Polarisation of the Vacuum

Dark Energy and Cosmological Inflation

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G.S.

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Gauge field theory vacuum and cosmological inflation without scalar field

Friedman Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p). \qquad < 0$$

The matter equation of state in the universe

$$p = p(\epsilon)$$

defines the behaviour of the solutions of the Friedmann equations.

Consider the equation of state

$$p = w\epsilon$$

where w is barotropic parameter

Friedman Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0, \longrightarrow \dot{\epsilon} = 0,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p). \longrightarrow \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon - 3\epsilon) = +\frac{8\pi G}{3c^4}\epsilon > 0$$

If

$$w = -1 \qquad \rightarrow \qquad p = -\epsilon$$

Then the dark energy density ε = constant
and
the Universe is accelerating > 0

With negative pressure - We have antigravity

Is there a Physical Matter which has w=-1 ?

A negative pressure fluid - Scalar field driven inflation

$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \rightarrow \quad \epsilon + p = \dot{\phi}^2 \geq 0$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad \rightarrow \quad \epsilon + 3p = 2\dot{\phi}^2 - 2V(\phi)$$

In the Metastable Vacuum

$$V^{'}(\phi_0) = 0, \; V(\phi_0) > 0, \; \dot{\phi}_0 = 0$$

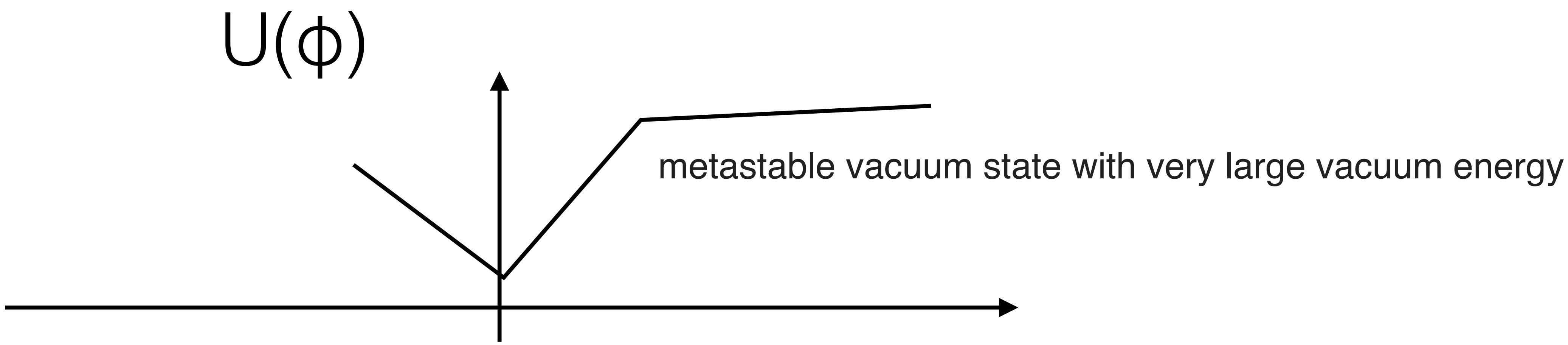
$$p = -\epsilon = -V(\phi_0) < 0 \qquad \frac{\ddot{a}}{a} = \frac{8\pi G}{3c^4} V(\phi_0) > 0$$

The problem is

how

to get out of Inflation ?

When inflation starts it is difficult to stop it



Google

A popular method of controlling inflation -

Reduce the money supply by increasing interest rates.

Enqvist

Scalar-field-driven inflation is a generic idea but as of yet,

there is no compelling, particle physics motivated theory of inflation.

Instead, there exists a vast number of different models.

There are models with many inflaton fields, models based on extra dimensions,

models based on the Higgs field with a non-minimal coupling to gravity,

models where the the superluminal expansion and the primordial perturbation are generated by different fields.

What is the Influence of the Vacuum Energy Density on the Cosmological Evolution?

Vacuum

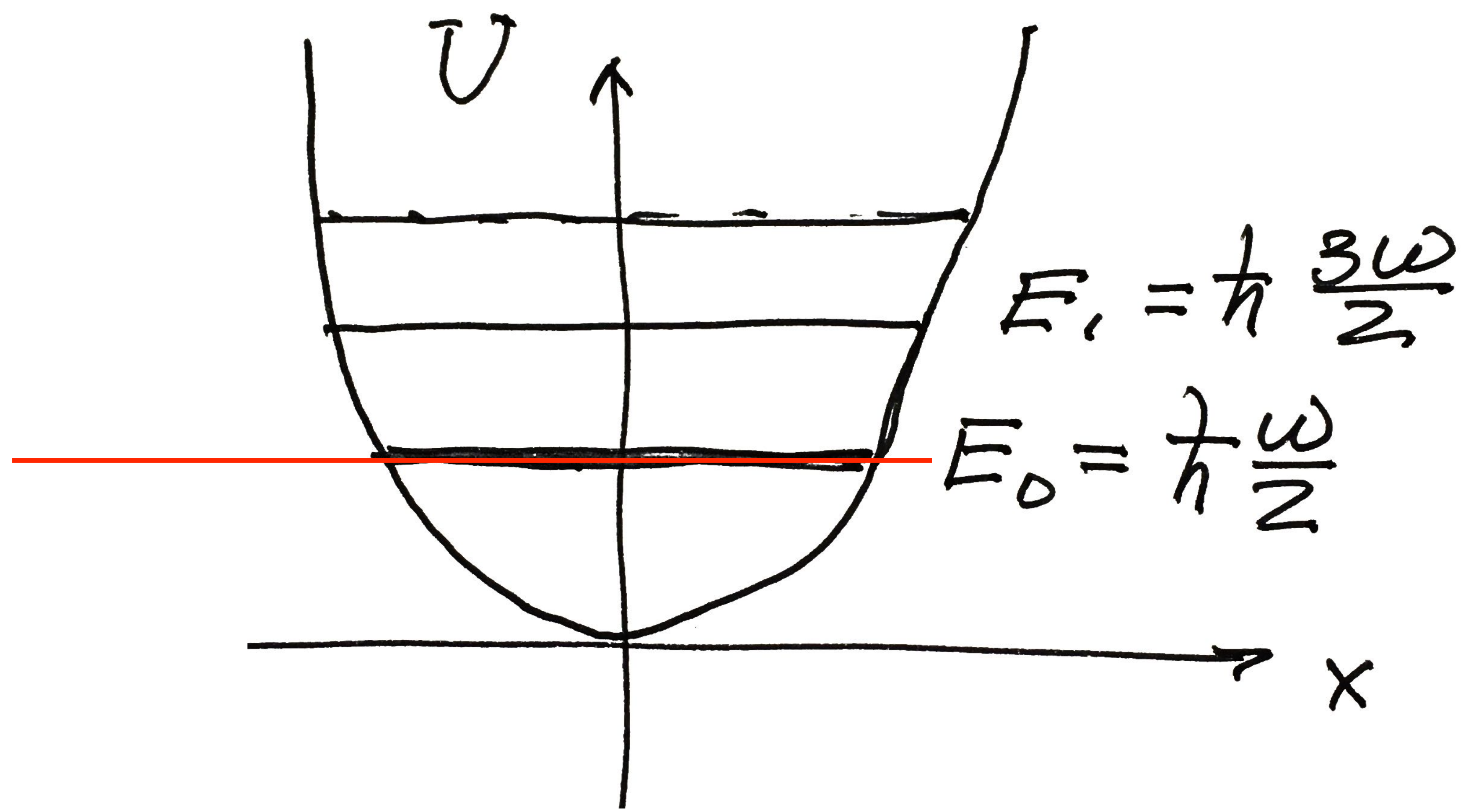
Y. B. Zel'dovich, *The Cosmological constant and the theory of elementary particles*, Sov. Phys. Usp. **11** (1968) 381

S. Weinberg, *The Cosmological constant problem*, Rev. Mod. Phys. **61** (1989) 1-23

V. Mukhanov, Physical Foundations of Cosmology, Cambridge University Press, New York, 2005.

Is there Energy Density in the Vacuum ?

Zero Point Energy of a Quantised Field

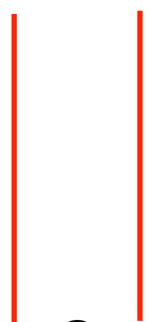


$$U_{vacuum} = \hbar \sum_k \frac{\omega_k}{2}$$

There is Energy Density in the Vacuum, it is Zero Point Energy

Lamb shift - 1947

Casimir effect 1948



a

$$U_\gamma^\infty = \sum \frac{1}{2}\hbar\omega_k e^{-\gamma\omega_k}$$

$$\lim_{\gamma\to 0}[\ U_\gamma^\infty(J) - U_\gamma^\infty(0)\ ] = U_{phys}$$

$$U_{phys} = \hbar\ c\pi^2 \frac{Area}{720a^3}$$

The Cosmological vacuum energy density from a quantum field

$$E_0 = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\omega_p \sim \frac{1}{16\pi^2}\Lambda^4 \qquad \approx 1.44\times 10^{110}\frac{g}{s^2cm}$$

Critical Energy Density in Universe

$$\epsilon_{crit} = 3\frac{c^4}{8\pi G}\Big(\frac{H_0}{c}\Big)^2 \approx 7.67\times 10^{-9}\frac{g}{s^2cm}$$

Vacuum energy contribution to the energy density of the universe

$$\epsilon_{crit} = 3\frac{c^4}{8\pi G}\left(\frac{H_0}{c}\right)^2 \approx 7.67 \times 10^{-9}\frac{g}{s^2 cm}$$

100%

$$\epsilon_{\Lambda} = 3\frac{c^4}{8\pi G}\left(\frac{H_0}{c}\right)^2 \Omega_{\Lambda} \approx 5.28 \times 10^{-9}\frac{g}{s^2 cm}$$

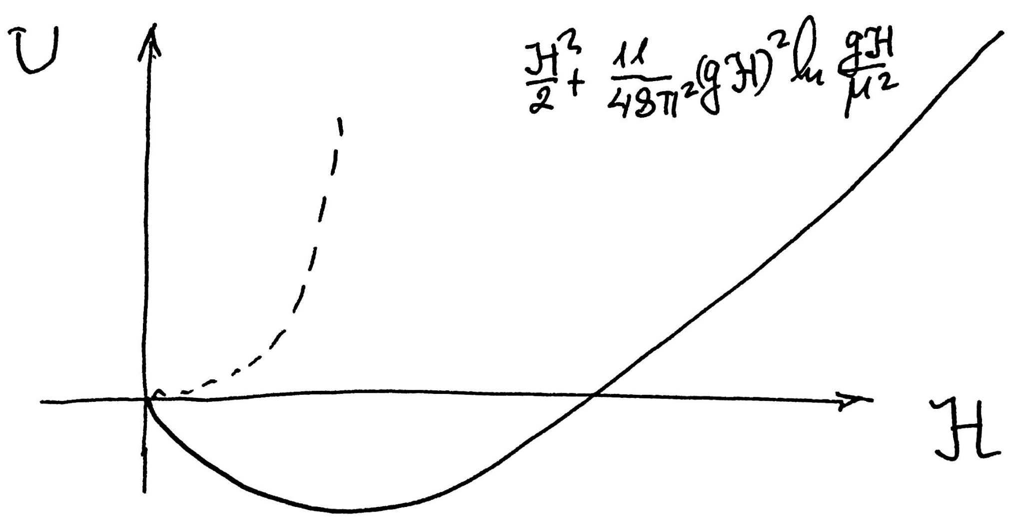
68%

The Yang-Mills Theory Vacuum Energy Density

G.S. 1977, 2020

$$\mathcal{L}_g = -\mathcal{F} - \frac{11N}{96\pi^2} g^2 \mathcal{F}\Big(\ln \frac{2g^2\mathcal{F}}{\mu^4} - 1\Big) \ , \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}_a^2 - \vec{\mathcal{E}}_a^2}{2} > 0, \quad \mathcal{G} = \vec{\mathcal{E}}_a \vec{\mathcal{H}}_a = 0 \ .$$

$$\mathcal{L}_q = -\mathcal{F} + \frac{N_f}{48\pi^2} g^2 \mathcal{F}\Big[\ln(\frac{2g^2\mathcal{F}}{\mu^4}) - 1\Big]$$



$$2g^2\mathcal{F}_{vac} = \mu^4 \exp\big(-\frac{96\pi^2}{b\ g^2(\mu)}\big) = \Lambda^4_{YM},$$

where $b = 11N - 2N_f$.

$$T_{\mu\nu} = T^{YM}_{\mu\nu}\Big[1 + \frac{b\ g^2}{96\pi^2} \ln \frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0.$$

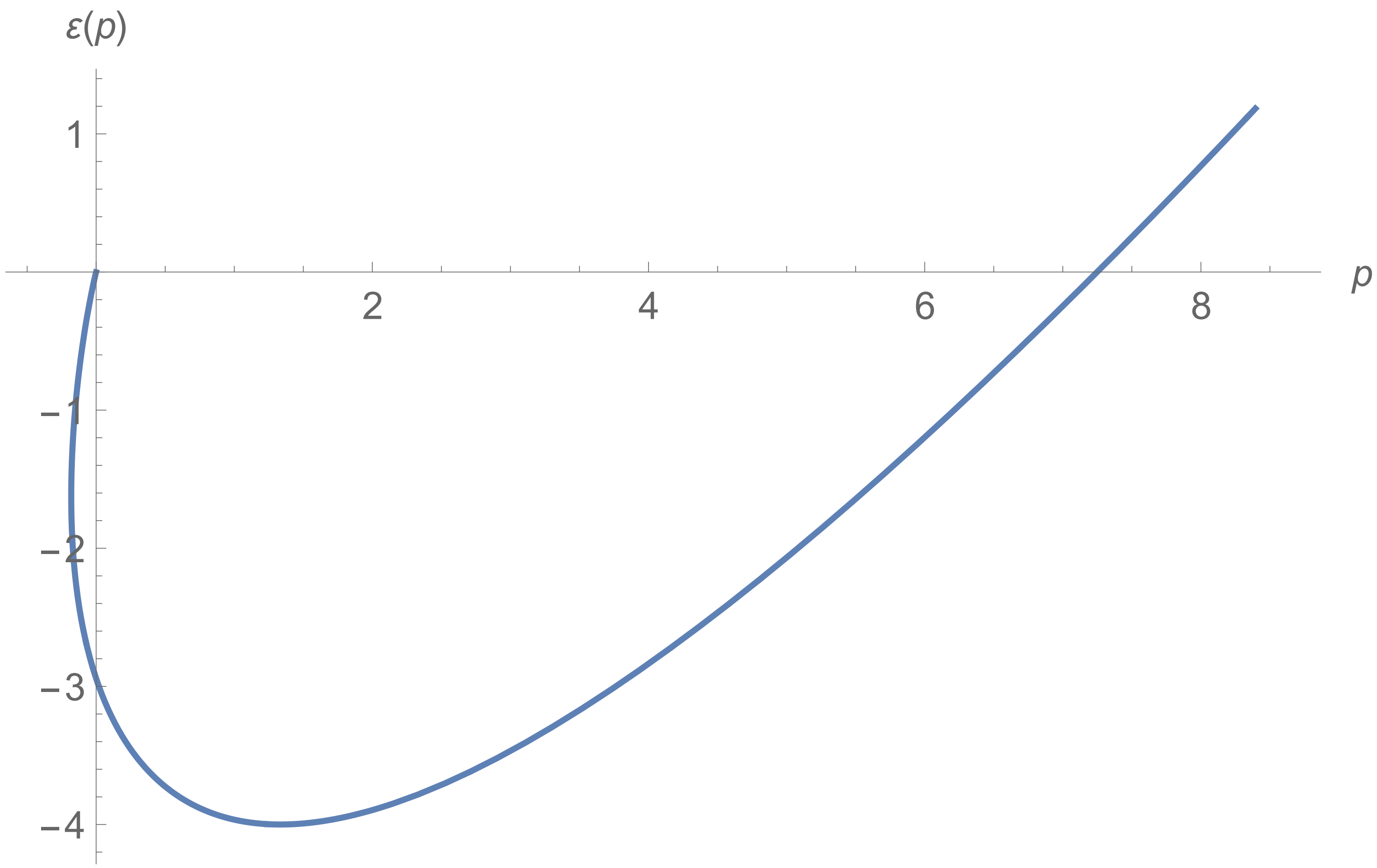
YM Quantum Energy Momentum Tensor

$$T_{\mu\nu} = T_{\mu\nu}^{YM}\left[1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\right] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0,$$

$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

$$\mathcal{F} = \tfrac{1}{4}g^{\alpha\beta}g^{\gamma\delta}G^a_{\alpha\gamma}G_{\beta\delta} \geq 0 \qquad \mathcal{G} = G^*_{\mu\nu}G^{\mu\nu} = 0$$

Yang-Mills Quantum Equation of State



$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

Yang-Mills Quantum Equation of State

$$p = \frac{1}{3}\epsilon + \frac{4}{3}\frac{b\ g^2\mathcal{F}}{96\pi^2}\Lambda_{YM}^4 \qquad \text{and} \qquad w = \frac{p}{\epsilon} = \frac{\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 3}{3\Big(\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} - 1\Big)}$$

general parametrisation of the equation of state $p = w\epsilon$

Friedman Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p).$$

Friedmann Evolution Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0, \longrightarrow \quad \epsilon + p = \frac{4\mathcal{A}}{3}\,(2g^2\mathcal{F})\,\log\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p). \longrightarrow \quad \epsilon + 3p = 2\mathcal{A}\,(2g^2\mathcal{F})\,\left(\log\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 1\right).$$

the first equation can be solved for the field strength

$$2g^2\dot{\mathcal{F}} + 4(2g^2\mathcal{F})\frac{\dot{a}}{a} = 0 \qquad\qquad 2g^2\mathcal{F}\;a^4 = const \equiv \Lambda_{YM}^4\;a_0^4,$$

General Relativity and Yang-Mills Vacuum Energy Density

$$S = -\frac{c^3}{16\pi G}\int R\sqrt{-g}d^4x + \int \left(\mathcal{L}_q + \mathcal{L}_g\right)\sqrt{-g}d^4x.$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\Big[T^{YM}_{\mu\nu}\Big(1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big) - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big].$$

The contribution of the YM vacuum field to the energy balance of the universe

Friedmann Evolution Equations in YM, QCD Vacuum

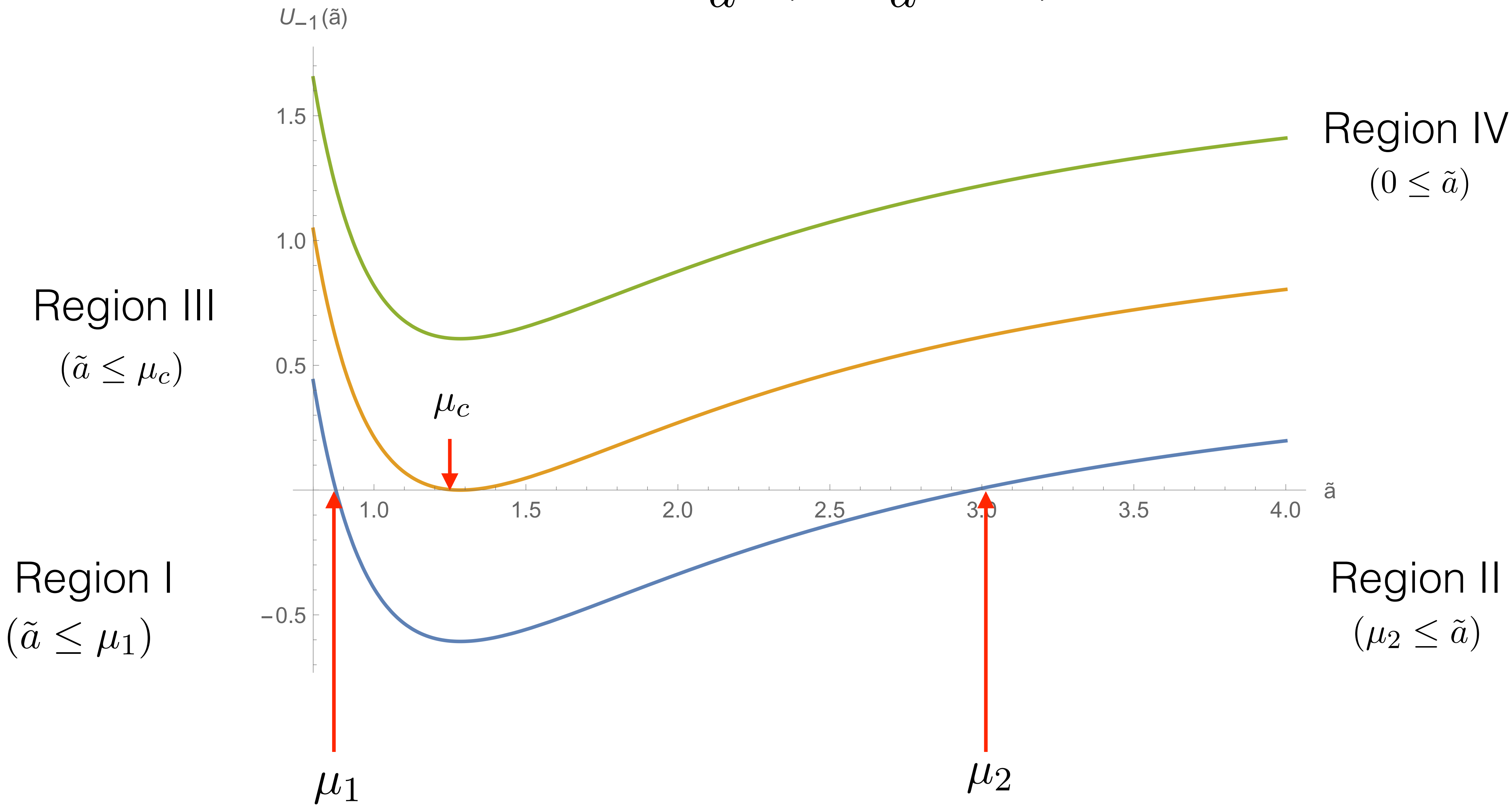
$$a(\tau) = a_0 \; \tilde{a}(\tau), \qquad ct = L \; \tau,$$

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

$$\frac{1}{L^2} = \frac{8\pi G}{3c^4} \; \mathcal{A} \; \Lambda_{YM}^4 \equiv \Lambda_{eff} \;\; ,$$

$$\mathcal{A} = \frac{b}{192\pi^2} = \frac{11N - 2N_f}{192\pi^2}.$$

$$U_{-1}(\tilde{a}) \equiv \frac{1}{\tilde{a}^2}\left(\log\frac{1}{\tilde{a}^4} - 1\right) + \gamma^2.$$



$$0 \leq \gamma^2 < \gamma_c^2$$

$$\gamma^2 = \gamma_c^2 = \frac{2}{\sqrt{e}}$$

$$\gamma_c^2 < \gamma^2$$

$$U_{-1}(\tilde{a}) \equiv \frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) + \gamma^2.$$

$k = -1, \quad 0 \le \gamma^2 < \gamma_c^2$ Regions I ($\tilde{a} \le \mu_1$) and II ($\mu_2 \le \tilde{a}$)

$k = -1, \quad \gamma^2 = \gamma_c^2 = \dfrac{2}{\sqrt{e}}$ Region III (separatrix, $\tilde{a} \le \mu_c$)

$k = -1, \quad \gamma_c^2 < \gamma^2$ Regions IV ($0 \le \tilde{a}$)

$k = 0,$

$k = 1, \quad 0 \le \gamma^2.$

Polarisation of the YM vacuum and the Effective Lagrangians

$$\epsilon_{YM} = 3\frac{c^4}{8\pi G}\frac{1}{L^2}, \qquad \frac{1}{L^2} = \frac{8\pi G}{3c^4}\,\frac{11N - 2N_f}{196\pi^2}\,\Lambda^4_{YM}$$

Λ^4_{YM} is the dimensional transmutation scale of YM theory

$$\epsilon_{YM} = 3\frac{c^4}{8\pi G}\frac{1}{L^2} = \begin{cases} 9.31 \times 10^{-3} & eV \\ 9.31 \times 10^{29} & QCD \\ 9.31 \times 10^{97} & GUT \\ 9.31 \times 10^{110} & Planck \end{cases} \qquad \frac{g}{s^2 cm}$$

the YM vacuum energy density is well defined and is finite

G.S.
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Type II Solution — Initial Acceleration of Finite Duration

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

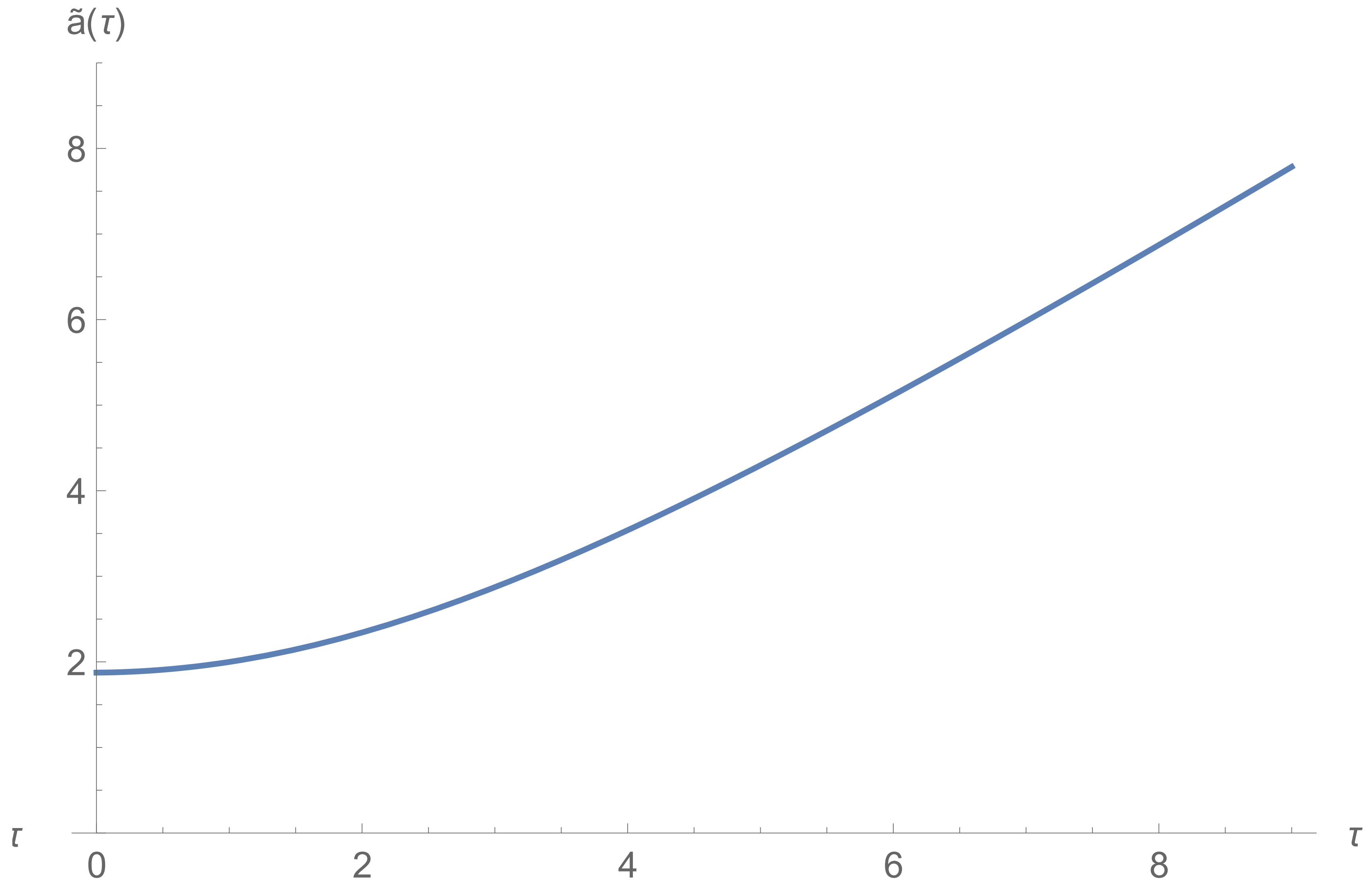
$$\tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\ e^{-\frac{b^2}{2}}\Big(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}} - 1) - 1\Big)^{1/2}.$$

$$\mu_2^2 = -\frac{2}{\gamma^2}\ W_-\Big(-\frac{\gamma^2}{2\sqrt{e}}\Big), \qquad 0 \le \gamma^2 < \frac{2}{\sqrt{e}} \text{ and } \tilde{a} \ge \mu_2.$$

Type II Solution *Initial Acceleration of Finite Duration*

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\; e^{-\frac{b^2}{2}}\Big(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}}-1)-1\Big)^{1/2}. \qquad \tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b\in[0,\infty],$$

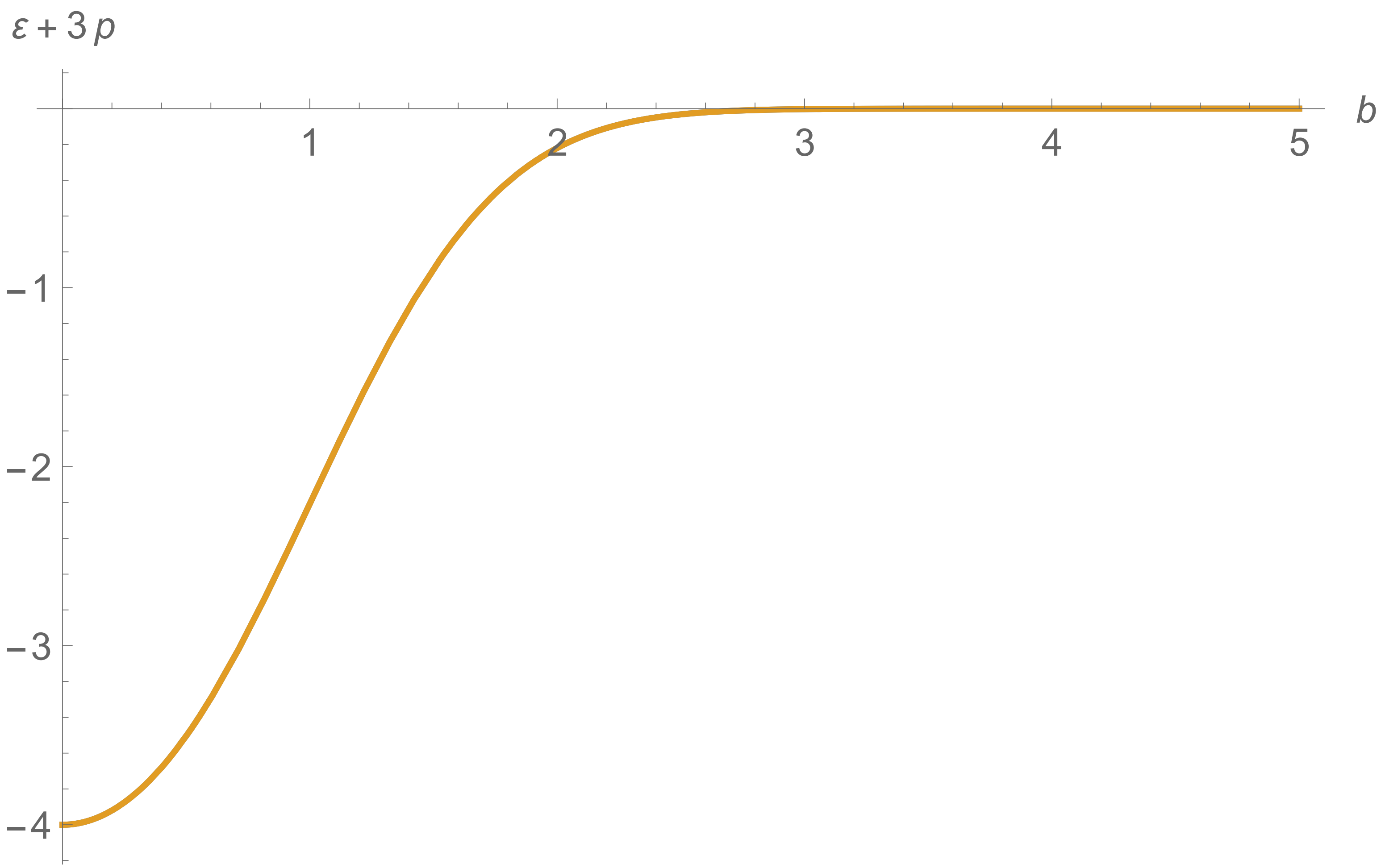


The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq \; ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

Type II Solution — Initial Acceleration of Finite Duration

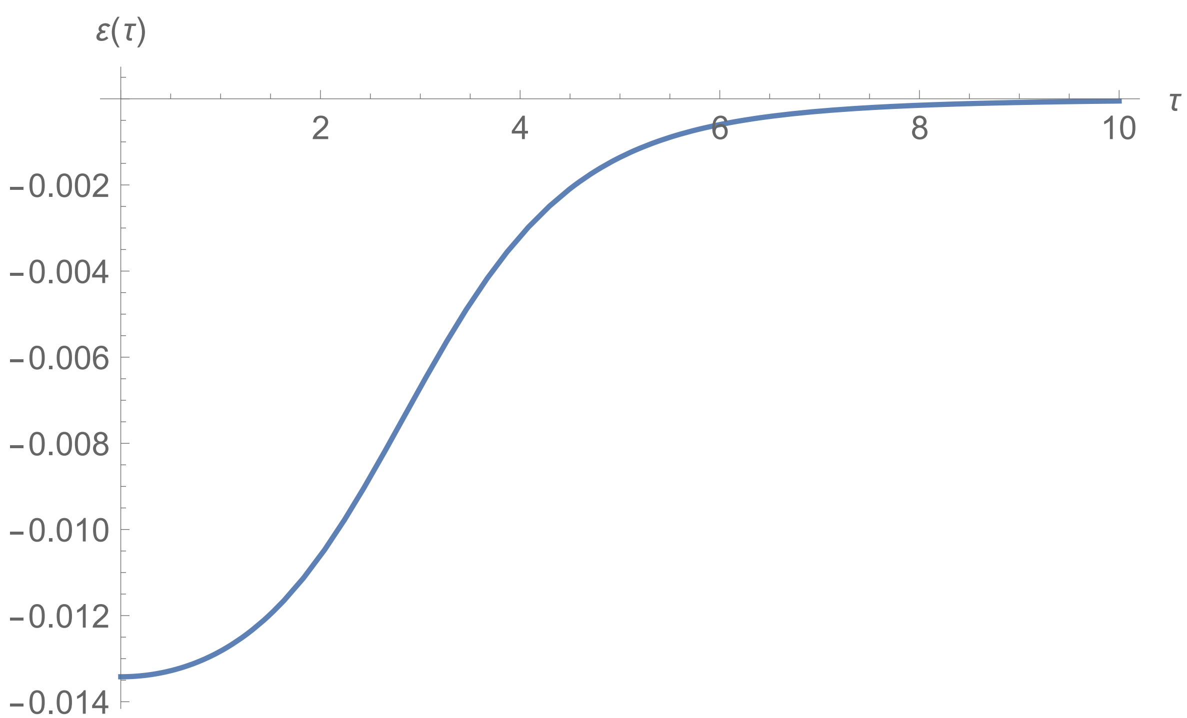
$$\epsilon + 3p = -\frac{2\mathcal{A}}{\mu_2^4}\ e^{-b^2(\tau)}(b^2(\tau) + \gamma^2\mu_2^2 - 2)\Lambda_{YM}^4, \qquad b \in [0, +\infty],$$

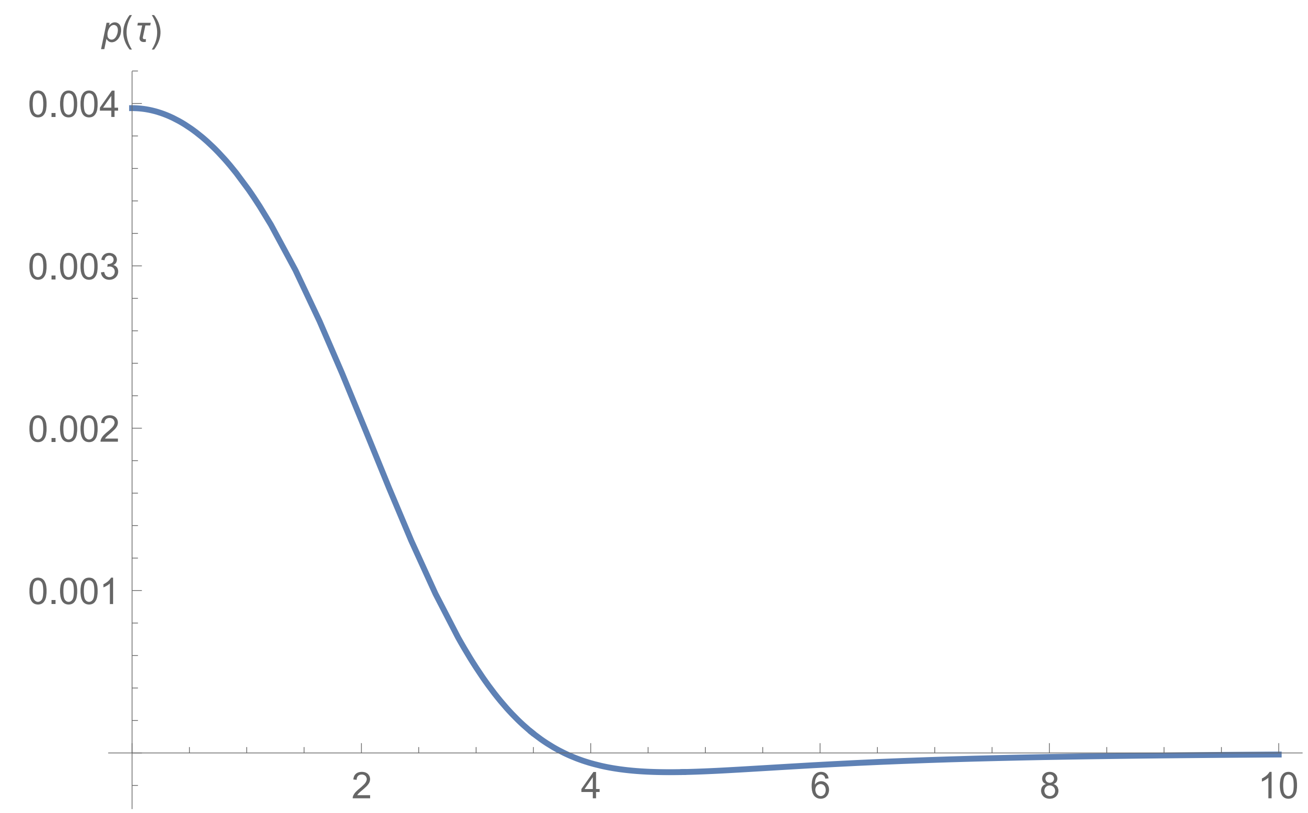


The r.h.s $\epsilon + 3p$ of the Friedmann acceleration equation (1.4) always negative

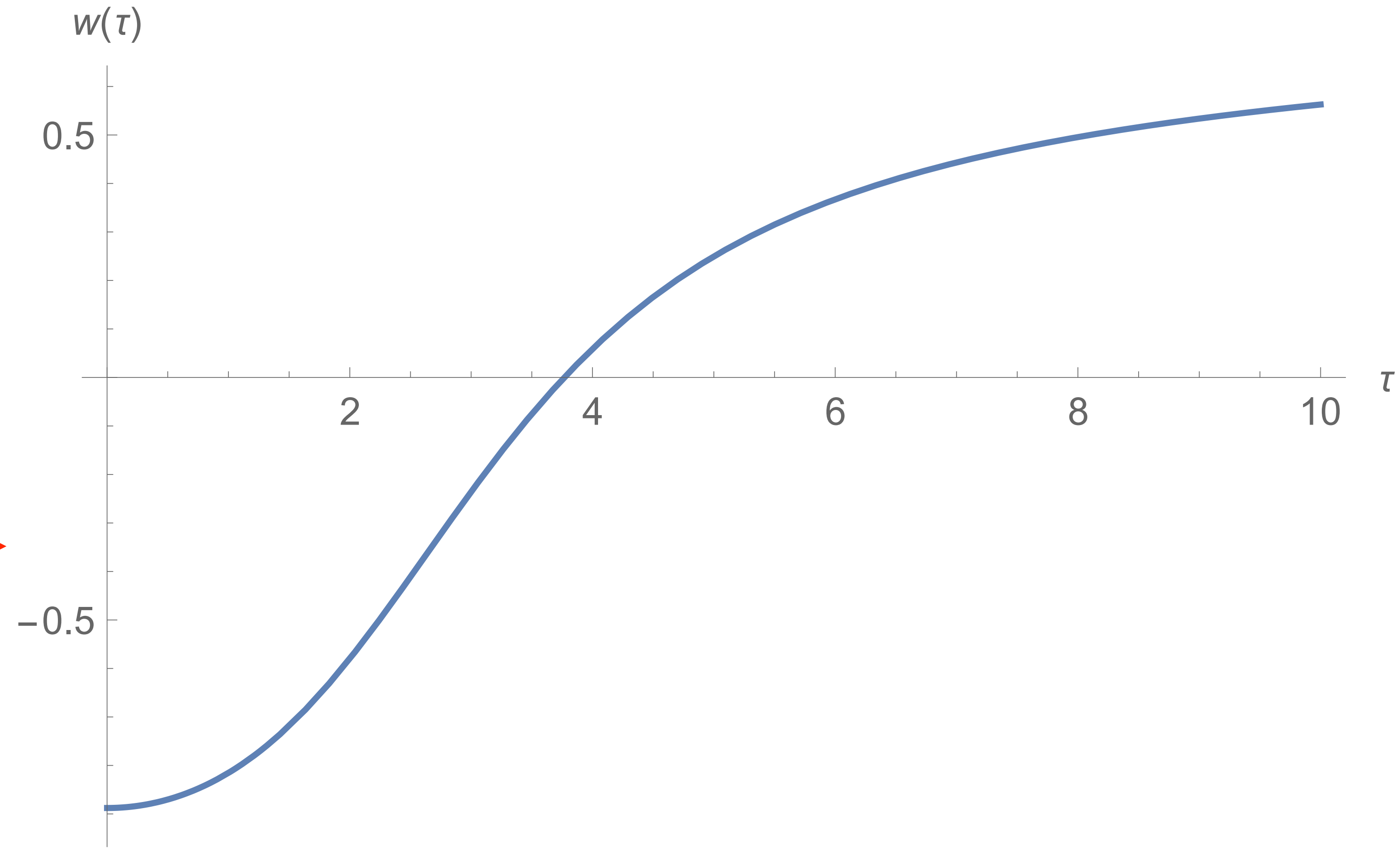
Evolution of Energy Density and Pressure

$$\epsilon = \frac{\mathcal{A}}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)\Lambda_{YM}^4, \qquad p = \frac{\mathcal{A}}{3\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} + 3\Big)\Lambda_{YM}^4.$$





Type II Solution — Effective Parameter w

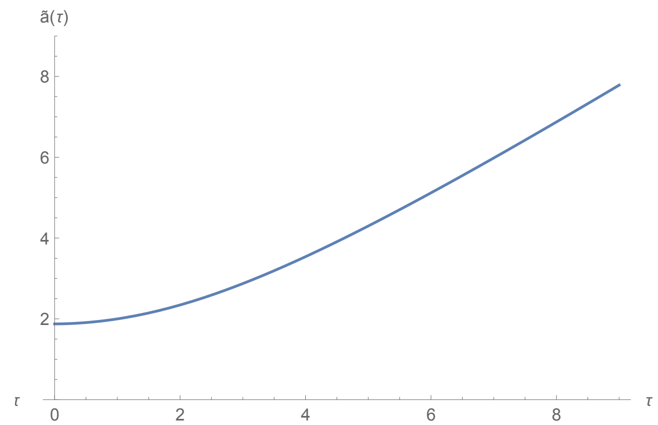


For the equation of state $p = w\epsilon$ one can find the behaviour of the effective parameter w

$$w_{II} = \frac{b^2(\tau) + \gamma^2\mu_2^2 - 4}{3\Big(b^2(\tau) + \gamma^2\mu_2^2\Big)}, \qquad\qquad -1 \leq w_{II},$$

$$w = \frac{p}{\epsilon} = \frac{\log\frac{1}{\tilde{a}^4(\tau)} + 3}{3\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)}.$$

Type II Solution   Initial Acceleration of Finite Duration



The number of e-foldings

typical parameters around $\gamma^2 = 1.211$, $\mu_2^2 \simeq 1.75$ we get $\tau_s = 10^{23}$ and $\mathcal{N} \simeq 53$. $\mathcal{N} = \ln \frac{a(\tau_s)}{a(0)}$.

$$t_s^{GUM} = \frac{L_{GUM}}{c}\tau_s \simeq 4.2 \times 10^{-13}\ sec, \qquad \text{where } L_{GUM} \simeq 1.25 \times 10^{-25} cm$$

$$a(0) = L_{GUM}\frac{\mu_2}{\gamma} \simeq 1.5 \times 10^{-25} cm, \qquad a(t_s) = L_{GUM}\frac{\mu_2}{\gamma}e^{\mathcal{N}} \simeq 1.25 \times 10^{-2} cm,$$

The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq \ ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq \; ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

At the late stages of the inflation the asymptotic behaviour of the scale factor became linear in time $a(t) \approx ct$ and corresponds to a flat geometry. The metric

$$ds^2 = c^2 dt^2 - c^2 t^2 (d\chi^2 + \sin^2 \chi d\Omega^2)$$

transformation $r = ct \sinh \chi$, $\tau = t \cosh \chi$ reduce metric to flat metric $ds^2 = c^2 d\tau^2 - (dr^2 + r^2 d\Omega^2)$.

Type IV Solution - Late time Acceleration

The type IV solution is defined in the region $\gamma^2 > \gamma_c^2$ where the equation

$$U_{-1}(\mu) = \frac{1}{\mu^2}\Big(\log\frac{1}{\mu^4} - 1\Big) + \gamma^2 = 0$$

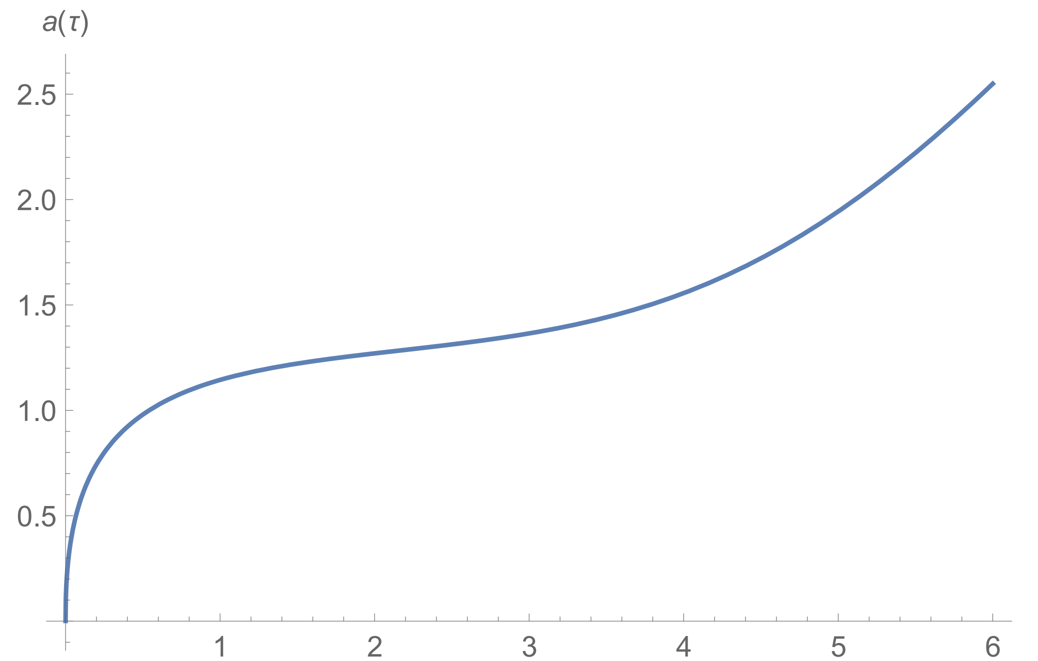
$$\tilde{a} = \mu_c e^b, \qquad b \in [-\infty, \infty], \qquad 2 < \gamma^2 \mu_c^2, \qquad \gamma_c^2 = \frac{2}{\sqrt{e}},$$

$$\frac{db}{d\tau} = \sqrt{\frac{2}{e}}\; e^{-2b}\Big(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b\Big)^{1/2}.$$

$$2g^2\mathcal{F} = e^{-4b(\tau)-1}\Lambda_{YM}^4,$$

$$\epsilon = 2\mathcal{A}e^{-4b(\tau)-1}\Big(- 2b(\tau) - 1\Big)\Lambda_{YM}^4, \qquad p = \frac{2\mathcal{A}}{3}e^{-4b(\tau)-1}\Big(- 2b(\tau) + 1\Big)\Lambda_{YM}^4.$$

Type IV Solution - Late time Acceleration



$$q_{IV} \simeq -\frac{2}{\gamma^2 \mu_c^2} b e^{-2b} \to 0.$$

$$H = \sqrt{\frac{2}{e}} \frac{e^{-2b}}{L} \left(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \right)^{1/2} \simeq \frac{1}{ct}.$$

$$\Omega_{vac} = 1 - \frac{\gamma^2}{\left(\frac{d\tilde{a}}{d\tau}\right)^2} = 1 - \frac{\gamma^2 e^{2b}}{\gamma_c^2 \left(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \right)} \to 0.$$

Primordial Gravitational Waves

The coefficient of amplification of primordial gravitational waves obtained by Grishchuk

$$K = \frac{1}{2}\Big(\frac{\beta}{n\eta_0}\Big)^2, \qquad \beta = \frac{1-3w}{1+3w}$$

the equation of state $p = w\epsilon$ is parametrised in terms of the barotropic parameter and n is a wave number and the wavelength is $\lambda = 2\pi a/n$.

The amplification tends to zero if equation of state is purely relativistic $w = 1/3$

The relation between energy density and the pressure is of the form

$$p = \frac{1}{3}\epsilon + \frac{4}{3}\frac{\mathcal{A}}{\tilde{a}^4(\tau)}\Lambda_{YM}^4$$

$$h^i_j = h(\eta)\ Y^i_j\ e^{inx} = \frac{\theta(\eta)}{a(\eta)}\ Y^i_j\ e^{inx}, \qquad \theta^{''} + \theta(n^2 - \frac{a^{''}}{a}) = 0$$

Primordial Gravitational Waves

The Freidmann equation in the gauge field theory vacuum

$$\Big(\frac{\tilde{a}^{'}}{\tilde{a}}\Big)^2 = \frac{1}{\gamma^2}\frac{1}{\tilde{a}^2}\Big(\ln\frac{1}{\tilde{a}^4} - 1\Big) - k$$

together with the acceleration equation

$$\frac{\tilde{a}^{''}}{\tilde{a}} - \Big(\frac{\tilde{a}^{'}}{\tilde{a}}\Big)^2 = -\frac{1}{\gamma^2}\frac{1}{\tilde{a}^2}\Big(\ln\frac{1}{\tilde{a}^4} + 1\Big).$$

gives

$$\tilde{a}^{''} = -\frac{2}{\gamma^2}\frac{1}{\tilde{a}} - k\tilde{a}$$

and the linear perturbation equation will take the form

$$\theta^{''} + \theta\Big(n^2 + \frac{2}{\gamma^2}\frac{1}{\tilde{a}^2} + k\Big) = 0.$$

In case of Type II solution with $\tilde{a}(0) = \mu_2$ the system avoids a singular behaviour in vicinity $\eta = 0$.

The amplification of the primordial gravitational waves is due to the second term when $n^2 < 2/\gamma^2\mu_2^2$.

Primordial Gravitational Waves

$$\epsilon\ a^{3(1+w)} = const, \qquad a \sim t^{\frac{2}{3(1+w)}}, \qquad a \sim \eta^{\frac{2}{1+3w}} \quad \text{when} \quad k = 0.$$

$$\theta'' + \theta\left(n^2 - \begin{cases} \frac{2(1-3w)}{(1+3w)^2}\frac{1}{\eta^2}, & \eta \geq \eta_0 \\ 0, & \eta < \eta_0 \end{cases}\right) = 0.$$

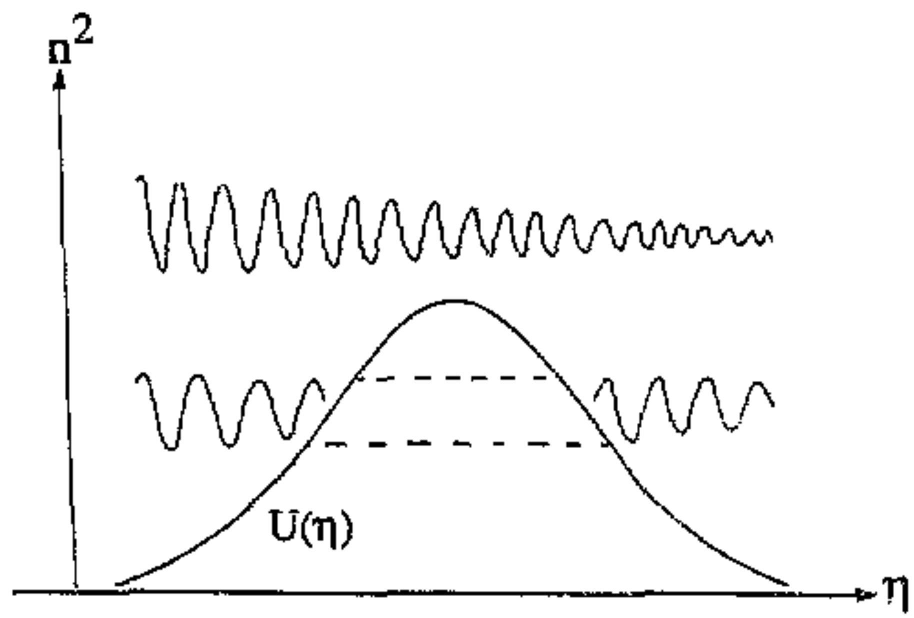


Figure 2. Parametric (superadiabatic) amplification of waves.

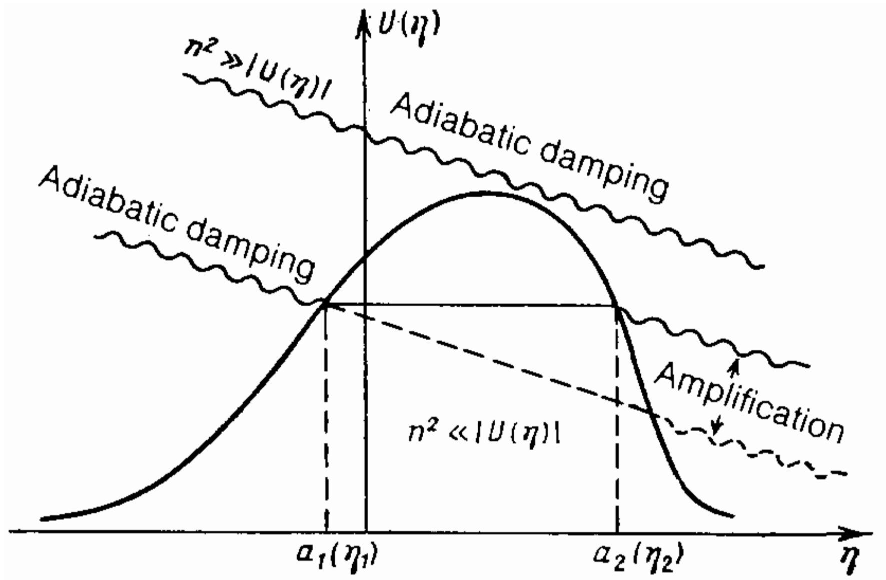


FIG. 3. Amplification of waves.

Astron. Astrophys. 160, 203–210 (1986)

ASTRONOMY
AND
ASTROPHYSICS

Collective relaxation of stellar systems

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$$\tau \simeq \left(\frac{T}{a^{4/3}\langle M\rangle N}\right)^{1/2} = \left(\frac{15}{4}\right)^{2/3} \frac{1}{2\pi\sqrt{2}} \frac{\langle v\rangle}{G\langle M\rangle n^{2/3}}. \tag{39}$$

The relaxation time (39) normalized by using characteristic values of the parameters of stellar systems like globular clusters and galaxies is

$$\tau \simeq 10^8\,\mathrm{yr} \left(\frac{\langle v\rangle}{10\,\frac{\mathrm{km}}{\mathrm{s}}}\right) \left(\frac{n}{1\,\mathrm{pc}^{-3}}\right)^{-2/3} \left(\frac{\langle M\rangle}{M_\odot}\right)^{-1}. \tag{40}$$

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Maximally chaotic dynamical systems of Anosov–Kolmogorov and fundamental interactions*

Discussions with:

V.Arnold
V.Ambartzumian
S.Chandrasekhar
Y.Zeldovich

Thank You !