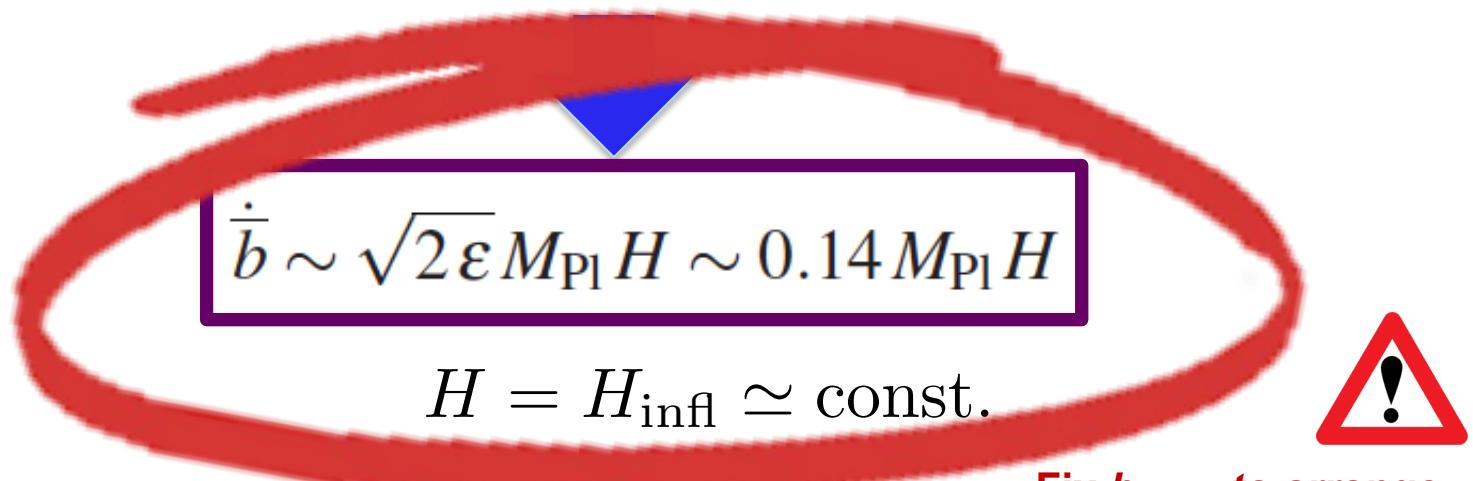
$\approx$  constant torsion

## Solutions (backgrounds) to the Eqs of Motion

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Using **slow-roll assumption**  $b$

$$\varepsilon = \frac{1}{2} \frac{1}{(H M_{\text{Pl}})^2} \dot{\bar{b}}^2 \sim 10^{-2} \quad \text{Planck Data}$$



@ end of  
Inflationary  
era

$$b_{\text{end}} \sim b_{\text{initial}} + 0.14 M_{\text{Pl}} H_{\text{infl}} t_{\text{end}},$$

$$t_{\text{end}} H_{\text{infl}} \sim \mathcal{N} = e - \text{foldings}$$

$\sim 55\text{-}70$

Fix  $b_{\text{initial}}$  to arrange  
approx. constant  
condensate  
during appropriate  
time period (**inflation**)

## Gravitational Anomaly Condensates $\rightarrow$ Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x) R_{\mu\mu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7 \epsilon \mathcal{N} H^4 > 0$$

*e-foldings*

Positive  
Cosmological  
Constant-like

Positive total energy density since  $\Lambda$ -term dominates

$$\rho_{\text{total}} = \rho_b + \rho_g c_s + \rho_\Lambda \simeq 3M_{\text{Pl}}^4 \left[ -1.7 \times 10^{-3} \left( \frac{H}{M_{\text{Pl}}} \right)^2 + (1.17 - 1.37) \times 10^7 \left( \frac{H}{M_{\text{Pl}}} \right)^4 \right] > 0$$

## Gravitational Anomaly Condensates → Dynamical Inflation

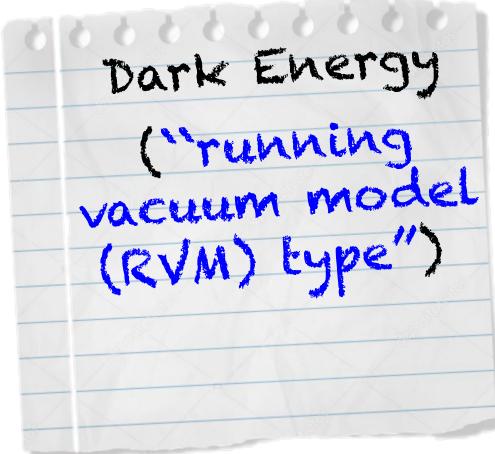
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## Gravitational Anomaly Condensates $\rightarrow$ Dynamical Inflation

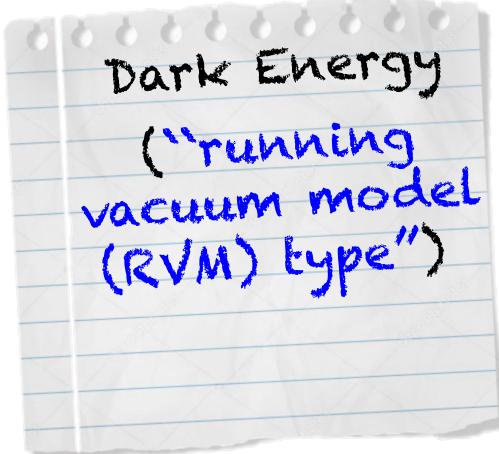
NEM, Sola

$$\Lambda \equiv \langle b(x) R_{\mu\mu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7 \epsilon \mathcal{N} H^4 > 0$$

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Positive total energy density since  $\Lambda$ -term dominates

$$\rho_{\text{total}} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\text{Pl}}^4 \left[ -1.7 \times 10^{-3} \left( \frac{H}{M_{\text{Pl}}} \right)^2 + (1.17 - 1.37) \times 10^7 \left( \frac{H}{M_{\text{Pl}}} \right)^4 \right] > 0$$



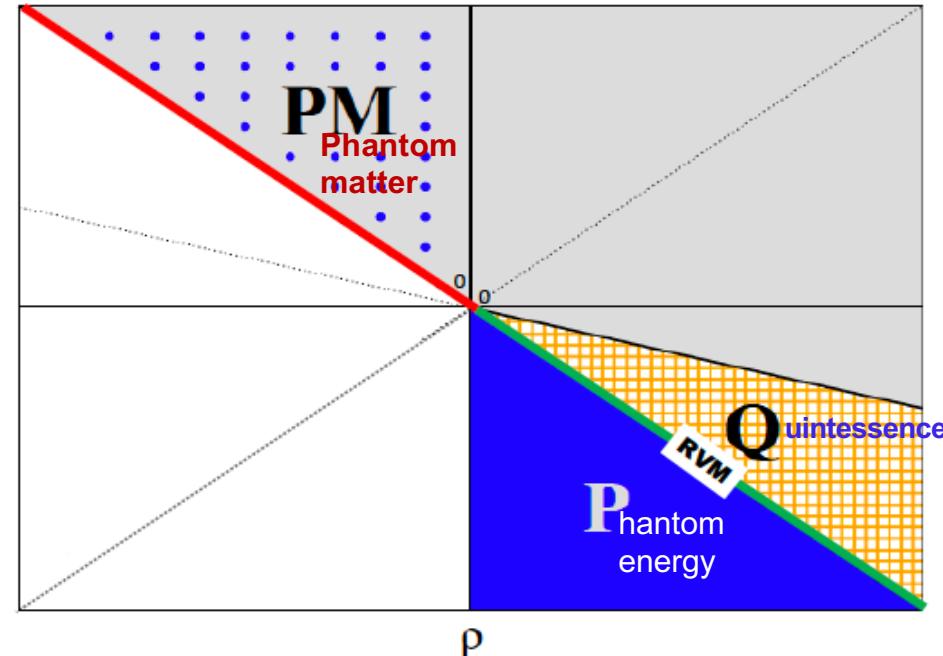
Equation of state :

$$0 > \rho_b + \rho_{gCS} = -(\rho_b + \rho_{gCS}) \text{ cf. phantom "matter"}$$

$$0 < \rho_\Lambda = -p_\Lambda \rightarrow \text{dominates} \rightarrow$$

$$0 < \rho_b + \rho_{gCS} + \rho_\Lambda = -(\rho_b + \rho_{gCS} + p_\Lambda) \text{ true RVM vacuum}$$

## Gravitational Anomaly Condensates → Dynamical Inflation

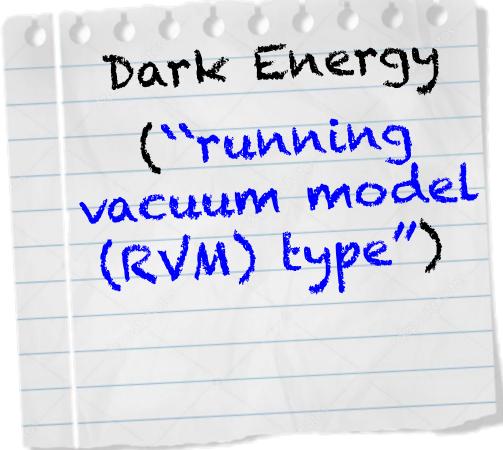


NEM, Sola

$$10^7 \epsilon \mathcal{N} H^4 > 0$$

Positive  
Cosmological  
Constant-like

$$\left[ -\frac{1}{1} \right]^2 + \left( 1.17 - 1.37 \right) \times 10^7 \left( \frac{H}{M_{Pl}} \right)^4 > 0$$



Equation of state :

$$0 > \rho_b + \rho_{gcs} = - (p_b + p_{gcs}) \text{ cf. phantom "matter"}$$

$$0 < \rho_\Lambda = -p_\Lambda \rightarrow \text{dominates} \rightarrow$$

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## Gravitational Anomaly Condensates → Dynamical Inflation

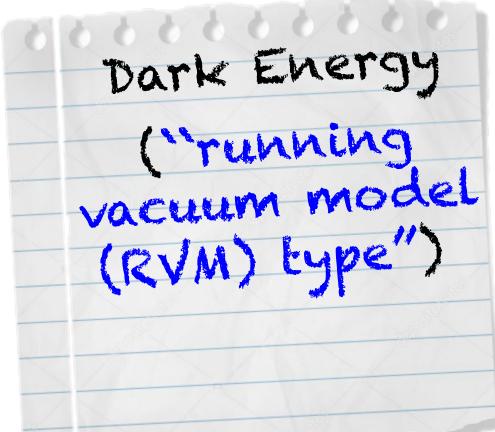
Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x) R_{\mu\mu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7 \epsilon \mathcal{N} H^4 > 0$$

Positive  
Cosmological  
Constant-like

Positive total energy density since  $\Lambda$ -term dominates

$$\rho_{\text{total}} = \rho_b + \rho_g c_s + \rho_\Lambda \simeq 3M_{\text{Pl}}^4 \left[ -1.7 \times 10^{-3} \left( \frac{H}{M_{\text{Pl}}} \right)^2 + (1.17 - 1.37) \times 10^7 \left( \frac{H}{M_{\text{Pl}}} \right)^4 \right] > 0$$



RVM-like terms  
drive inflation  
contain scalar d.o.f.  
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But slow roll is due to the KR axion field  $\epsilon \sim \frac{1}{2} \frac{1}{(HM_{\text{Pl}})^2} \dot{b}^2 \sim 10^{-2}$



# Running Vacuum Model (RVM)

Shapiro + Solà  
Solà, ...

Dark Energy  
("running  
vacuum model  
(RVM) type")

$$\rho_{\Lambda}^{\text{RVM}} = \kappa^{-2} \Lambda + c_1 H^2 + c_2 H^4 + \dots$$

$$\equiv \kappa^{-2} \Lambda(t)$$

$$\Lambda \equiv 3 c_0 \quad c_1 = 3\nu\kappa^{-2}, \quad c_2 = 3\alpha\kappa^{-2} H_I^{-2},$$

$$H_I \sim 10^{-5} \kappa^{-1} \text{ (current pheno)}$$

Vacuum energy density assumed de Sitter like but with time-dependent Cosmological parameter  $\Lambda(t)$  :

$$\rho_{\text{RVM}}^{\Lambda}(t) = \Lambda(t)/\kappa^2 \quad \kappa = \sqrt{8\pi G} = M_{\text{Pl}}^{-1}$$

$$p(t)_{\text{RVM}} = -\rho_{\text{RVM}}^{\Lambda(t)}(t)$$

**Renormalization-Group-like** equation for the evolution of **vacuum energy density**  
**Hubble parameter  $H(t) \leftrightarrow$  RG scale  $\mu$**

$$\frac{d\rho_{\Lambda}^{\text{RVM}}(t)}{d\ln H^2} = \frac{1}{(4\pi)^2} \sum_{i=F,B} \left[ a_i M_i^2 H^2 + b_i H^4 + \mathcal{O}\left(\frac{H^6}{M_i^2}\right) \right]$$

general covariance →  
even powers of  $H$



## Cosmological Evolution of RVM

Basilakos, Lima,  
Sola + Gomez Valent  
+ ... (2013 - 2018 )

$$\omega = \rho_m/p_m \quad m = \text{matter, radiation}$$

$$\nabla^\mu T_{\mu\nu} = 0 \quad \rightarrow \quad \dot{\rho}_m + 3(1 + \omega)H\rho_m = -\dot{\rho}_{\text{RVM}}^{\Lambda}$$

$$\boxed{\dot{H} + \frac{3}{2}(1 + \omega)H^2 \left( 1 - \nu - \frac{c_0}{H^2} - \alpha \frac{H^2}{H_I^2} \right) = 0}$$

**Solution**

$$H(a) = \left( \frac{1 - \nu}{\alpha} \right)^{1/2} \frac{H_I}{\sqrt{D a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$$D > 0$$

**Early de Sitter  
(unstable)**

$$D a^{4(1-\nu)} \ll 1 \quad \rightarrow \quad H^2 = (1 - \nu)H_I^2/\alpha$$

**Radiation**

$$D a^{4(1-\nu)} \gg 1 \quad \rightarrow \quad H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4} \\ \omega = 1/3$$

**Late dark-Energy  
dominated era**

$$H^2(a) = H_0^2 \left[ \tilde{\Omega}_{m0} a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda0} \right] \quad \tilde{\Omega}_{\Lambda0} \text{ dominant}$$

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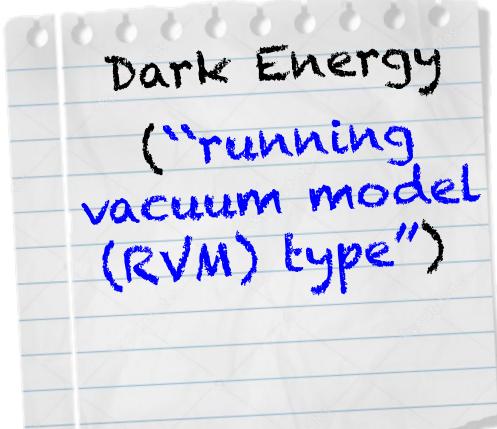
## Gravitational Anomaly Condensates → Dynamical Inflation

Cannot obtain such terms  
in ordinary Quantum Field Theories  
You need the **condensate of  
the gravitational anomalies**  
which have **CP-violating couplings**  
with the **gravitational axions**



NEM, Solà

$$\rho_{\text{total}} = \rho_b + \rho_g c_s + \rho_\Lambda \simeq 3M_{\text{Pl}}^4 \left[ -1.7 \times 10^{-3} \left( \frac{H}{M_{\text{Pl}}} \right)^2 + (1.17 - 1.37) \times 10^7 \left( \frac{H}{M_{\text{Pl}}} \right)^4 \right] > 0$$



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You need the condensate of  
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Another important  
role of CP-violation  
in Early Universe

$$\rho_{\text{total}} = \rho_b + \rho_g c_s + \rho_\Lambda \simeq 3M_{\text{Pl}}^4 \left[ -1.7 \times 10^{-3} \left( \frac{H}{M_{\text{Pl}}} \right)^2 + (1.17 - 1.37) \times 10^7 \left( \frac{H}{M_{\text{Pl}}} \right)^4 \right] > 0$$

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Positive  
Cosmological  
Constant-like

Positive total energy density since  $\Lambda$ -term dominates

$$\rho_{\text{total}} = \rho_b + \rho_g CS + \rho_\Lambda \simeq 3M_{\text{Pl}}^4 \left[ -1.7 \times 10^{-3} \left( \frac{H}{M_{\text{Pl}}} \right)^2 + (1.17 - 1.37) \times 10^7 \left( \frac{H}{M_{\text{Pl}}} \right)^4 \right] > 0$$



Negative coefficient  $v < 0$   
due to CS anomaly  
in early Universe, unlike  
late-era RVM

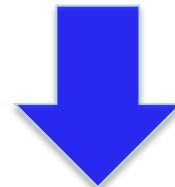
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## Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha \left[ \sqrt{-g} \left( \partial^\alpha \bar{b} - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \mathcal{K}^\alpha(t) \right) \right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \mathcal{K}^0 \sim \text{constant}$$

Undiluted KR axion background  
at the end of Inflation



@ end of  
Inflationary  
era

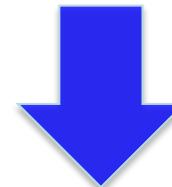
$$\dot{\bar{b}} \sim \sqrt{2\varepsilon} M_{\text{Pl}} H \sim 0.14 M_{\text{Pl}} H$$

$$H = H_{\text{infl}} \simeq \text{const.}$$

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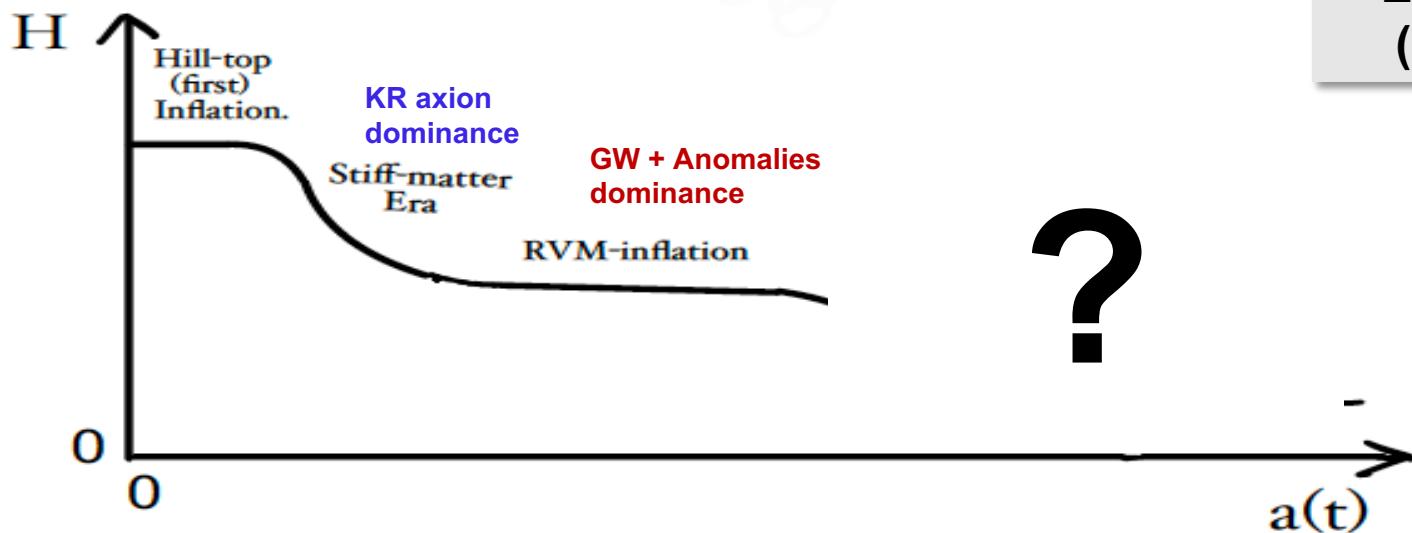
$$H = H_{\text{infl}} \simeq \text{const.}$$

Important for Leptogenesis @ radiation era



# **6. Post Inflationary Eras & Cosmic Evolution of the **stringy RVM****

# Post-RVM-Inflation Eras & Evolution



NEM,Sola  
EPJ-ST  
(2020)

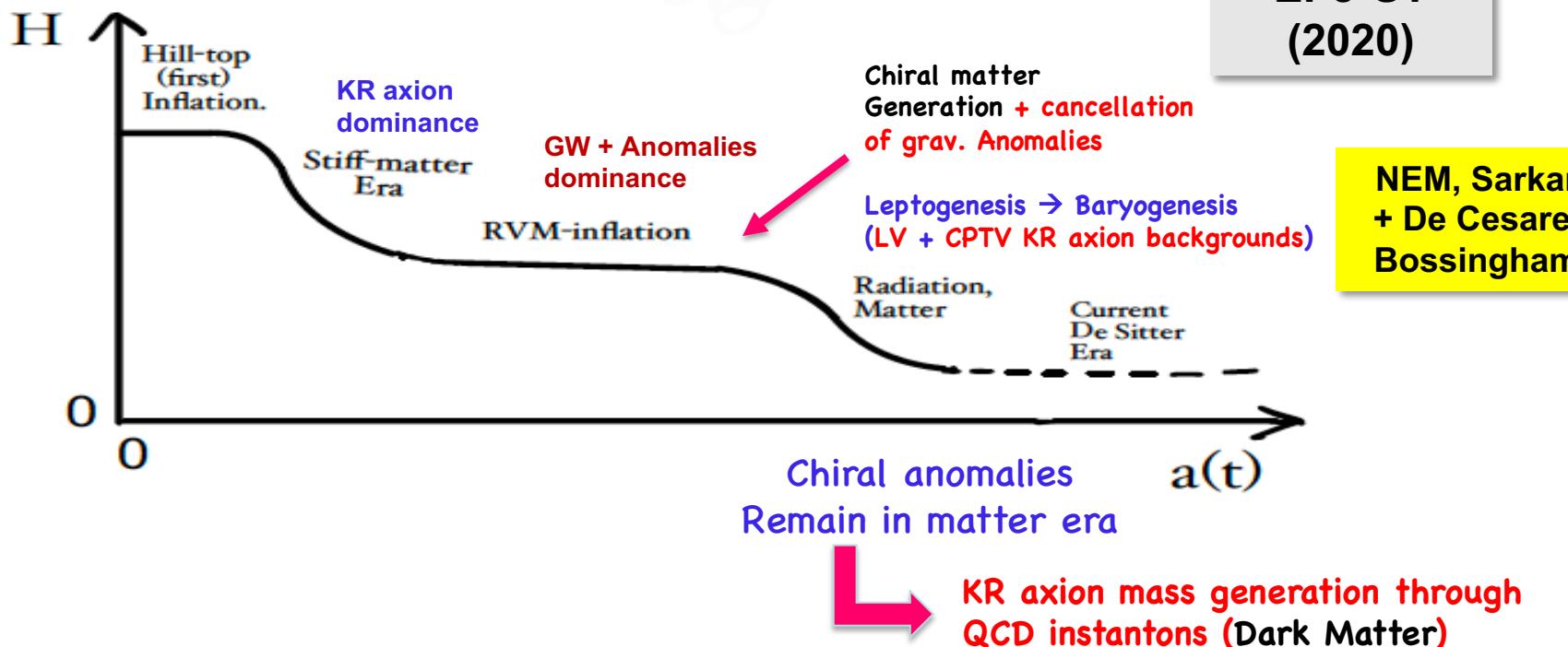
# Cancellation of Gravitational Anomalies in Radiation Era

by:

## Chiral Fermionic Matter generation @ end of Inflation

Required by consistency of quantum theory  
of matter and radiation (**diffeomorphism invariance**)

Basilakos, NEM,Solà (2019-20)



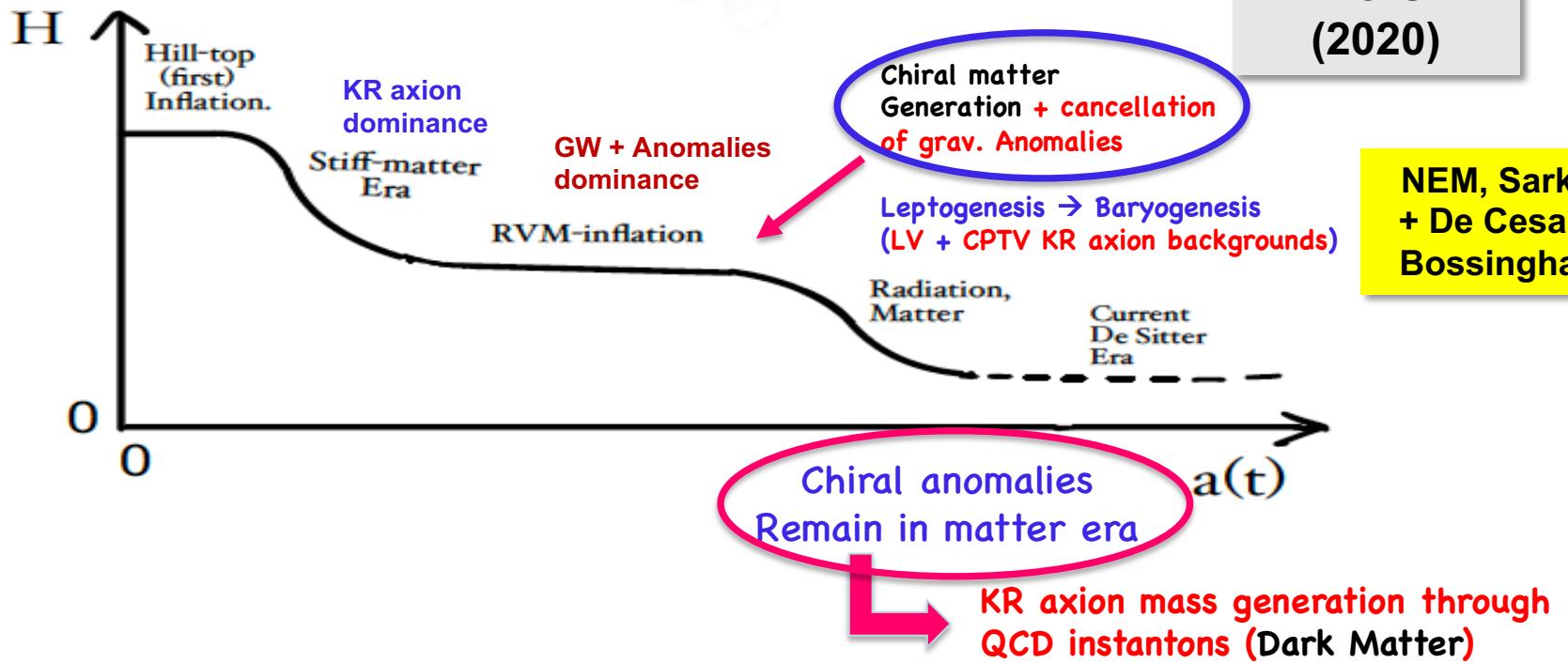
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# Summary of (stringy-RVM) Cosmological Evolution

Cosmic

Time

**Big-Bang, pre-inflationary phase (broken Sugra)**

Basilakos, NEM, Solà

## RVM Inflationary (de Sitter) Phase

Primordial  
Gravitational  
Waves



Gravitational  
anomaly (GA)



**From a pre-inflationary  
era after Big-Bang**

### Radiation Era

$$B_0 \propto T^3$$

**Leptogenesis induced by  
RHN (tree-level) decays**

$$N_I \rightarrow \bar{\phi} \ell, \phi \bar{\ell}$$

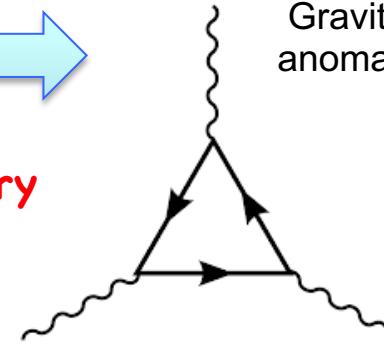
$\Delta L$  In the (approx.) constant LV + CPTV background

$$B_\mu = M_{\text{Pl}}^{-1} \dot{\bar{b}} \delta_{\mu 0}$$

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon} M_{\text{Pl}} H \sim 0.14 M_{\text{Pl}} H$$

chiral matter  
generation  
@ inflation exit

NEM, Sarkar  
+ De Cesare,  
Bossingham



Cancellation of GA

**B-L conserving sphaleron processes → Baryogenesis**

### Matter Era

Possible potential (mass) generation for  $b \rightarrow$  axion Dark matter

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Gravitational  
anomaly (GA)

Undiluted constant  
KR axial background

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chiral matter  
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## Post-RVM-Inflationary Era

Cancellation of  
Gravitational Anomalies

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Required by consistency of quantum theory  
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Basilakos, NEM,Solà (2019-20)

$$S^{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b + \frac{\alpha'}{\kappa} b(x) \nabla_\mu \left( \sqrt{\frac{2}{3}} \frac{1}{96} \mathcal{K}^\mu - \sqrt{\frac{3}{8}} J^{5\mu} \right) \right] + \dots,$$

$$J^{5\mu} = \sum_j \bar{\psi}_j \gamma^\mu \gamma^5 \psi_j; \quad \text{Chiral current, including RHN}$$

# Cancellation of Gravitational Anomalies in Radiation Era

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Basilakos, NEM,Solà (2019-20)

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$J^{5\mu} = \sum_j \bar{\psi}_j \gamma^\mu \gamma^5 \psi_j$       **Chiral current, including RHN**

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chiral U(1)

Gluon QCD

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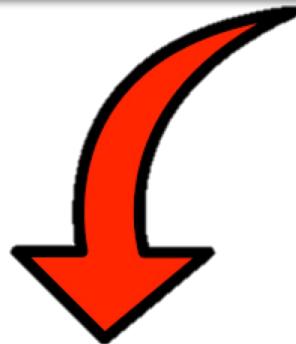
instanton generated potential for KR axion b-field  
during matter dominance  $\rightarrow$  axion Dark Matter

# Cancellation of Gravitational Anomalies in Radiation Era by:

## Chiral Fermionic Matter generation @ end of Inflation

Required by consistency of quantum theory  
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Basilakos, NEM,Solà (2019-20)



Scale factor  $a(t) \sim T^1$

Possibly also QCD

$$\dot{\bar{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}$$

sufficiently slowly varying during Leptogenesis  
**(brief) epoch**  $\rightarrow$  qualitatively similar to  
approximately const. background

Bossingham, NEM,  
Sarkar

Lorentz- & CPT-Violating

Leptogenesis →

→ Baryogenesis

in models with Massive  
Right-handed Neutrinos

## Models with Right-handed Majorana Neutrinos $N_I$ , $I=1,2,\dots$

$$L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}$$

Light Neutrino Masses through see saw

$$m_\nu = -M^D \frac{1}{M_I} [M^D]^T .$$

$$M_D = F_{\alpha I} v$$

$$v = \langle \phi \rangle \sim 175 \text{ GeV} \quad M_D \ll M_I$$



# Models with Right-handed Majorana Neutrinos $N_I$ , $I=1,2,\dots$



mass of lightest of  $N_I$ ,  
say  $M_1 = m$   
by agreement  
with Cosmological data

$$L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}$$

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Add interaction with  
approximately  
constant  
axial background  
 $B_\mu$  (e.g. generated  
by torsion)

+

$$\mathcal{L}_{\text{int}} = -\bar{N}_I B_\mu \gamma^\mu \gamma^5 N_I$$

**Isotropy  $\neq$  Homogeneity:**  $B_0 = \text{non trivial}$ ,  $B_i = 0$ ,  $i=1,2,3$

In our KR-torsion-induced axion background

$$B_\mu = M_{\text{Pl}}^{-1} \dot{\bar{b}} \delta_{\mu 0}$$

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon} M_{\text{Pl}} H \sim 0.14 M_{\text{Pl}} H$$

# CPT Violation



de Cesare, NEM, Sarkar  
Eur.Phys.J. C75, 514 (2015)

Early Universe  
 $T \gg T_{EW}$

$$\mathcal{L} = i\bar{N}\not{\partial}N - \frac{m}{2}(\bar{N^c}N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5N - Y_k\bar{L}_k\tilde{\phi}N + h.c.$$

Heavy Right-Handed-Neutrino ( $N$ ) interact with **axial (approx.) constant background** with only temporal component  $B_0 \propto \dot{b} \neq 0$

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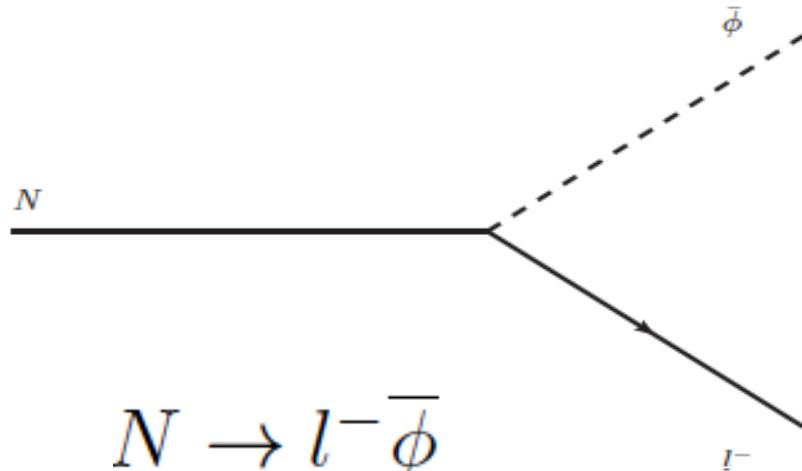
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Heavy Right-Handed-Neutrino ( $N$ ) interact with **axial (approx.) constant background** with only temporal component  $B_0 \propto \dot{b} \neq 0$

**Produce Lepton asymmetry**

Lepton number & CP Violations  
@ tree-level due to  
Lorentz/CPTV Background

$$N \rightarrow l^+ \phi$$



$$N \rightarrow l^- \bar{\phi}$$

$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2} \frac{m^2}{\Omega} \frac{\Omega + B_0}{\Omega - B_0} \quad \neq \quad \Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2} \frac{m^2}{\Omega} \frac{\Omega - B_0}{\Omega + B_0}$$

$B_0 \neq 0$

CPV &  
LV

$$\Omega = \sqrt{B_0^2 + m^2}$$

$$\mathcal{L} = i\bar{N}\not{\partial}N - \frac{m}{2}(\bar{N}^c N + m\bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \not{\phi}N + h.c.$$

Early Universe  
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# CPT Violation

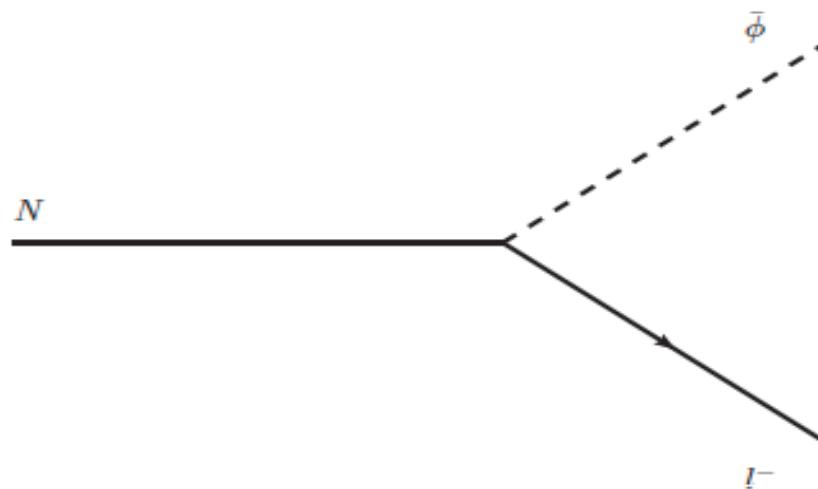
(approx.) Constant  $B_0$  Background



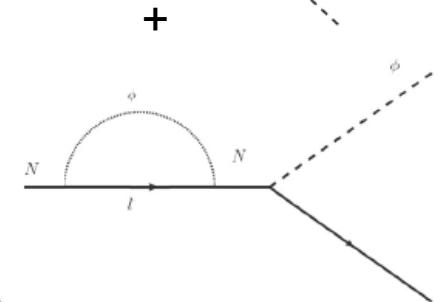
Lepton number & CP Violations @ tree-level  
due to Lorentz/CPTV Background

$$N_I \rightarrow \bar{\phi}\ell, \phi\bar{\ell}$$

Produce Lepton asymmetry



Contrast with one-loop conventional  
CPV Leptogenesis  
(in absence of H-torsion)



Fukugita, Yanagida,

# CPTV Thermal

$$\mathcal{L} = i\bar{N}\partial N - \frac{m}{2}(\bar{N^c}N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \not{\phi} N + h.c.$$

Early Universe  
T > 10<sup>5</sup> GeV

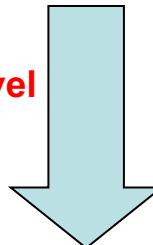
## CPT Violation



(approx.) Constant B<sup>0</sup> ≠ 0 background

Lepton number & CP Violations @ tree-level  
due to Lorentz/CPTV Background

$$N_I \rightarrow \bar{\phi}\ell, \phi\bar{\ell}$$



$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$



$$\frac{B_0}{m} \simeq 10^{-8}$$

*Produce Lepton asymmetry*

Solving  
system  
of Boltzmann  
eqs

$$\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7e(2\pi)^{3/2}} \frac{B_0}{m} \simeq 0.007 \frac{B_0}{m}$$

$$Y_k \sim 10^{-5}$$

$$m \geq 100 \text{TeV} \rightarrow$$

$$B^0 \sim 1 \text{MeV}$$

$$T_D \simeq m \sim 100 \text{ TeV}$$

Similar order of magnitude estimates  
if B<sup>0</sup> ~ T<sup>3</sup> during Leptogenesis era

Bossingham, NEM,  
Sarkar

# CPTV Thermal

$$\mathcal{L} = i\bar{N}\partial N - \frac{m}{2}(\bar{N^c}N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \not{\phi} N + h.c.$$

Early Universe  
T > 10<sup>5</sup> GeV

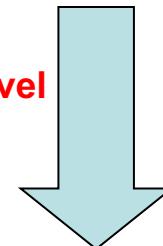
## CPT Violation



(approx.) Constant B<sup>0</sup> ≠ 0 background

Lepton number & CP Violations @ tree-level  
due to Lorentz/CPTV Background

$$N_I \rightarrow \bar{\phi} \ell, \phi \bar{\ell}$$



$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$



$$\frac{B_0}{m} \simeq 10^{-8}$$

*Produce Lepton asymmetry*

Equilibrated electroweak  
B+L violating sphaleron interactions

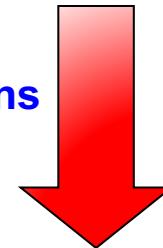
B-L conserved

$$L = \frac{2}{M} l_L l_L \phi \phi + \text{H.c.}$$

where

$$l_L = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L, \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix}_L, \begin{bmatrix} \nu_\tau \\ \tau \end{bmatrix}_L$$

*Environmental  
Conditions Dependent*



*Observed Baryon Asymmetry  
In the Universe (BAU)*

Fukugita, Yanagida,

$$\frac{n_B - \bar{n}_B}{n_B + \bar{n}_B} \sim \frac{n_B - \bar{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

# Summary of (stringy-RVM) Cosmological Evolution

Cosmic

Time

**Big-Bang, pre-inflationary phase (broken Sugra)**

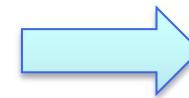
Basilakos, NEM, Solà

## RVM Inflationary (de Sitter) Phase

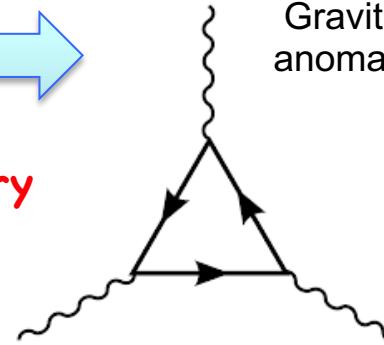
Primordial  
Gravitational  
Waves



Gravitational  
anomaly (GA)



**From a pre-inflationary  
era after Big-Bang**



**Undiluted constant  
KR axial background**

$$B_\mu = M_{\text{Pl}}^{-1} \dot{\bar{b}} \delta_{\mu 0}$$

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon} M_{\text{Pl}} H \sim 0.14 M_{\text{Pl}} H$$

**chiral matter  
generation  
@ inflation exit**

## Radiation Era

$$B_0 \propto T^3$$

**Leptogenesis induced by  
RHN (tree-level) decays**

$$N_I \rightarrow \bar{\phi} \ell, \phi \bar{\ell} \quad \Delta L \text{ In the (approx.) constant LV + CPTV background} \quad B_\mu = M_{\text{Pl}}^{-1} \dot{\bar{b}} \delta_{\mu 0}$$

**B-L conserving sphaleron processes → Baryogenesis**

## Matter Era

Possible potential (mass) generation for  $b \rightarrow$  axion Dark matter

**Chiral anomalies @ QCD era (instantons)**

forward direction

# Summary of (stringy-RVM) Cosmological Evolution

Cosmic

Time

**Big-Bang, pre-inflationary phase (broken Sugra)**

Basilakos, NEM, Solà

## RVM Inflationary (de Sitter) Phase

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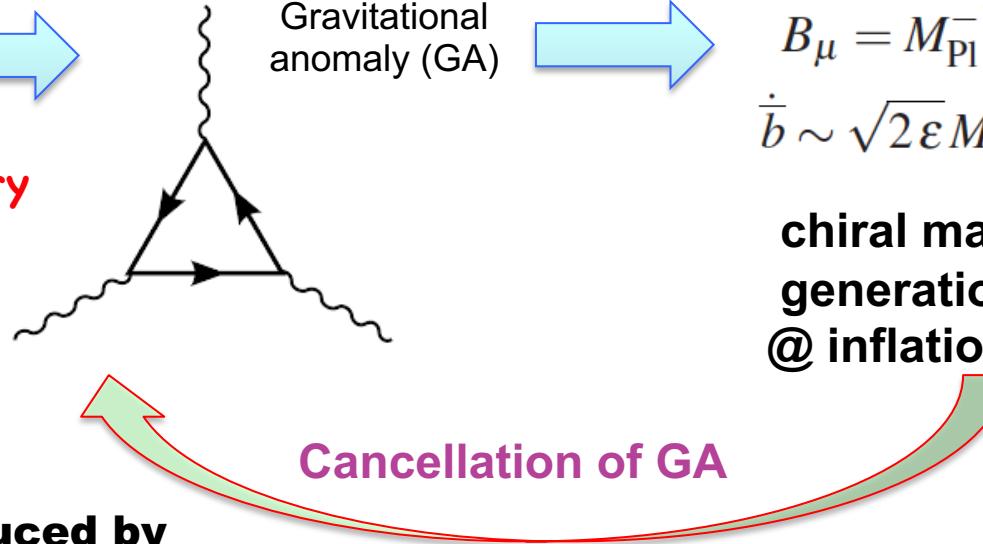
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**chiral matter  
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forward direction

# Summary of (stringy-RVM) Cosmological Evolution

Cosmic  
Time

Basilakos, NEM, Solà

**Collider bound**

$$10 \text{ TeV} = \mathcal{O}(10^{-14}) M_{\text{Pl}} < M_s \leq M_{\text{Pl}}$$

$$V_b^{\text{QCD}} \simeq \Lambda_{\text{QCD}}^4 \left( 1 - \cos\left(\frac{b}{f_b}\right) \right),$$

$$f_b = 96 \sqrt{\frac{3}{2}} \frac{M_s^2}{M_{\text{Pl}}}$$

$$\Lambda_{\text{QCD}} \sim 218 \text{ MeV}$$

**@ QCD  
Era**

T ~ 200 MeV

$$S_b^{\text{eff}} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \partial_\mu b \partial^\mu b - \frac{\alpha'}{\kappa} \sqrt{\frac{3}{8}} \frac{\alpha_s}{8\pi} b(x) G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \right]$$

**Remaining chiral anomalies**

Instanton-effects-induced  
KR-axion potential and mass  
due to QCD chiral anomaly

**Matter Era**

Possible potential (mass) generation for  $b \rightarrow$  axion Dark matter

$$2 \times 20^{-13} \text{ eV} < m_b = \frac{\Lambda_{\text{QCD}}^2}{f_b} < 2 \times 10^{15} \text{ eV}$$

# Summary of (stringy-RVM) Cosmological Evolution

Cosmic  
Time

Basilakos, NEM, Solà

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Cosmic  
Time

Basilakos, NEM, Solà

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**@ QCD  
Era**

T ~ 200 MeV

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*Remaining chiral anomalies*

**Matter Era**

Possible poten

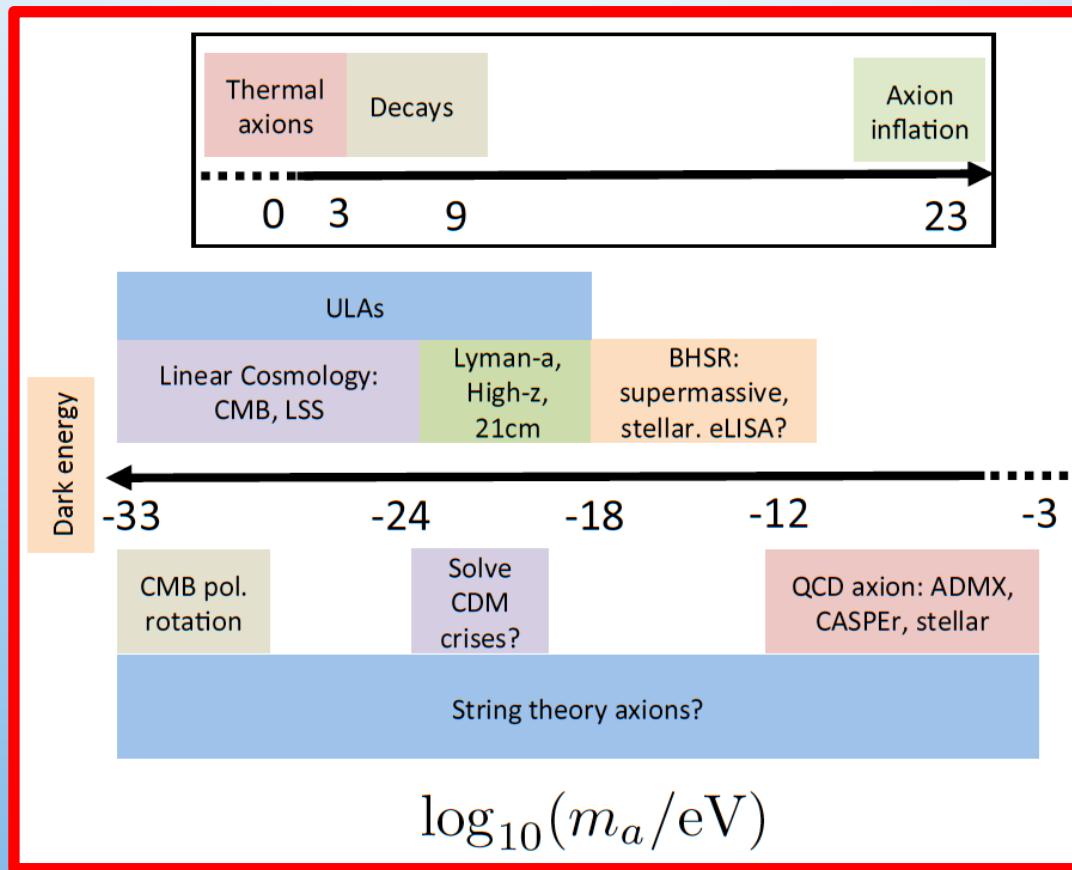
Mass upper bound restricted  
further (severely) by cosmological  
& other constraints

$$2 \times 20^{-13} \text{ eV} < m_b = \frac{\Lambda_{\text{QCD}}^2}{f_b} < 2 \times 10^{15} \text{ eV}$$

**D.J.E. Marsh,**  
Phys. Rept. 643, 1  
(2016)  
[arXiv:1510.07633  
[astro-ph.CO]].

# Axion Cosmology

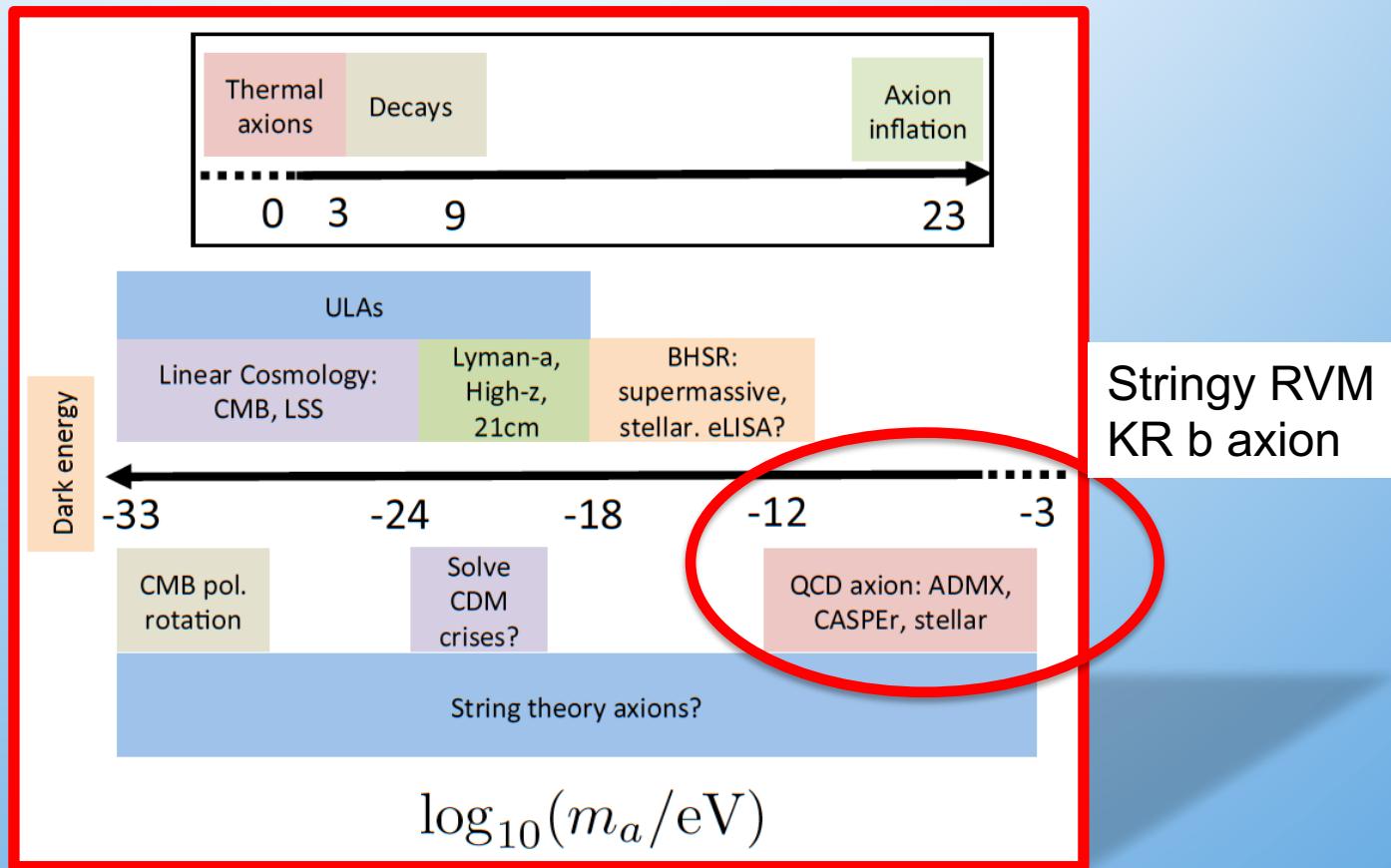
## Cosmological Constraints & probes of axions



**D.J.E. Marsh,**  
Phys. Rept. 643, 1  
(2016)  
[arXiv:1510.07633  
[astro-ph.CO]].

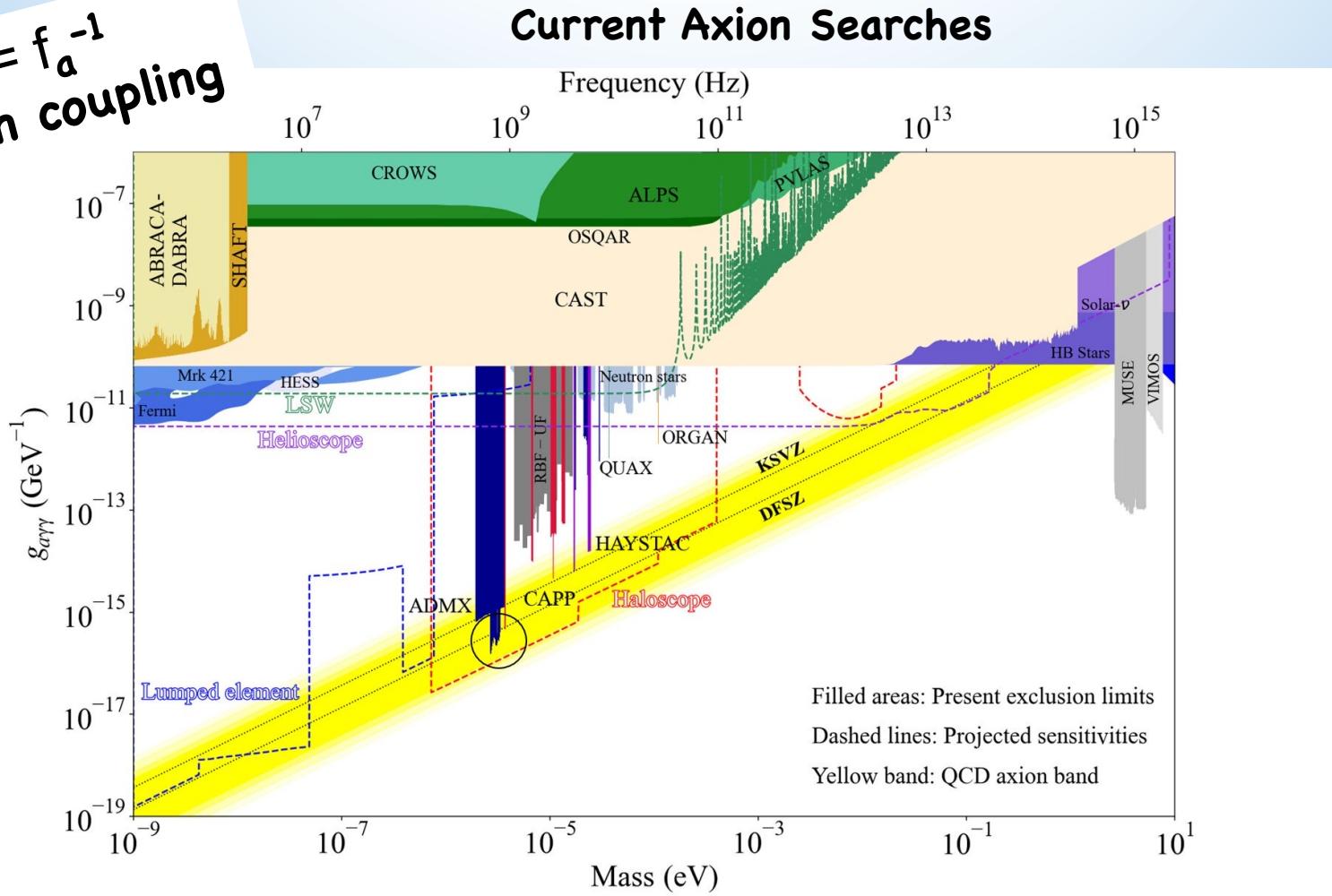
# Axion Cosmology

**Cosmological  
Constraints  
& probes of  
axions**



$$g_{a\gamma\gamma} = f_a^{-1}$$

axion coupling



These bounds are consistent with BBN Constraints on  $f_a$

$$\mathcal{L}_{\text{int}} = -\frac{g_{\phi\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{g_{\phi N}}{2m_N}\partial_\mu\phi(\bar{N}\gamma^\mu\gamma_5N) + \frac{g_{\phi e}}{2m_e}\partial_\mu\phi(\bar{e}\gamma^\mu\gamma_5e) - \frac{i}{2}g_d\phi\bar{N}\sigma_{\mu\nu}\gamma_5NF^{\mu\nu},$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

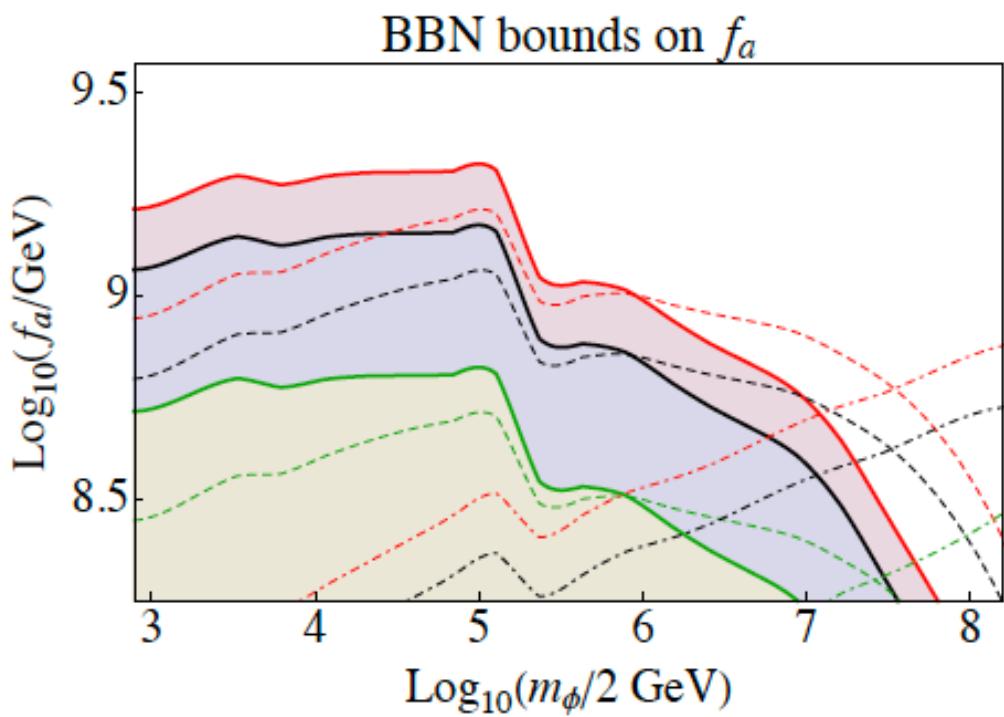
Nucleon  
or in general  
massive fermion  
 $\Psi$  of mass  $m_f$

$\varphi$  = axion, Use massive fermion equations of motion so as  
to obtain effective axion-fermion interactions :

$$\mathcal{L}_f = c_f m_f \phi \bar{\psi} \gamma^5 \psi / f_a$$

Implying production of heavy fermions  $f$ , via  $a + \gamma \rightarrow f + \bar{f}$   
which can alter the proton to neutron ratio during BBN

$$c_f = 1 \quad \mathcal{L}_f = c_f m_f \phi \bar{\psi} \gamma^5 \psi / f_a$$



$\Delta N_{\text{eff}} = 0.1, 0.5, 1$  (green, black, red)

J. P. Conlon and M. C. D. Marsh, JHEP10, 214 (2013), 1304.1804.

**BBN constraints rule out**  
 $f_a \leq 10^9 \text{ GeV}$  for a  
wide range of Masses  $m_\phi$

**For KR axion coupling**

$$f_b = 96 \sqrt{\frac{3}{2}} \frac{M_s^2}{M_{\text{Pl}}} < 10^9 \text{ GeV}$$

**Excludes**

$$m_b = \Lambda_{\text{QCD}}^2 / f_b > 4 \times 10^{-2} \text{ eV}$$

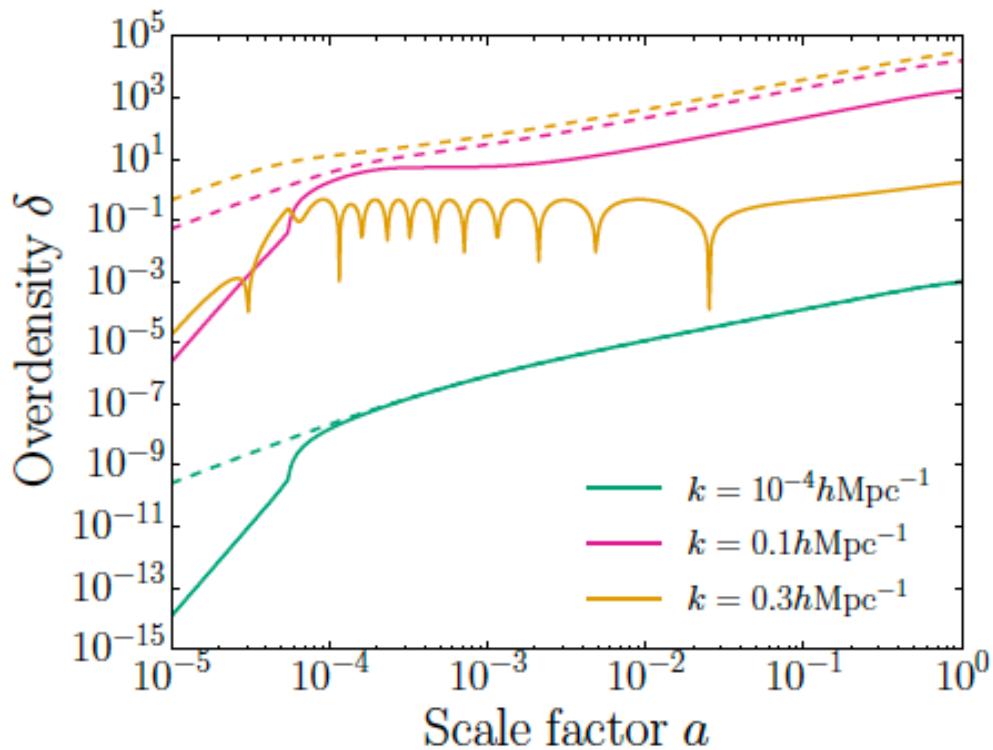
**D.J.E. Marsh,**  
Phys. Rept. 643, 1  
(2016)  
[arXiv:1510.07633  
[astro-ph.CO]].

**NB**

# Ultra Light Axion (ULA) DM (allowed in string theory)

Compactification actions, **NOT  $f_b$**  in stringy RVM

Contribution to galactic growth if dominant DM species



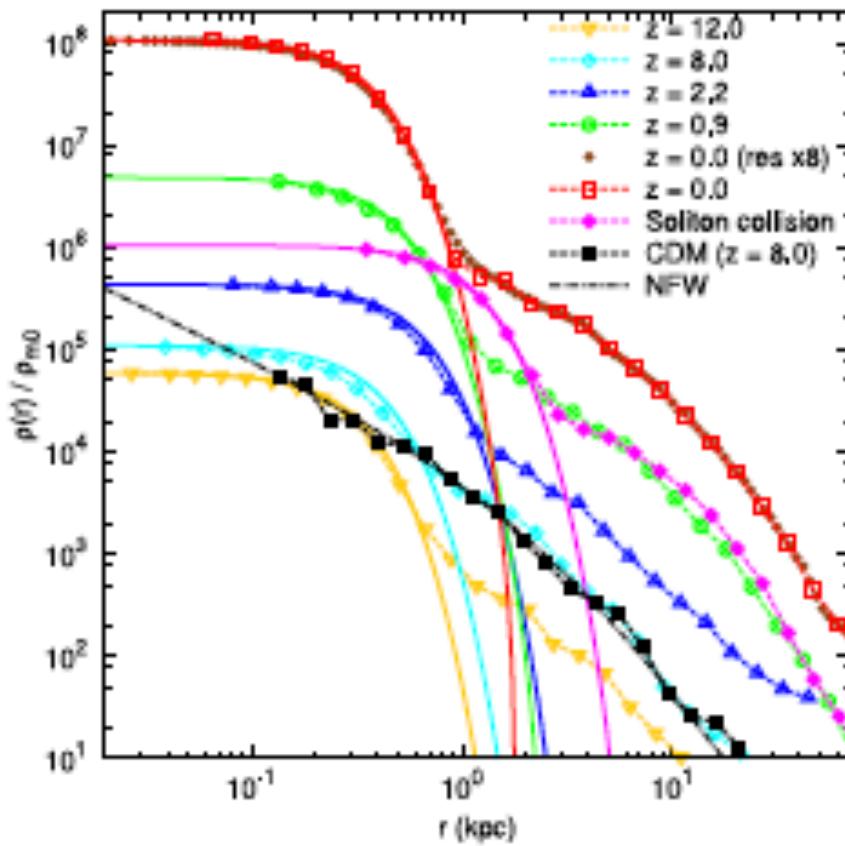
D.J.E. Marsh,  
Phys. Rept. 643, 1  
(2016)  
[arXiv:1510.07633  
[astro-ph.CO]].

$$m_a = 10^{-26} \text{ eV}$$

R. Hlozek, D. Grin, D. J. E. Marsh, and P. G. Ferreira, Phys. Rev. D91, 103512 (2015), 1410.2896.

**NB**Compactification actions, NOT  $f_b$  in stringy RVM

## Halo Density Profiles and ULA

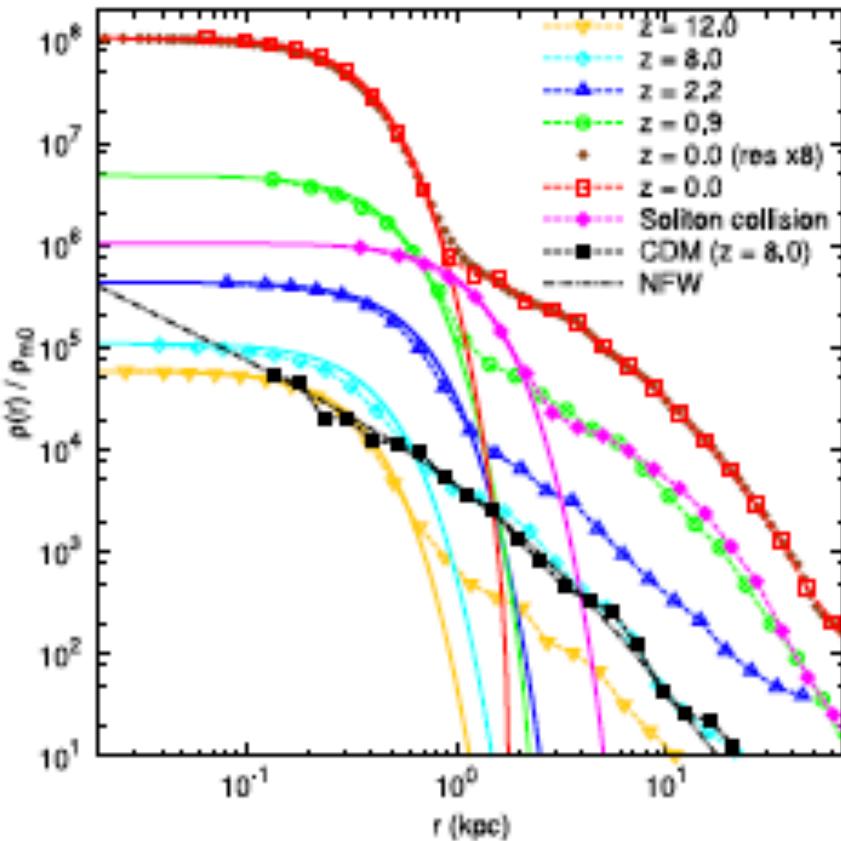


$$m_a = 8.1 \times 10^{-23} \text{ eV}$$

D.J.E. Marsh,  
Phys. Rept. 643, 1  
(2016)  
[arXiv:1510.07633  
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**NB**Compactification actions, NOT  $f_b$  in stringy RVM

## Halo Density Profiles and ULA



$$m_a = 8.1 \times 10^{-23} \text{ eV}$$

For examples of effective models in SUSY & strings with such axions see, e.g.  
Halverson, Long, Nath,  
**PRD96 (2017) 056025**

D.J.E. Marsh,  
Phys. Rept. 643, 1  
(2016)  
[arXiv:1510.07633  
[astro-ph.CO]].

# Summary of (stringy-RVM) Cosmological Evolution

Cosmic

Time

**Big-Bang, pre-inflationary phase (broken Sugra)**

Basilakos, NEM, Solà

## RVM Inflationary (de Sitter) Phase

Primordial  
Gravitational  
Waves



Gravitational  
anomaly (GA)



**From a pre-inflationary  
era after Big-Bang**

### Radiation Era

$$B_0 \propto T^3$$

**Leptogenesis induced by  
RHN (tree-level) decays**

$$N_I \rightarrow \bar{\phi} \ell, \phi \bar{\ell}$$

**B-L conserving sphaleron processes → Baryogenesis**

### Matter Era

Possible potential (mass) generation for  $b \rightarrow$  axion Dark matter

### Modern de-Sitter Era

**GA resurfacing**

$$\dot{b}_{\text{today}} \sim \sqrt{2\varepsilon'} M_{\text{Pl}} H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

**Phenomenology**

**Undiluted constant  
KR axial background**

$$B_\mu = M_{\text{Pl}}^{-1} \dot{\bar{b}} \delta_{\mu 0}$$

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon} M_{\text{Pl}} H \sim 0.14 M_{\text{Pl}} H$$

**chiral matter  
generation  
@ inflation exit**

**Cancellation of GA**



forward direction

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Cosmic

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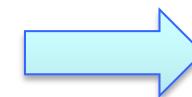
Basilakos, NEM, Solà

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**chiral matter  
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@ inflation exit**

**Consistent with current  
bounds on LV & CPTV**  
 $B_0 < 10^{-2} \text{ eV},$   
 $B_i < 10^{-22} \text{ eV}$

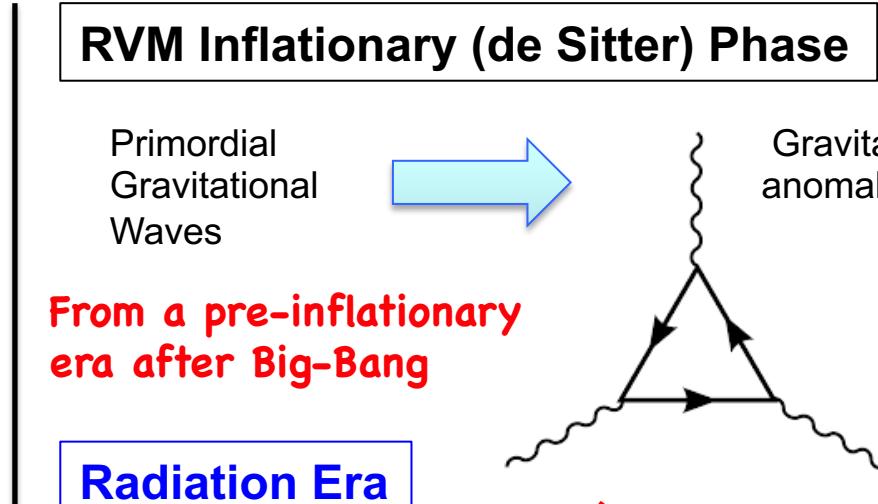
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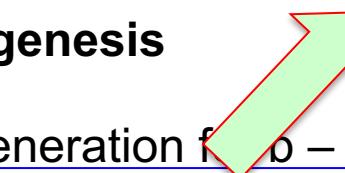
$$H_0 \sim 10^{-42} \text{ GeV} \approx 10^{-60} M_{\text{Pl}} \approx 10^{-33} \text{ eV}$$

**Phenomenology**

forward direction



**Cancellation of GA**



**Phenomenology**

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Basilakos, NEM, Solà

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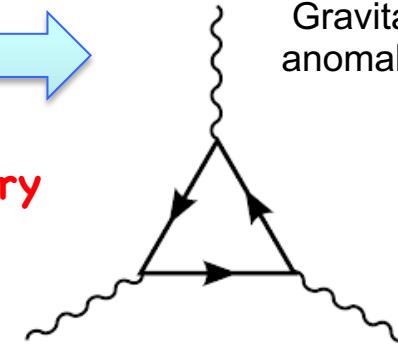
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**chiral matter  
generation  
@ inflation exit**

**Radiation Era**



$$B_0 \propto \dot{\bar{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}$$

$$B_0|_{\text{today}} \sim 2.435 \times 10^{-34} \text{ eV}$$

**Consistent with current  
bounds on LV & CPTV**  
 $B_0 < 10^{-2} \text{ eV},$   
 $B_i < 10^{-22} \text{ eV}$

**Matter Era**

Possible potential (mass) generation from  $\phi \rightarrow$  axion Dark matter

**Modern de-Sitter Era**

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forward direction

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Basilakos, NEM, Sola

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chiral matter  
generation  
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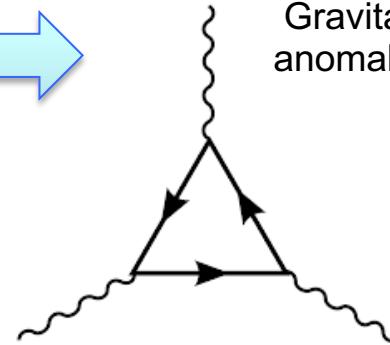
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B-L conserving sphaleron

Matter era

**Modern de-Sitter Era**



Cancellation of GA

Need to understand  
Modern Era better

Consistent with current  
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Dark matter

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**Phenomenology**

forward direction



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Basilakos, NEM, Solà

## RVM Inflationary (de Sitter) Phase

Primordial  
Gr  
Wa

Gravitational

Distinguishing feature from  $\Lambda$ CDM  
Alleviate data tensions

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chiral matter  
generation

Inflation exit

Rad

$B_0$

Lei  
RH

$N_I$

B-L

Ma

Modern de-Sitter Era

$$\text{today } \rho_{\text{RVM}}(H) = 3M_{\text{Pl}}^4 \left( c_0 + \nu_0 \left( \frac{H_0}{M_{\text{Pl}}} \right)^2 \right)$$

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\frac{3}{\kappa^2} c_0 \simeq 10^{-122} M_{\text{Pl}}^4$$

GA resurfacing

$$\text{today } -\varepsilon' M_{\text{Pl}} H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

Gómez-Valent  
Solà

RVM-type  
Running Dark Energy

forward direction

Could  
Alleviate  
Tensions in  
Data, e.g.  
 $H_0$ , growth ( $\sigma_8$ )  
tensions

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \beta \lesssim \mathcal{O}(1)$$

$$\frac{3}{2} c_0 \simeq 10^{-122} M_{\text{Pl}}^4$$

$$\rho_{\text{RVM}}(H) = 3M_{\text{Pl}}^4 \left( c_0 + \nu_0 \left( \frac{H_0}{M_{\text{Pl}}} \right)^2 + \beta \frac{H^4}{M_{\text{Pl}}^4} \right), \quad \beta > 0.$$

Running RVM  
Dark Energy

Not dominant today

Could  
ALleviate  
Tensions in  
Data, e.g.  
 $H_0$ , growth ( $\sigma_8$ )  
tensions

Saridakis  
talk

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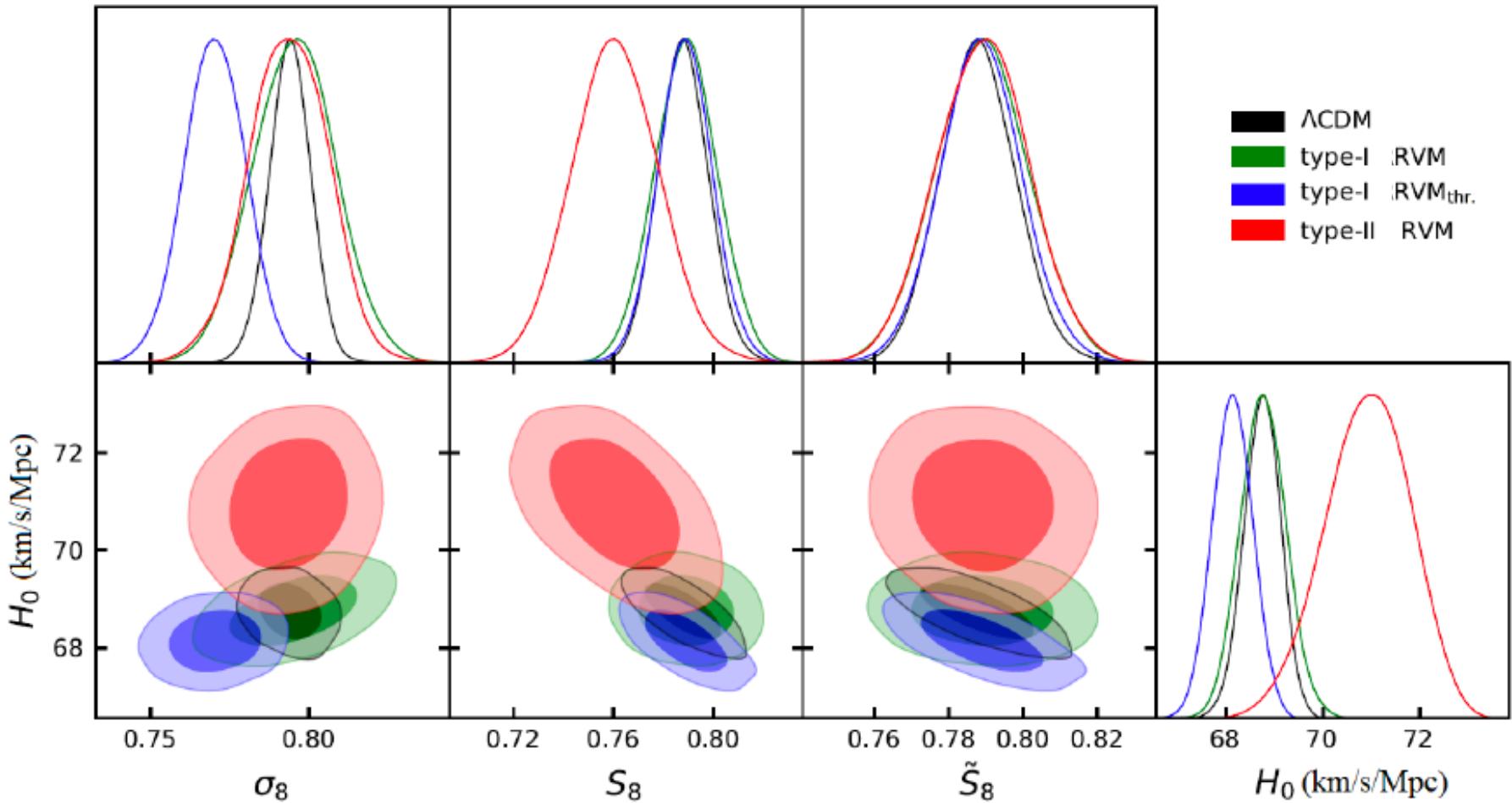
Running RVM  
Dark Energy

Not dominant today

If tensions  
are not due  
to statistics

Solà, Gómez-Valent,  
De Cruz Perez, Moreno-Pulido,  
**(Planck 2018 data)**

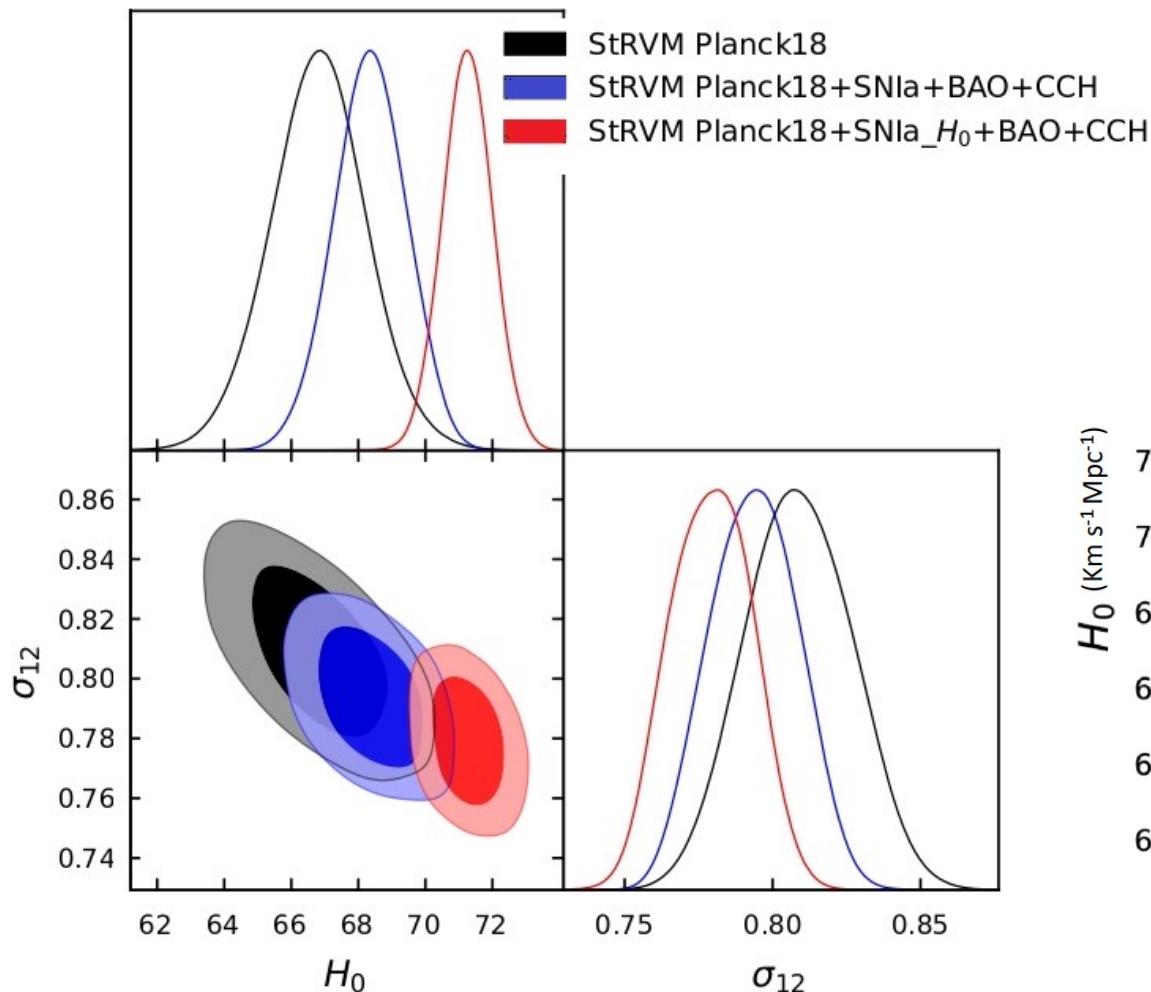
## Alleviation of the $H_0$ , $\sigma_8$ tension by RVM model



Gomez Valent,  
NEM, Solà (2023)

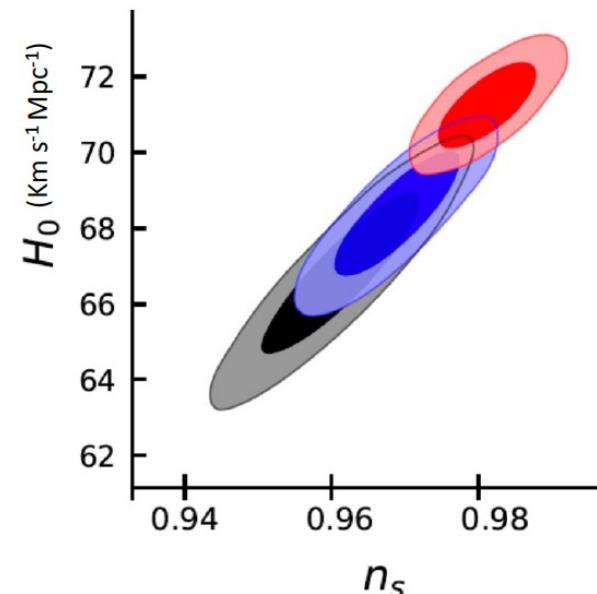
Integrating out graviton or matter flcts

$$\rho \propto (c_1 + c_2 \ln H) H^2 + (c_3 + c_4 \ln H) H^4 + \Lambda$$



Almost-Type II RVM  
in our stringy RVM

Alleviation of  $H_0$  &  
 $\sigma_{12}$  growth tensions



# **7. Conclusions & Outlook**

Deviations from  $\Lambda$ CDM  
Resolution of tensions ?

# The Basic "Cosmic Cycle"

Dark Energy

("running  
vacuum model  
(RVM) type")

current  
epoch

Dark Matter

Lorentz-  
Violating  
Leptogenesis

≠  
matter-  
antimatter  
Asymmetry

Stringy  
gravitational  
Axions  
KR axion  
Mass  
+  
torsion

geometric  
origin

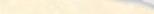
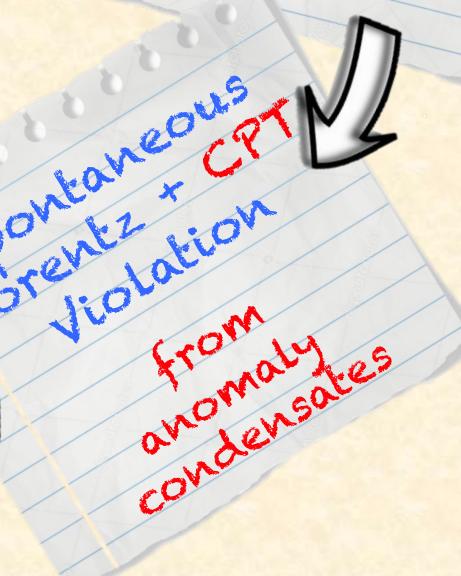
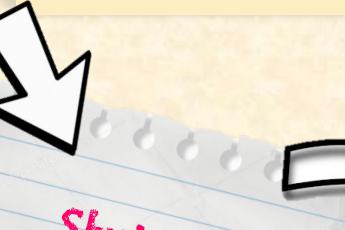
Gravitational  
anomalies

Primordial  
gravitational  
waves

Dynamical  
Inflation  
of RVM type  
without  
external  
inflatons

Spontaneous  
Lorentz + CPT  
Violation

from  
anomaly  
condensates



Deviations from  $\Lambda$ CDM  
Resolution of tensions ?

# The Parts/the Whole

Dark Energy

("running  
vacuum model  
(RVM) type")

current  
epoch

Dark Matter

Lorentz  
Violating  
Leptogenesis

matter-  
antimatter  
Asymmetry

Stringy  
gravitational  
Axions  
KR axion  
Mass  
+  
torsion

geometric  
origin

Gravitational  
anomalies

Primordial  
gravitational  
waves

**STRINGY RVM**

Dynamical  
Inflation  
of RVM type  
without  
external  
inflatons

Spontaneous  
Lorentz + CPT  
Violation

from  
anomaly  
condensates

# Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic  
Time

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial  
Gravitational  
Waves

Gravitational  
anomaly (GA)

Undiluted constant  
KR axial background



Paraphrasing  
C. Sagan:  
we are  
anomalously  
made of star  
stuff !

We exist because  
of Anomalies !

Leptogenesis induced by  
RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type  
Running Dark Energy

# Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic  
Time

Pre RVM-Inflationary era

## RVM Inflationary (de Sitter) Phase

Primordial  
Gravitational  
Waves

Gravitational  
anomaly (GA)

Undiluted constant  
KR axial background

We exist because  
of Anomalies!

Leptogenesis induced by  
RHN (tree-level) decays

Spontaneous

OUTLOOK: (i) Incorporate other  
model-dependent stringy  
axions → Axiverse  
Interesting Cosmology  
(eg Marsh 2015)  
could be ultralight → AION etc

Matter Era

Modern de-Sitter Era

axion Dark matter

RVM-type  
Running Dark Energy

forward direction

# Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic  
Time

Pre RVM-Inflationary era

## RVM Inflationary (de Sitter) Phase

Primordial  
Gravitational  
Waves

Gravitational  
anomaly (GA)

Undiluted constant  
KR axial background

exist because  
anomalies!

OUTLOOK: (ii) Look for imprints of the  
LV & CPTV KR axial background in CMB  
in early eras.

Leptogenesis induced by  
RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type  
Running Dark Energy

forward direction

# Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic  
Time

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial  
Gravitational  
Waves

Gravitational  
anomaly (GA)

Undiluted constant  
KR axial background

We  
or

OUTLOOK: (iii) Can we also get evidence of  
 $v < 0$  coefficient of  $H^2$  during RVM inflation?

$$\rho_{\text{RVM}}^{\text{string}} \simeq 3 M_{\text{Pl}}^4 \left[ -1.7 \times 10^{-3} \left( \frac{H}{M_{\text{Pl}}} \right)^2 + \mathcal{O}(10^7) \left( \frac{H}{M_{\text{Pl}}} \right)^4 \right]$$

Leptogenesis induced by  
RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type  
Running Dark Energy

## References:

# Thank you!



a microscopic  
(string-  
inspired)  
model for  
**RVM Universe...**

Links with :  
spontaneous Lorentz violation  
(via (gravitational axion)  
backgrounds)  
and  
Matter-Antimatter Asymmetry  
in theories with  
Right-Handed Neutrinos

- Basilakos, NEM, Solà  
(i) JCAP 12 (2019) 025  
(ii) IJMD28 (2019) 1944002  
(iii) Phys.Rev.D 101 (2020) 045001  
(iv) Phys.Lett.B 803 (2020) 135342  
(v) Universe 2020, 6(11), 218  
NEM, Solà  
(vi) EPJST 230 (2020), 2077  
(vii) EPJPlus 136 (2021), 1152  
NEM  
(viii) arXiv:2205.07044  
(ix) Universe 7 (2021), 480  
(x) Phil. Trans. A380 (2022) 2222  
NEM, Spanos, Stamou,  
(xi) Phys. Rev. D106 (2022), 063532

- (i) NEM & Sarben Sarkar, EPJC 73  
(2013), 2359  
(ii) John Ellis, NEM & Sarkar, PLB 725  
(2013), 407  
(iii) De Cesare, NEM & Sarkar, EPJC 75  
(2015), 514  
(iv) Bossingham, NEM & Sarkar,  
EPJC 78 (2018), 113; 79 (2019), 50  
(v) NEM & Sarben Sarkar, EPJC 80  
(2020), 558

**SPARES**

## **8. Enhanced cosmic perturbations and densities of primordial black holes and Gravitational Waves**

Anomaly condensate  $\rightarrow$  **linear axion potential**  $V_{\text{eff}} \ni \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle b(x)$

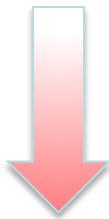
approximately de Sitter provided during the duration of inflation

$$b(t) = \bar{b}(0) + 0.14M_{\text{Pl}} H t_{\text{end}} \simeq \bar{b}(0) \quad \text{order of magnitude}$$

$< 0$

N=e-folds

beginning  
of inflation



$$|\bar{b}(0)| \gtrsim \mathcal{O}(10) M_{\text{Pl}}$$

Distance-swampland  
conjectures?

Anomaly condensate  $\rightarrow$  **linear axion potential**  $V_{\text{eff}} \supset \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle b(x)$

$$V(b) \simeq b \tilde{\Lambda}_0^4 \sqrt{\frac{2}{3}} \frac{M_{\text{Pl}}}{96 M_s^2} \equiv b \frac{\tilde{\Lambda}_0^4}{f_b} \equiv b \Lambda_0^3$$

Such a potential can also arise in appropriate brane compactifications  
(eg type IIB strings)

L. McAllister, E. Silverstein and A. Westphal,  
Phys. Rev. D 82 (2010), 046003  
[arXiv:0808.0706 [hep-th]].

We may extend the model to include other **stringy axions** arising from **compactification**

$$V_{a_I}^{\text{lin}} = a_I(x) \frac{f_b}{f_a} \Lambda_0^3 \quad \Lambda_0 = 8.4 \times 10^{-4} M_{\text{Pl}}. \quad f_a = \text{axion coupling}$$

**canonical kinetic  
terms for a-axions**

$$f_b \equiv \left( \sqrt{\frac{2}{3}} \frac{M_{\text{Pl}}}{96 M_s^2} \right)^{-1} \stackrel{Eq.(9)}{\simeq} 5.3 \times 10^{-6} M_{\text{Pl}}$$

# World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480,  
e-Print: 2111.05675 [hep-th]

NEM, Spanos, Stamou  
PRD106 (2022), 063532

Anomaly condensate  $\rightarrow$  linear axion potential  $V_{\text{eff}} \ni \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle b(x)$   
 world-sheet (non-perturbative) instantons  $\rightarrow$  periodic potential perturbations

$$V_{\text{wsinst}}^b \simeq \Lambda_b^4 \cos\left(\frac{b}{f_b}\right) \quad \Lambda_b^4 \sim M_s^4 e^{-S_{\text{wsinst}}} \quad \rightarrow \quad \Lambda_b \ll \Lambda_0.$$

$$V_{\text{wsinst}}^{a_I} \simeq \Lambda_I^4 \cos\left(\frac{a_I}{f_{a_I}}\right) \quad \Lambda_0 \gg \Lambda_I \neq \Lambda_b, \quad \text{Restrict to } I = 1 : a_1 \equiv a$$

$$V_{\text{brane-compact.-effects}}(a) \ni \Lambda_2^4 \frac{1}{f_a} a + \Lambda_I^4 \left(1 + \xi_a \frac{a}{f_a}\right) \cos\left(\frac{a}{f_a}\right)$$

warp factor

$$\frac{\Lambda_2^4}{f_a} \sim \frac{\epsilon}{L} \sqrt{\frac{3}{(2\pi)^3}} M_s^3$$

L. McAllister, E. Silverstein and A. Westphal,  
Phys. Rev. D 82 (2010), 046003  
[arXiv:0808.0706 [hep-th]].

## World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

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world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a, b) = \Lambda_1^4 \left( 1 + f_a^{-1} \tilde{\xi}_1 a(x) \right) \cos(f_a^{-1} a(x)) + \frac{1}{f_a} \left( f_b \Lambda_0^3 + \Lambda_2^4 \right) a(x) + \Lambda_0^3 b(x)$$

Case I  $\left( \frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a \Lambda_0^3} \right)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$

NEM, Sola + Basilakos

NEM, Spanos, Stamou  
PRD106 (2022), 063532

Case II  $\Lambda_0 \ll \left( \frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a \Lambda_0^3} \right)^{1/3} \Lambda_0 < \Lambda_1$

Zhou, Jiang, Cai, Sasaki, Pi,  
Phys. Rev. D 102 (2020) no.10, 103527

# World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential**  $V_{\text{eff}} \ni \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle b(x)$

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Case I  $\left( \frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a \Lambda_0^3} \right)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$

Case Enhancement of cosmic perturbations  $\Lambda_0 \ll \left( \frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a \Lambda_0^3} \right)^{1/3} \Lambda_0 < \Lambda_1$



NEM, Sola + Basilakos

NEM, Spanos, Stamou  
PRD106 (2022), 063532

Zhou, Jiang, Cai, Sasaki, Pi,  
Phys. Rev. D 102 (2020) no.10, 103527

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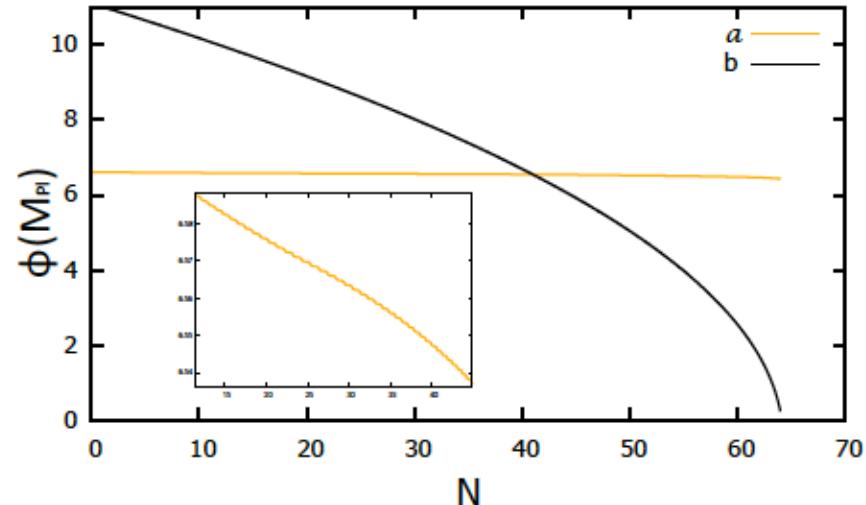
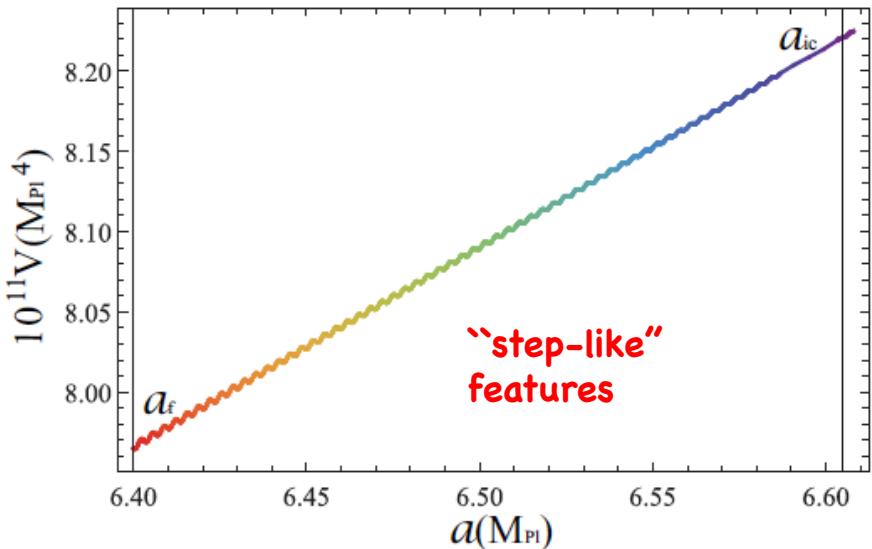
$$V(a, b) = \Lambda_1^4 \left( 1 + f_a^{-1} \tilde{\xi}_1 a(x) \right) \cos(f_a^{-1} a(x)) + \frac{1}{f_a} \left( f_b \Lambda_0^3 + \Lambda_2^4 \right) a(x) + \Lambda_0^3 b(x)$$

Case I

$$\left( \frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a \Lambda_0^3} \right)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$$

NEM, Spanos, Stamou  
PRD106 (2022), 063532

**b-field + condensate** drive inflation, **a-axion ends inflation**



$$V(a, b) = \Lambda_1^4 \left( 1 + f_a^{-1} \tilde{\xi}_1 a(x) \right) \cos(f_a^{-1} a(x)) + \frac{1}{f_a} \left( f_b \Lambda_0^3 + \Lambda_2^4 \right) a(x) + \Lambda_0^3 b(x)$$

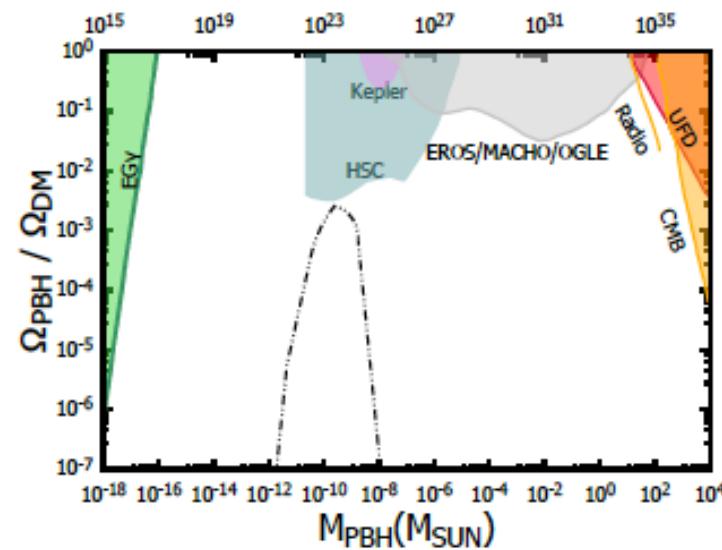
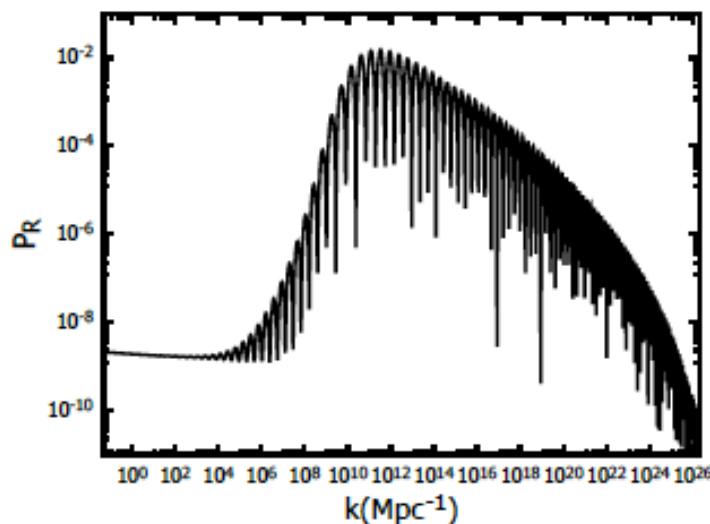
$$n_s = 1 + \frac{d \ln P_R}{d \ln k} \quad r = \frac{P_T}{P_R} \quad P_T = \frac{2}{\pi^2} H^2$$

SET	$g_1$	$g_2$	$\xi$	$f(M_{Pl})$	$\Lambda_0(M_{Pl})$	$\Lambda_1(M_{Pl})$	$\Lambda_3(M_{Pl})$
1	0.021	0.904	-0.15	$2.5 \times 10^{-4}$	$8.4 \times 10^{-4}$	$8.19 \times 10^{-4}$	$2.32 \times 10^{-4}$
2	0.026	0.774	-0.20	$2.5 \times 10^{-4}$	$8.4 \times 10^{-4}$	$7.89 \times 10^{-4}$	$2.49 \times 10^{-4}$

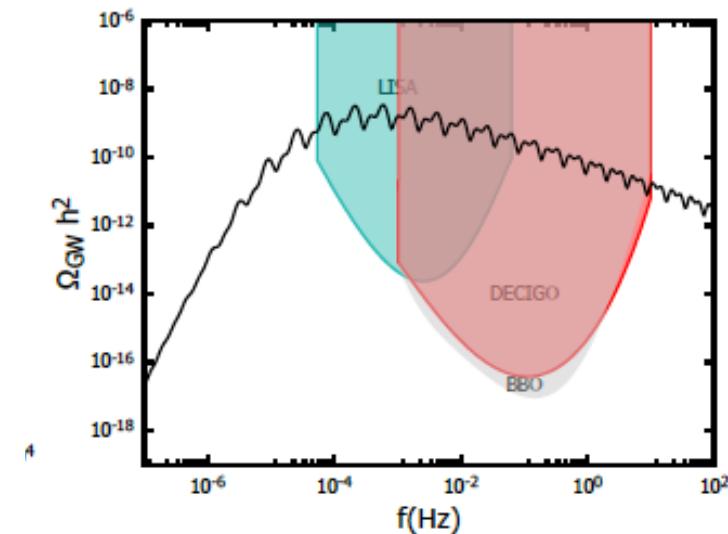
SET	$a_{ic}$	$b_{ic}$	$n_s$	$r$
1	6.605	11.1	0.9638	0.062
2	4.932	11.4	0.9619	0.060

# Primordial Black Hole (PBH) and GW enhanced production during inflation

NEM, Spanos, Stamou  
PRD106 (2022), 063532



**SET 1**



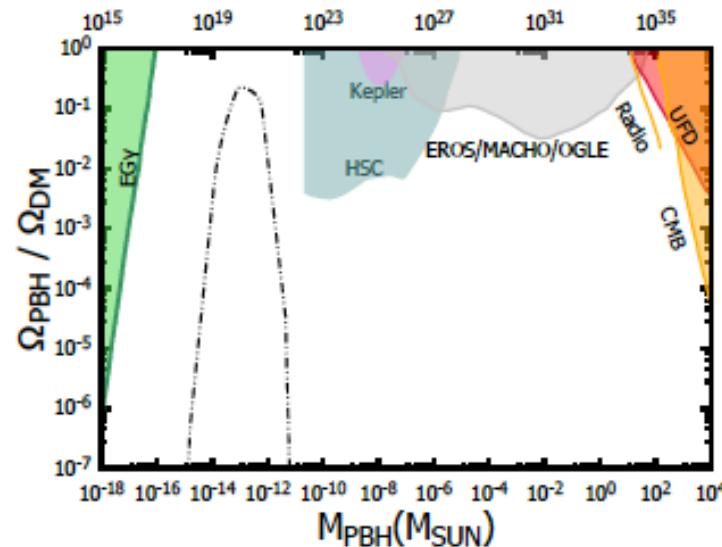
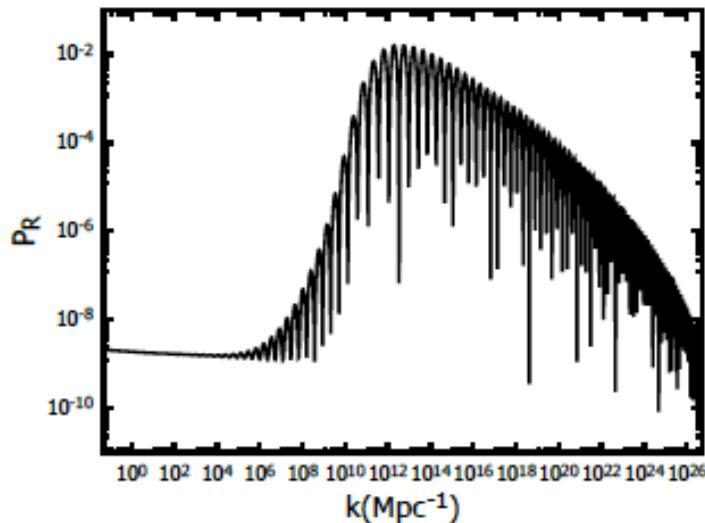
fractional PBH abundance

$$f_{PBH} = \int_k dM_{PBH}(k) \frac{1}{M_{PBH}(k)} \frac{\Omega_{PBH}}{\Omega_{DM}}(M_{PBH}(k))$$

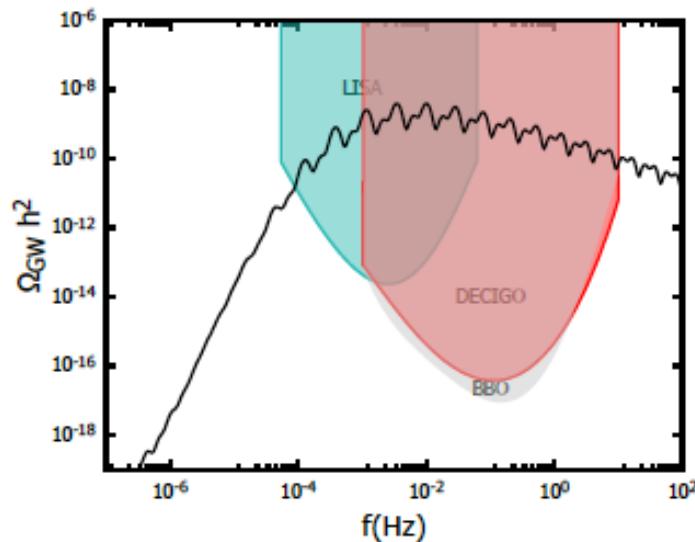
$$f_{PBH} = 0.01$$

# Primordial Black Hole (PBH) and GW enhanced production during inflation

NEM, Spanos, Stamou  
PRD106 (2022), 063532



**SET 2**



fractional PBH abundance

$$f_{PBH} = \int_k dM_{PBH}(k) \frac{1}{M_{PBH}(k)} \frac{\Omega_{PBH}}{\Omega_{DM}}(M_{PBH}(k))$$

$$f_{PBH} = 0.80.$$

Anomaly condensate → **linear axion potential**  $V_{\text{eff}} \ni \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a, b) = \Lambda_1^4 \left( 1 + f_a^{-1} \tilde{\xi}_1 a(x) \right) \cos(f_a^{-1} a(x)) + \frac{1}{f_a} \left( f_b \Lambda_0^3 + \Lambda_2^4 \right) a(x) + \Lambda_0^3 b(x)$$

Case II

$$\Lambda_0 \ll \left( \frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a \Lambda_0^3} \right)^{1/3} \Lambda_0 < \Lambda_1$$

Zhou, Jiang, Cai, Sasaki, Pi,  
Phys. Rev. D 102 (2020) no.10, 103527

Anomaly condensate  $\rightarrow$  **linear axion potential**  $V_{\text{eff}} \ni \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle b(x)$

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**specific set of parameters**  
 enhancement due to **inflection points** in the potential  $\rightarrow$   
 different enhancement mechanism than in

Anomaly condensate  $\rightarrow$  **linear axion potential**  $V_{\text{eff}} \supset \langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle b(x)$

world-sheet (non-perturbative) instantons  $\rightarrow$  periodic potential perturbations

$$V(a, b) = \Lambda_1^4 \left( 1 + f_a^{-1} \tilde{\xi}_1 a(x) \right) \cos(f_a^{-1} a(x)) + \frac{1}{f_a} \left( f_b \Lambda_0^3 + \Lambda_2^4 \right) a(x) + \Lambda_0^3 b(x)$$

$$\Lambda_0 = 8.4 \times 10^{-4} M_{\text{Pl}}, \quad g_1 = 110, \quad g_2 = 1.779 \times 10^4, \quad \xi = -0.09, \quad f = 0.09 M_{\text{Pl}}.$$

**SET 3**  $(a_{ic}, b_{ic}) = 7.5622, 0.522,$

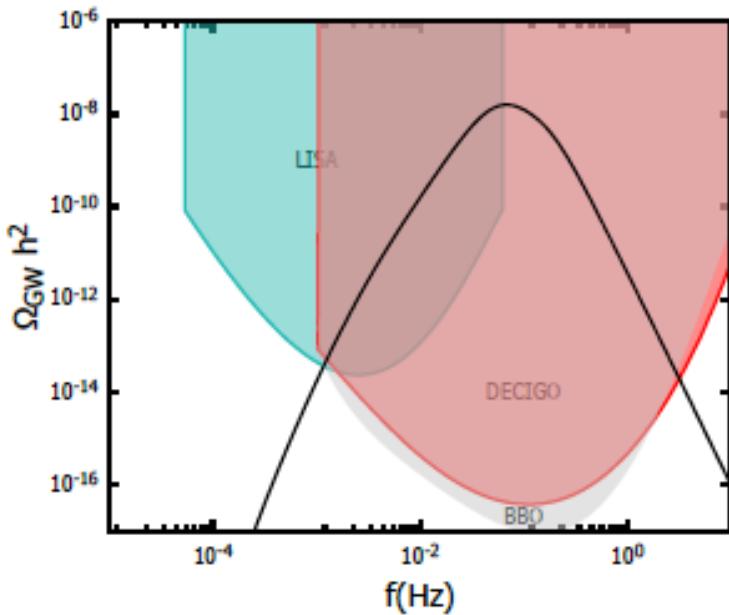
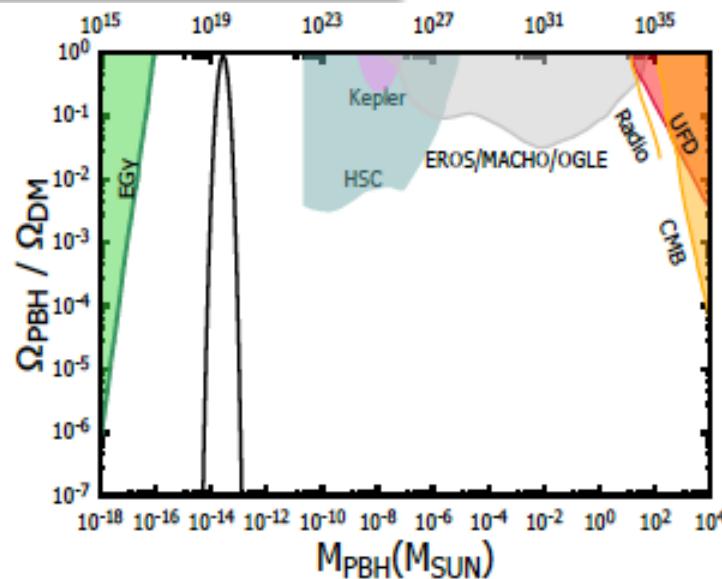
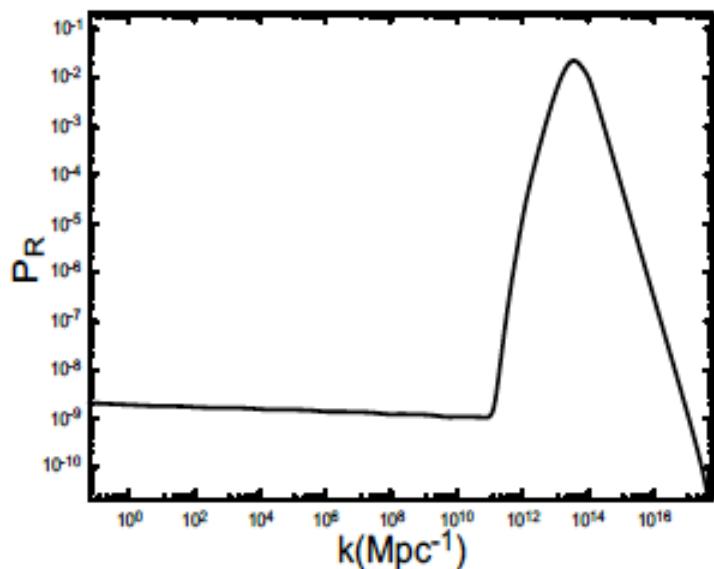
Case II  $\Lambda_0 \ll \left( \frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a \Lambda_0^3} \right)^{1/3} \Lambda_0 < \Lambda_1$



specific set of parameters  
 enhancement due to **inflection points** in the potential  $\rightarrow$   
 different enhancement mechanism than in

# Primordial Black Hole (PBH) and GW enhanced production during inflation in Case 2

NEM, Spanos, Stamou  
PRD106 (2022), 063532



fractional PBH abundance

$$f_{PBH} = \int_k dM_{PBH}(k) \frac{1}{M_{PBH}(k)} \frac{\Omega_{PBH}}{\Omega_{DM}}(M_{PBH}(k))$$

$$f_{PBH} = 0.762$$

**SET 3**

**SUMMARY:** Primordial Black Hole (PBH) and GW enhanced production during inflation in Cases 1 + 2

NEM, Spanos, Stamou  
PRD106 (2022), 063532

SET	$P_R^{peak}$	$M_{PBH}^{peak}(M_\odot)$	$f_{PBH}$
1	$1.466 \times 10^{-2}$	$2.394 \times 10^{-10}$	0.009
2	$1.365 \times 10^{-2}$	$8.313 \times 10^{-14}$	0.799
3	$2.24 \times 10^{-2}$	$1.791 \times 10^{-14}$	0.762

Hence in both hierarchies of scales :

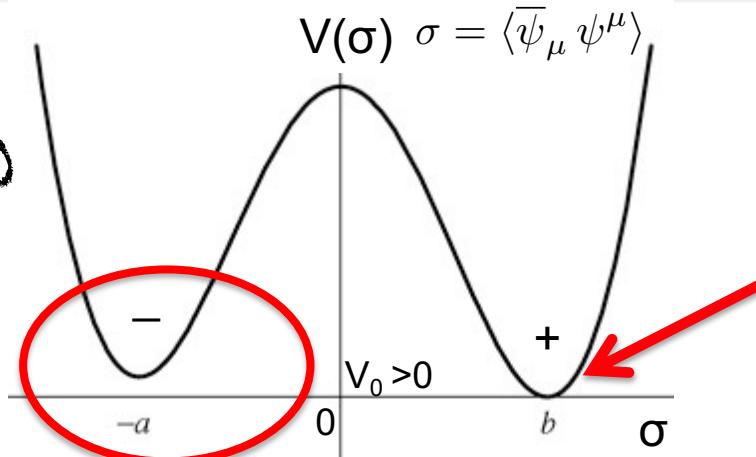
$$1: \left( \frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a \Lambda_0^3} \right)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0 \quad , \quad 2: \quad \Lambda_0 \ll \left( \frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a \Lambda_0^3} \right)^{1/3} \Lambda_0 < \Lambda_1$$

one may get **significant enhancement** of cosmic perturbations, and PBH production, and thus a **significant portion** of PBH could play **the role of DM**, also, as a result, **profiles of GW** could **change** during radiation, in principle **falsifiable predictions** at **interferometers**.

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ , $\psi_\mu$ )

Basilakos, NEM,  
Solà (2019-20)

Role of (Local)  
Supersymmetry



SUGRA broken  
gravitino  
Condensate  
stabilised →

RVM GW-induced Inflation

Statistical bias (percolation) in  
occupation probabilities of the +,- vacua

Lalak, Ovrut,  
Lola, G. Ross,  
Thomas

## Primordial Gravitational Waves Potential Origins in pre-inflationary era?

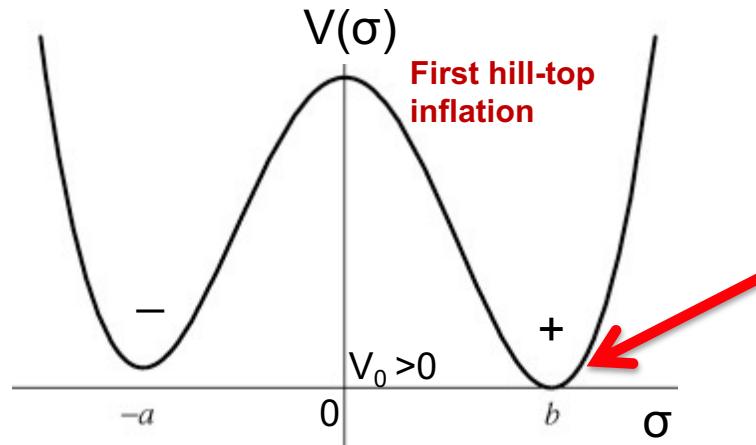
Collapse/collisions of Domain walls formed in  
theories with (approximate) discrete symmetry  
breaking, e.g. via bias in double-well potentials of  
some condensate (gravitino  $\psi_\mu$  or gaugino)

NEM,Solà  
EPJ-ST  
(2020)

Ellis, NEM,  
Alexandre,  
Houston

# The Model in Early Universe: only gravitational d.o.f. ( $b$ , $g_{\mu\nu}$ , $\psi_\mu$ )

Basilakos, NEM,  
Solà (2019-20)



SUGRA broken  
gravitino  
Condensate  
stabilised →  
**RVM GW-induced Inflation**

**Pre-RVM inflationary phase:** superstring/supergravity  
Effective action → **Imaginary parts** → **instabilities**

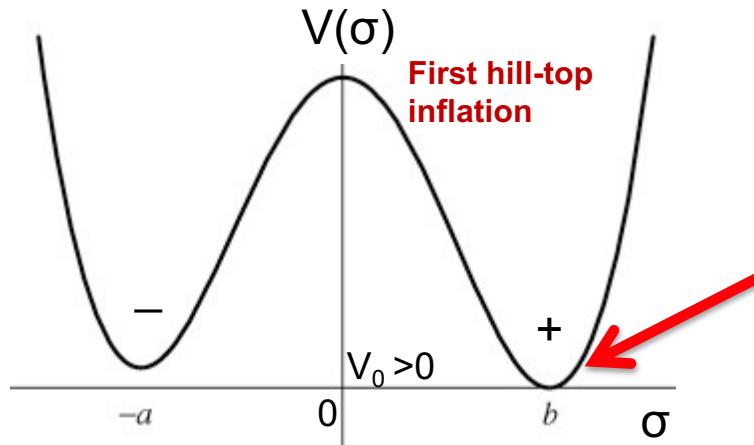
**First Hill-top inflation** = finite life –time →  
System **tunnels** to **RVM inflationary vacuum (GW condense)**

NEM, Solà  
EPJ-ST  
(2020)

Ellis, NEM,  
Alexandre,  
Houston

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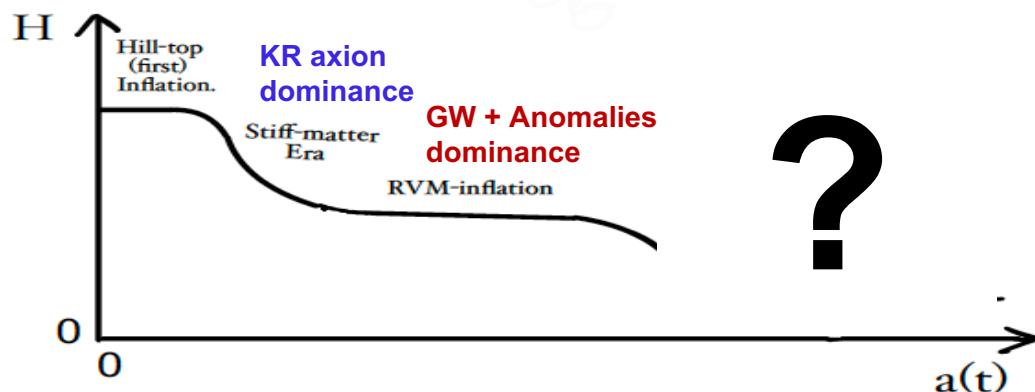
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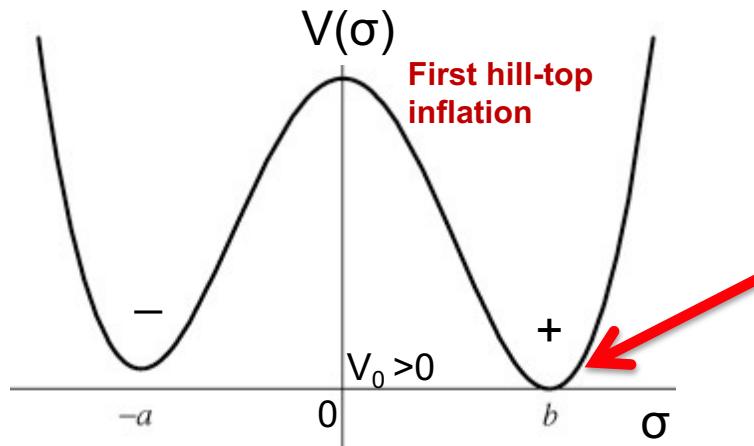


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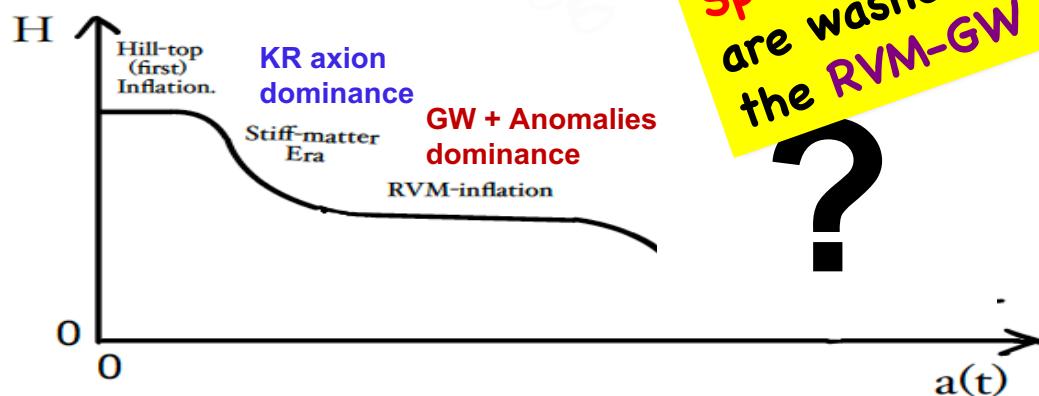
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First inflation ensures any  
Spatial inhomogeneities  
are washed out before  
the RVM-GW inflation

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