Dissipative Simulation of Quantum Dynamics with Tensor Networks

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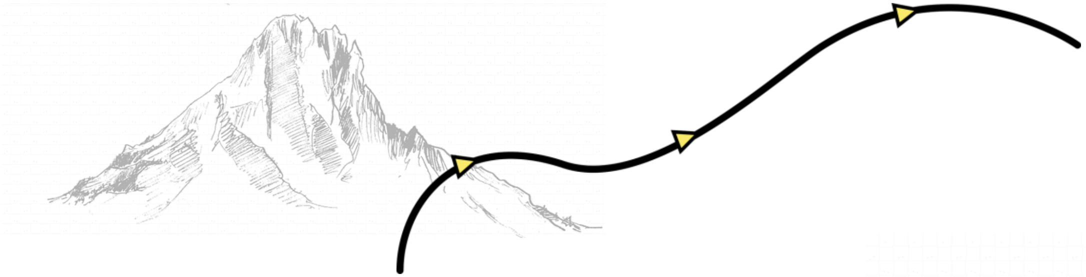
"la Caixa" Foundation

Roadmap

1. Quantum evolution
in tensor network language

Roadmap



2. Entanglement barrier

1. Quantum evolution
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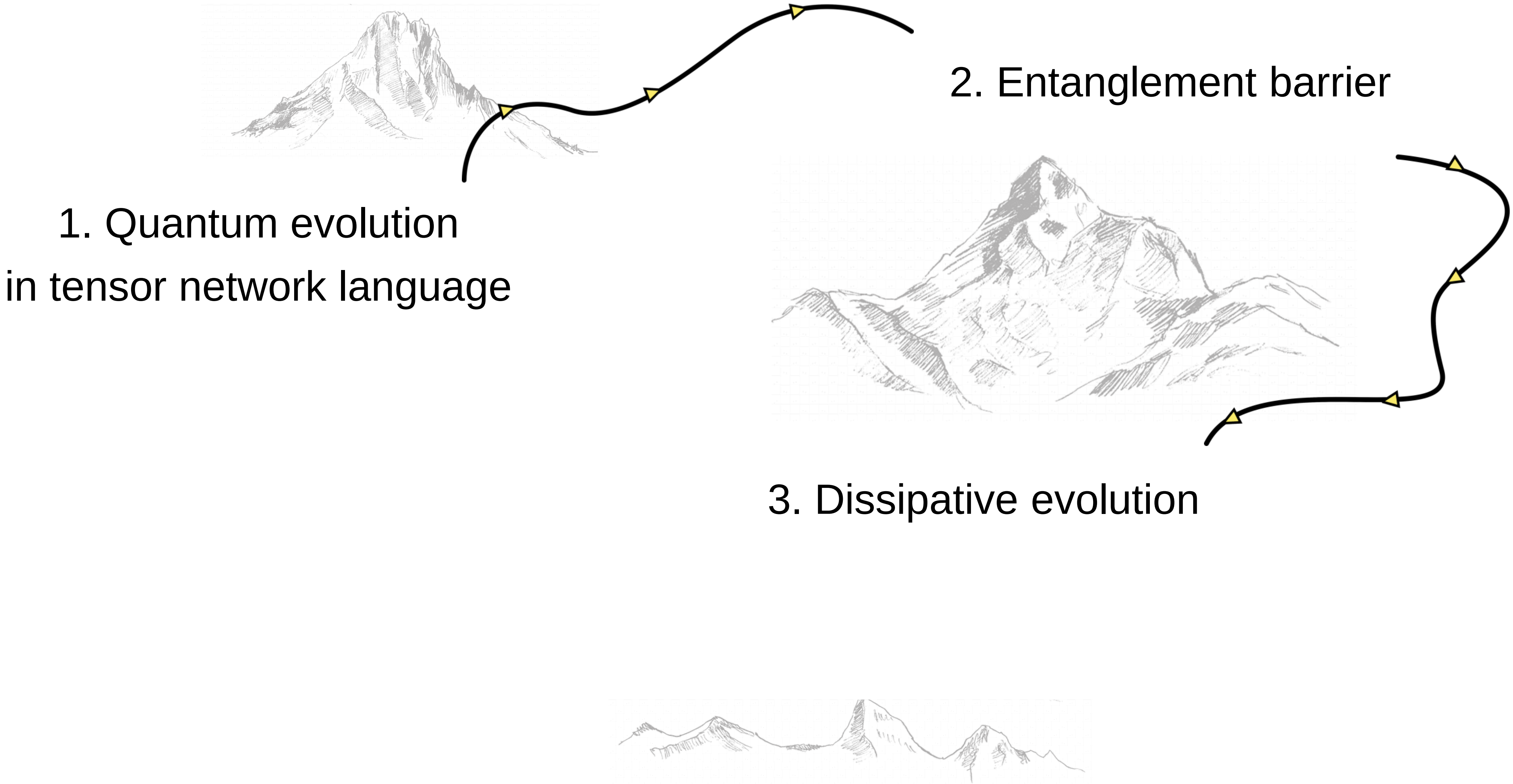

1. Quantum evolution
in tensor network language
2. Entanglement barrier
3. Dissipative evolution

Roadmap

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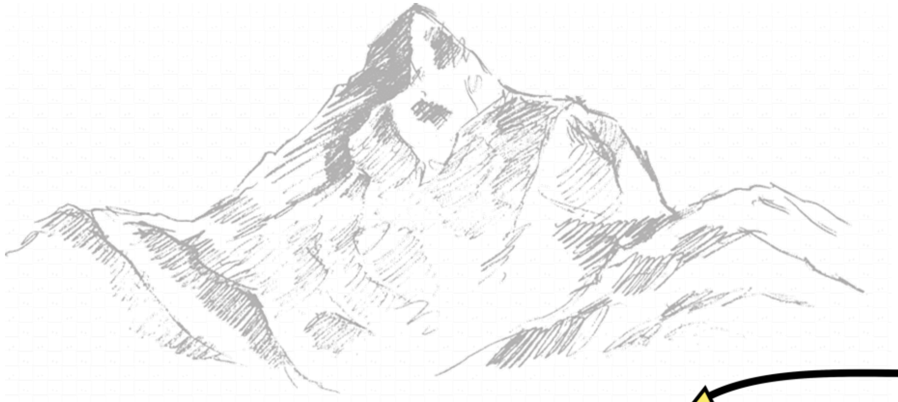

3. Dissipative evolution

4. Preliminary results

1. Quantum Evolution in Tensor Network Language

Given an initial state/density matrix

$\rho(t = 0)$

$\mathcal{O}$

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the evolution is generated by

$\delta\mathcal{U} = e^{-i \cdot \delta t \cdot \mathcal{H}}$

forward

$\delta\mathcal{U}^{\dagger} = e^{i \cdot \delta t \cdot \mathcal{H}}$

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The expectation value of the operator in the evolved state reads

$$\mathcal{O}(t = T) = \mathrm{tr}\{\mathcal{U}\ \rho(t = 0)\ \mathcal{U}^{\dagger}\ \mathcal{O}\}$$

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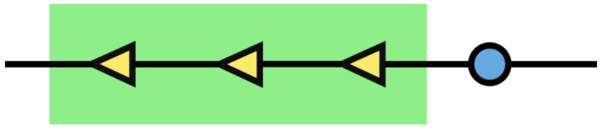
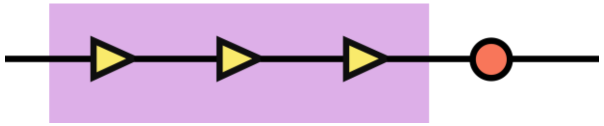


$$\Delta\mathcal{U}\ \rho(t = 0)$$

backward



$$\Delta\mathcal{U}^\dagger\mathcal{O}$$

1. Quantum Evolution in Tensor Network Language

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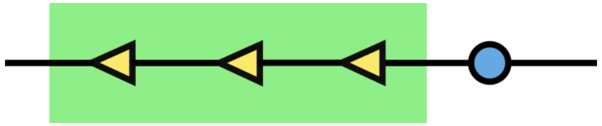

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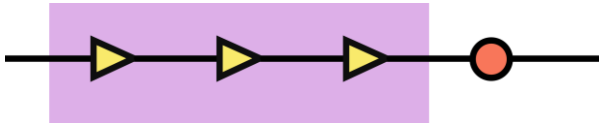
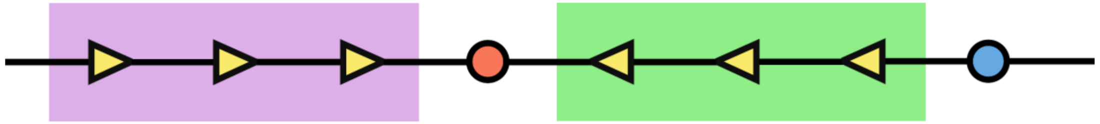


$$\Delta\mathcal{U}^\dagger\mathcal{O}$$

permute under trace



$$\mathcal{U}^\dagger\ \mathcal{O}\ \mathcal{U}\ \rho(t = 0)$$

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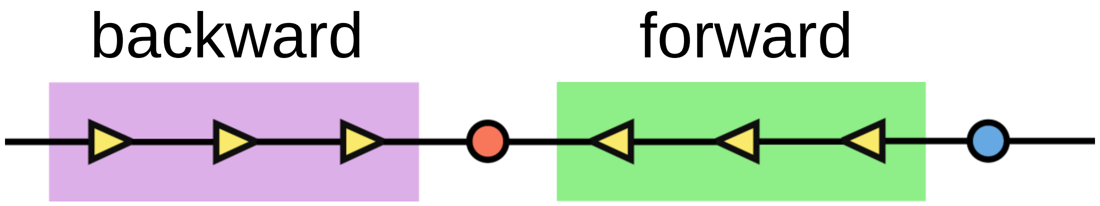

$$\mathcal{U}^\dagger \, \mathcal{O} \, \mathcal{U} \, \rho(t=0)$$

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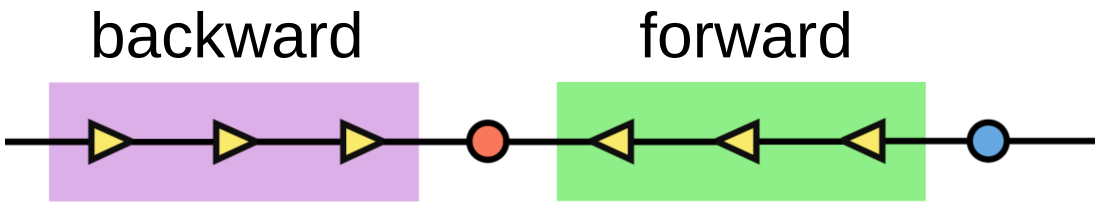
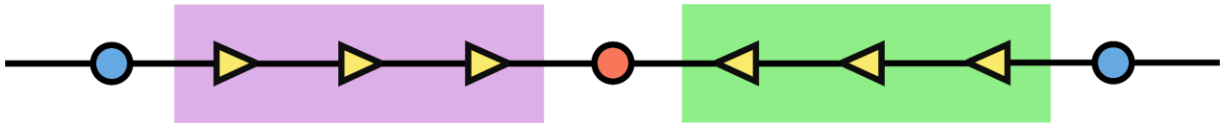


$$\mathcal{U}^\dagger \, \mathcal{O} \, \mathcal{U} \; \rho(t = 0)$$

$$\downarrow$$

pure initial state



$$\mathcal{U}^\dagger \, \mathcal{O} \, \mathcal{U} \; |\Psi(t = 0)\rangle\langle\Psi(t = 0)|$$

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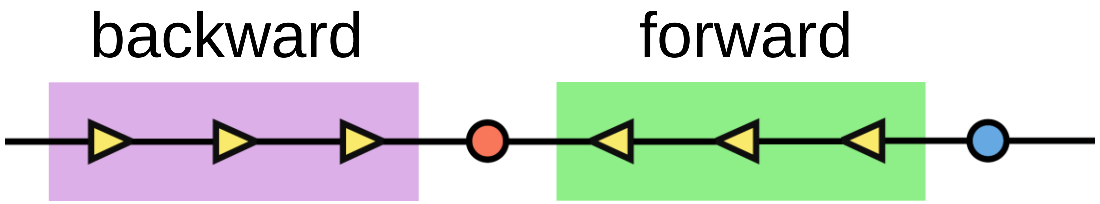

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↓

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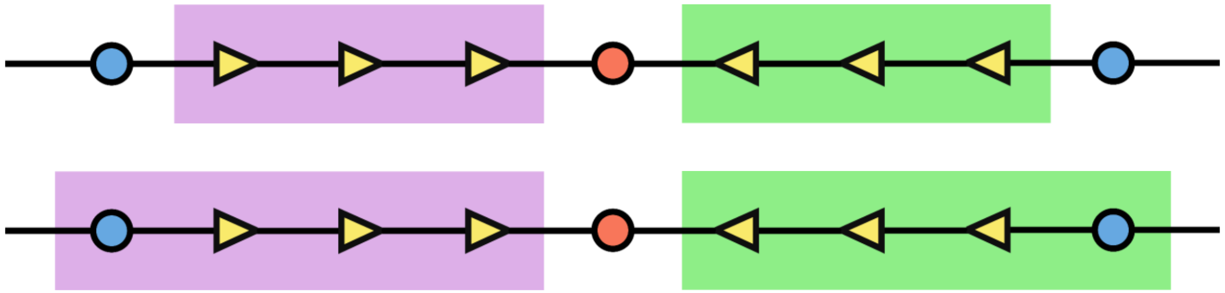

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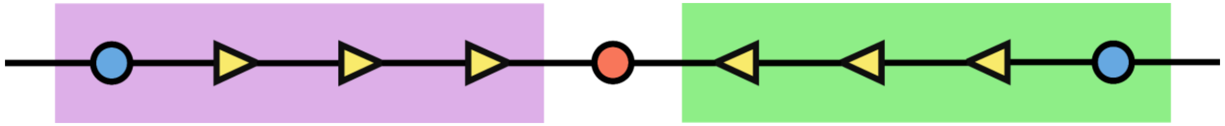

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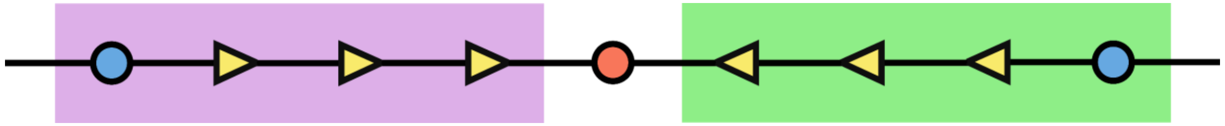
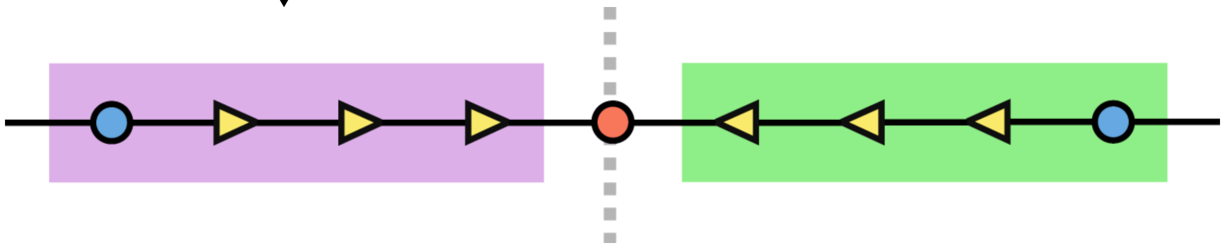


$$\mathcal{U}^\dagger\ \mathcal{O}\ \mathcal{U}\ |\Psi(t=0)\rangle\langle\Psi(t=0)|$$

fold the network



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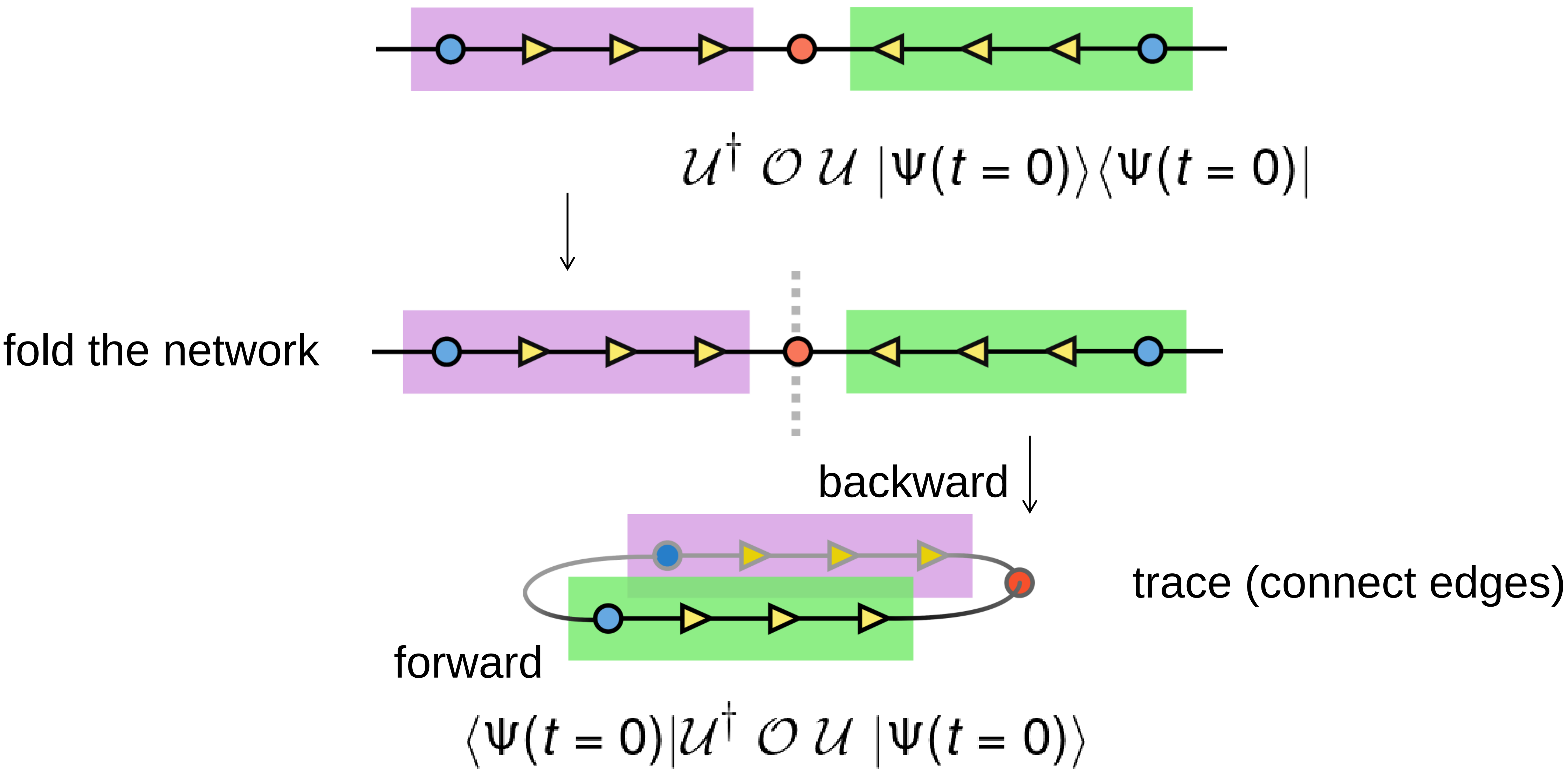
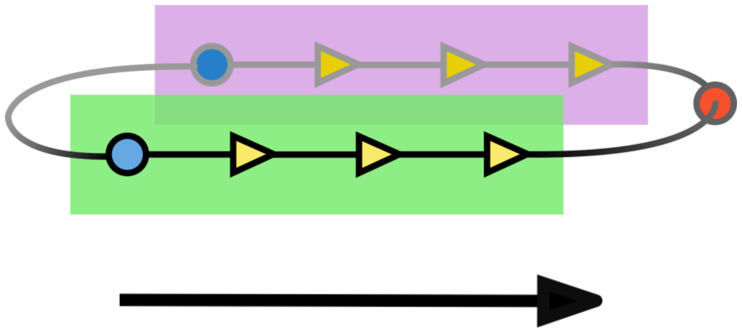


1. Quantum Evolution in Tensor Network Language

Time Contour

(field theory)

$\mathcal{O}(t = T)$



time flow

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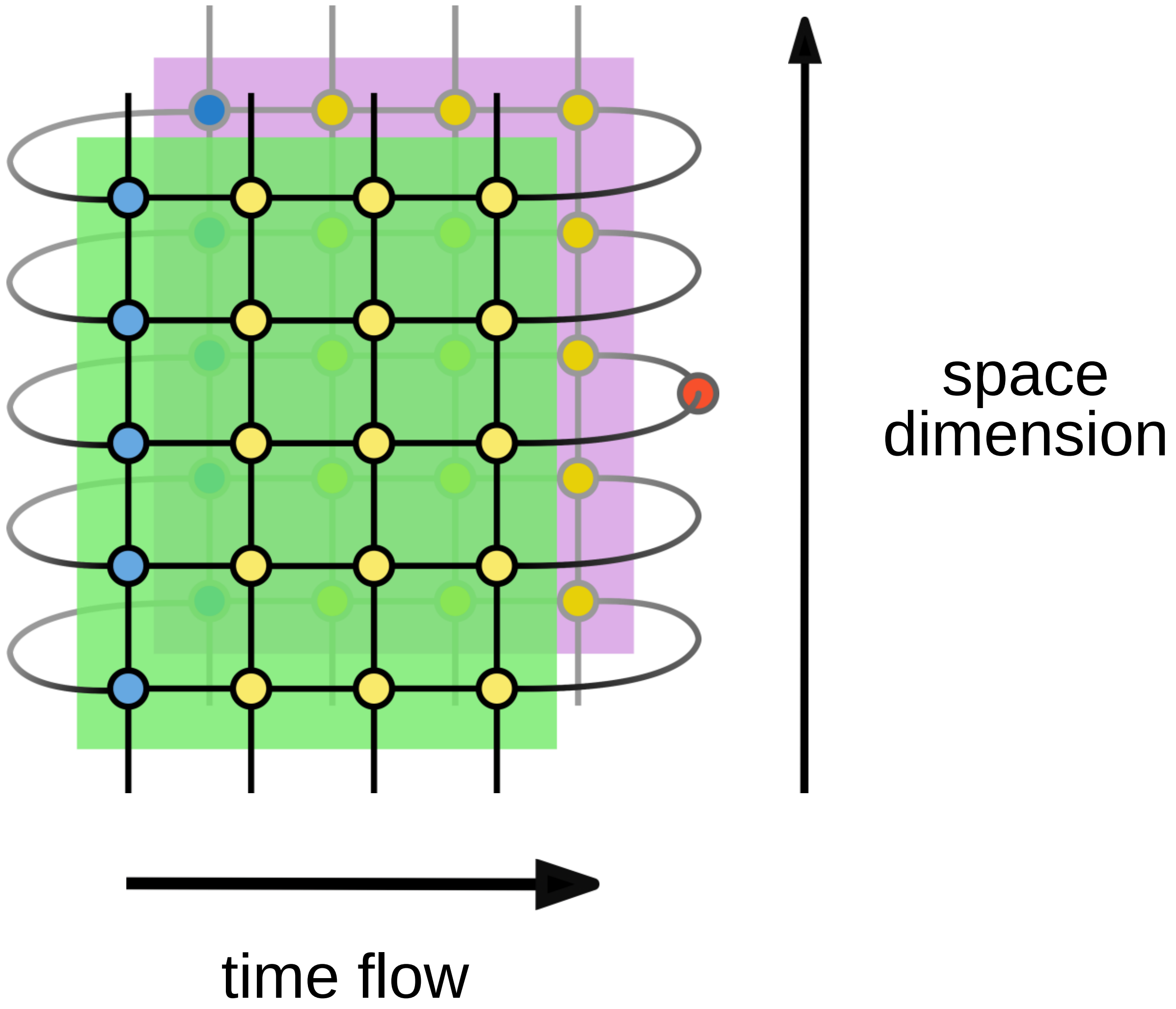

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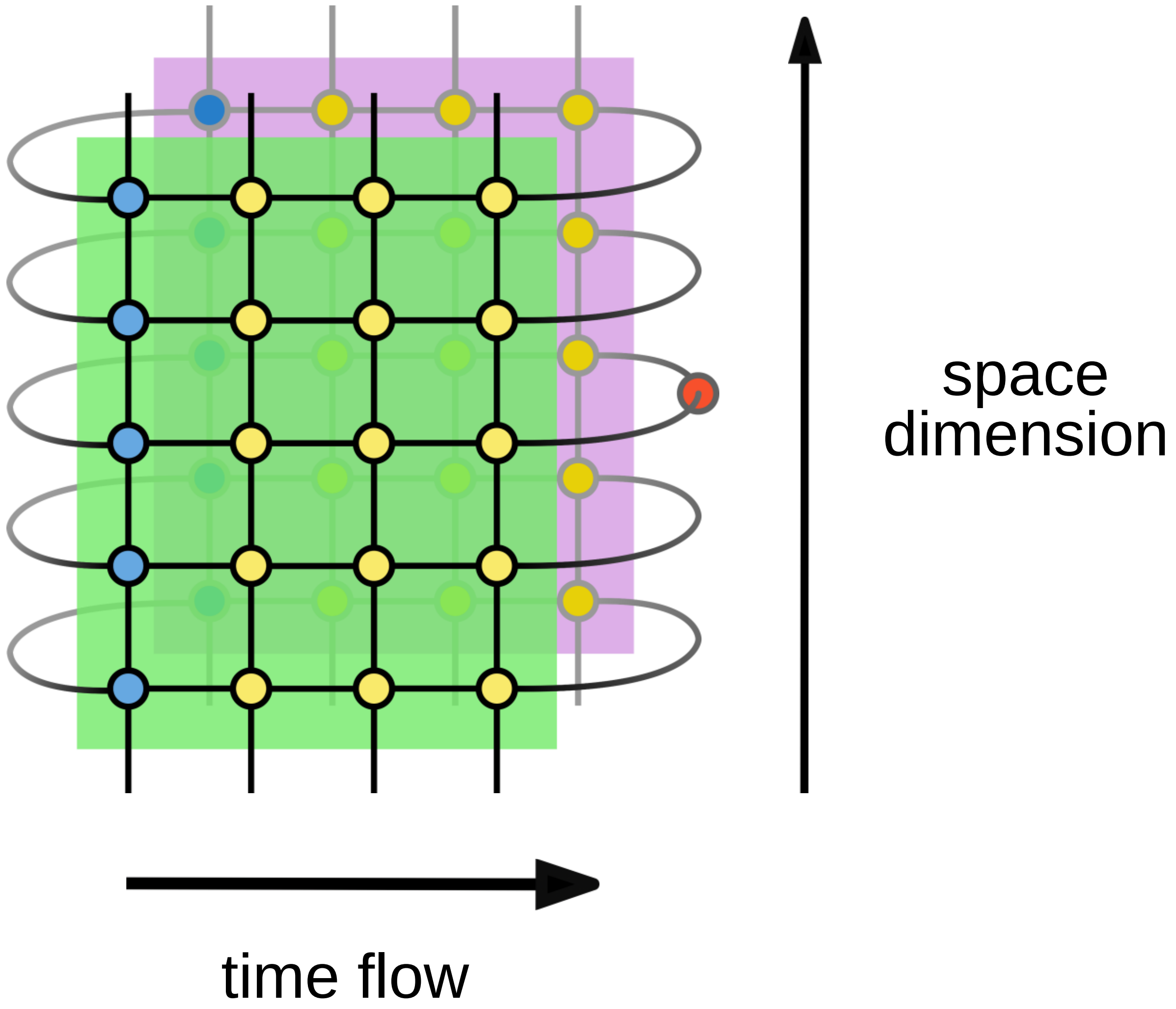

Can reach big sizes by **efficiently factoring** the operators into connected **local** tensors

2. Entanglement Barrier

Swept under the carpet:

TNs work while **spatial entanglement is low**

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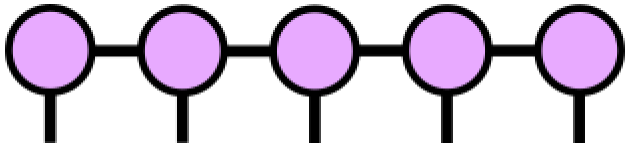

Matrix Product State

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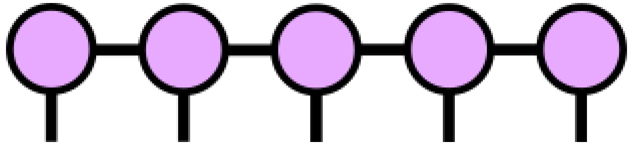
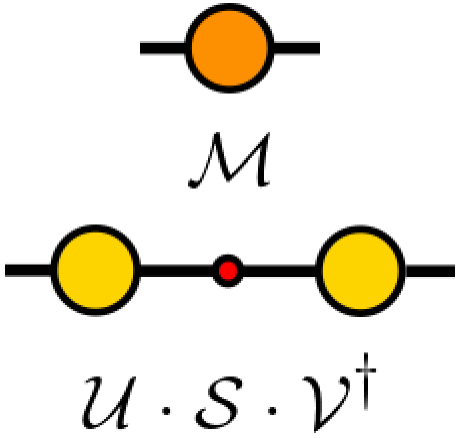


Matrix Product State

Singular
Value
Decomposition



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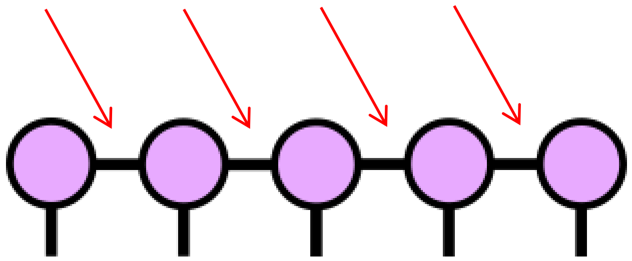

Matrix Product State

←

Singular

Value

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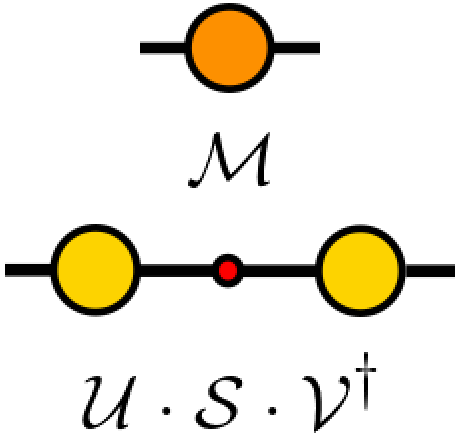

numerically: need **bounded number of singular values** per link

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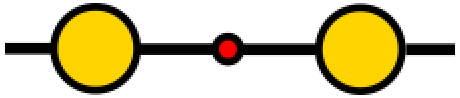

$$\left\{ s_i(\ell) \right\}_i^{\chi}$$

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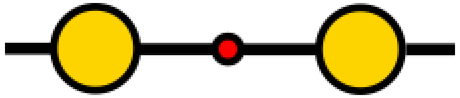
Bipartite Entropy ←



$$S(\ell) = -\sum_{i}^{\chi} s_i^2(\ell) \cdot \log\left(s_i^2(\ell)\right)$$

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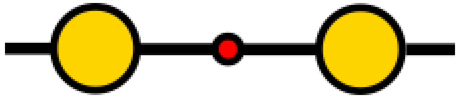
Bipartite Entropy ⟵ 

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physically: **subextensive entropy, no volume law**

2. Entanglement Barrier

Nevertheless, evolution generically entangles

i.e. the sum of singular values **grows exponentially**

arXiv:0706.2480 **arXiv:0903.2432**

3. Dissipative Evolution

Some works found that dissipation could lower entanglement

arXiv:2004.05177

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$\longleftrightarrow$

Our proposal is based on decoherence

arXiv:2004.05177

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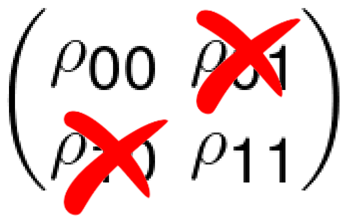

We introduce a quantum channel targetting **unnecessary coherence** for the description of **local observables**

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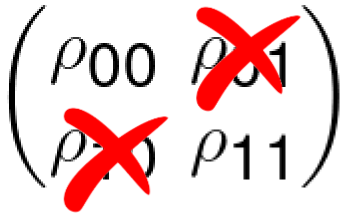
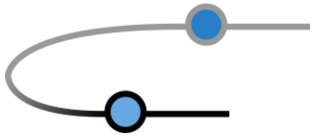


We introduce a quantum channel targetting **unnecessary coherence** for the description of **local observables**

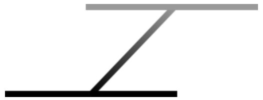
Connection to **thermalization**: reduced system is expected to decohere

3. Dissipative Evolution

Engineer the quantum channel:

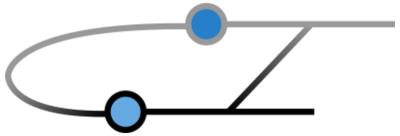


$$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \longrightarrow$$



copy tensor

$$\longrightarrow$$



$$\begin{pmatrix} \rho_{00} & 0 \\ 0 & \rho_{11} \end{pmatrix}$$

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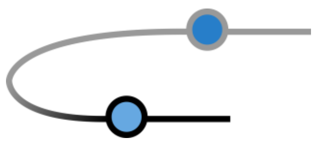

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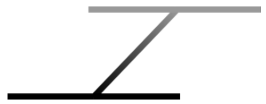

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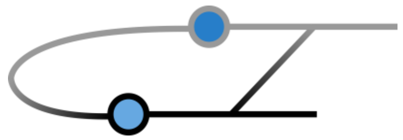
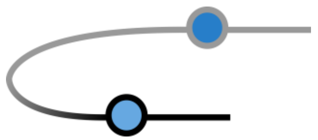


$$\begin{pmatrix} \rho_{00} & 0 \\ 0 & \rho_{11} \end{pmatrix}$$

Can decide its intensity:



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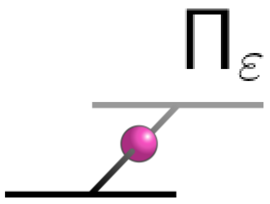
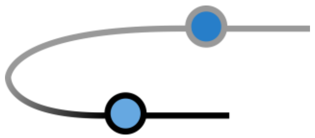


core copy tensor

$$\begin{pmatrix} \rho_{00} & \varepsilon\rho_{01} \\ \varepsilon\rho_{10} & \rho_{11} \end{pmatrix}$$

3. Dissipative Evolution

Engineer the quantum channel:



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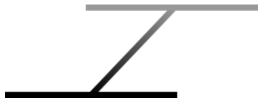

copy tensor

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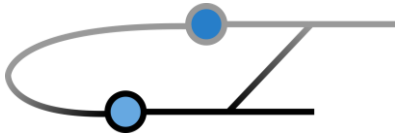

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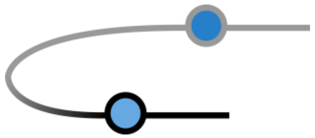

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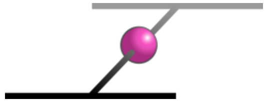
Π_ε



$$\begin{pmatrix} \rho_{00} & \varepsilon\rho_{01} \\ \varepsilon\rho_{10} & \rho_{11} \end{pmatrix}$$

core copy tensor

choose a mask, like σ_X

weight ε

3. Dissipative Evolution

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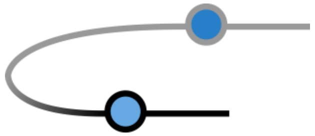

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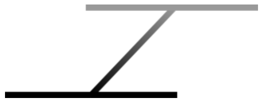

copy tensor

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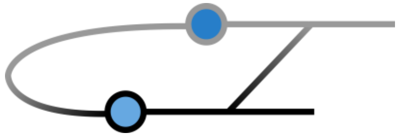

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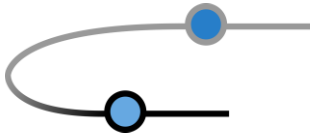

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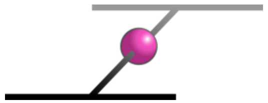


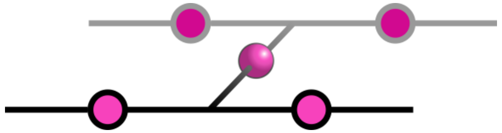
$$\begin{pmatrix} \rho_{00} & \varepsilon\rho_{01} \\ \varepsilon\rho_{10} & \rho_{11} \end{pmatrix}$$

core copy tensor

↙ choose a mask, like σ_X

↘ weight ε

More freedom: the copy tensor is **not basis covariant**, can introduce rotations

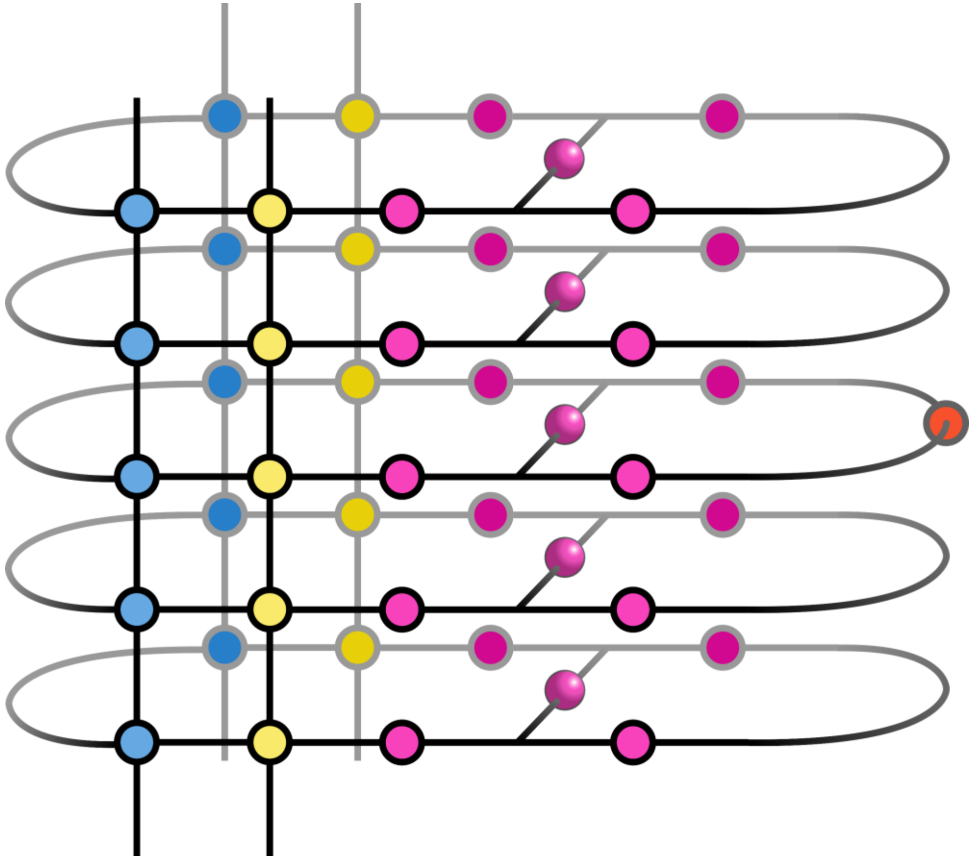


$\mathcal{W}$ $\mathcal{W}^\dagger$

3. Dissipative Evolution

Design an auxiliary evolution expected to retrieve the right local observables:

one-step contour:



$|\Psi(t=0)\rangle$

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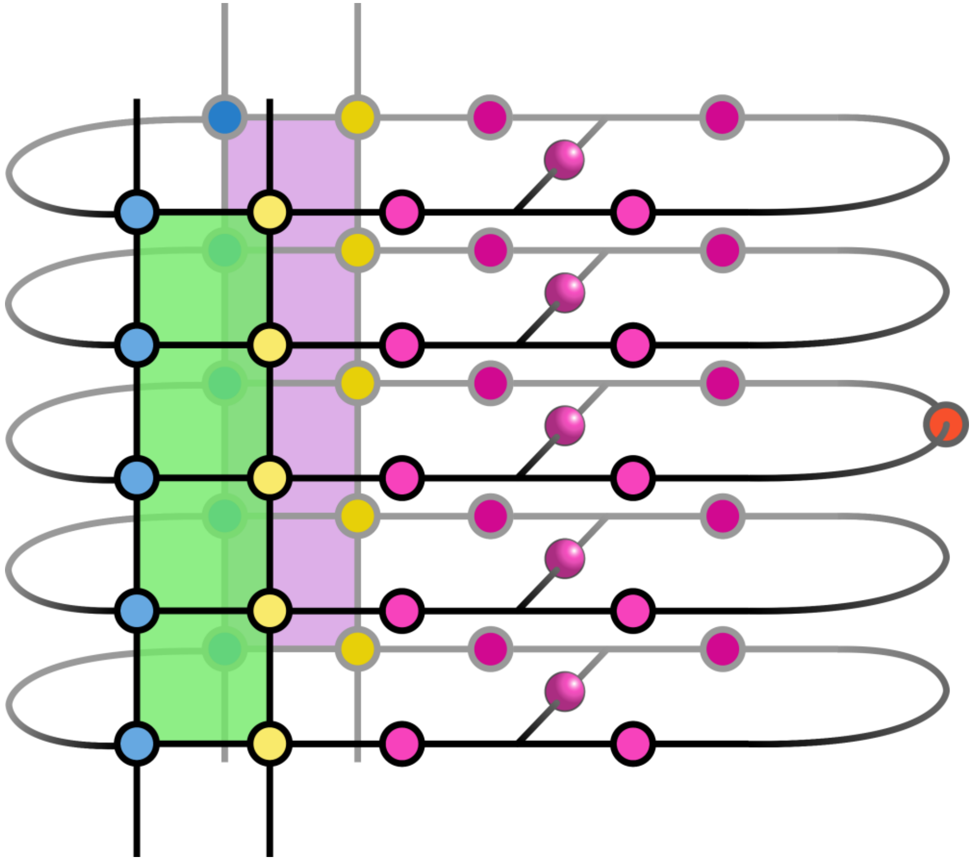
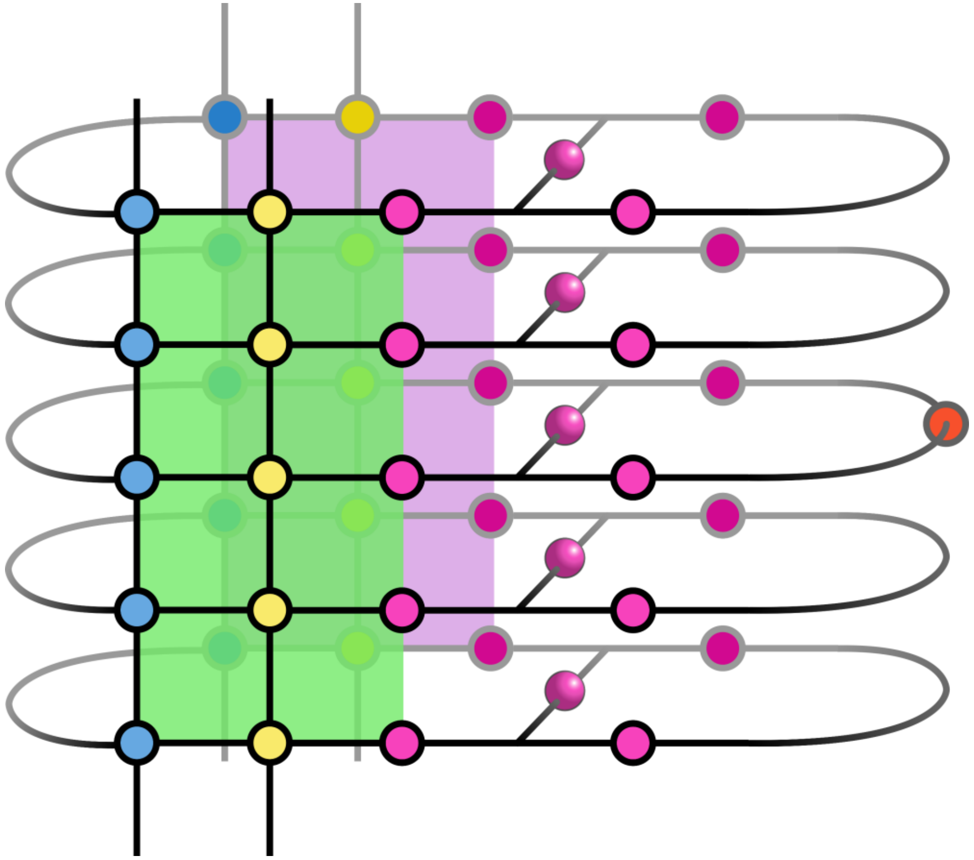


$$\delta\mathcal{U}\ |\Psi(t=0)\rangle$$

3. Dissipative Evolution

Design an auxiliary evolution expected to retrieve the right local observables:

one-step contour:



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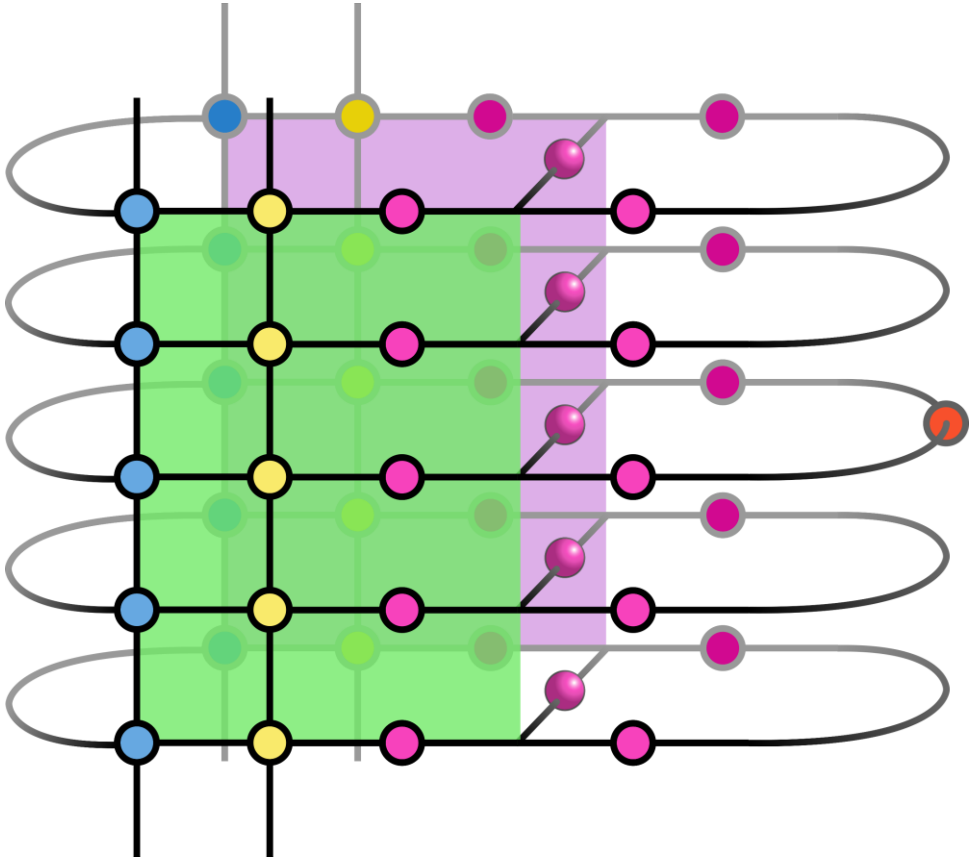

$$\Pi_{\varepsilon}\{\, \mathcal{W} \;\; \delta\mathcal{U} \;\; |\Psi(t=0)\rangle \,\}$$

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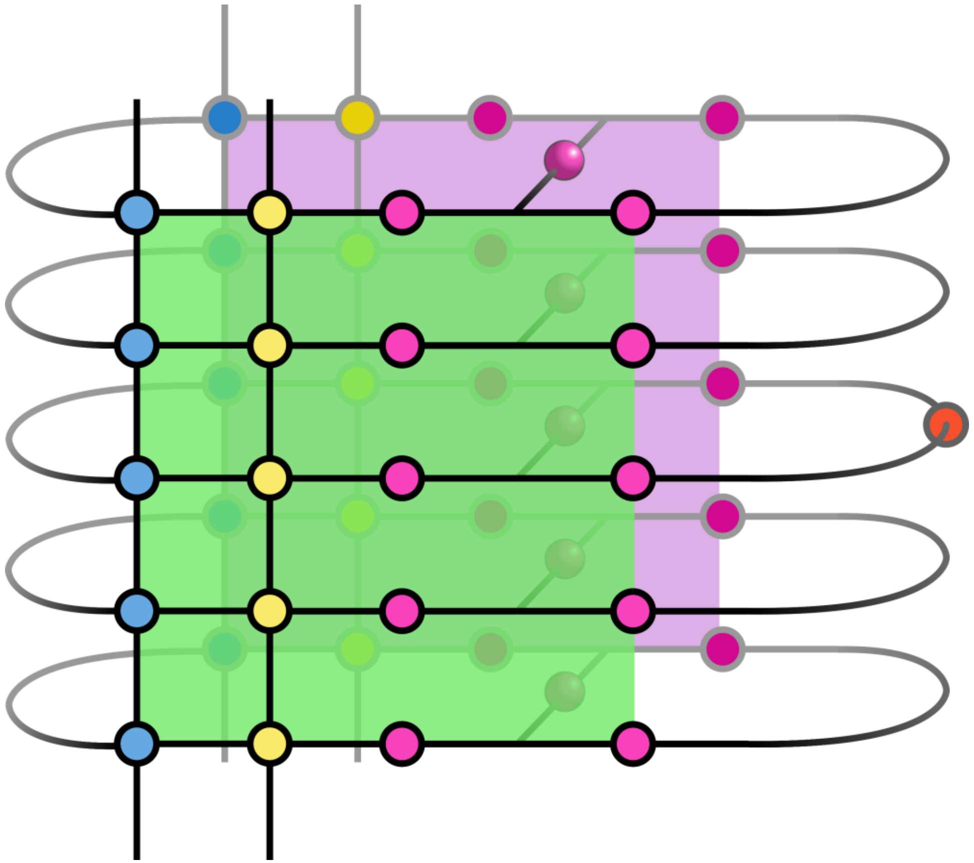

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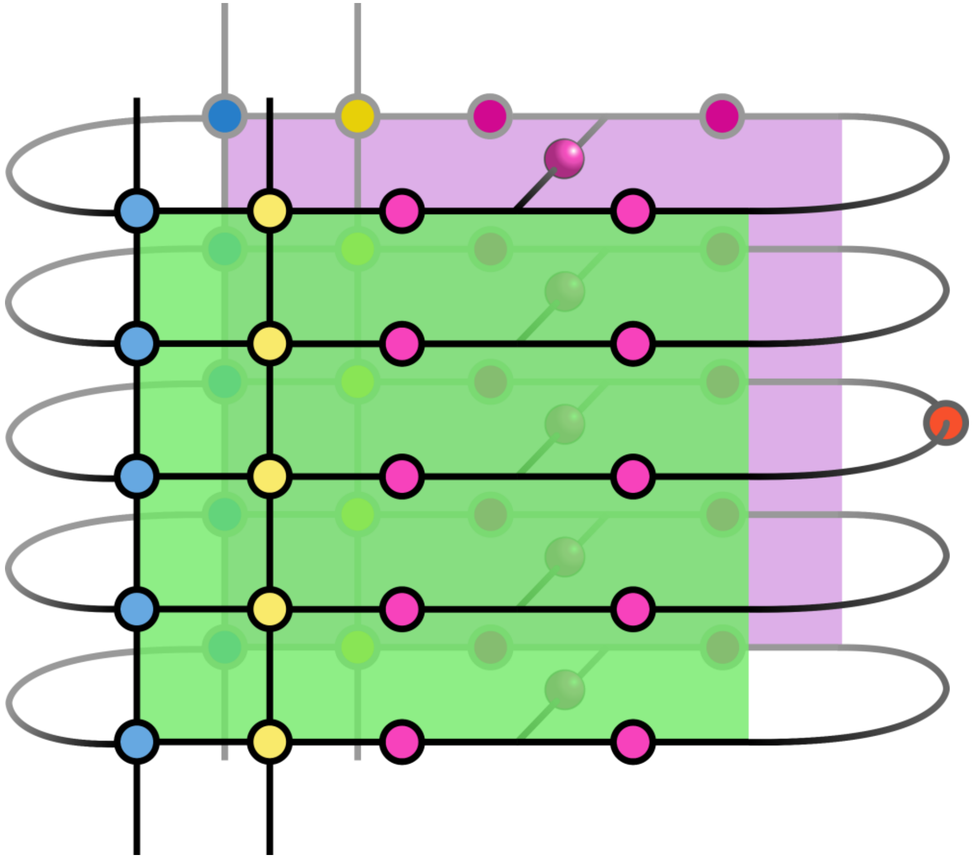


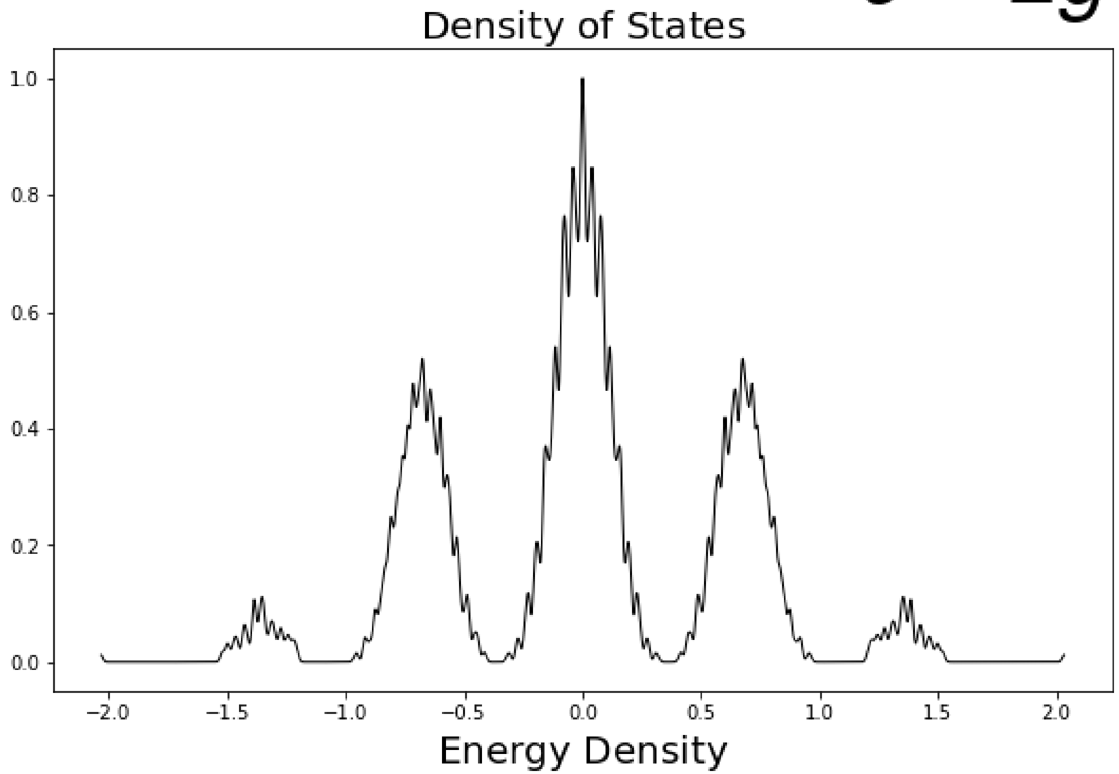
$$\mathcal{O}\,\mathcal{W}^{\dagger}\,\Pi_{\varepsilon}\{\,\mathcal{W}\;\;\delta\mathcal{U}\;|\Psi(t=0)\rangle\,\}$$

4. Preliminary Results

Currently exploring **small systems** $L \leq 12$ for the **Ising model**

$$\mathcal{H} = -J \sum_i \sigma_i^z \sigma_{i+1}^z - g \sum_i \sigma_i^x$$

$J = 2g$



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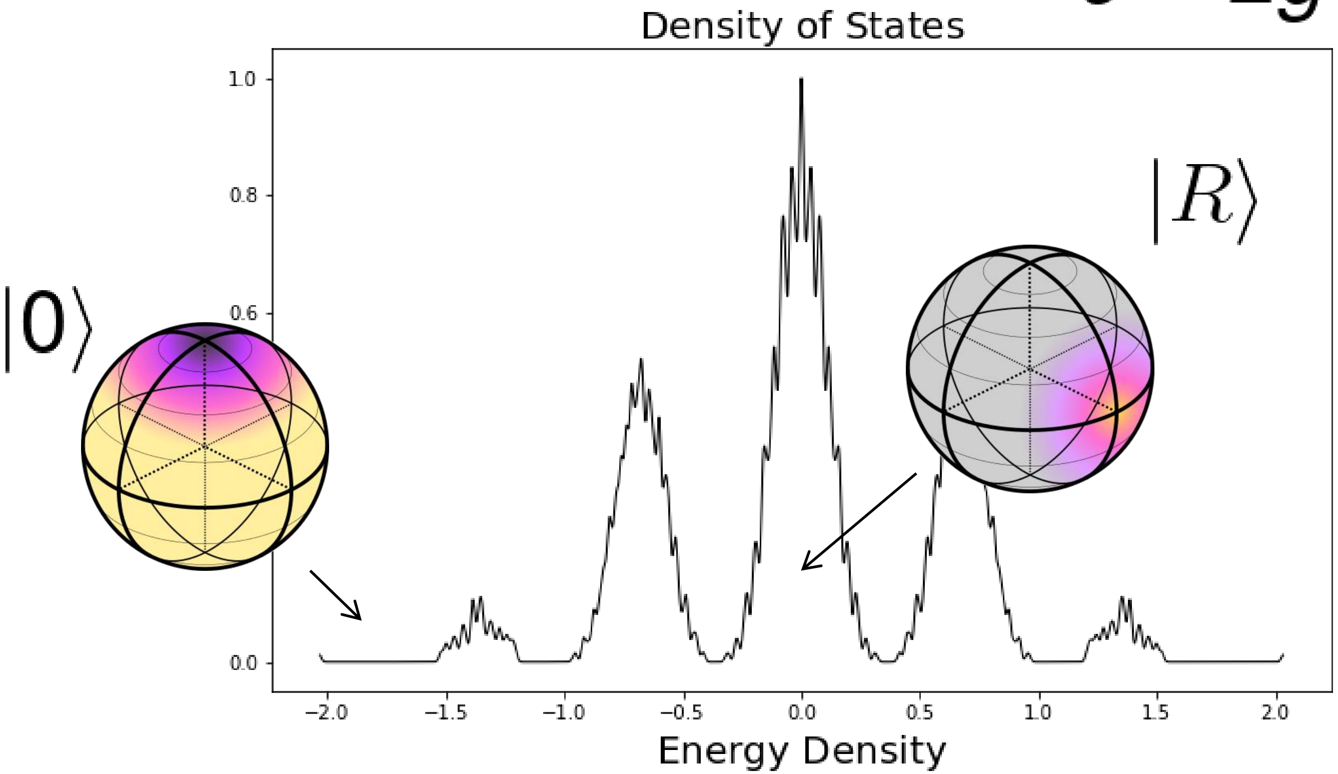

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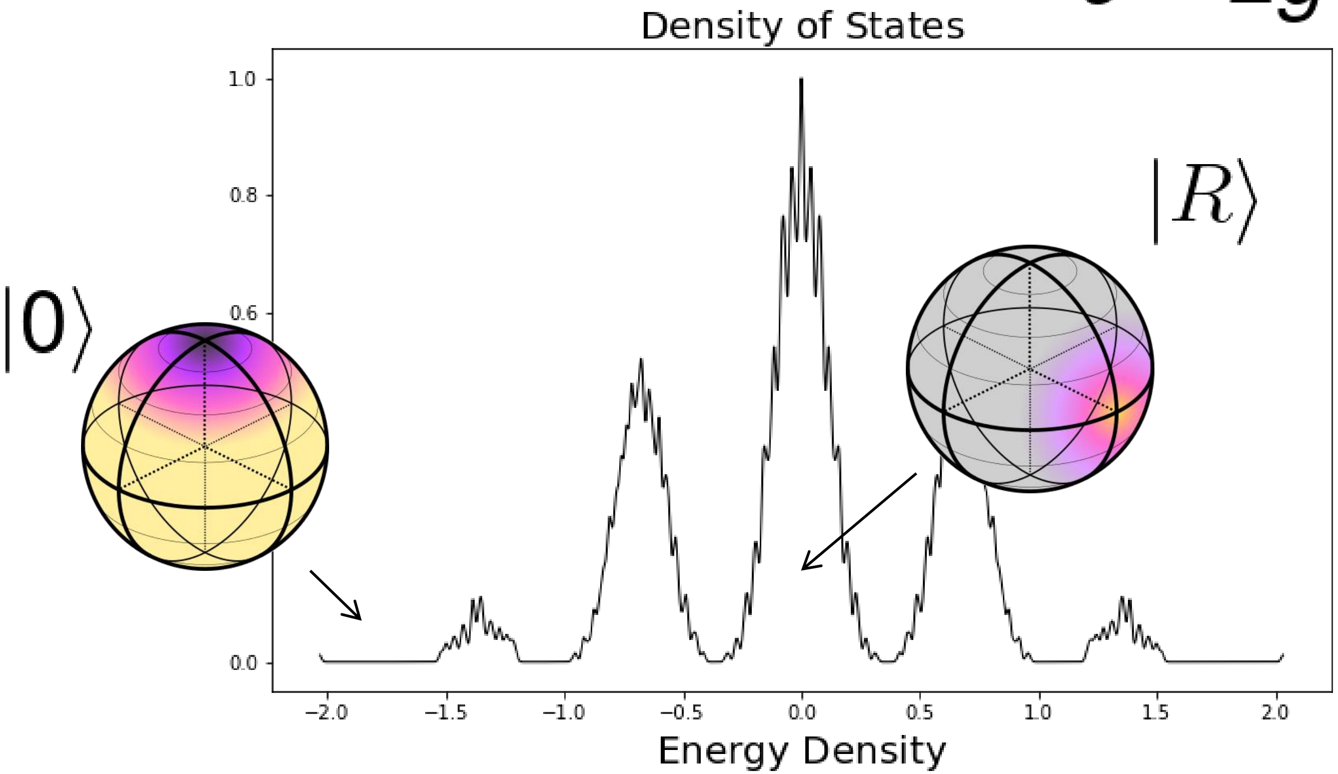
corresponding to many energies (temperatures)



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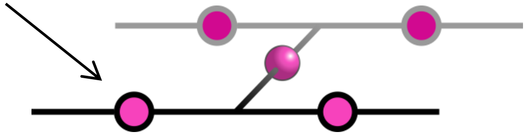

Basis election: **parallel magnetization** $\mathbb{M}^{||} = \sum_i \sigma_i^{||}$

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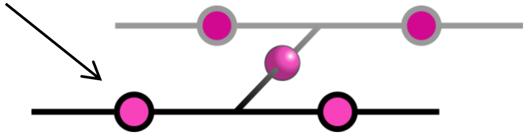

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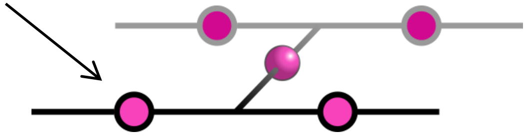

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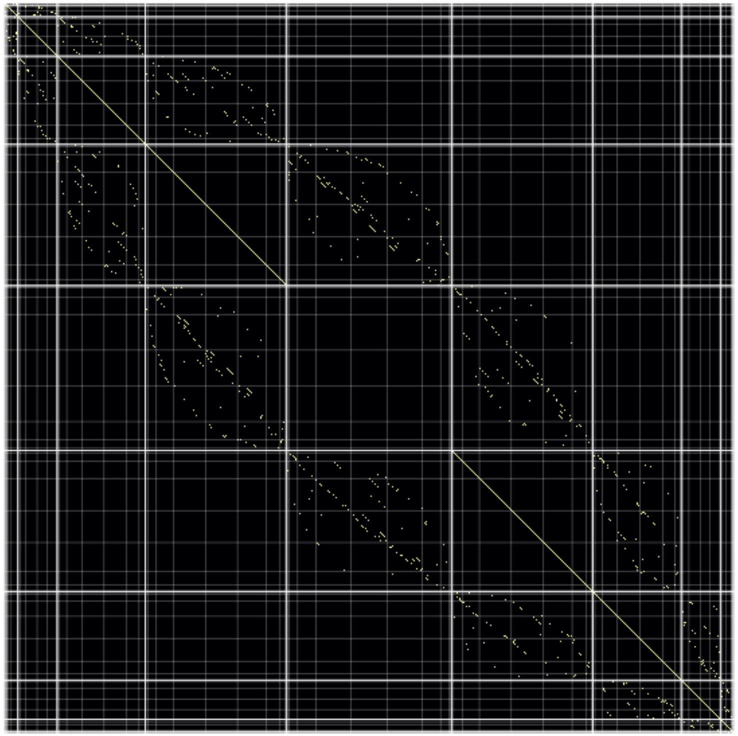
(2) magnetization sectors: **tidy and target** particular coherences

$\mathbb{M}_x^{||} + \mathbb{M}_z^{||}$



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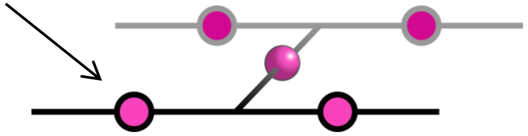

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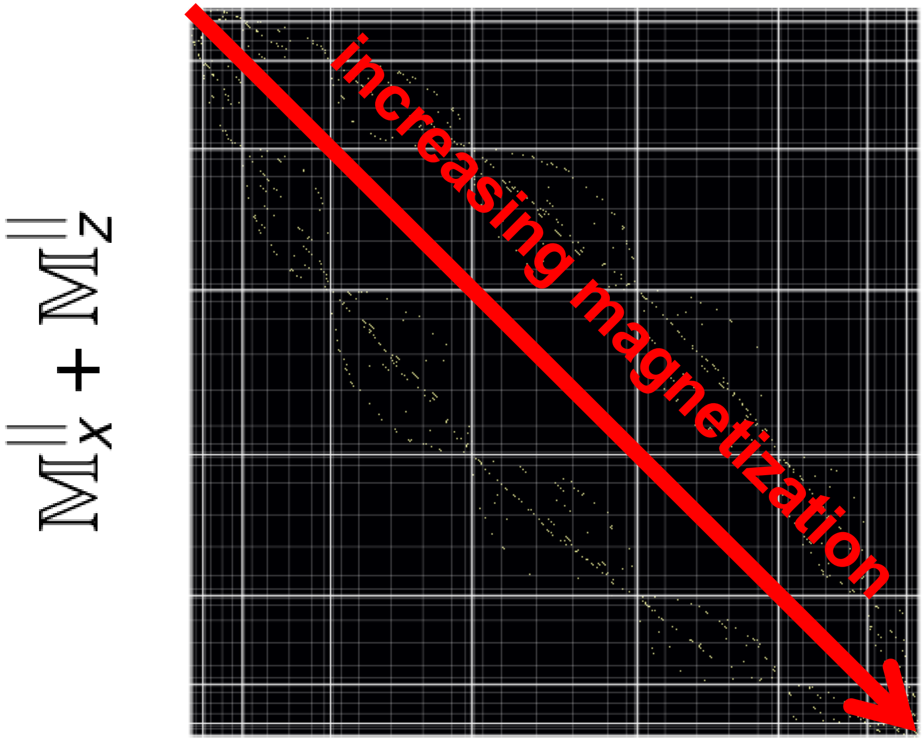

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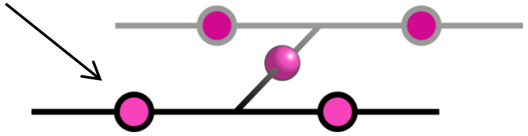

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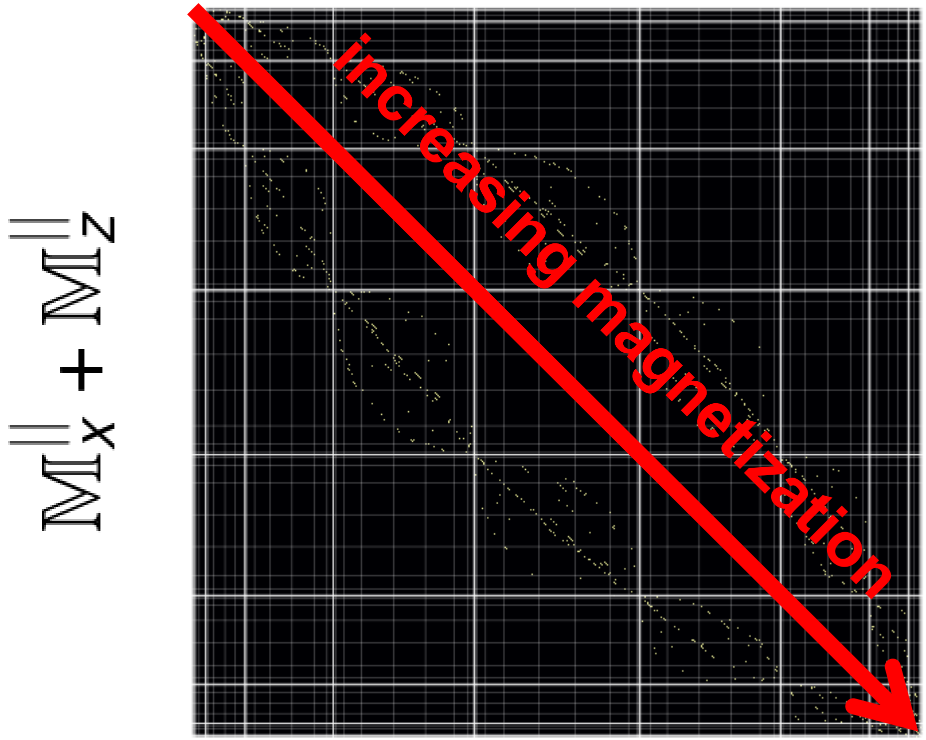
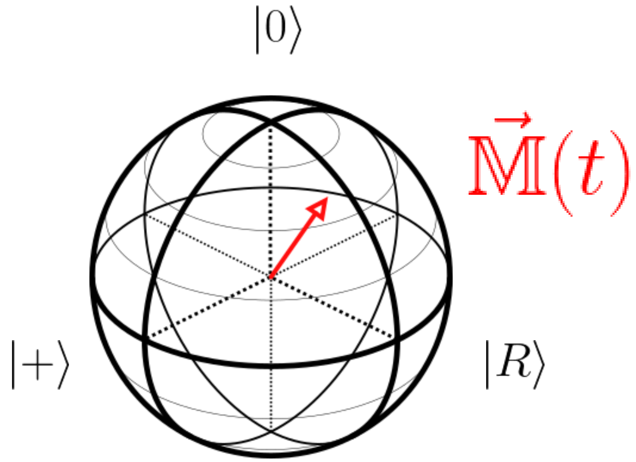


(3) expect strong overlap with instantaneous state for short times



0-th order approximation, **system entangles**

4. Preliminary results

Core election: action on magnetization eigenstate $|\Psi_i\rangle$

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$$
\begin{cases}
1 & \text{generically,} \\
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\end{cases}
$$

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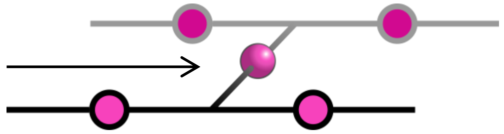

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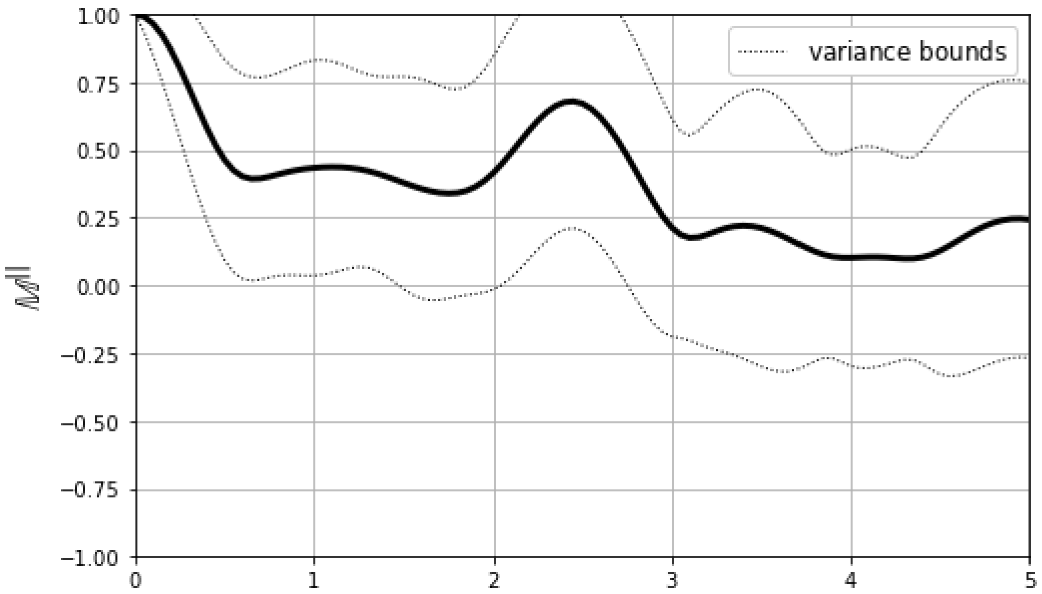

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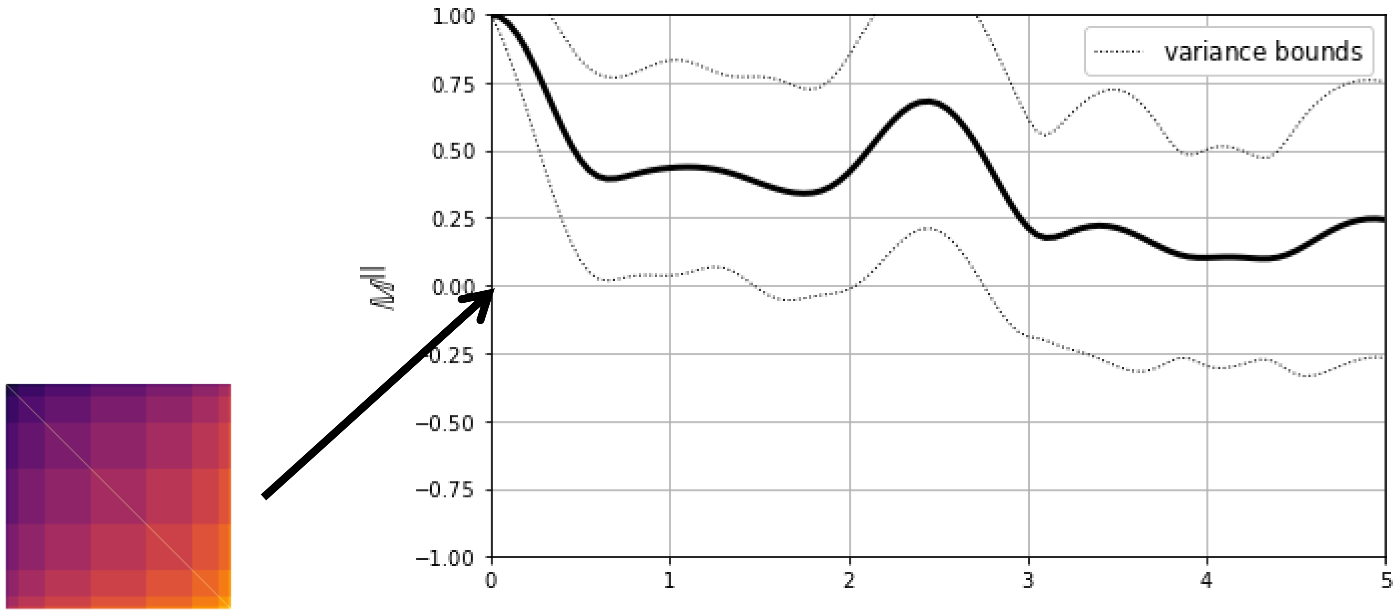

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the quantum channel **masks** the coherences

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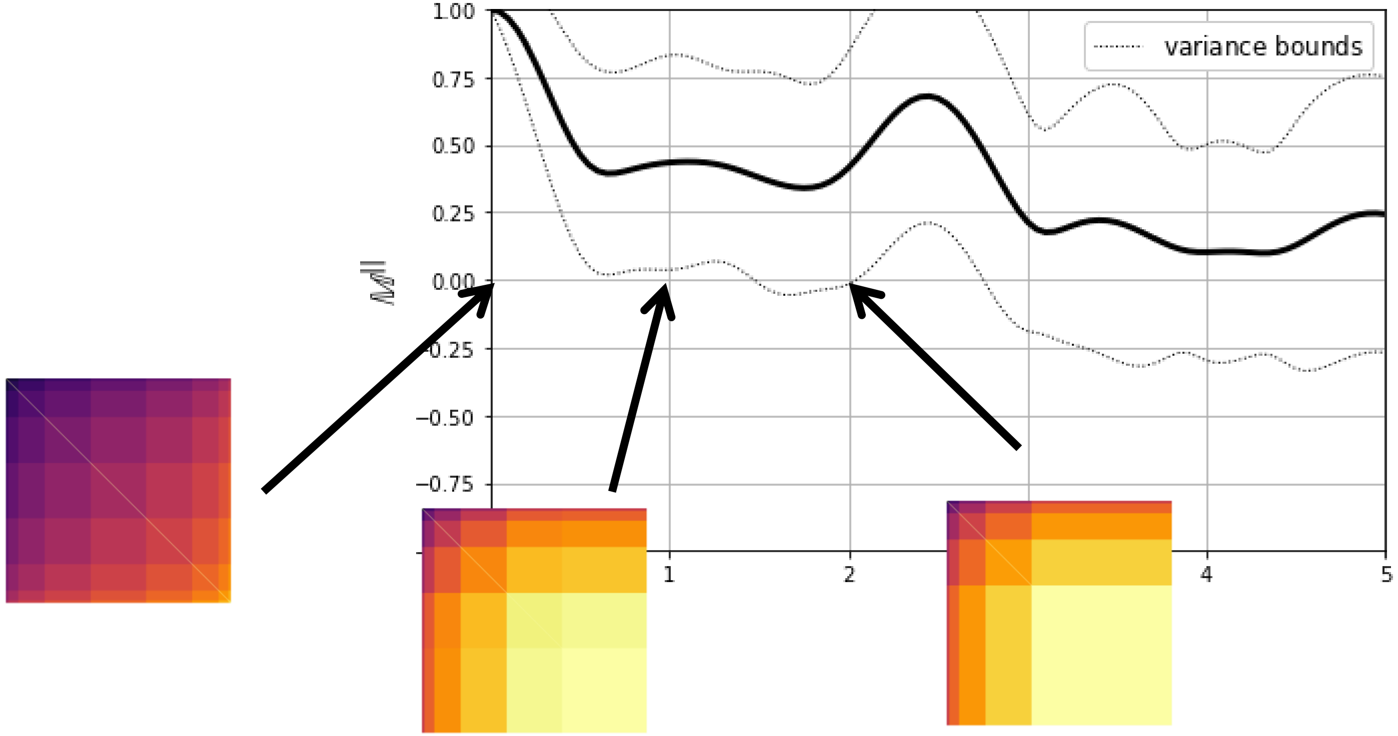


evolution of $|R\rangle$ for $J = 2g$

the quantum channel **masks** the coherences

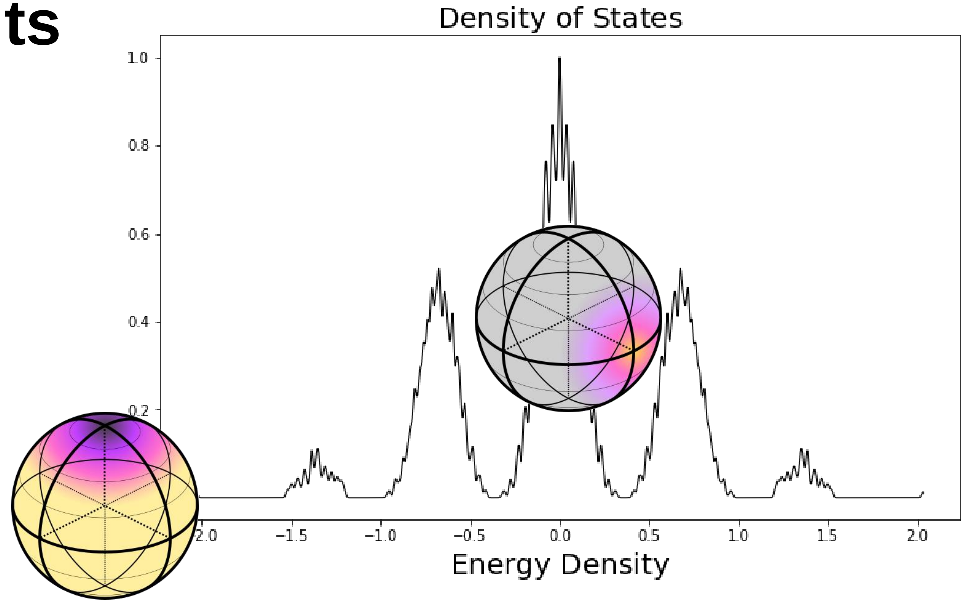
time dependent!

follow the change in variance

4. Preliminary Results

Two extreme cases:

close to **GS** and to **infinite temperature**



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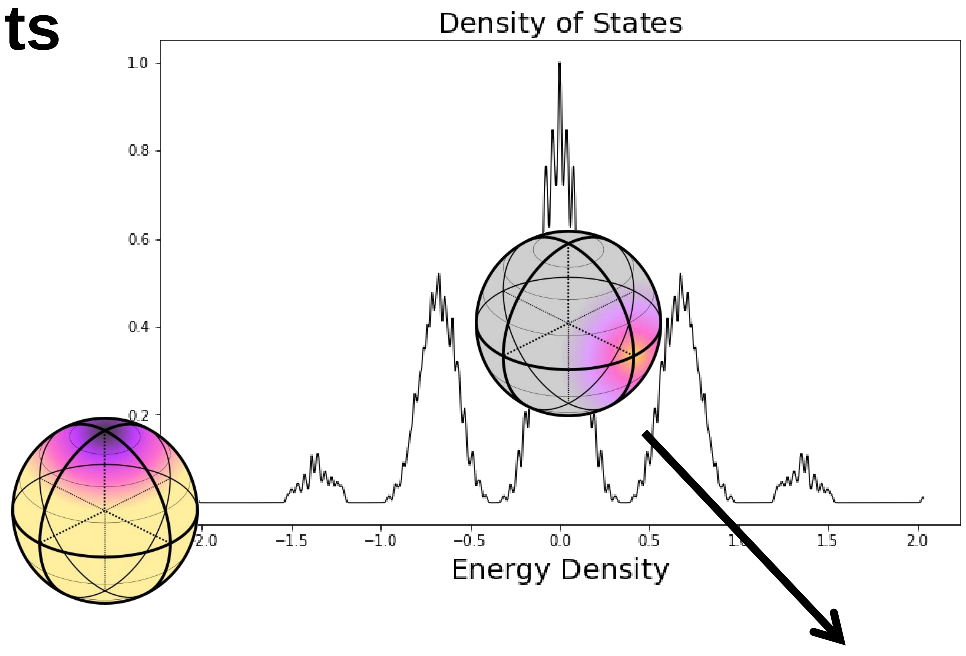

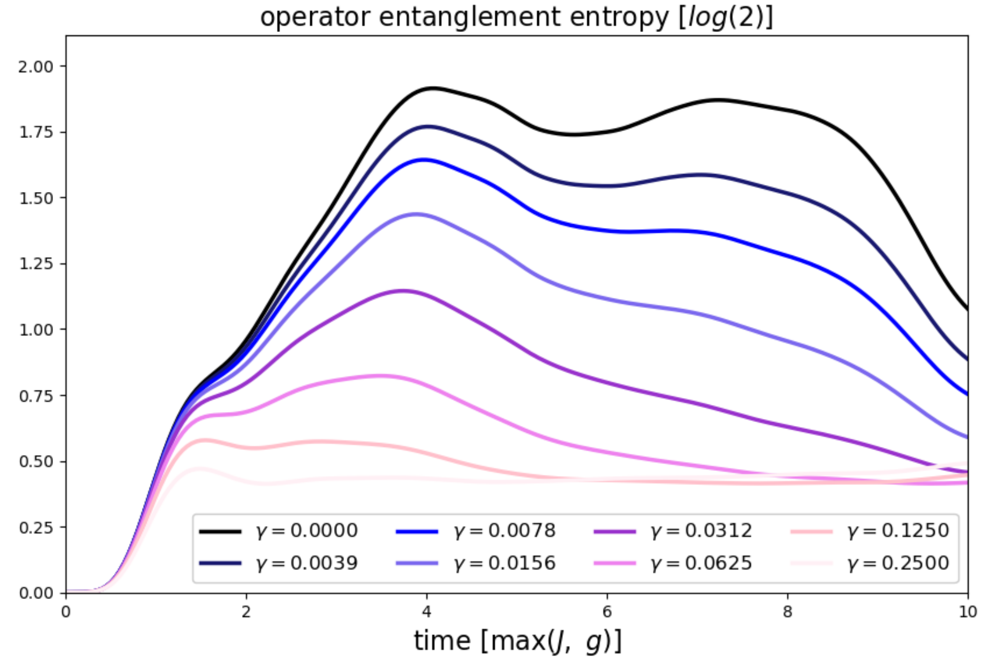

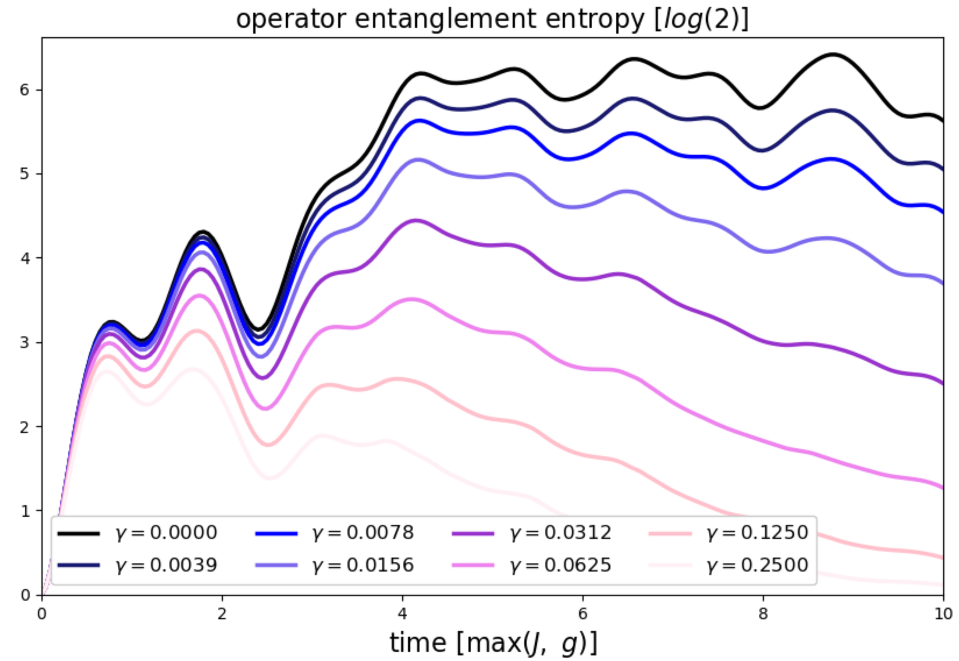

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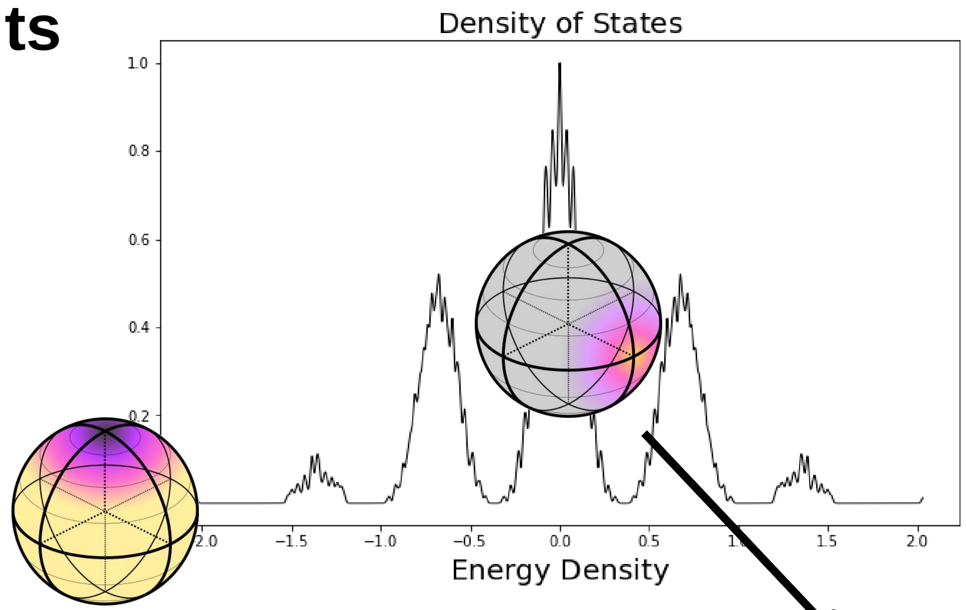
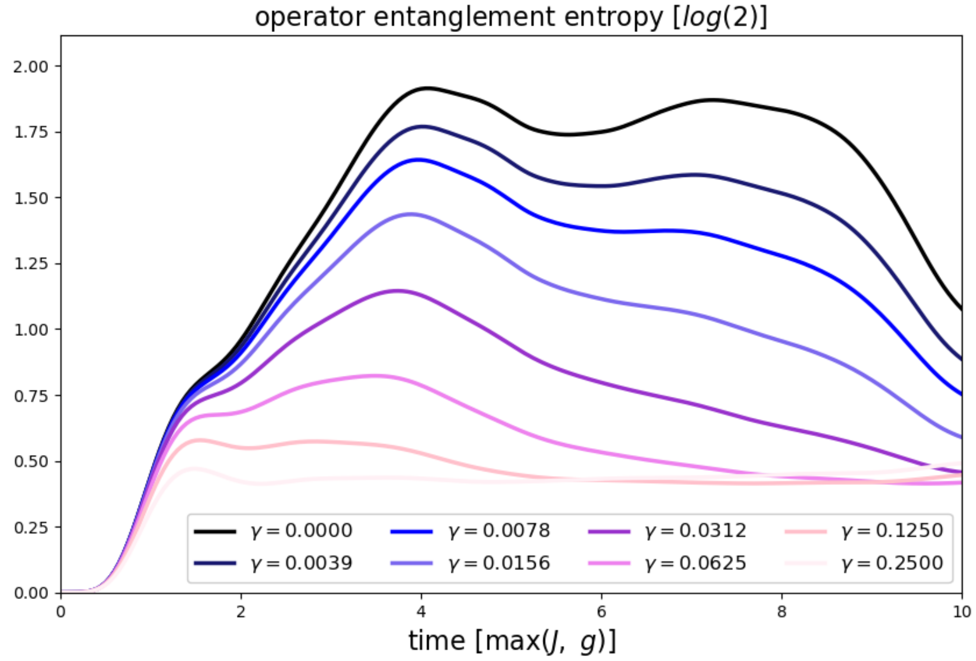
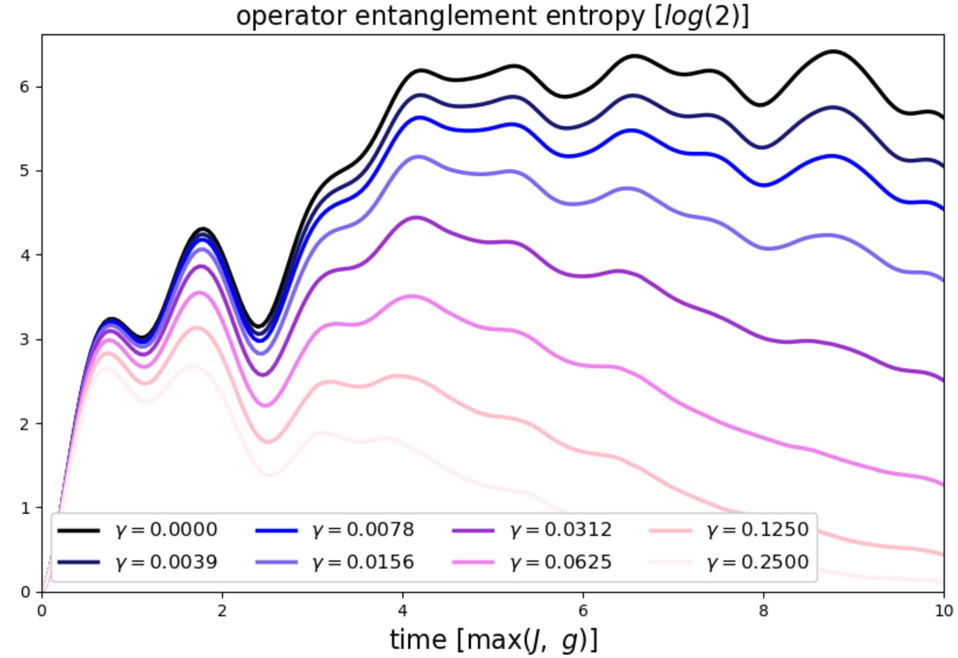


Observe a **reduction of entanglement** with increasing strength of decoherence

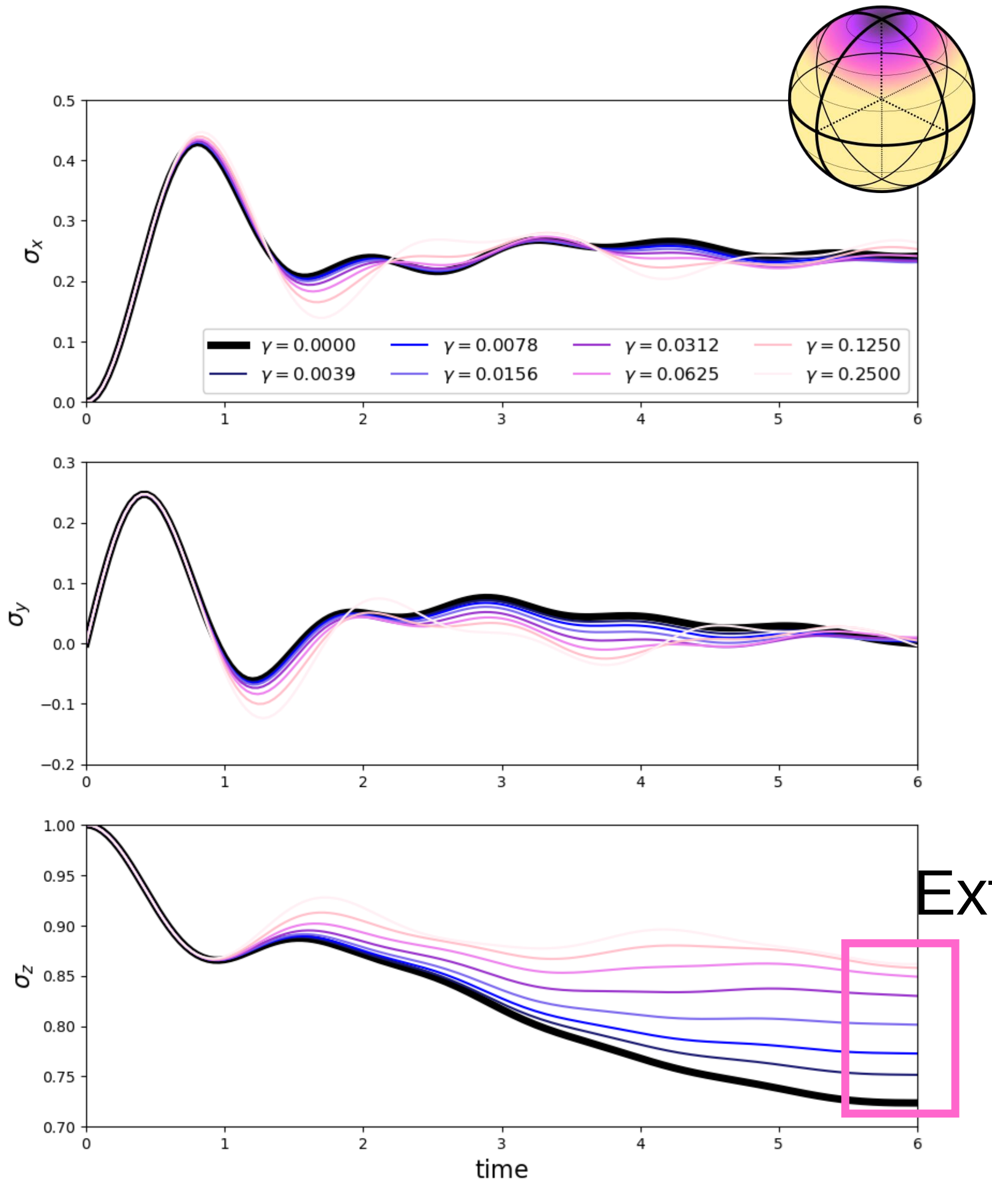
might improve simulability





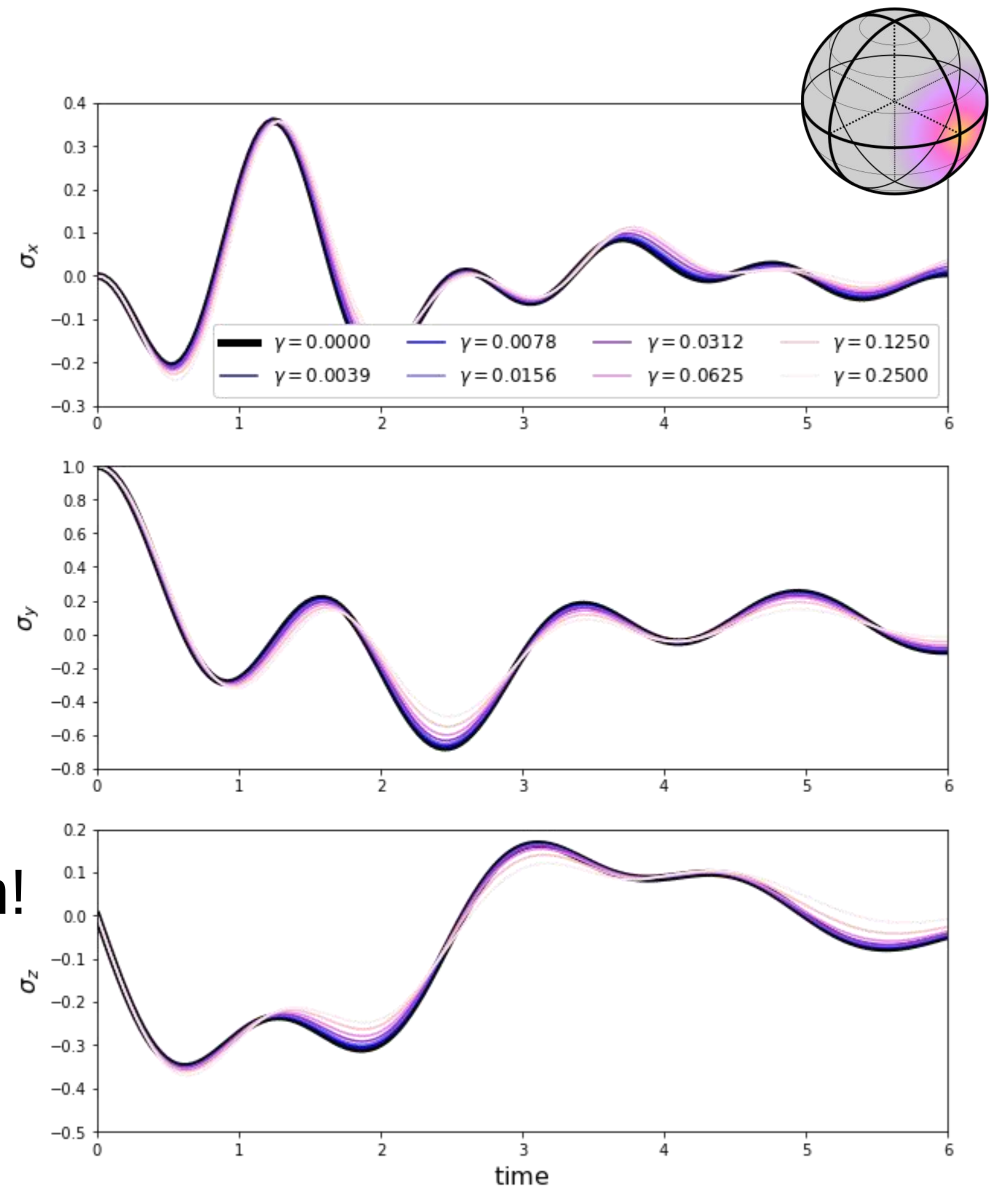
4. Preliminary Results

‘Good performance’



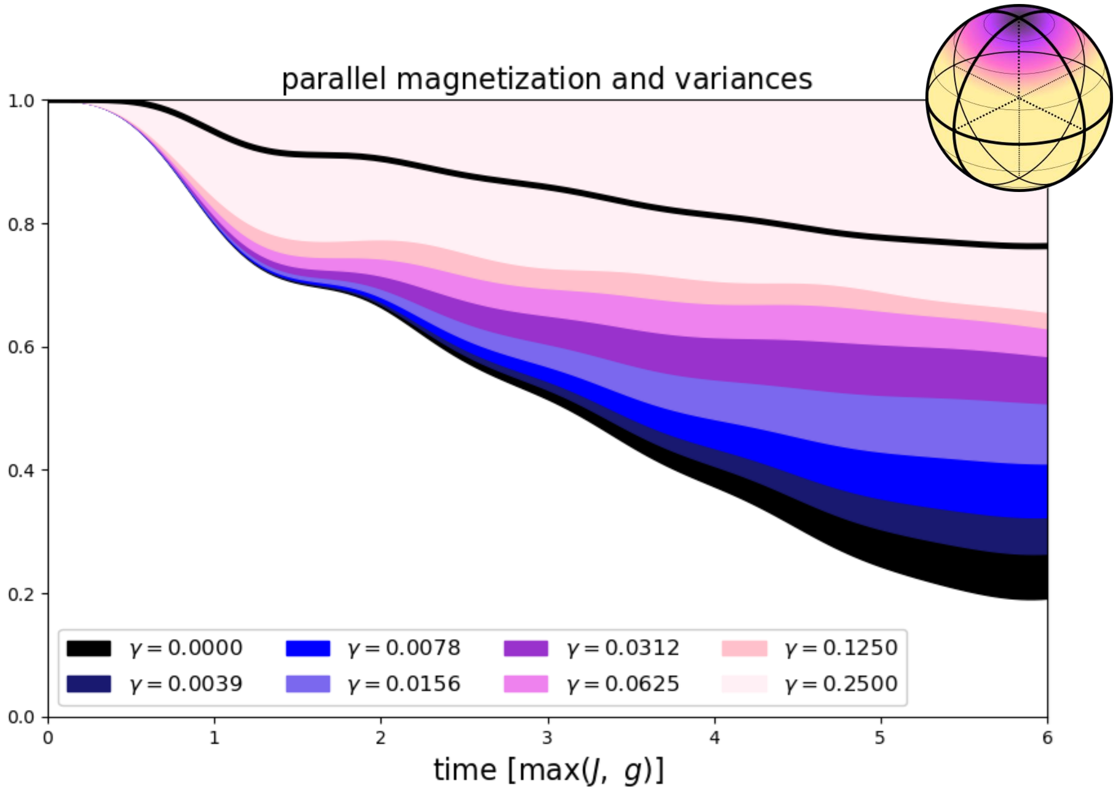
Extrapolation!

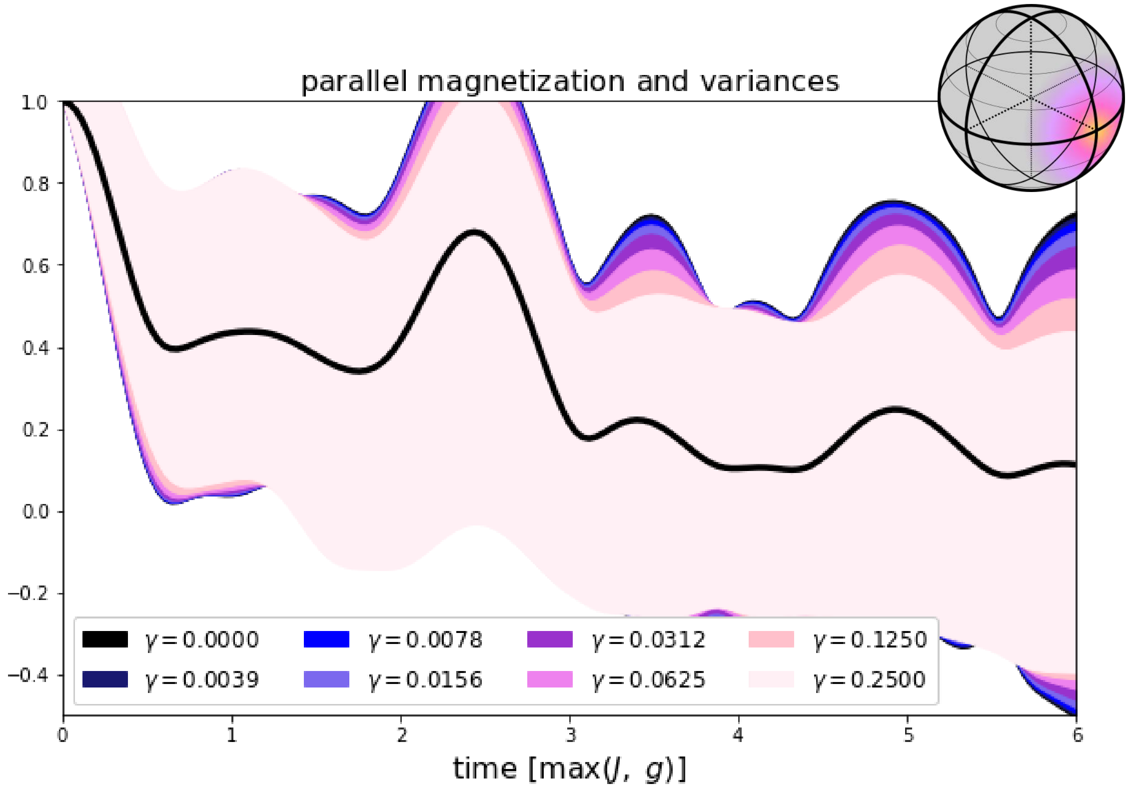
$$\gamma \to 0$$



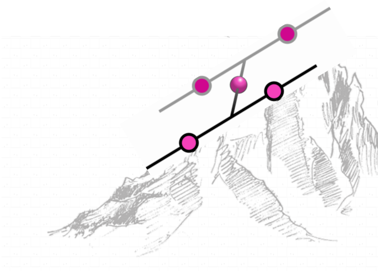
4. Preliminary Results

Physically: filtering with operator having a **small variance** on the current state





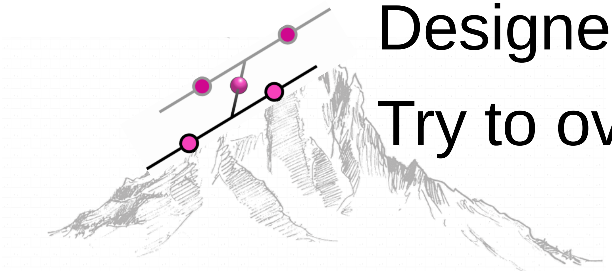
Concluding remarks



Designed a **flexible algorithm** targeting local dynamics.

Try to overcome the entanglement barrier.

Concluding remarks



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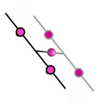

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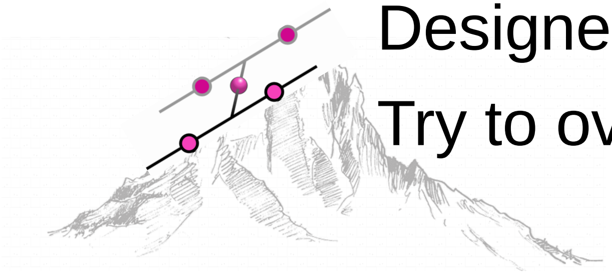

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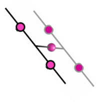

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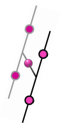

The use of dissipative coupling can **decrease the operator entanglement** (range of applicability to be determined)

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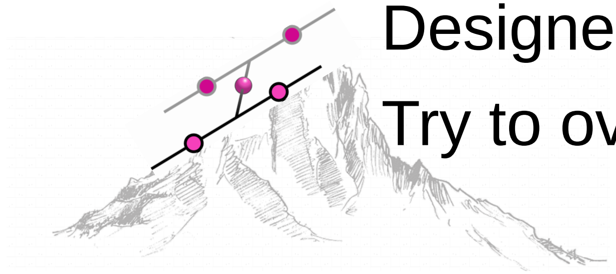

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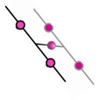

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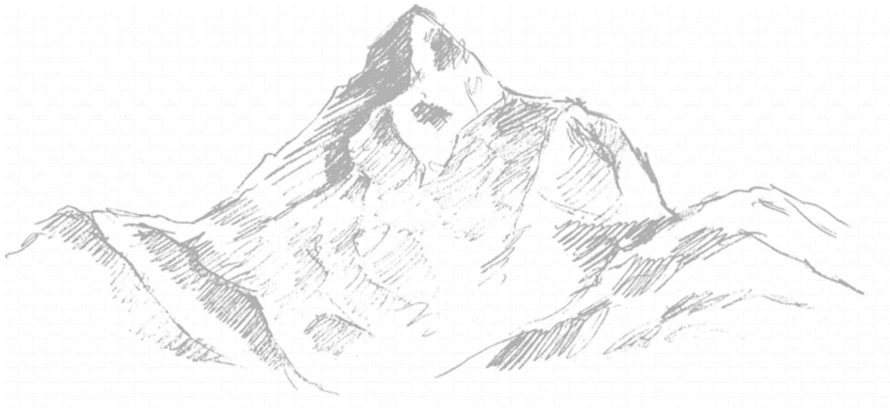

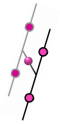

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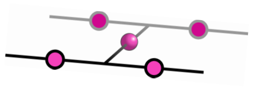
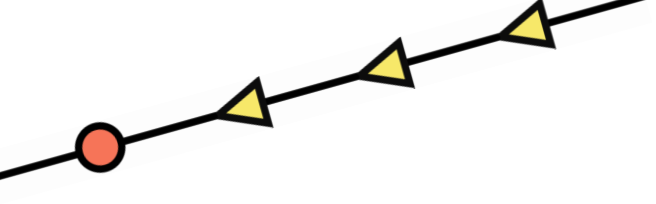
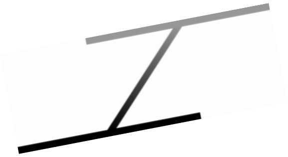
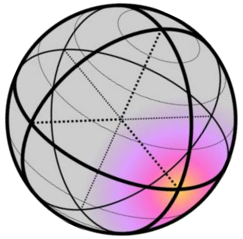
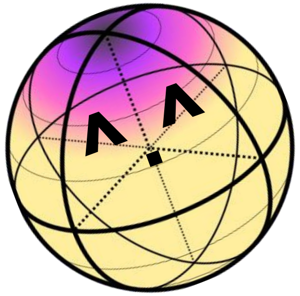
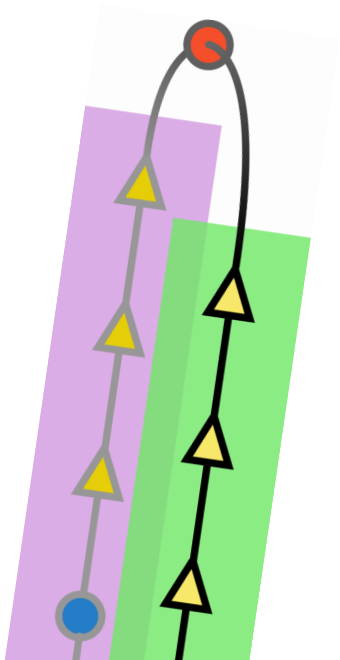
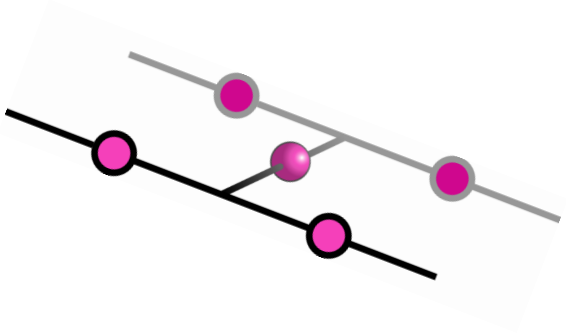
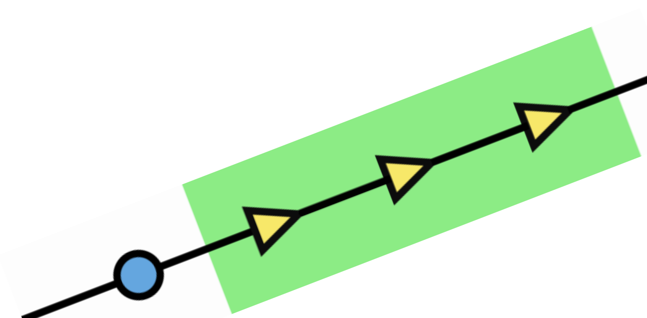


In case of proven efficiency: clear **extensions to non-local** observables (yet to come)

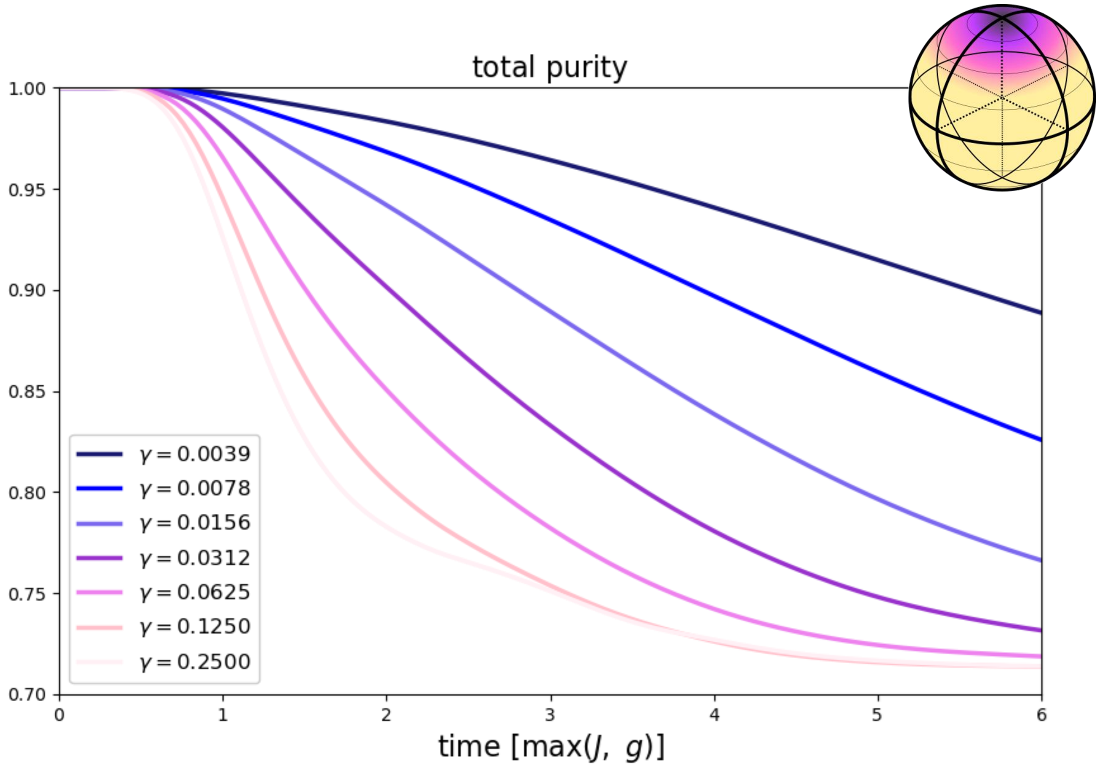





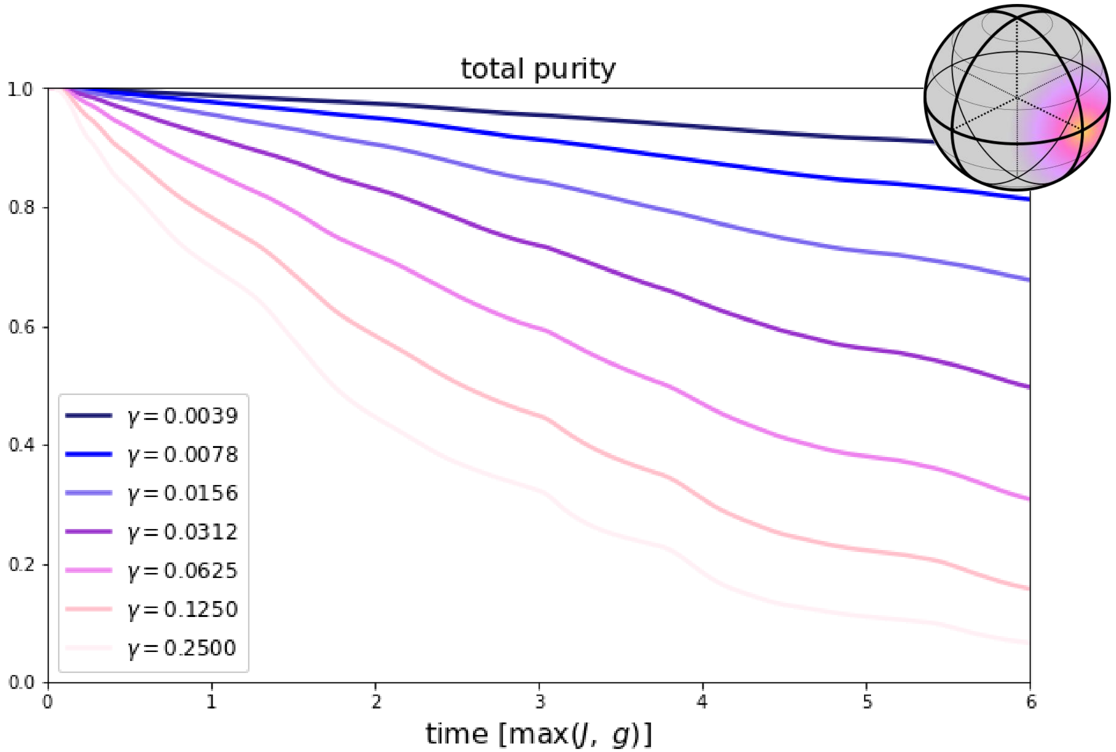
Thank you!

5. Backup: depurification

Dissipation depurifies total state…





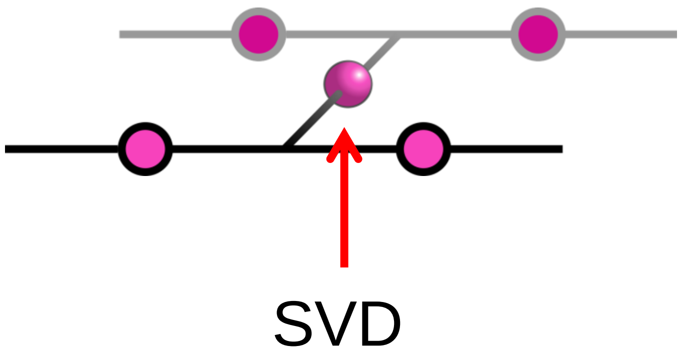
…this hinders the simulability in case of excessive loss of purity.

This bounds dissipation intensity from above (while memory does it from below).

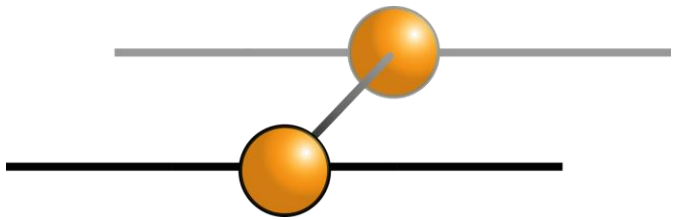
5. Backup: connection to measurement

Reshaping the core copy tensor:

$$\sum_q^{\chi_Q} M_q\{\ \cdot\ \}M_q^\dagger$$



$\longrightarrow$



$$\mathbb{I} = \sum_q^{\chi_Q} M_q M_q^\dagger$$

if weak: could think on thermalization by connection to a reservoir $\longleftarrow$ generalized quantum measurement

5. Backup: fine decoherence

$$\mathbb{M}^{||}_{(1)} = \sum_i \sigma^{||}_i$$

generate extensive set of commuting operators in which we can decohere

$$\mathbb{M}^{||}_{(2)} = \sum_i \sigma^{||}_i \sigma^{||}_{i+1}$$

$$\mathbb{M}^{||}_{(3)} = \sum_i \sigma^{||}_{i-1} \sigma^{||}_i \sigma^{||}_{i+1}$$

new filter with extensive number of

rates ~ Lagrange multipliers ~ temperatures

$$e^{-\sum_{i=1}^{L} \lambda_i \left(\mathbb{M}^{||}_{(i)} - \langle \mathbb{M}^{||}_{(i)}(t) \rangle \right)^2}$$

numerically: allow for **multidimensional extrapolation** $\lambda_i \to 0$

physically: **generalized thermalization?**

5. Backup: variational formulation

Generic formulation of the algorithm:

(1) find local operators whose variance is minimal at instantaneous state

$$\mathcal{C}(\{\theta_i\}) = \mathrm{tr}\left\{\rho^\dagger(t) Q^2(\{\theta_i\})\right\} - \mathrm{tr}^2\left\{\rho^\dagger(t) Q(\{\theta_i\})\right\}$$

written as MPO

(2) filter in the operator eigenbasis

generate mixture of similar operators
with the proper local information

frustration free operator!

$$\mathbb{M}^{||}_{(1)} = \sum_i \sigma_i^{||}$$