Quantum reservoir computing in finite dimensions

Physical Review E 107 (3), 035306

Rodrigo Martínez-Peña (IFISC)
Juan-Pablo Ortega (NTU, Singapore)

PhD supervisors:
Roberta Zambrini and Miguel C. Soriano

June 1st 2023

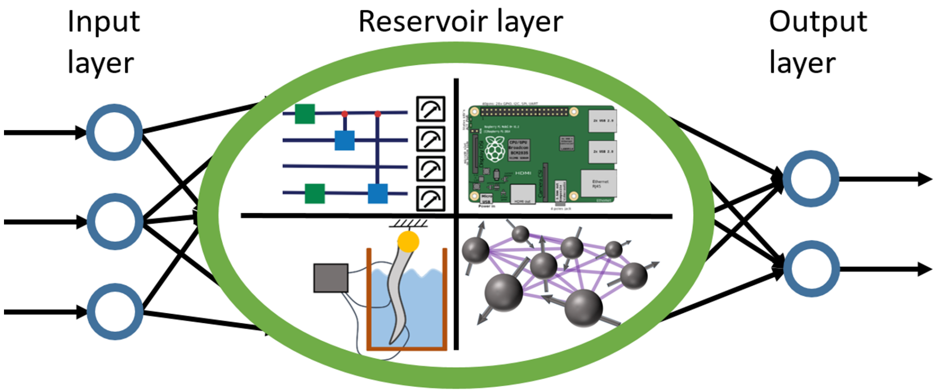
QuaReC
PRD2018/47

EXCELENCIA MARÍA DE MAEZTU
MDM-2017-0711

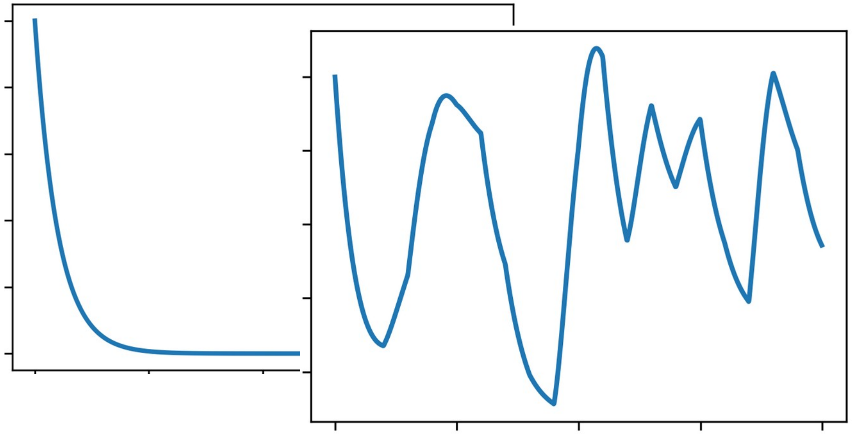
QuaResC
PID2019-109094GB-C21

- Introduction

- (Quantum) Reservoir Computing



- Motivation of this work



- Result 1
- Result 2 and examples
- Conclusions and outlook

Machine Learning is mainly developed for **digital computers** due to its **accessibility**:

- Ease of use
- Unified theory
- Universal (can emulate Turing machines)

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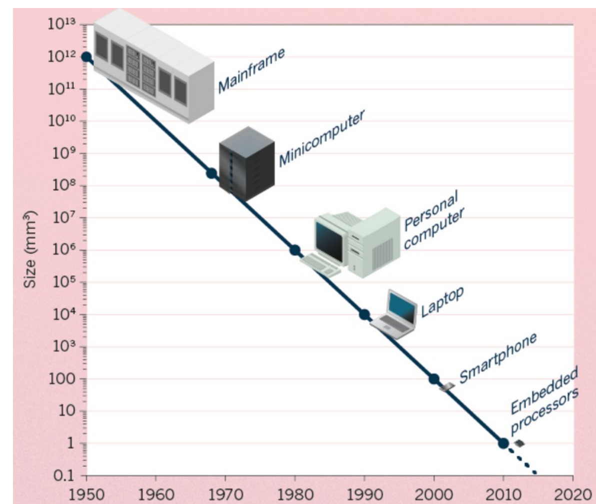
- Miniaturization (end’s of Moore’s law)



Mitchell Waldrop *Nature* 2016

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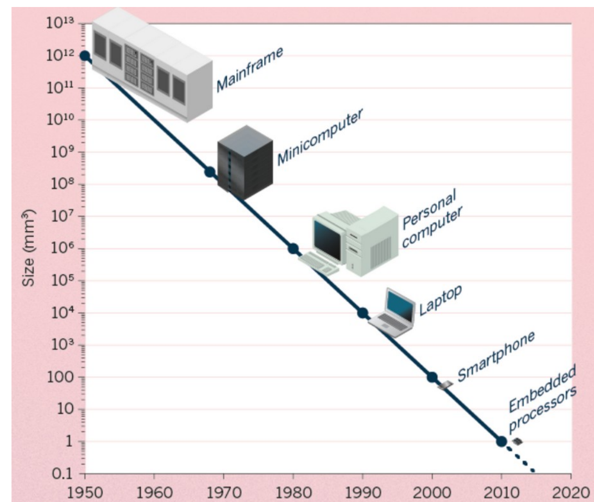

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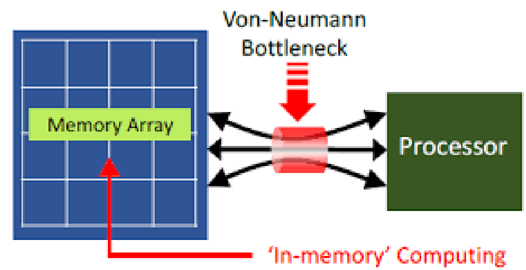
- Von Neumann Bottleneck



G. Indiveri *et. al. Proceedings of the IEEE*, 2015

Tom Dillinger

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Machine learning is already taking advantage of this progress. Example: Reservoir Computing

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Short answer: machine learning technique that **exploits dynamical systems**

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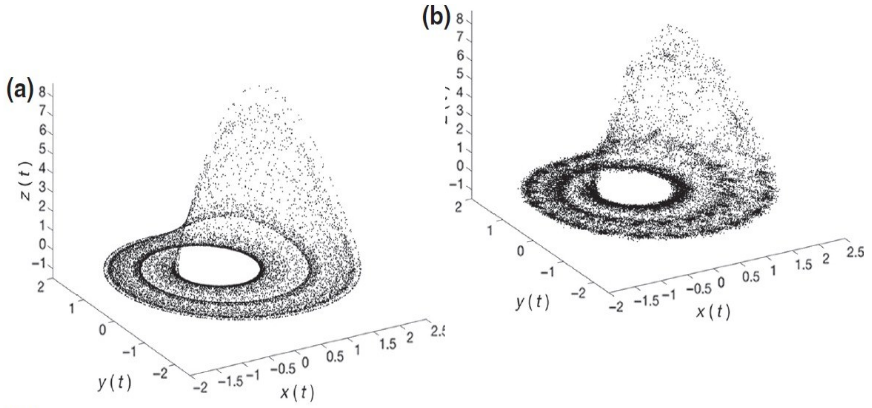
- Dynamical systems
- Noisy processes
- Input/output maps



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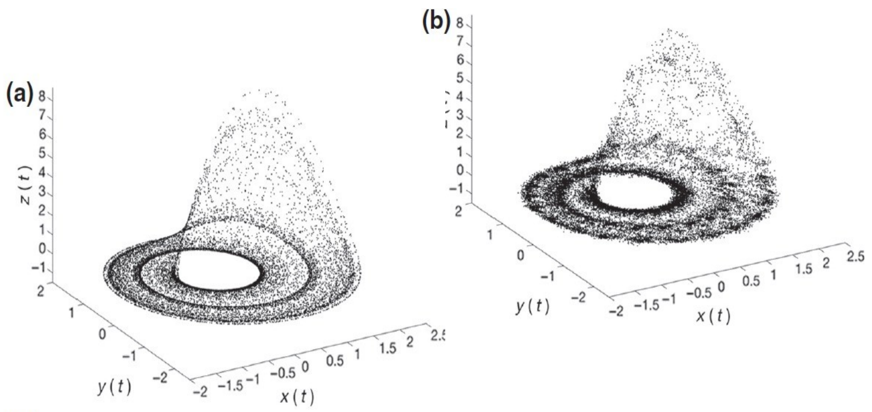
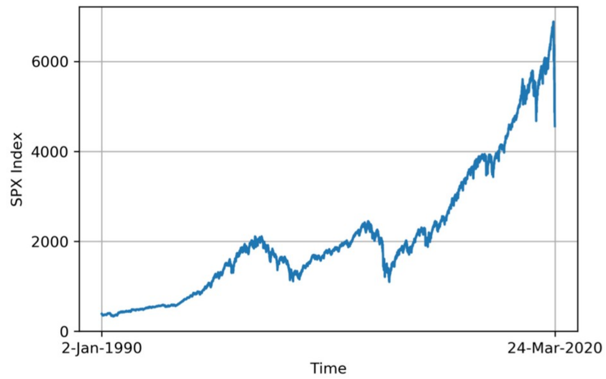


Applications:

- Biomedicine
- Engineering
- Financial
- Communication...

Gouhei Tanaka et al. *Neural Networks* 2019



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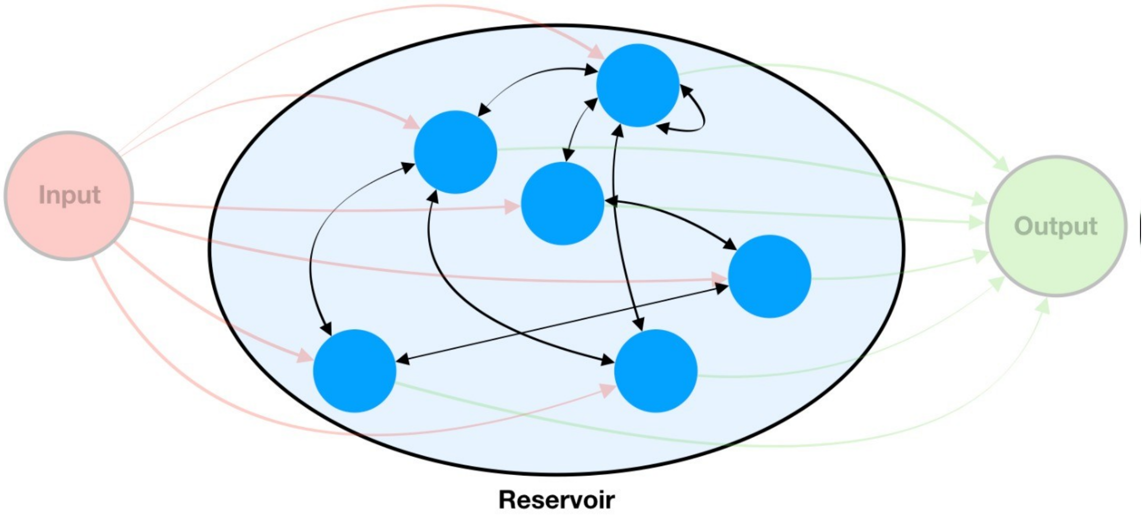

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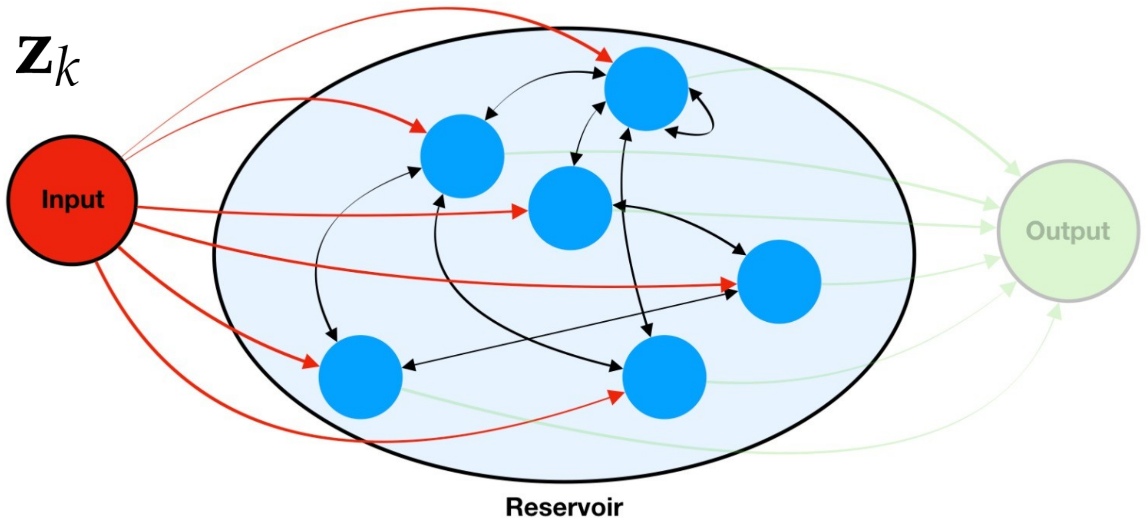

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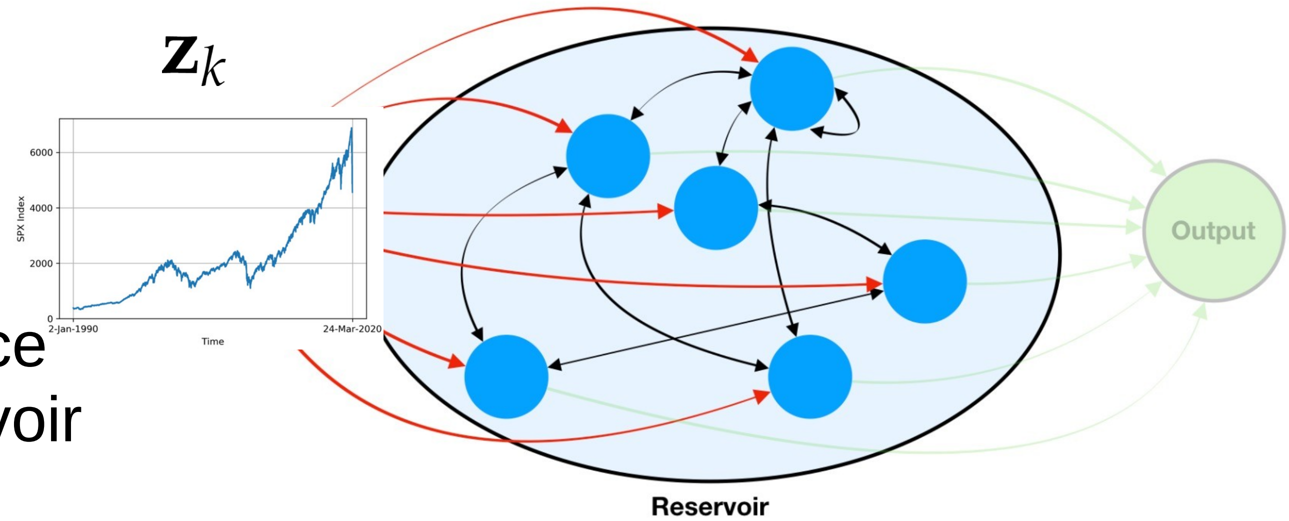

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$$\mathbf{x}_k = T(\mathbf{x}_{k-1}, \mathbf{z}_k)$$

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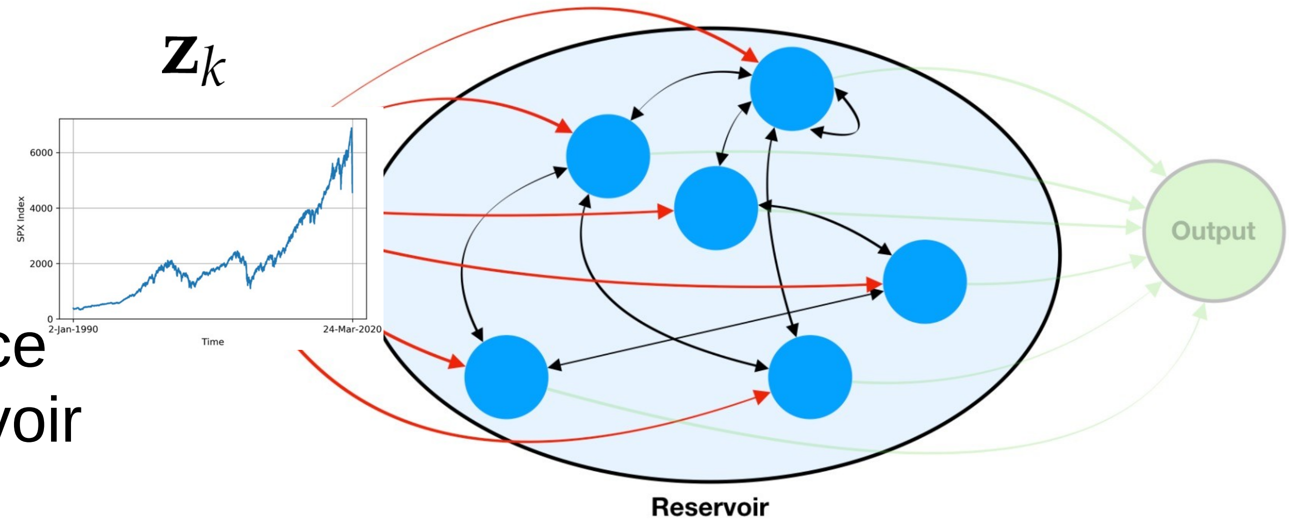

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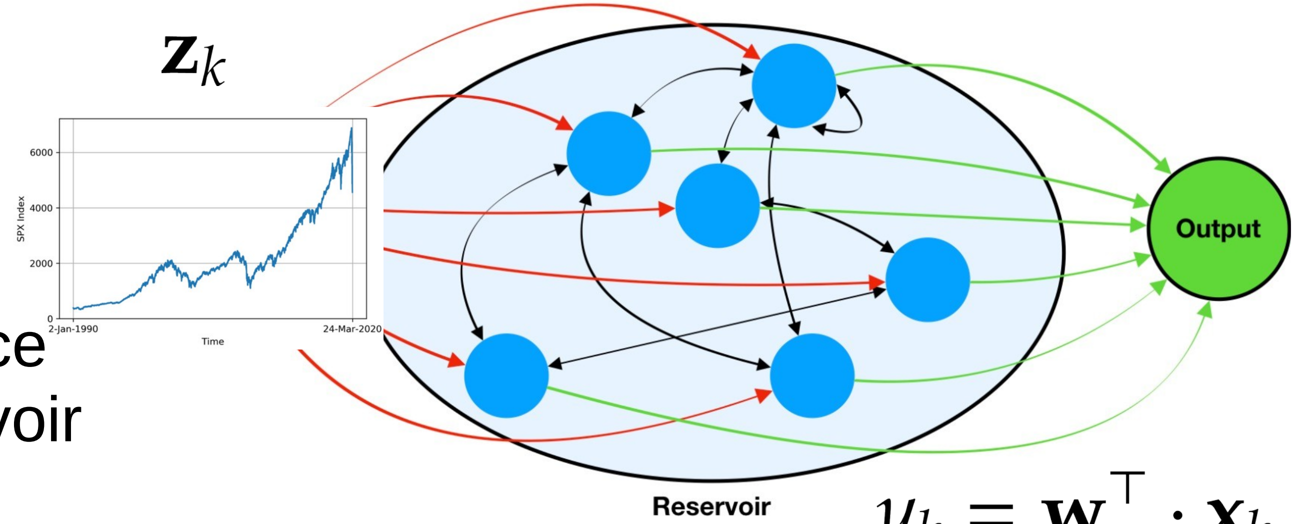

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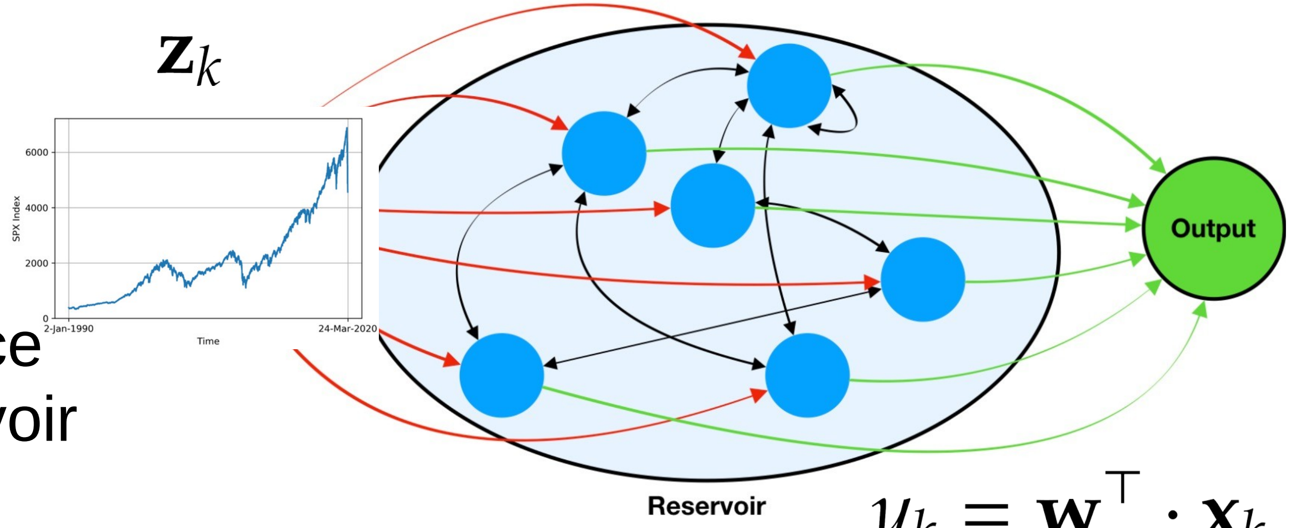

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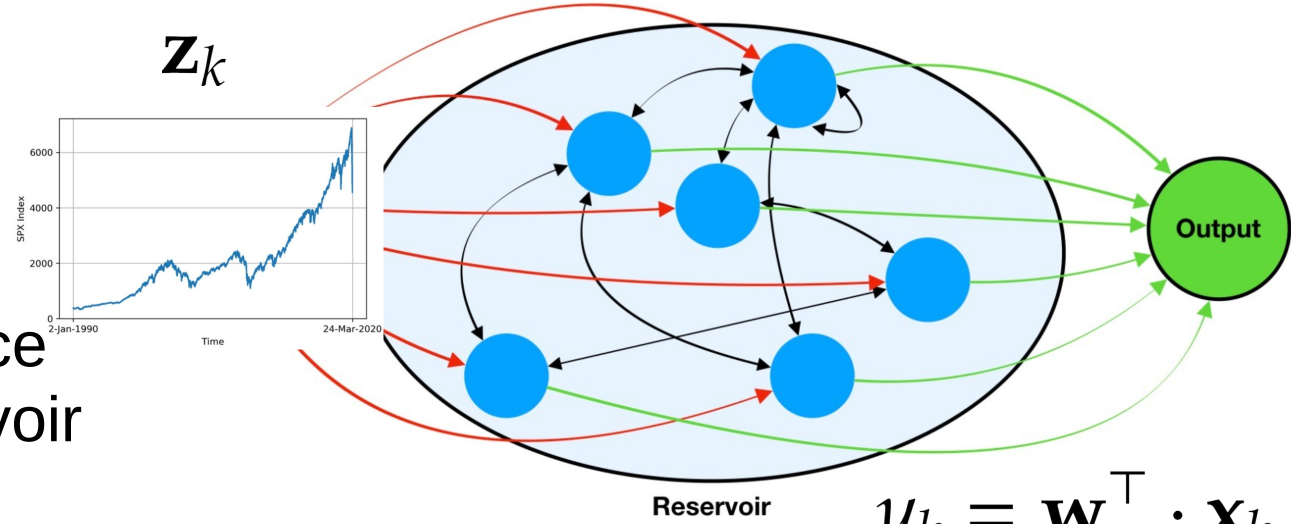

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Ready to use!

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- **Fast training** (linear regression is enough!) $y_k = \mathbf{w}^\top \cdot \mathbf{x}_k$
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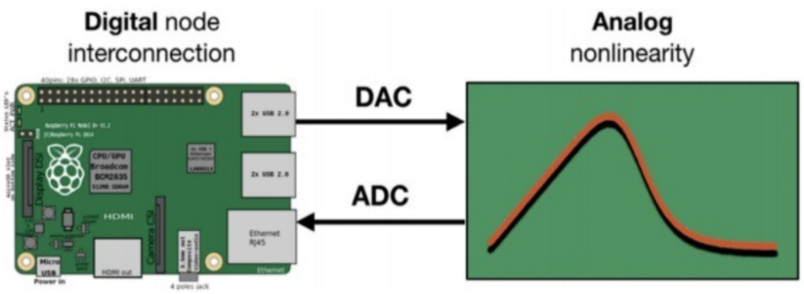

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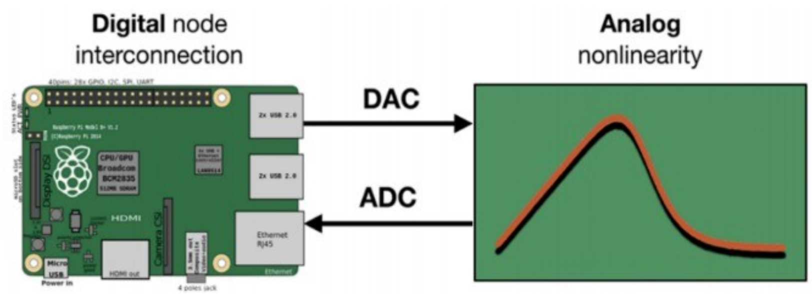

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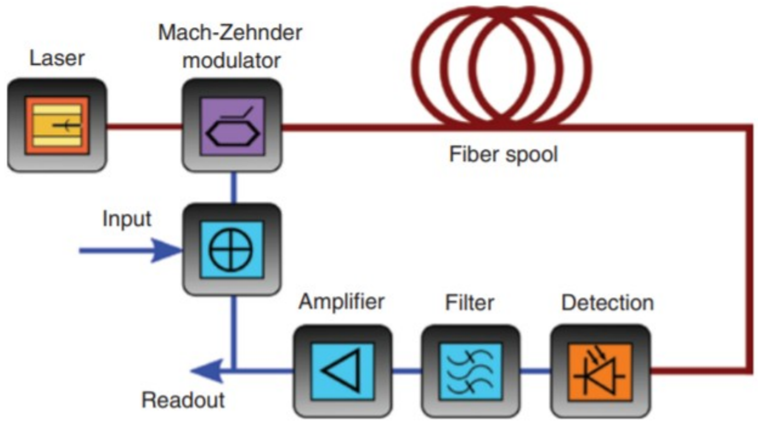

Van der Sande et al. *Nanophotonics* 2017

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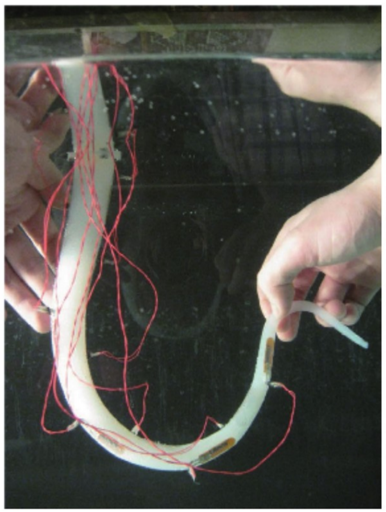

Nakajima et al.
Scientific Reports
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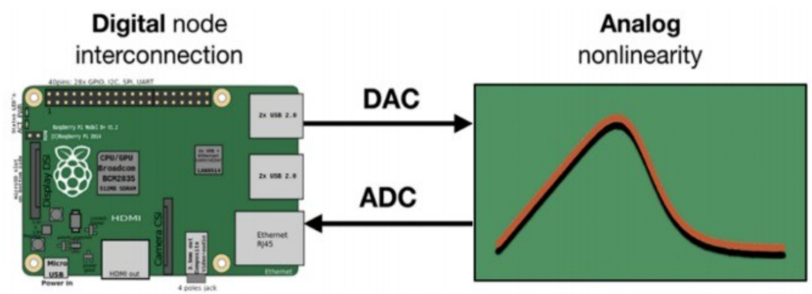

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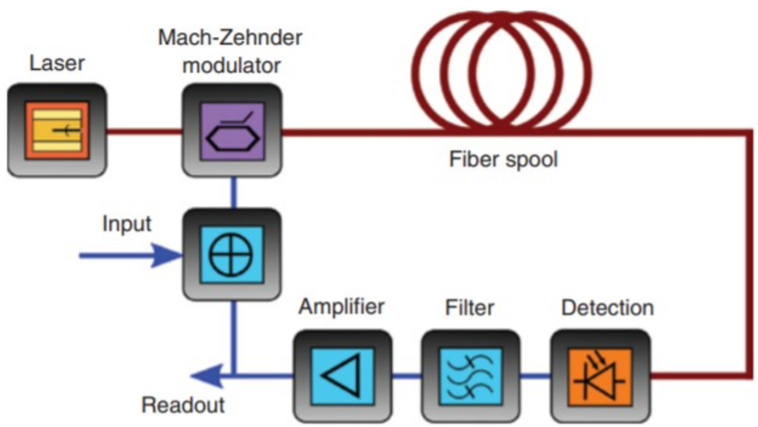

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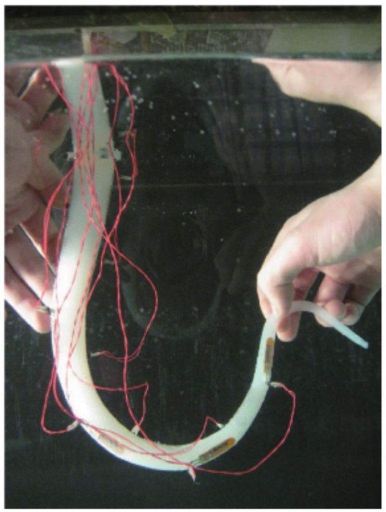

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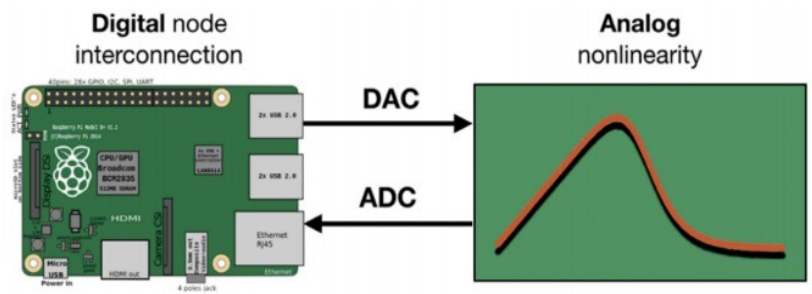

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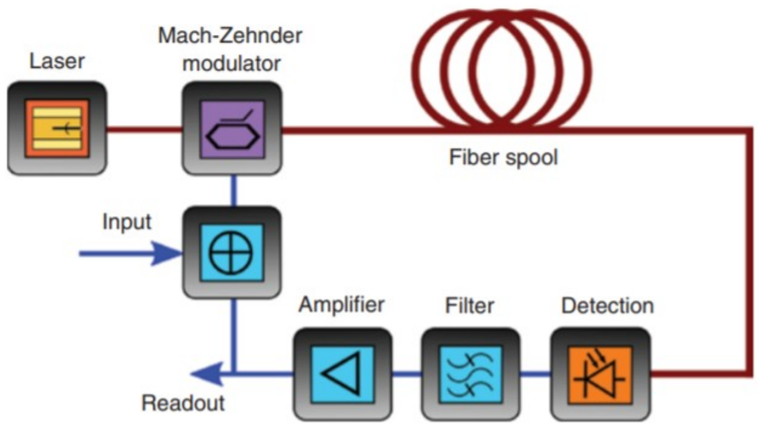

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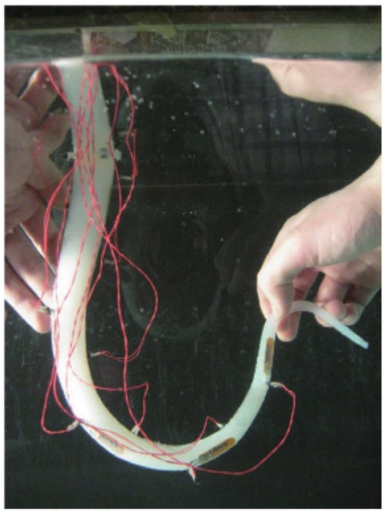

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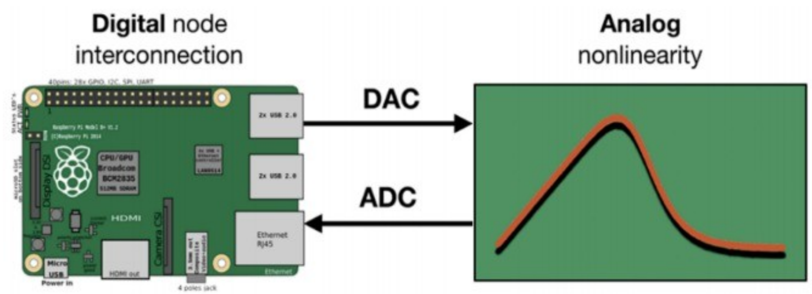

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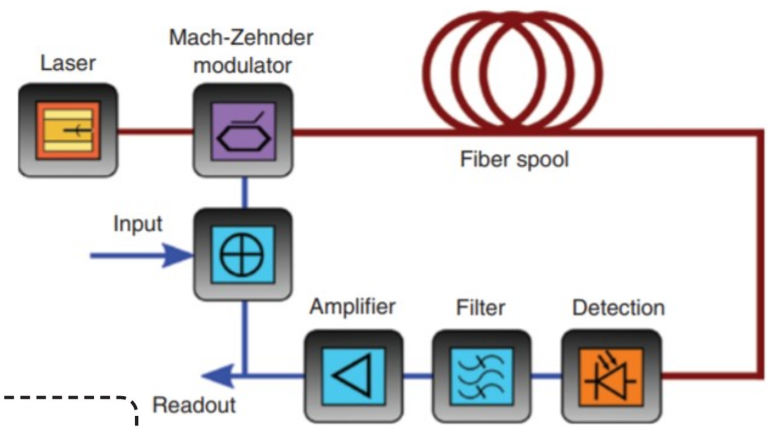

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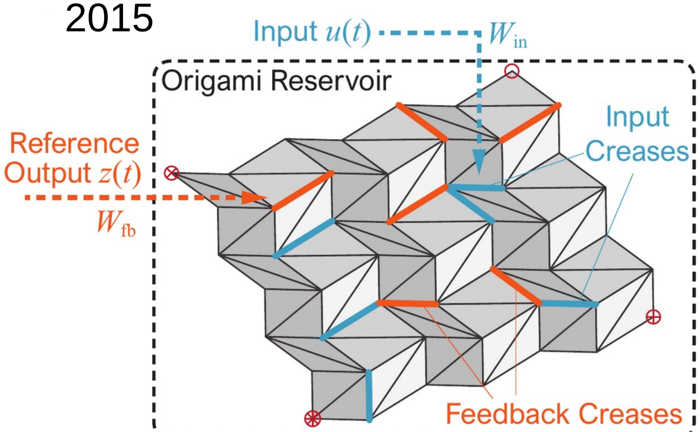

P. Bhovad and S. Li.
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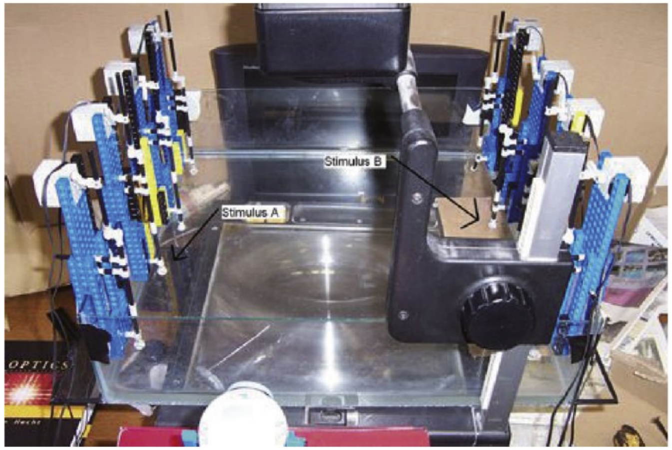

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But what about **quantum systems**?

What is **Quantum Reservoir Computing**?

PERSPECTIVE

ADVANCED QUANTUM TECHNOLOGIES
www.advquantumtech.com

Opportunities in Quantum Reservoir Computing and Extreme Learning Machines

2021!

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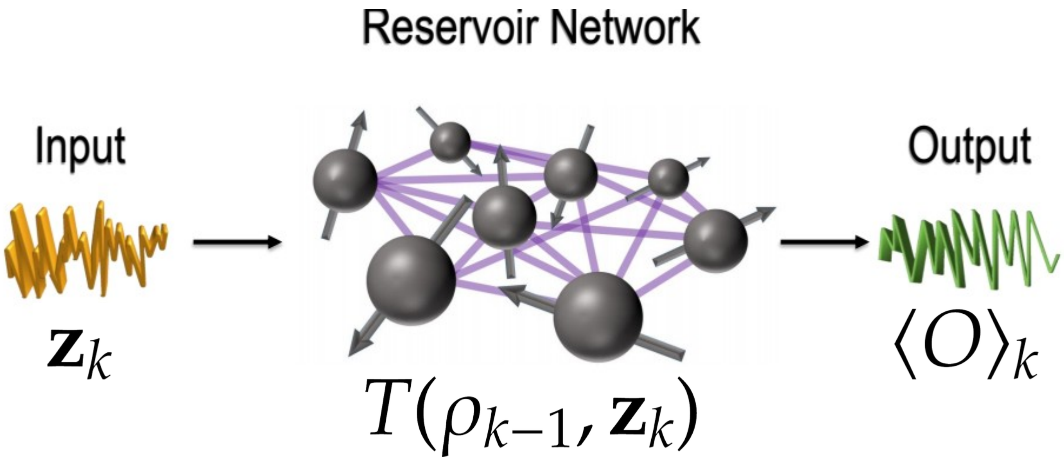

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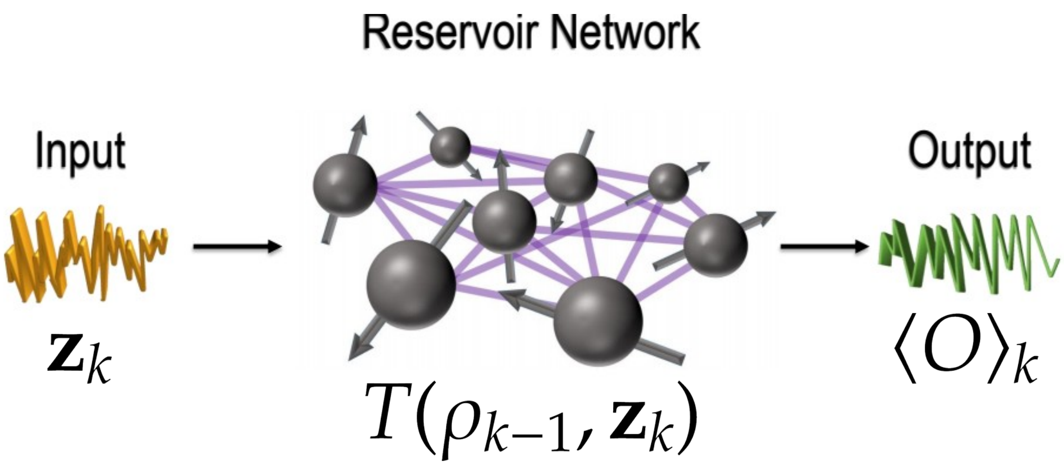

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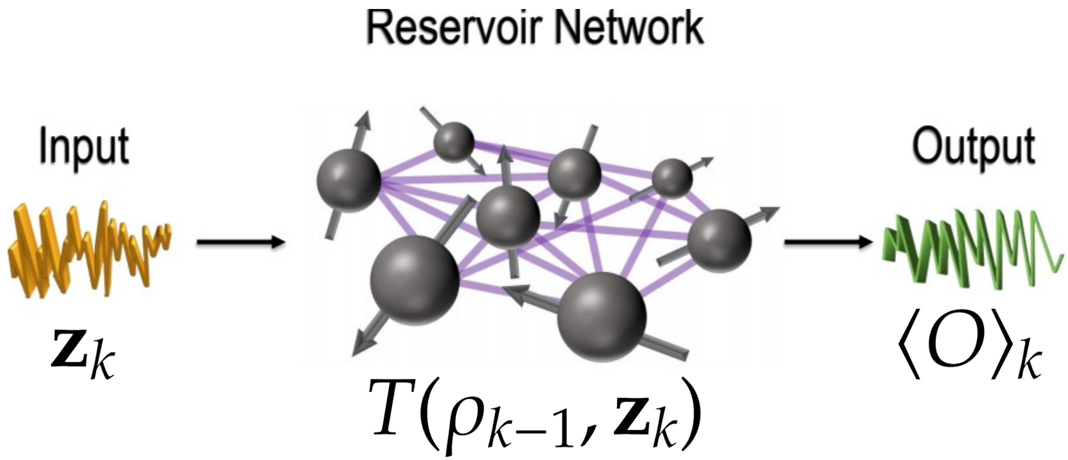

Why quantum?

- Number of degrees of freedom **increases exponentially in few body systems**
- Accessible on **noisy quantum devices**
- Extension to process **quantum data**

- **Context**: only a **few theoretical results** in the field. Mainly **sufficient conditions for useful reservoir systems**.

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Quantum Information Processing (2019) 18:198
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Learning nonlinear input–output maps with dissipative quantum systems

Jiayin Chen[1] · Hendra I. Nurdin[1]

ARTICLE
https://doi.org/10.1038/s42005-021-00556-w OPEN
Communication Physics, 2021
Check for updates

Gaussian states of continuous-variable quantum systems provide universal and versatile reservoir computing

Johannes Nokkala[1✉], Rodrigo Martínez-Peña[1], Gian Luca Giorgi[1], Valentina Parigi[2], Miguel C. Soriano[1] & Roberta Zambrini[1✉]

PHYSICAL REVIEW LETTERS **127,** 260401 (2021)

Learning Temporal Quantum Tomography

Quoc Hoan Tran[1,*] and Kohei Nakajima[1,2,†]
[1]*Graduate School of Information Science and Technology, The University of Tokyo, Tokyo 113-8656, Japan*
[2]*Next Generation Artificial Intelligence Research Center, The University of Tokyo, Tokyo 113-8656, Japan*

(Received 5 April 2021; revised 11 October 2021; accepted 30 November 2021; published 22 December 2021)

Dissipation as a resource for Quantum Reservoir Computing

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ArXiv: 2212.12078

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- **<u>What we did</u>**: find **necessary and sufficient conditions** of **useful reservoir systems.**

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- **What we did**: find **necessary and sufficient conditions** of **useful reservoir systems.**

- **Why it is important**: **universal approximation property** and connection with **experimental design.**

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- Finite dimensional system
- Classical input information
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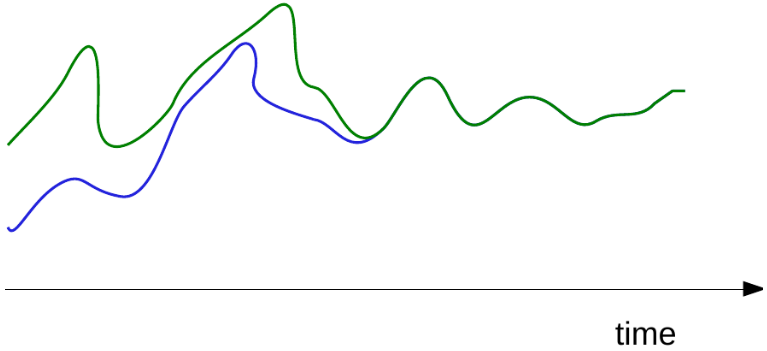
Result:

A quantum reservoir is **useful if and only** if the dynamics of the quantum channel **converges** towards **input-dependent** fixed points



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- Ideal measurements (Some experimental works can be approximated by this limit)

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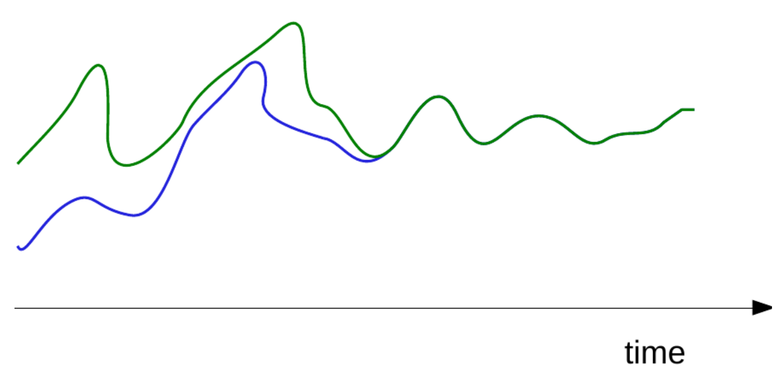
A quantum reservoir is **useful if and only** if the dynamics of the quantum channel **converges** towards **input-dependent** fixed points

Result 2



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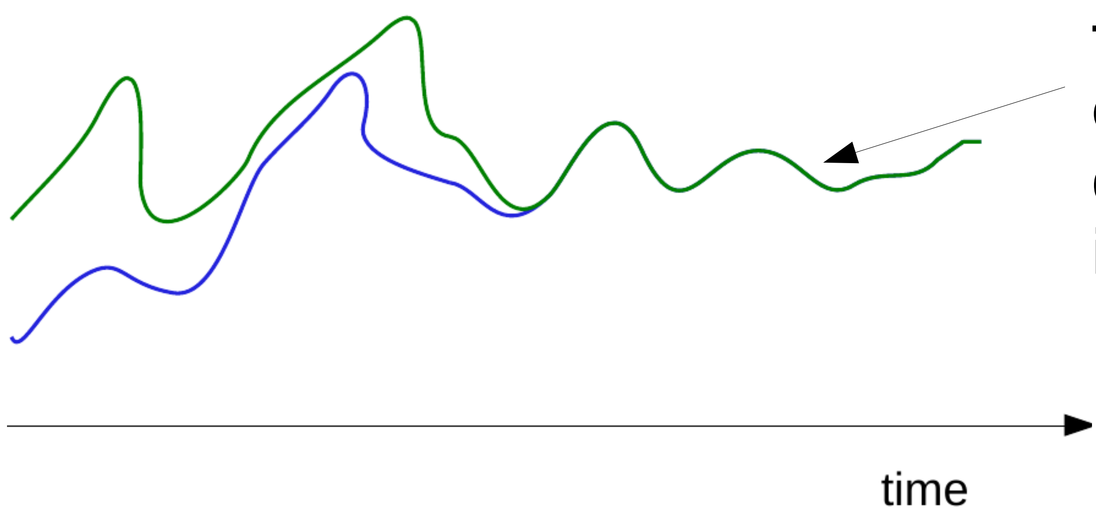
- Fading Memory Property (FMP): **memory** of past inputs.



Reservoir dynamics **forgets the initial conditions and dissipates past inputs**

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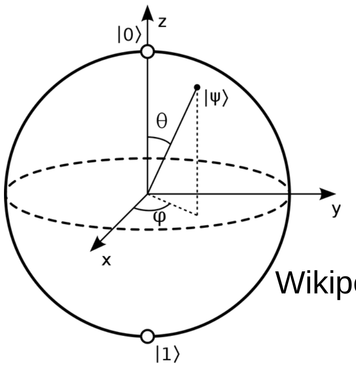
Example of one qubit:

$$\rho \Rightarrow \mathbf{x} = (1, \langle \sigma^x \rangle, \langle \sigma^y \rangle, \langle \sigma^z \rangle)$$

$$\{B_i\} = \{I, \sigma^x, \sigma^y, \sigma^z\}$$



Wikipedia

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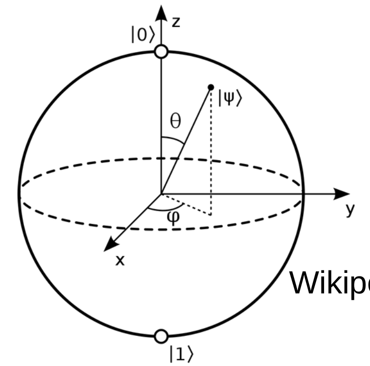

Quantum channel in observables form:

$$x_{k,j} = (\widehat{T}(\mathbf{z}_k)|_{\mathcal{H}_0})_{i,j} x_{k-1,j} + \widehat{T}(\mathbf{z}_k)_{0j}$$

$\mathbf{z}_k$: input

$$\mathcal{H}_0 = \{\sigma^x, \sigma^y, \sigma^z\}$$

$$\widehat{T}(\mathbf{z}_k)_{ij} = \frac{1}{2}\mathrm{tr}(B_i T(B_j, \mathbf{z}_k))$$

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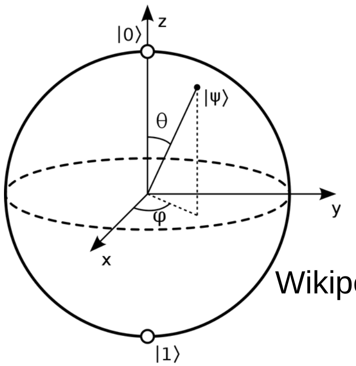

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$$\widehat{T}(\mathbf{z}_k)_{ij} = \frac{1}{2}\mathrm{tr}(B_i T(B_j, \mathbf{z}_k))$$

$$|||\widehat{A}||| := \sup_{||\mathbf{x}|| \neq 0} \frac{||\widehat{A\mathbf{x}}||}{||\mathbf{x}||}$$

Induced norm $|||\widehat{T}(\mathbf{z}_k)|_{\mathcal{H}_0}||| < 1$

D. J. Hartfiel, *Nonhomogeneous matrix products*, 2002

There is an equivalent condition in density matrix form

- **<u>Result 2:</u>** a quantum reservoir becomes **input-dependent (useful in the long-term) if and only if** the quantum channel have a unique **input-dependent fixed point**.

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$$T(\rho^*(\mathbf{z}_k), \mathbf{z}_k) = \rho^*(\mathbf{z}_k)$$

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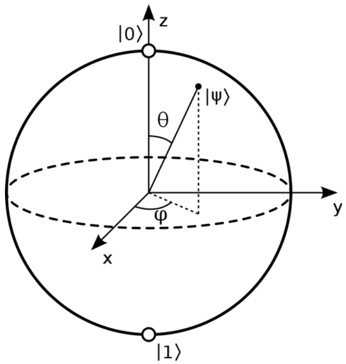
- **Intuition:** convergence makes the system approach an attractor. To **drive** a system **over time steps** you need **attractors that depends on the input.**

Is example 1 a useful quantum reservoir?

Example 1:

$$\dot{\rho} = -i[H,\rho] + \gamma L\rho L^\dagger - \frac{\gamma}{2}\{L^\dagger L, \rho\}$$

$$H = \frac{h(\mathbf{z}_k)}{2}\sigma^z \qquad L = \sigma^-$$



Solution 1:

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Does this system fading memory property?

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But...

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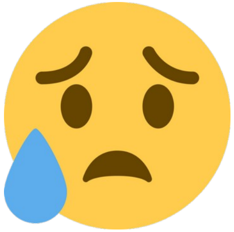
<u>Solution 1</u>:

$$H = \frac{h(\mathbf{z}_k)}{2}\sigma^z \qquad L = \sigma^-$$

Input-independent fixed point:

$$\rho^* = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$



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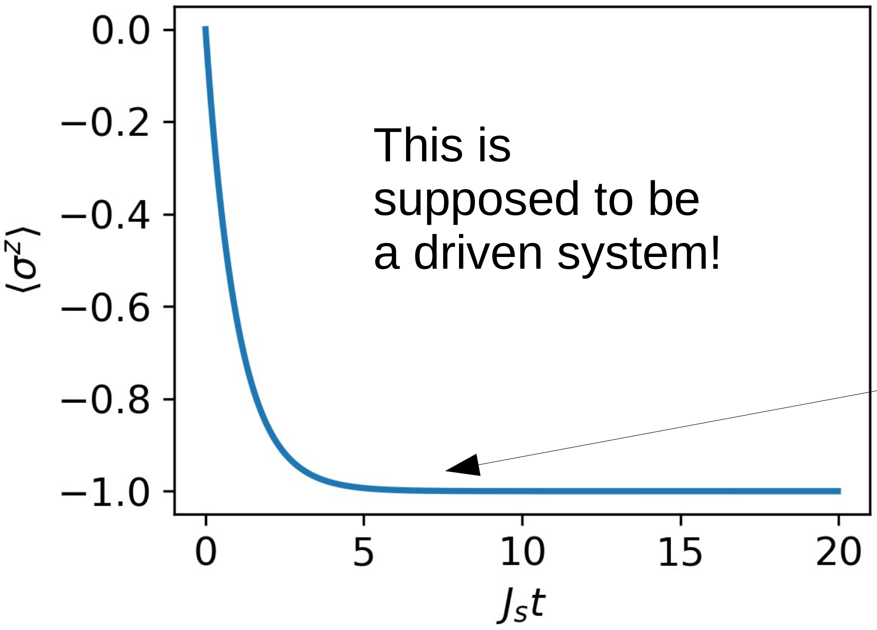

Result 2 says that this system is **useless** for long-input sequences (or not that long...)

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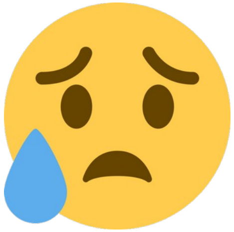

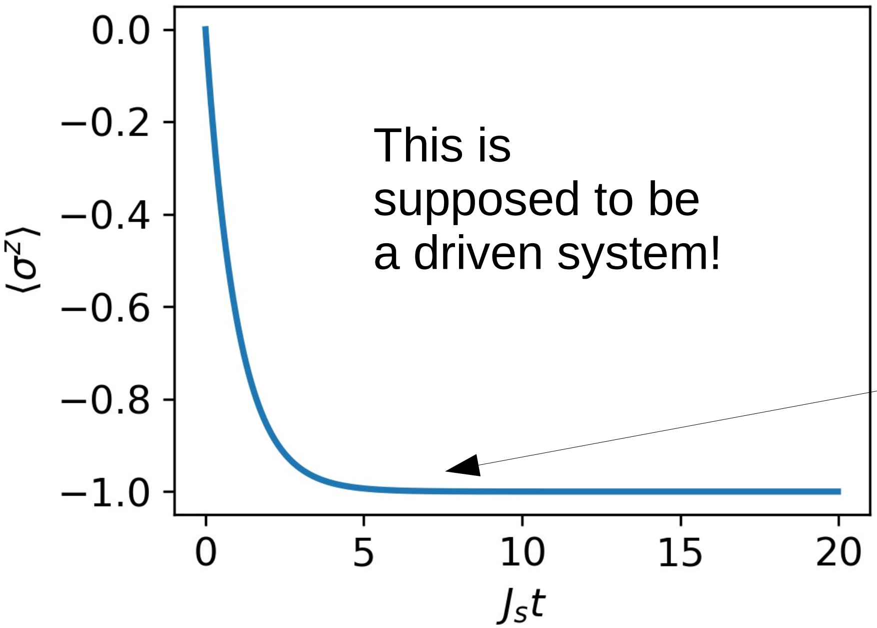

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<u>Conclusion:</u> fading memory is important, but the design is very important as well!

Is example 2 a useful quantum reservoir?

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$$\dot{\rho} = -i[H,\rho] + \gamma L \rho L^\dagger - \frac{\gamma}{2}\{L^\dagger L, \rho\}$$

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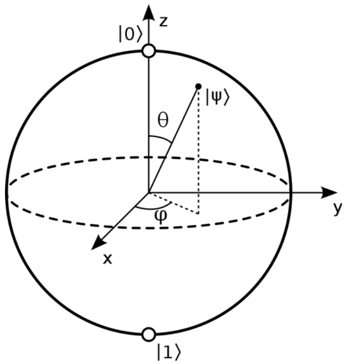

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$$\widehat{T}_{22} = e^{-\frac{\gamma\Delta t}{2}}$$

$$\widehat{T}_{33} = e^{-\frac{3\gamma\Delta t}{4}}\left(\cosh\left(\frac{\Delta t}{4}\sqrt{\gamma^2-16h_k^2}\right) + \frac{\gamma}{\sqrt{\gamma^2-16h_k^2}}\sinh\left(\frac{\Delta t}{4}\sqrt{\gamma^2-16h_k^2}\right)\right)$$

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$$\widehat{T}_{43} = -\widehat{T}_{34}$$

$$\widehat{T}_{31} = \frac{2\gamma h_k}{\gamma^2+2h_k^2}\left\{-1 + e^{-\frac{3\gamma\Delta t}{4}}\left(\cosh\left(\frac{\Delta t}{4}\sqrt{\gamma^2-16h_k^2}\right) + \frac{3\gamma}{\sqrt{\gamma^2-16h_k^2}}\sinh\left(\frac{\Delta t}{4}\sqrt{\gamma^2-16h_k^2}\right)\right)\right\}$$

$$\widehat{T}_{41} = \frac{\gamma}{\gamma^2+2h_k^2}\left\{-\gamma + e^{-\frac{3\gamma\Delta t}{4}}\left(\gamma\cosh\left(\frac{\Delta t}{4}\sqrt{\gamma^2-16h_k^2}\right) - \frac{\gamma^2+8h_k^2}{\sqrt{\gamma^2-16h_k^2}}\sinh\left(\frac{\Delta t}{4}\sqrt{\gamma^2-16h_k^2}\right)\right)\right\}$$

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$$s_1 = e^{-\gamma\Delta t/2} < 1$$

Bloch vector representation:

$$\begin{pmatrix} 1 \\ \langle\sigma^x\rangle_k \\ \langle\sigma^y\rangle_k \\ \langle\sigma^z\rangle_k \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \widehat{T}_{22} & 0 & 0 \\ \widehat{T}_{31} & 0 & \widehat{T}_{33} & \widehat{T}_{34} \\ \widehat{T}_{41} & 0 & \widehat{T}_{43} & \widehat{T}_{44} \end{pmatrix} \begin{pmatrix} 1 \\ \langle\sigma^x\rangle_{k-1} \\ \langle\sigma^y\rangle_{k-1} \\ \langle\sigma^z\rangle_{k-1} \end{pmatrix}$$

$\widehat{T}|_{\mathcal{H}_0}$

?

$$|||\widehat{T}(\mathbf{z}_k)|_{\mathcal{H}_0}||| < 1$$

Induced Frobenius norm:
maximum singular value

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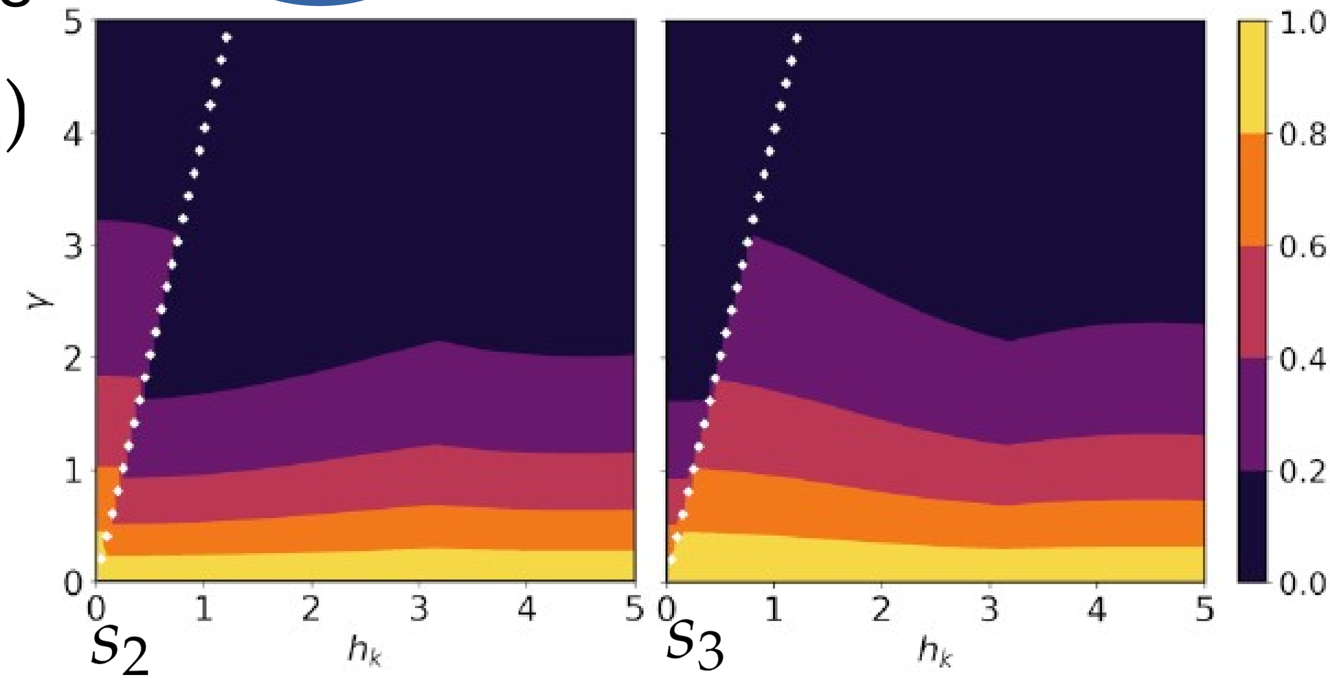

Solution 2

Bloch vector representation:

$$\begin{pmatrix} 1 \\ \langle\sigma^x\rangle_k \\ \langle\sigma^y\rangle_k \\ \langle\sigma^z\rangle_k \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \widehat{T}_{22} & 0 & 0 \\ \widehat{T}_{31} & 0 & \widehat{T}_{33} & \widehat{T}_{34} \\ \widehat{T}_{41} & 0 & \widehat{T}_{43} & \widehat{T}_{44} \end{pmatrix} \begin{pmatrix} 1 \\ \langle\sigma^x\rangle_{k-1} \\ \langle\sigma^y\rangle_{k-1} \\ \langle\sigma^z\rangle_{k-1} \end{pmatrix}$$

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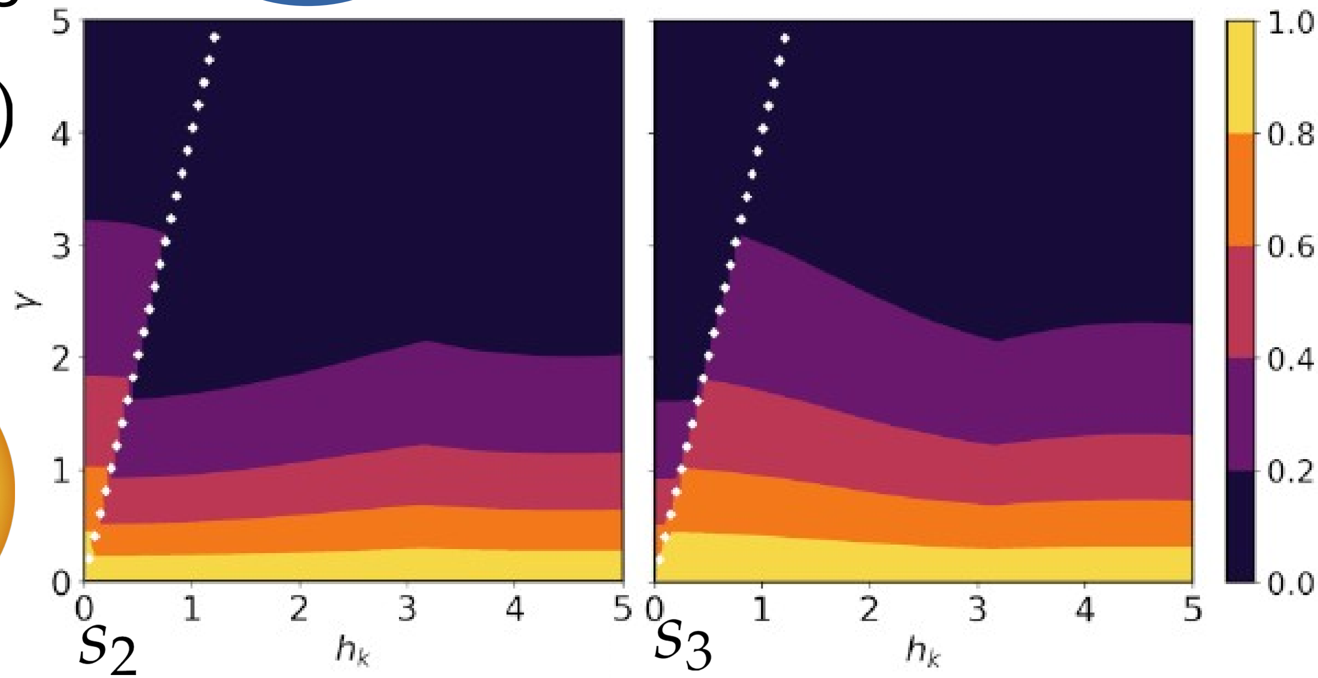
This system has fading memory!!



Solution 2:

$$\dot{\rho} = -i[H, \rho] + \gamma L \rho L^{\dagger} - \frac{\gamma}{2}\{L^{\dagger}L, \rho\}$$

Input-dependent fixed point:

$$H = \frac{h(\mathbf{z}_k)}{2}\sigma^x \qquad L = \sigma^-$$

$$\rho^* = \frac{1}{\gamma^2 + 2h_k^2}\begin{pmatrix} h_k^2 & i\gamma h_k \\ -i\gamma h_k & \gamma^2 + h_k^2 \end{pmatrix}$$

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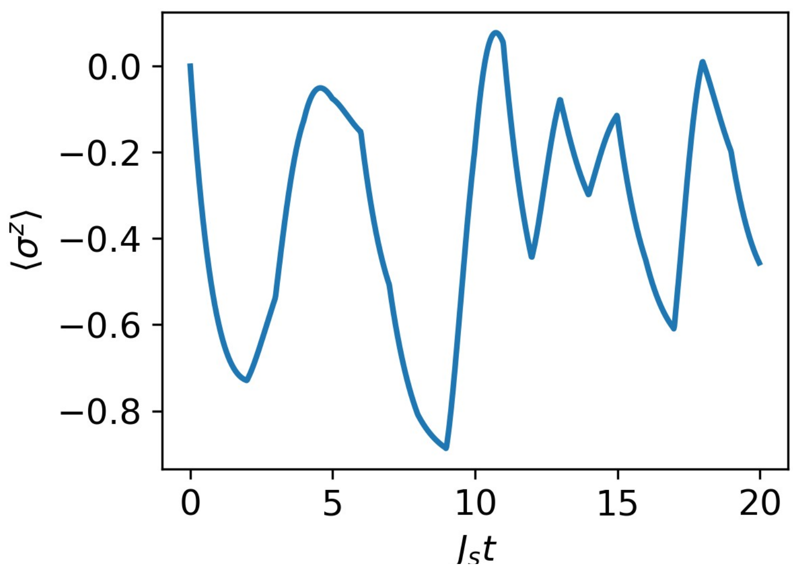

Result 2 states that this system will be always **input-dependent**!

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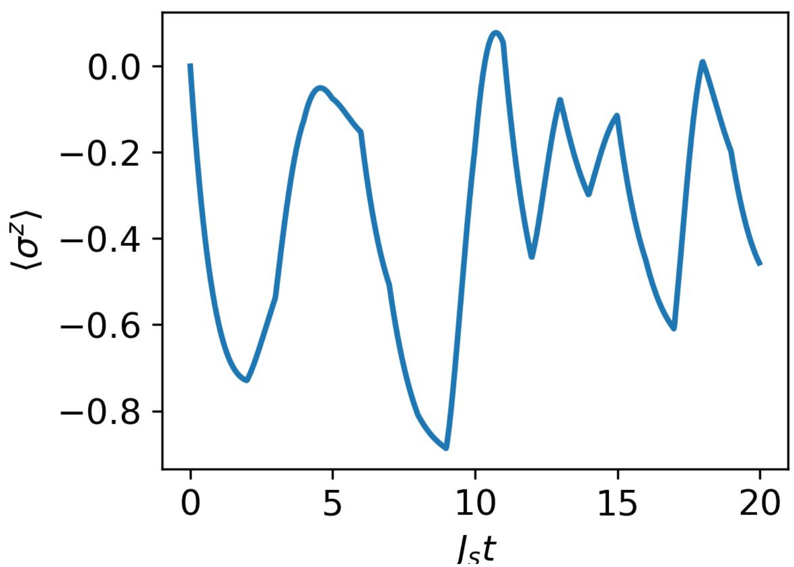
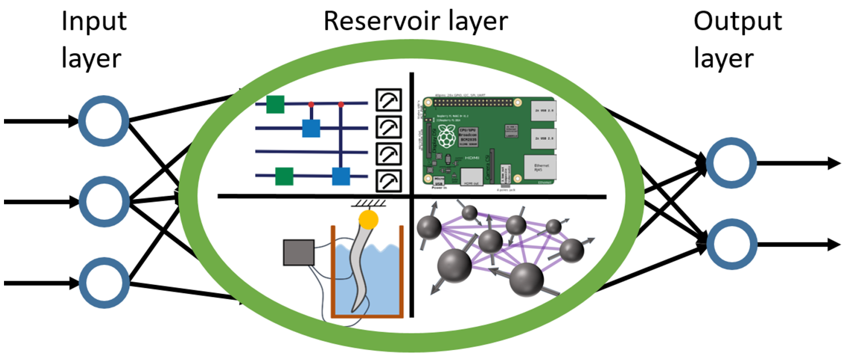


Result 2 states that this system will be always **input-dependent**!

<u>Conclusion:</u> we **don't know how good** the reservoir is for specific tasks, **but it works!**

Conclusions

(Quantum) Reservoir Computing might be a good **alternative** to **conventional computers** to process **temporal** data



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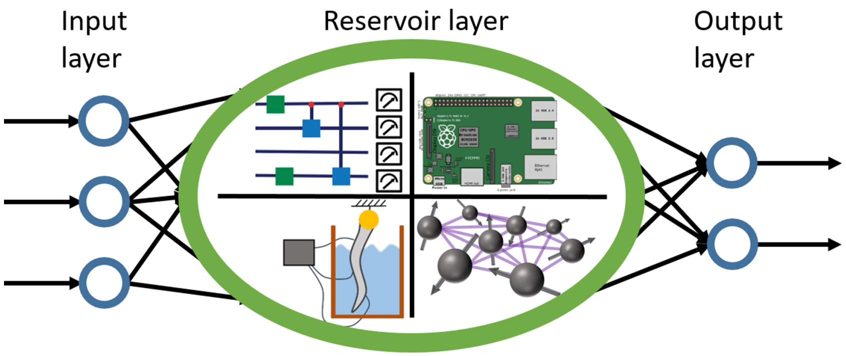
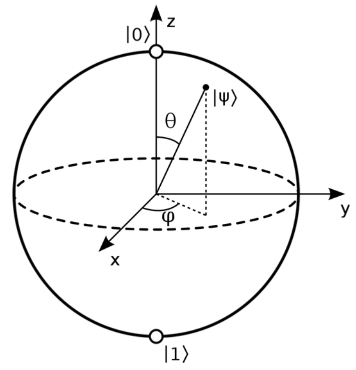


Different representations of quantum systems can provide a **better insight** of **reservoir computing properties**



Conclusions

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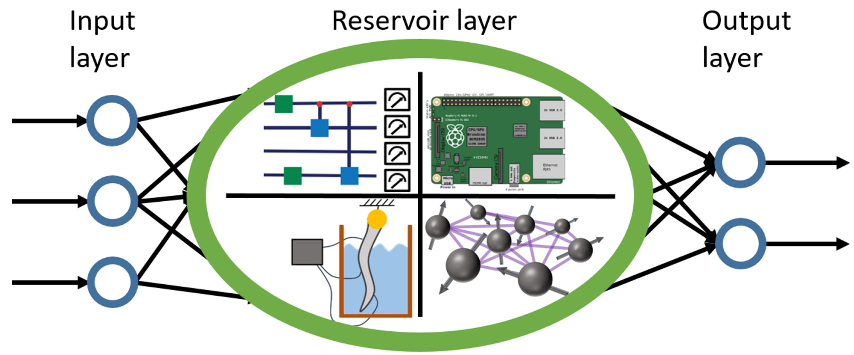

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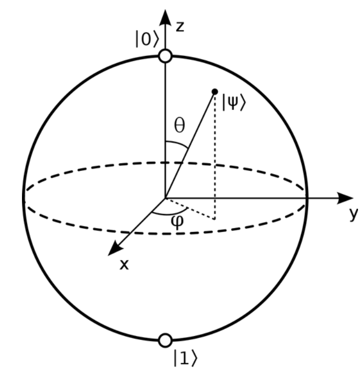
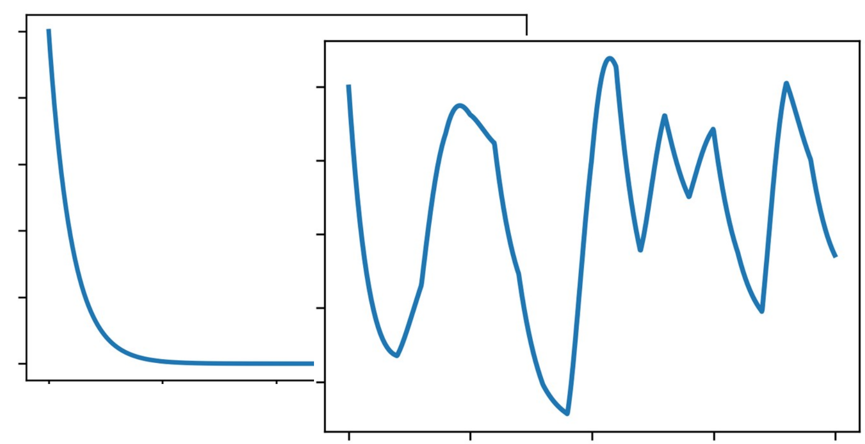


A quantum reservoir is **useful if and only if** the dynamics **converges** towards **input-dependent** fixed points



Outlook & Thanks!

Studying infinite dimensional systems

Extending the theory to non-ideal situations: finite number of measurements, POVMs...

Finding the most general conditions for universal approximation property

R. Martínez-Peña, J. P. Ortega, *Quantum reservoir computing in finite dimensions*, Physical Review E 107 (3), 035306

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THANK YOU

for your attention

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