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INSTITUTO DE FÍSICA FUNDAMENTAL
CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

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S. Carignano (BSC), C. Ramos (UB)

previous work with: M. Piani (evolutionQ) Niall Robertson (IBM) Jacopo Surace (ICFO) Erik Tonni (SISSA)

Ongoing effort to simulate the out-of-equilibrium dynamics with TN

# Entanglement and coherence out of equilibrium

Plan de Recuperación, Transformación y Resiliencia

GOBIERNO DE ESPAÑA MINISTERIO DE CIENCIA E INNOVACIÓN

Phys. Rev. B 99, 235115 (2019)

M. Frias-Perez (LT) MC.- Bañuls

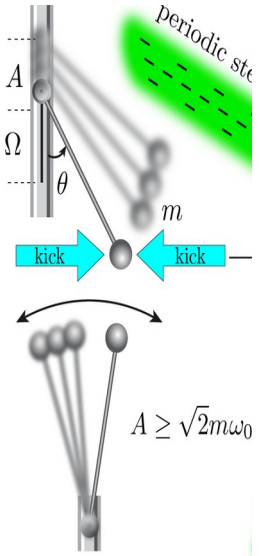
arXiv:2306.xxxx

PGC2018-095862-B-C22
PID2021-127968NB-I00
TED2021-130552B-C22

- Out of equilibrium
- Entanglement barrier
- Quasi-particles
- Trading entanglement for mixture
  - Unveiling local equilibration
  - Identifying fast degrees of freedom
  - Relation with decoherence

Classical:

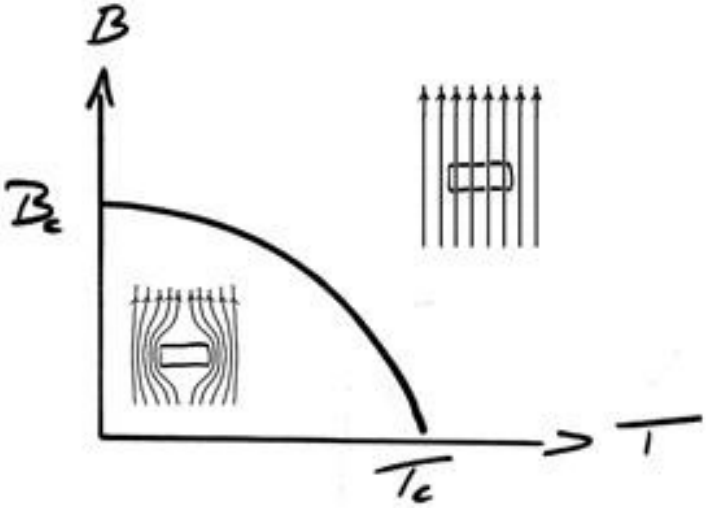
Bukov



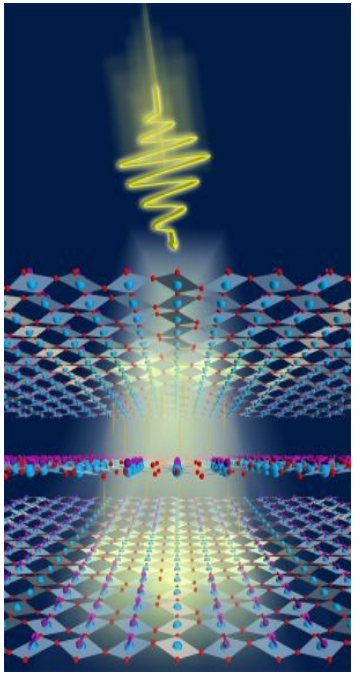

# Kapitsa pendulum

Quantum:

# Light induced superconductor




Cavallieri



- Out of equilibrium
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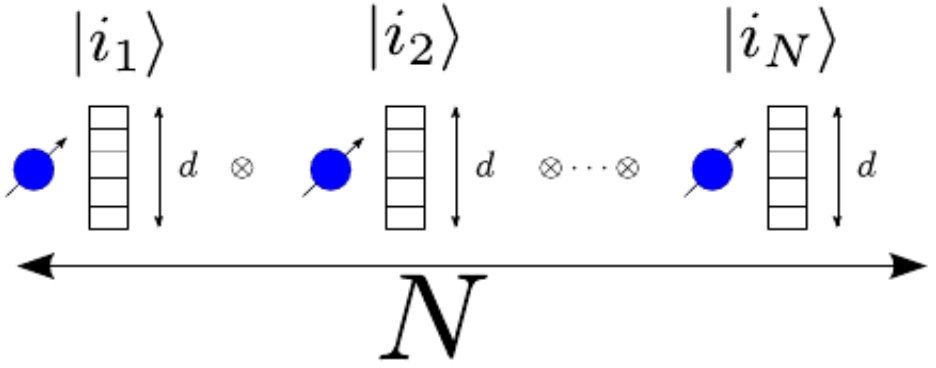
# A state of a many body systems

$$|\psi\rangle = \sum_{i_1 \cdots i_N} c^{i_1 \cdots i_N} |i_1 \cdots i_N\rangle$$

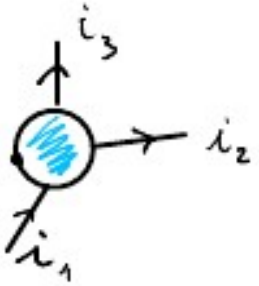
$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \cdots \otimes \mathcal{H}_N$$



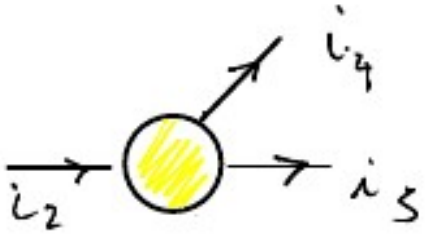

$c^{i_1 \cdots i_N}$ contains d^N parameters

- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- ...
- $2^{40} = 1.0995116e+12$

Tensors:

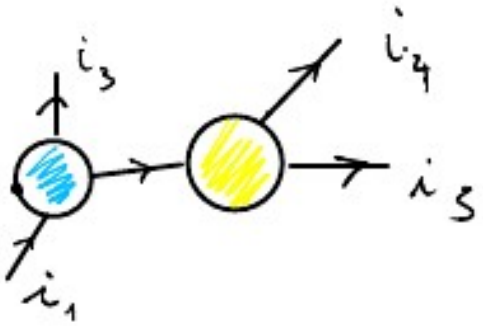



$$T_{i_1}^{i_2\, i_3}$$




$$S_{i_2}^{i_5\, i_4}$$

Operations among tensors:

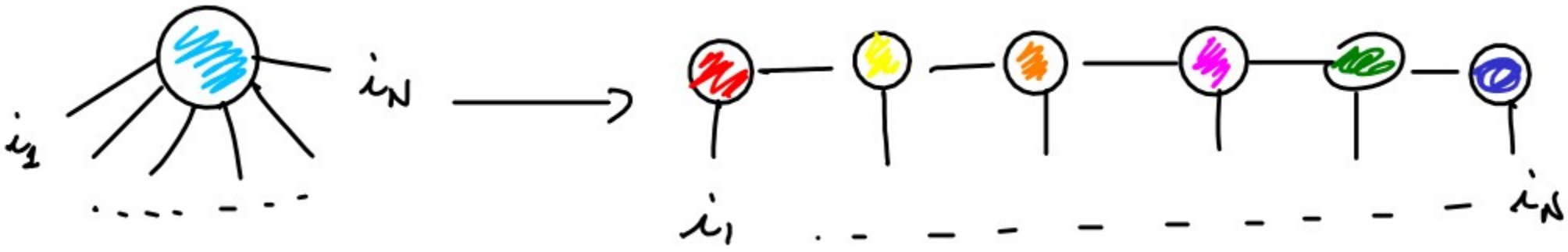



$$(TS)_{i_1}^{i_3\, i_4\, i_5} = \sum_{i_2} T_{i_1}^{i_3\, i_2}\, S_{i_2}^{i_4\, i_5}$$

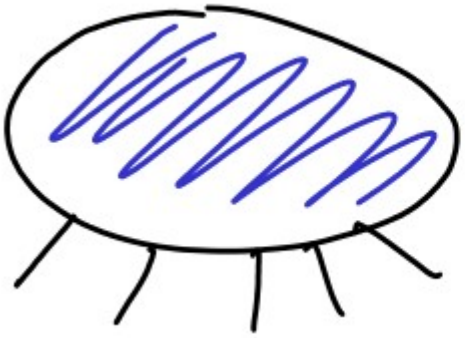
Express exponentially large tensors

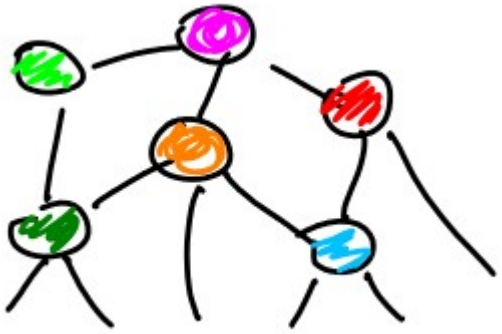
$$C^{i_1 \cdots i_N}$$

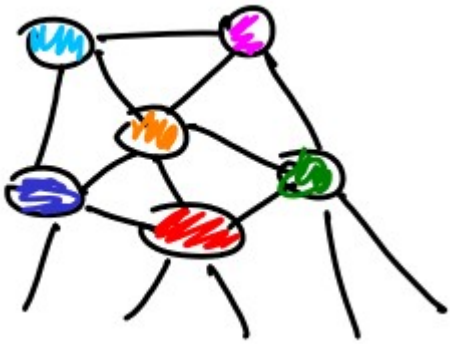
as contraction of small elementary tensors, TENSOR NETWORK



$$|\psi\rangle_A \otimes |\psi\rangle_B$$

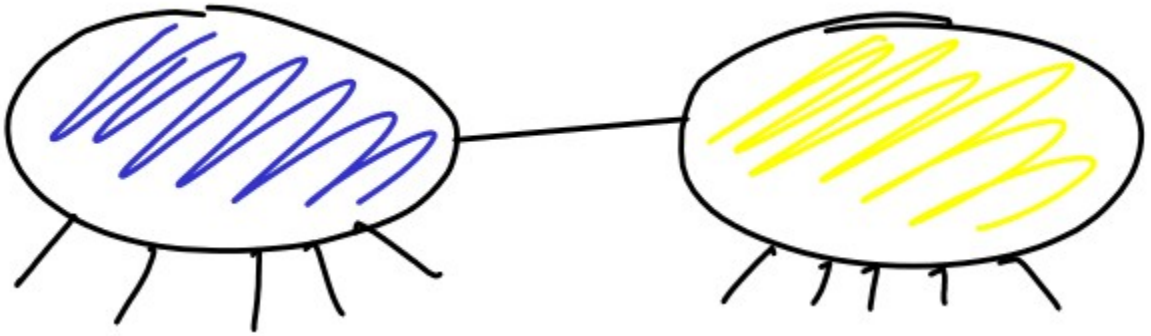


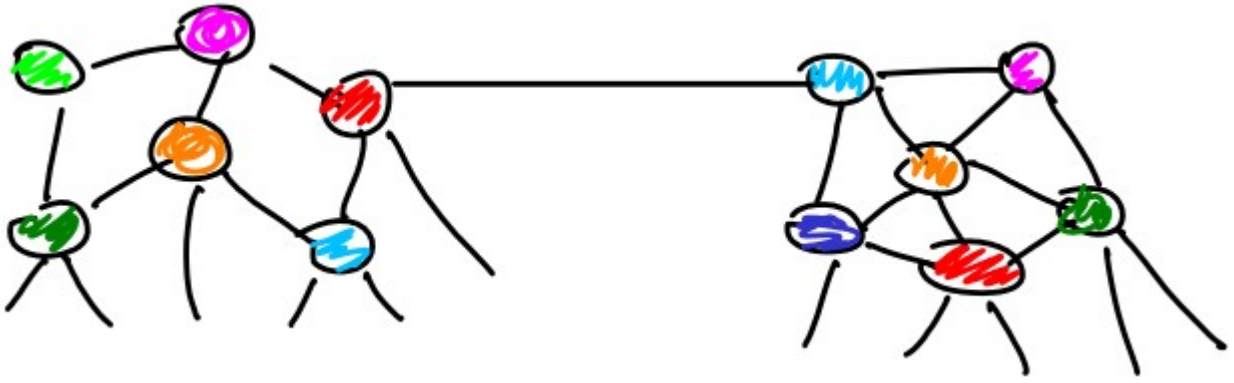




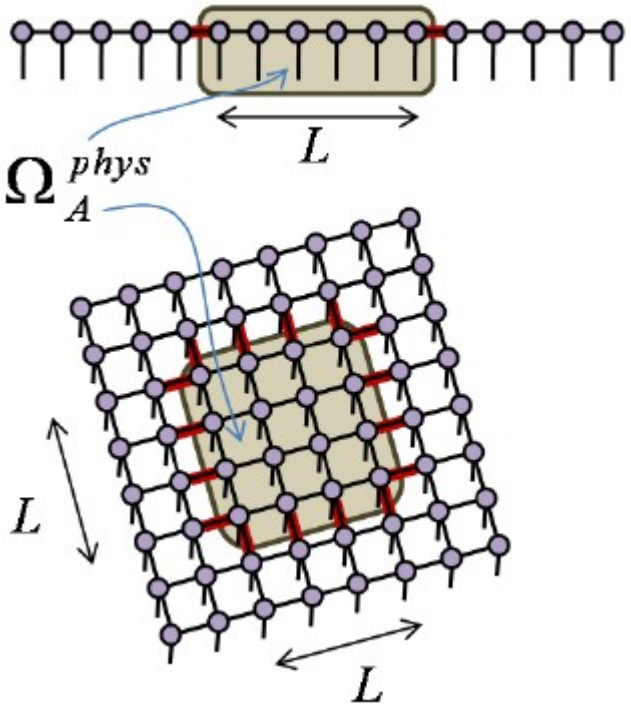
$$|\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

$$|\psi\rangle_{AB} = \sum_i c^i |\psi_i\rangle_A \otimes |\psi_i\rangle_B$$

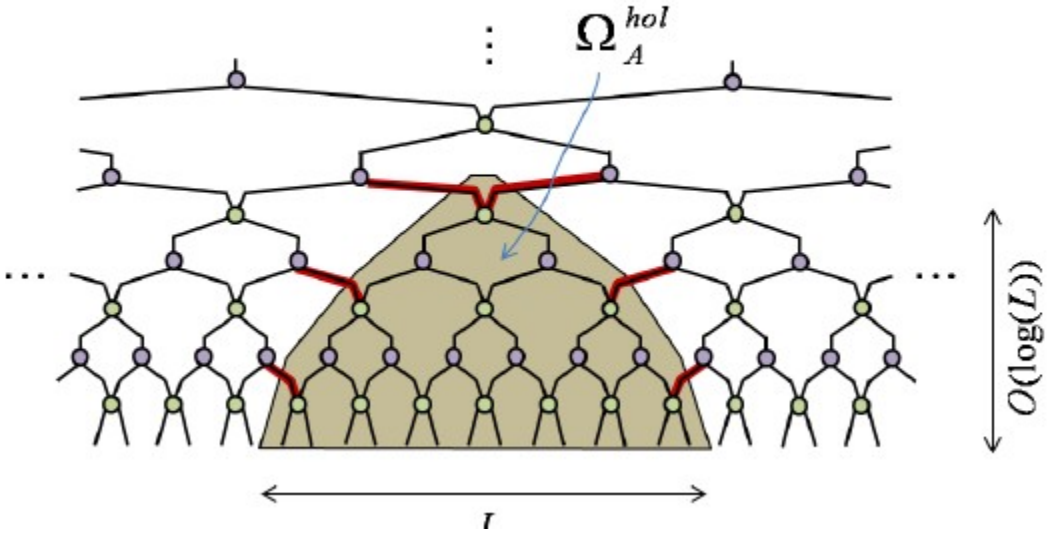




$$S \leq n_{AB} \log(\chi)$$




Entanglement area law states




Entanglement beyond area law states (Evenbly Vidal 2011)

States and maybe space-time are Tensor Networks

Hilbert space is a “convenient illusion”

Large many-body systems become

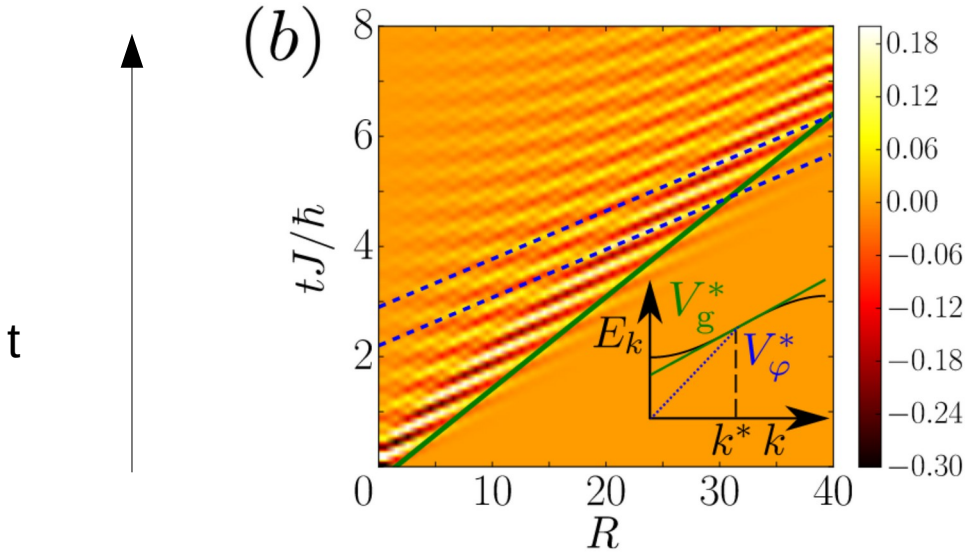
NUMERICALLY TRACTABLE

LT Optimal simulation of quantum dynamics
Nat Phys 2022

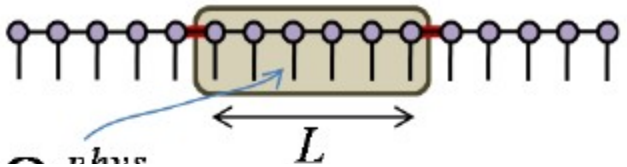
**ground state** $|\psi_0\rangle$ of $H_0$

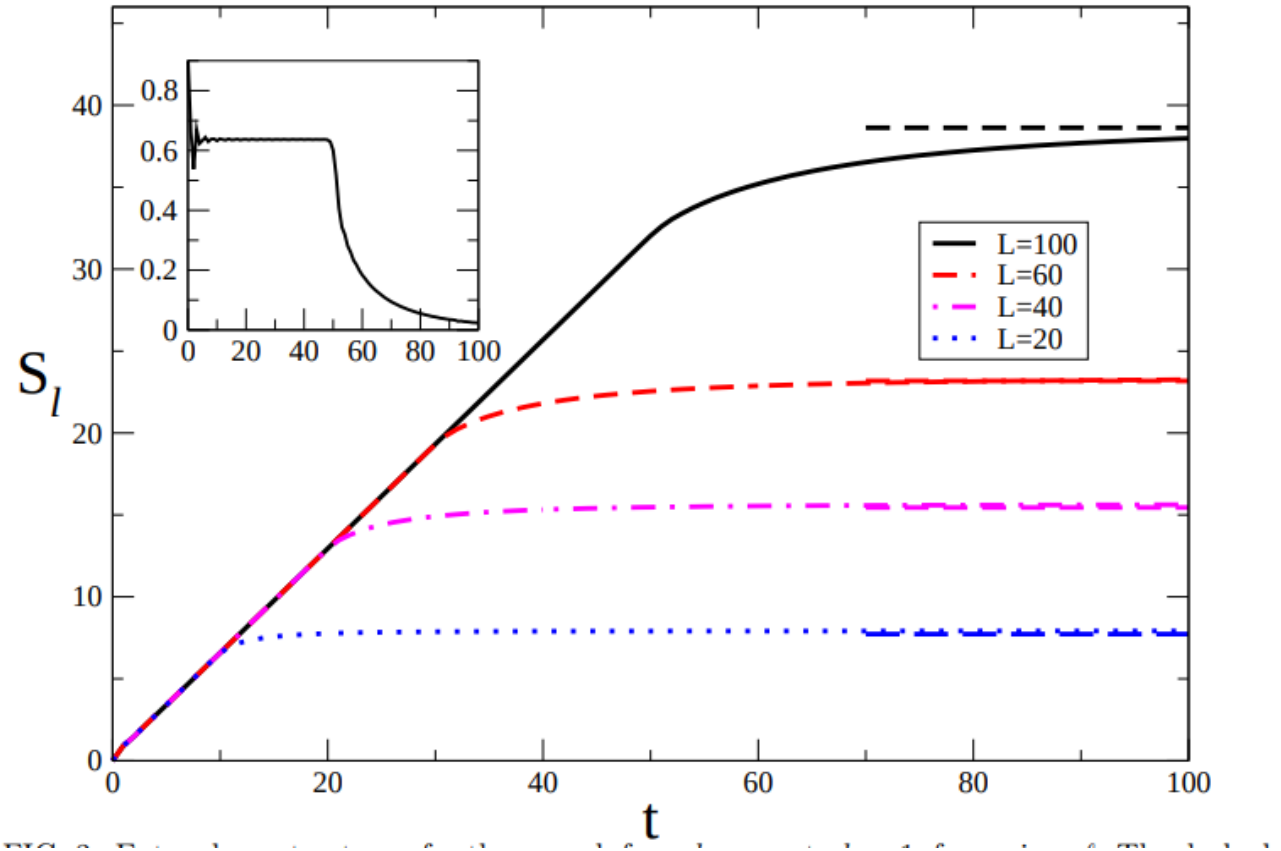
out of equilibrium by **quenching** the Hamiltonian to $H_1$

$$|\psi(t)\rangle = \exp\left[-iH_1 t\right]|\psi_0\rangle$$




Cevolani et al (LT) PRB 2017




from Calabrese Cardy 2005

$$S \leq n_{AB} \log(\chi)$$

$$\chi^{n_{AB}} \geq \exp S \propto \exp(t)$$

- Out of equilibrium
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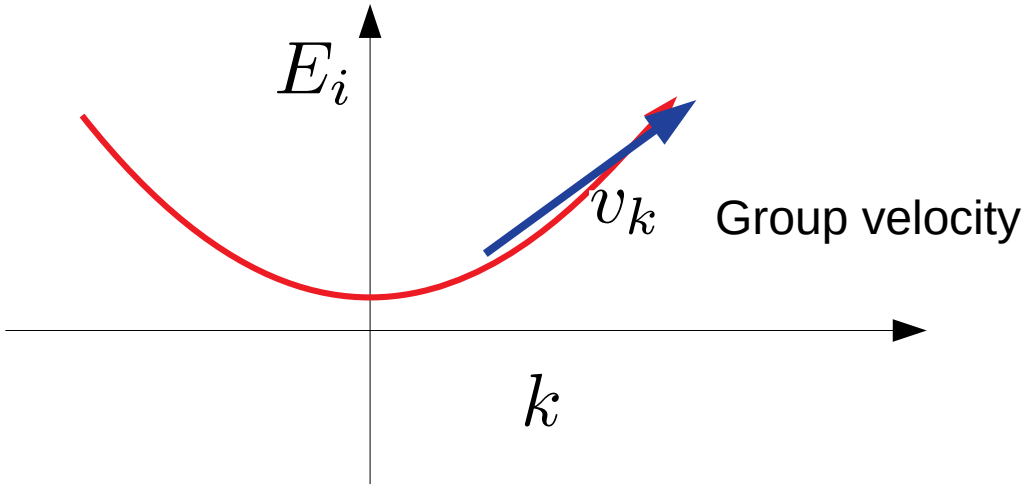
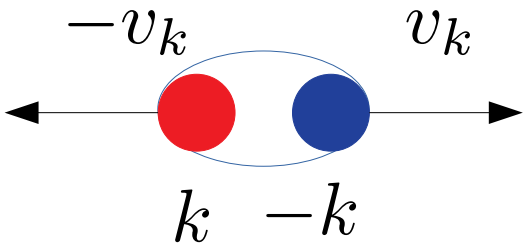
- The Hamiltonian is translational invariant,
- Momentum is a good quantum number

$$H = \oplus_k H_k$$

- We have bands, **quasi-particles** $E_i(k)$

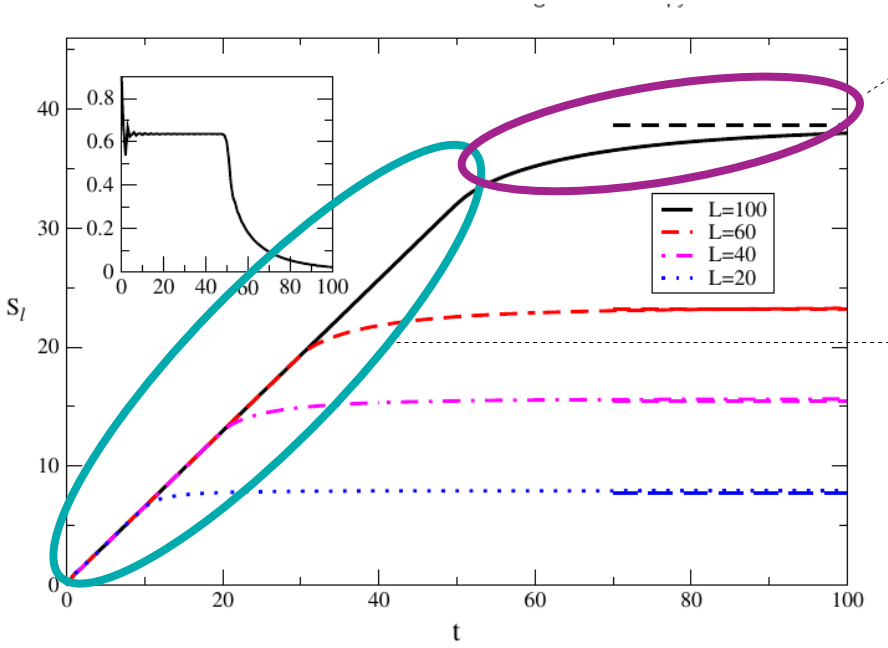


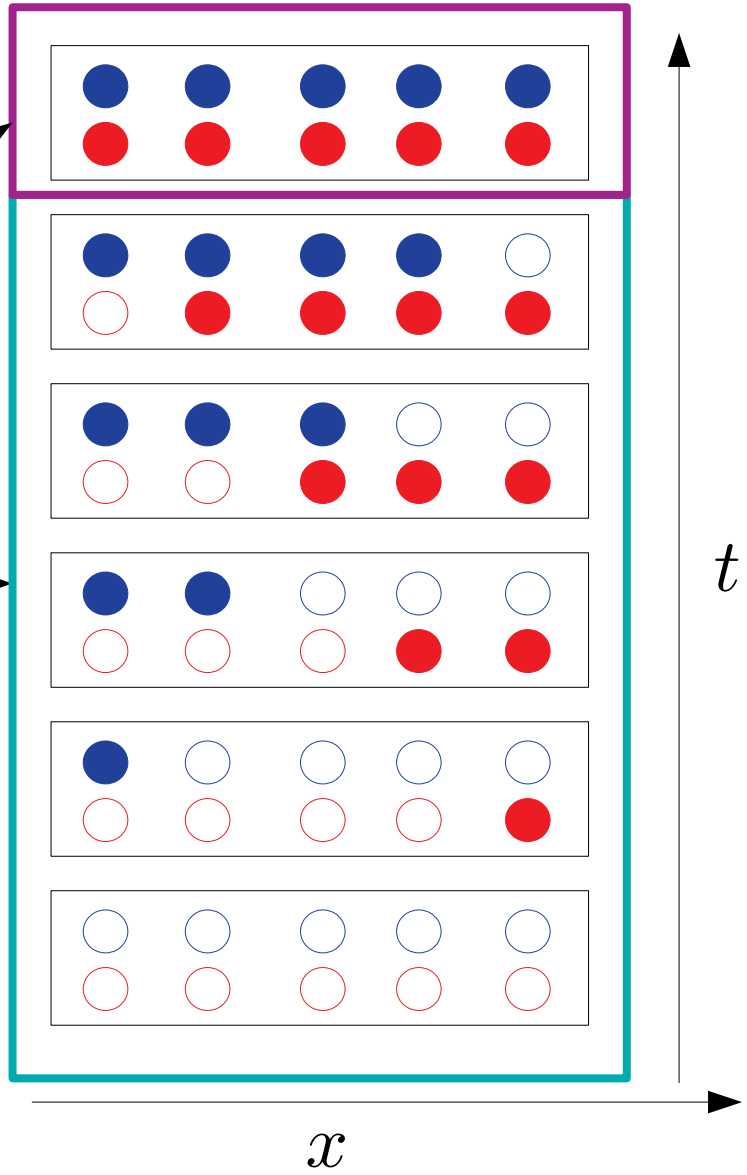
The **extra energy** in the quench is radiated by entangled **quasi-particle** with opposite momenta $k$ and velocities $v_k$



# Entanglement after a quench

Calabrese Cardy 05



- Improve TN ansatz
- Fight entanglement with universality and scaling
- Trade entanglement for mixture (decoherence)

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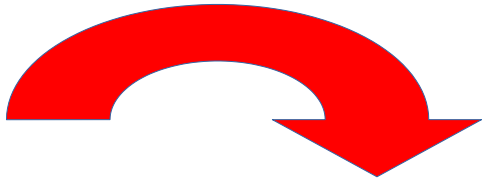
# Trading entanglement for mixture, decoherence

- Surace Piani Tagliacozzo PRB2019

# The quench protocol

$$H(\theta) = -\sin(\theta) \sum_{i=0}^{N-1} \sigma_i^x \sigma_{i+1}^x - \cos(\theta) \sum_{i=0}^{N-1} \sigma_i^z$$



$\pi/4$

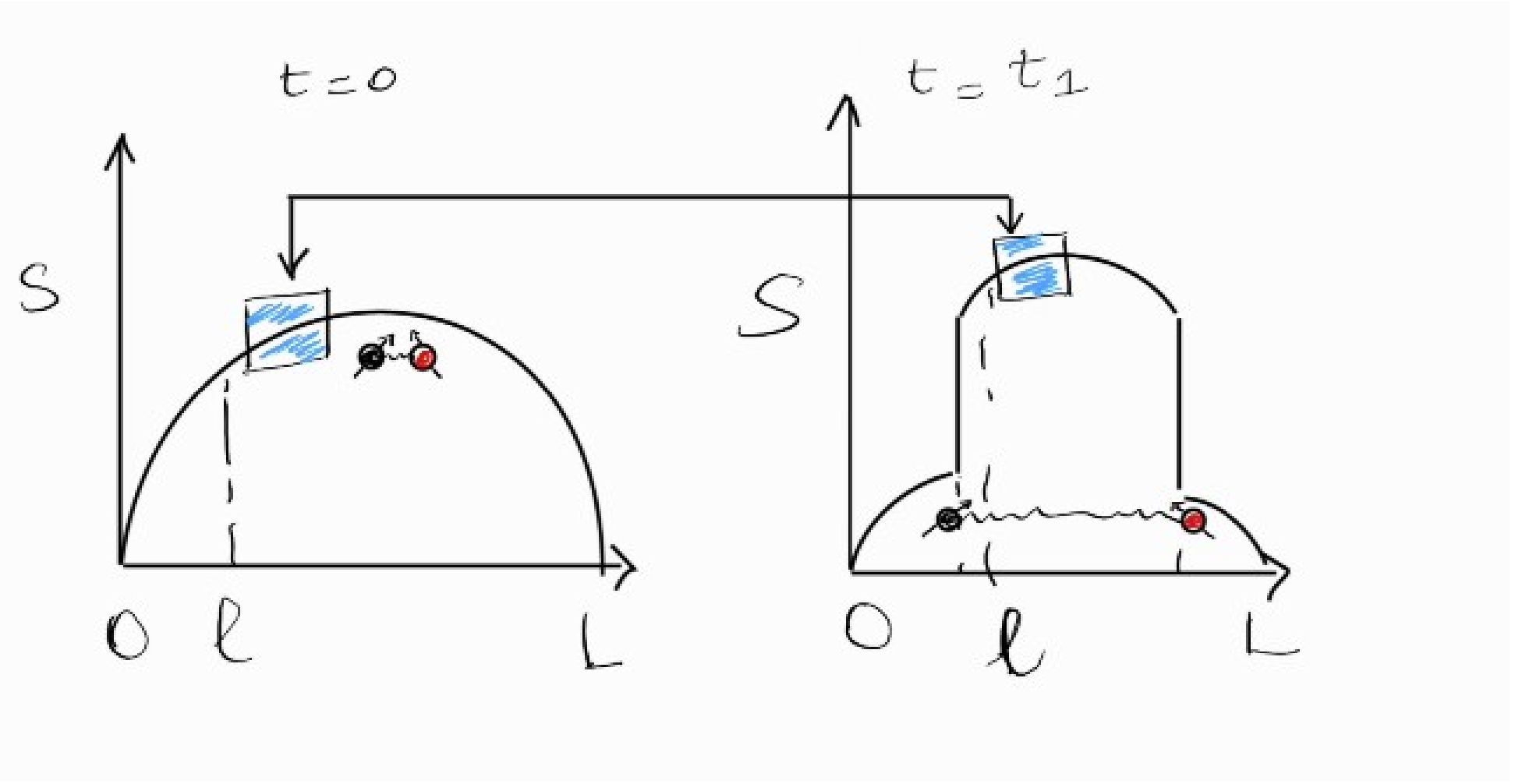
$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

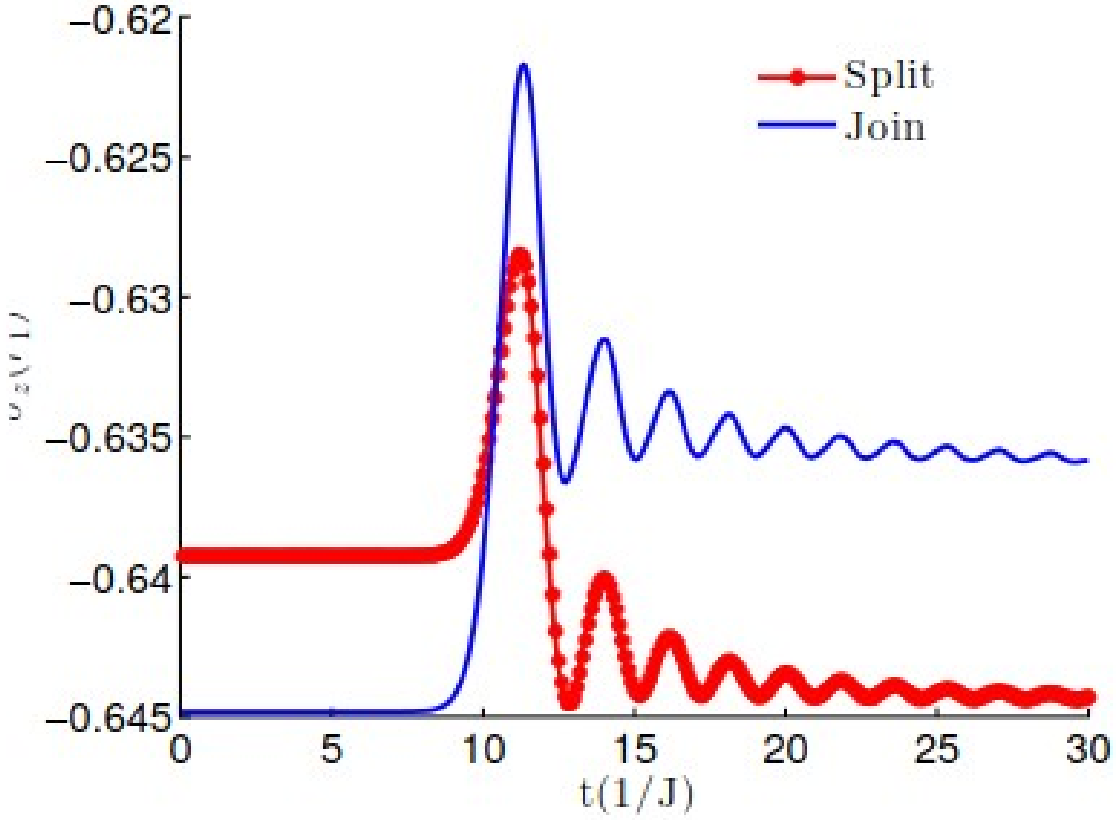
$[\mathcal{O}, \rho] \neq 0 \iff$ $\mathcal{O}$ has quantum fluctuations if the system is in the state $\rho$

$$\lambda_{dB} = \frac{h}{mv}$$

$$mv^2 \sim k_B T$$

$$\lambda_{dB} \sim \frac{h}{\sqrt{m k_B T}} \, .$$


t = 0
t = t1
S
S
O
l
L
O
l
L




Zamora (LT) JSTAT 2014

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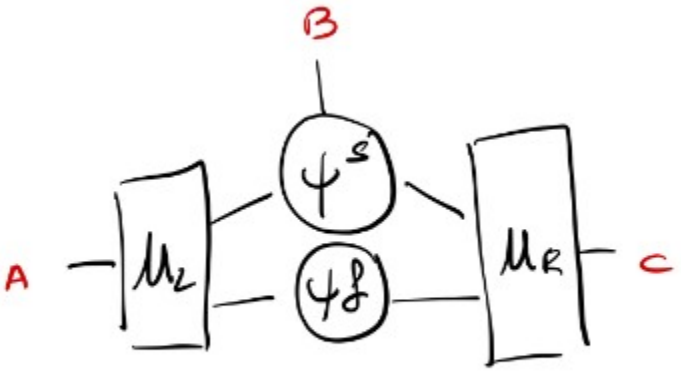
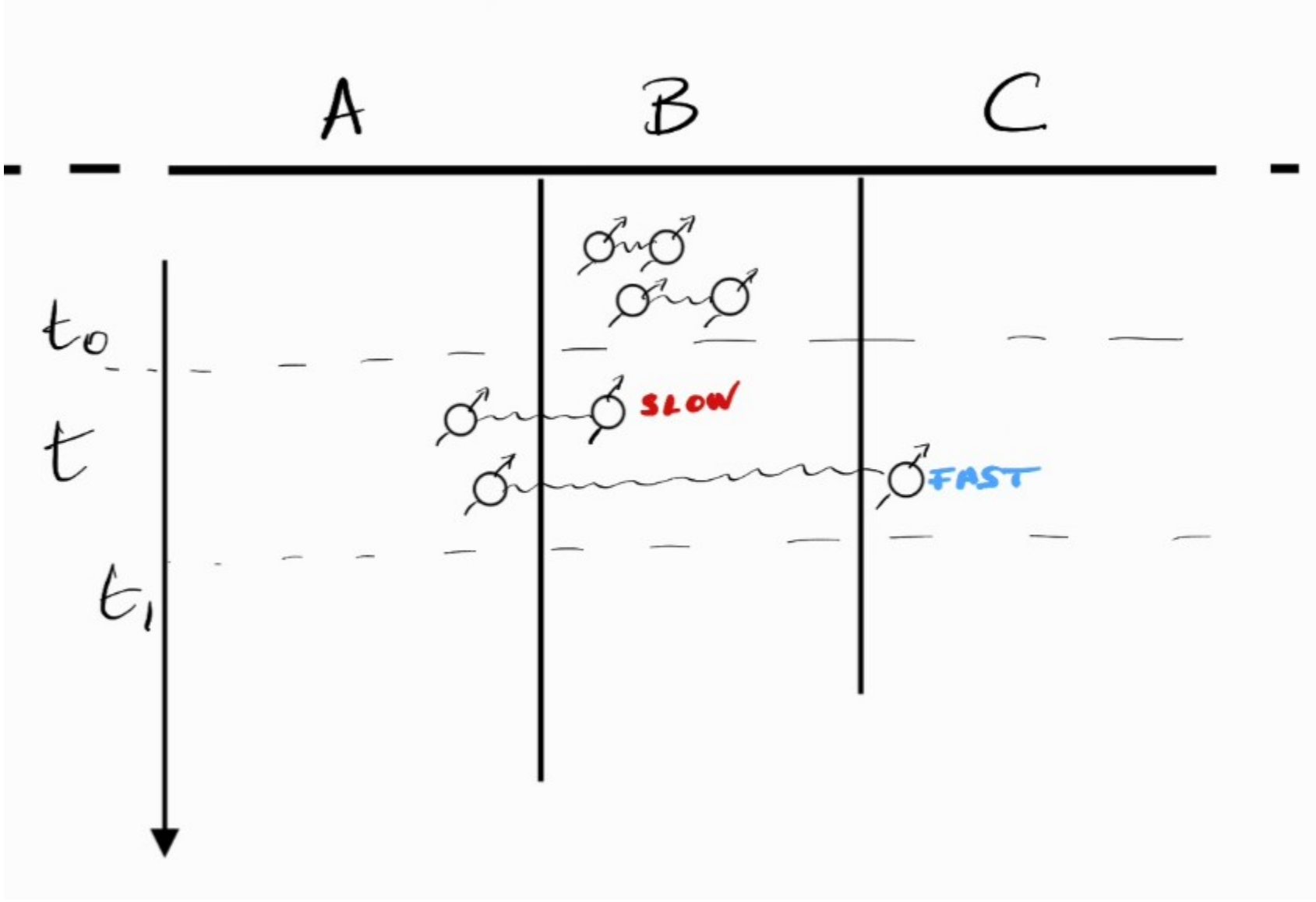
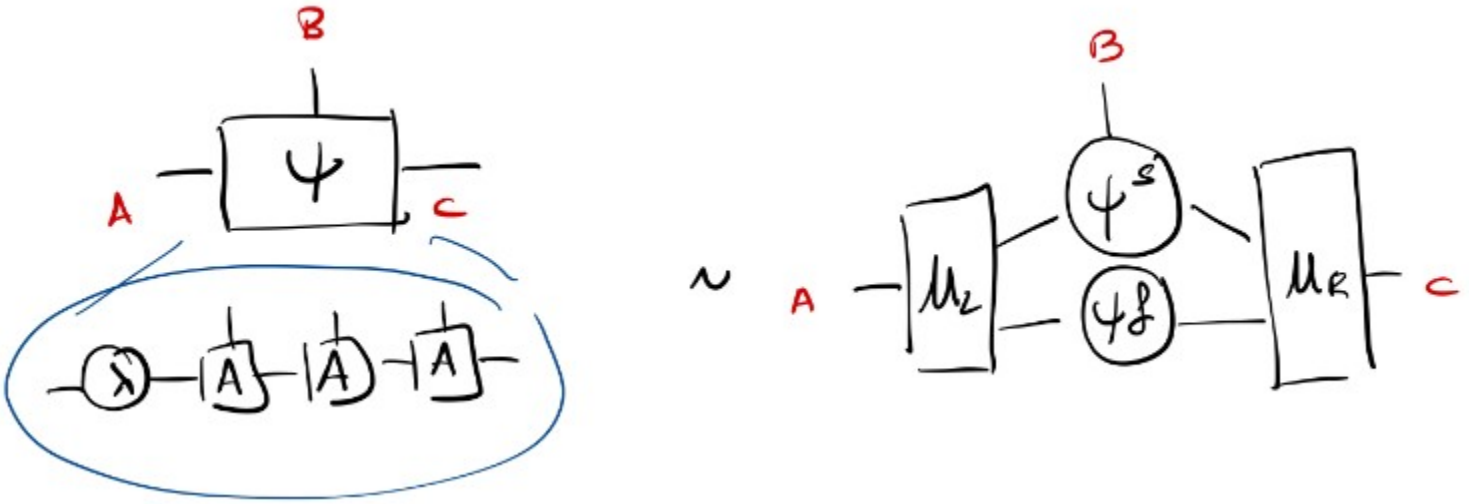



# Unveiling local equilibration in TN

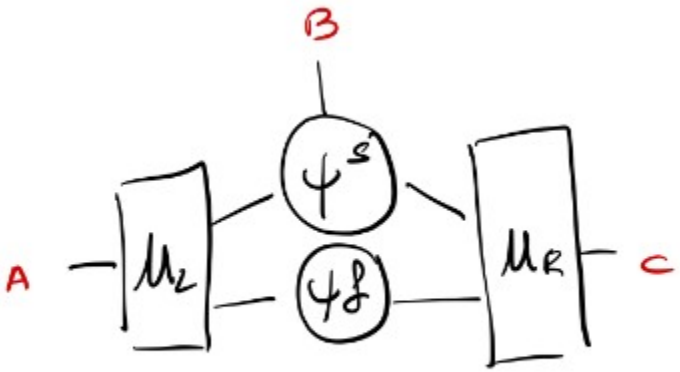
Work in progress with M Frias-Perez and MC Bañuls (MPQ), arXiv2306xxxx

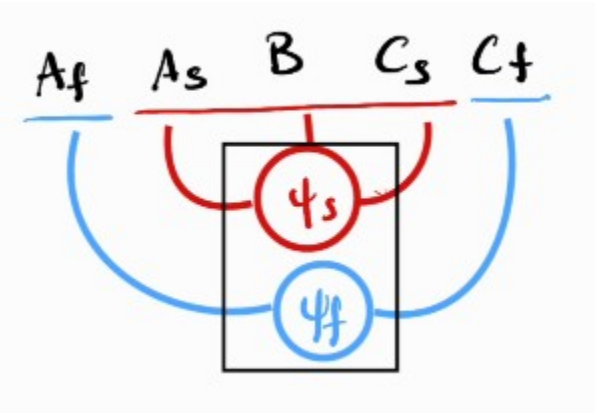
$$|\psi(t)\rangle \simeq |\psi_{A_s BC_s}\rangle \otimes |\psi_{A_f BC_f}\rangle$$

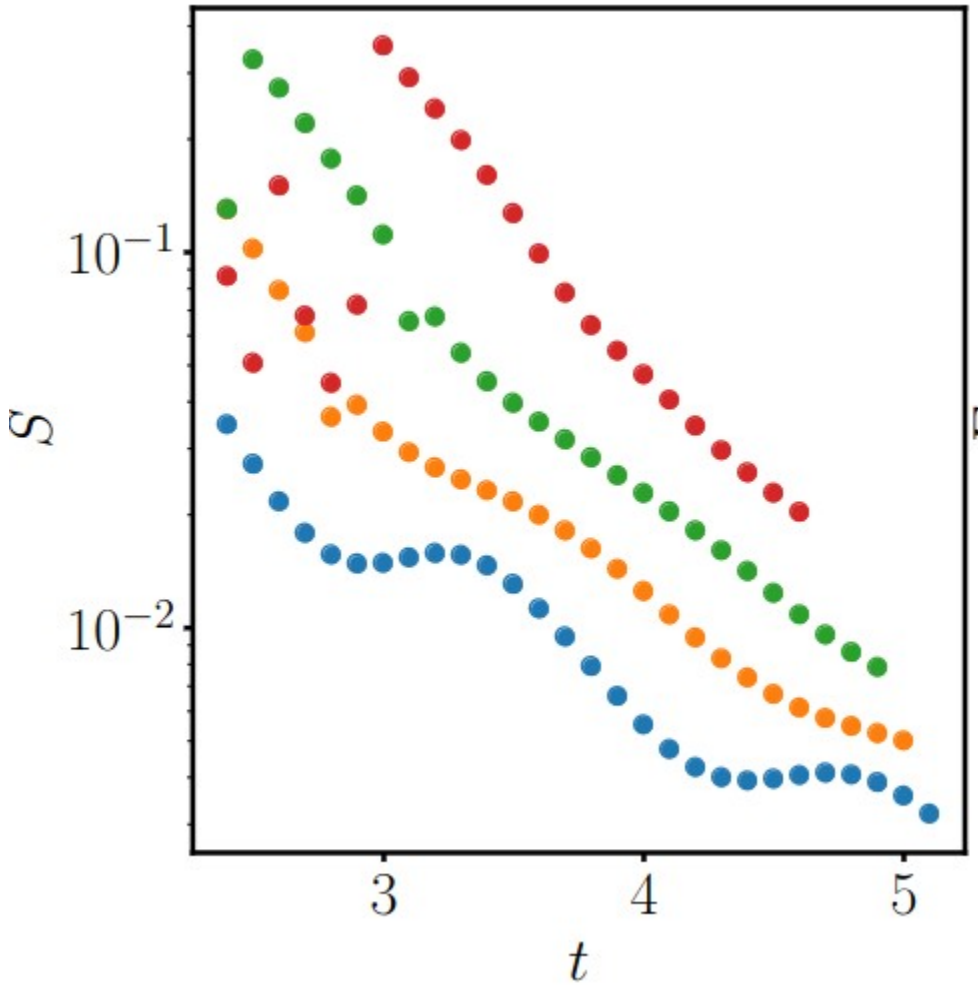


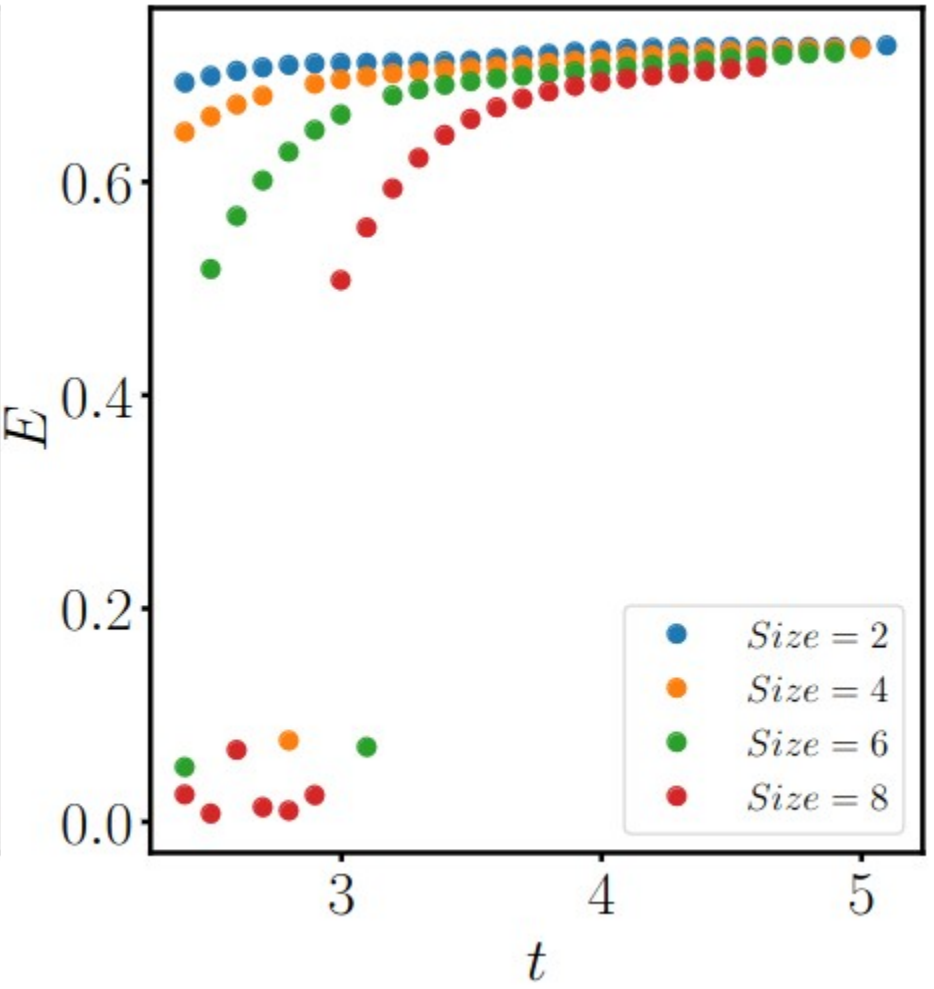



Frias-Perez (LT) Bañuls in prep


B
A
C


Af
As
B
Cs
Cf


$10^{-1}$
$10^{-2}$
S
3
4
5
t


0.6
0.4
0.2
0.0
E
3
4
5
t
Size = 2
Size = 4
Size = 6
Size = 8

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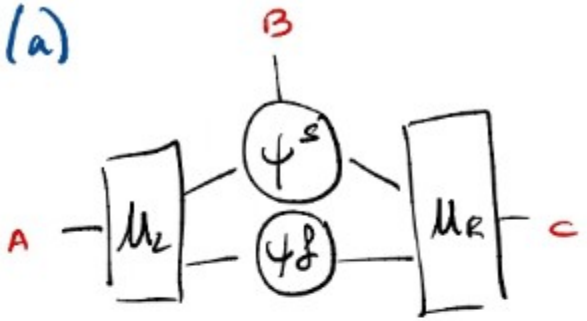
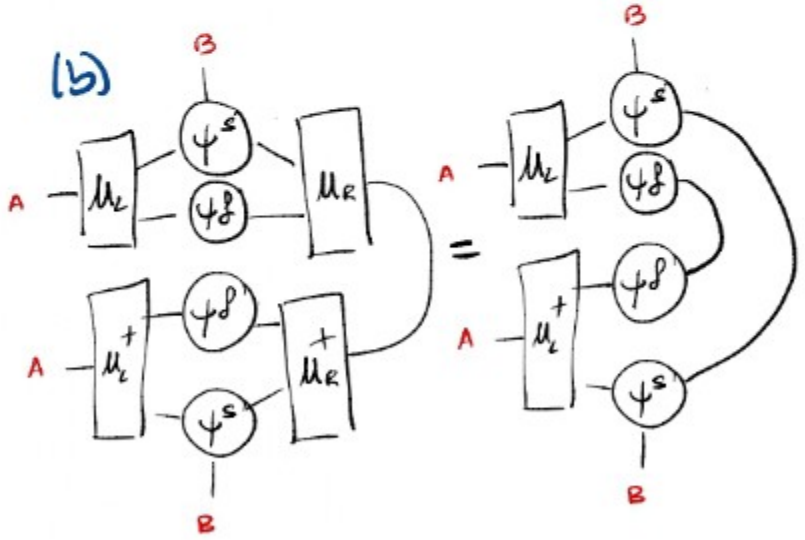
(a)

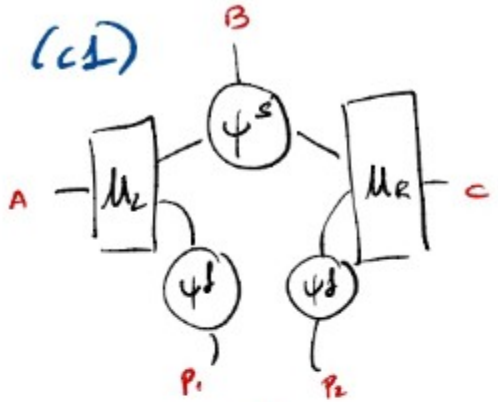



$|\psi_{ABC}\rangle$

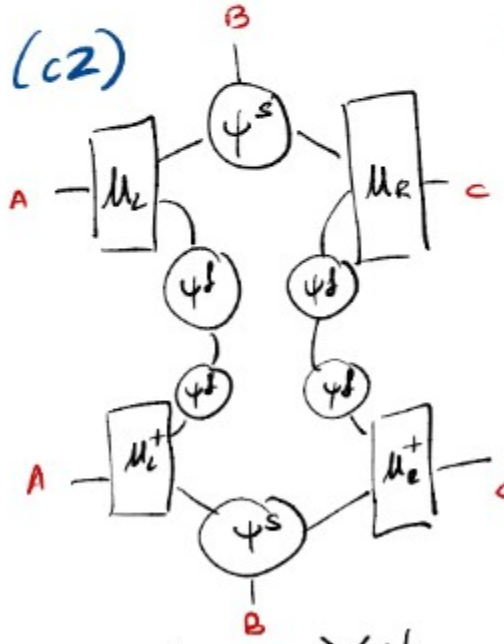
(b)



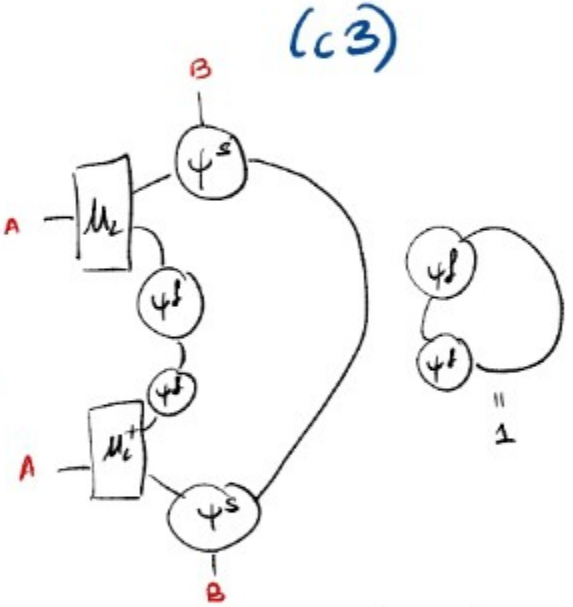

(c1)




(c2)



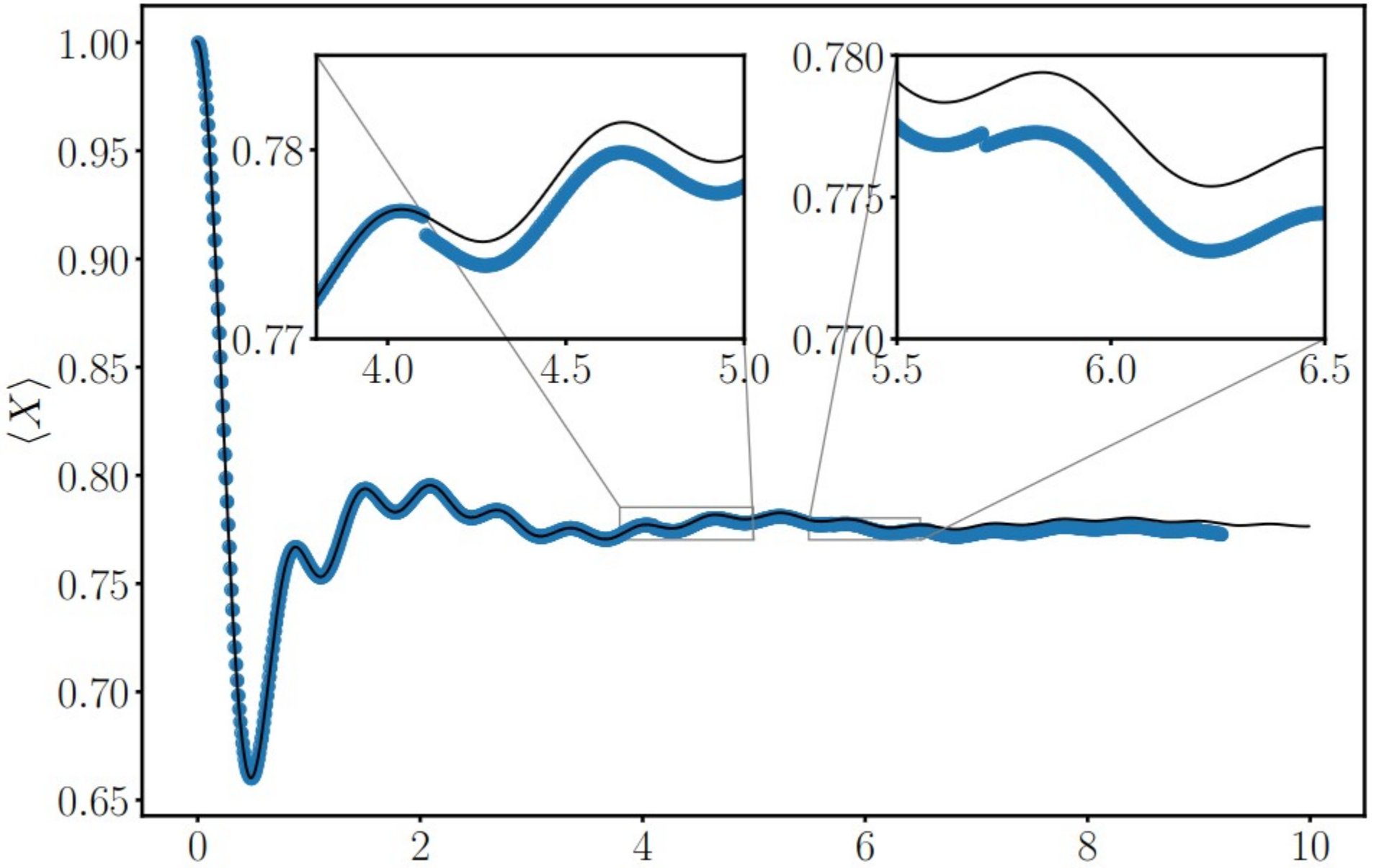

(c3)




$\rho_{AB} = Tr_C |\psi_{ABC}\rangle\langle\psi_{ABC}|$

$\rho_{ABC} = Tr_{P_1 P_2} |\psi_{ABC,P_1P_2}\rangle\langle\psi_{ABC,P_1P_2}|$

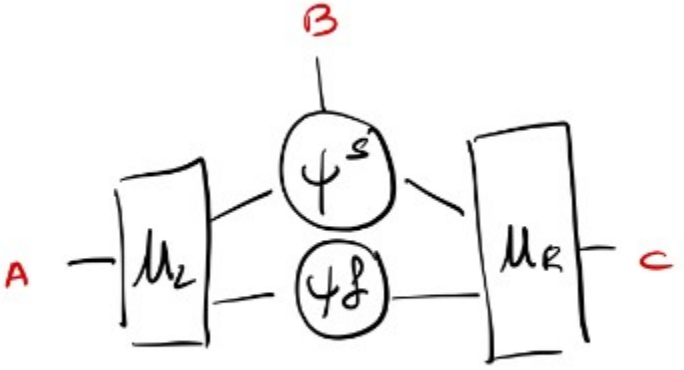
$\rho_{AB} = Tr_{C,P_1P_2} |\psi_{ABC,P_1P_2}\rangle\langle\psi_{ABC,P_1P_2}|$




Frias-Perez (LT) Bañuls in prep



# Different patterns of decoherences

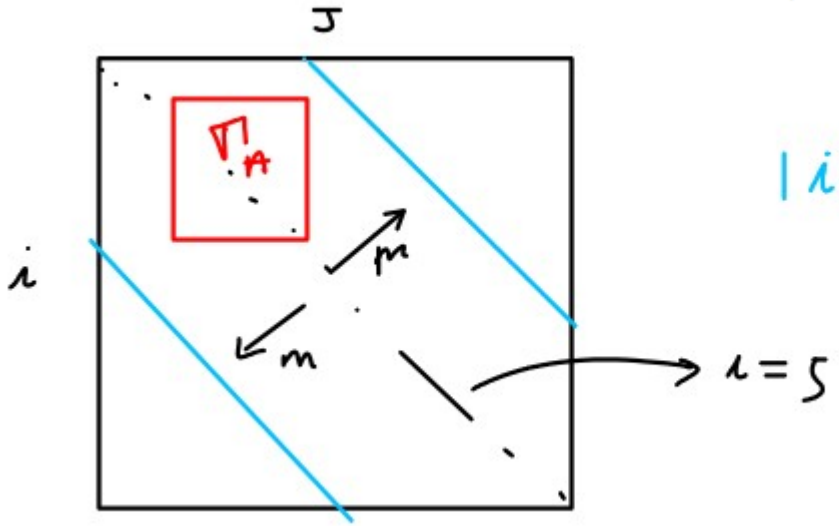



Frias-Perez (LT) Bañuls in prep

# Gaussian states, Surace (LT) 2019

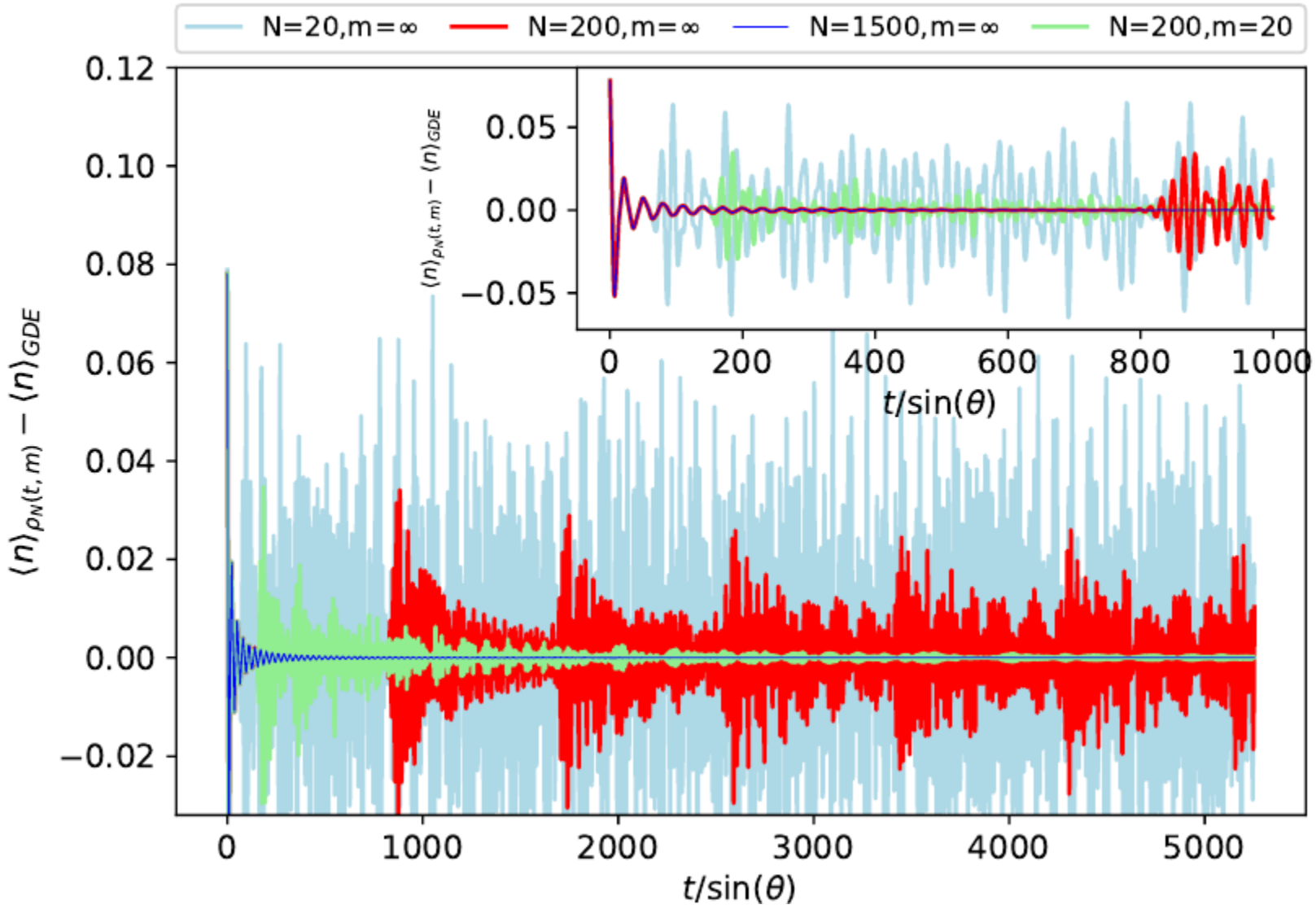
- The relevant correlations are inside a band of size m in the correlation matrix

$$\Gamma_{i,j} = \langle \vec{\alpha}_i \vec{\alpha}_j^{\dagger} \rangle,$$

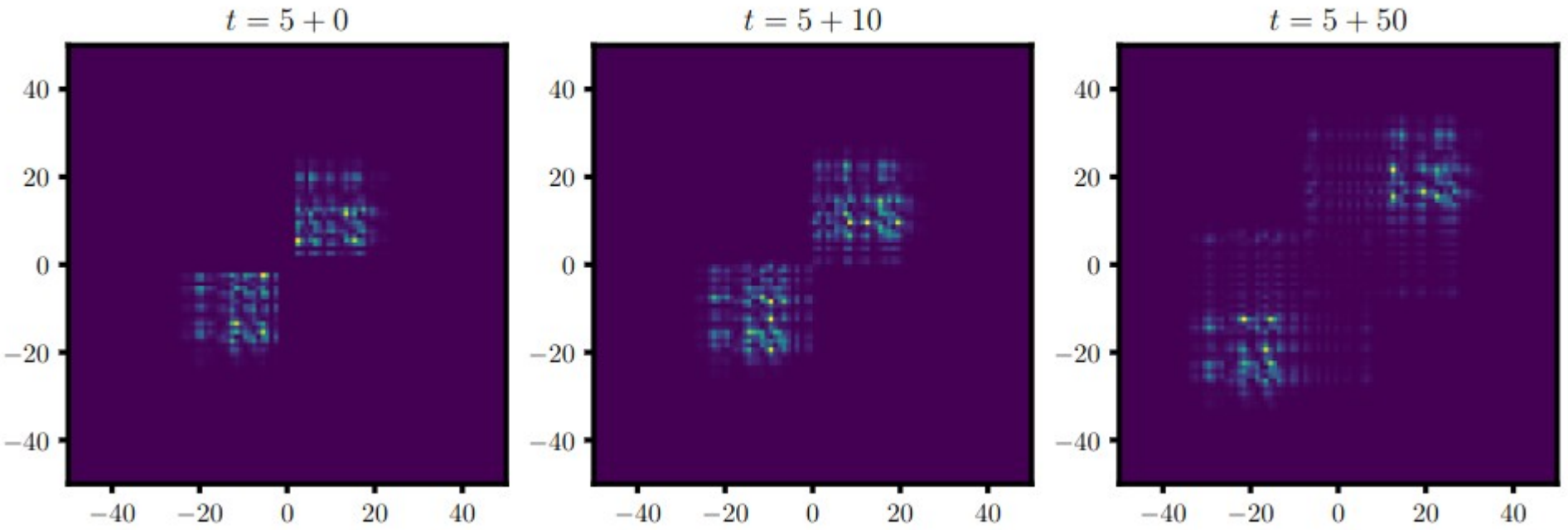



- At the truncation stage all correlation that are outside the band are zeroed

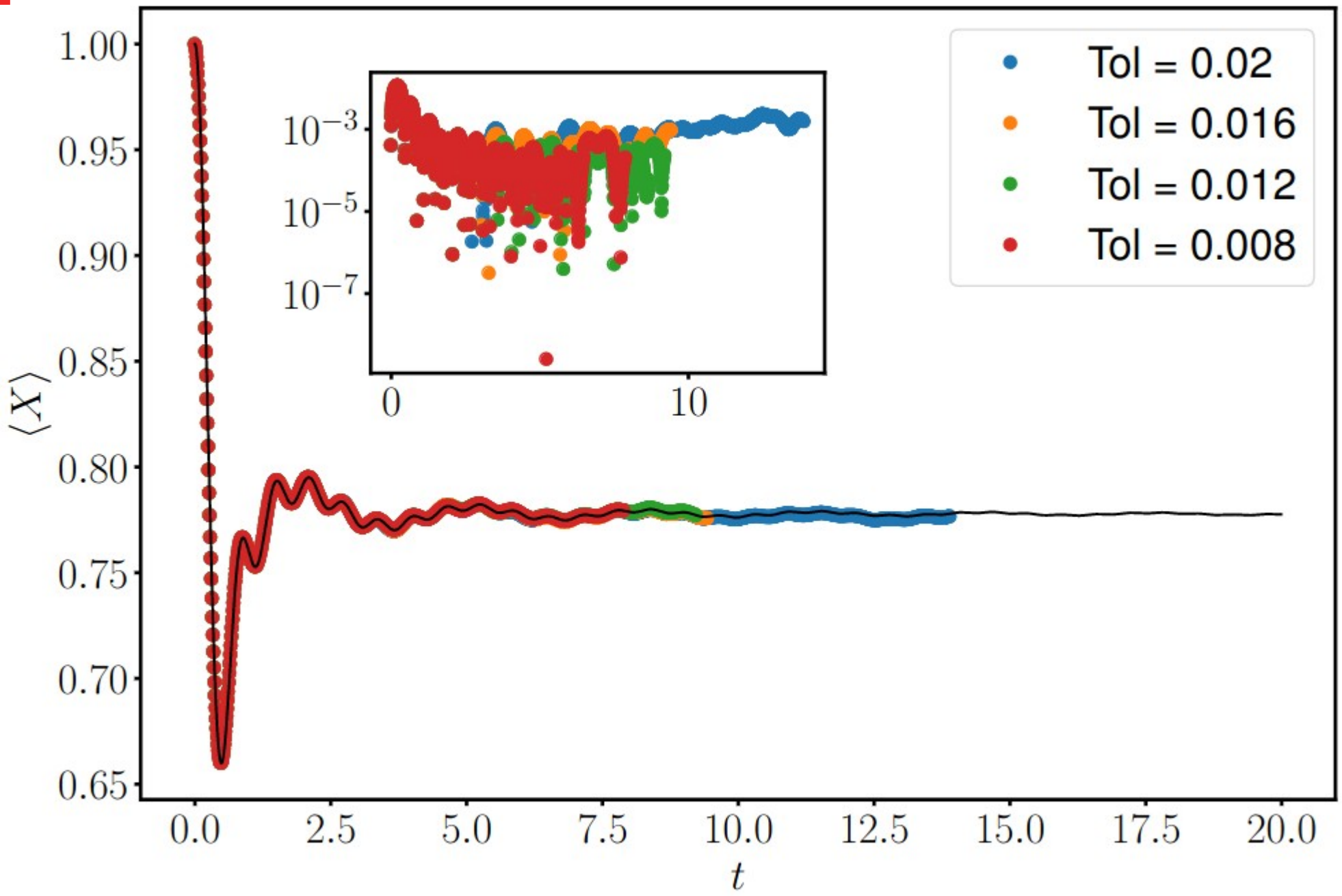
# Quenches in Ising, using only local correlations



# de-cohere only
# fast degrees of freedom




Frias-Perez (LT) Bañuls in prep




Frias-Perez (LT) Bañuls in prep

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- The entanglement barrier **can be circumvented**
- We need to **better understand the physics** of quenches to design better tensor network algorithms,
- **Decoherence** as the main ingredients allows to reach longer times

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