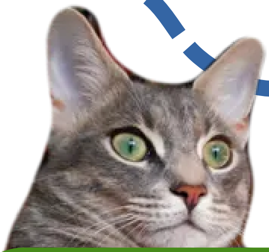
Bounding the minimum time of a quantum measurement

Nathan Shettel, Federico Centrone, Luis Pedro Garcia-Pintos

Quantum mechanics is hard to digest



"If all of this is true, then it means the end of physics"

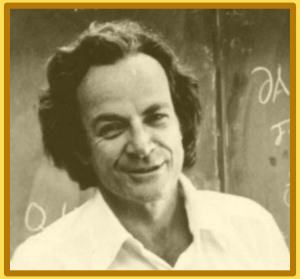
– Albert Einstein

"I don't like it, and I'm sorry I ever had anything to do with it."

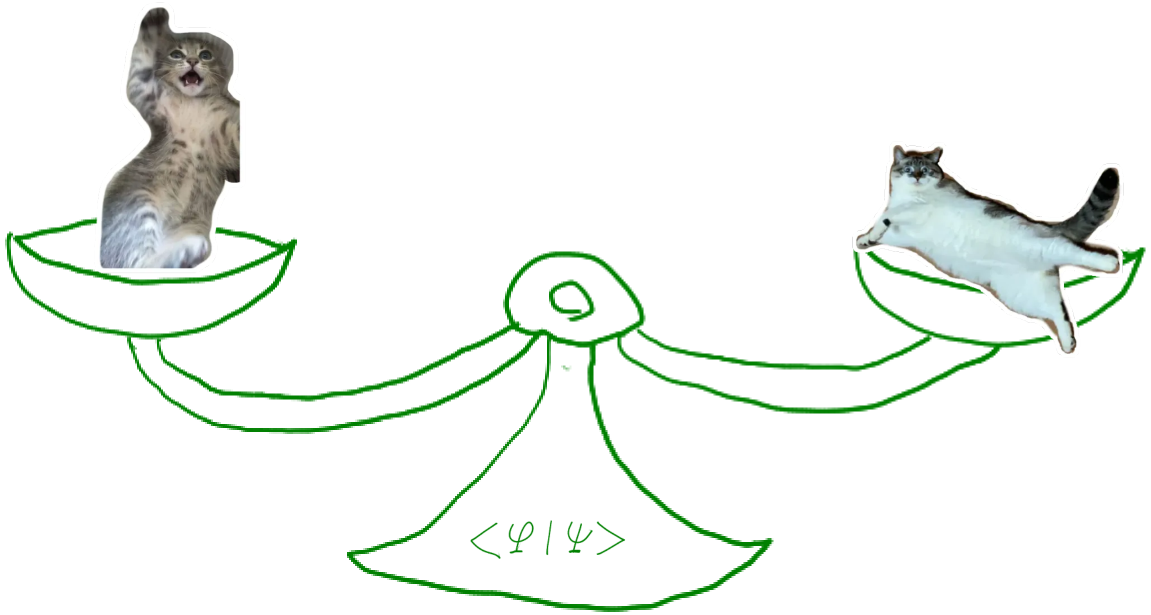
- Erwin Schrödinger

"I think I can safely say that nobody understands quantum mechanics."

- Richard Feynman



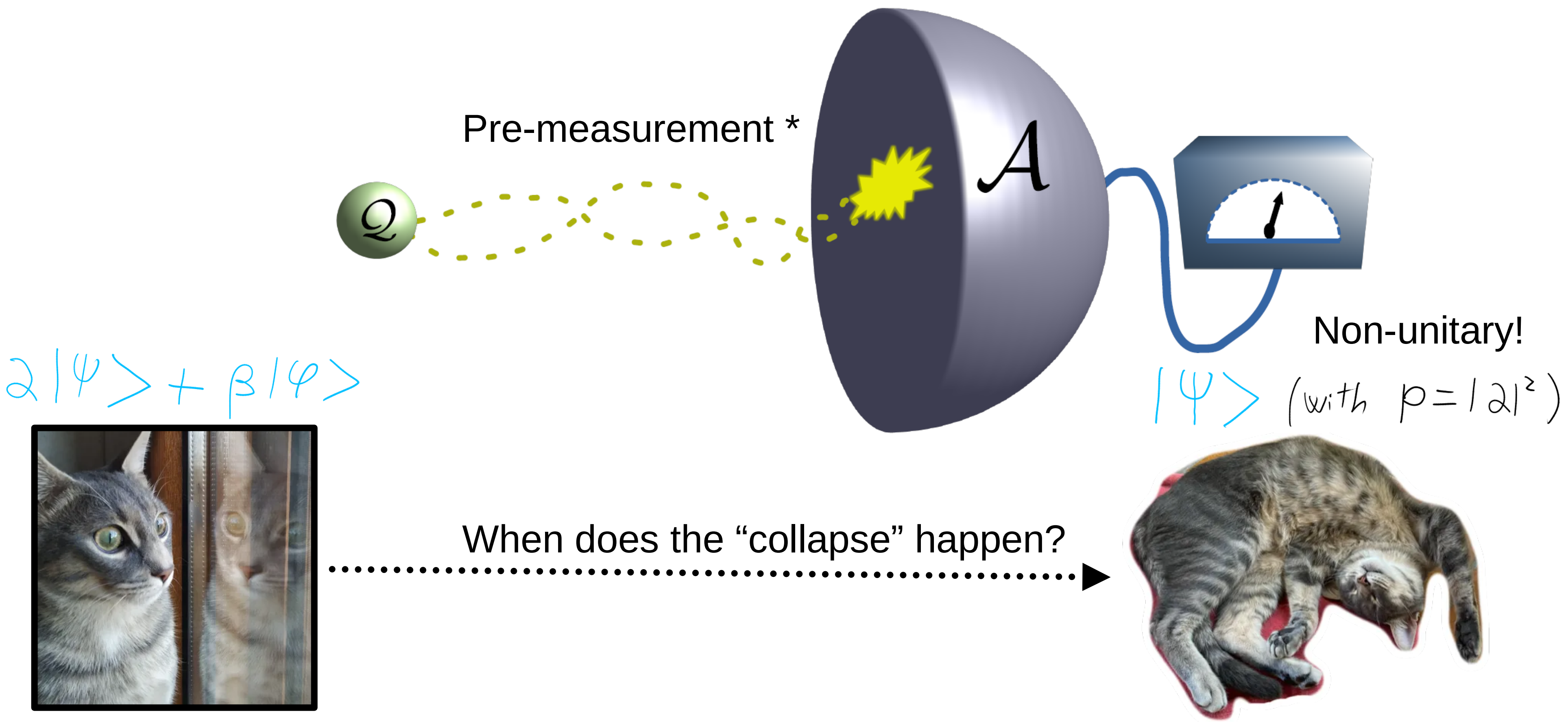
The measurement problem



Can measurement itself be described by a physical process?

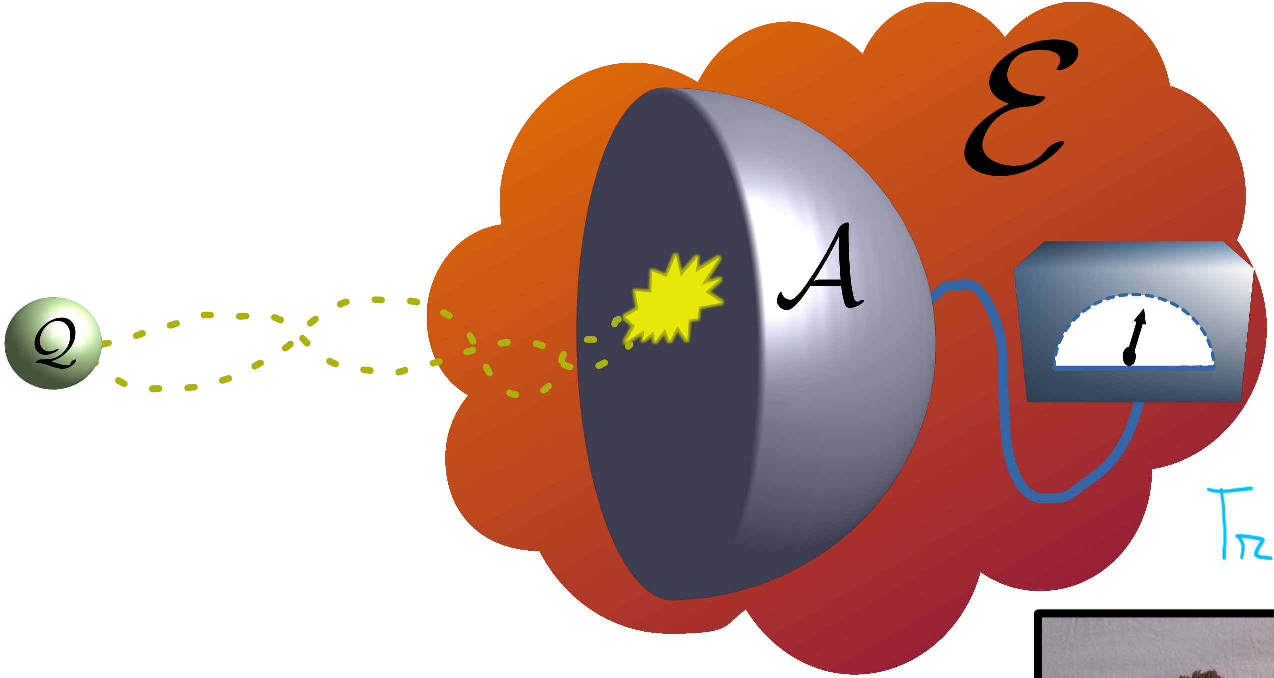
Dourdent, Hippolyte. "A Quantum Godelian Hunch." arXiv preprint arXiv:2005.04274 (2020).

Collapse of the wavefunction



* Strasberg, Philipp, Kavan Modi, and Michalis Skotiniotis. "How long does it take to implement a projective measurement?." European Journal of Physics 43.3 (2022): 035404.

Decoherence



Unitary!

$\hat{\rho}_{SA} \otimes \hat{\rho}_E$

$\mathrm{Tr}_E[\hat{\rho}_{SAE}]$

Entangled with the environment

Schlosshauer, Maximilian. "Decoherence, the measurement problem, and interpretations of quantum mechanics." Reviews of Modern physics 76.4 (2005): 1267.

Heisenberg's uncertainty relations :

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Conjugate observables
(well understood!)

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Energy-time
(wtf?)

Say my name...

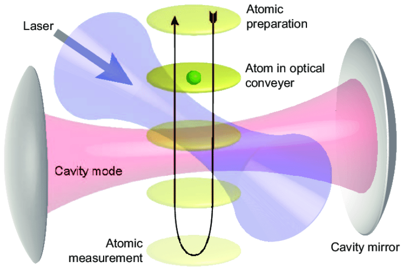
Time scale of the evolution: **Quantum Speed Limits**

Mandelstam and Tamm

$$\Delta_\rho H \Delta_\rho M \geq \frac{|\langle[\hat{H},\hat{M}]\rangle_\rho|}{2} = \frac{\hbar}{2}\left|\frac{\partial}{\partial t}\langle\hat{M}\rangle_\rho\right|$$

$$\Rightarrow \Delta H \Delta \tau \geq \frac{\hbar}{2}$$

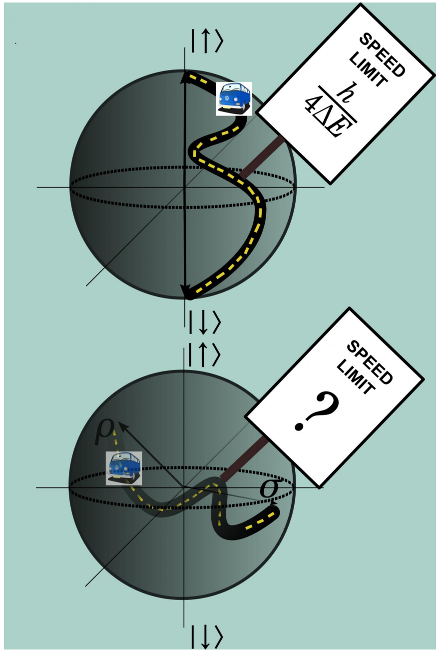
QSL have been used to explore the limits of computation and complexity. [1]



Non-Markovian dynamics can speed up quantum processes :
Verified in a cavity QED experiment. [2]

[1] Lloyd, Seth. "Ultimate physical limits to computation." Nature 406.6799 (2000): 1047-1054.

[2] Deffner, Sebastian, and Eric Lutz. "Quantum speed limit for non-Markovian dynamics." Physical review letters 111.1 (2013): 010402.



Geometric approach:

The shortest path between distinguishable quantum states.

$$v(t) := \lim_{\delta t \to 0} \frac{D_B(\rho(t+\delta t), \rho(t))}{\delta t}$$

$$= \frac{1}{2\hbar}\sqrt{I_Q(\rho(t))}$$

Information-Time Uncertainty relation:

Fast observable dynamics requires large fluctuations.

$$\Delta\tau\sqrt{I_Q} \geq 1$$

Taddei, Márcio M., et al. "Quantum speed limit for physical processes." Physical review letters 110.5 (2013): 050402.

Garcia-Pintos, Luis, et al. "Unraveling quantum and classical speed limits on observables." APS March Meeting Abstracts. Vol. 2021. 2021.

Timescale bound

How much time τ does it take for a car to run a distance d?

Need to make some assumptions and solve complex differential equation to know the dynamics

Timescale bound

τ is at least d/v_{max}

Timescale bound

This bound can be generalized from cars to quantum states and observables

$$\tau \geq \frac{d}{V_{max}}$$

- Entropic distance from the measured state

Information transferred to the environment

- Max speed of the observable in the evolution of the system

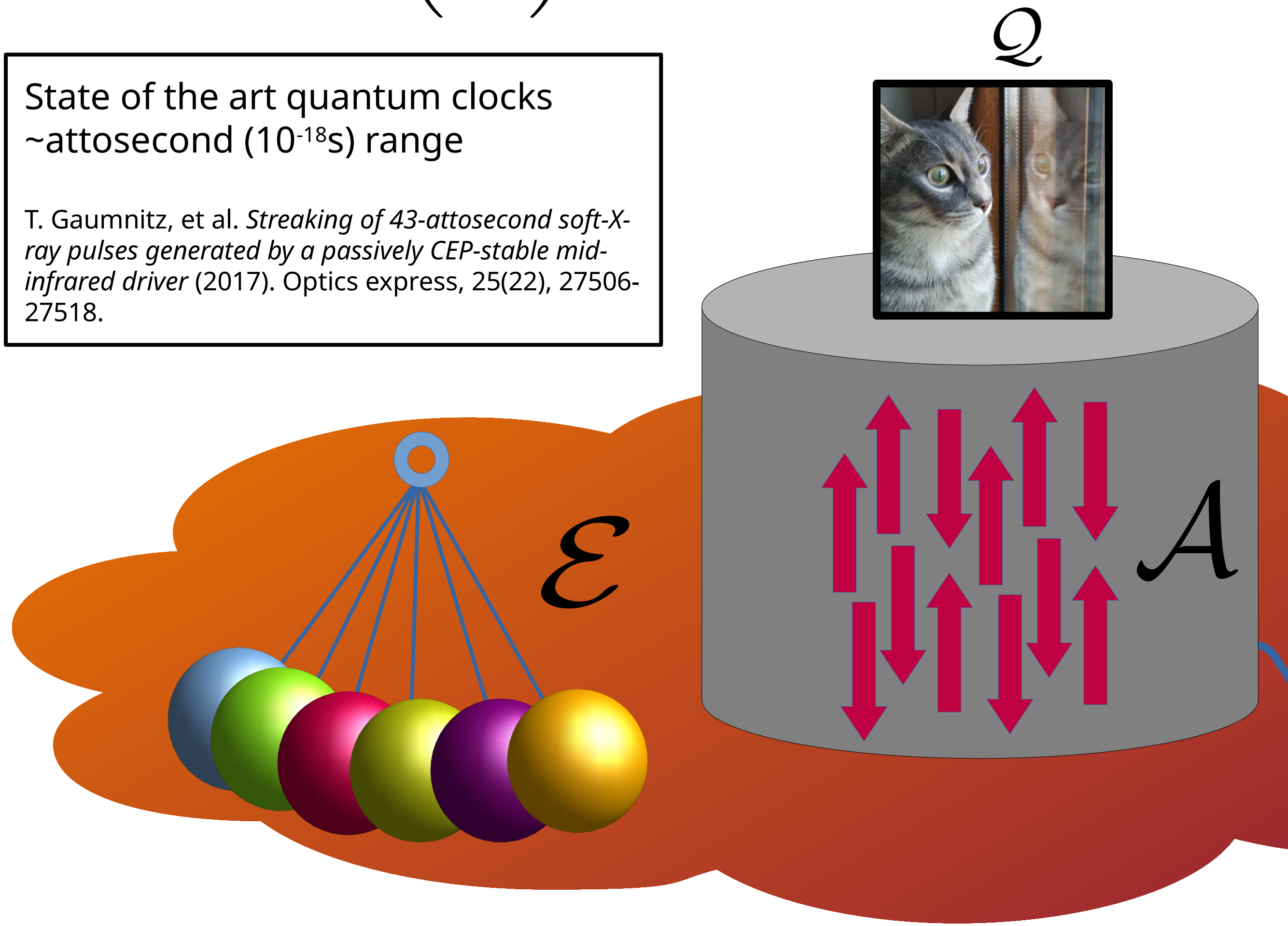
Grows with the energy of the environment and the number of outcomes

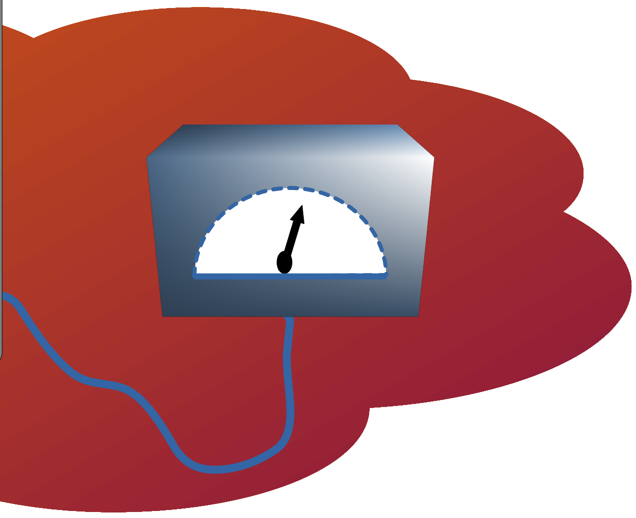
Example: spin-boson

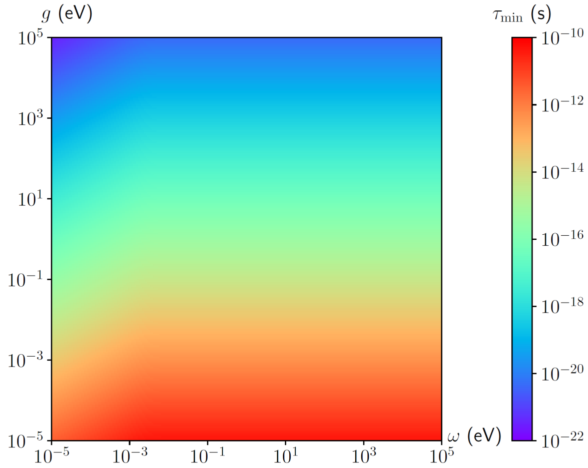
$$\tau = O\left(\frac{1}{N}\right)$$

State of the art quantum clocks ~attosecond (10^{-18}s) range

T. Gaumnitz, et al. *Streaking of 43-attosecond soft-X-ray pulses generated by a passively CEP-stable mid-infrared driver* (2017). Optics express, 25(22), 27506-27518.

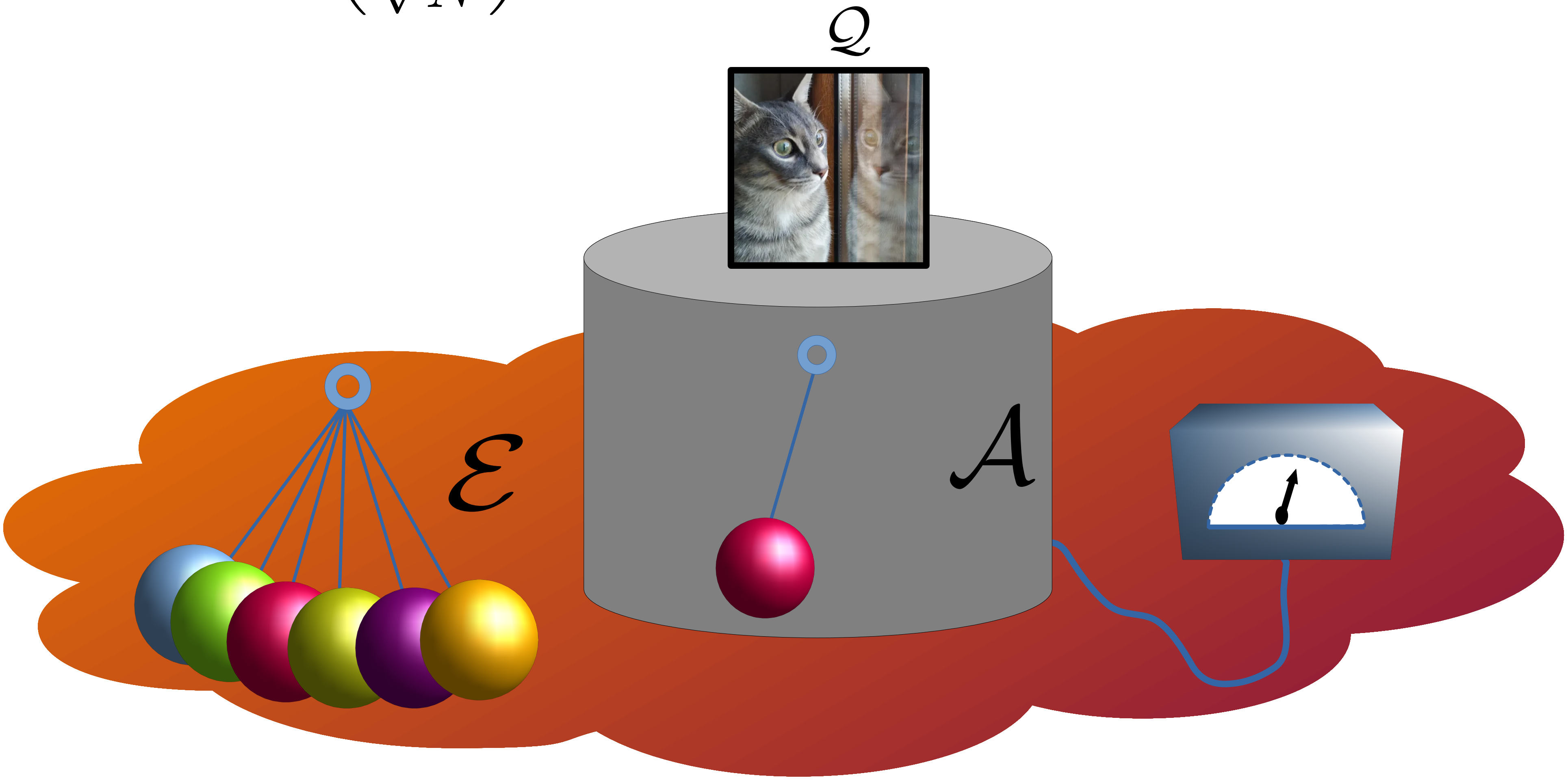






$$\tau = O\left(\frac{1}{\sqrt{N}}\right)$$

Example: boson-boson



To sum up:

- The decoherence model allows to compute a **non-vanishing** measurement time.
- Quantum speed limits can be used to find a bound in very **general scenarios**.
- Our bound does not require to solve **complex differential equations**.
- Applicable to **more than just measurement**.
- The bound is loose but **experimentally testable**.

Outlook:

- Tightening the bound.
- Testbed for non-linear Schrodinger equation.
- Influence from exotic environments.
- Physics of quantum information (Bremmerman-Bekenstein bound).

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$\langle \varphi | \psi \rangle$

Decoherence measurement

$$\Big(\sum_j \alpha_j |\psi_j^{\mathcal{Q}}\rangle\Big) \otimes |A^{\mathcal{A}}\rangle \rightarrow \sum_j \alpha_j |\psi_j^{\mathcal{Q}}\rangle \otimes |a_j^{\mathcal{A}}\rangle \rightarrow \sum_j |\alpha_j|^2 |\psi_j^{\mathcal{Q}}\rangle\langle\psi_j^{\mathcal{Q}}| \otimes |a_j^{\mathcal{A}}\rangle\langle a_j^{\mathcal{A}}|$$

Pre-measurement
(t=0)

Time scale bound

$$\hbar\left|\frac{dS}{dt}\right| \leq 2\Delta S \Delta H_{\text{int}}{}^{*} \longrightarrow \tau \geq \frac{S(\rho^{\mathcal{QA}}||\rho^{\mathcal{QA}}_{\mathcal{M}})|_{t=0}}{\max_t |\partial_t S(\rho^{\mathcal{QA}}||\rho^{\mathcal{QA}}_{\mathcal{M}})|} \geq \frac{\hbar}{2 f_A}\frac{S(\rho^{\mathcal{QA}}||\rho^{\mathcal{QA}}_{\mathcal{M}})|_{t=0}}{\max_t \Delta H_{\text{int}}}$$

Kullback–Leibler divergence

$$\Delta S \leq \sqrt{\frac{1}{4}\ln^2(A-1)+1} = f_A.$$

Approximately time independent in the strong coupling regime

* Garcia-Pintos, Luis, et al. "Unraveling quantum and classical speed limits on observables." APS March Meeting Abstracts. Vol. 2021. 2021.

Example: Spin-Boson

$$|\psi^{\mathcal{QA}}\rangle = x|0\rangle|\downarrow\rangle^{\otimes N} + y|1\rangle|\uparrow\rangle^{\otimes N}$$

$$\rho^{\mathcal{E}} = \bigotimes_k \frac{1}{Z_k} e^{-\beta\omega_k a_k^\dagger a_k}$$

- Q: 2-level system
- A: N spins
- E: multi-mode thermalized bosonic states (harmonic oscillators)

Interaction couples spins to E-modes

$$H_{\text{int}}^{\mathcal{AE}} = \sum_{i=1}^{N} \sum_k \sigma_Z^{(i)} \otimes g_k (a_k + a_k^\dagger)$$

Time scales inversely with N and coupling strength

$$\tau \geq \frac{\hbar \ln 2}{2N} \left(\int_0^\infty J(\omega) \coth(\beta\omega/2) \mathrm{d}\omega \right)^{-1/2}$$

Continuum of modes:

$$\sum_k |g_k|^2 \to \int_0^\infty J(\omega) \mathrm{d}\omega$$

Spin-Boson: Analytic expression

Born-Markov approximation:

$$\rho^{\mathcal{QA}} = \begin{pmatrix} |x|^2 & xy^* e^{-\Gamma} \\ x^* y e^{-\Gamma} & |y|^2 \end{pmatrix}$$

$$\Gamma = 4N \int_0^\infty \frac{J(\omega)}{\omega^2}(1 - \cos(\omega t/\hbar)) \coth(\beta\omega/2) \mathrm{d}\omega$$

Solution for relative entropy to equal

$$t = \frac{\hbar\sqrt{\ln(1/\varepsilon)}}{2\sqrt{N}} \Big(\int_0^\infty J(\omega) \coth(\beta\omega/2) \mathrm{d}\omega \Big)^{-1/2}$$

Our bound

$$\tau \geq \frac{\hbar \ln 2}{2N} \Big(\int_0^\infty J(\omega) \coth(\beta\omega/2) \mathrm{d}\omega \Big)^{-1/2}$$

Example: Boson-Boson

$$|\psi^{\mathcal{QA}}\rangle = x|0\rangle|\alpha\rangle + y|1\rangle|-\alpha\rangle$$

- Q: 2-level system
- A: Cat state
- E: multi-mode thermalized bosonic states (harmonic oscillators)

$$\rho^{\mathcal{E}} = \bigotimes_k \frac{1}{Z_k} e^{-\beta\omega_k a_k^\dagger a_k}$$

Interaction couples A-modes to E-modes

$$H_{\text{int}}^{\mathcal{AE}} = \sum_k g_k (a_k b^\dagger + a_k^\dagger b)$$

Our bound:

Scales inversely with $|\boldsymbol{\alpha}|$

Experimental paper [1]:

Scales inversely with $|\boldsymbol{\alpha}|^2$

Due to different Hamiltonian

[1] M. Brune,, et al. *Observing the progressive decoherence of the "meter" in a quantum measurement* (1996). Physical review letters, 77(24) 4887.

Tightening the bound

$$\tau \geq \frac{S(\rho^{\mathcal{QA}}||\rho^{\mathcal{QA}}_{\mathcal{M}})|_{t=0}}{\max_t |\partial_t S(\rho^{\mathcal{QA}}||\rho^{\mathcal{QA}}_{\mathcal{M}})|} \geq \frac{\hbar}{2f_A} \frac{S(\rho^{\mathcal{QA}}||\rho^{\mathcal{QA}}_{\mathcal{M}})|_{t=0}}{\max_t \Delta H_{\text{int}}}$$

Middle inequality 'fixes' N vs √N discrepancy in spin-boson example

$$|\partial_t S(\rho^{\mathcal{QA}}||\rho^{\mathcal{QA}}_{\mathcal{M}})| \leq \Delta S \sqrt{I} \leq 2 \Delta S \Delta H^{\mathcal{AE}}_{\text{int}}$$ [1]

$$\max(\Delta S) = f_A$$

Cannot be saturated

[1] L. P. García-Pintos, et al. *Unifying quantum and classical speed limits on observables* (2022). Physical Review X, 12(1), 011038.