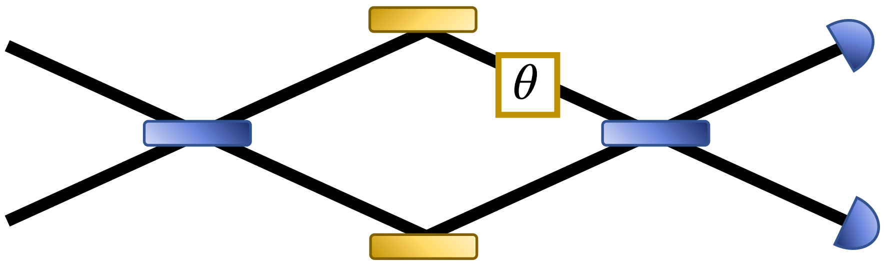
Quantum Metrology with non-Gaussian Spin States

Manuel Gessner
IFIC-Instituto de Física Corpuscular
Universidad de Valencia

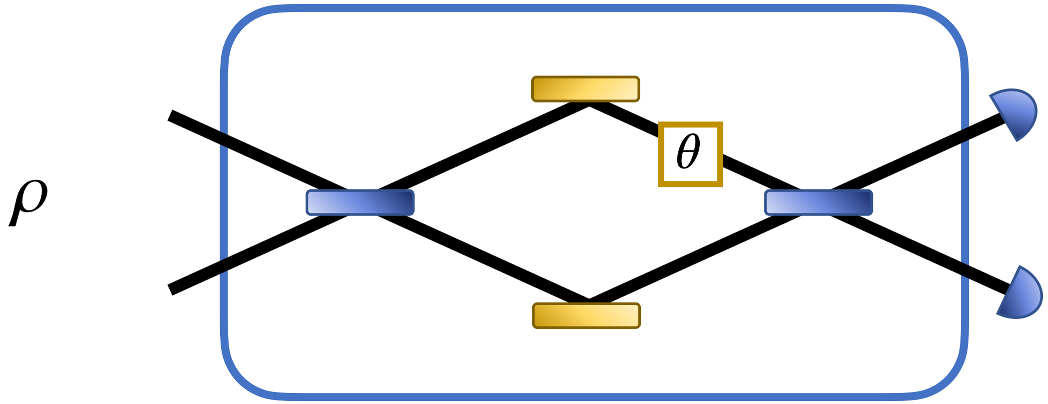
Quantum Information in Spain (ICE-8)
Santiago de Compostela
30/05/2023

Quantum Parameter Estimation



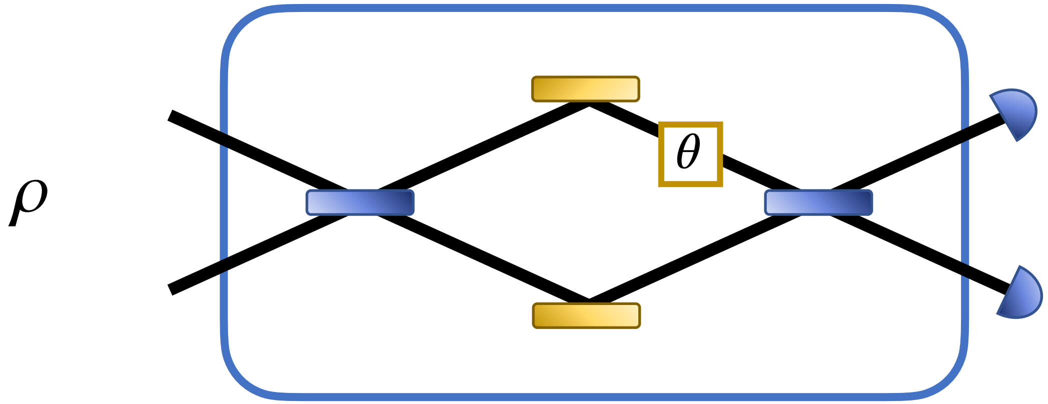
Quantum Parameter Estimation

State preparation **Parameter imprinting** **Measurement**



Quantum Parameter Estimation

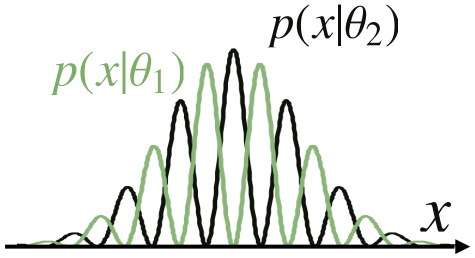
State preparation **Parameter imprinting** **Measurement**



Results:

$x_1, x_2, \ldots, x_\mu$

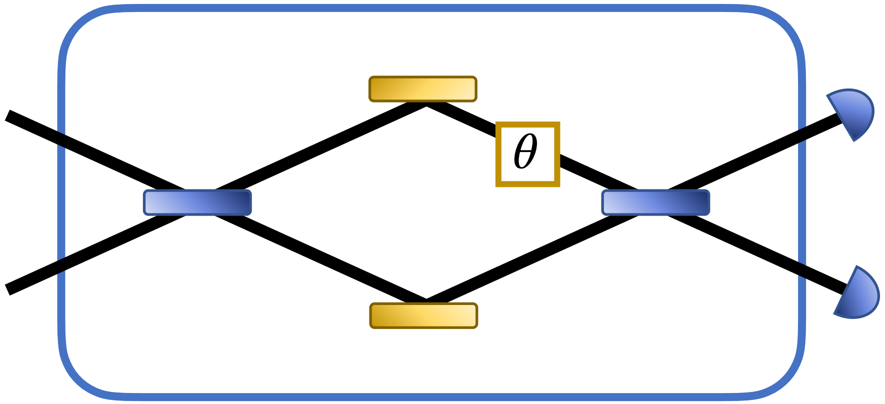
Distribution: $p(x|\theta)$



Quantum Parameter Estimation

State preparation **Parameter imprinting** **Measurement** **Estimation**

ρ

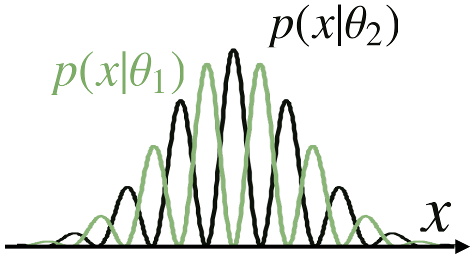


Results:

$x_1, x_2, \ldots, x_\mu$

Distribution: $p(x|\theta)$

$\theta_{\mathrm{est}}(x_1, x_2, \ldots, x_\mu)$

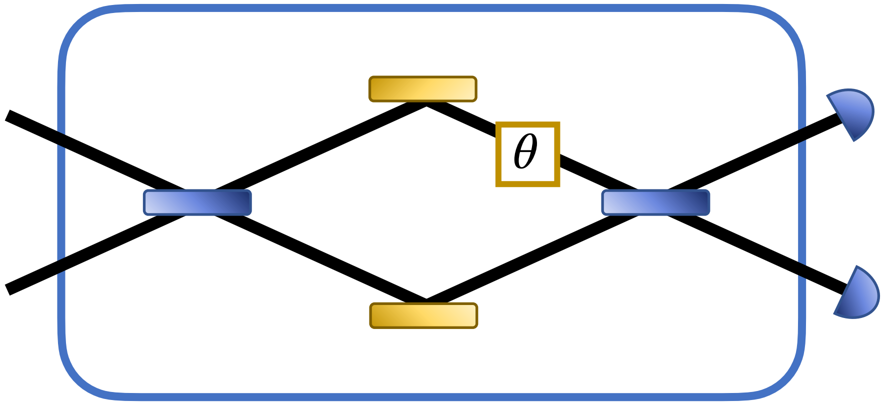


QUANTUM PARAMETER ESTIMATION

State preparation

ρ

Parameter imprinting

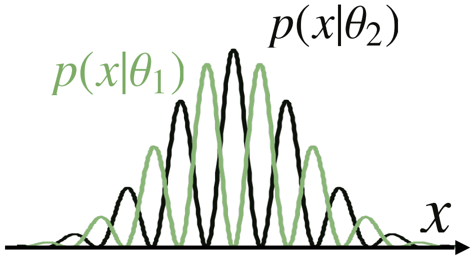


Measurement

Results:

$x_1, x_2, \ldots, x_\mu$

Distribution: $p(x|\theta)$



Estimation

$\theta_{\text{est}}(x_1, x_2, \ldots, x_\mu)$

Objective:

$\langle \theta_{\text{est}} \rangle = \theta$

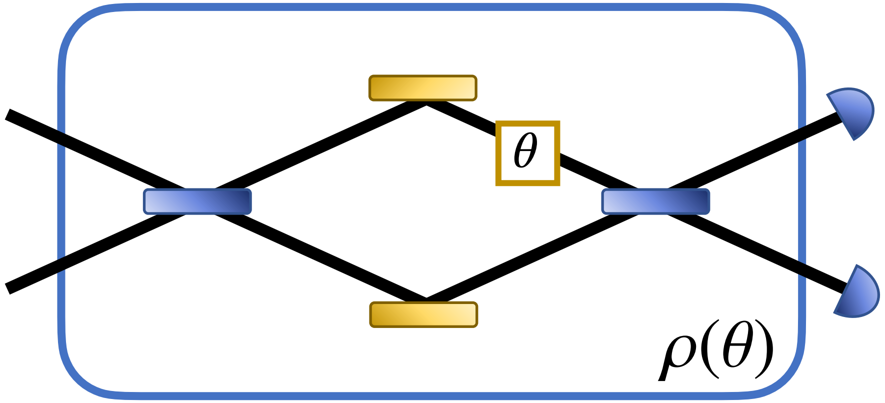
Minimize $(\Delta\theta_{\text{est}})^2$

Quantum Parameter Estimation

State preparation

ρ

Parameter imprinting

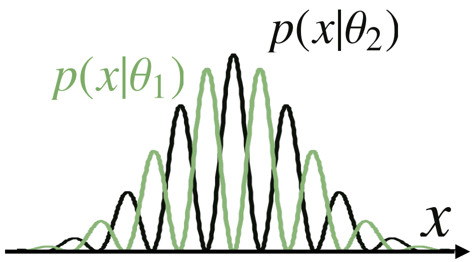


Measurement

Results:

$x_1, x_2, \ldots, x_\mu$

Distribution: $p(x|\theta)$



Quantum system:

$$p(x|\theta) = \mathrm{Tr}\{\rho(\theta)\Pi_x\}$$

Estimation

$$\theta_{\mathrm{est}}(x_1, x_2, \ldots, x_\mu)$$

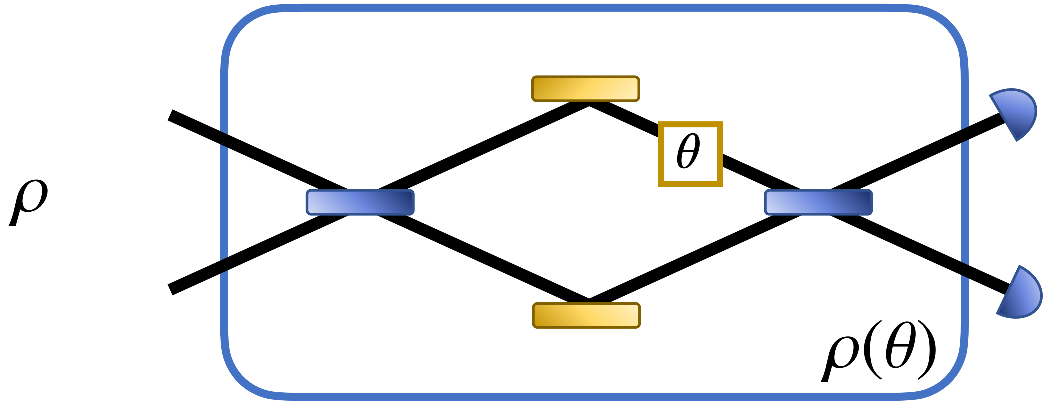
Objective:

$$\langle \theta_{\mathrm{est}} \rangle = \theta$$

Minimize $(\Delta\theta_{\mathrm{est}})^2$

Quantum Parameter Estimation

State preparation **Parameter imprinting** **Measurement** **Estimation**

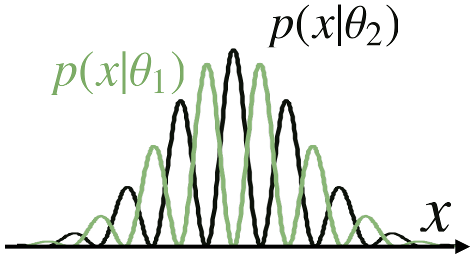


Results:

$x_1, x_2, \ldots, x_\mu$

Distribution: $p(x|\theta)$

$\theta_{\mathrm{est}}(x_1, x_2, \ldots, x_\mu)$



Objective:

$\langle \theta_{\mathrm{est}} \rangle = \theta$

Minimize $(\Delta\theta_{\mathrm{est}})^2$

Quantum system:

$$p(x|\theta) = \mathrm{Tr}\{\rho(\theta)\Pi_x\}$$

Quantum strategies

Making optimal choices for

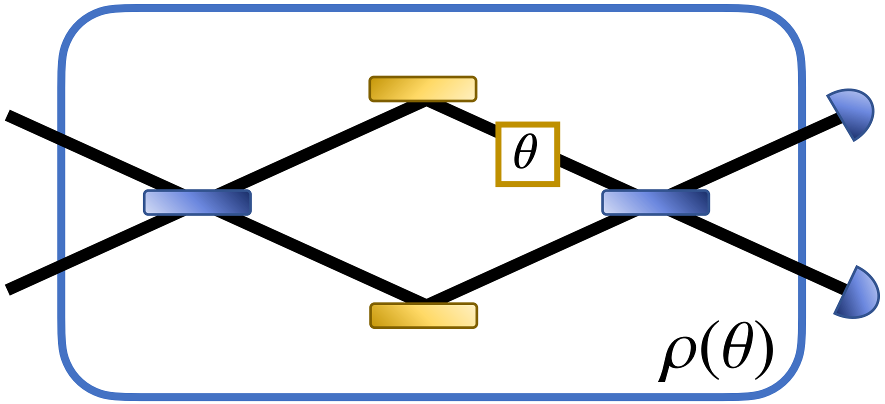
- measurement observable
- initial state

Quantum Parameter Estimation

State preparation

ρ

Parameter imprinting

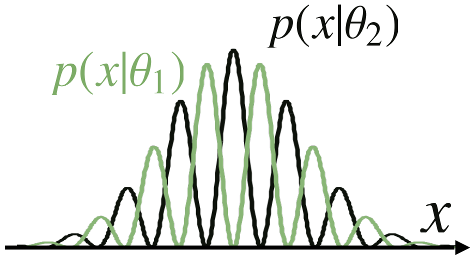


Measurement

Results:

$x_1, x_2, \ldots, x_\mu$

Distribution: $p(x|\theta)$



Quantum system:

$$p(x|\theta) = \mathrm{Tr}\{\rho(\theta)\Pi_x\}$$

Estimation

$$\theta_{\mathrm{est}}(x_1, x_2, \ldots, x_\mu)$$

Objective:

$$\langle\theta_{\mathrm{est}}\rangle = \theta$$

Minimize $(\Delta\theta_{\mathrm{est}})^2$

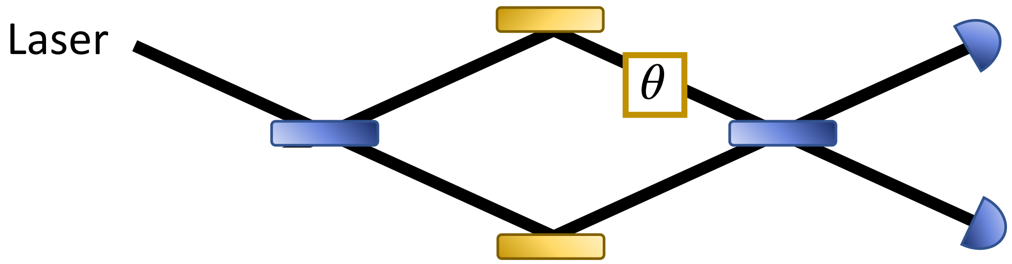
Quantum strategies

Making optimal choices for

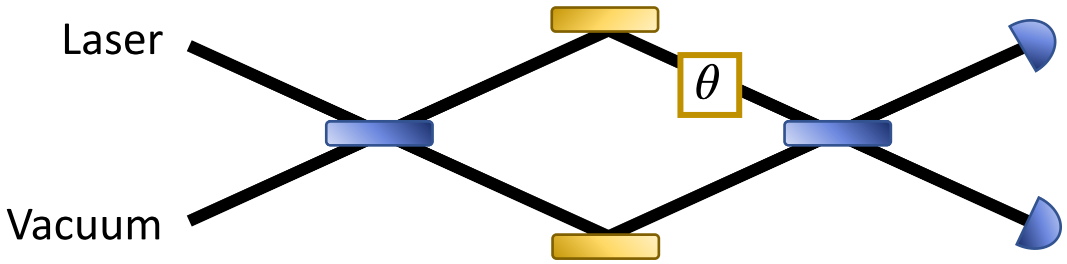
- measurement observable
- initial state

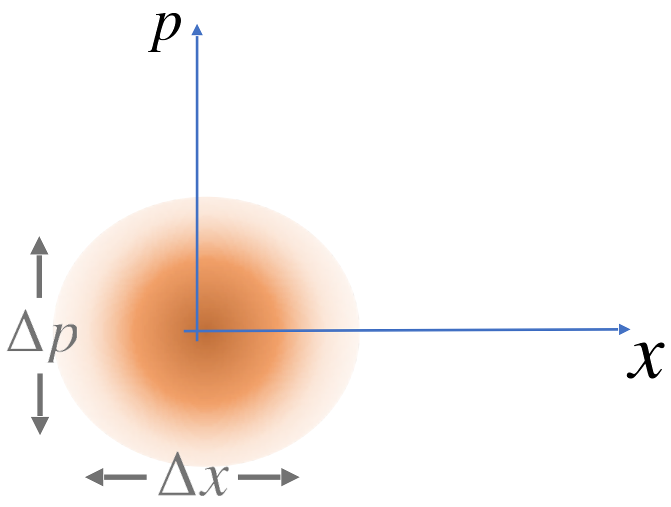
Fundamental limitation:
Quantum fluctuations

SQUEEZING IN INTERFEROMETRY

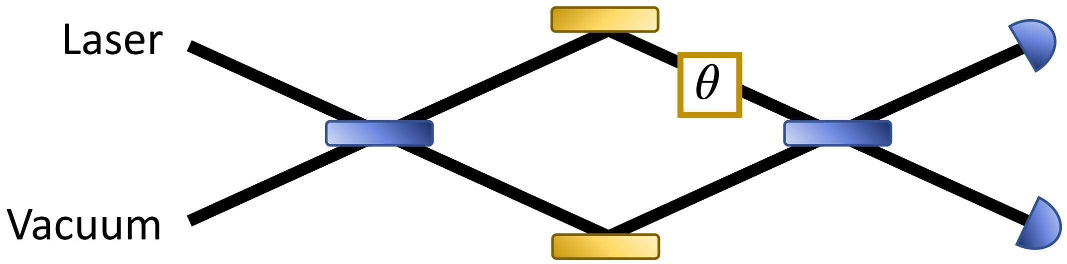


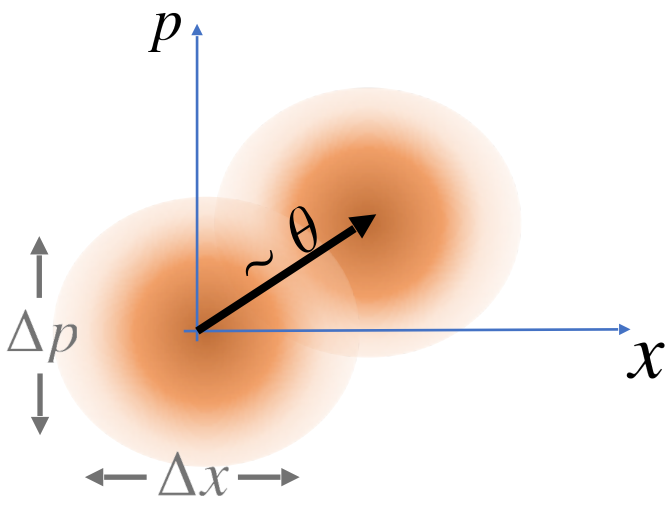
Squeezing in Interferometry



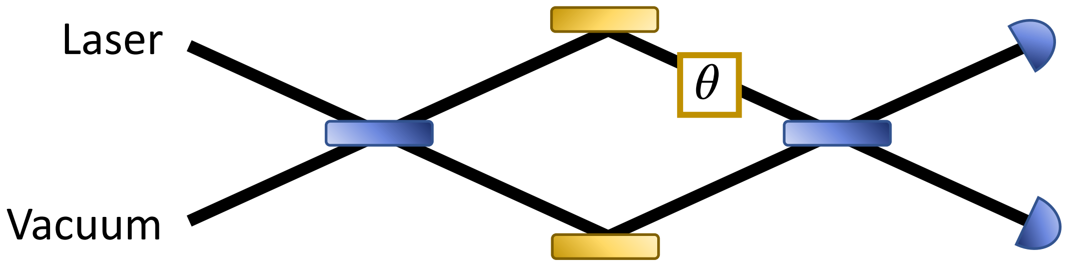


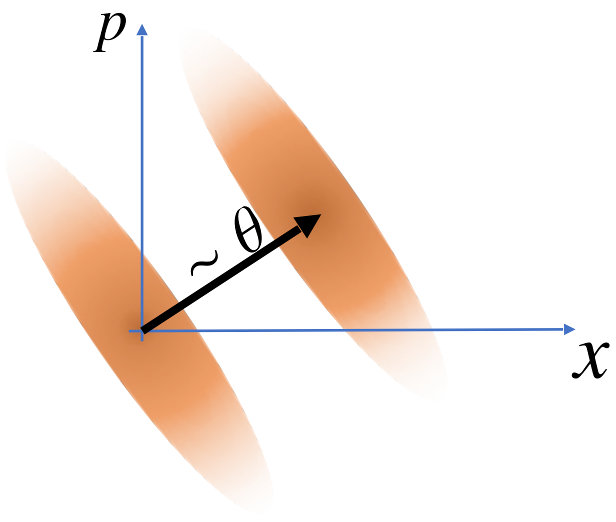
Squeezing in Interferometry





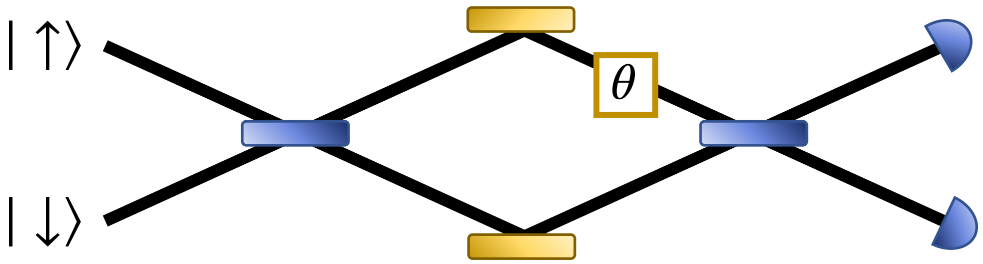
Squeezing in Interferometry

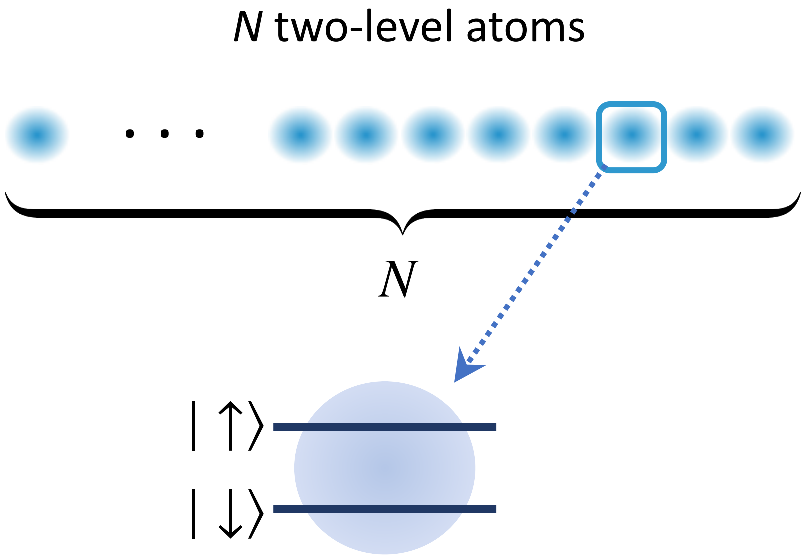




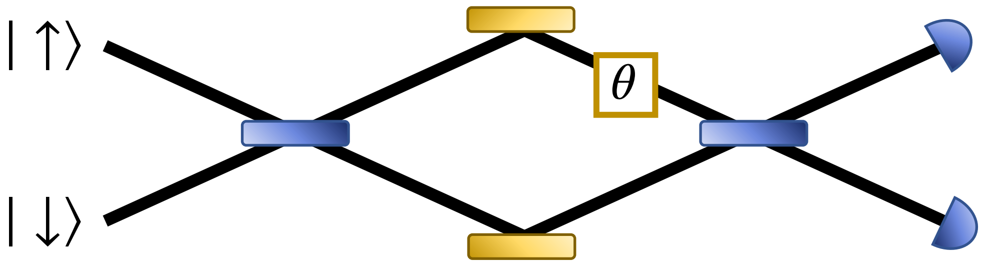
C.M. Caves, Phys. Rev. D **23**, 1693 (1981)

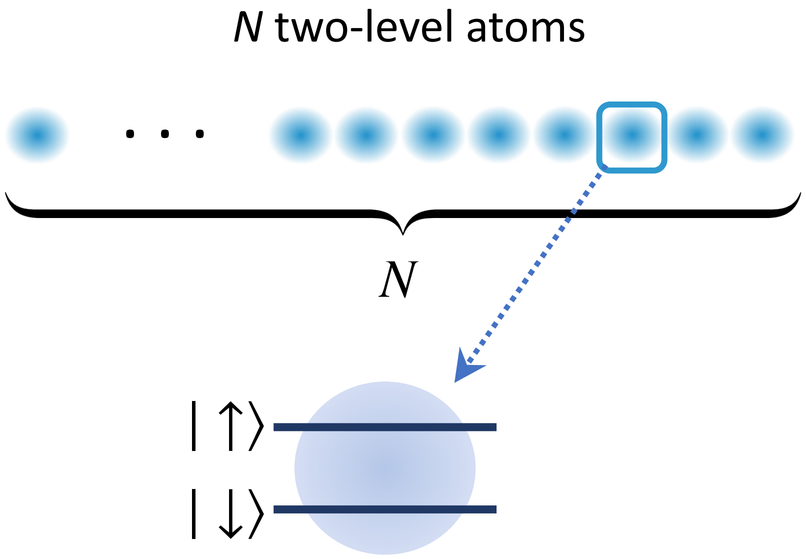
Atomic Interferometry: “Ramsey Spectroscopy”





Atomic Interferometry: "Ramsey Spectroscopy"



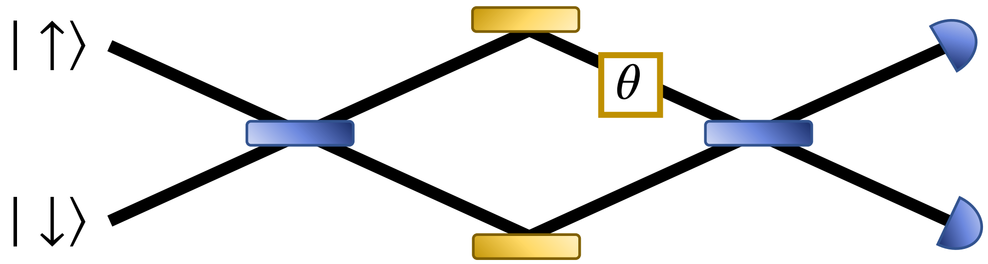


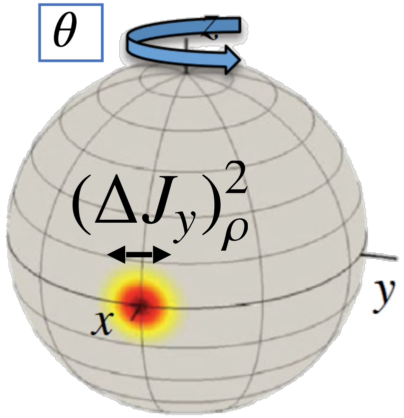
Collective angular momentum observables: *SU(2)*

$$J_x = \frac{1}{2}\sum_{i=1}^{N}\sigma_x^{(i)} \qquad J_y = \frac{1}{2}\sum_{i=1}^{N}\sigma_y^{(i)} \qquad J_z = \frac{1}{2}\sum_{i=1}^{N}\sigma_z^{(i)}$$

Pauli matrices

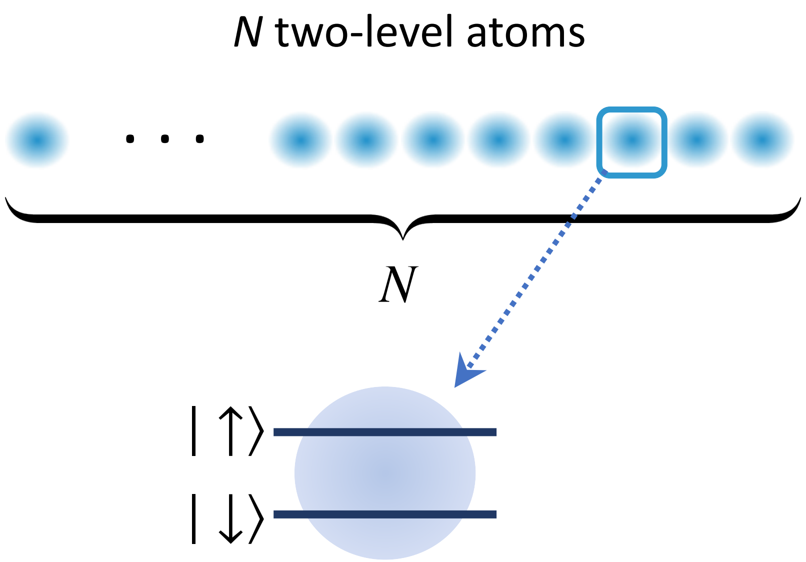
Atomic Interferometry: "Ramsey Spectroscopy"





Coherent spin state

Separable:
No correlations

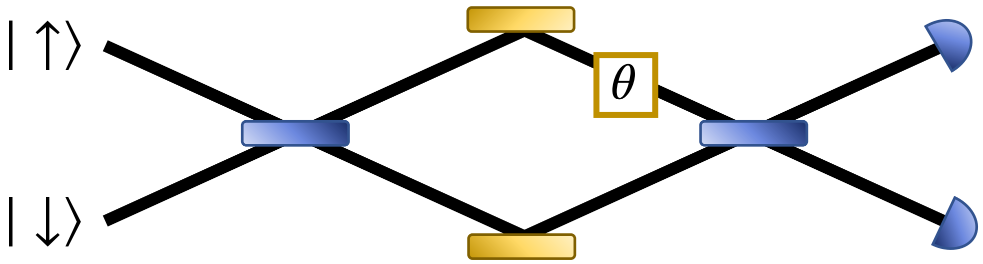


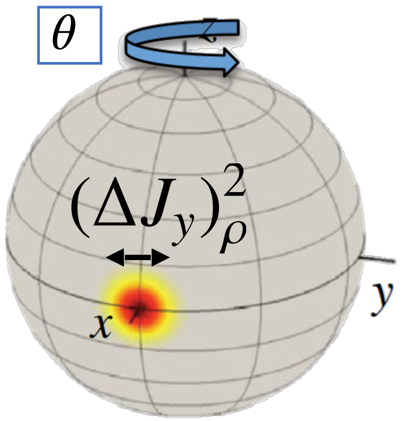
Collective angular momentum observables: *SU(2)*

$$J_x = \frac{1}{2}\sum_{i=1}^{N}\sigma_x^{(i)} \qquad J_y = \frac{1}{2}\sum_{i=1}^{N}\sigma_y^{(i)} \qquad J_z = \frac{1}{2}\sum_{i=1}^{N}\sigma_z^{(i)}$$

↑ Pauli matrices

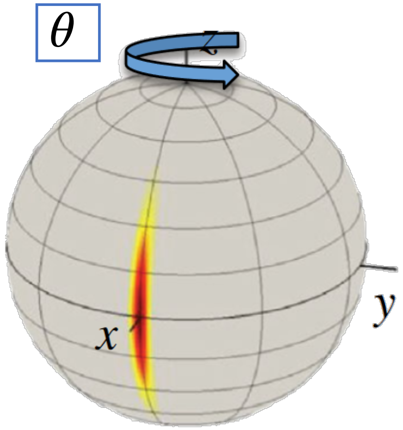
Atomic Interferometry: "Ramsey Spectroscopy"





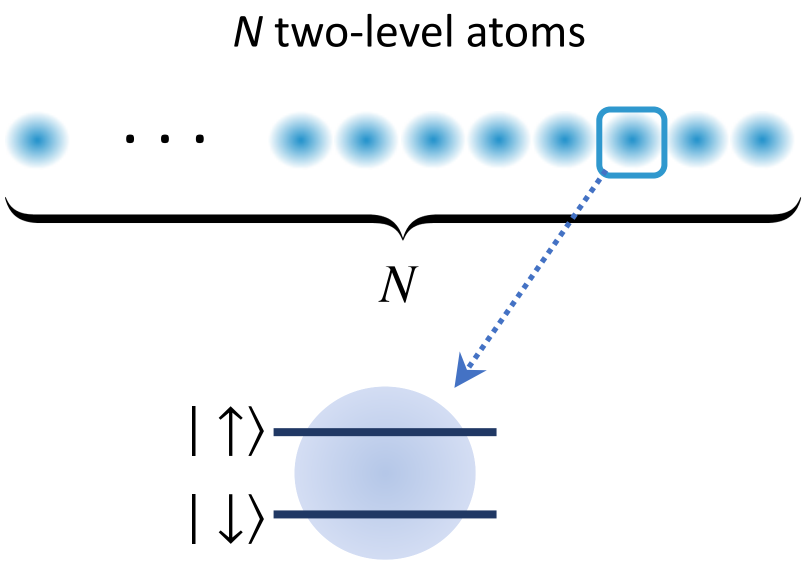
Coherent spin state

Separable:
No correlations



Squeezed spin state

Entangled:
Requires quantum correlations



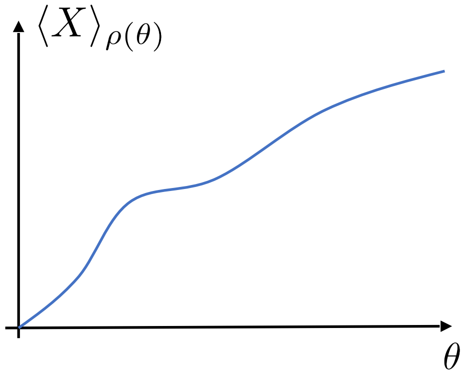
Collective angular momentum observables: *SU(2)*

$$J_x = \frac{1}{2}\sum_{i=1}^{N}\sigma_x^{(i)} \qquad J_y = \frac{1}{2}\sum_{i=1}^{N}\sigma_y^{(i)} \qquad J_z = \frac{1}{2}\sum_{i=1}^{N}\sigma_z^{(i)}$$

Pauli matrices

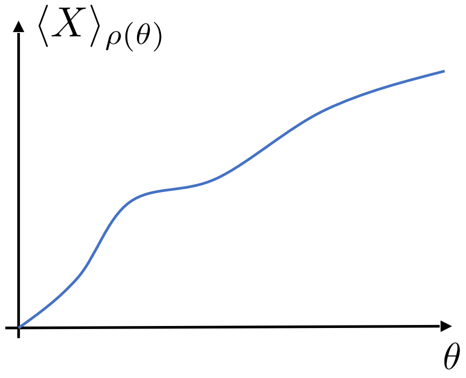
Quantum Noise and Squeezing

Calibration



Quantum Noise and Squeezing

Calibration

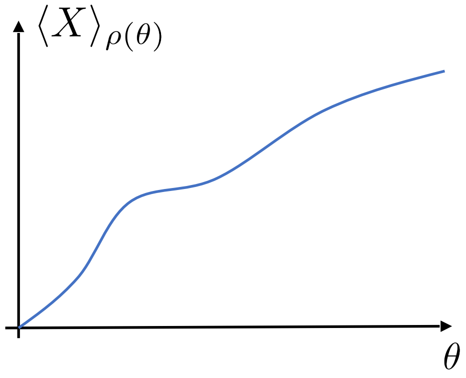


Measurement

$$x_1, \ldots, x_\mu$$

Quantum Noise and Squeezing

Calibration

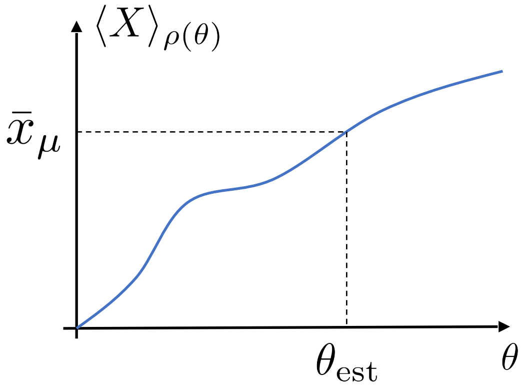


Measurement

$$x_1, \ldots, x_\mu \longrightarrow \bar{x}_\mu = \frac{1}{\mu}\sum_{i=1}^{\mu} x_i$$

Quantum Noise and Squeezing

Calibration

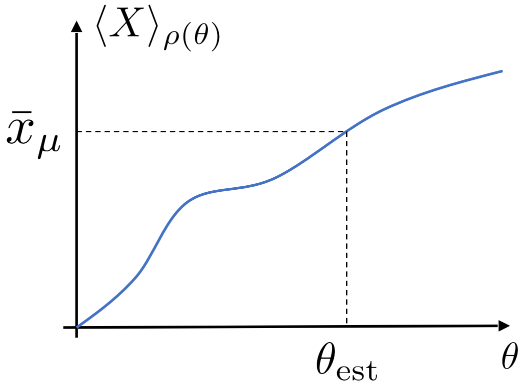


Measurement

$x_1, \ldots, x_\mu \longrightarrow \bar{x}_\mu = \frac{1}{\mu} \sum_{i=1}^{\mu} x_i$

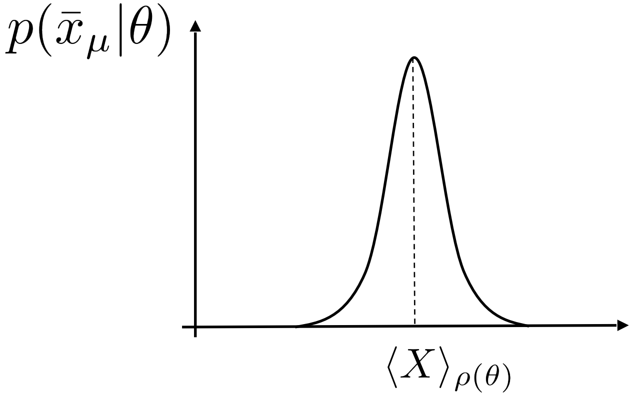
Quantum Noise and Squeezing

Calibration



Uncertainty

In the central limit, $\mu \gg 1$

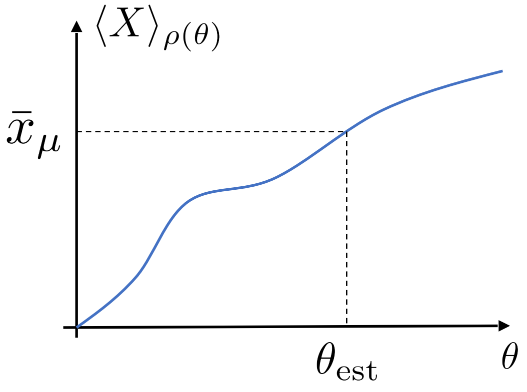


Measurement

$x_1, \ldots, x_\mu \longrightarrow \bar{x}_\mu = \frac{1}{\mu}\sum_{i=1}^{\mu} x_i$

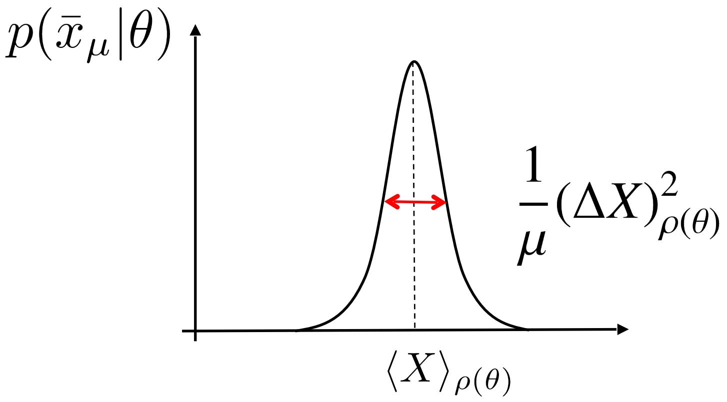
Quantum Noise and Squeezing

Calibration



Uncertainty

In the central limit, $\mu \gg 1$

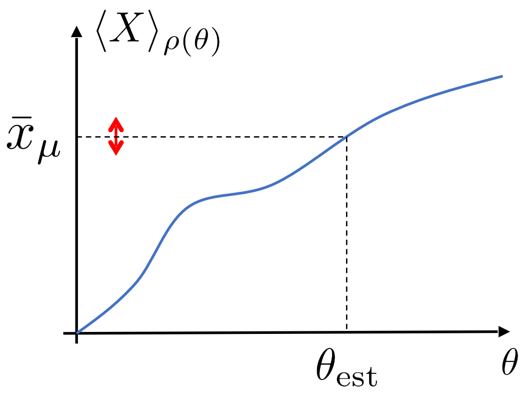


Measurement

$$x_1, \ldots, x_\mu \longrightarrow \bar{x}_\mu = \frac{1}{\mu}\sum_{i=1}^{\mu} x_i$$

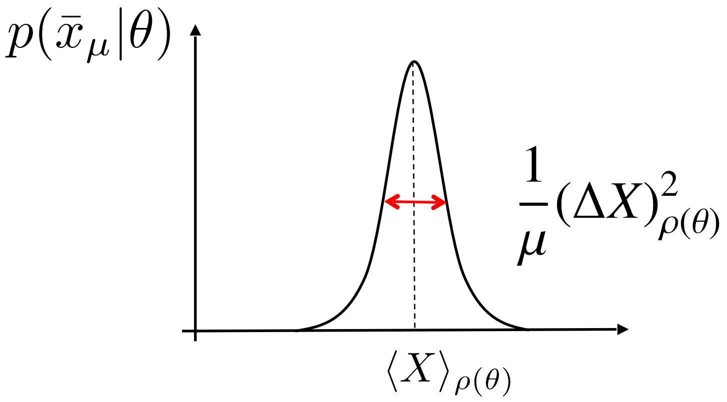
Quantum Noise and Squeezing

Calibration



Uncertainty

In the central limit, $\mu \gg 1$

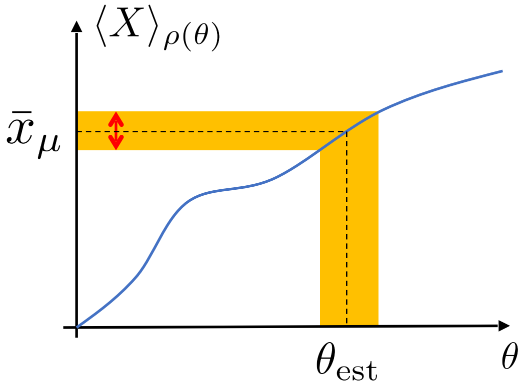


Measurement

$x_1, \ldots, x_\mu \longrightarrow \bar{x}_\mu = \frac{1}{\mu}\sum_{i=1}^{\mu} x_i$

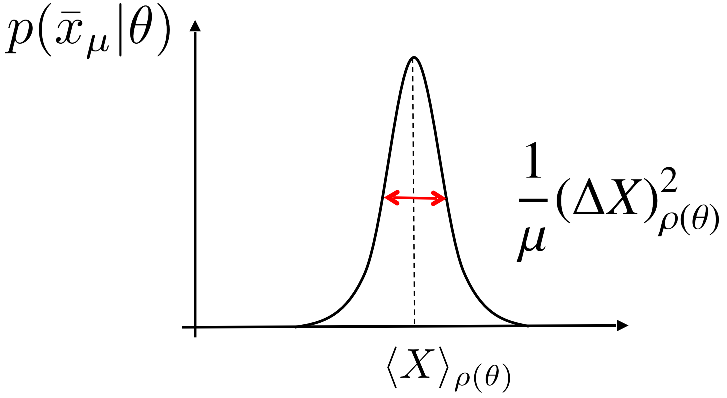
Quantum Noise and Squeezing

Calibration



Uncertainty

In the central limit, $\mu \gg 1$

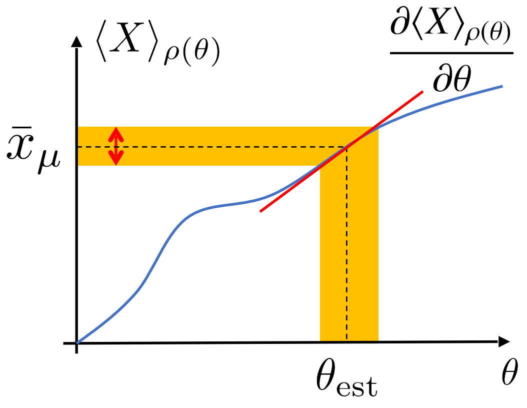


Measurement

$x_1, \ldots, x_\mu \longrightarrow \bar{x}_\mu = \frac{1}{\mu}\sum_{i=1}^{\mu} x_i$

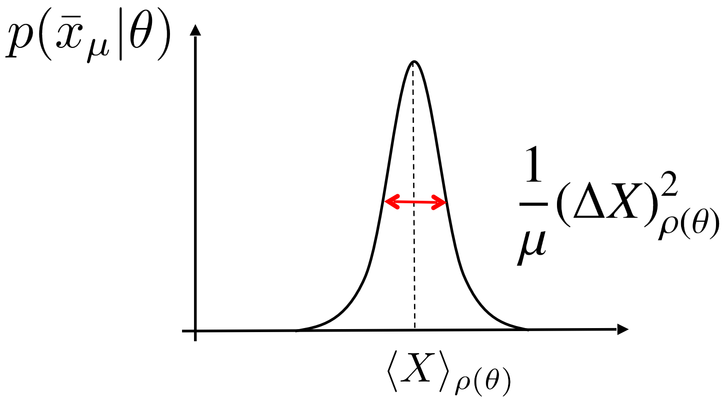
Quantum Noise and Squeezing

Calibration



Uncertainty

In the central limit, $\mu \gg 1$

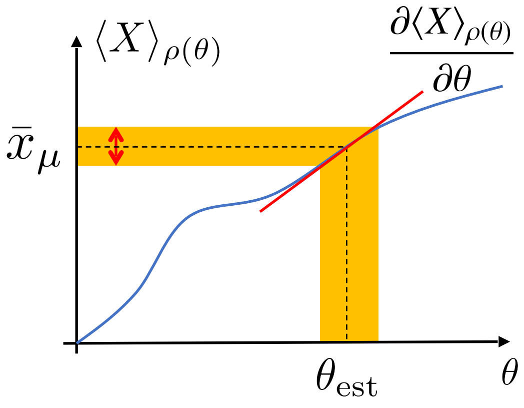


Measurement

$$x_1, \ldots, x_\mu \longrightarrow \bar{x}_\mu = \frac{1}{\mu}\sum_{i=1}^{\mu} x_i$$

Quantum Noise and Squeezing

Calibration

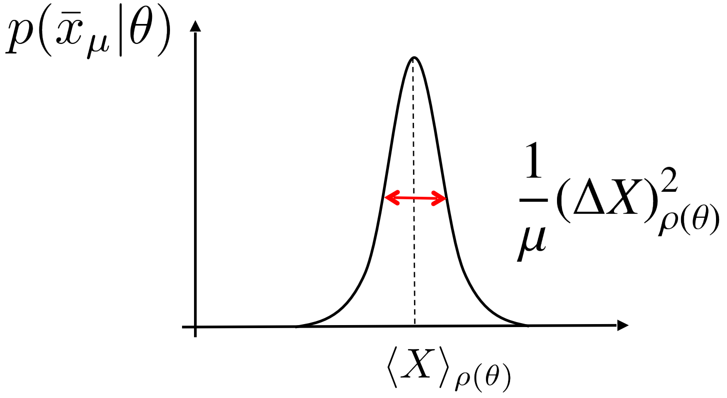


Measurement

$$x_1, \ldots, x_\mu \longrightarrow \bar{x}_\mu = \frac{1}{\mu} \sum_{i=1}^{\mu} x_i$$

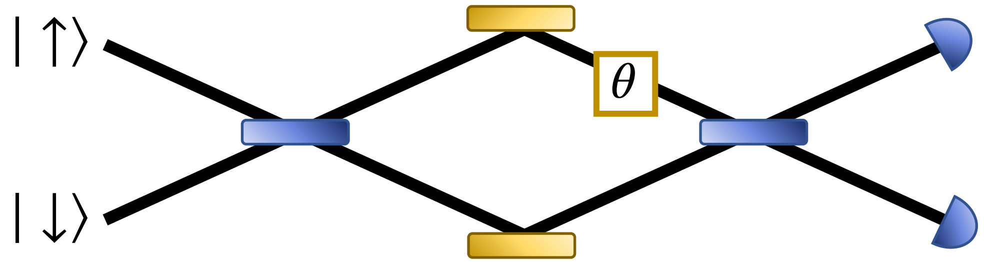
Uncertainty

In the central limit, $\mu \gg 1$

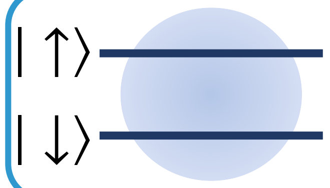


$$(\Delta\theta_{\text{est}})^2 = \frac{1}{\mu} \frac{(\Delta X)^2_{\rho(\theta)}}{\left| \frac{\partial \langle X \rangle_{\rho(\theta)}}{\partial \theta} \right|^2}$$

Quantum Noise and Squeezing



state preparation



rotation

$$e^{iJ_z\theta}$$

measurement

$$J_y$$

Ramsey spectroscopy

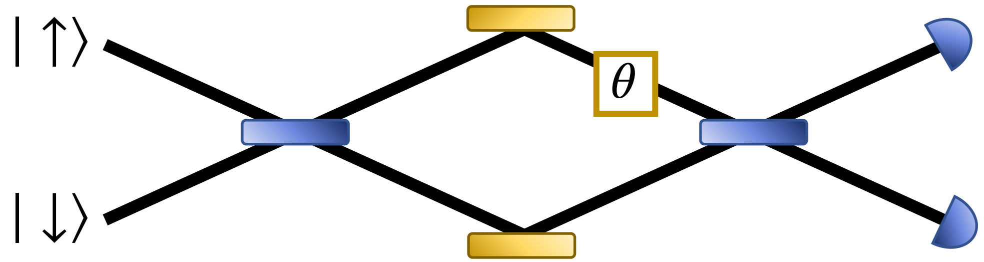
$$(\Delta\theta_{\text{est}})^2 = \frac{1}{\mu}\frac{(\Delta X)^2_{\rho(\theta)}}{\left|\frac{\partial\langle X\rangle_{\rho(\theta)}}{\partial\theta}\right|^2}$$

$$X = J_y$$

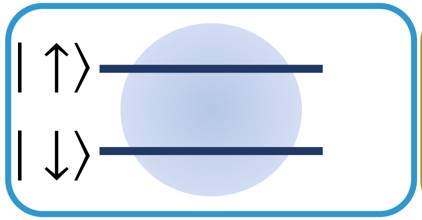
Schrödinger's equation

$$\frac{\partial\langle X\rangle_\rho}{\partial\theta} = i\langle[J_z, J_y]\rangle_\rho = i\langle J_x\rangle_\rho$$

Quantum Noise and Squeezing



state preparation



rotation

$$e^{iJ_z\theta}$$

measurement

$$J_y$$

Sensitivity

$$\chi^{-2} = \frac{|\langle J_x\rangle_\rho|^2}{(\Delta J_y)^2_\rho}$$

„Spin Squeezing“

D. J. Wineland *et al.*,
Phys. Rev. A **46**, R6797 (1992)

Ramsey spectroscopy

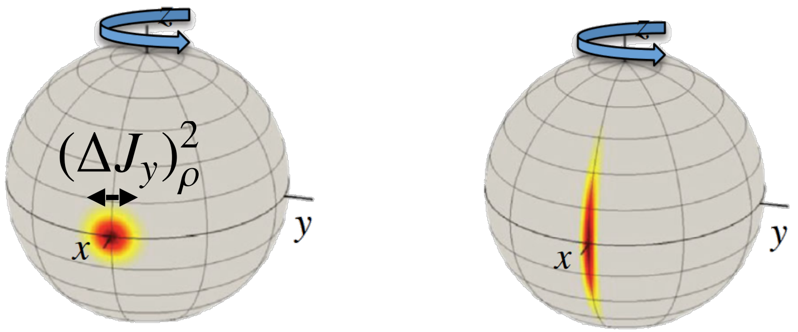
$$(\Delta\theta_{\mathrm{est}})^2 = \frac{1}{\mu}\frac{(\Delta X)^2_{\rho(\theta)}}{\left|\frac{\partial\langle X\rangle_{\rho(\theta)}}{\partial\theta}\right|^2}$$

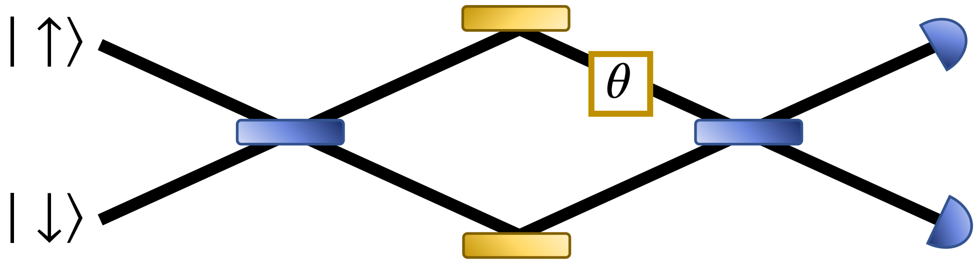
$$X = J_y$$

Schrödinger’s equation

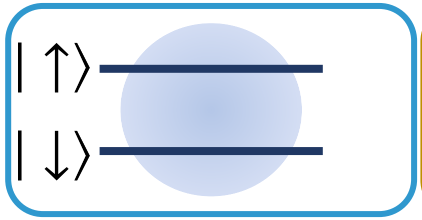
$$\frac{\partial\langle X\rangle_\rho}{\partial\theta} = i\langle[J_z, J_y]\rangle_\rho = i\langle J_x\rangle_\rho$$

Quantum Noise and Squeezing





state preparation



rotation

$$e^{iJ_z\theta}$$

measurement

$$J_y$$

Sensitivity

$$\chi^{-2} = \frac{|\langle J_x\rangle_\rho|^2}{(\Delta J_y)^2_\rho}$$

„Spin Squeezing“

D. J. Wineland *et al.*,
Phys. Rev. A **46**, R6797 (1992)

Ramsey spectroscopy

$$(\Delta\theta_{\mathrm{est}})^2 = \frac{1}{\mu}\frac{(\Delta X)^2_{\rho(\theta)}}{\left|\frac{\partial\langle X\rangle_{\rho(\theta)}}{\partial\theta}\right|^2}$$

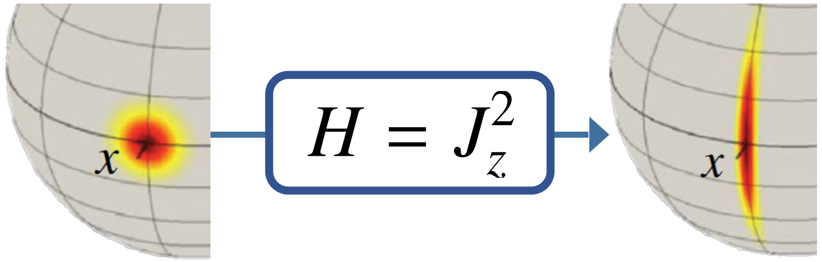
$$X = J_y$$

Schrödinger’s equation

$$\frac{\partial\langle X\rangle_\rho}{\partial\theta} = i\langle[J_z, J_y]\rangle_\rho = i\langle J_x\rangle_\rho$$

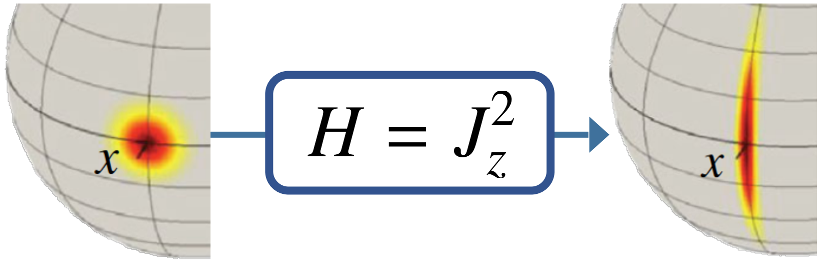
QUANTUM NOISE AND SQUEEZING: EXPERIMENTS

Kitagawa & Ueda, PRA **47** 5138 (1993)



Quantum Noise and Squeezing: Experiments

Kitagawa & Ueda, PRA **47** 5138 (1993)

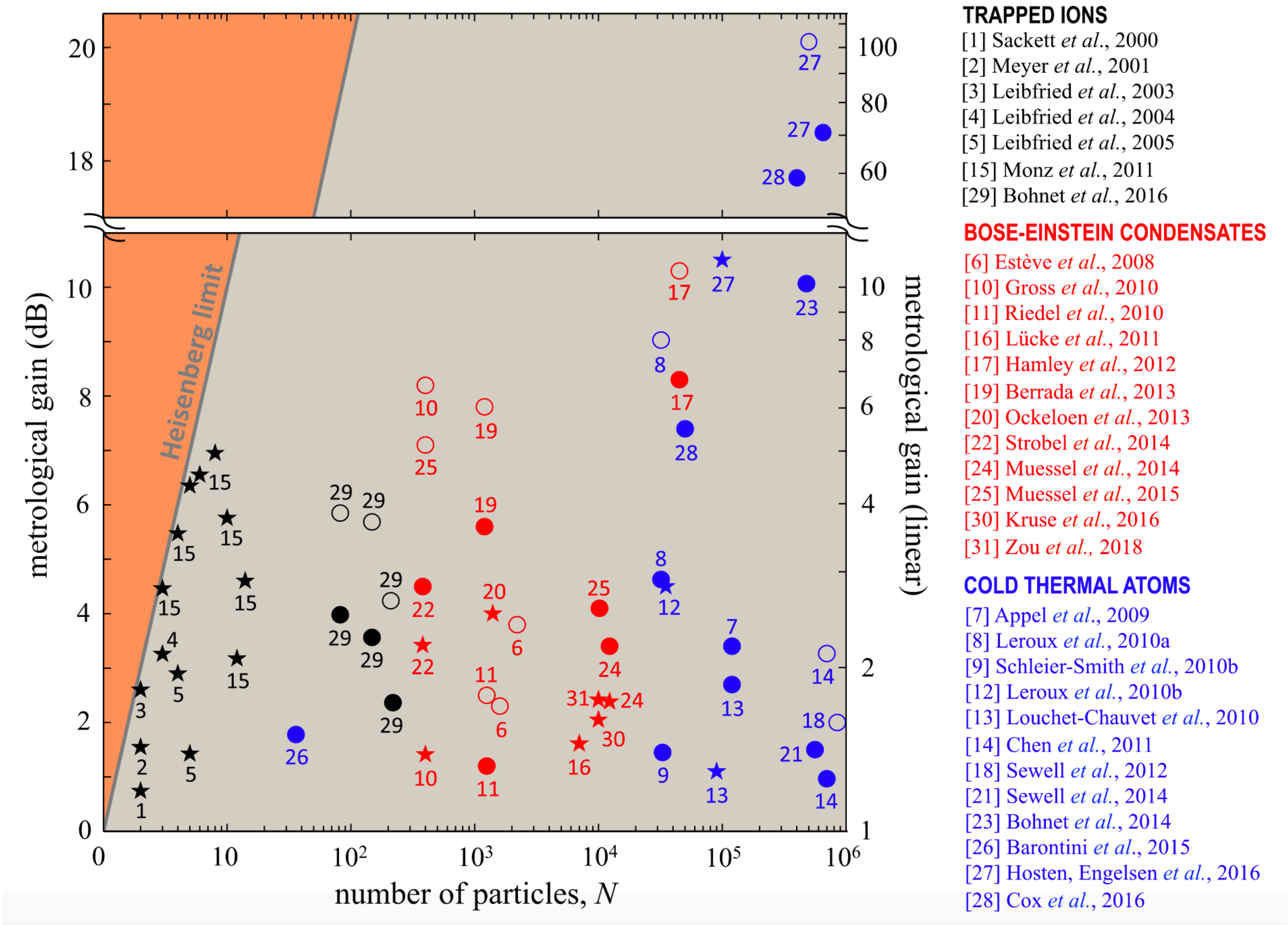


Proposals for implementation

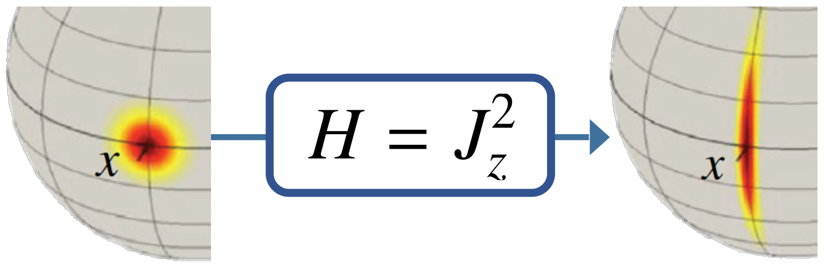
Trapped ions:
Mølmer & Sørensen PRL **82** 1835 (1999)

Bose-Einstein condensates:
Sørensen, Duan, Cirac & Zoller,
Nature **409**, 63 (2001)

Quantum Noise and Squeezing: Experiments



Kitagawa & Ueda, PRA **47** 5138 (1993)



Proposals for implementation

Trapped ions:
Mølmer & Sørensen PRL **82** 1835 (1999)

Bose-Einstein condensates:
Sørensen, Duan, Cirac & Zoller, Nature **409**, 63 (2001)

Review:
Pezzè, Smerzi, Oberthaler, Schmied, Treutlein Rev. Mod. Phys. **90**, 035005 (2018)

Quantifying Quantum Sensitivity Enhancements

Sensitivity limit: quantum Cramér-Rao bound $$(\Delta\theta_{\mathrm{est}})^2 \geq \frac{1}{F_Q[\rho, J_z]}$$

Quantifying Quantum Sensitivity Enhancements

Sensitivity limit: quantum Cramér-Rao bound $$(\Delta\theta_{\text{est}})^2 \geq \frac{1}{F_Q[\rho, J_z]}$$

Maximal achievable sensitivity

measurement of an optimal observable

$$F_Q[\rho, J_z]$$

Quantifying Quantum Sensitivity Enhancements

Sensitivity limit: quantum Cramér-Rao bound $$(\Delta\theta_{\text{est}})^2 \geq \frac{1}{F_Q[\rho, J_z]}$$

Precision from a measurement of *X*

Maximal achievable sensitivity

measurement of an optimal observable

$$\frac{|\langle [J_z, X] \rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, J_z]$$

Quantifying Quantum Sensitivity Enhancements

Sensitivity limit: quantum Cramér-Rao bound $$(\Delta\theta_{\mathrm{est}})^2 \geq \frac{1}{F_Q[\rho, J_z]}$$

Precision from a measurement of *X*

Maximal achievable sensitivity

measurement of an optimal observable

$$\frac{|\langle [J_z, X] \rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, J_z]$$

Spin Squeezing

$$X = J_y$$

D. J. Wineland et al.,
Phys. Rev. A **46**, R6797 (1992)

Quantifying Quantum Sensitivity Enhancements

Sensitivity limit: quantum Cramér-Rao bound $$(\Delta\theta_{\text{est}})^2 \geq \frac{1}{F_Q[\rho, J_z]}$$

Precision from a measurement of *X*

Maximal achievable sensitivity

measurement of an optimal observable

$$\frac{|\langle [J_z, X] \rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, J_z]$$

Spin Squeezing

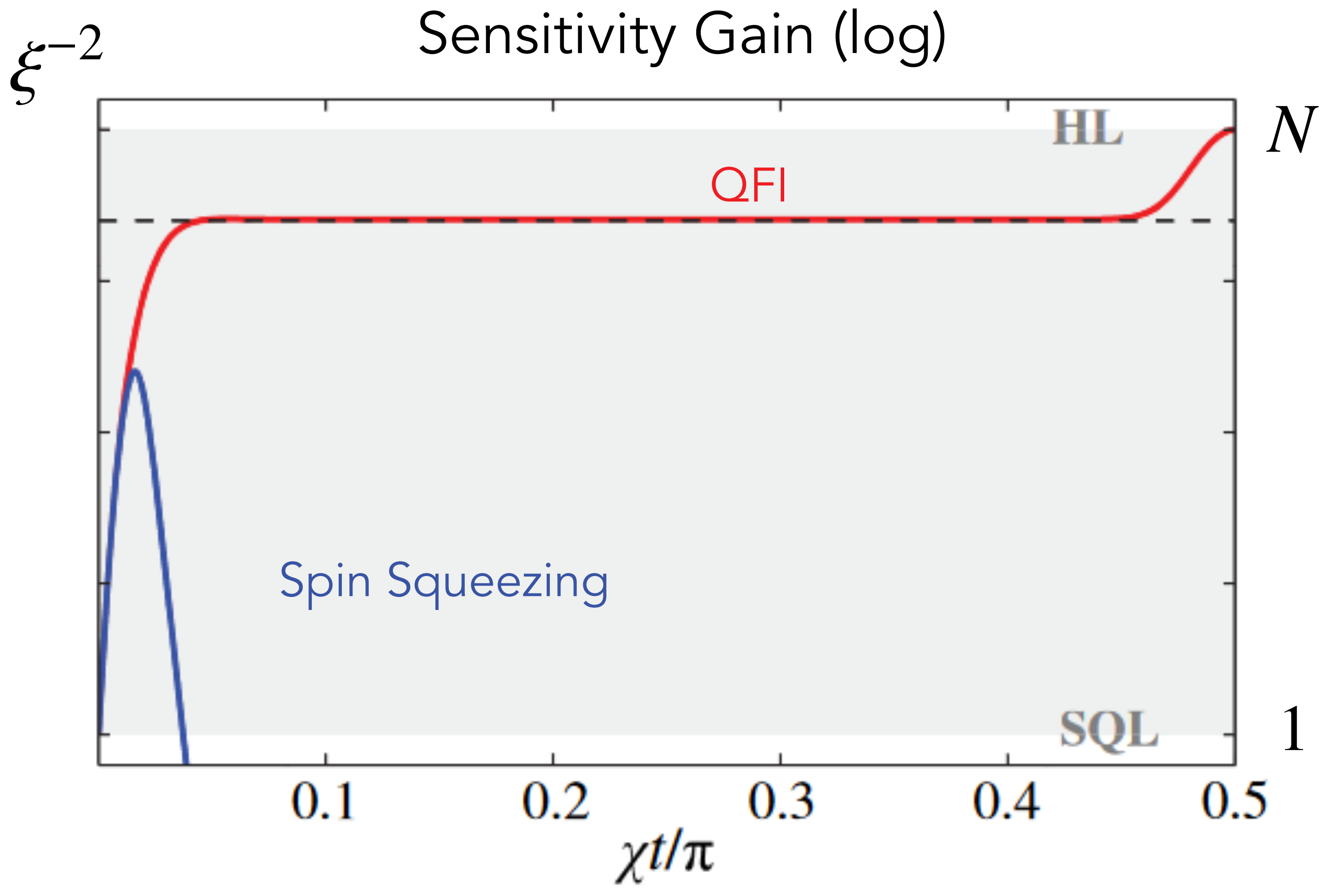
$$X = J_y$$

D. J. Wineland et al.,
Phys. Rev. A **46**, R6797 (1992)

- Simple observable
- Gaussian approximation of the QFI

One-Axis Twisting

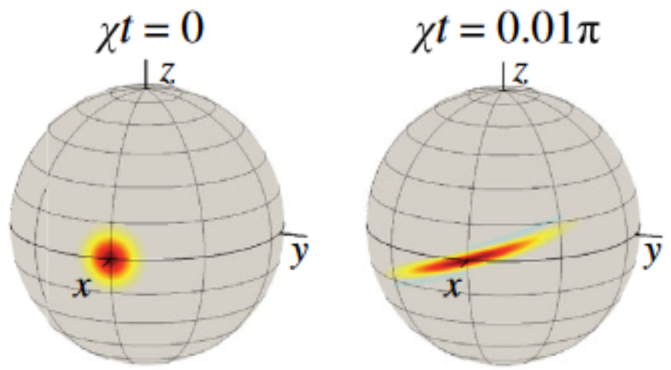
$$H_{\mathrm{NL}} = \chi J_z^2$$

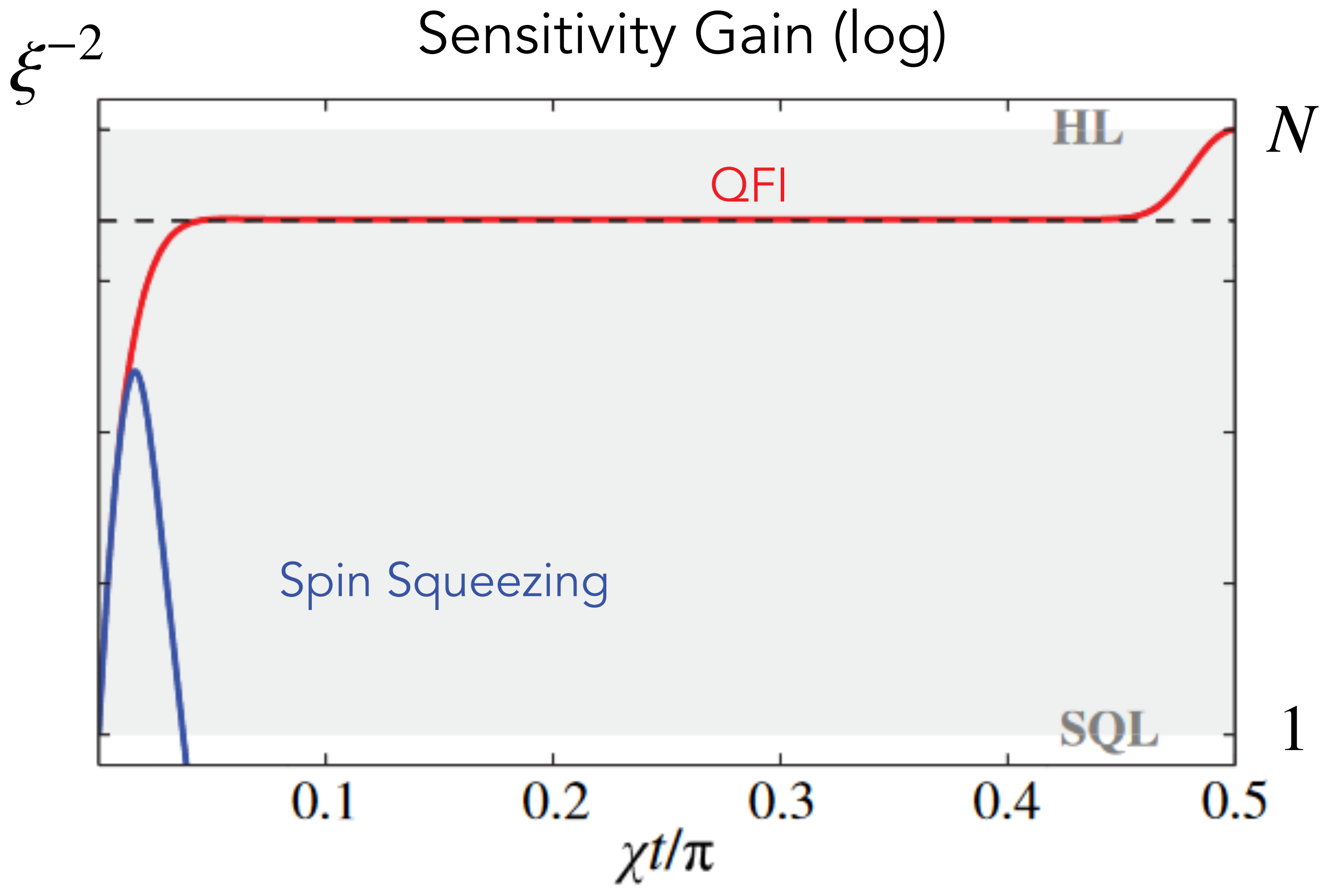


Kitagawa & Ueda, PRA **47** 5138 (1993)
Pezzé *et al.*, RMP **90**, 035005 (2018)

One-Axis Twisting

$$H_{\mathrm{NL}} = \chi J_z^2$$

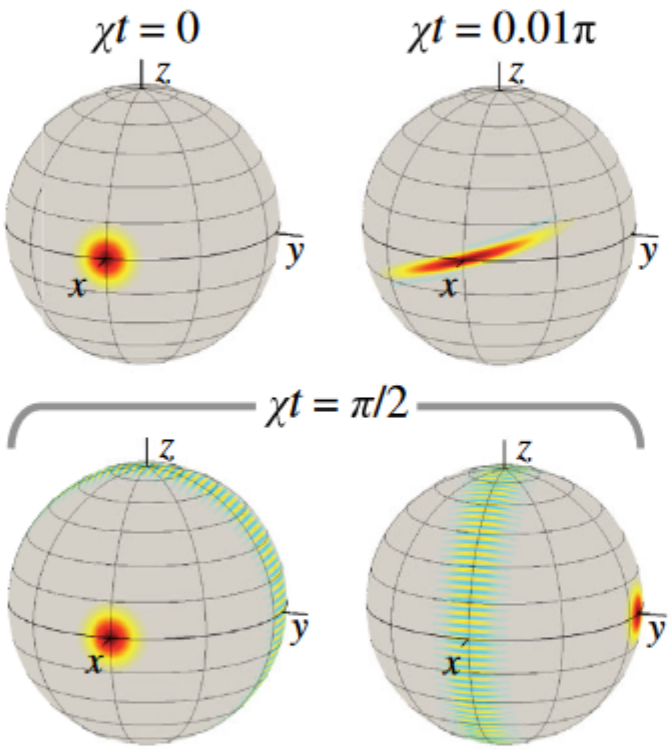


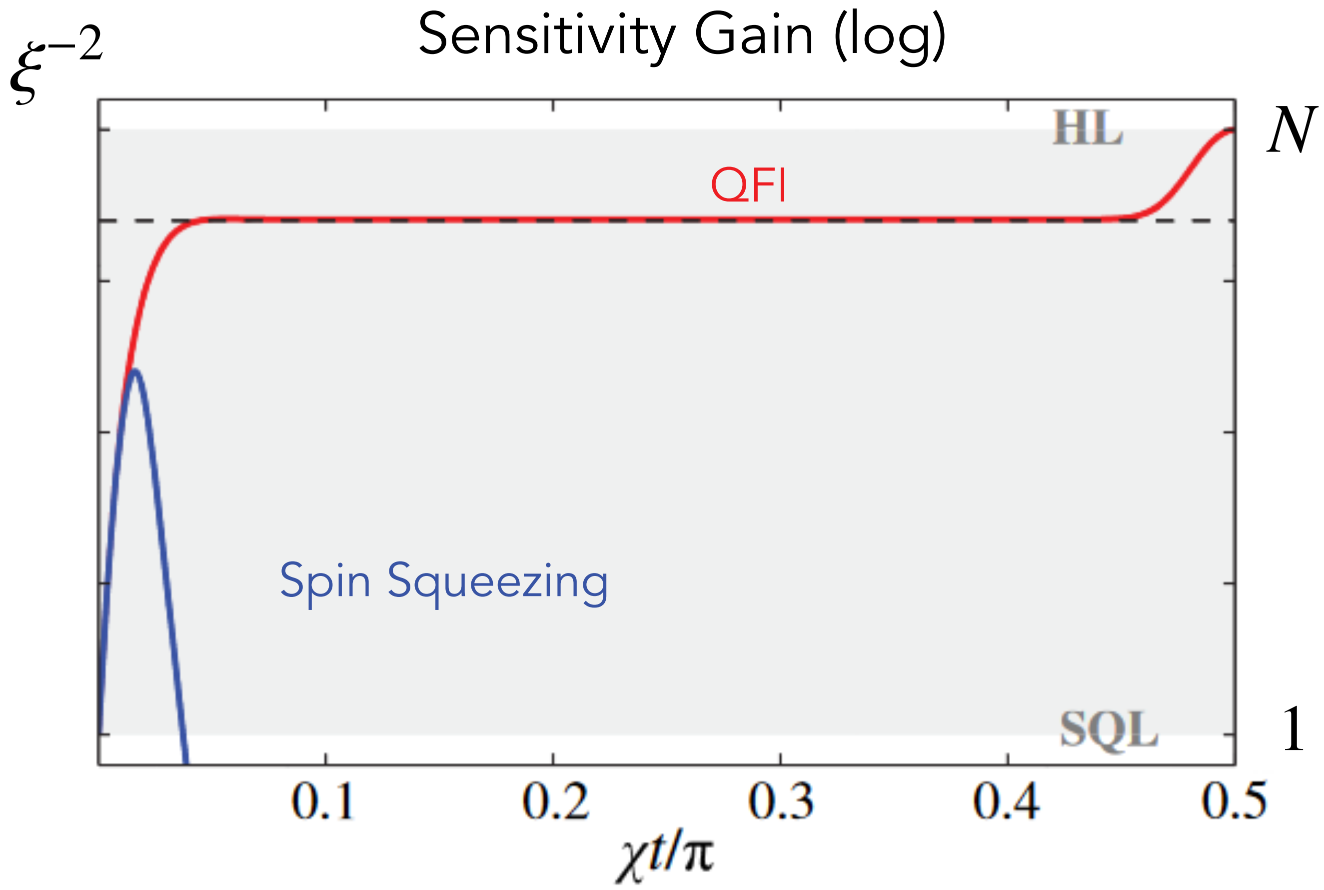


Kitagawa & Ueda, PRA **47** 5138 (1993)
Pezzé *et al.*, RMP **90**, 035005 (2018)

One-Axis Twisting

$$H_{\mathrm{NL}} = \chi J_z^2$$

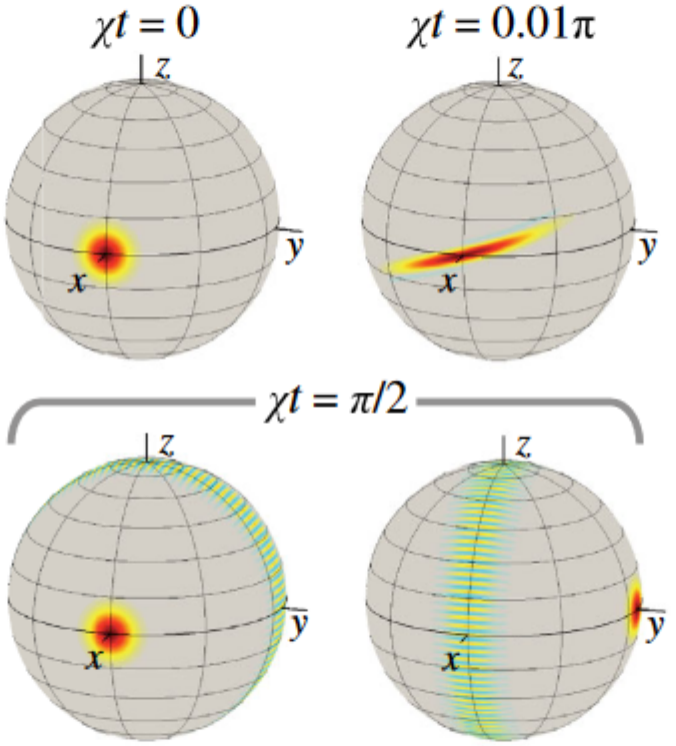


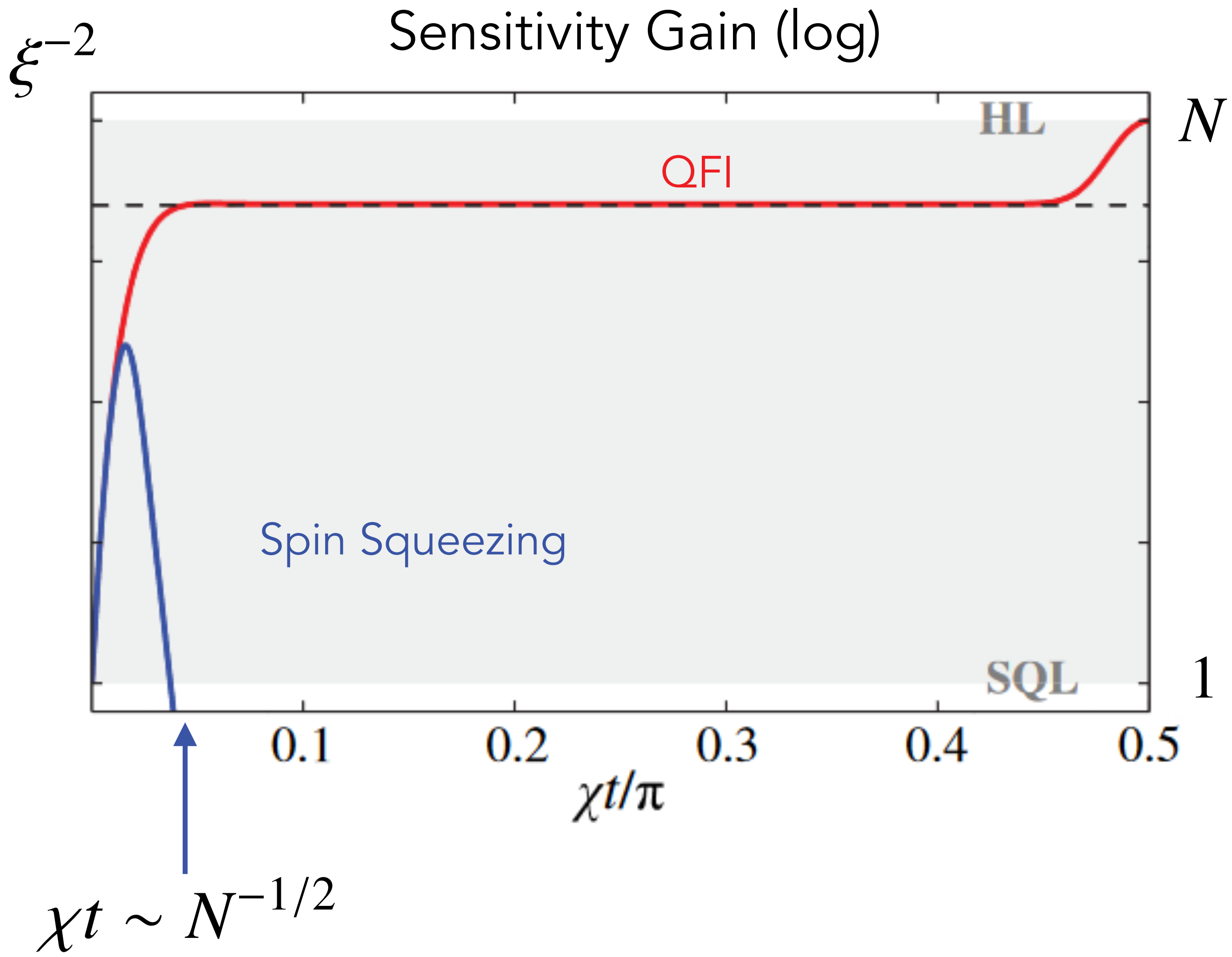


Kitagawa & Ueda, PRA **47** 5138 (1993)
Pezzé *et al.*, RMP **90**, 035005 (2018)

One-Axis Twisting

$$H_{\mathrm{NL}} = \chi J_z^2$$

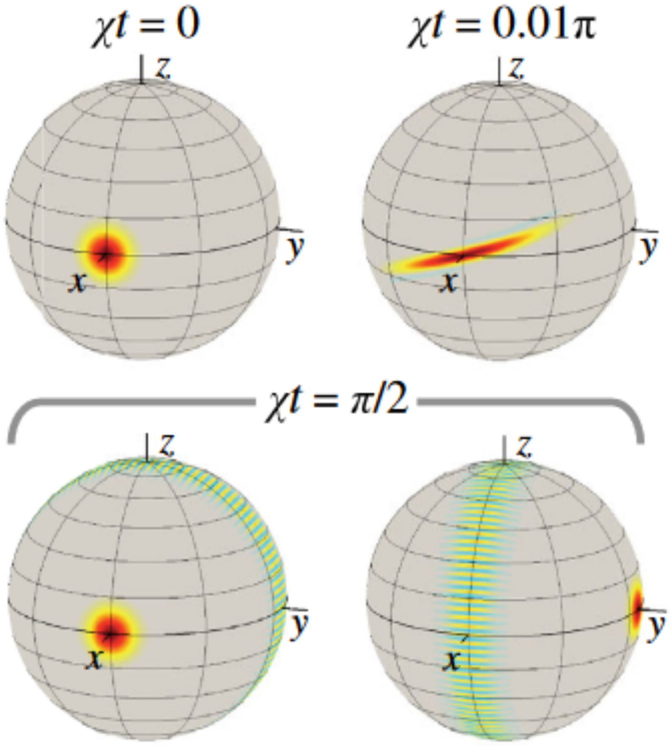




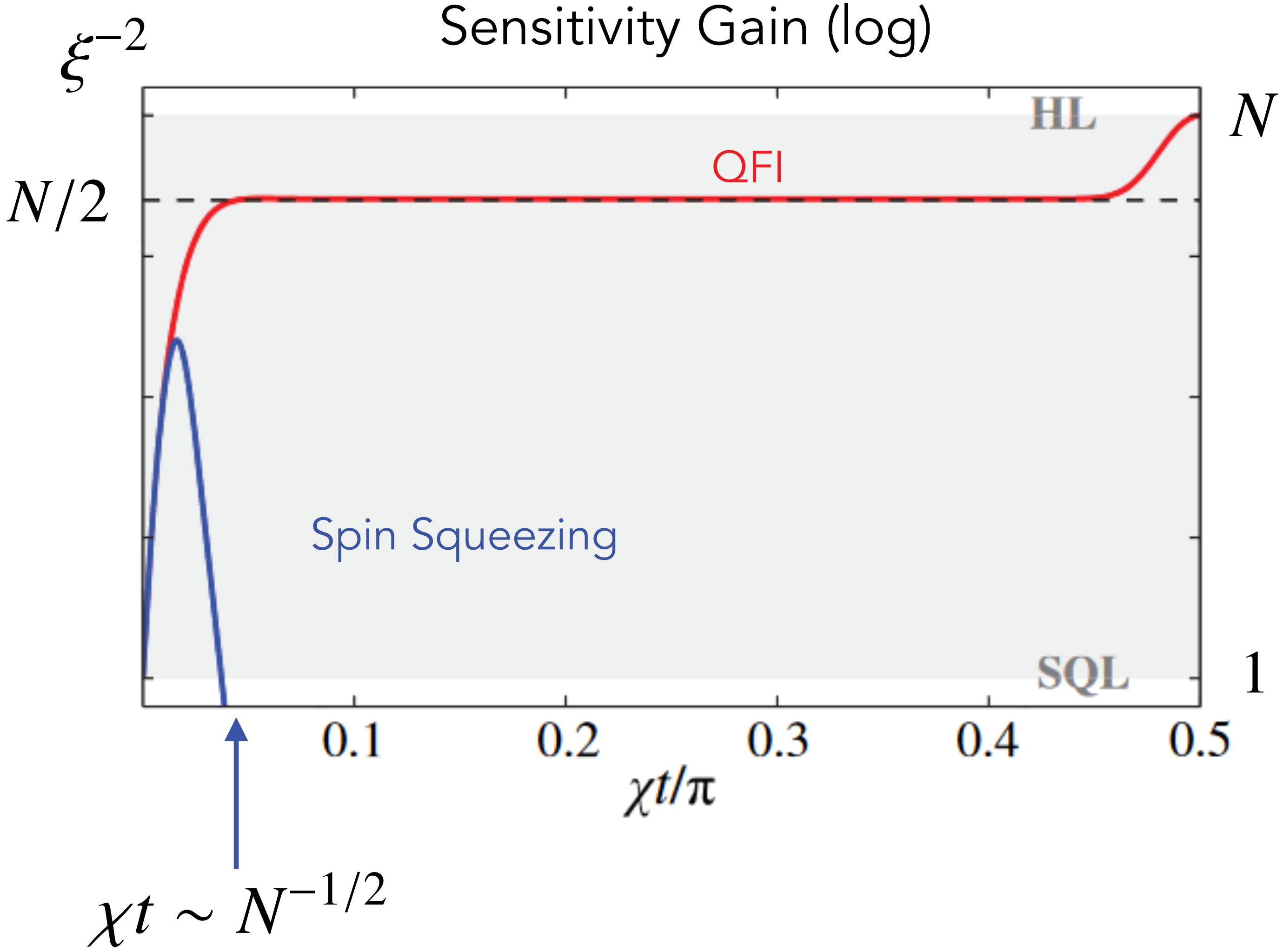
Kitagawa & Ueda, PRA **47** 5138 (1993)
Pezzé *et al.*, RMP **90**, 035005 (2018)

One-Axis Twisting

$$H_{\mathrm{NL}} = \chi J_z^2$$



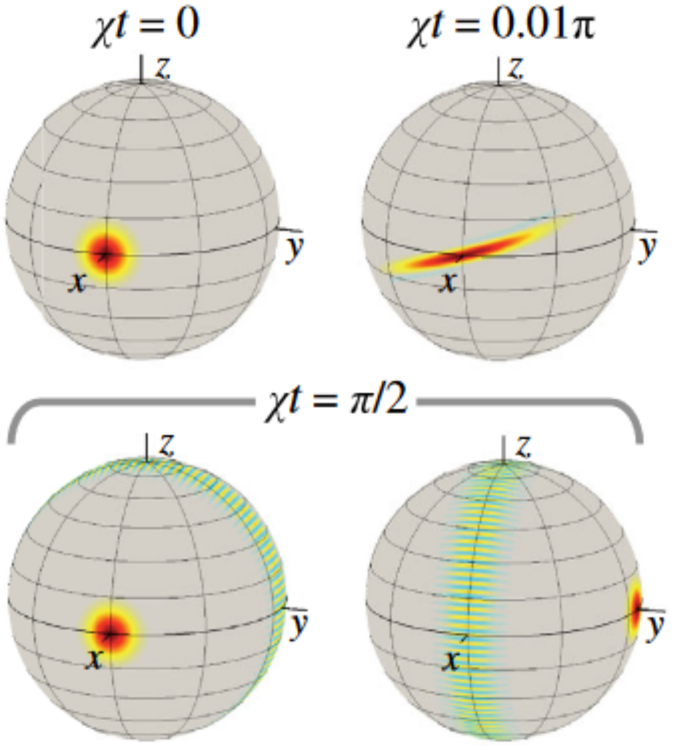
Sensitivity Gain (log)

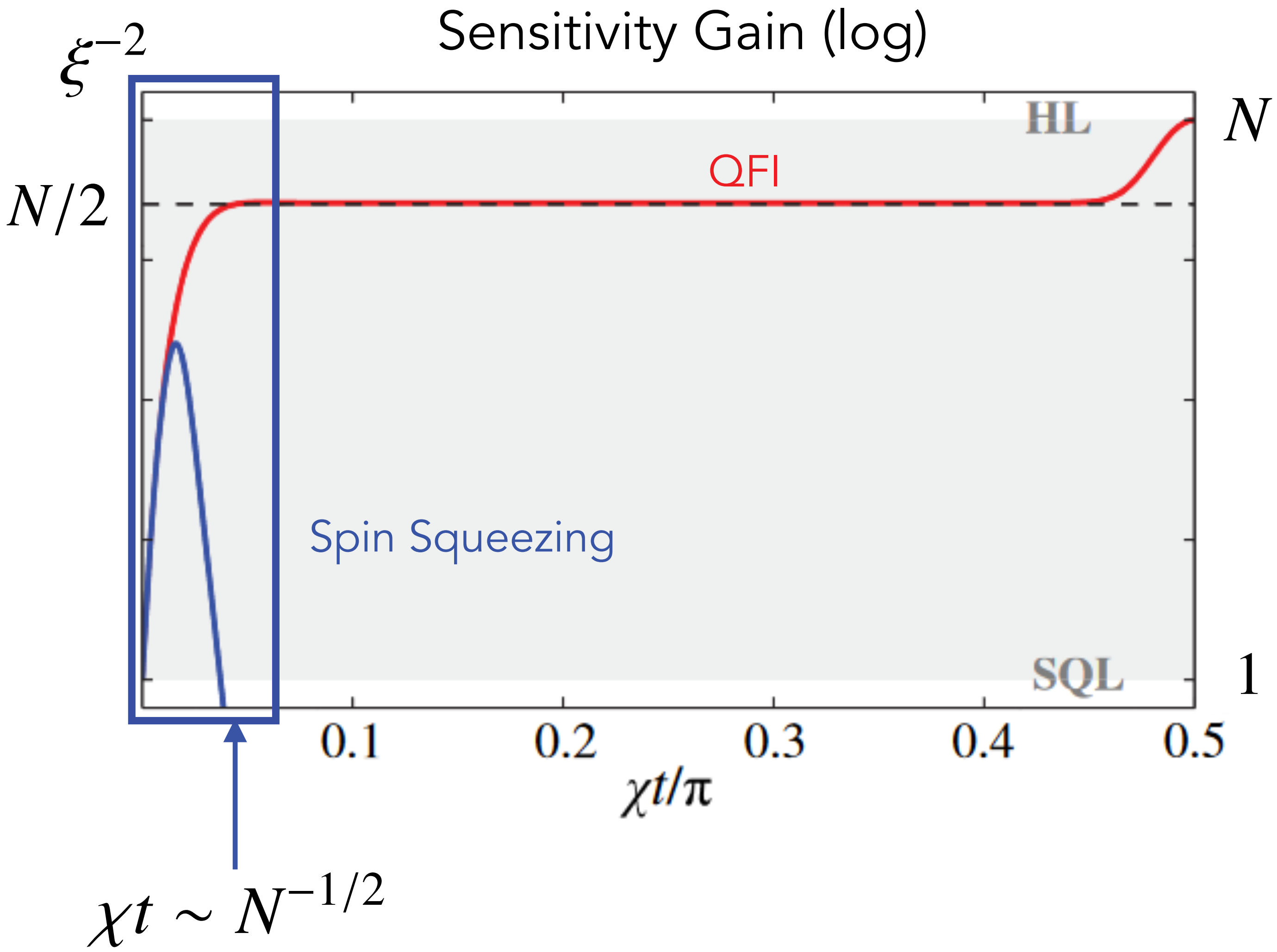


Kitagawa & Ueda, PRA **47** 5138 (1993)
Pezzé *et al.*, RMP **90**, 035005 (2018)

One-Axis Twisting

$$H_{\mathrm{NL}} = \chi J_z^2$$





Kitagawa & Ueda, PRA **47** 5138 (1993)
Pezzé *et al.*, RMP **90**, 035005 (2018)

QUANTIFYING QUANTUM SENSITIVITY ENHANCEMENTS

Sensitivity limit: quantum Cramér-Rao bound $$(\Delta\theta_{\text{est}})^2 \geq \frac{1}{F_Q[\rho, J_z]}$$

Precision from a measurement of *X*

Maximal achievable sensitivity

measurement of an optimal observable

$$\frac{|\langle [J_z, X] \rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, J_z]$$

Spin Squeezing

$$X = J_y$$

D. J. Wineland et al.,
Phys. Rev. A **46**, R6797 (1992)

QUANTIFYING QUANTUM SENSITIVITY ENHANCEMENTS

Sensitivity limit: quantum Cramér-Rao bound $$(\Delta\theta_{\mathrm{est}})^2 \geq \frac{1}{F_Q[\rho, J_z]}$$

Precision from a measurement of *X*

Maximal achievable sensitivity

measurement of an optimal observable

$$\frac{|\langle[J_z, X]\rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, J_z]$$

Spin Squeezing

$$X = J_y$$

D. J. Wineland et al.,
Phys. Rev. A **46**, R6797 (1992)

Nonlinear Spin Squeezing

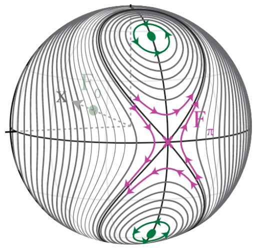
$$X = c_1 J_{\mathbf{n}_1} + c_2 J^2_{\mathbf{n}_2} + c_3 J^3_{\mathbf{n}_3} + \cdots$$

M. Gessner, A. Smerzi, and L. Pezzè,
Phys. Rev. Lett. **122**, 090503 (2019)

Squeezing of Non-Gaussian States

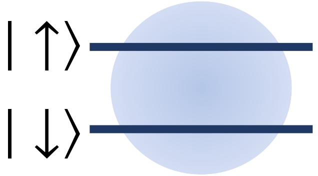
Sensitivity

$$\chi^{-2} = \frac{|\langle [J_{\mathbf{n}}, J_{\mathbf{m}}] \rangle|^2}{(\Delta J_{\mathbf{n}})^2}$$



$$H_{\mathrm{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

state preparation



rotation

$$e^{-iJ_{\mathbf{m}}\theta}$$

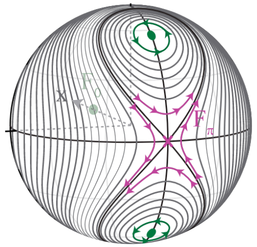
measurement

$$J_{\mathbf{n}}$$

Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle [J_{\mathbf{n}}, J_{\mathbf{m}}] \rangle|^2}{(\Delta J_{\mathbf{n}})^2}$$



$$H_{\mathrm{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

state preparation

$$e^{-iH_{\mathrm{NL}}\tau}$$

Nonlinear evolution

rotation

$$e^{-iJ_{\mathbf{m}}\theta}$$

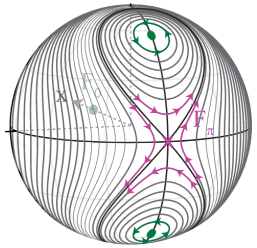
measurement

$$J_{\mathbf{n}}$$

Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle [J_{\mathbf{n}}, J_{\mathbf{m}}] \rangle|^2}{(\Delta J_{\mathbf{n}})^2}$$

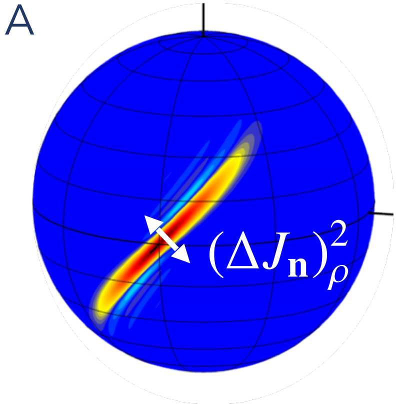


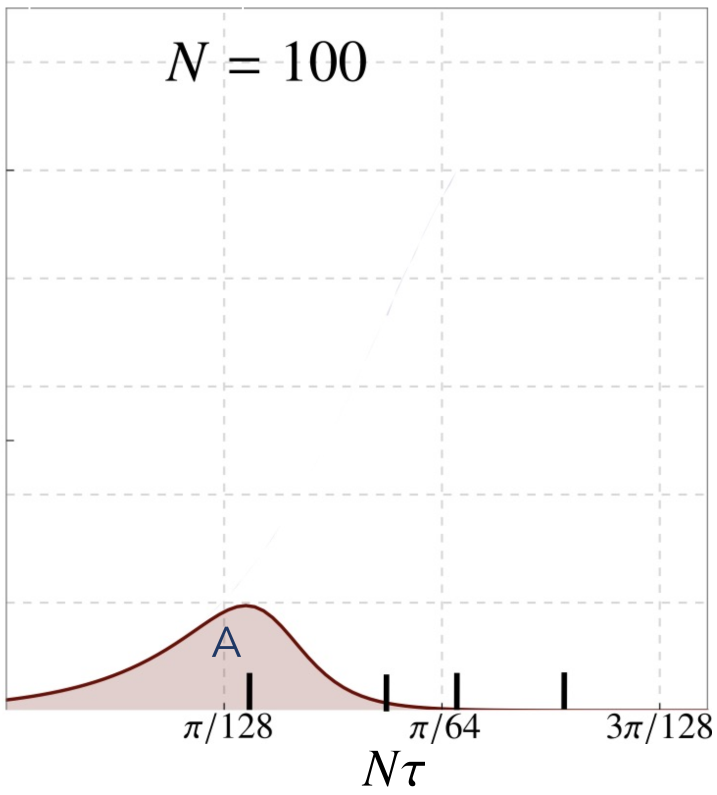
$$H_{NL} = \frac{1}{N} J_z^2 + \lambda J_x$$

state preparation — rotation — measurement

$$e^{-iH_{NL}\tau} \quad e^{-iJ_{\mathbf{m}}\theta} \quad J_{\mathbf{n}}$$

Nonlinear evolution

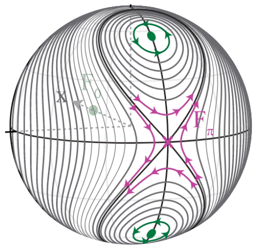




Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle[J_{\mathbf{n}}, J_{\mathbf{m}}]\rangle|^2}{(\Delta J_{\mathbf{n}})^2}$$



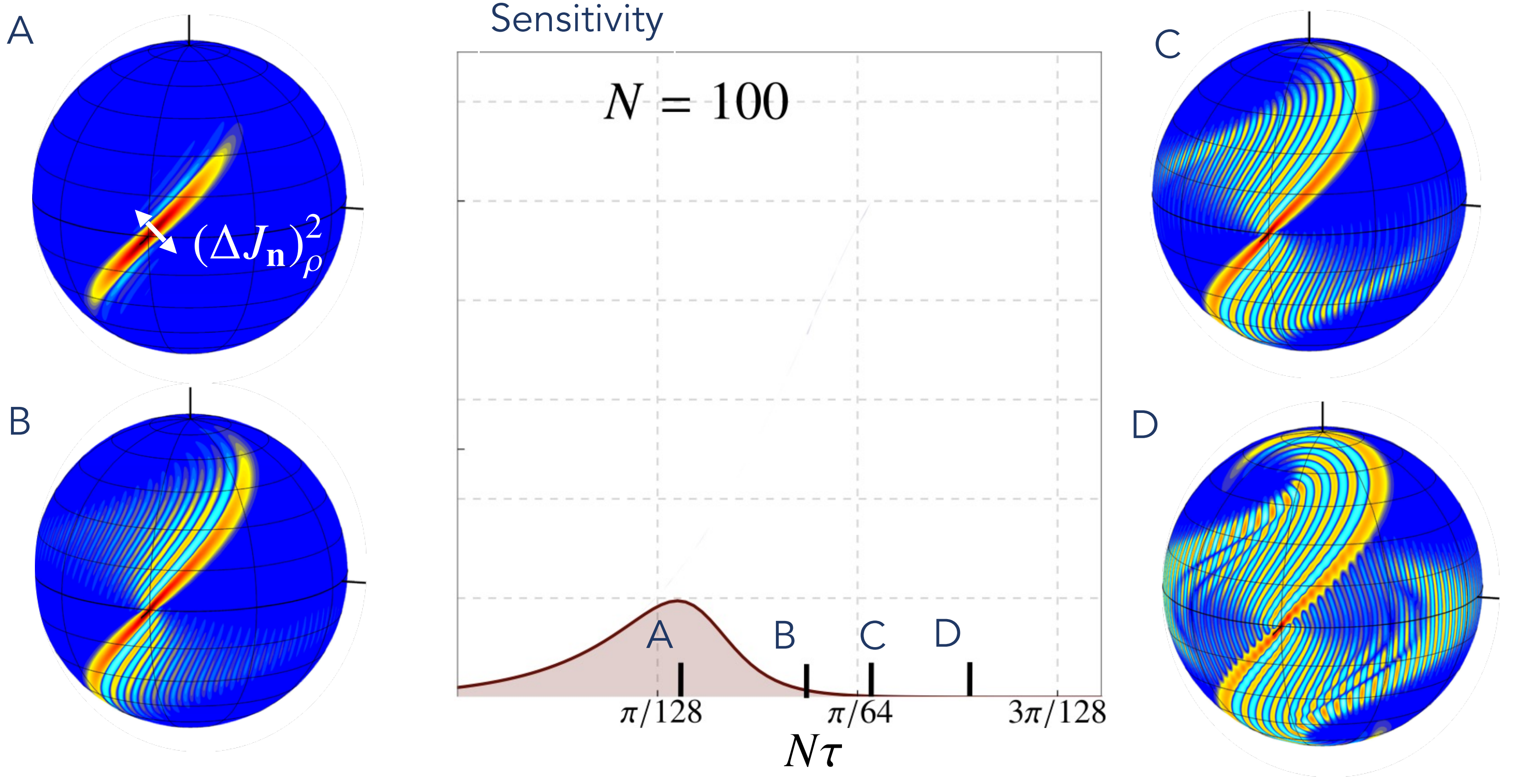
$$H_{\mathrm{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

state preparation: $e^{-iH_{\mathrm{NL}}\tau}$

Nonlinear evolution

rotation: $e^{-iJ_{\mathbf{m}}\theta}$

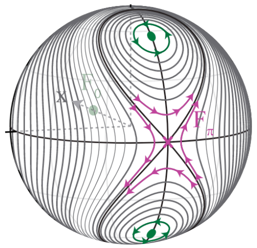
measurement: $J_{\mathbf{n}}$



Squeezing of Non-Gaussian States

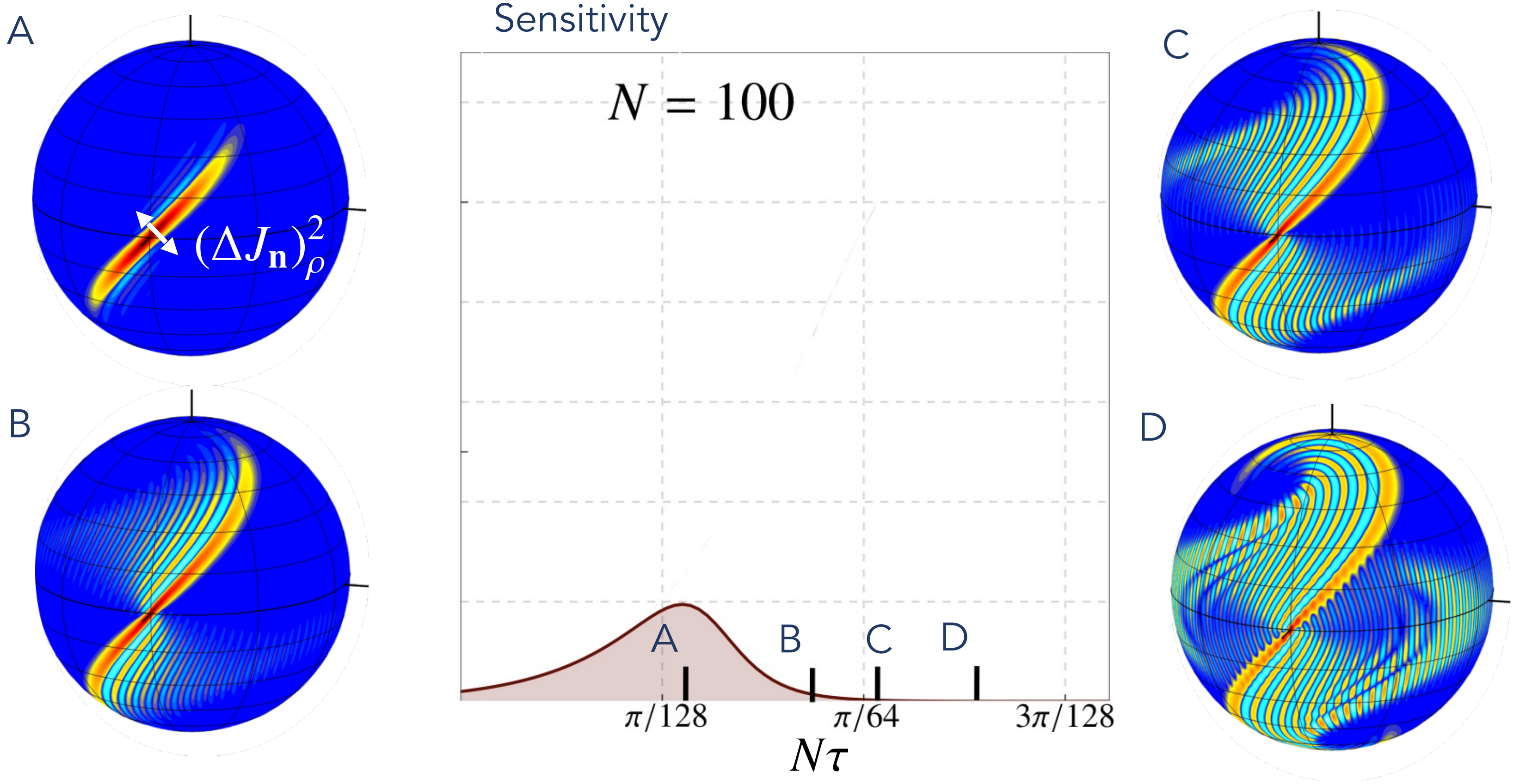
Sensitivity

$$\chi^{-2} = \frac{|\langle [J_{\mathbf{n}}, J_{\mathbf{m}}] \rangle|^2}{(\Delta J_{\mathbf{n}})^2}$$



$$H_{\mathrm{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

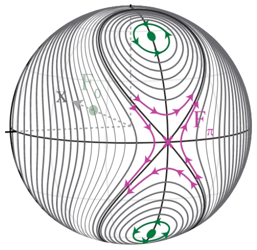
state preparation	rotation	measurement
$e^{-iH_{\mathrm{NL}}\tau}$	$e^{-iJ_{\mathbf{m}}\theta}$	$J_{\mathbf{n}}$
Nonlinear evolution		Nonlinear observable



Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle [X, J_{\mathbf{m}}] \rangle|^2}{(\Delta X)^2}$$

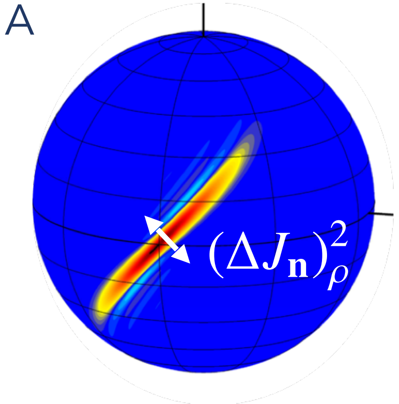


$$H_{\rm NL} = \frac{1}{N} J_z^2 + \lambda J_x$$

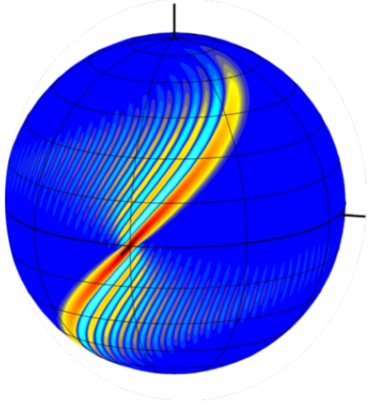
state preparation	rotation	measurement
$e^{-iH_{\rm NL}\tau}$	$e^{-iJ_{\mathbf{m}}\theta}$	X
Nonlinear evolution		Nonlinear observable

M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019)

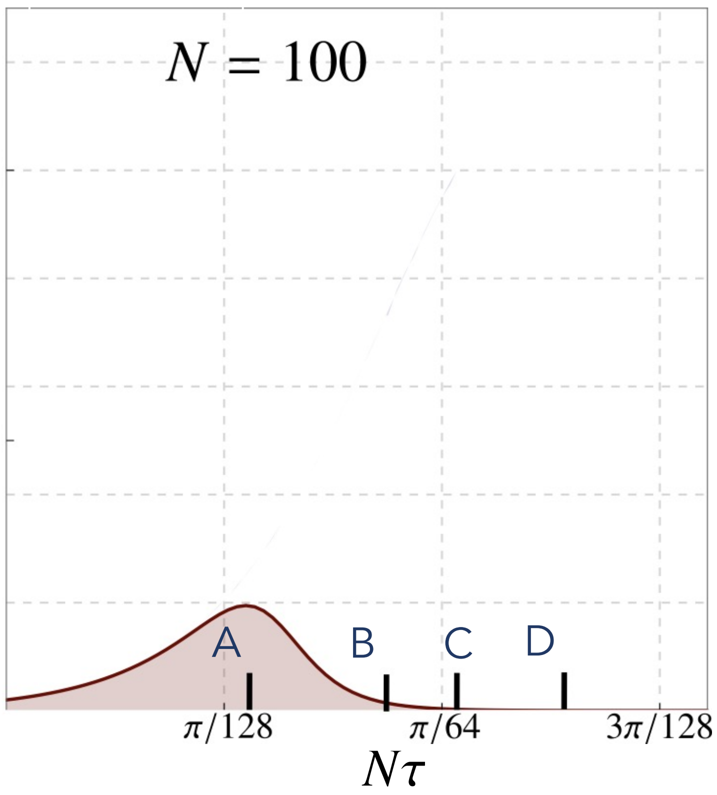
A



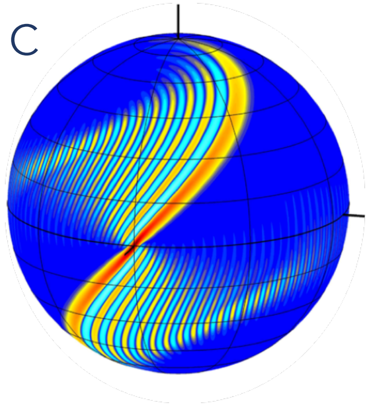
B



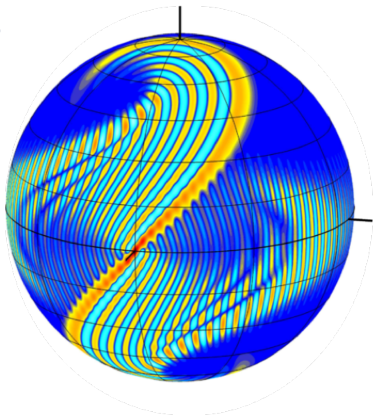
Sensitivity



C



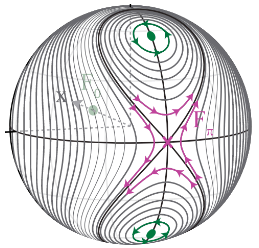
D



Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle [X, J_{\mathbf{m}}] \rangle|^2}{(\Delta X)^2}$$



$$H_{\mathrm{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

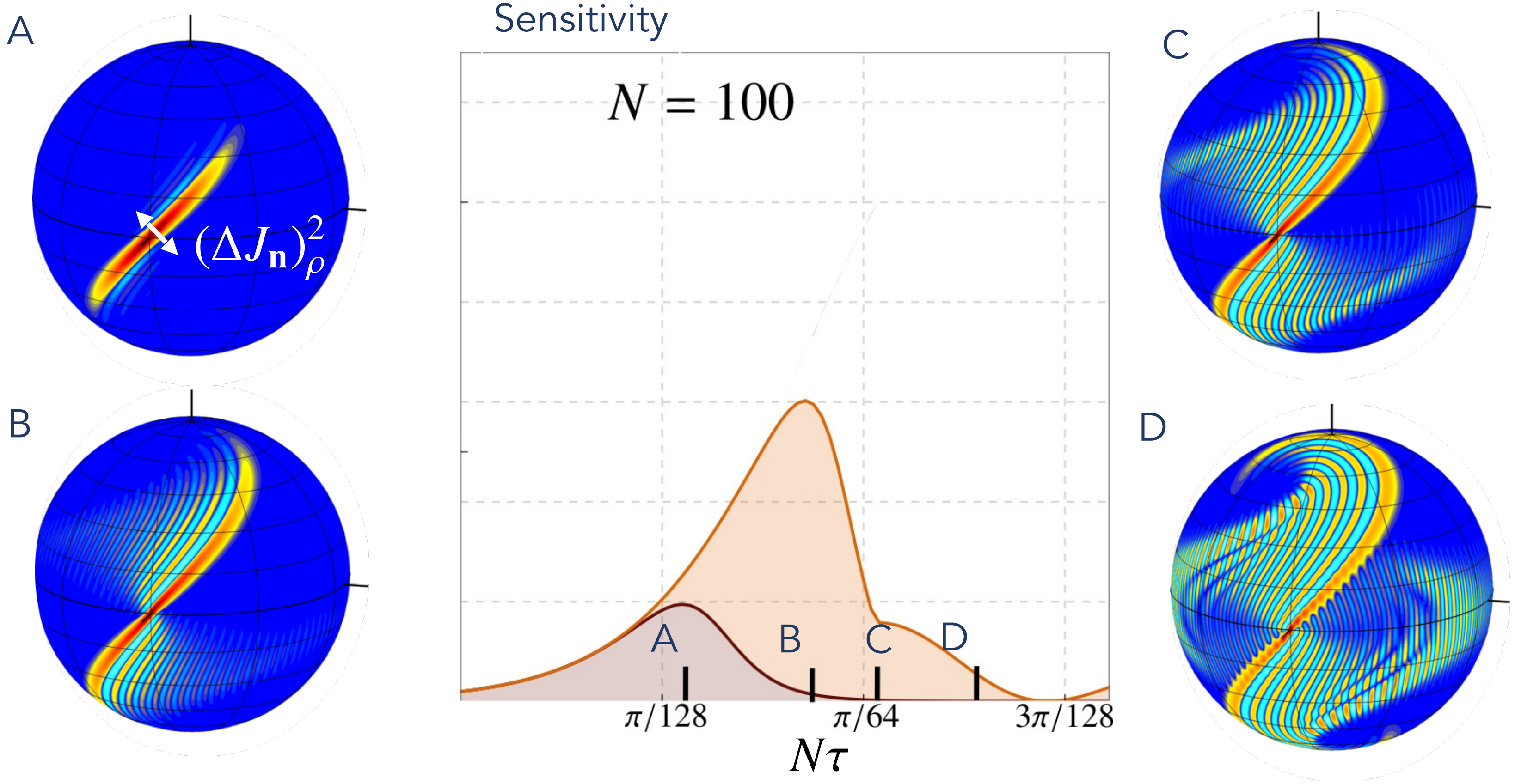
state preparation: $e^{-iH_{\mathrm{NL}}\tau}$ — Nonlinear evolution

rotation: $e^{-iJ_{\mathbf{m}}\theta}$

measurement: X — Nonlinear observable

M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019)

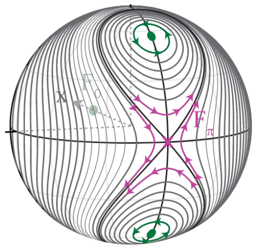
$$X = c_1 J_{\mathbf{n}_1} + c_2 J_{\mathbf{n}_2}^2$$



Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle [X, J_{\mathbf{m}}] \rangle|^2}{(\Delta X)^2}$$

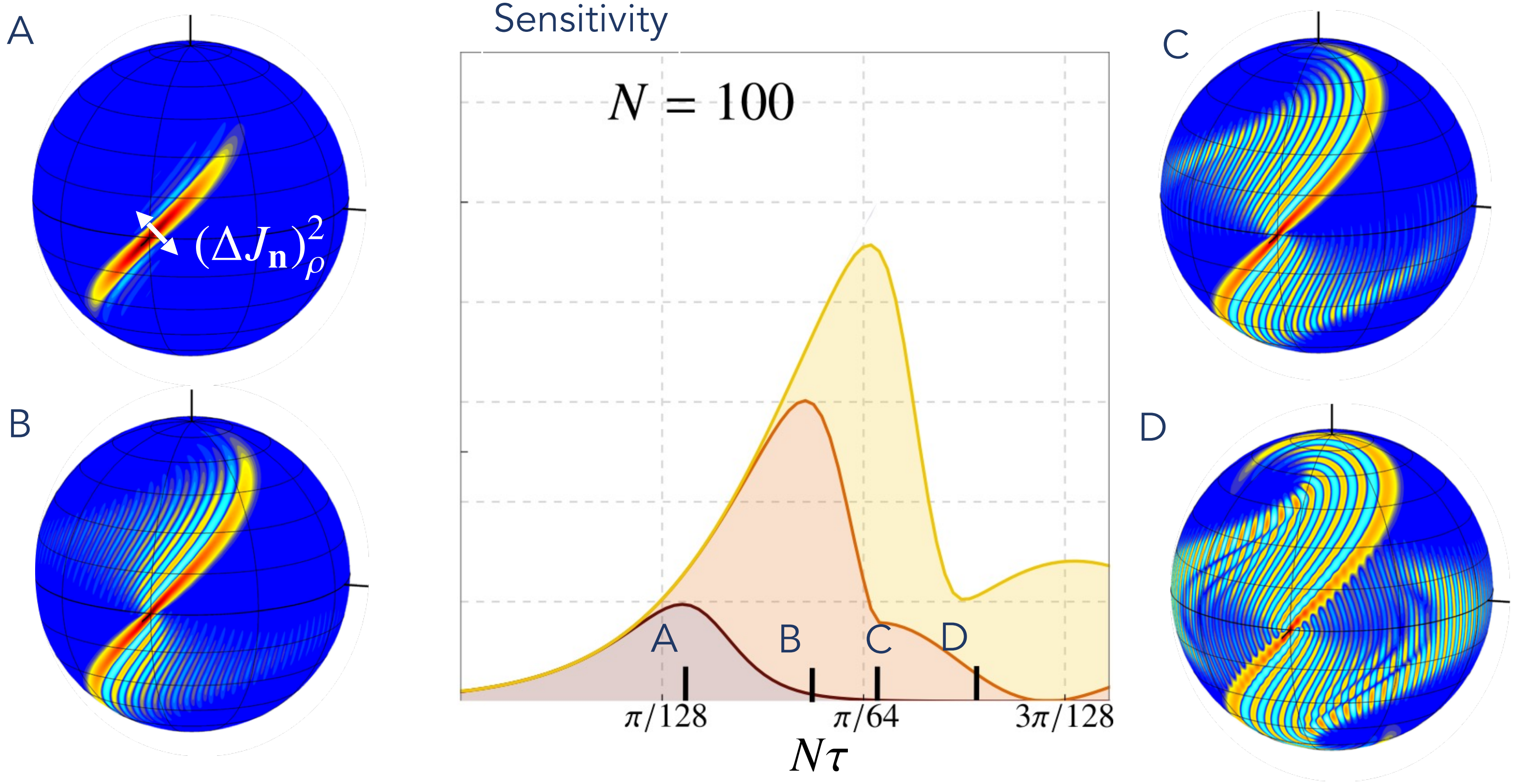


$$H_{\mathrm{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

state preparation	rotation	measurement
$e^{-iH_{\mathrm{NL}}\tau}$	$e^{-iJ_{\mathbf{m}}\theta}$	X
Nonlinear evolution		Nonlinear observable

M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019)

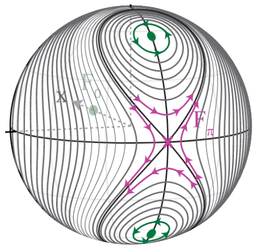
$$X = c_1 J_{\mathbf{n}_1} + c_2 J_{\mathbf{n}_2}^2 + c_3 J_{\mathbf{n}_3}^3$$



Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle [X, J_{\mathbf{m}}] \rangle|^2}{(\Delta X)^2}$$

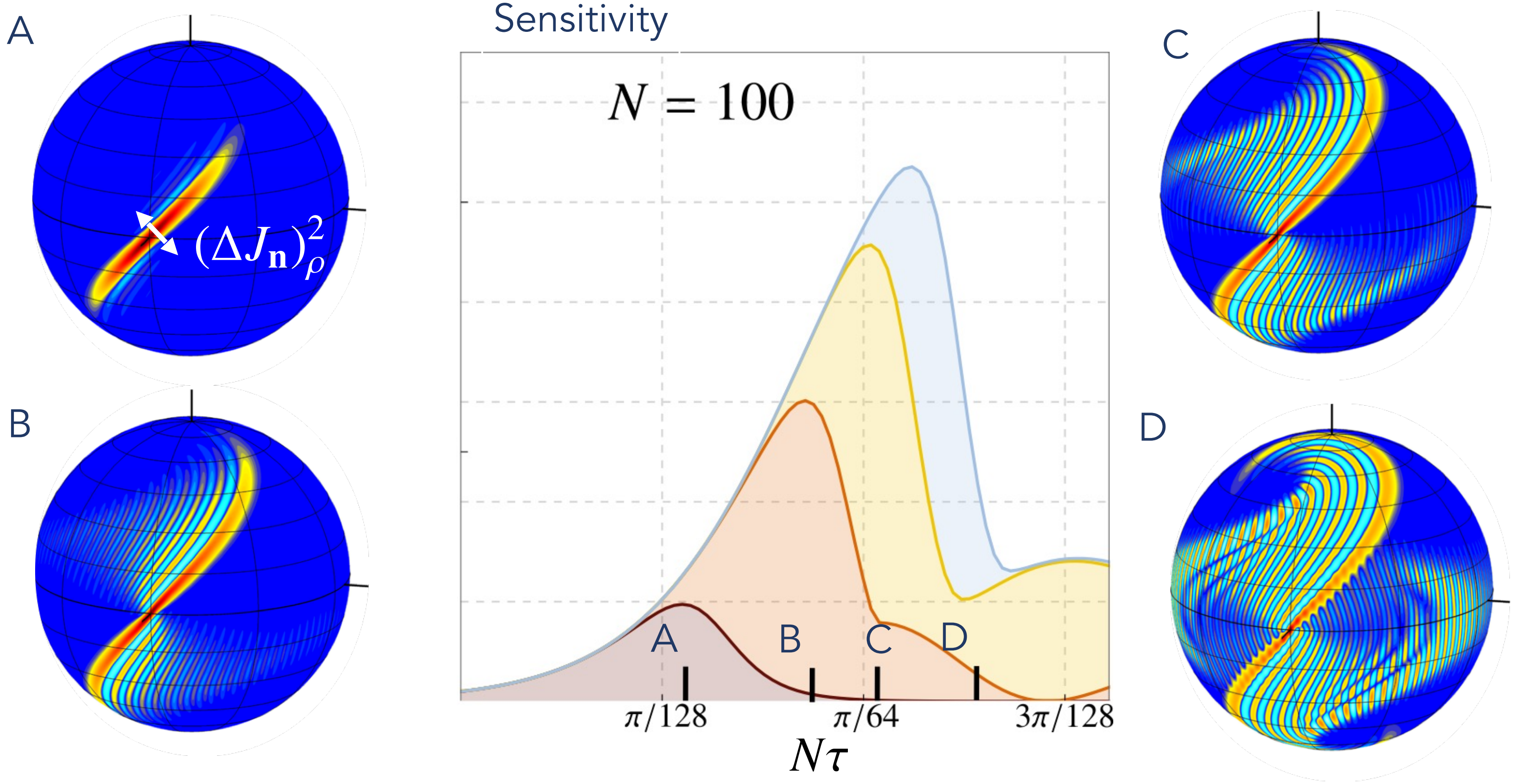


$$H_{\mathrm{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

state preparation	rotation	measurement
$e^{-iH_{\mathrm{NL}}\tau}$	$e^{-iJ_{\mathbf{m}}\theta}$	X
Nonlinear evolution		Nonlinear observable

M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019)

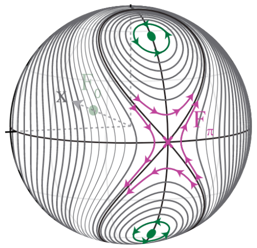
$$X = c_1 J_{\mathbf{n}_1} + c_2 J_{\mathbf{n}_2}^2 + c_3 J_{\mathbf{n}_3}^3 + \cdots$$



Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle [X, J_{\mathbf{m}}] \rangle|^2}{(\Delta X)^2}$$



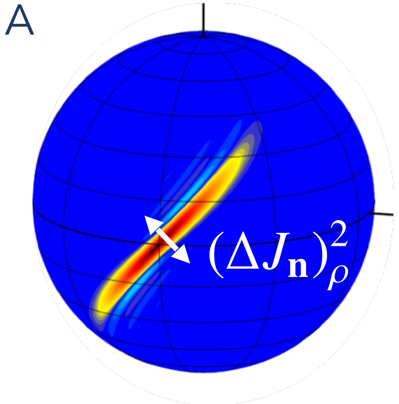
$$H_{\rm NL} = \frac{1}{N} J_z^2 + \lambda J_x$$

state preparation	rotation	measurement
$e^{-iH_{\rm NL}\tau}$	$e^{-iJ_{\mathbf{m}}\theta}$	X
Nonlinear evolution		Nonlinear observable

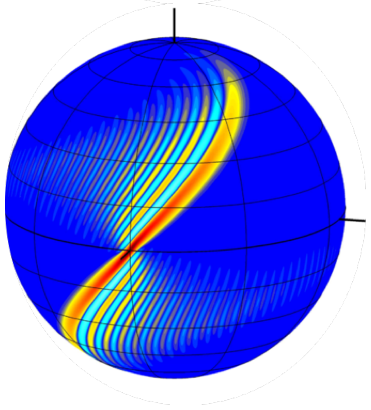
M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019)

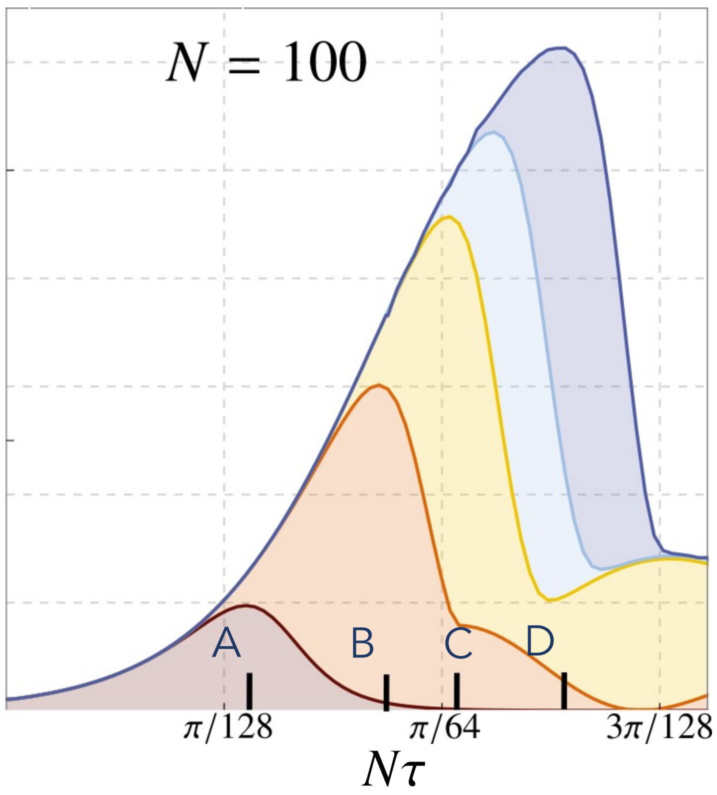
$$X = c_1 J_{\mathbf{n}_1} + c_2 J_{\mathbf{n}_2}^2 + c_3 J_{\mathbf{n}_3}^3 + \cdots$$

A

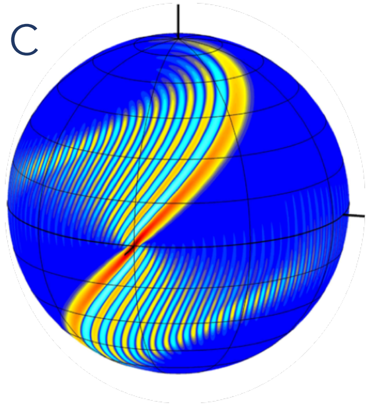


B

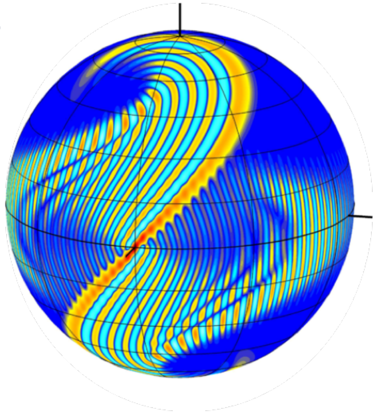




C



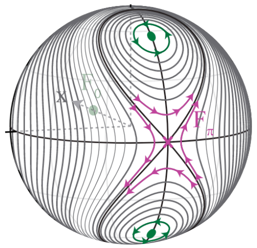
D



Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle [X, J_{\mathbf{m}}]\rangle|^2}{(\Delta X)^2}$$



$$H_{\mathrm{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

state preparation	rotation	measurement
$e^{-iH_{\mathrm{NL}}\tau}$	$e^{-iJ_{\mathbf{m}}\theta}$	X
Nonlinear evolution		Nonlinear observable

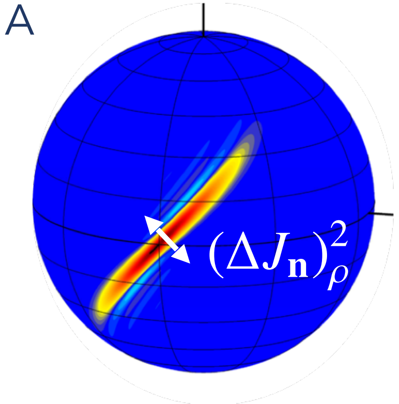
M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019)

$$X = c_1 J_{\mathbf{n}_1} + c_2 J_{\mathbf{n}_2}^2 + c_3 J_{\mathbf{n}_3}^3 + \cdots$$

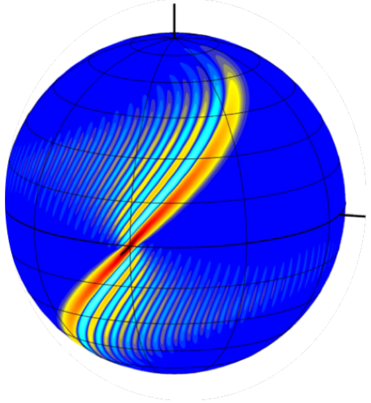
Optimize over *all* observables

$$\rightarrow \max_X \chi^{-2} = F_Q[\rho(\theta)]$$

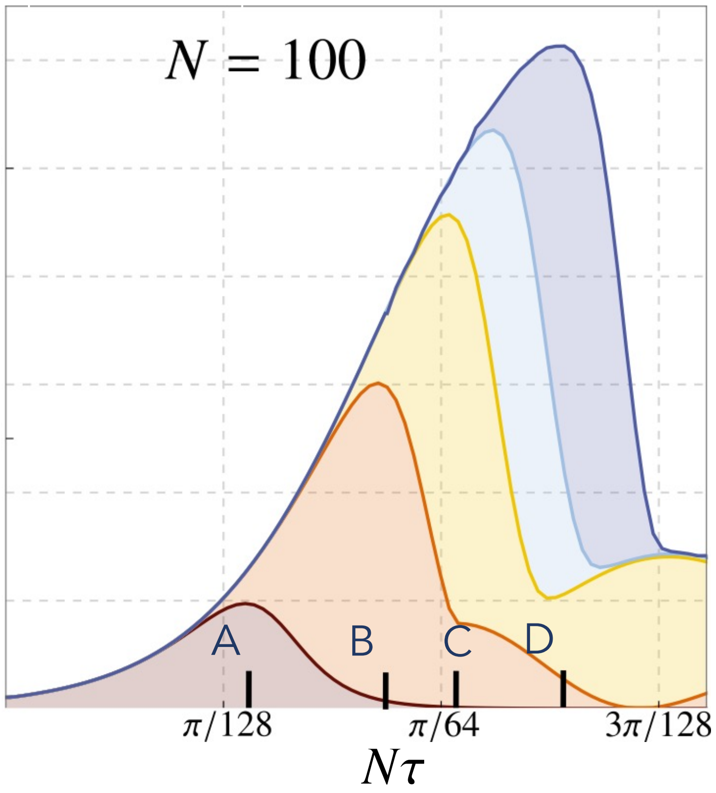
A



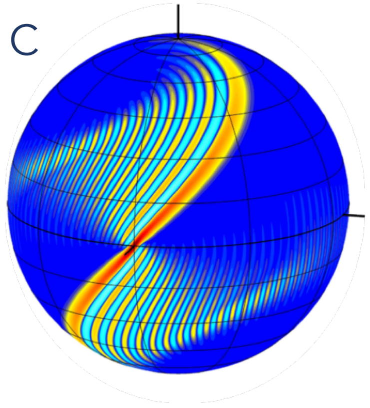
B



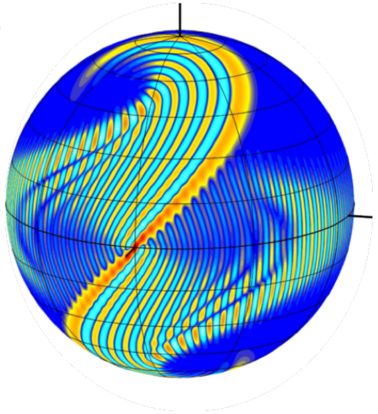
Sensitivity



C



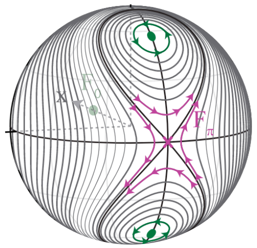
D



Squeezing of Non-Gaussian States

Sensitivity

$$\chi^{-2} = \frac{|\langle [X, J_{\mathbf{m}}]\rangle|^2}{(\Delta X)^2}$$

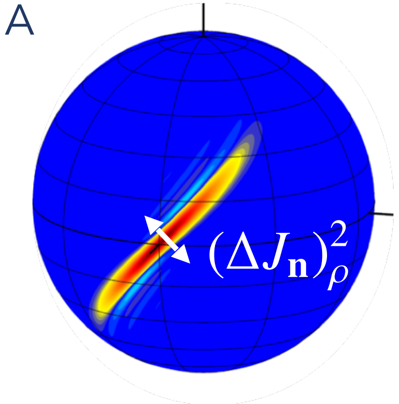


$$H_{\mathrm{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

state preparation	rotation	measurement
$e^{-iH_{\mathrm{NL}}\tau}$	$e^{-iJ_{\mathbf{m}}\theta}$	X
Nonlinear evolution		Nonlinear observable

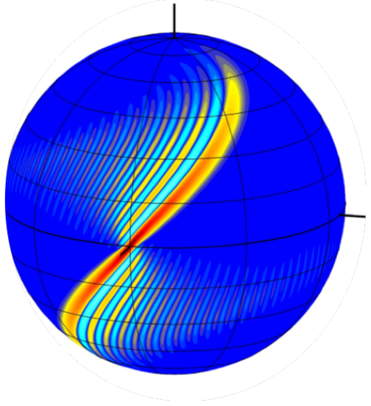
M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019)

A



$$X = c_1 J_{\mathbf{n}_1} + c_2 J_{\mathbf{n}_2}^2 + c_3 J_{\mathbf{n}_3}^3 + \cdots$$

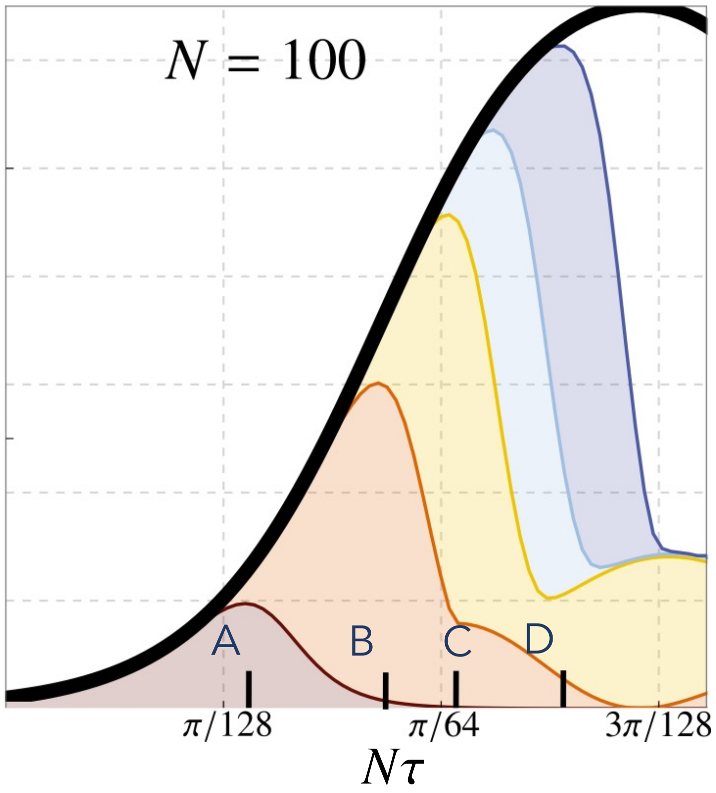
B



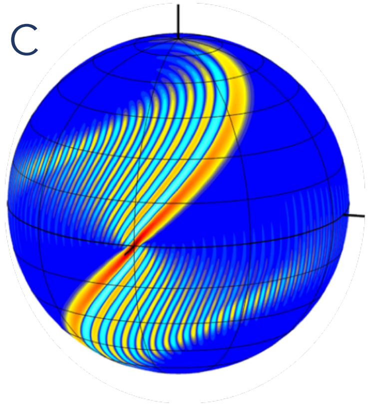
Optimize over *all* observables

$$\rightarrow \max_X \chi^{-2} = F_Q[\rho(\theta)]$$

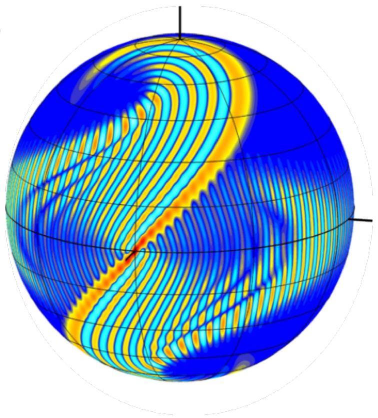
Sensitivity



C



D



Quantifying Quantum Sensitivity Enhancements

Classical limit

separable states

$$(\Delta\theta_{\text{est}})^2_{\text{SQL}} = \frac{1}{N}$$

"Standard quantum limit"

Quantifying Quantum Sensitivity Enhancements

Classical limit

separable states

$$(\Delta\theta_{\text{est}})^2_{\text{SQL}} = \frac{1}{N}$$

"Standard quantum limit"

Quantum limit

multipartite entangled states

$$(\Delta\theta_{\text{est}})^2_{\text{HL}} = \frac{1}{N^2}$$

"Heisenberg limit"

Quantifying Quantum Sensitivity Enhancements

Quantum enhancement

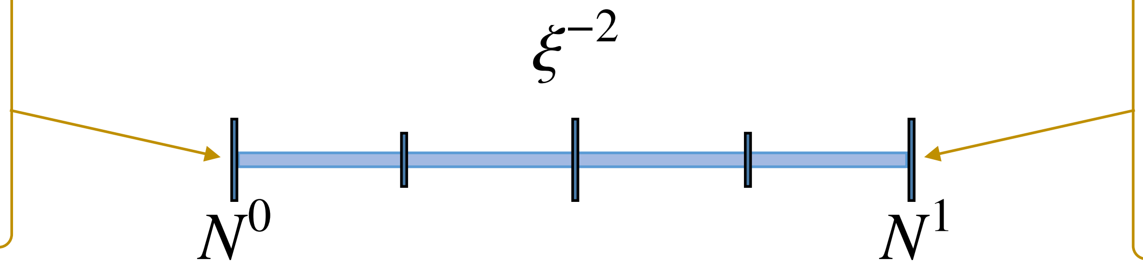
$$\xi^{-2} = \frac{(\Delta\theta_{\mathrm{est}})^2_{\mathrm{SQL}}}{(\Delta\theta_{\mathrm{est}})^2}$$

Classical limit

separable states

$$(\Delta\theta_{\mathrm{est}})^2_{\mathrm{SQL}} = \frac{1}{N}$$

"Standard quantum limit"



Quantum limit

multipartite entangled states

$$(\Delta\theta_{\mathrm{est}})^2_{\mathrm{HL}} = \frac{1}{N^2}$$

"Heisenberg limit"

QUANTIFYING QUANTUM SENSITIVITY ENHANCEMENTS

Quantum enhancement

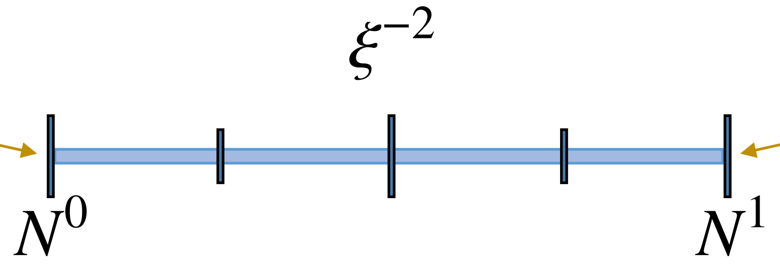
$$\xi^{-2} = \frac{(\Delta\theta_{\mathrm{est}})^2_{\mathrm{SQL}}}{(\Delta\theta_{\mathrm{est}})^2}$$

Classical limit

separable states

$$(\Delta\theta_{\mathrm{est}})^2_{\mathrm{SQL}} = \frac{1}{N}$$

"Standard quantum limit"



Quantum limit

multipartite entangled states

$$(\Delta\theta_{\mathrm{est}})^2_{\mathrm{HL}} = \frac{1}{N^2}$$

"Heisenberg limit"

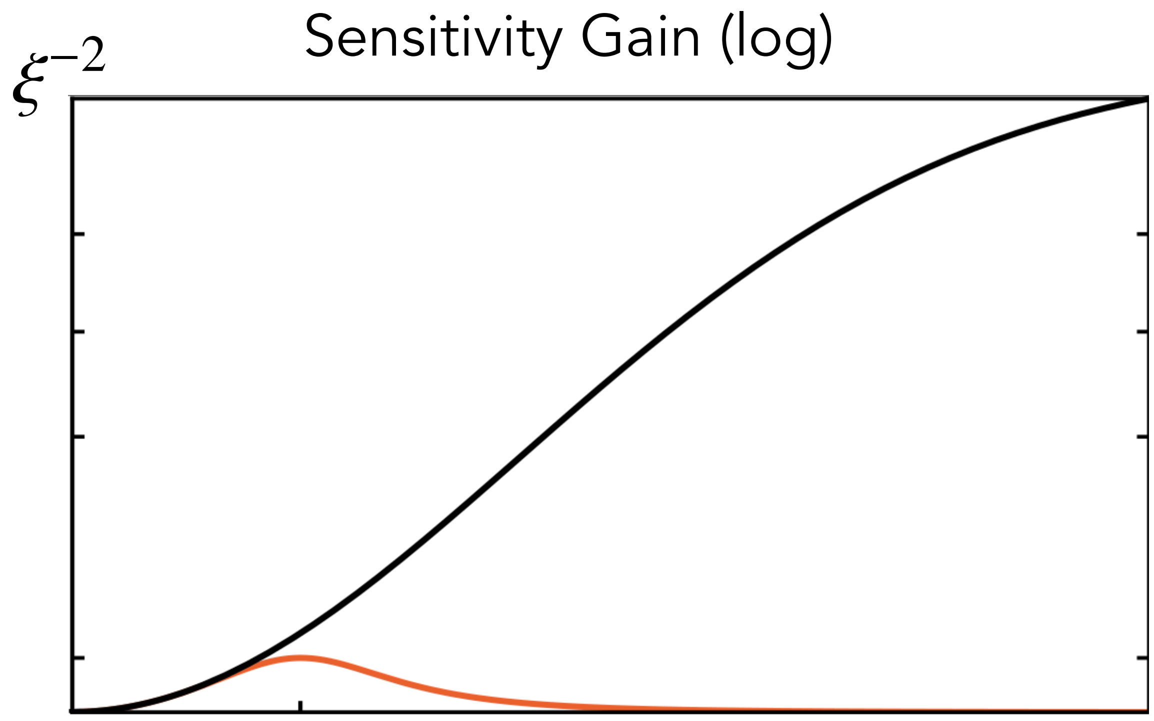
Witness for the degree of multipartite entanglement

L. Pezzé and A. Smerzi, PRL **102**, 100401 (2009)

P. Hyllus et al., PRA **85**, 022321 (2012)
G. Tóth, PRA **85**, 022322 (2012)

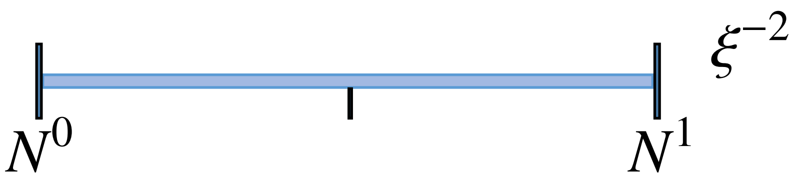
Z. Ren, W. Li, A. Smerzi, M. Gessner, PRL **126**, 080502 (2021)

Scaling of Quantum Sensitivity Enhancements



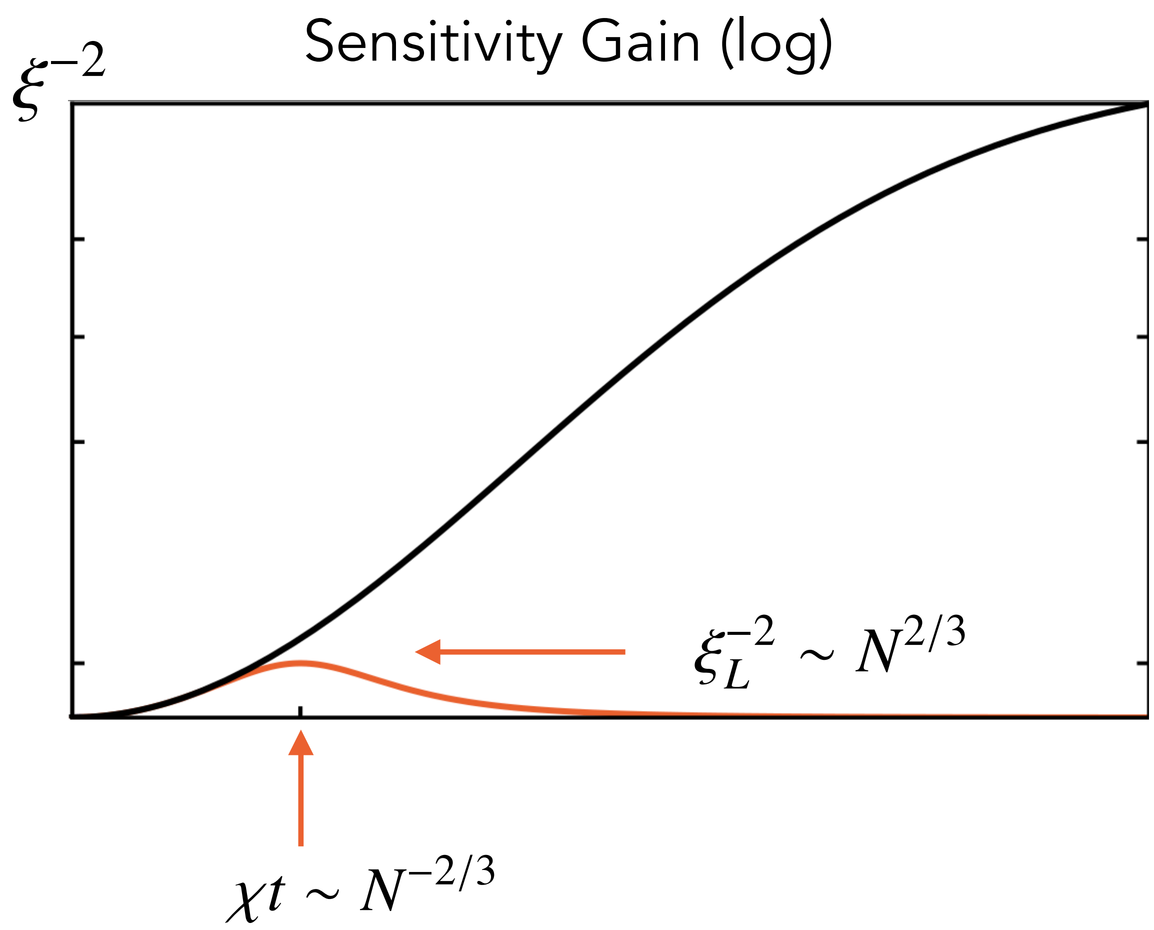
$$\xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

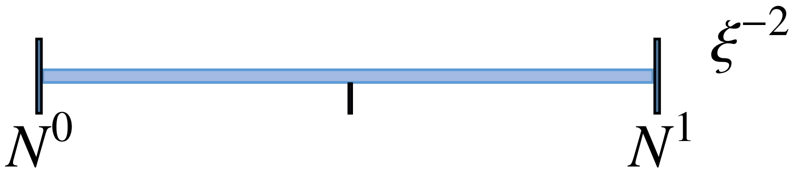
$$X = J_{\mathbf{n}}$$



Kitagawa & Ueda, PRA **47** 5138 (1993)

Scaling of Quantum Sensitivity Enhancements





$$\xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

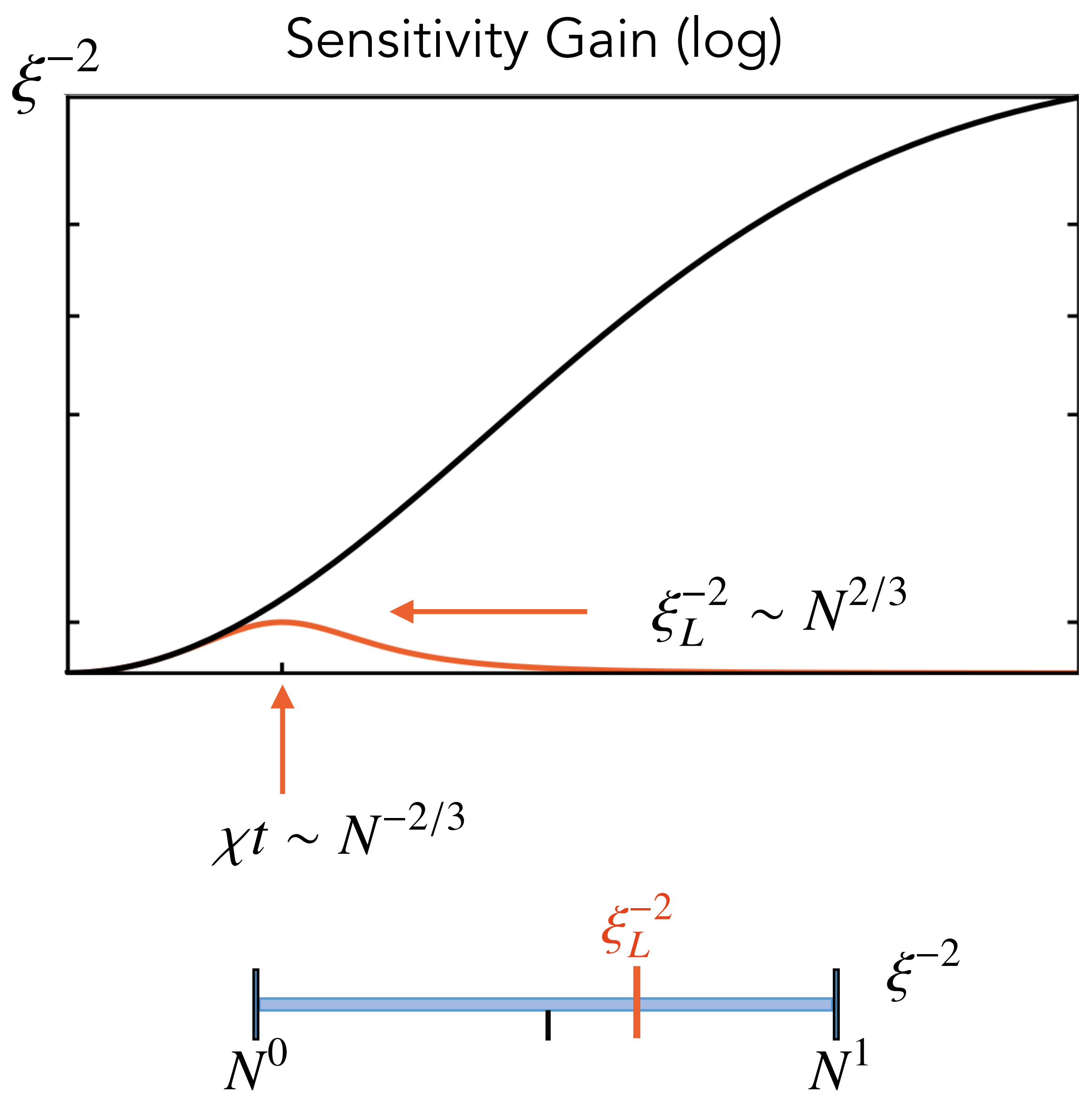
Fisher

L

$$X = J_{\mathbf{n}}$$

Kitagawa & Ueda, PRA **47** 5138 (1993)

Scaling of Quantum Sensitivity Enhancements

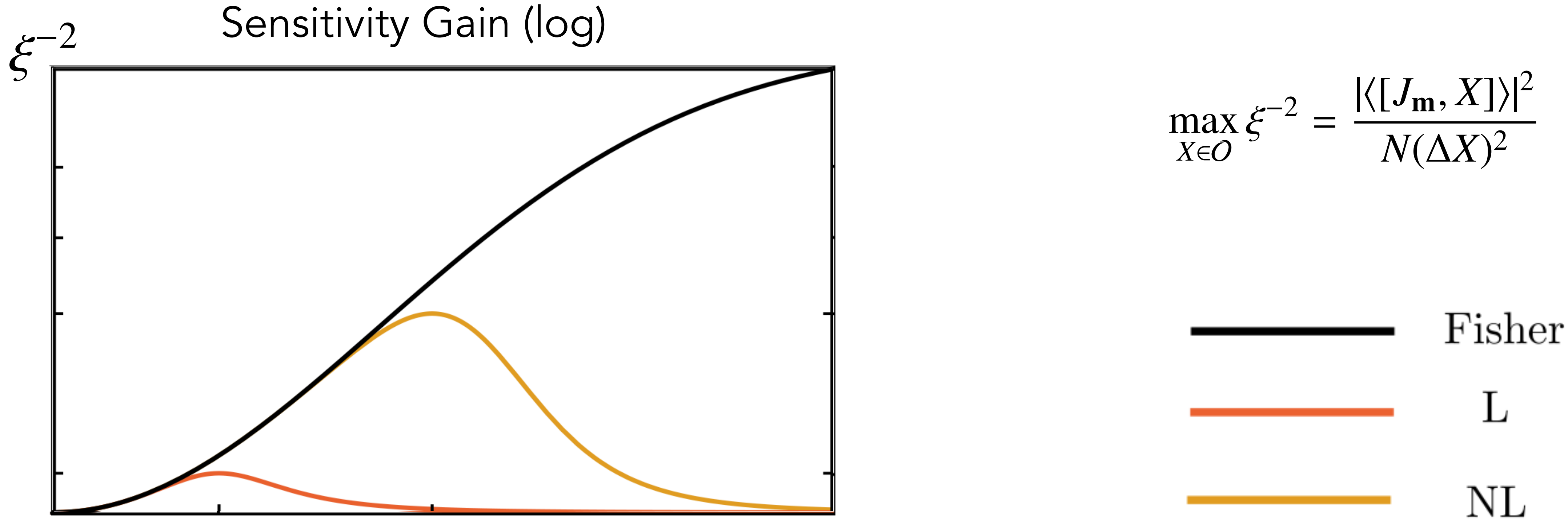


$$\xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

$$X = J_{\mathbf{n}}$$

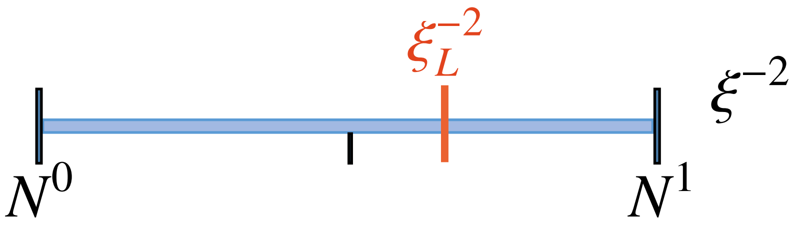
Kitagawa & Ueda, PRA **47** 5138 (1993)

Scaling of Quantum Sensitivity Enhancements



$$\max_{X\in\mathcal{O}} \xi^{-2} = \frac{|\langle[J_{\mathbf{m}}, X]\rangle|^2}{N(\Delta X)^2}$$

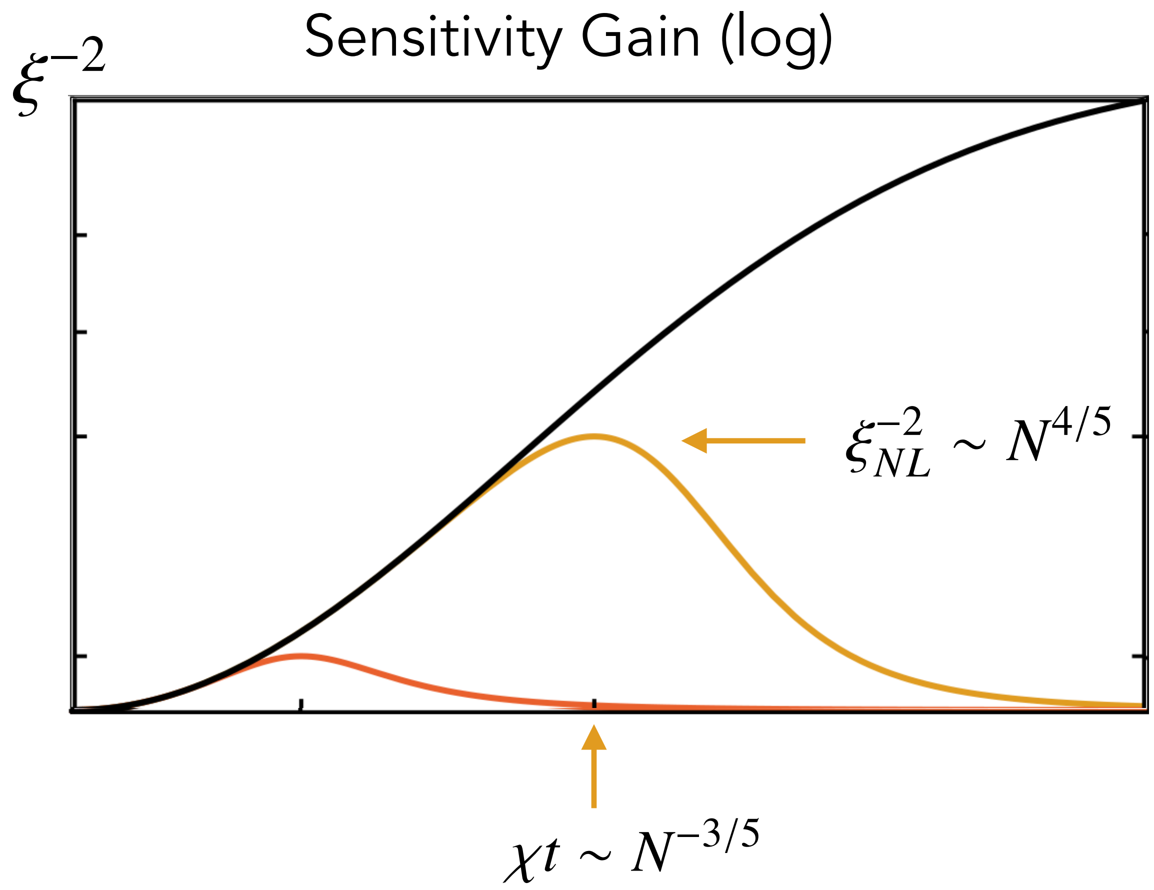
$$X = m_{\mathbf{n}} J_{\mathbf{n}} + m_{xz}\{J_x, J_z\}$$

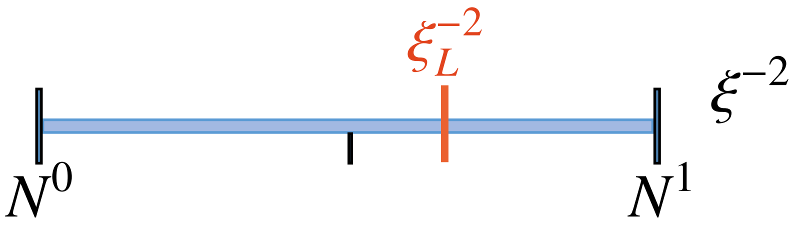


Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021);

Comptes Rendus. Phys. **23**, 1 (2022).

Scaling of Quantum Sensitivity Enhancements





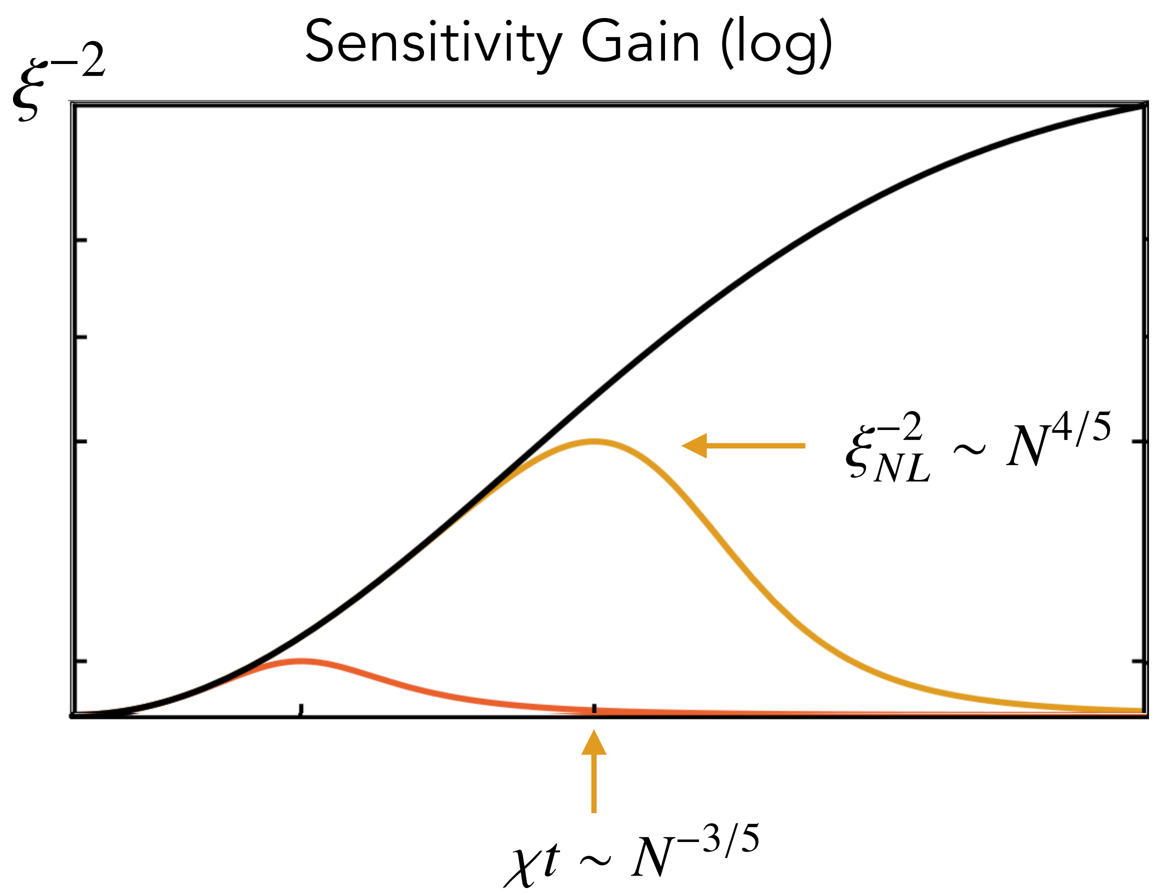
$$\max_{X \in \mathcal{O}} \xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

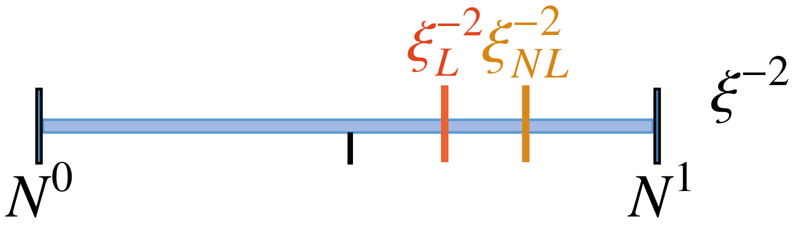
$$X = m_{\mathbf{n}} J_{\mathbf{n}} + m_{xz}\{J_x, J_z\}$$

Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021);

Comptes Rendus. Phys. **23**, 1 (2022).

Scaling of Quantum Sensitivity Enhancements





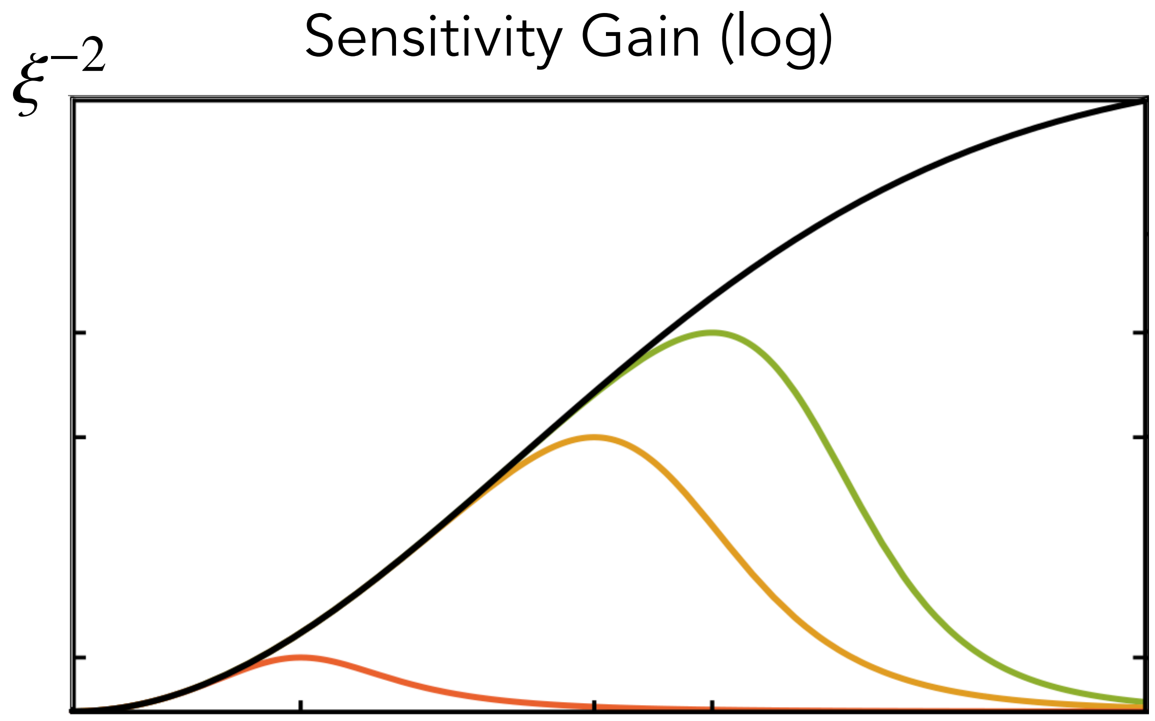
$$\max_{X \in \mathcal{O}} \xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

$$X = m_{\mathbf{n}} J_{\mathbf{n}} + m_{xz}\{J_x, J_z\}$$

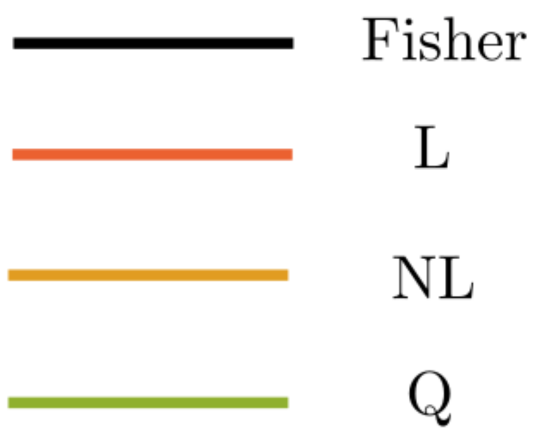
Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021);

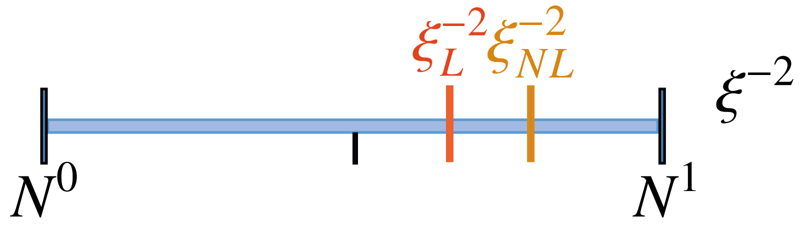
Comptes Rendus. Phys. **23**, 1 (2022).

Scaling of Quantum Sensitivity Enhancements



$$\max_{X \in \mathcal{O}} \xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N (\Delta X)^2}$$



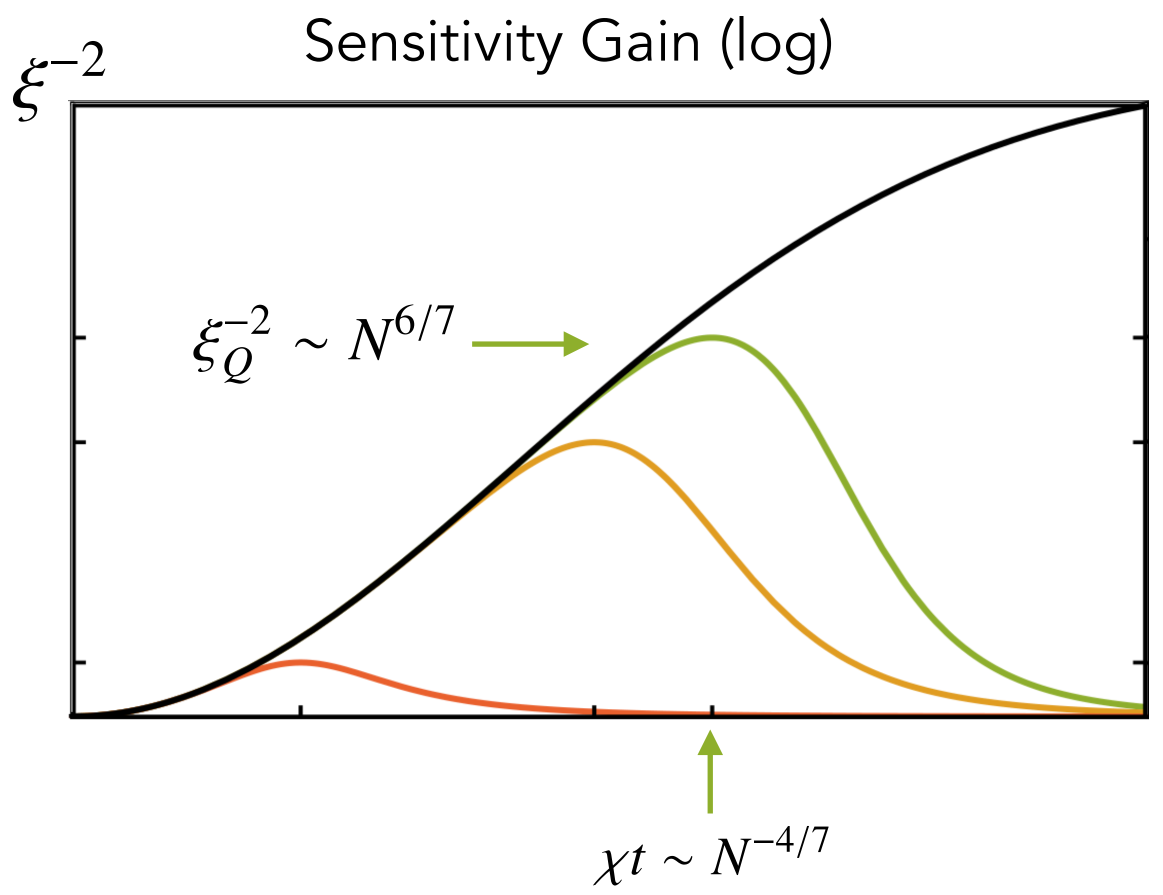


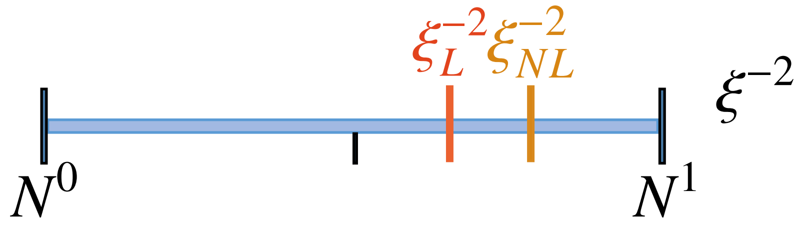
$$X \simeq m_{\mathbf{n}} J_{\mathbf{n}} + m_{\mathbf{n}'} J_{\mathbf{n}'}^2$$

Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021);

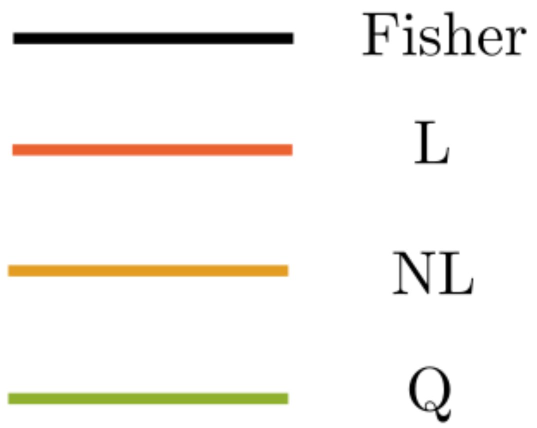
Comptes Rendus. Phys. **23**, 1 (2022).

Scaling of Quantum Sensitivity Enhancements





$$\max_{X \in \mathcal{O}} \xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

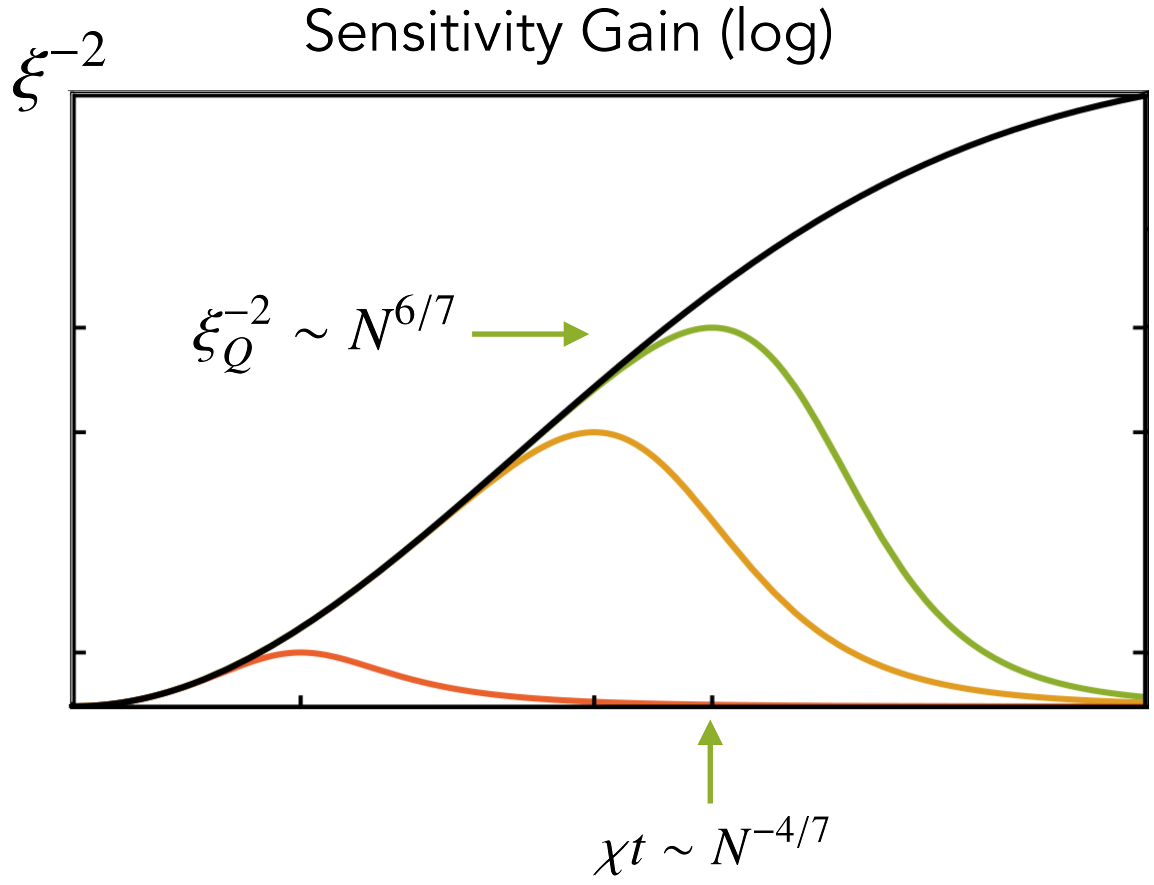


$$X \simeq m_{\mathbf{n}} J_{\mathbf{n}} + m_{\mathbf{n}'} J_{\mathbf{n}'}^2$$

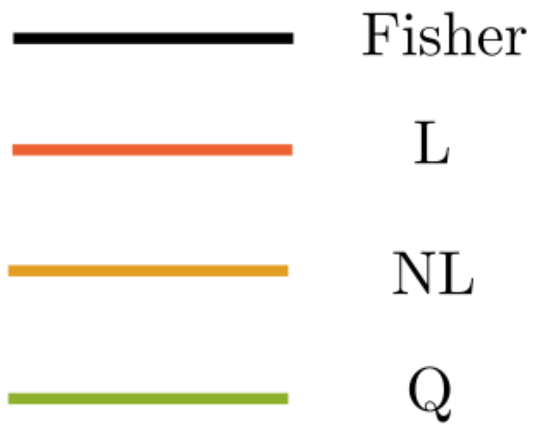
Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021);

Comptes Rendus. Phys. **23**, 1 (2022).

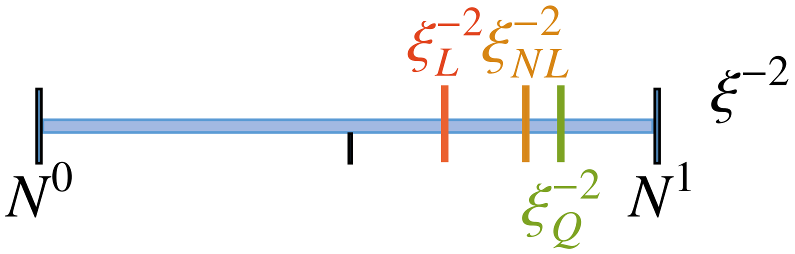
Scaling of Quantum Sensitivity Enhancements



$$\max_{X \in \mathcal{O}} \xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$



$$X \simeq m_{\mathbf{n}} J_{\mathbf{n}} + m_{\mathbf{n}'} J_{\mathbf{n}'}^2$$



Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021);

Comptes Rendus. Phys. **23**, 1 (2022).

Implementing Nonlinear Measurements

state preparation

$$e^{-iJ_z^2\chi t}$$

Nonlinear evolution

rotation

$$e^{-iJ_{\mathbf{m}}\theta}$$

measurement

$$J_{\mathbf{n}}$$

Implementing Nonlinear Measurements

state preparation

$$e^{-iJ_z^2\chi t}$$

Nonlinear evolution

rotation

$$e^{-iJ_{\mathbf{m}}\theta}$$

measurement

$$X$$

Nonlinear observable

Implementing Nonlinear Measurements

state preparation

$$e^{-iJ_z^2\chi t}$$

Nonlinear evolution

rotation

$$e^{-iJ_{\mathbf{m}}\theta}$$

$$e^{iJ_z^2\chi t}$$

Nonlinear evolution

measurement

$$J_{\mathbf{n}}$$

Implementing Nonlinear Measurements

state preparation	rotation		measurement
$e^{-iJ_z^2\chi t}$	$e^{-iJ_{\mathbf{m}}\theta}$	$e^{iJ_z^2\chi t}$	$J_{\mathbf{n}}$
Nonlinear evolution		Nonlinear evolution	

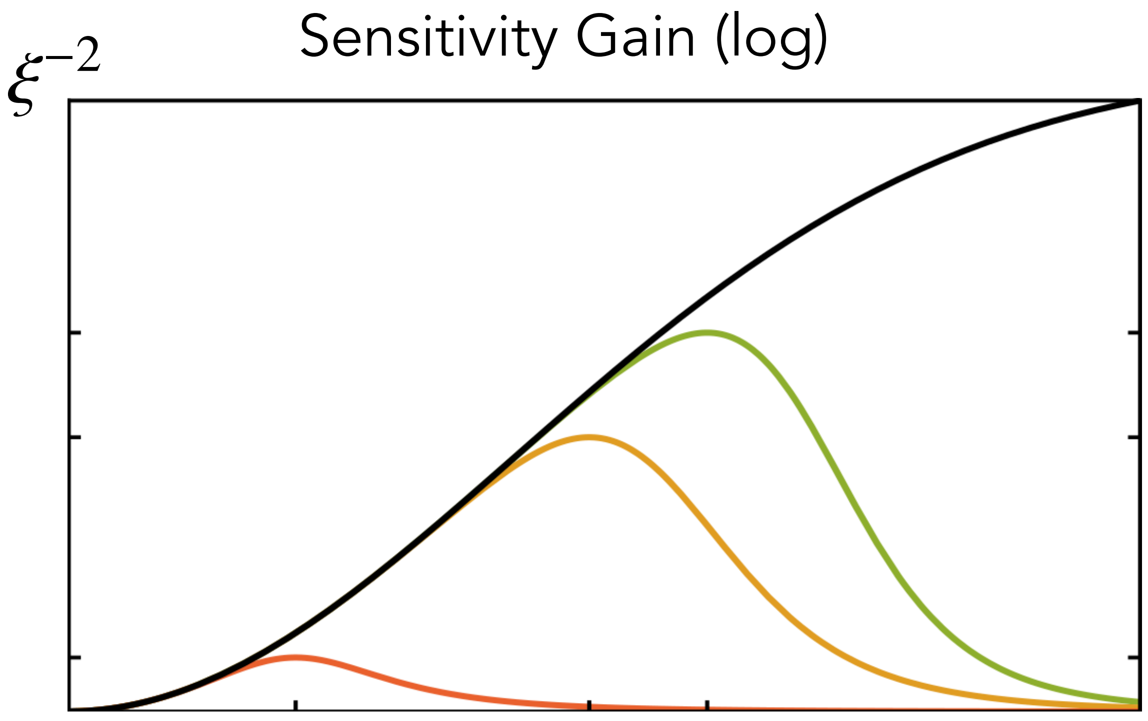
Measurement-after-interaction (MAI)

$$X_{\mathrm{MAI}} = e^{-iJ_z^2\chi t} J_{\mathbf{n}} e^{iJ_z^2\chi t}$$

"Squeezing echo"
“Interaction-based readout”

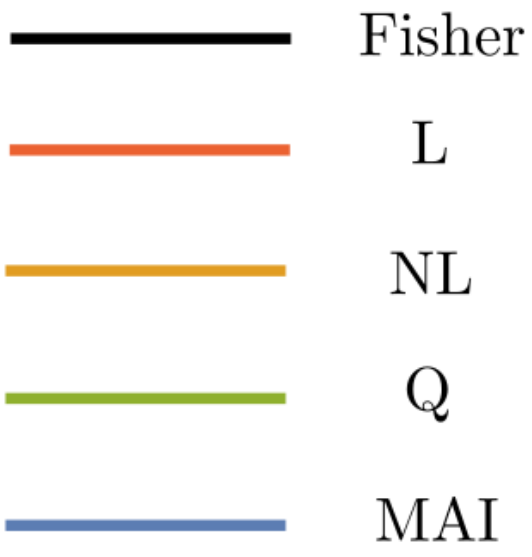
E. Davis *et al.* PRL **116**, 053601 (2016).
F. Fröwis *et al.* PRL **116**, 090801 (2016).
T. Macrì *et al.* PRA **94**, 010102(R) (2016).
S. P. Nolan *et al.* PRL **119**, 193601 (2017).
M. Schulte *et al.* Quantum **4**, 268 (2020).

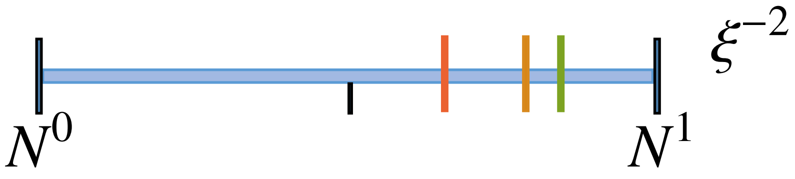
Scaling of Quantum Sensitivity Enhancements



$\chi t = \sigma N^{-\alpha} \qquad (\alpha > 1/2)$

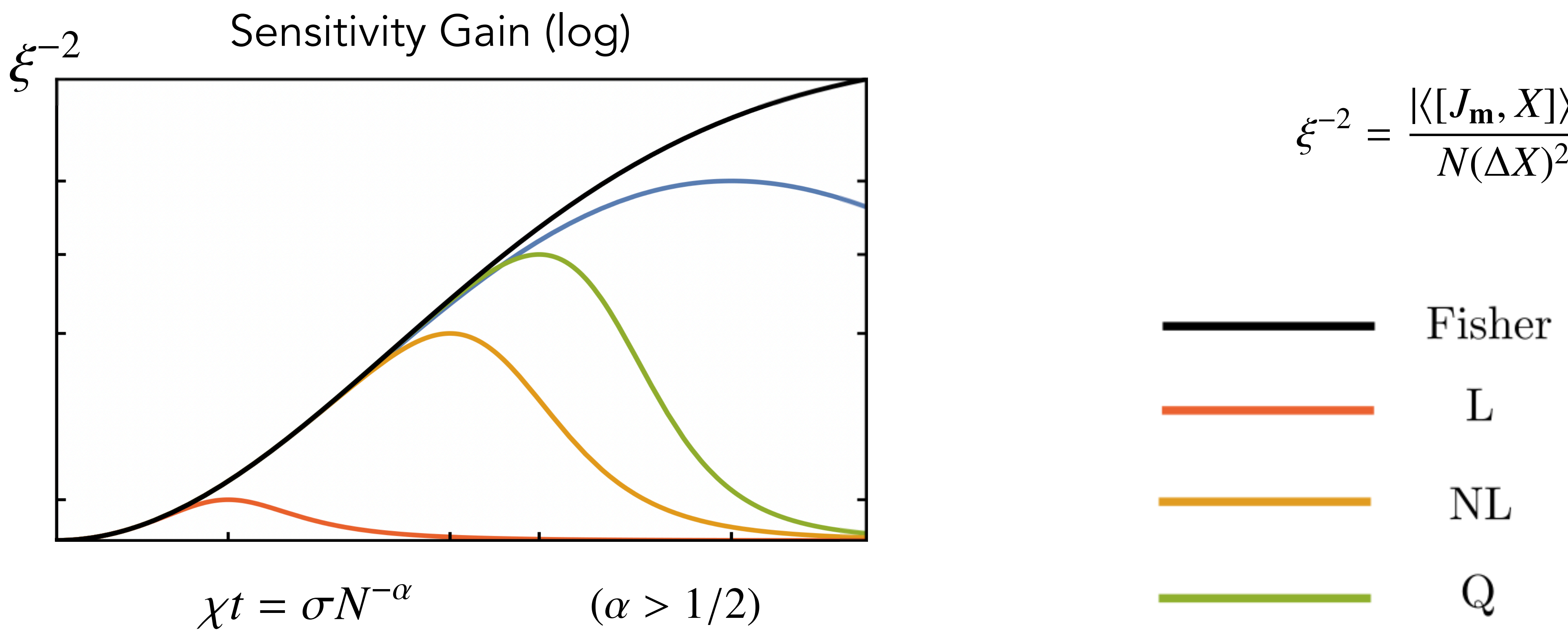
$$\xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$



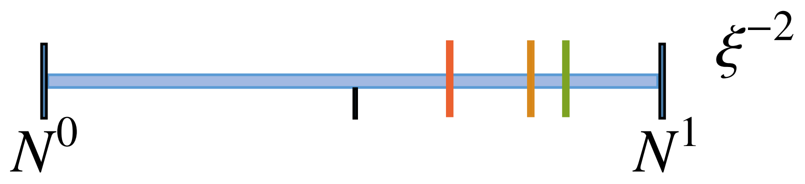


Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

Scaling of Quantum Sensitivity Enhancements

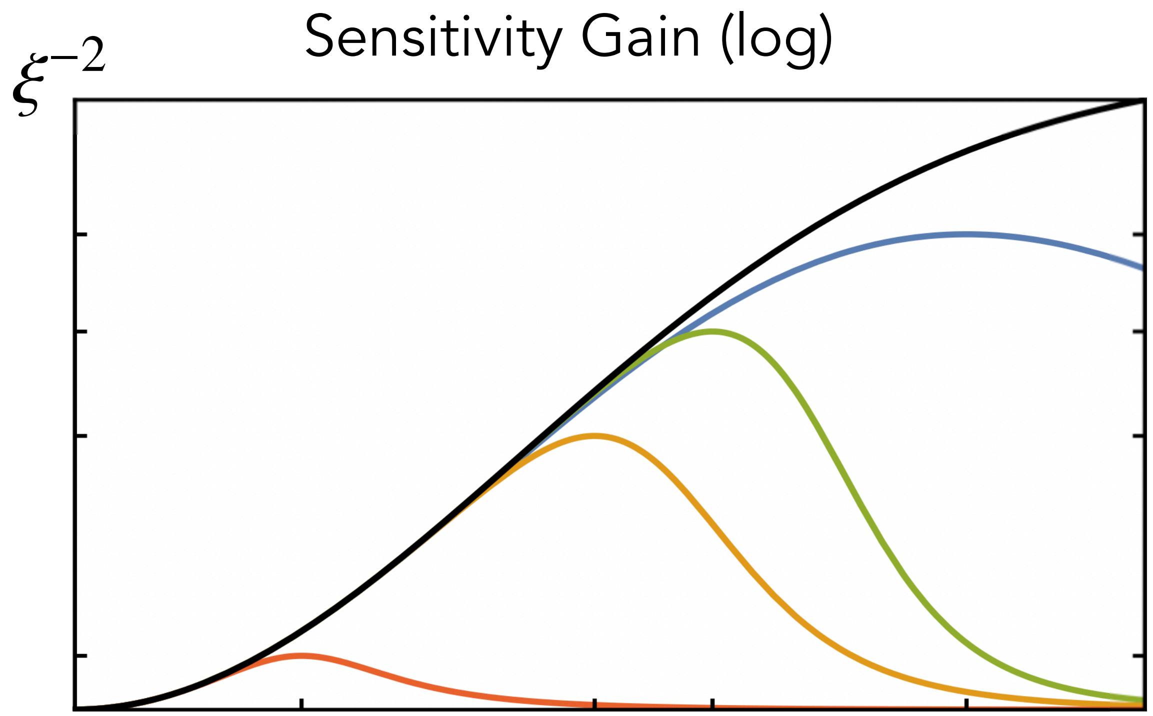


$$\xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

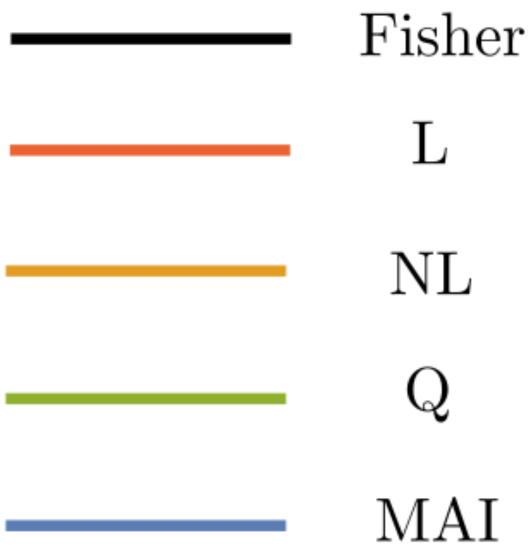


Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

Scaling of Quantum Sensitivity Enhancements

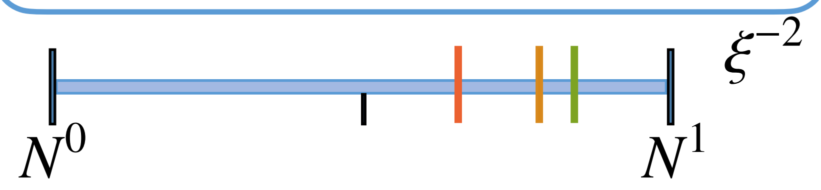


$$\xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$



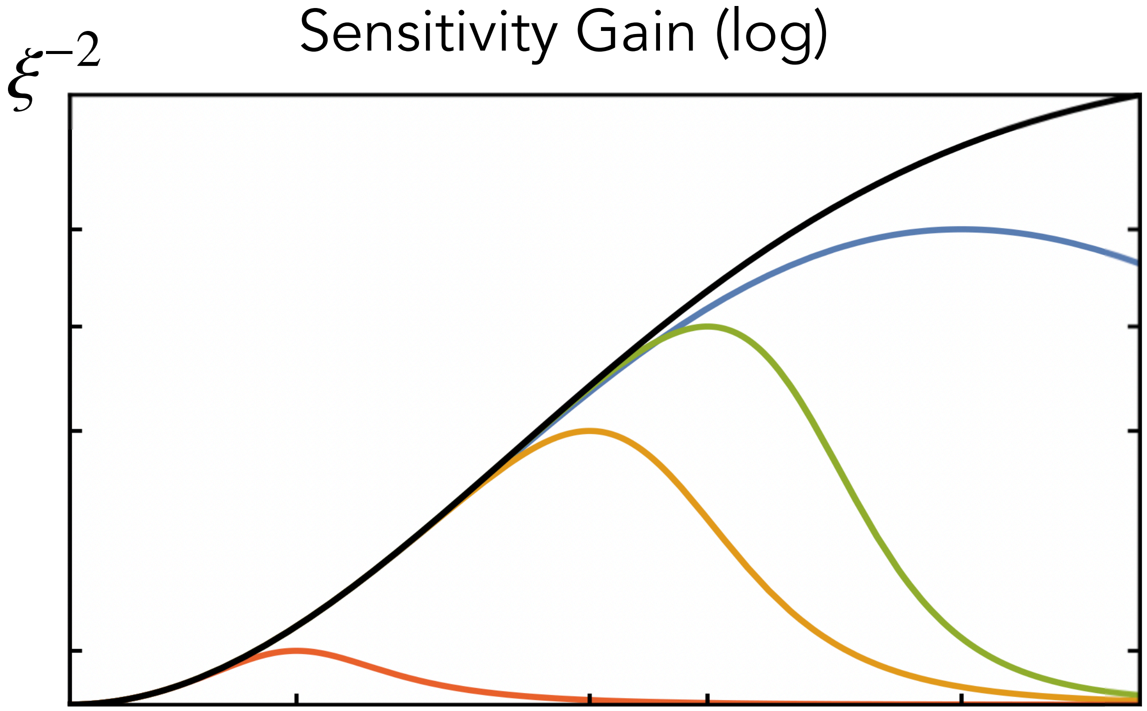
$$\chi t = \sigma N^{-\alpha} \qquad (\alpha > 1/2)$$

$$\xi^{-2}_{\mathrm{MAI}} = \sigma^2 N^{2-2\alpha}$$



Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

Scaling of Quantum Sensitivity Enhancements



$$\xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

Fisher

$\chi t \sim N^{-2/3}$ L $\xi_L^{-2} \sim N^{2/3}$

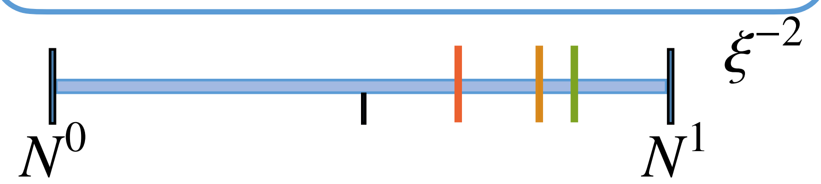
$\chi t \sim N^{-3/5}$ NL $\xi_{NL}^{-2} \sim N^{4/5}$

$\chi t \sim N^{-4/7}$ Q $\xi_Q^{-2} \sim N^{6/7}$

MAI

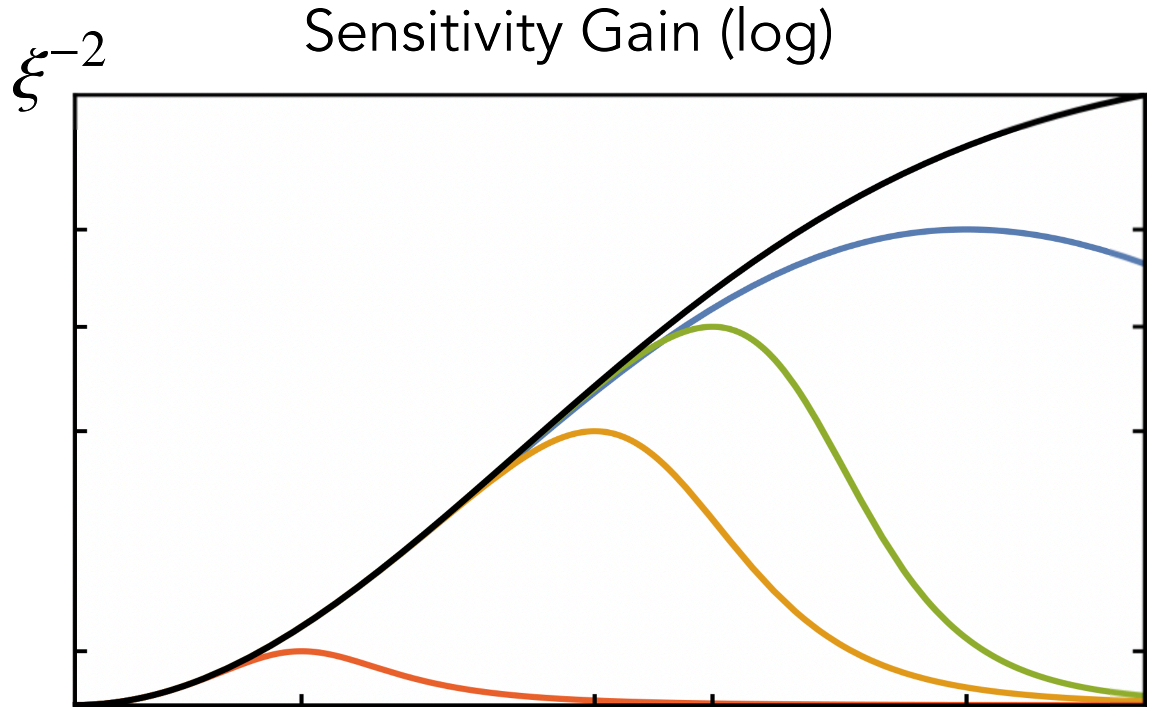
$\chi t = \sigma N^{-\alpha}$ $(\alpha > 1/2)$

$$\xi_{\mathrm{MAI}}^{-2} = \sigma^2 N^{2-2\alpha}$$



Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

Scaling of Quantum Sensitivity Enhancements

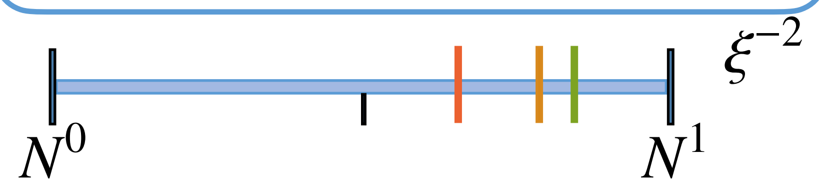


$$\xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

$\chi t = \sigma N^{-\alpha}$	Fisher	$\xi^{-2}_{\mathrm{QFI}} = \sigma^2 N^{2-2\alpha}$
$\chi t \sim N^{-2/3}$	L	$\xi^{-2}_{L} \sim N^{2/3}$
$\chi t \sim N^{-3/5}$	NL	$\xi^{-2}_{NL} \sim N^{4/5}$
$\chi t \sim N^{-4/7}$	Q	$\xi^{-2}_{Q} \sim N^{6/7}$
	MAI	

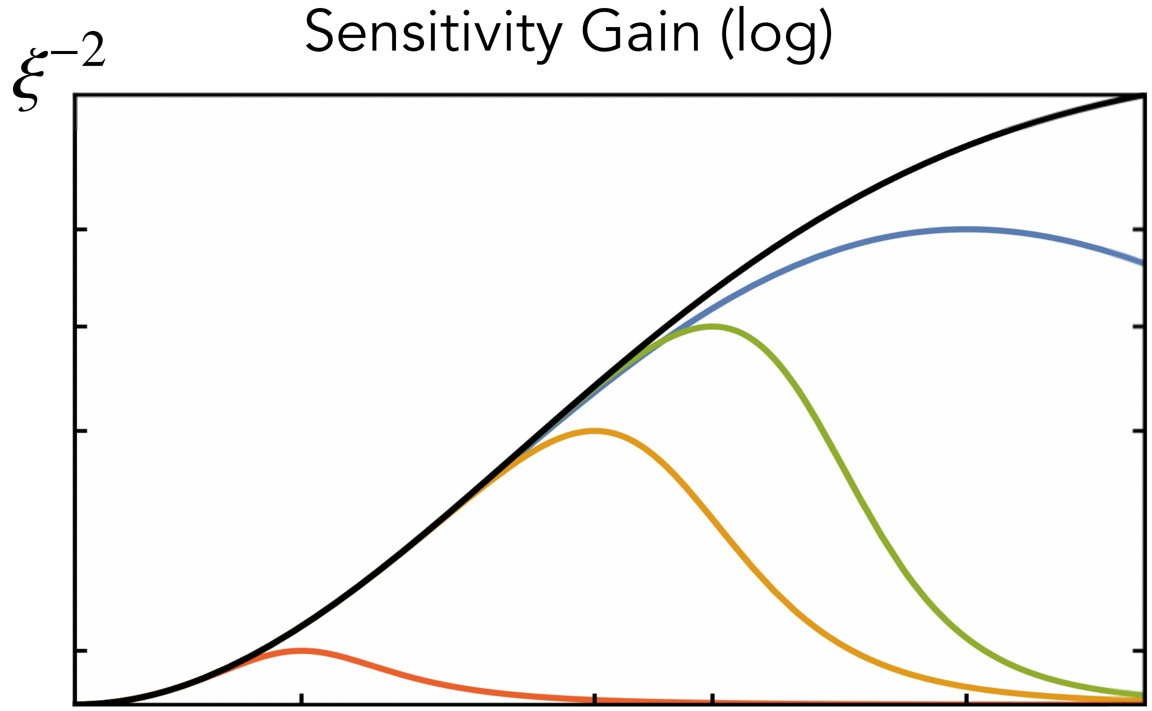
$\chi t = \sigma N^{-\alpha}$ $(\alpha > 1/2)$

$$\xi^{-2}_{\mathrm{MAI}} = \sigma^2 N^{2-2\alpha}$$



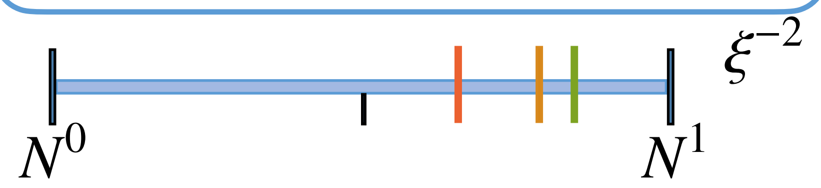
Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

Scaling of Quantum Sensitivity Enhancements



$$\chi t = \sigma N^{-\alpha} \qquad (\alpha > 1/2)$$

$$\xi^{-2}_{\text{MAI}} = \sigma^2 N^{2-2\alpha}$$



$$\xi^{-2} = \frac{|\langle [J_{\mathbf{m}}, X] \rangle|^2}{N(\Delta X)^2}$$

$\chi t = \sigma N^{-\alpha}$	Fisher	$\xi^{-2}_{\text{QFI}} = \sigma^2 N^{2-2\alpha}$
$\chi t \sim N^{-2/3}$	L	$\xi^{-2}_{L} \sim N^{2/3}$
$\chi t \sim N^{-3/5}$	NL	$\xi^{-2}_{NL} \sim N^{4/5}$
$\chi t \sim N^{-4/7}$	Q	$\xi^{-2}_{Q} \sim N^{6/7}$
	MAI	

Optimal strategy?

➡ Depends on the noise!

Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

DECOHERENCE

Noiseless evolution:

Quantum enhancement $\xi_{\mathrm{MAI}}^{-2} = \sigma^2 N^{2-2\alpha}$ Preparation time $\chi t = \sigma N^{-\alpha}$

Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

DECOHERENCE

Noiseless evolution:

Quantum enhancement $\xi_{\mathrm{MAI}}^{-2} = \sigma^2 N^{2-2\alpha}$

Preparation time $\chi t = \sigma N^{-\alpha}$

Lindblad Master Equation

$$\frac{\partial \rho}{\partial t} = -i[\chi J_z^2, \rho] + \gamma_C \left(J_z \rho J_z - \frac{1}{2}\{J_z^2, \rho\} \right)$$

“Diffusive dephasing"

$$\xi_{\mathrm{MAI,dif}}^{-2} = \sigma^2 N^{1-\alpha}$$

Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

Decoherence

Noiseless evolution:

Quantum enhancement $\xi_{\mathrm{MAI}}^{-2} = \sigma^2 N^{2-2\alpha}$

Preparation time $\chi t = \sigma N^{-\alpha}$

Lindblad Master Equation

$$\frac{\partial \rho}{\partial t} = -i[\chi J_z^2, \rho] + \gamma_C \left(J_z \rho J_z - \frac{1}{2}\{J_z^2, \rho\} \right)$$

"Diffusive dephasing"

$$\xi_{\mathrm{MAI,dif}}^{-2} = \sigma^2 N^{1-\alpha}$$

Fluctuating Hamiltonian

$$H = \chi(J_z^2 + D J_z)$$

Gaussian random variable $\langle D^2 \rangle = \epsilon N^{\gamma}$

"Ballistic dephasing"

$$\xi_{\mathrm{MAI,bal}}^{-2} = \frac{\sigma^2 N^{2-2\alpha}}{1 + 4\epsilon\sigma^2 N^{1+\gamma-2\alpha}}$$

Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

BALLISTIC DEPHASING

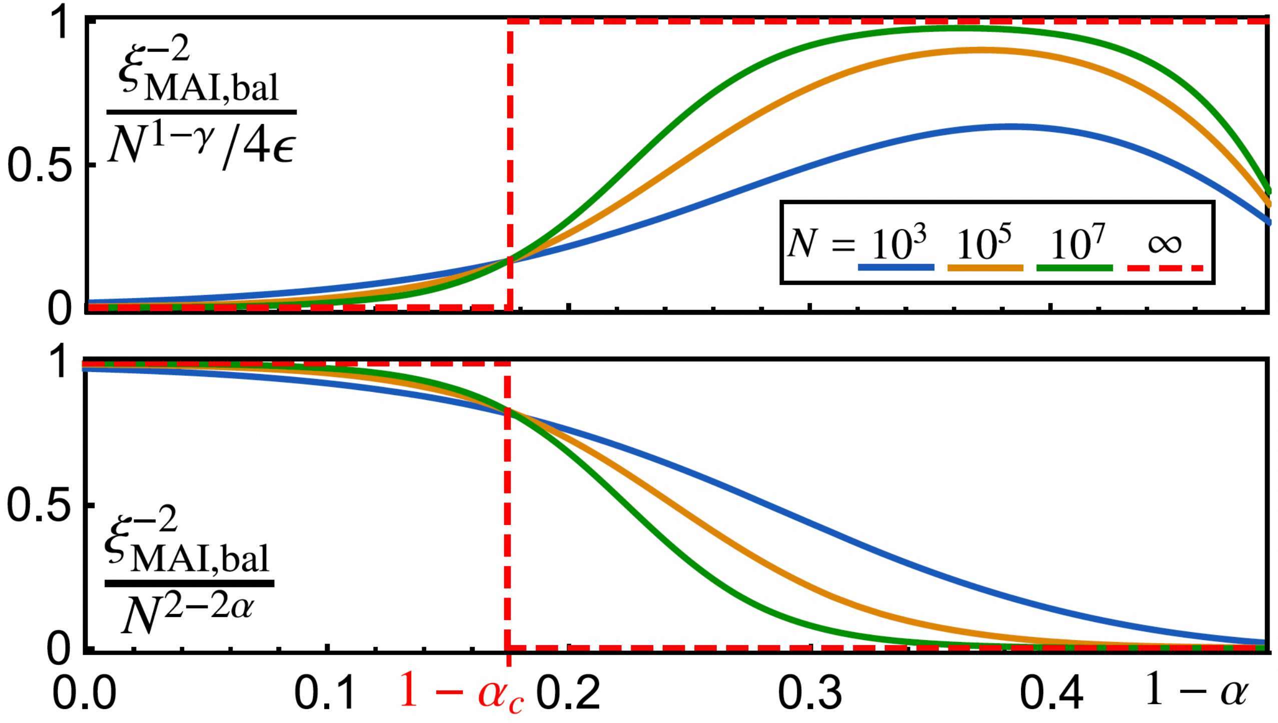
$$\xi_{\mathrm{MAI,bal}}^{-2} = \frac{\sigma^2 N^{2-2\alpha}}{1 + 4\epsilon\sigma^2 N^{1+\gamma-2\alpha}}$$

Critical preparation time $\alpha_c = \frac{1+\gamma}{2}$

$$H = \chi(J_z^2 + DJ_z)$$

Preparation time $\chi t = \sigma N^{-\alpha}$

Gaussian random variable $\langle D^2 \rangle = \epsilon N^{\gamma}$



Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

Ballistic Dephasing

$$\xi^{-2}_{\mathrm{MAI,bal}} = \frac{\sigma^2 N^{2-2\alpha}}{1 + 4\epsilon\sigma^2 N^{1+\gamma-2\alpha}}$$

Critical preparation time $\alpha_c = \frac{1+\gamma}{2}$

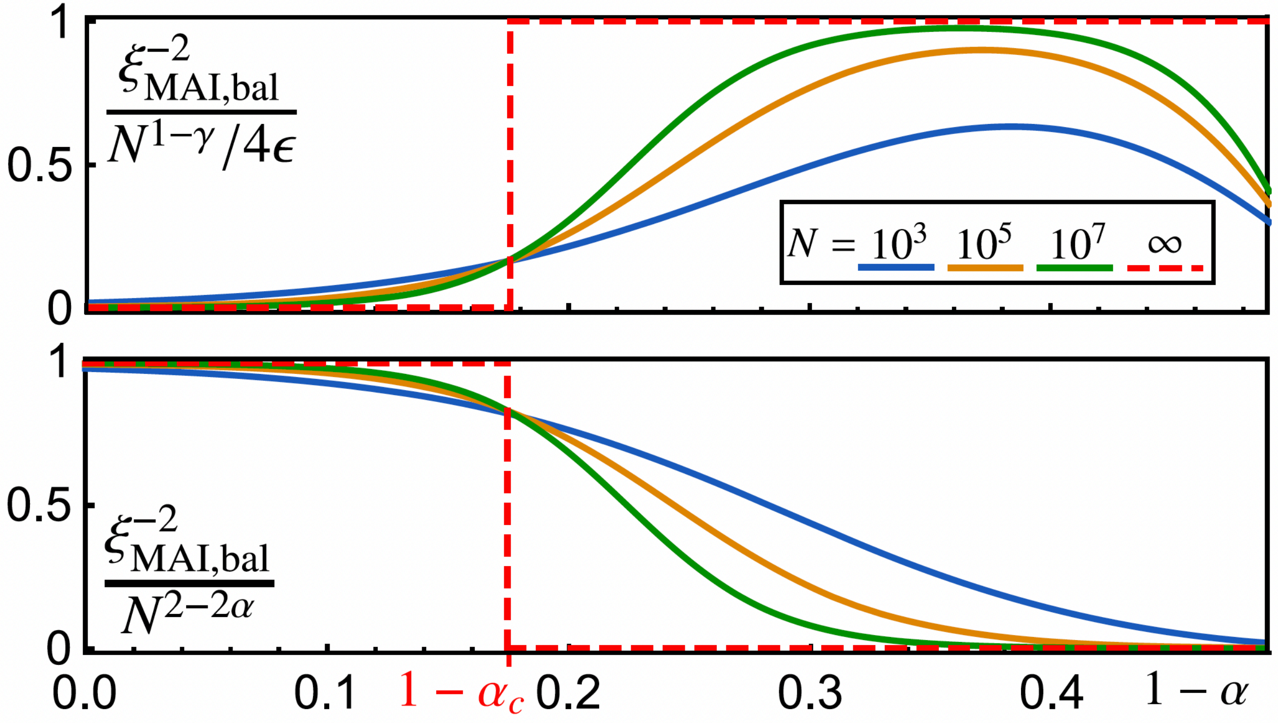
Shorter times $\alpha \geq \alpha_c$

$$\xi^{-2}_{\mathrm{MAI,bal}} = \sigma^2 N^{2-2\alpha}$$

$$H = \chi(J_z^2 + DJ_z)$$

Preparation time $\chi t = \sigma N^{-\alpha}$

Gaussian random variable $\langle D^2 \rangle = \epsilon N^{\gamma}$



Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

Ballistic Dephasing

$$\xi^{-2}_{\mathrm{MAI,bal}} = \frac{\sigma^2 N^{2-2\alpha}}{1 + 4\epsilon\sigma^2 N^{1+\gamma-2\alpha}}$$

Critical preparation time $\alpha_c = \frac{1+\gamma}{2}$

Shorter times $\alpha \geq \alpha_c$

$$\xi^{-2}_{\mathrm{MAI,bal}} = \sigma^2 N^{2-2\alpha}$$

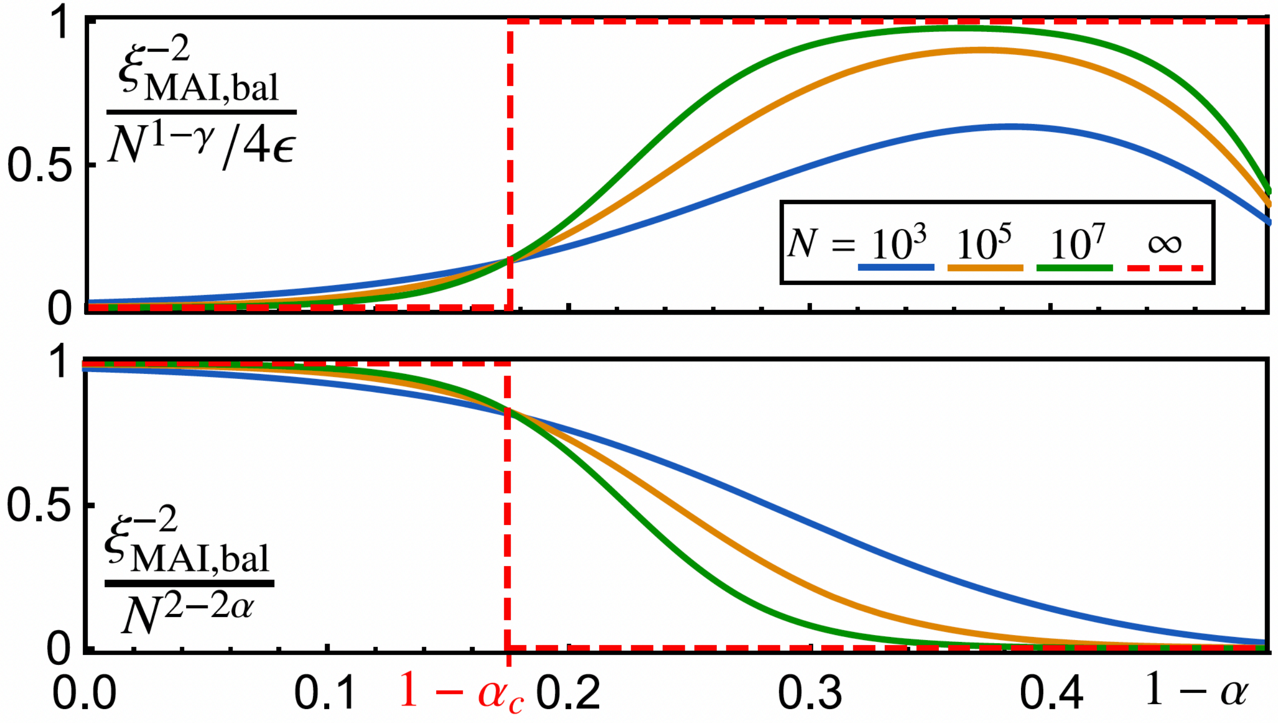
Longer times $\alpha < \alpha_c$

$$\xi^{-2}_{\mathrm{MAI,bal}} = \frac{1}{4\epsilon} N^{1-\gamma}$$

$$H = \chi(J_z^2 + DJ_z)$$

Preparation time $\chi t = \sigma N^{-\alpha}$

Gaussian random variable $\langle D^2 \rangle = \epsilon N^\gamma$



Y. Baamara, A. Sinatra, and M. Gessner, Phys. Rev. Lett. **127**, 160501 (2021).

Particle Loss

- Sensitivity gain is limited by lost fraction.

Particle Loss

- Sensitivity gain is limited by lost fraction.
- Nonlinear squeezing can still improve the precision if linear squeezing does not reach the limit.

$$\xi_{\text{Loss}}^{-2} \leq \frac{3}{\gamma^{(1)} t}$$

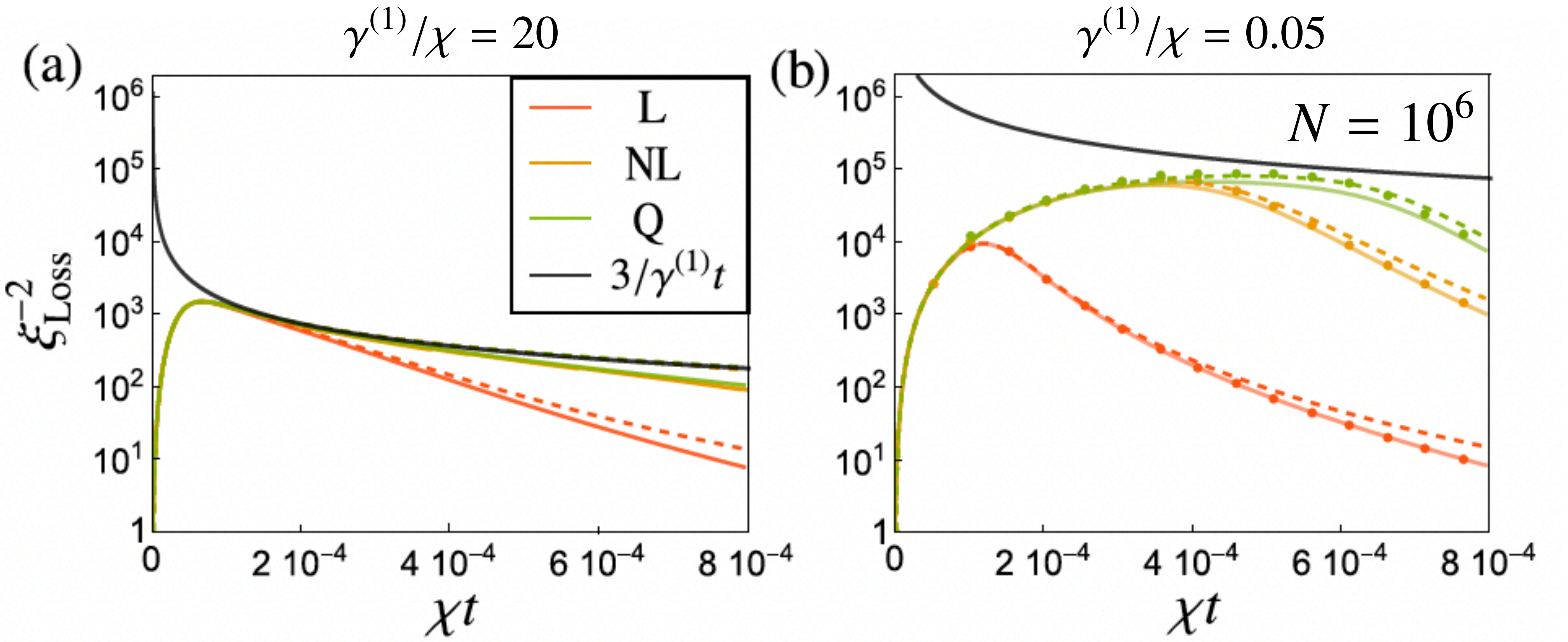
fraction of lost atoms

Particle Loss

- Sensitivity gain is limited by lost fraction.
- Nonlinear squeezing can still improve the precision if linear squeezing does not reach the limit.

$$\xi^{-2}_{\text{Loss}} \leq \frac{3}{\gamma^{(1)} t}$$

fraction of lost atoms



Conclusions

- **Spin squeezing is a Gaussian approximation**
  to the full sensitivity (Fisher information)

$$\frac{|\langle [H, X] \rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, H]$$

Conclusions

- **Spin squeezing is a Gaussian approximation**
  to the full sensitivity (Fisher information)

$$\frac{|\langle [H, X] \rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, H]$$

- **Non-Gaussian squeezing**:
  Systematic optimizations over **nonlinear** observables X

CONCLUSIONS

- **Spin squeezing is a Gaussian approximation**
  to the full sensitivity (Fisher information)

$$\frac{|\langle [H, X] \rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, H]$$

- **Non-Gaussian squeezing**:
  Systematic optimizations over **nonlinear** observables X

- Scaling of non-Gaussian quantum enhancements
  are **far beyond the reach of spin squeezing**

- Optimal implementations depend on the noise

M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019).

Y. Baamara, A. Sinatra, and M. Gessner
Phys. Rev. Lett. **127**, 160501 (2021);
Comptes Rendus. Physique **23**, 1 (2022).

CONCLUSIONS

- **Spin squeezing is a Gaussian approximation** to the full sensitivity (Fisher information)

$$\frac{|\langle [H,X]\rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, H]$$

- **Non-Gaussian squeezing**:
  Systematic optimizations over **nonlinear** observables X

- Scaling of non-Gaussian quantum enhancements are **far beyond the reach of spin squeezing**

- Optimal implementations depend on the noise

M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019).

Y. Baamara, A. Sinatra, and M. Gessner
Phys. Rev. Lett. **127**, 160501 (2021);
Comptes Rendus. Physique **23**, 1 (2022).

Acknowledgments

LKB, Paris

- Youcef Baamara
- Alice Sinatra

LENS, Florence

- Luca Pezzè
- Augusto Smerzi

Conclusions

- **Spin squeezing is a Gaussian approximation** to the full sensitivity (Fisher information)

$$\frac{|\langle [H, X]\rangle_\rho|^2}{(\Delta X)^2_\rho} \leq F_Q[\rho, H]$$

- **Non-Gaussian squeezing**: Systematic optimizations over **nonlinear** observables X
- Scaling of non-Gaussian quantum enhancements are **far beyond the reach of spin squeezing**
- Optimal implementations depend on the noise

M. Gessner, A. Smerzi, and L. Pezzè
Phys. Rev. Lett. **122**, 090503 (2019).

Y. Baamara, A. Sinatra, and M. Gessner
Phys. Rev. Lett. **127**, 160501 (2021);
Comptes Rendus. Physique **23**, 1 (2022).

Acknowledgments

LKB, Paris
- Youcef Baamara
- Alice Sinatra

LENS, Florence
- Luca Pezzè
- Augusto Smerzi

Universitat de València
IFIC Institut de Física Corpuscular
Quantum Spain
Gobierno de España – Ministerio de Ciencia e Innovación

Thank you for your attention!