Heavier tail likelihoods for robustness against data outliers; Applications to the analysis of Gravitational Wave data

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LISA CONSORTIUM

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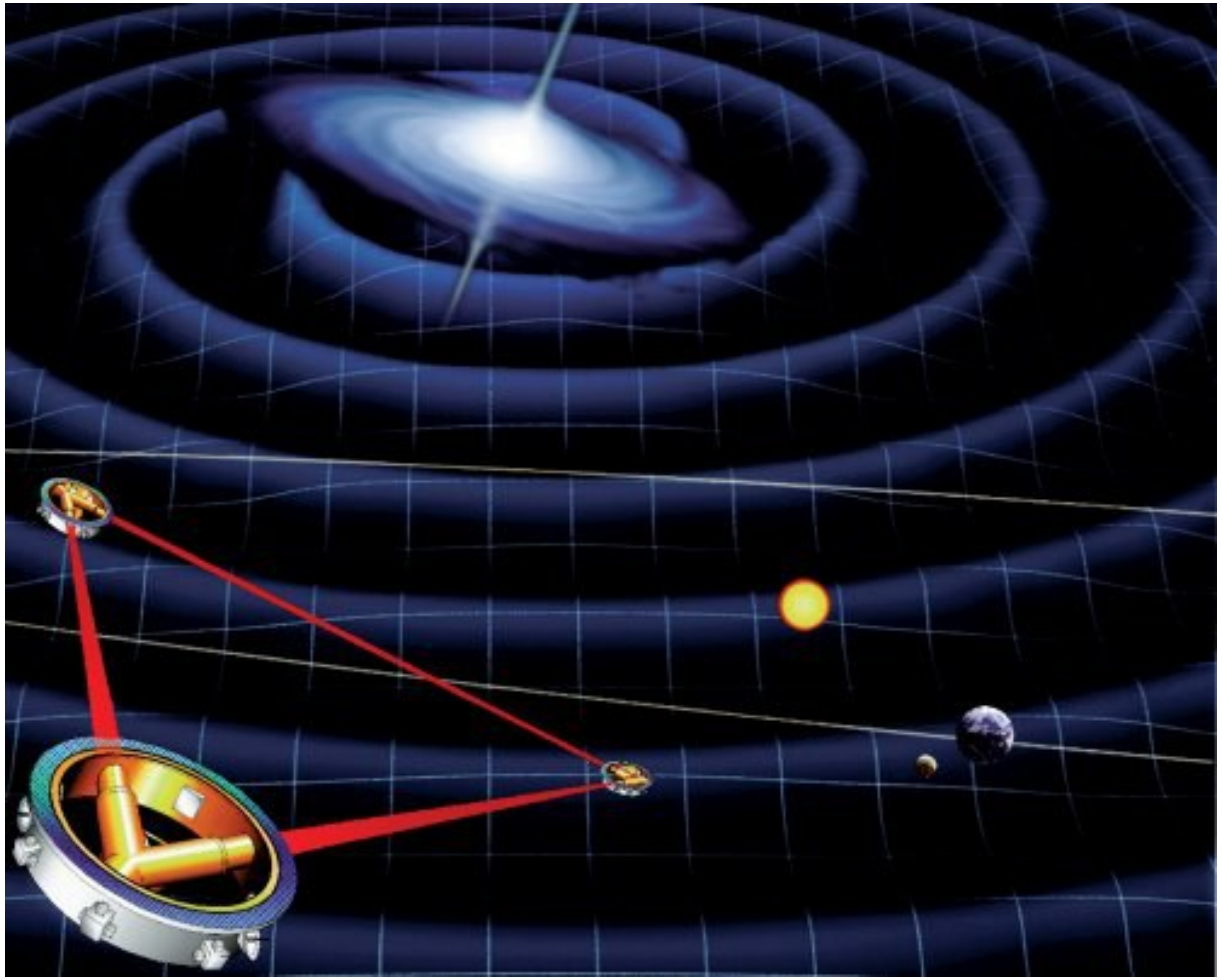
A DA framework for LISA

LISA, Obstacles & Approaches

Previously mentioned by Karnesis

- Overlapping signals from different type sources (UCBs, SMBBHs, EMRIs, Stochastic background …)
- The instrumental noise properties are not completely known
- Community attempts to overcome these issues with different methods and approaches



A Data Analysis framework for LISA

What we normally do...

- First, we assume $y = h(\vec{\theta}) + n$
- Then, assuming Gaussian properties of the noise, the likelihood of the measurement y given a parameter set $\vec{\theta}$

$$\pi(y|\vec{\theta}) = C \times e^{-\frac{1}{2}\left(y - h(\vec{\theta})\middle|y - h(\vec{\theta})\right)} = C \times e^{-\chi^2/2}$$

$$(a|b) = 2\int_0^{\infty} \mathrm{d}f \left[\tilde{a}^*(f)\tilde{b}(f) + \tilde{a}(f)\tilde{b}^*(f)\right] / \tilde{S}_n(f)$$

posterior

prior

$$\pi(\vec{\theta}|y) \propto \pi(y|\vec{\theta})p(\vec{\theta})$$

A Data Analysis framewor… LISA

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posterior

$\pi(\vec{\theta}|y)$ … $)$

How easy is that when dealing with LISA data?

- Depending on the frequency band, also accounting for other types of sources, how well the noise is estimated …. more …. and … more

What we can do/try?

Introduction

Generalized Hyperbolic (GH)

GH Probability Density Function (PDF)

tail shape asymmetry

$$\mathrm{gh}(x|\lambda,\alpha,\beta,\delta,\mu)) = a(\lambda,\alpha,\beta,\delta,\mu)\left(\delta^2+(x-\mu)^2\right)^{(\lambda-\frac{1}{2})/2} \times K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)\exp\left[\beta(x-\mu)\right]$$

scale location

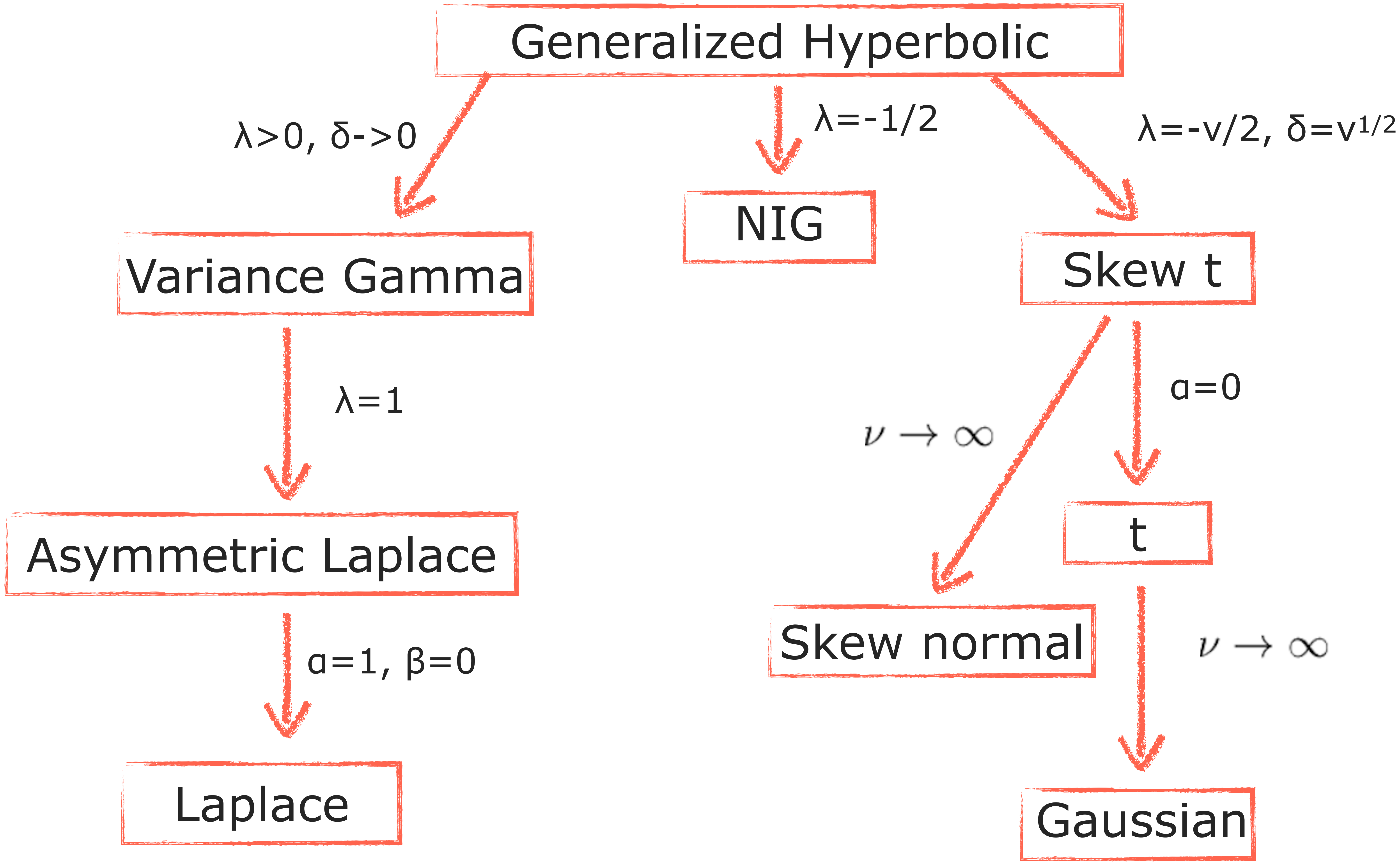
where

$$a(\lambda,\alpha,\beta,\delta,\mu) = \frac{(\alpha^2-\beta^2)^{\lambda/2}}{\sqrt{2\pi}\alpha^{\lambda-1/2}\delta^{\lambda}K_{\lambda}\left(\delta\sqrt{\alpha^2-\beta^2}\right)}$$

and K_λ is the modified Bessel function of the third kind.

Generalized Hyperbolic (GH)

Some limiting cases...



Toy models

Toy models

Shape Triangles

- Qualitatively characterisation of the distribution
- Parametrizations used by Prause K. (1999) yielding to the scale and location-invariant parameters {χ,ξ}:

$$\zeta = \sqrt{\alpha^2 - \beta^2}, \quad \varrho = \beta/\alpha$$

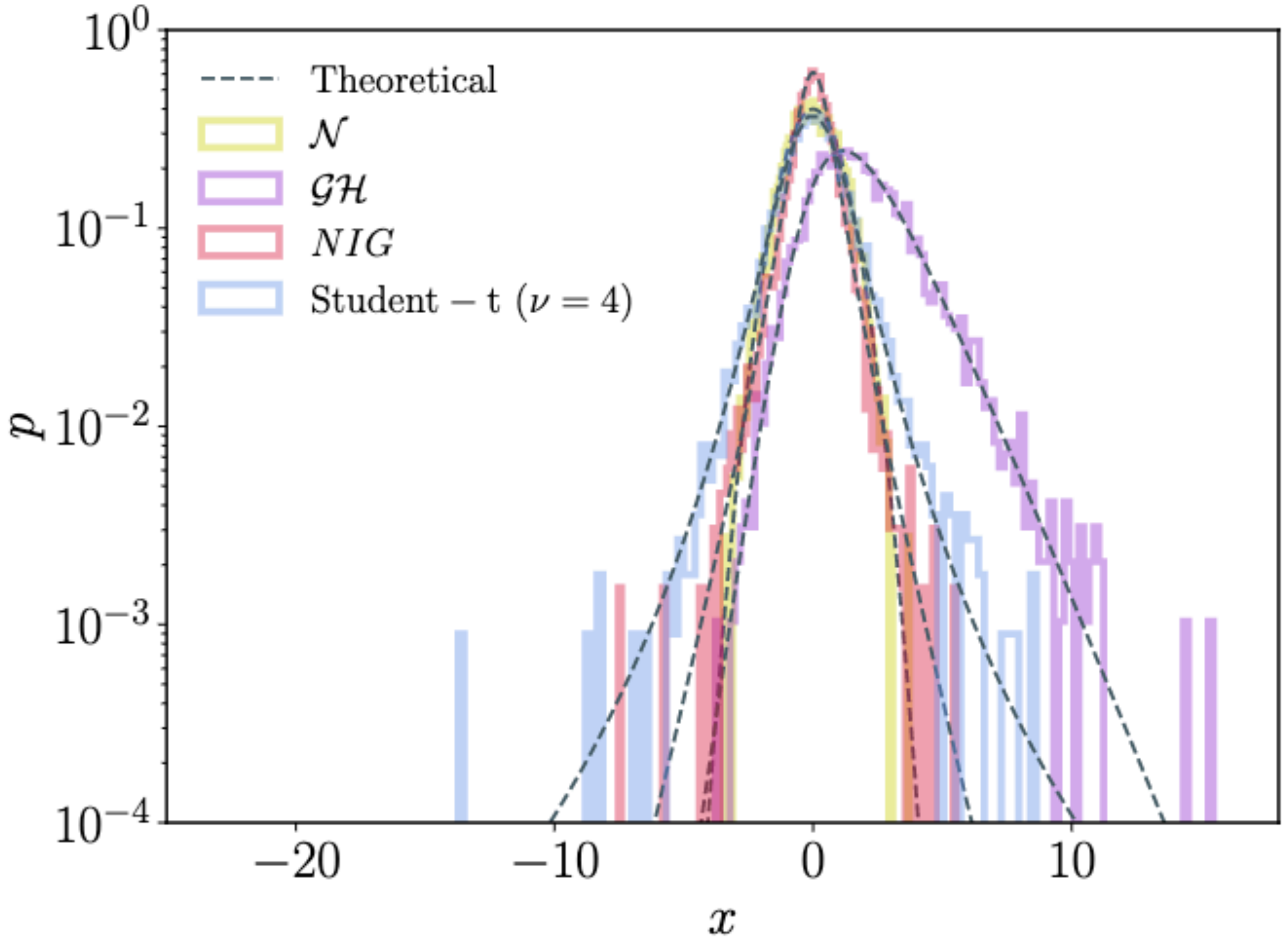
$$\xi = (1+\zeta)^{-1/2}, \quad \chi = \xi\varrho$$

[0,1] [-1,1]

Toy models

Recovering the underlying statistical properties…

1. Gaussian $\mathcal{GH}(\mu = 0, \lambda = 1, \beta = 0, \alpha \to \infty, \delta \to \infty)$
2. NIG $\mathcal{GH}(\mu = 0, \lambda = -1/2, \beta = 0, \alpha = 1, \delta = 0.8)$
3. Student-t $\mathcal{GH}(\mu = 0, \lambda = -\nu/2, \beta = 0, \alpha \to 0, \delta = \sqrt{\nu})$
4. Hyperbolic $\mathcal{GH}(\mu = 0, \lambda = 1, \beta = 0.75, \alpha = 1.5, \delta = 2)$



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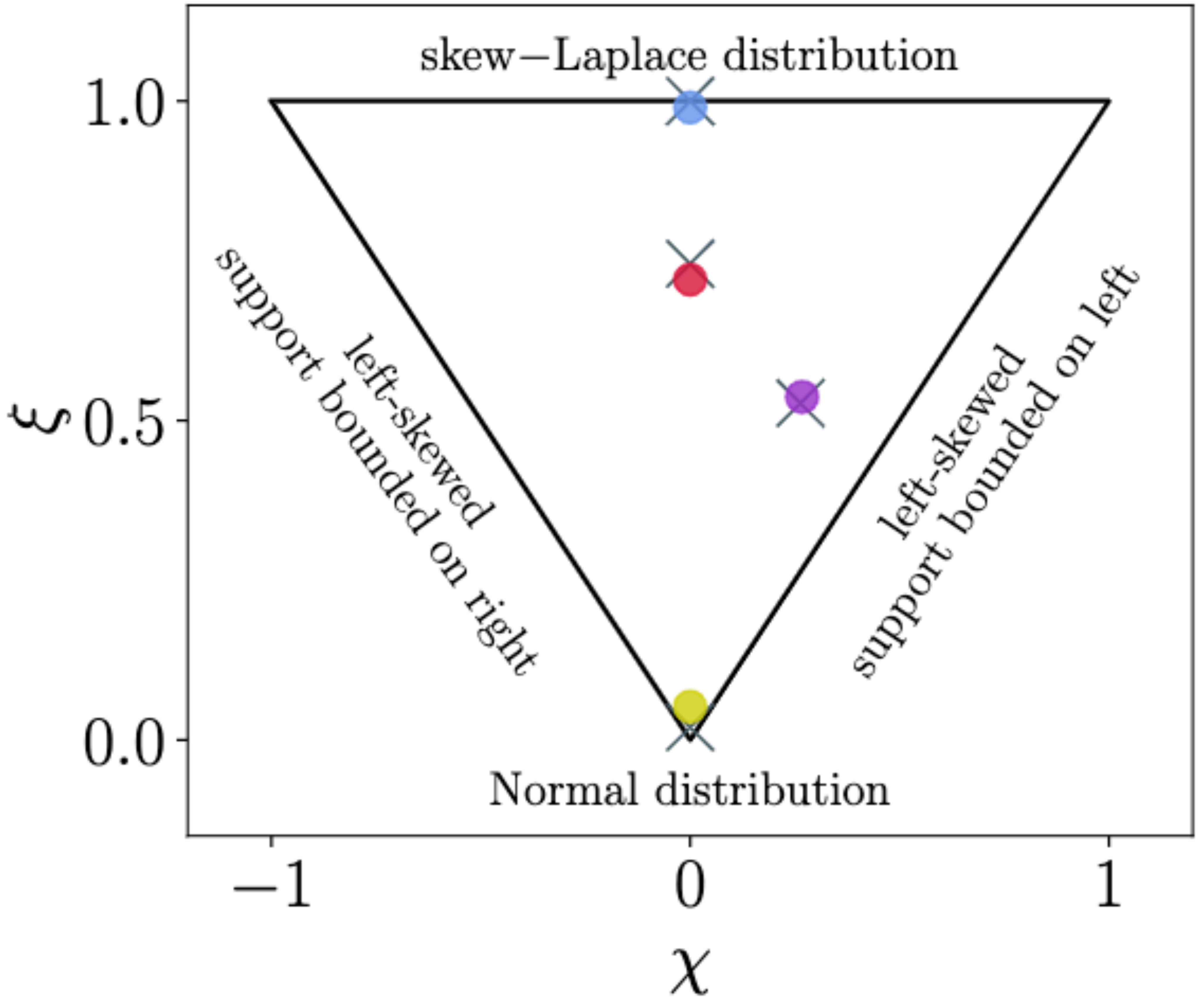


Applications to the Gravitational Waves data

Application to the GW data

Why we use the GH for our analysis?

- Multiple signals overlapping in time and frequency.
- The noise of the observatory will not be completely known.
- Unresolvable sources of UCBs will generate a non-stationary confusion signal.
- Glitches and data gaps.

→ Less information about the noise, when compared to ground-based observatories.

Application to the GW data

Working with the signals from UCBs

- **Why?** 1. Monochromatic nature, 2. very small computational time
- **How?** We simulate data given a particular instrument sensitivity, and then perform the analysis assuming the wrong levels of instrumental noise. We use three likelihood formulation:

Gaussian likelihood (Known noise, but wrong levels)

Gaussian (Fitting for the noise)

Hyperbolic likelihood (Known noise, but wrong levels)

Application to the GW data

Working with the signals from UCBs

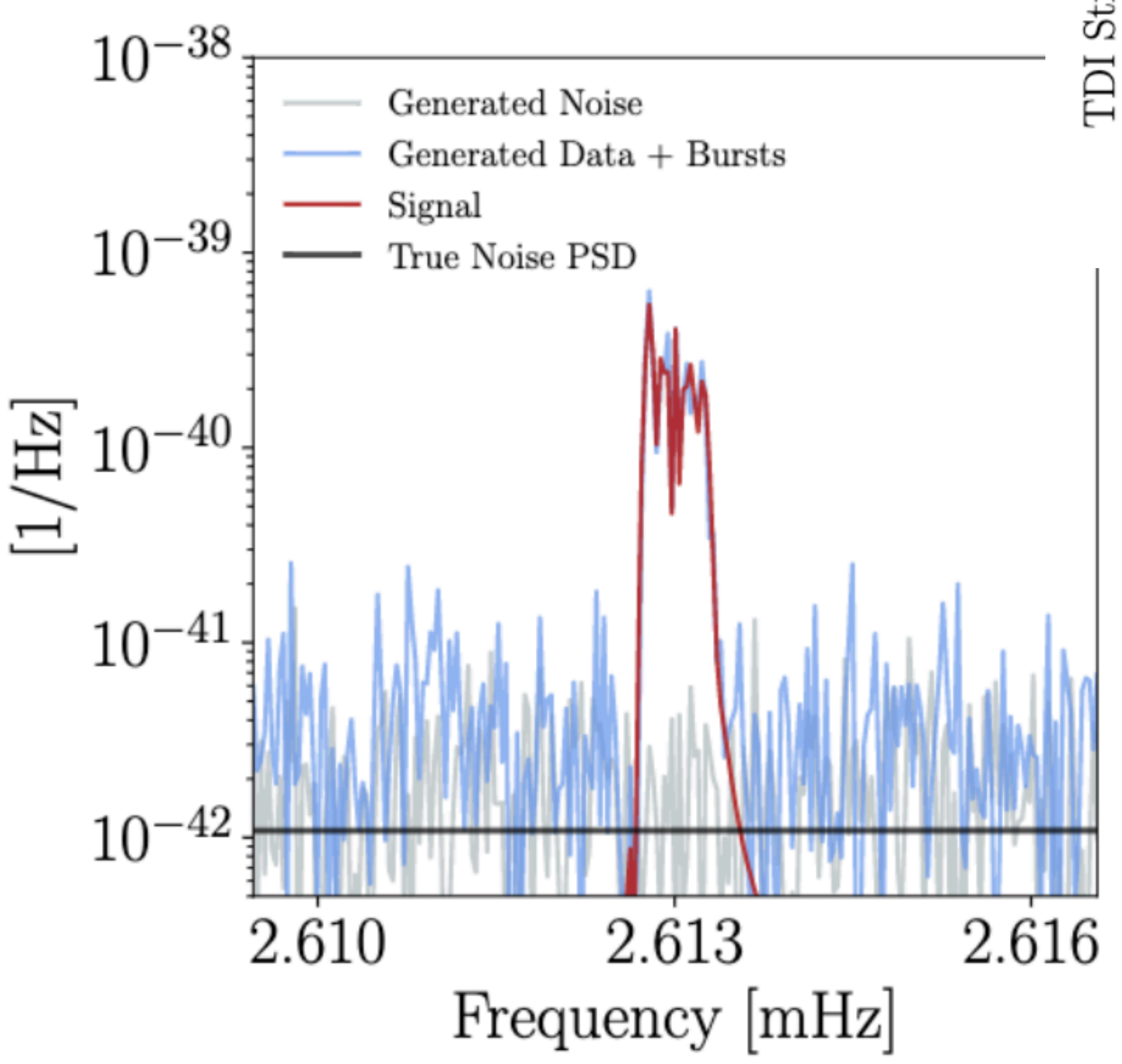
- **Assumptions?** 1. Uninterrupted data, 2. The space-craft interferometric noises are assumed to be equal for all three space-crafts.

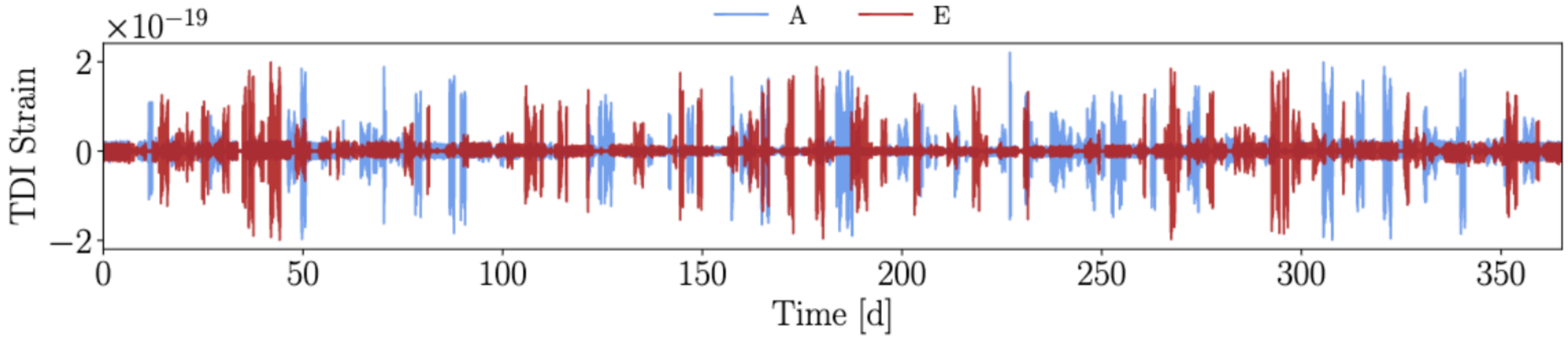
- Technical Information: Markov Chain Monte Carlo algorithms, enhanced with Parallel Tempering (30 temperatures, each running with 50 independent walkers).

- We inject the data with bursts of Gaussian noise placed randomly in the time series of the channels.

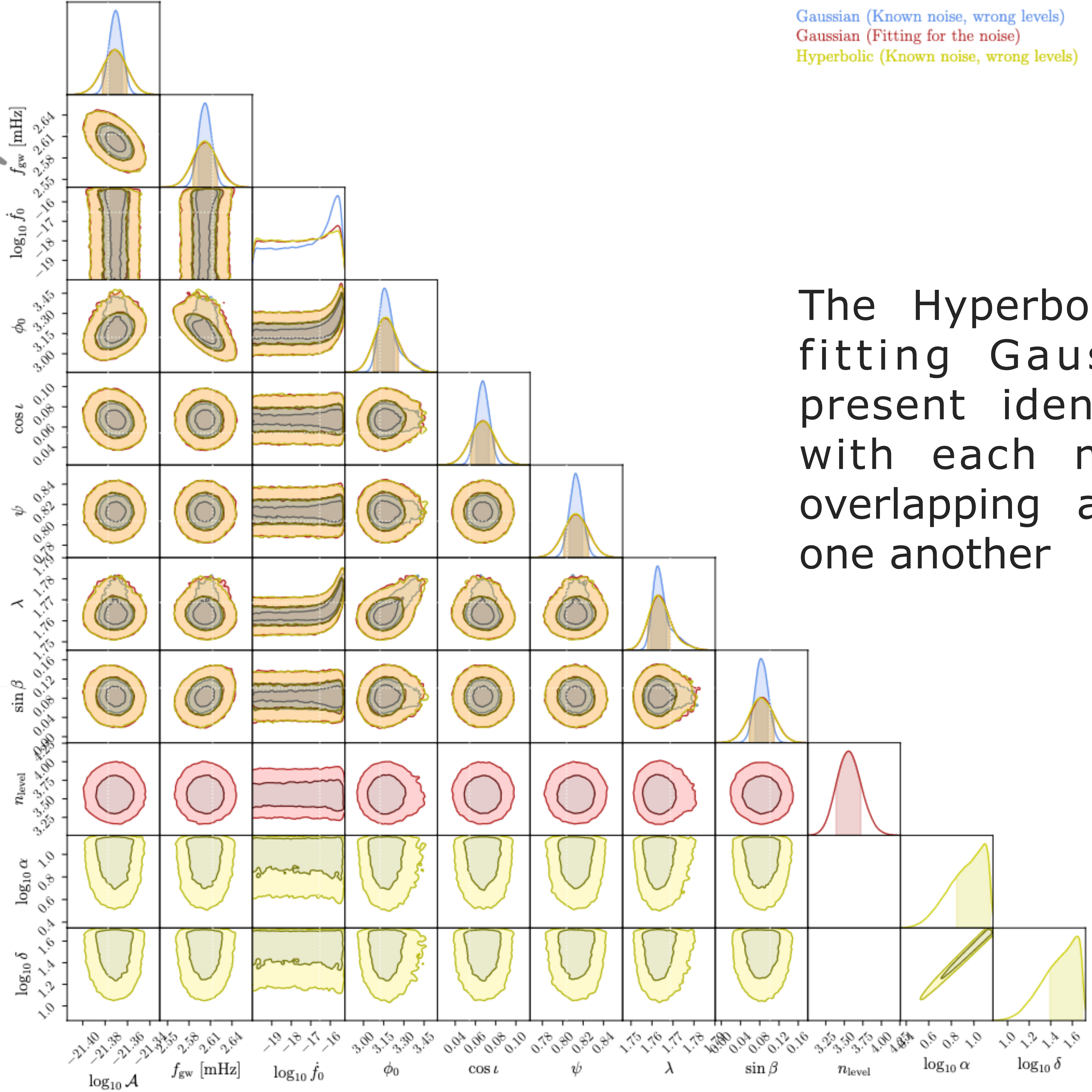
Application to the GW data

Working with the signals from UCBs - Recovering the true PSD





source parameters



The Hyperbolic and the noise-fitting Gaussian likelihoods present identical performance, with each marginal posterior overlapping almost perfectly on one another

Conclusions & Discussion

Conclusions & Discussion

- By adopting the GH likelihood we inferred its hyper-parameters, which essentially “tuned” its overall shape to that of the target distribution.

- Application to the GW data for the case of LISA (for which the noise is not completely known) shows that the distribution of the residuals asymptotically tend to the Gaussian (in agreement with theory).

- Different applications to the GW data confirm the versatility and robustness of the Hyperbolic filter, to parameter estimation situations where the instrumental noise properties are not completely known.

- Useful tool for analysing data with heavy tails due to its flexibility in the description of the data.

- Suggestions: 1. Using GH for the fluctuations responsible for Primordial Black Hole formation (arXiv:2112.04520 enhance the PBH formation probability, compared with the predictions of the perturbation theory). 2. A broader detailed analysis for LISA (e.g. glitches/gaps in the data, BBH detection etc).

Thank you!