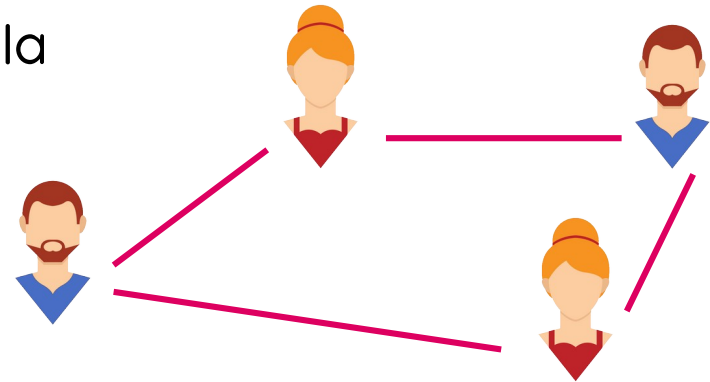


Topologically robust network nonlocality

arXiv:soon

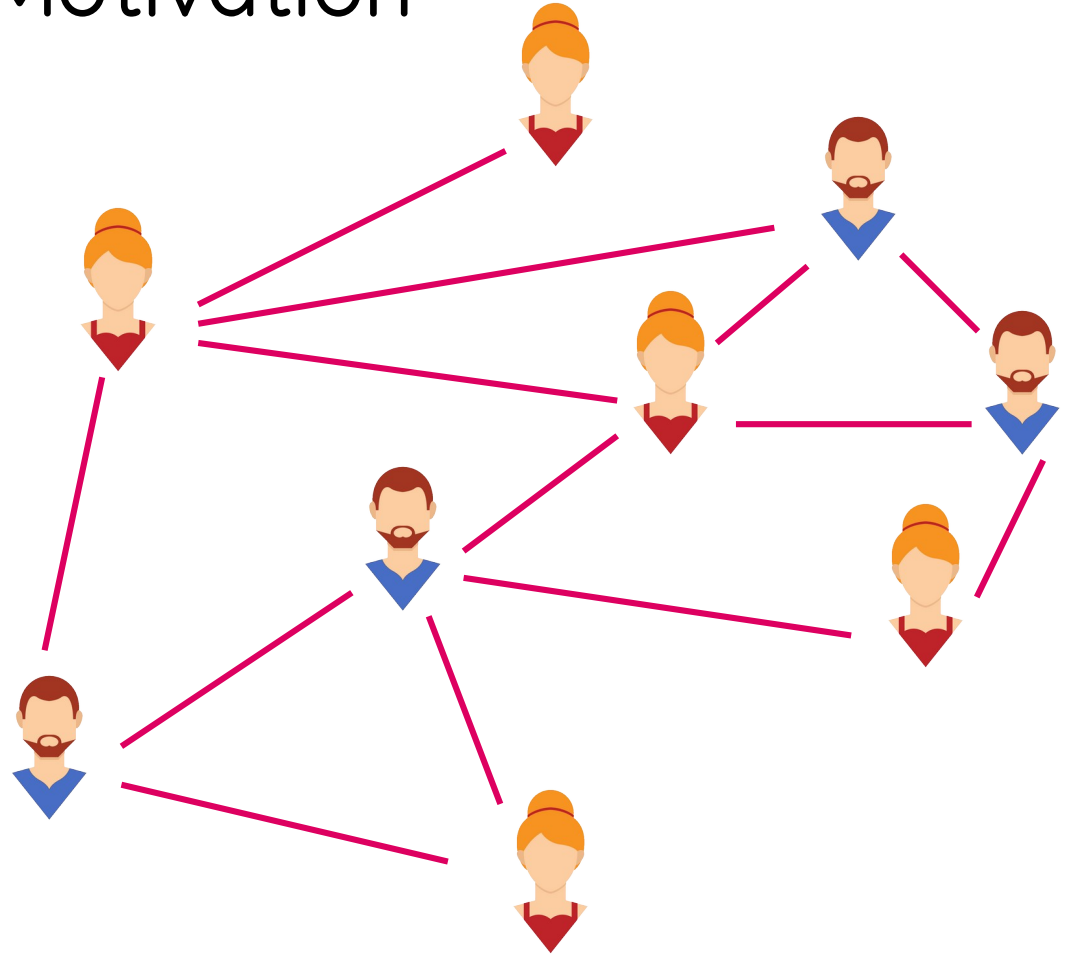
2023.05.29., Santiago de Compostela
Tamás Kriváchy

tamas.krivachy@gmail.com
 @KrivachyTamas



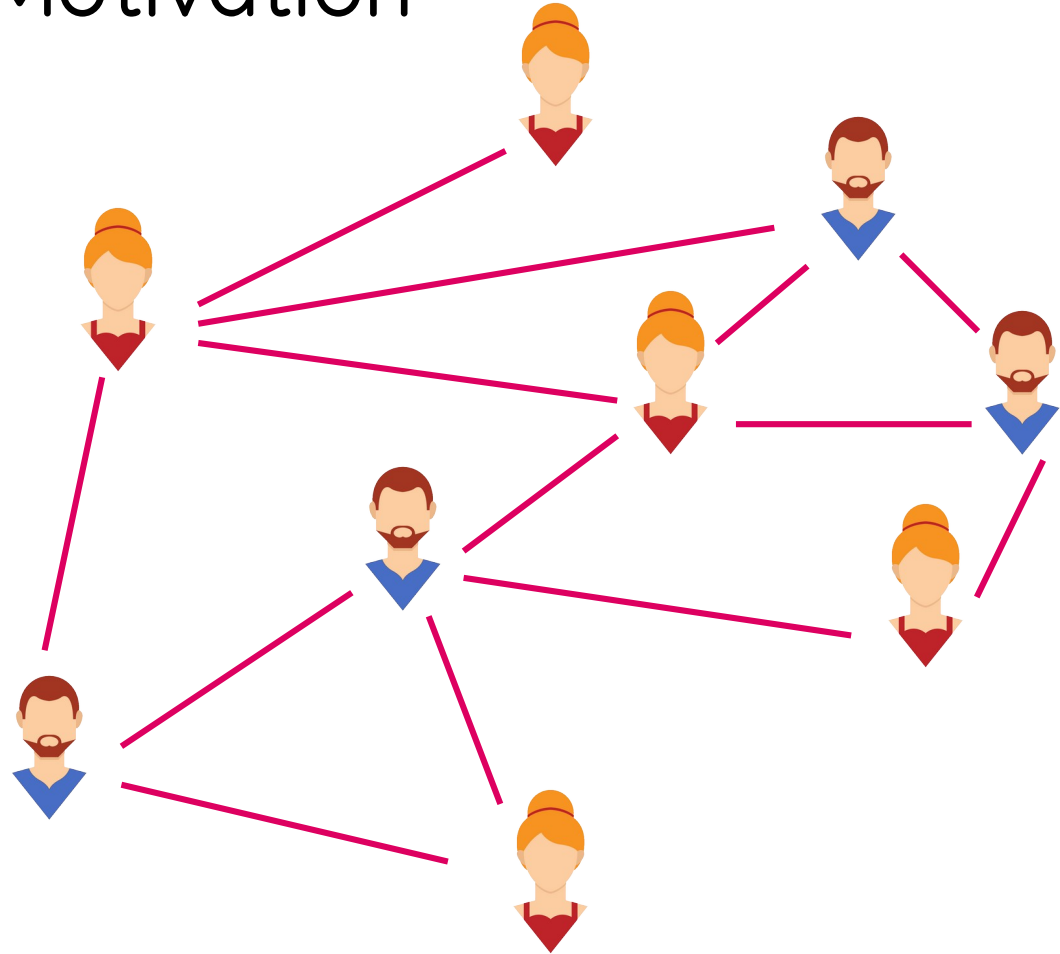
Motivation

- Decentralized networks



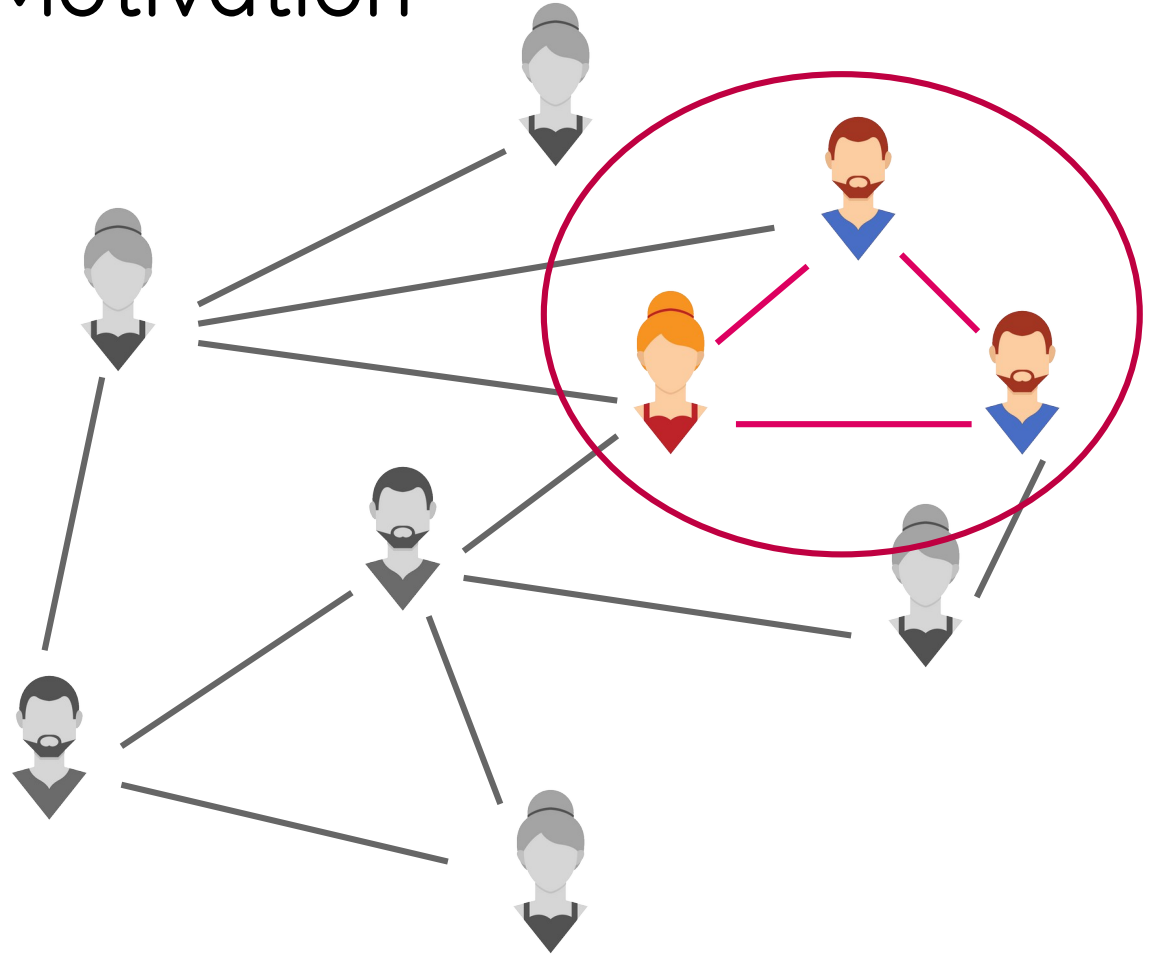
Motivation

- Decentralized networks
- Whole structure might
 - not be known
 - not be trusted



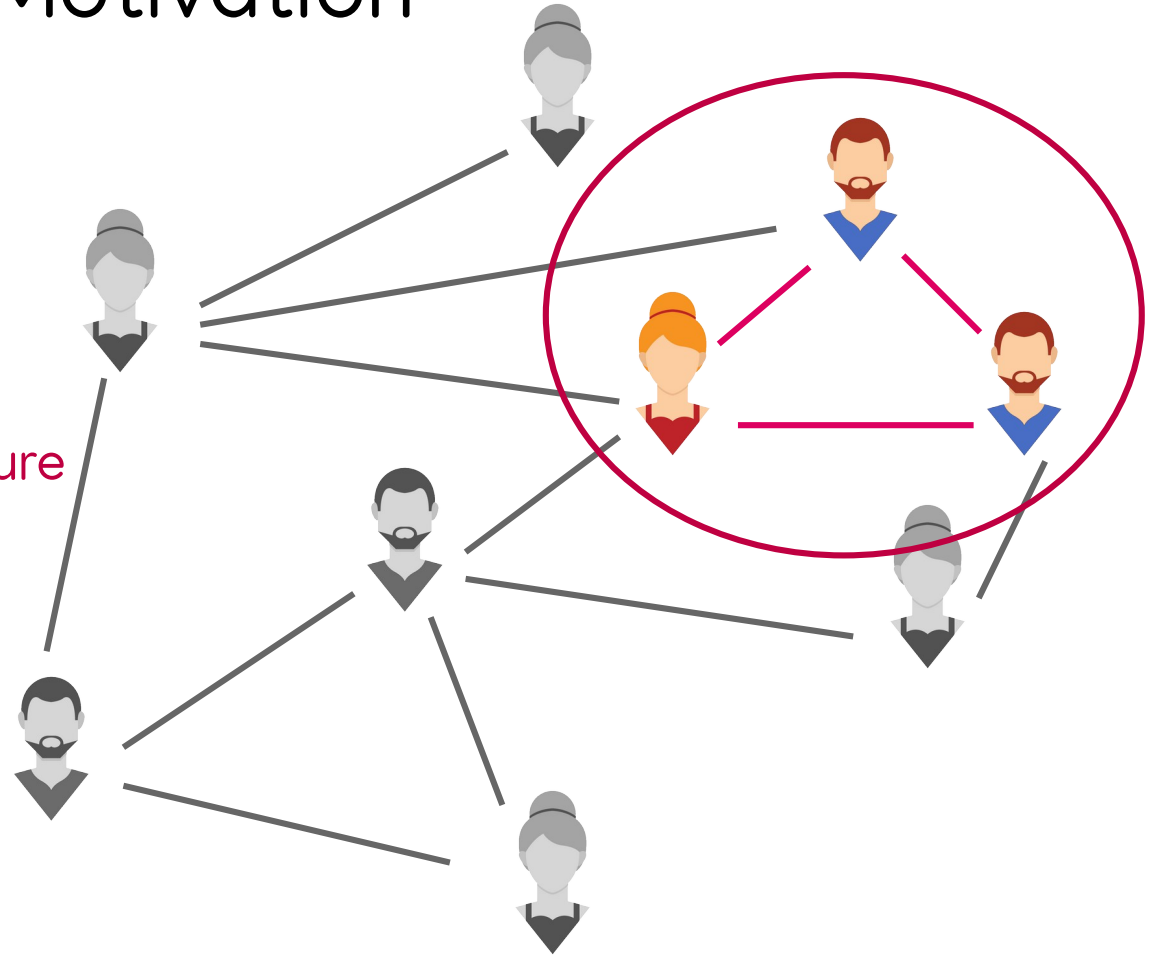
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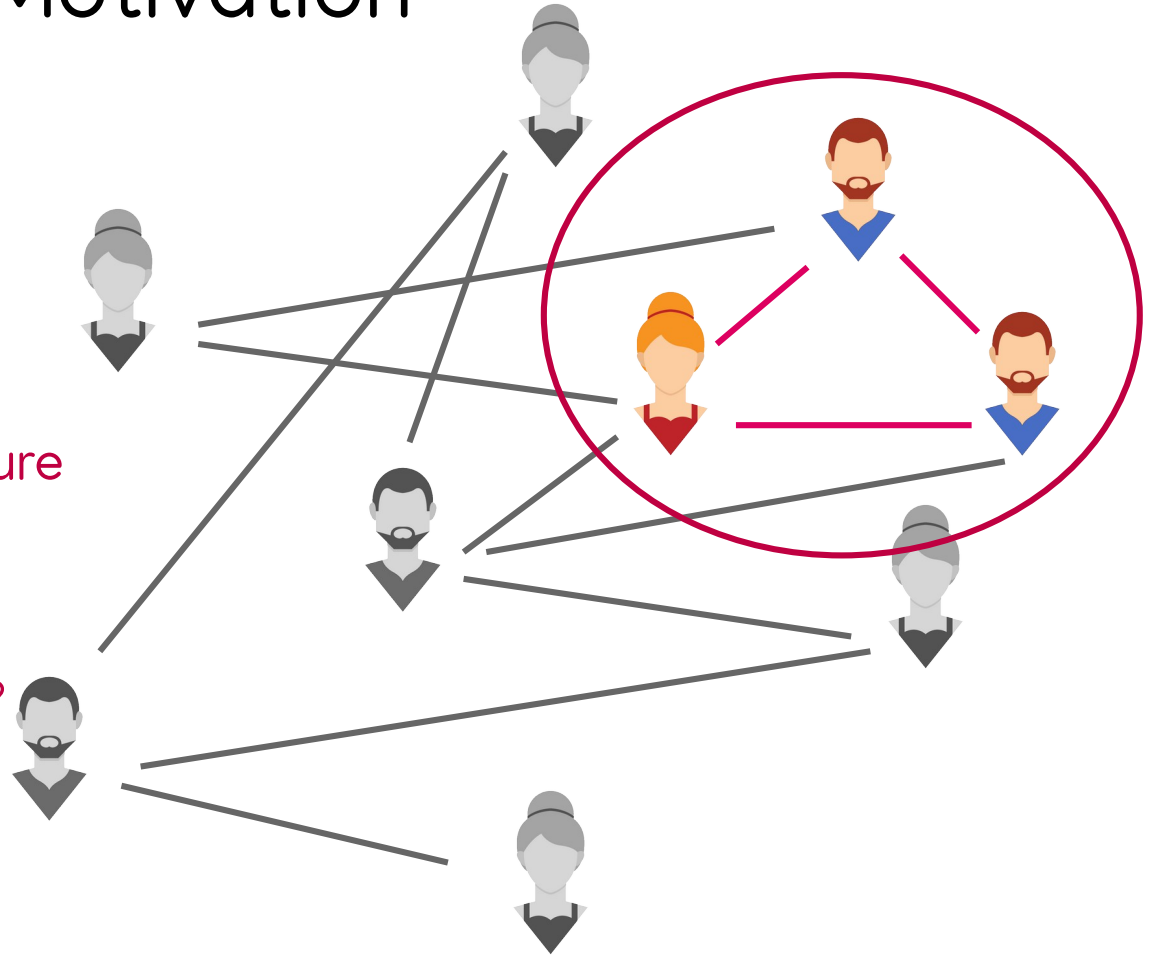
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- Previously: fixed network structure



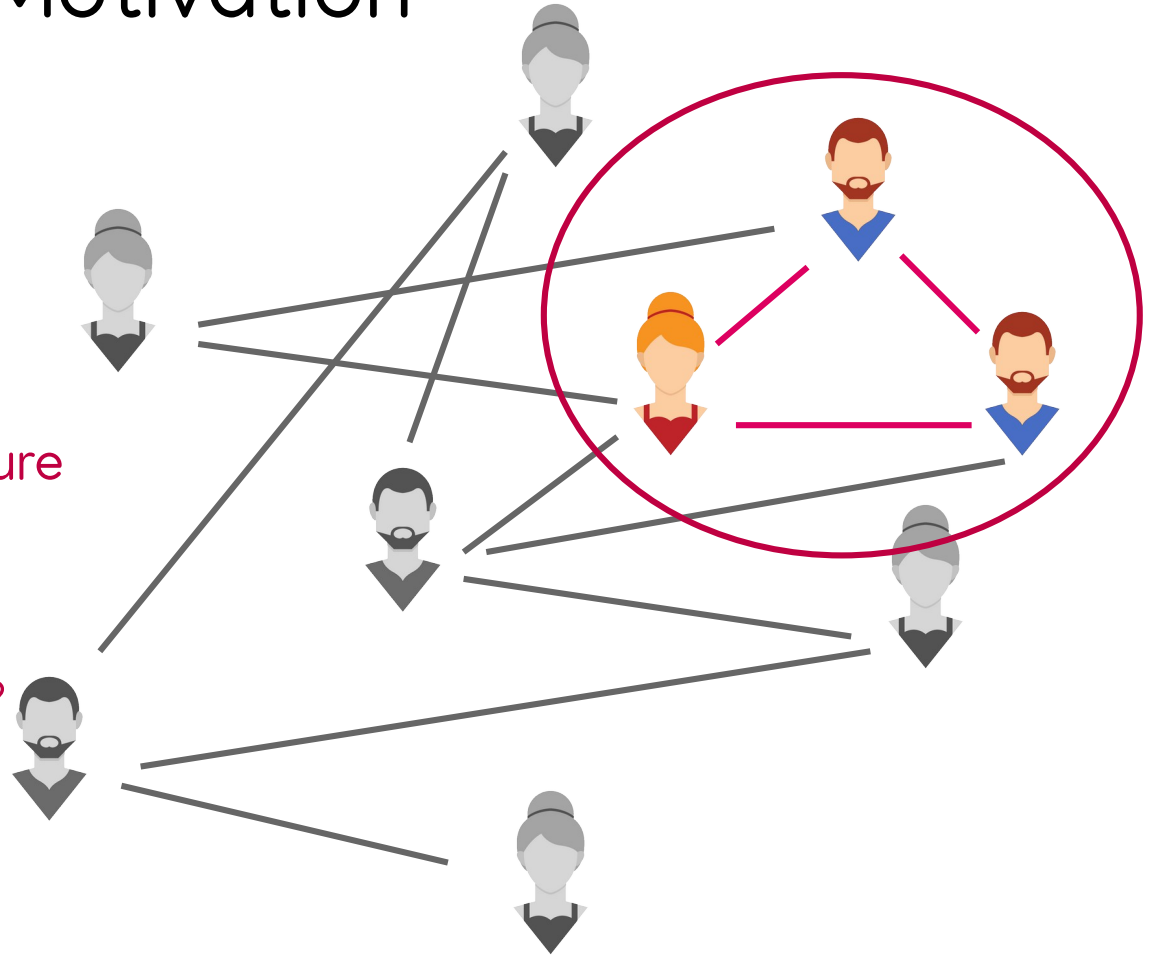
Motivation

- Decentralized networks
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 - not be **known**
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- Previously: **fixed** network **structure**
- This work: allow **variability** in network structure
 - Can we still **say something?**

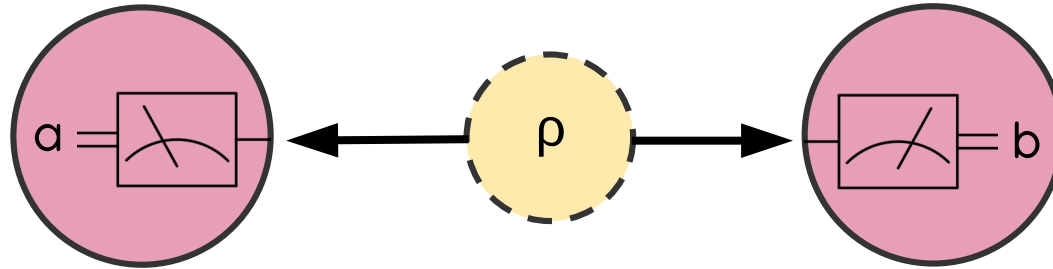


Motivation

- Decentralized networks
- Whole structure might
 - not be **known**
 - not be **trusted**
- Previously: **fixed** network **structure**
- This work: allow **variability** in network structure
 - Can we still **say something**?
- Interesting for **applications**?

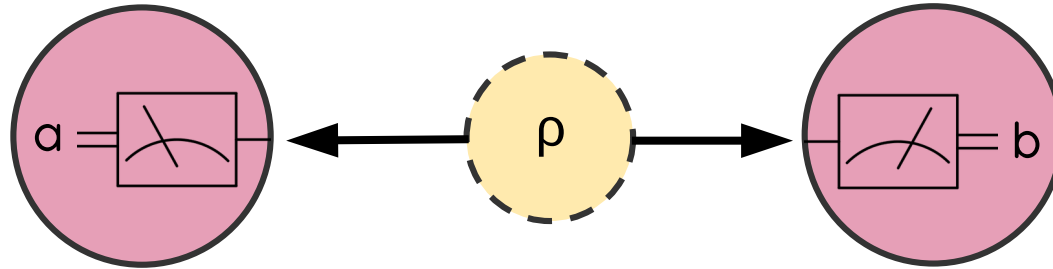


Quantum correlations

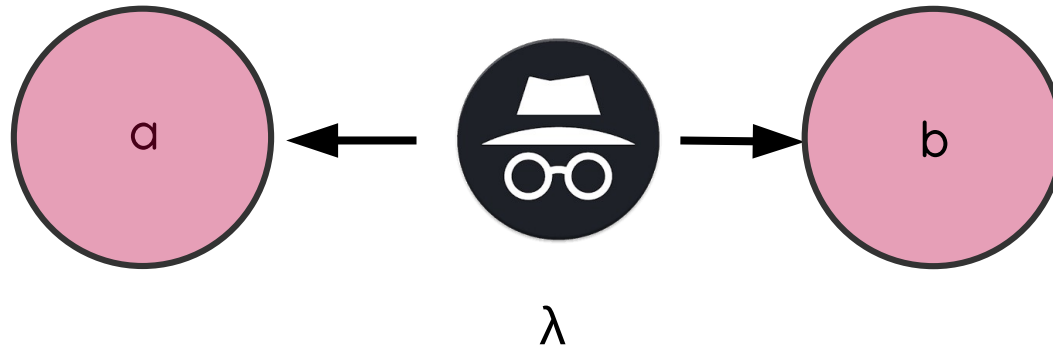


$p(ab)$ genuinely
quantum?

Quantum correlations

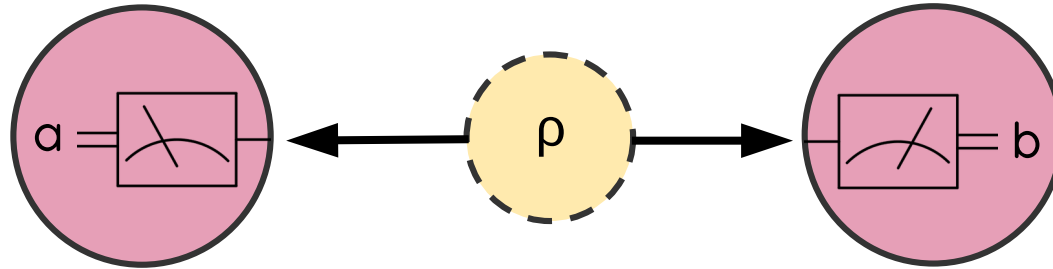


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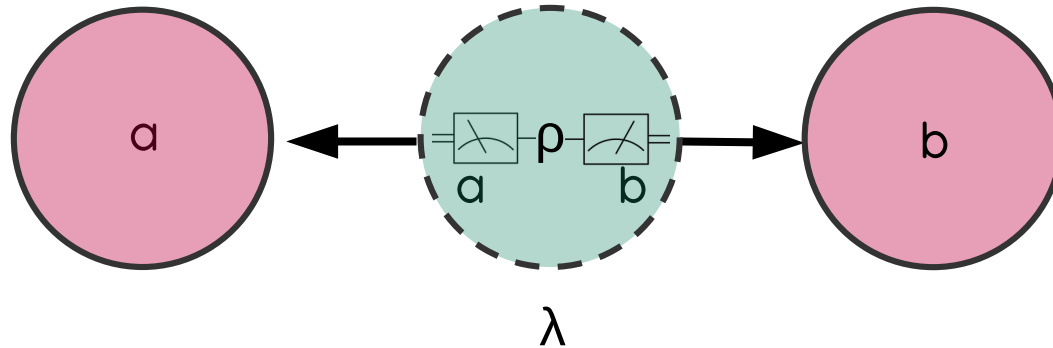


Classical explanation exists for $p(ab)$?

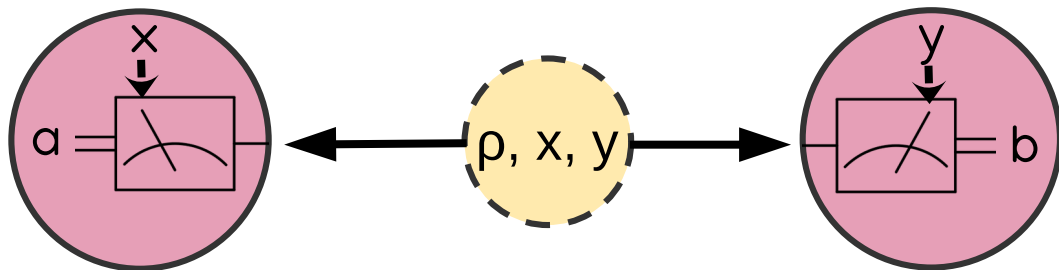
Quantum correlations



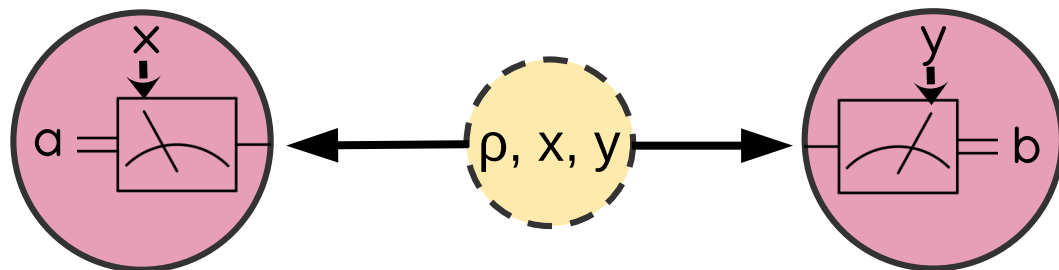
$p(ab)$ genuinely quantum?



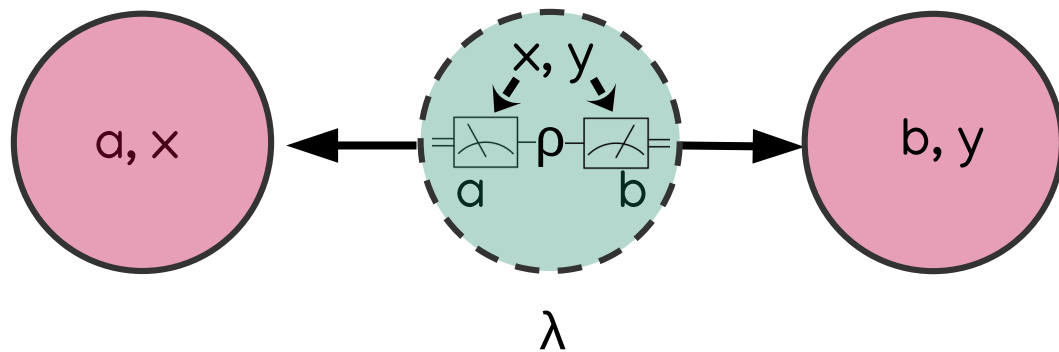
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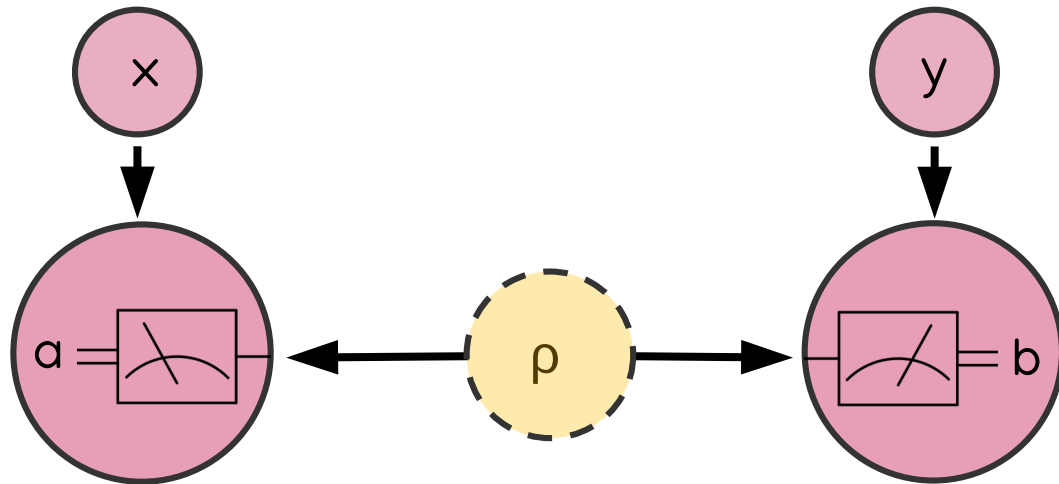
$p(ab|xy)$ genuinely
quantum?



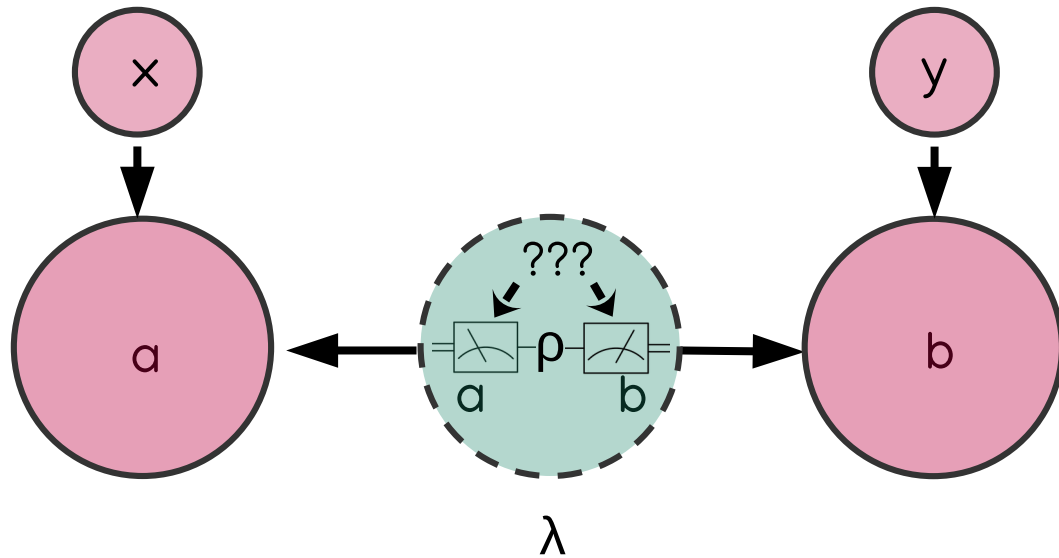
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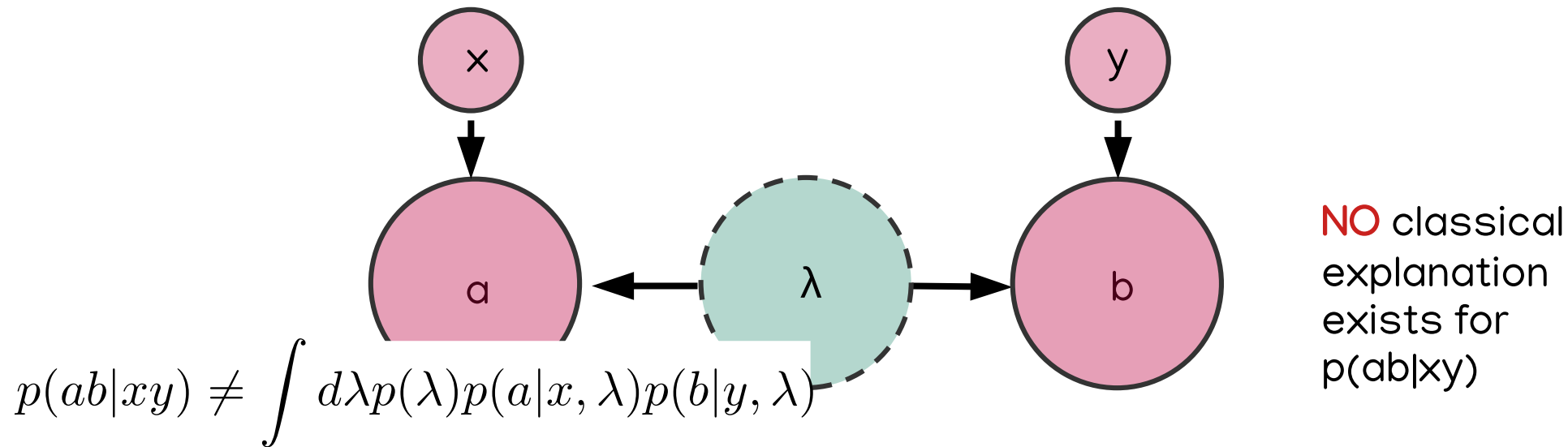
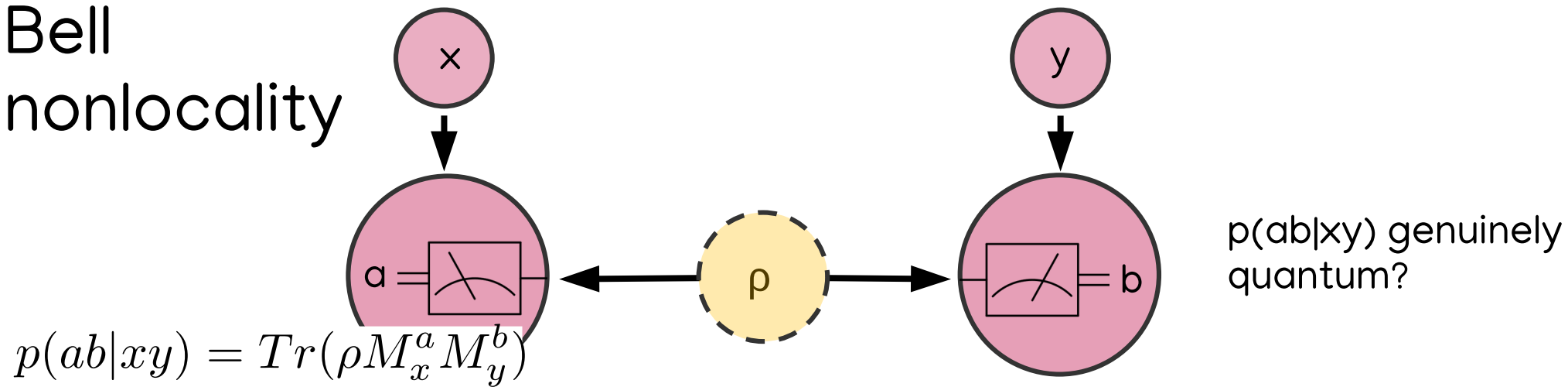


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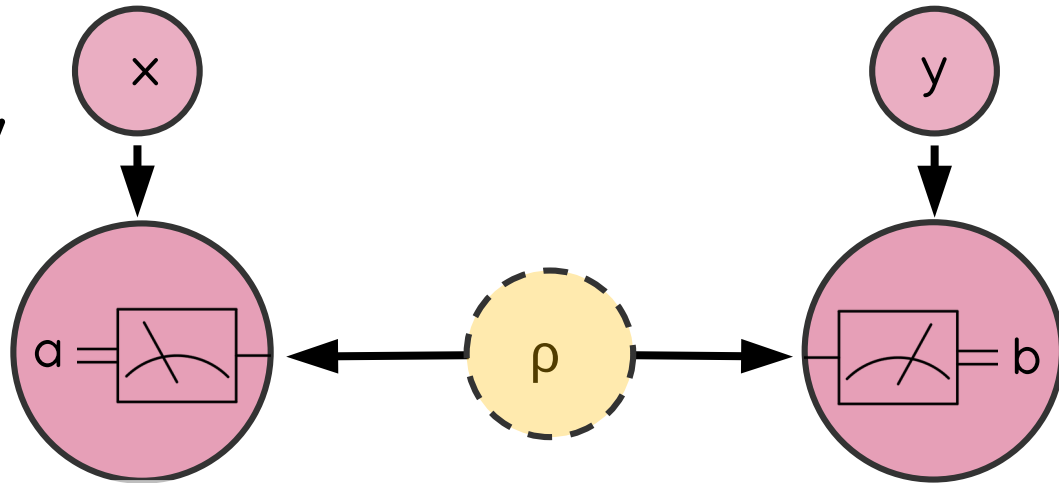


NO classical explanation exists for $p(ab|xy)$

Bell nonlocality

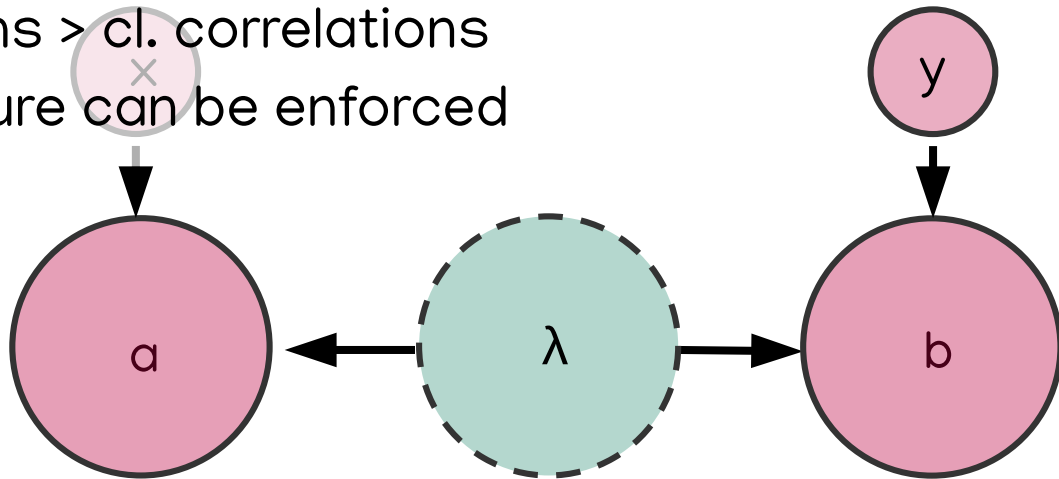


Bell nonlocality



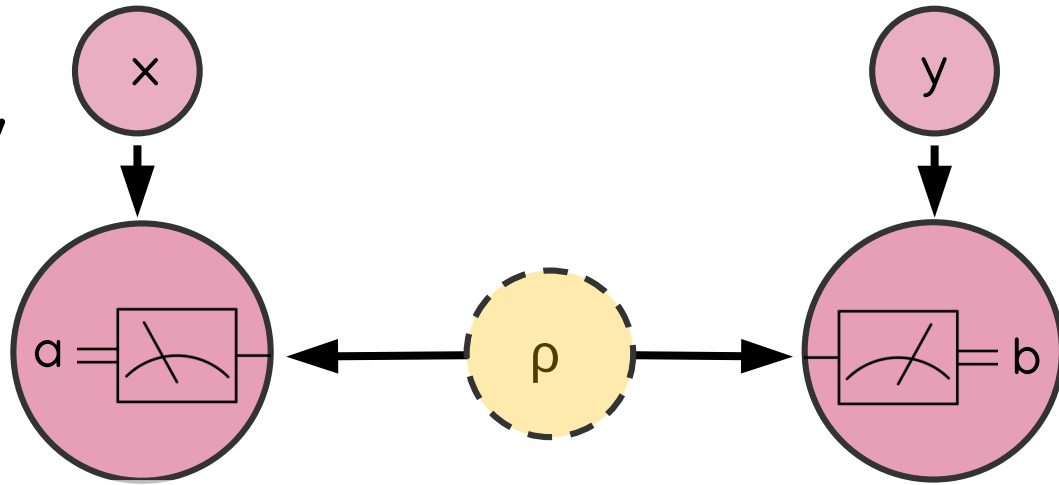
$p(ab|xy)$ genuinely quantum?

- qu. correlations $>$ cl. correlations
- causal structure can be enforced



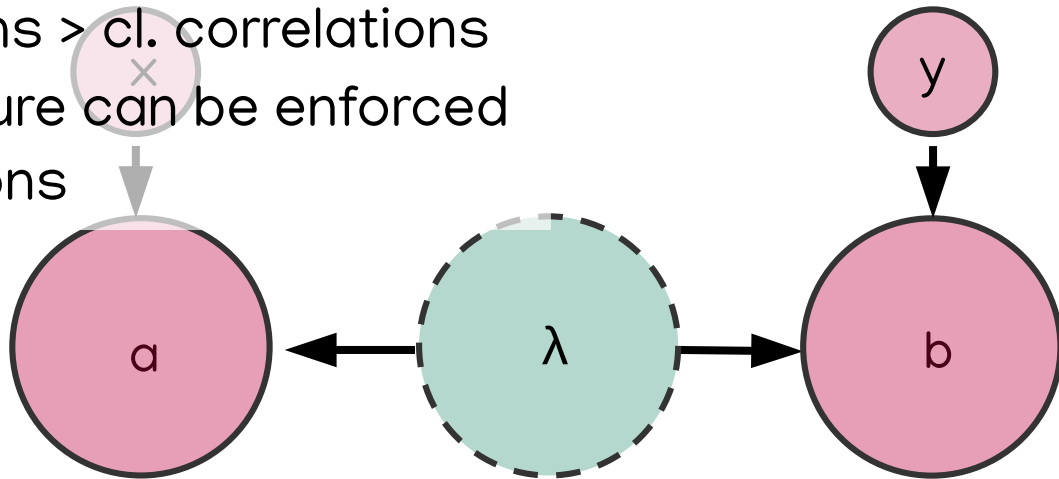
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Bell nonlocality



$p(ab|xy)$ genuinely quantum?

- qu. correlations $>$ cl. correlations
- causal structure can be enforced
- has applications

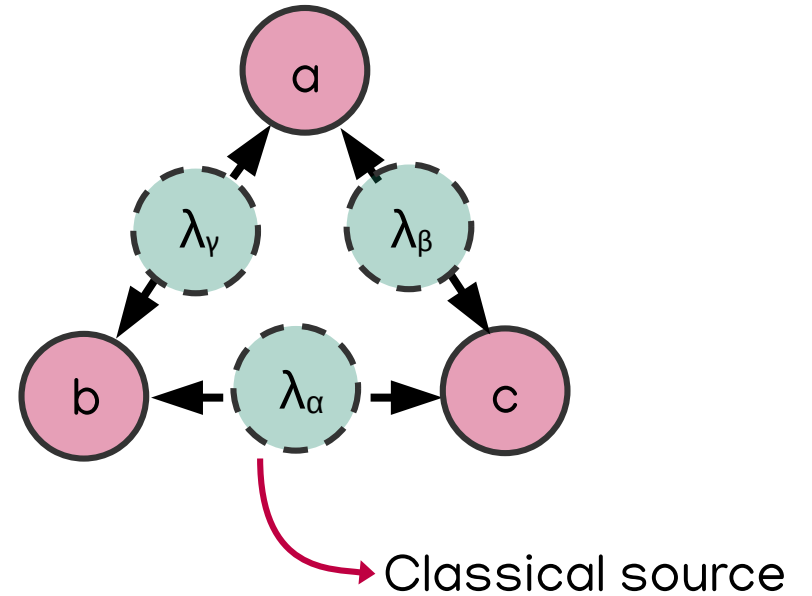
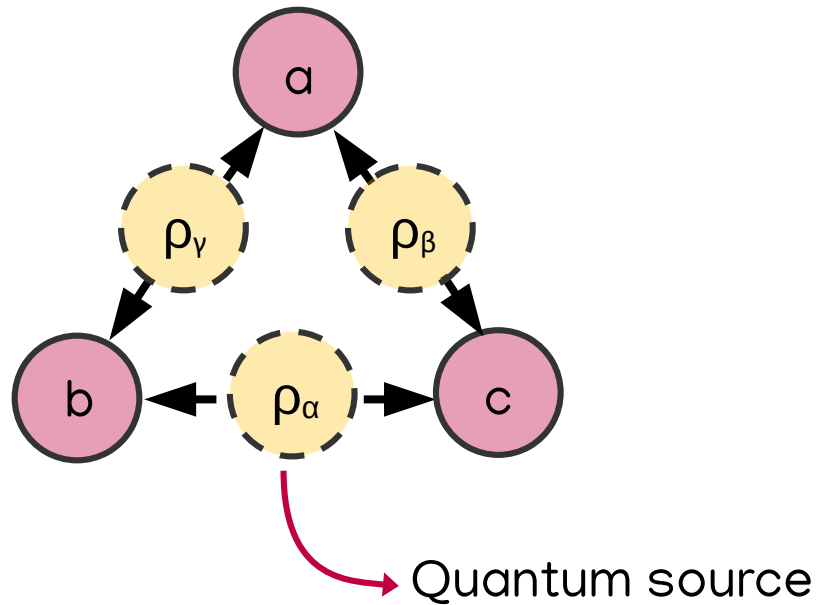


NO classical explanation exists for $p(ab|xy)$

Need inputs!!!!

Inputs should be independent from common history of A and B.

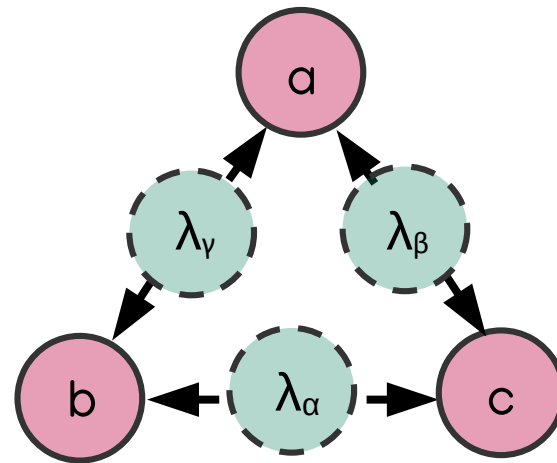
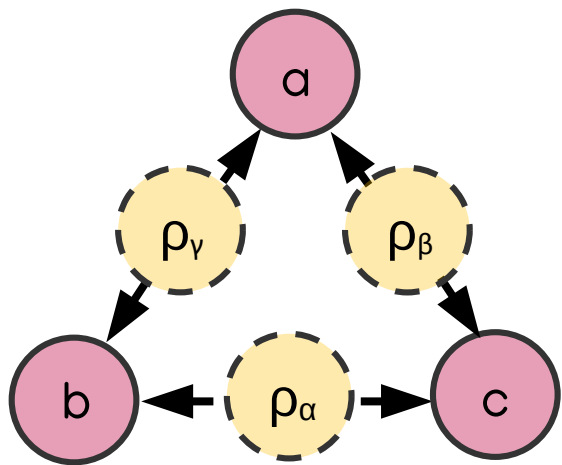
Bell nonlocality on networks



Review:

[Tavakoli et al. Rep. Prog. Phys. 85 056001 (2022)]

Bell nonlocality on networks



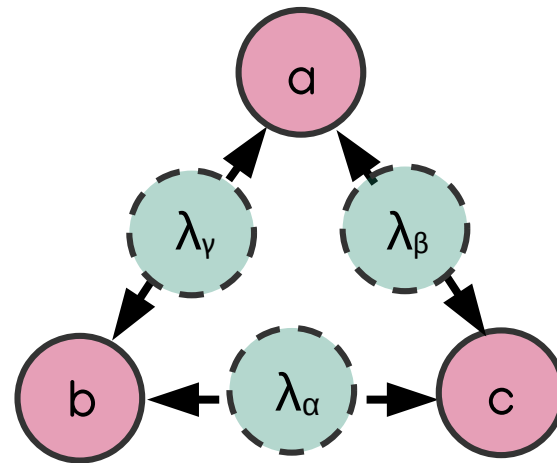
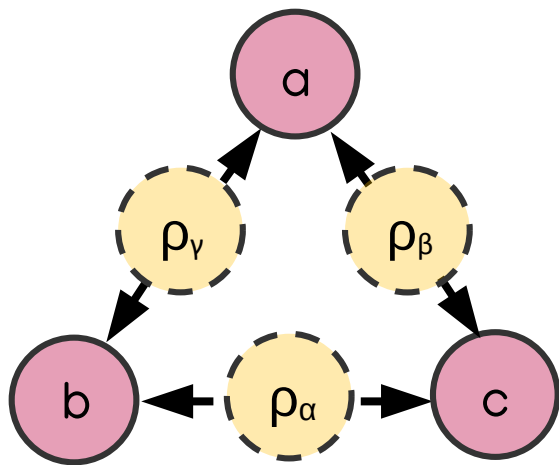
$$p(abc) = \text{Tr}(\rho_\gamma \otimes \rho_\alpha \otimes \rho_\beta M^a \otimes M^b \otimes M^c) \neq \int d\alpha d\beta d\gamma p_A(a|\beta\gamma)p_B(b|\gamma\alpha)p_C(c|\alpha\beta)p(\alpha)p(\beta)p(\gamma)$$

[Fritz, NJP 14, 103001 (2012)]

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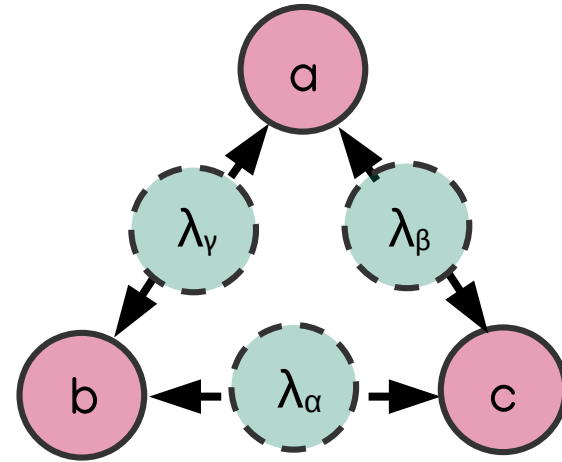
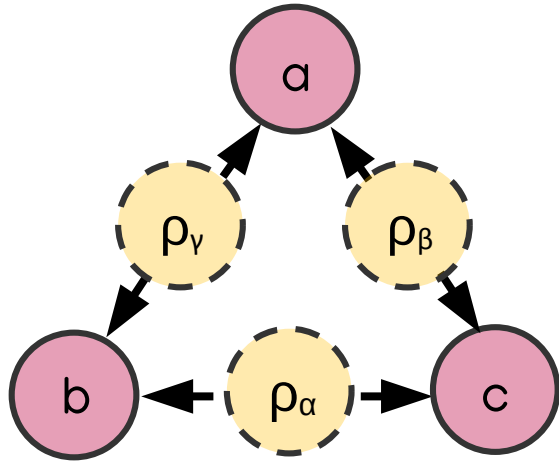
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad M : \{|00\rangle; |11\rangle; u|01\rangle + v|10\rangle; v|01\rangle - u|10\rangle\}$$

[Fritz, NJP 14, 103001 (2012)]
 [Renou et al., PRL 123, 14041 (2019)]

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Bell nonlocality on networks

No inputs needed!!



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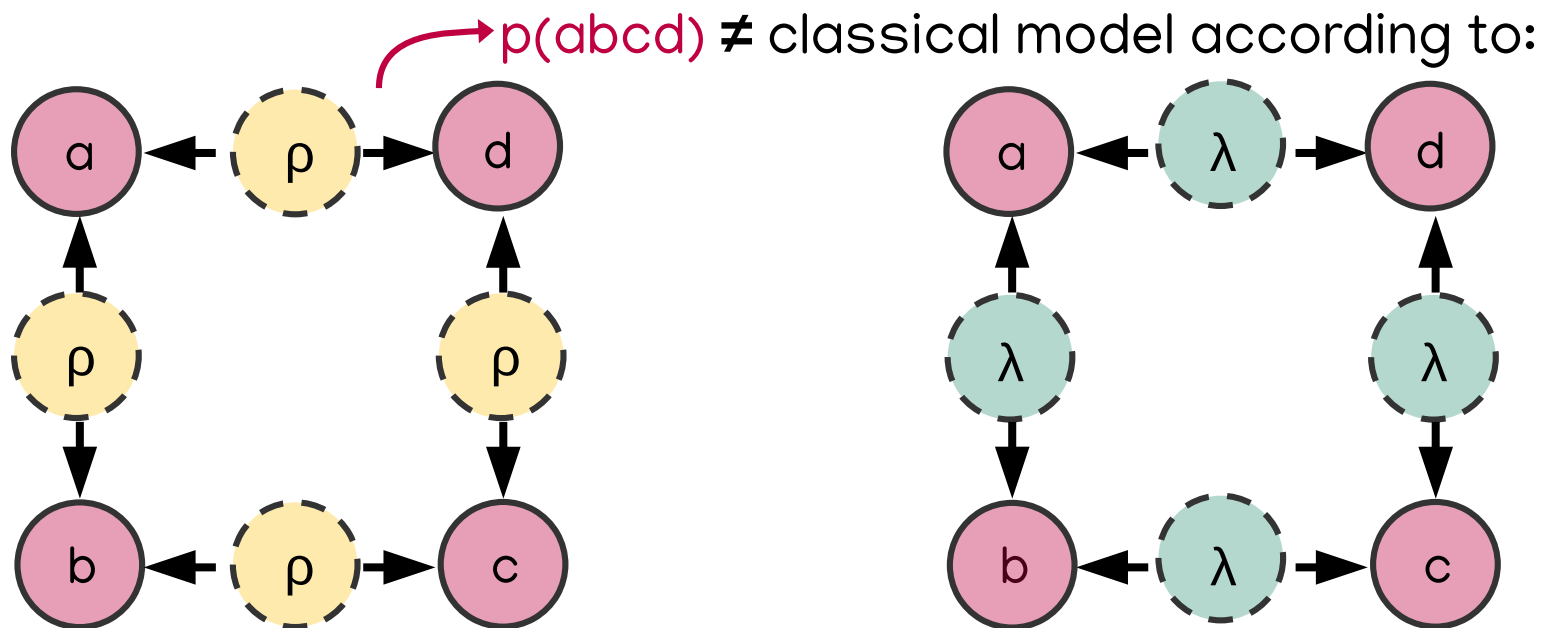
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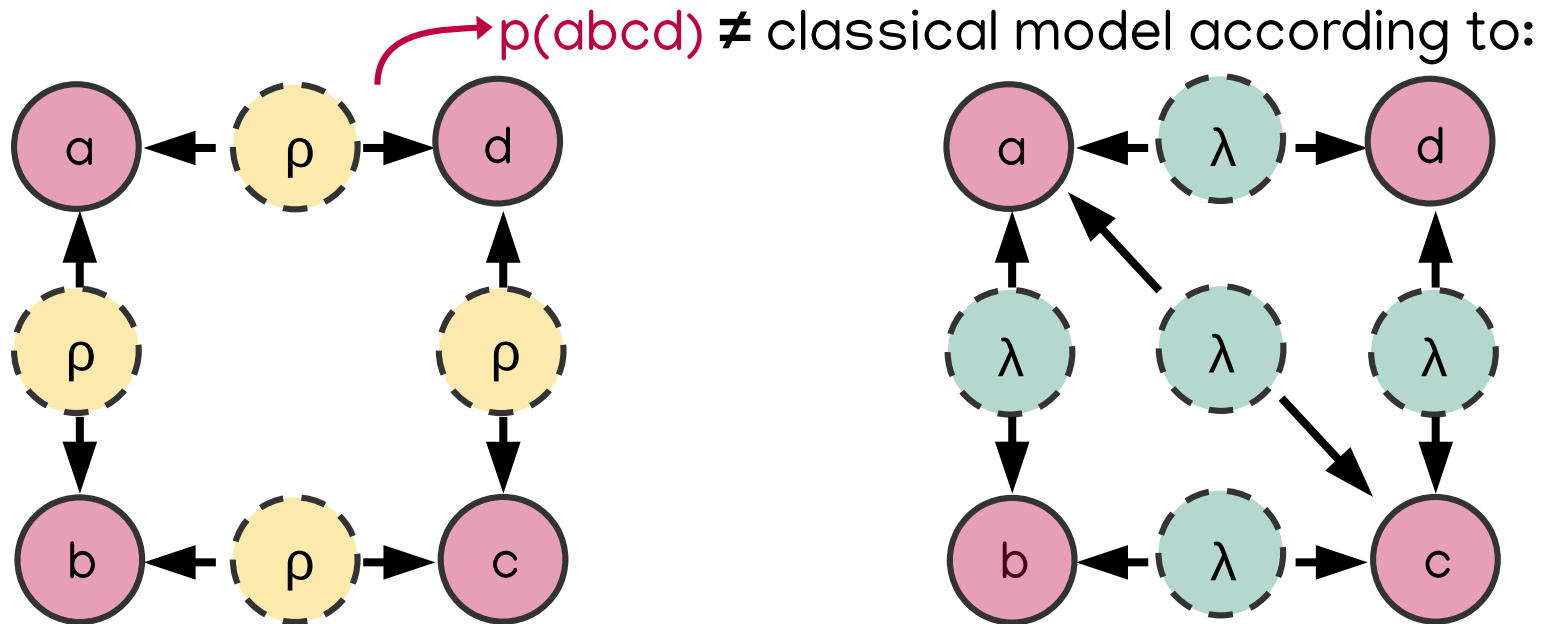
Topologically robust nonlocality

4 parties



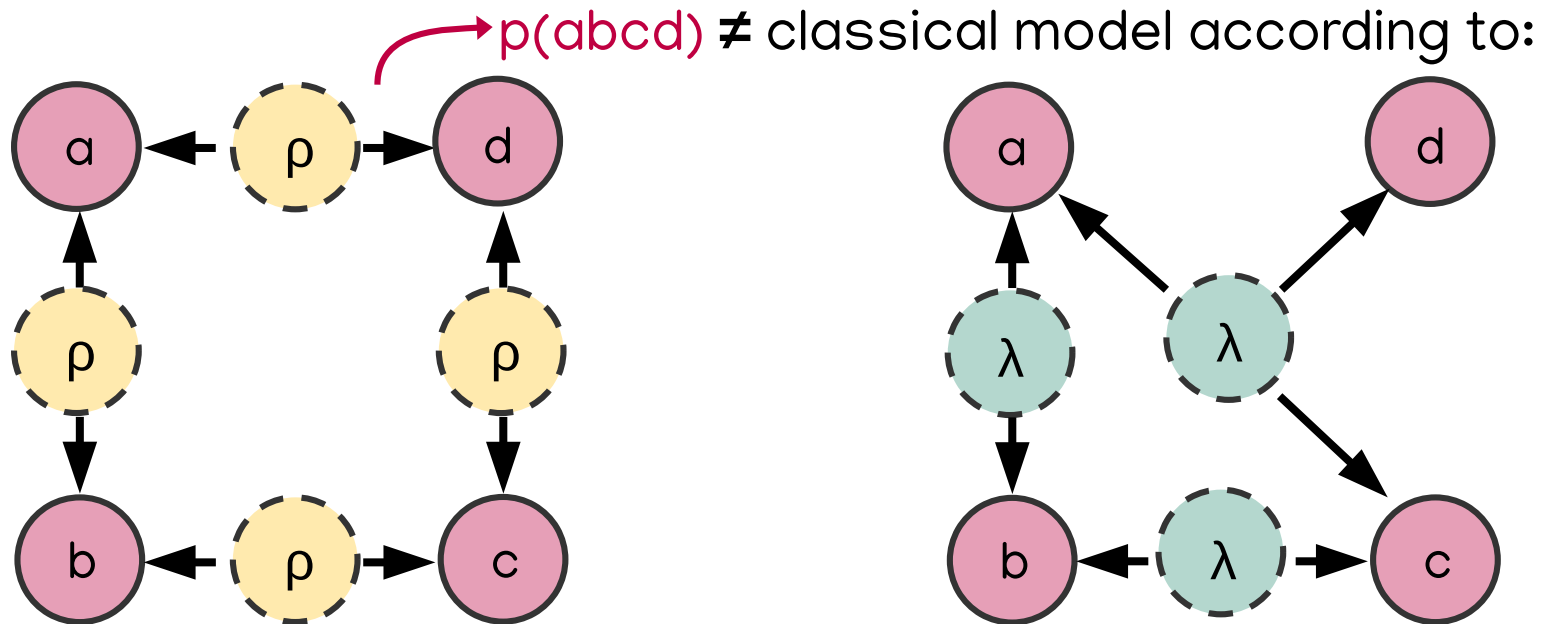
Topologically robust nonlocality

4 parties: Such strong correlations, that classical **extra link doesn't help!!**



Topologically robust nonlocality

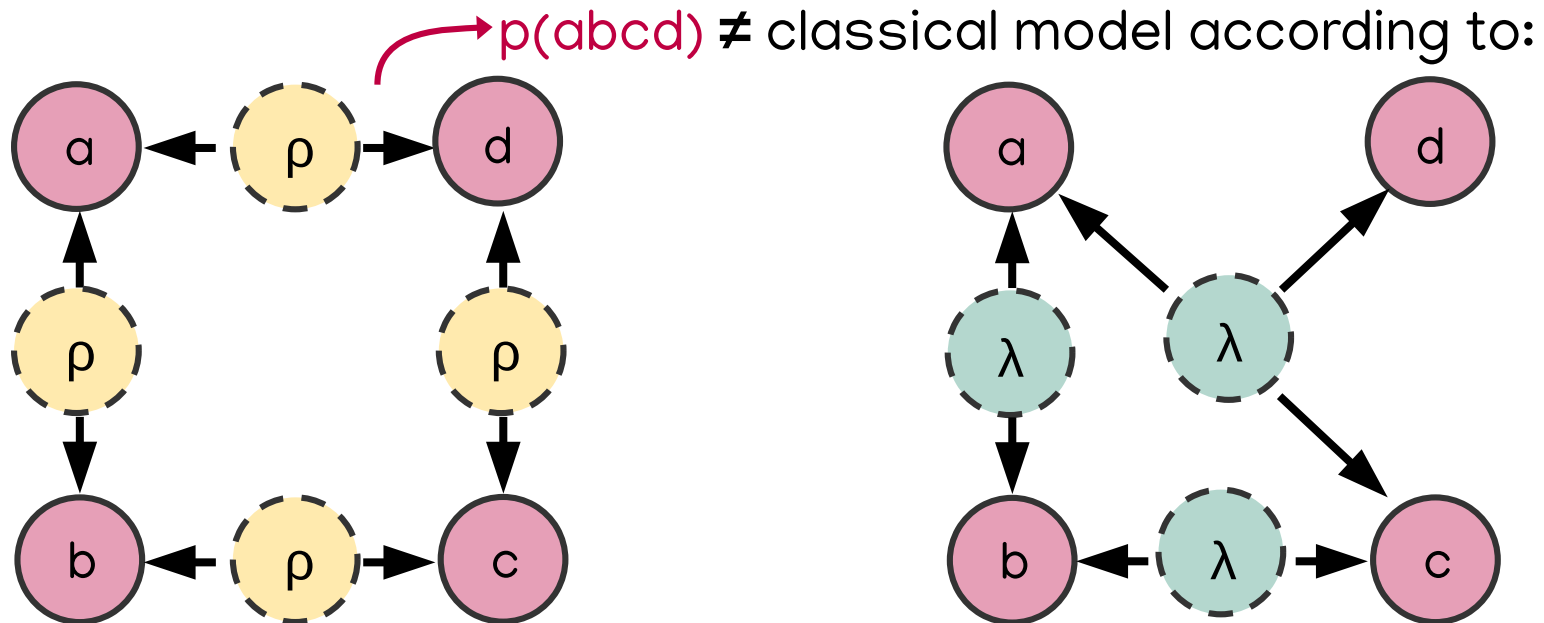
4 parties: Such strong correlations, that classical **extra link doesn't help!!**
Even a classical **tripartite source** doesn't help!



Topologically robust nonlocality

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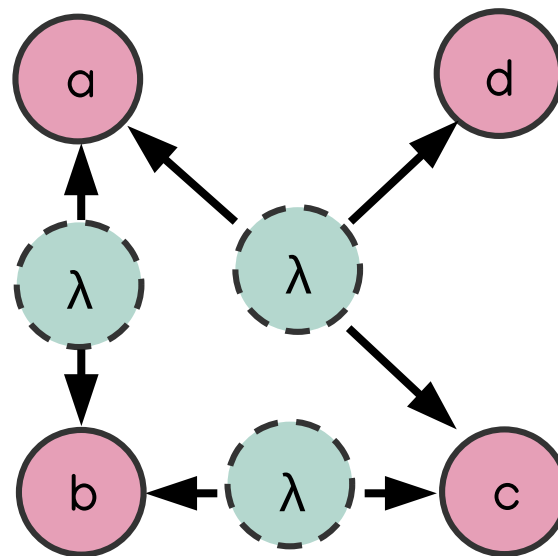
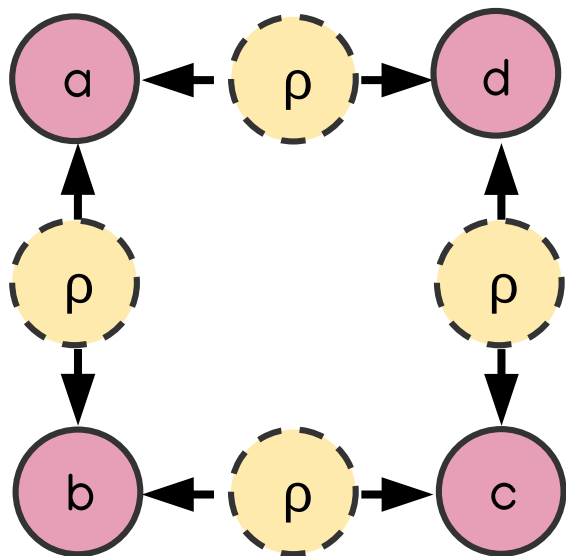
Topological robustness: nonlocality versus different (stronger) networks



Randomness

Adapt [Sekatski et al., arXiv:2209.09921 (2022)] for the following networks.

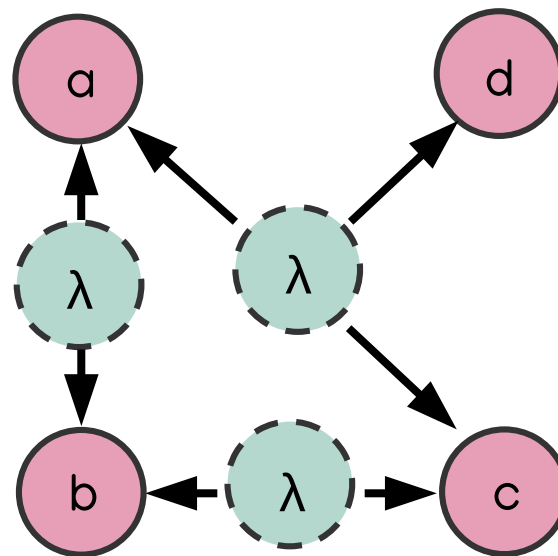
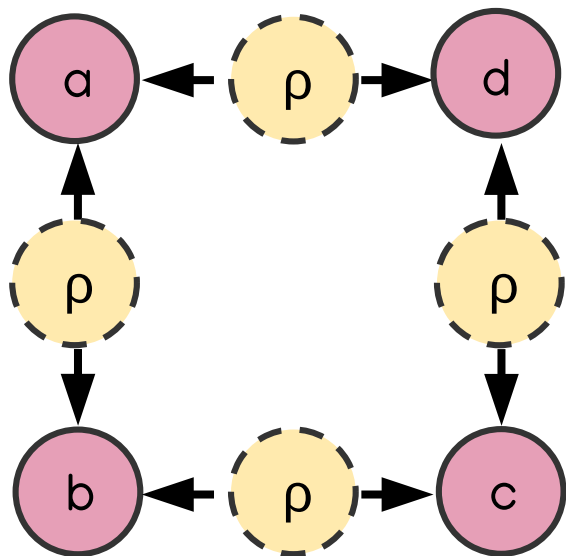
– must be a loop of entangled states and measurements



Randomness

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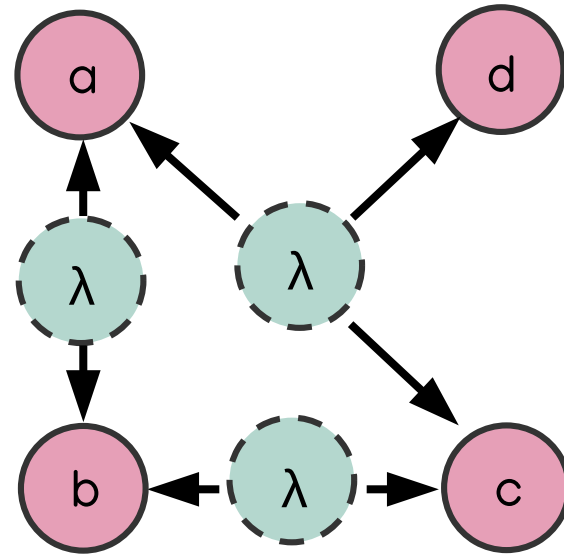
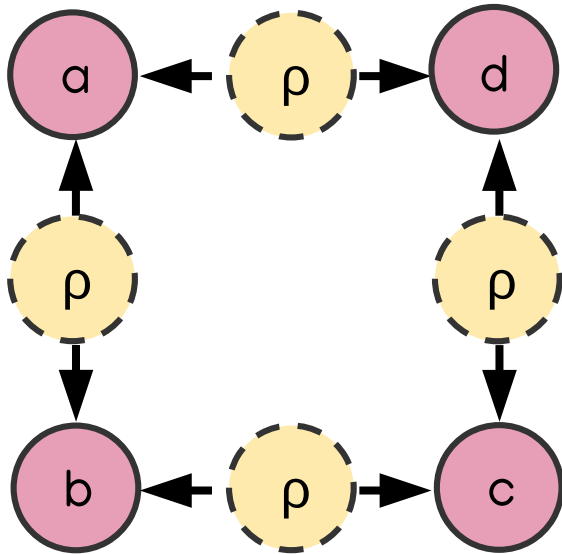
- must be a loop of **entangled states** and **measurements**
- there must be some **randomness** in outputs along loop



Randomness

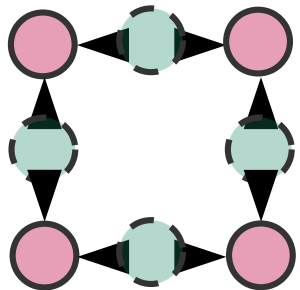
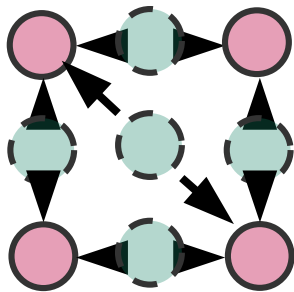
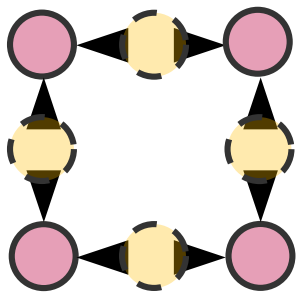
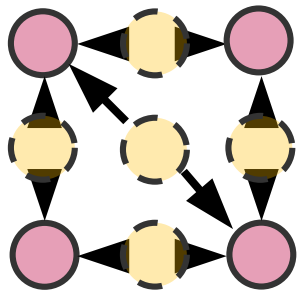
Adapt [Sekatski et al., arXiv:2209.09921 (2022)] for the following networks.

- must be a loop of **entangled states** and **measurements**
- there must be some **randomness** in outputs along loop
- ongoing: all parties must be in loop??

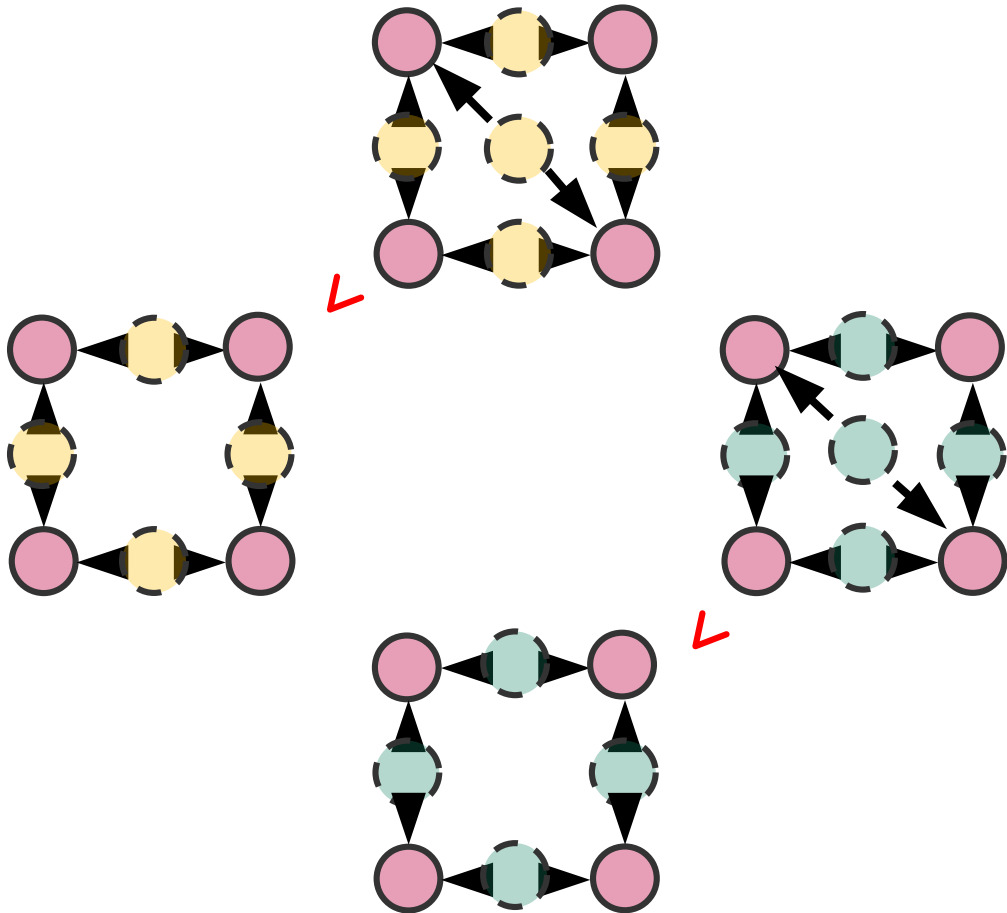


Foundational approach

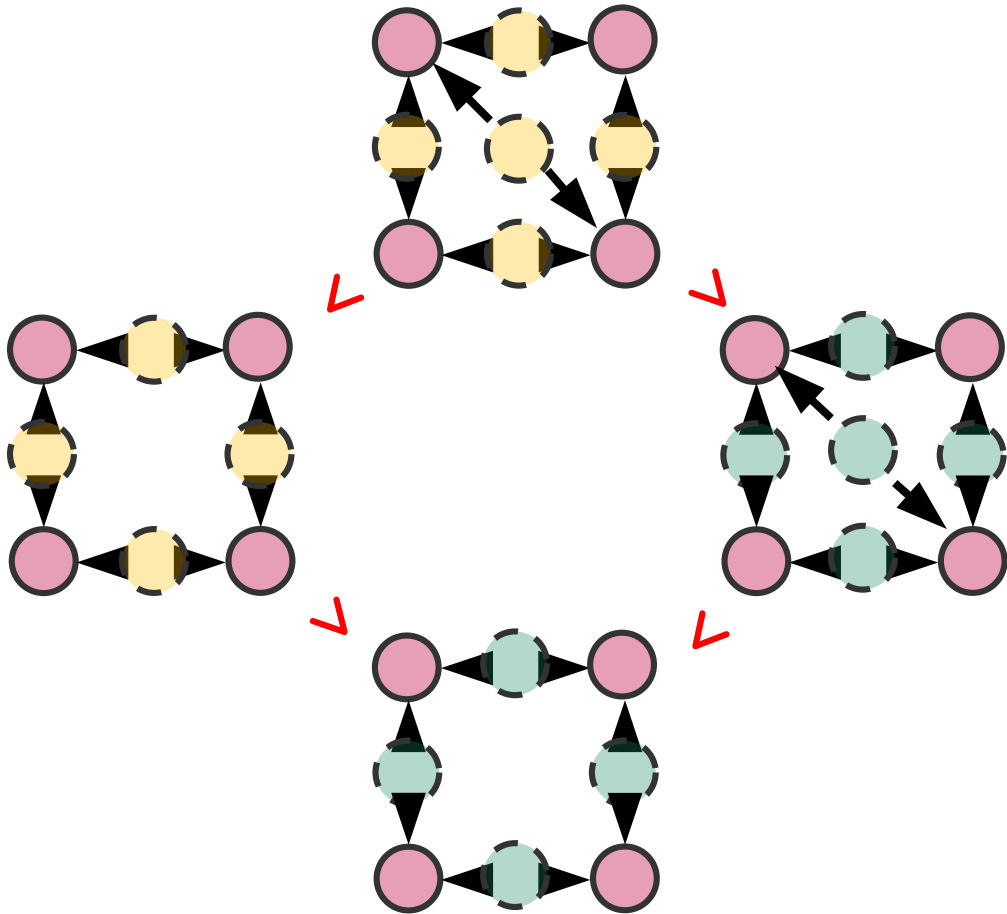
Pre-order of network structures



Pre-order of network structures



Pre-order of network structures

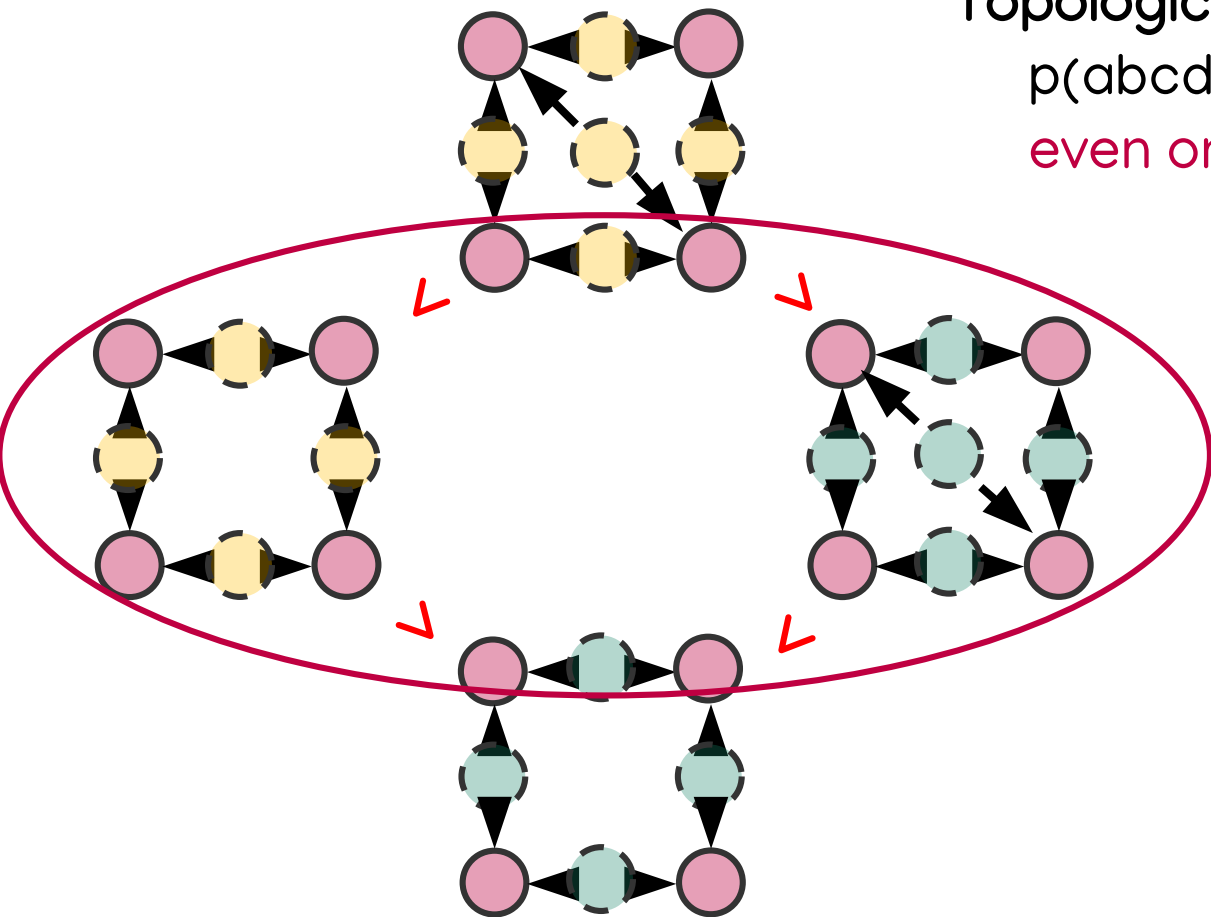


Pre-order of network structures

Topologically robust nonlocality:

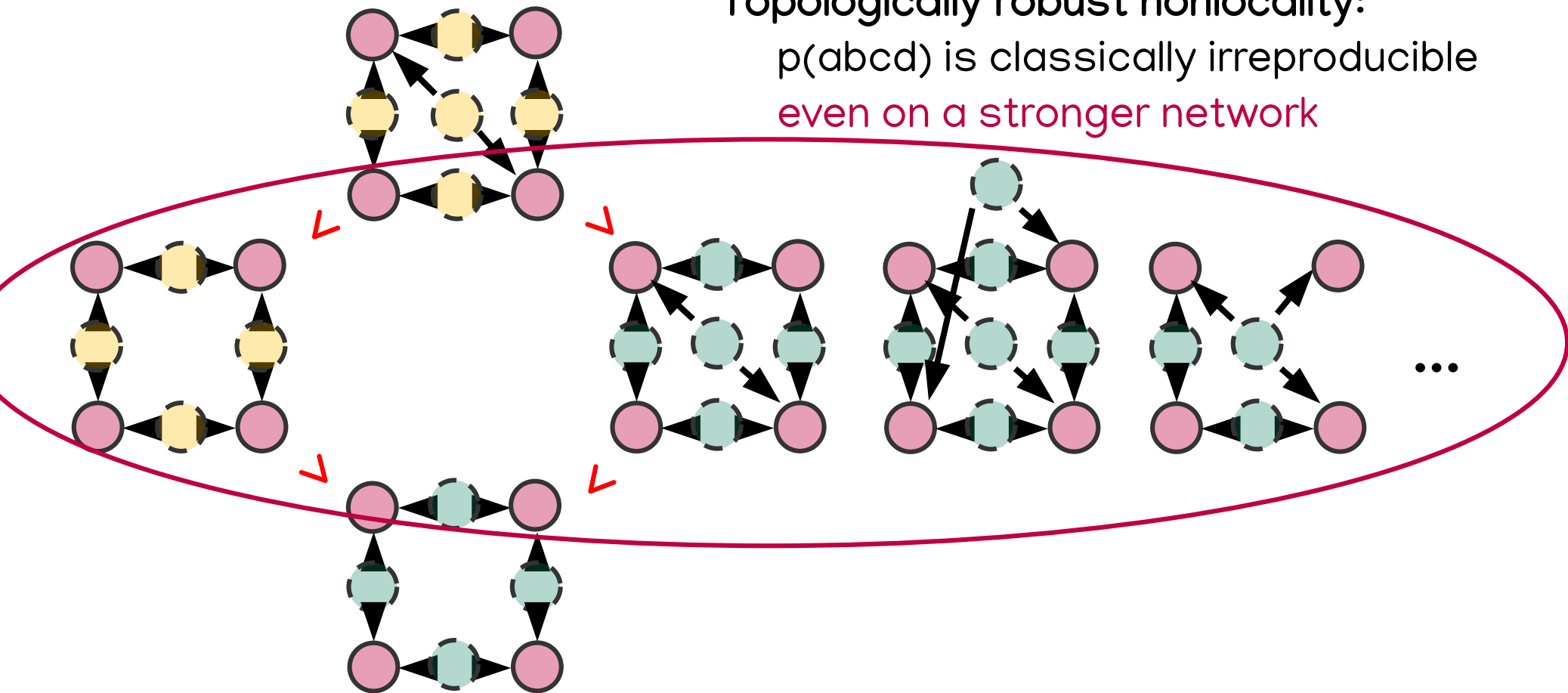
$p(abcd)$ is classically irreproducible

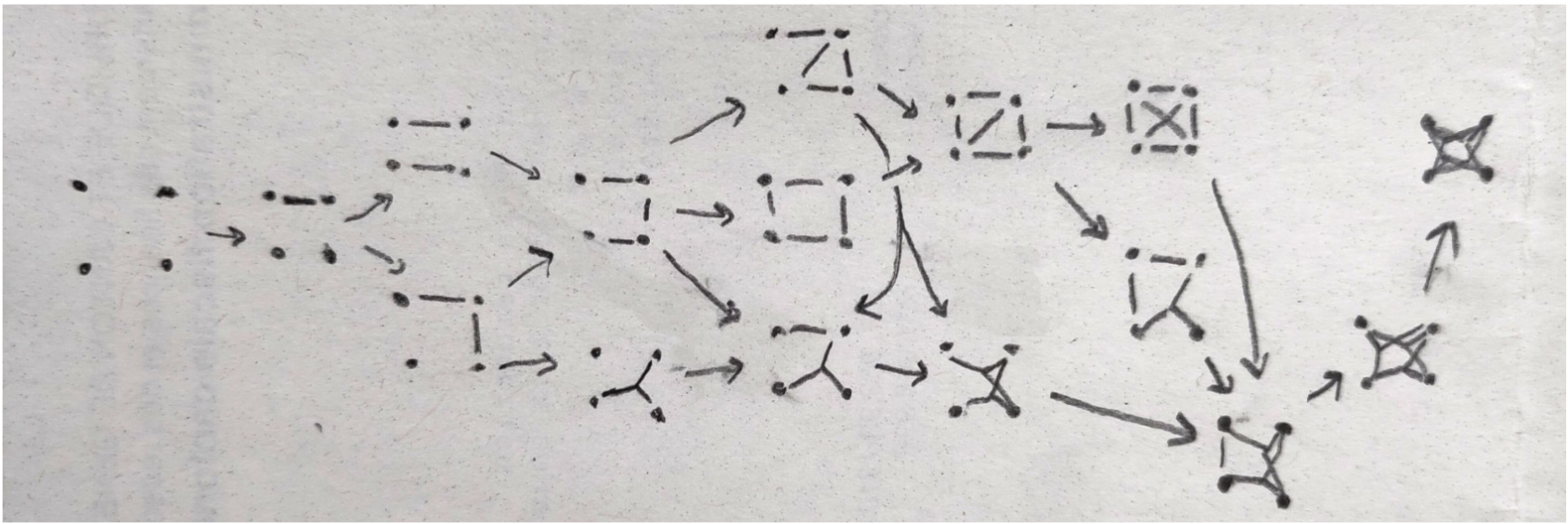
even on a stronger network



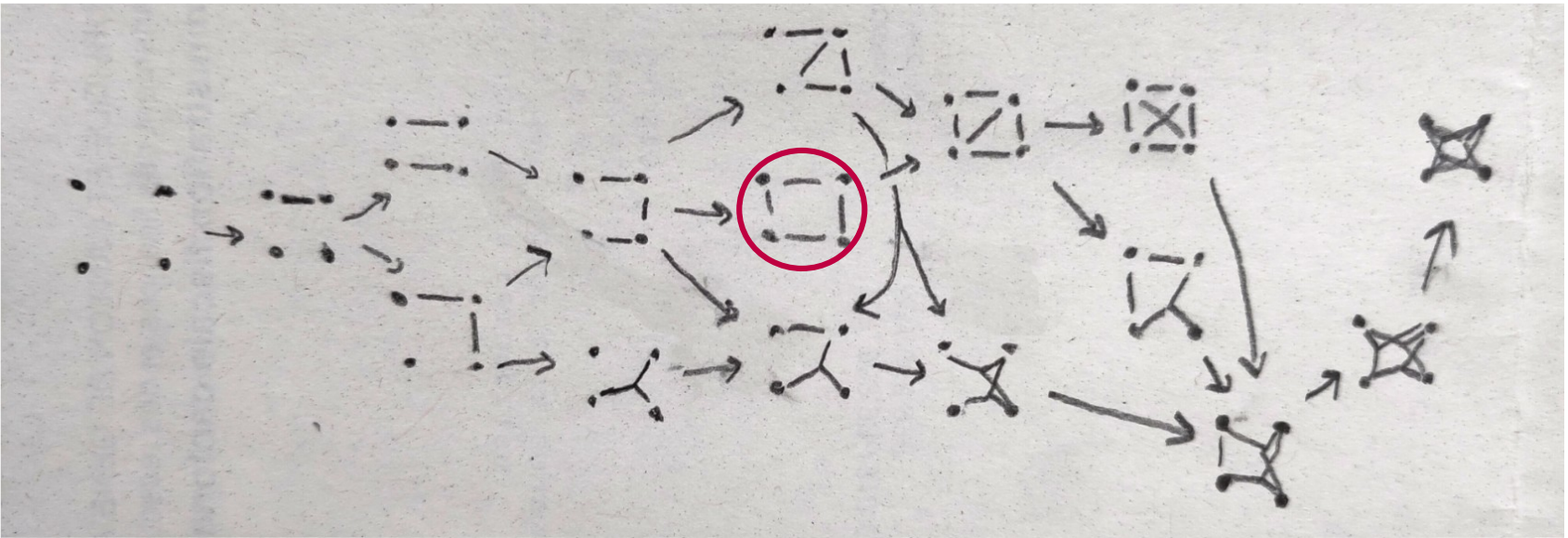
Pre-order of network structures

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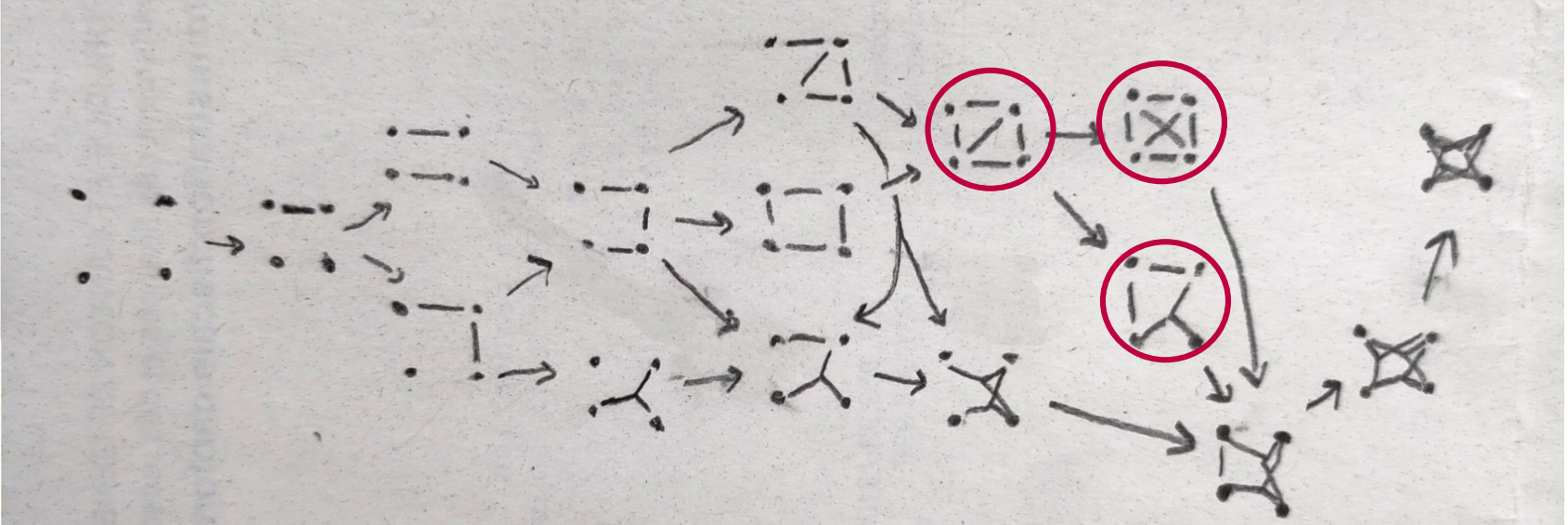




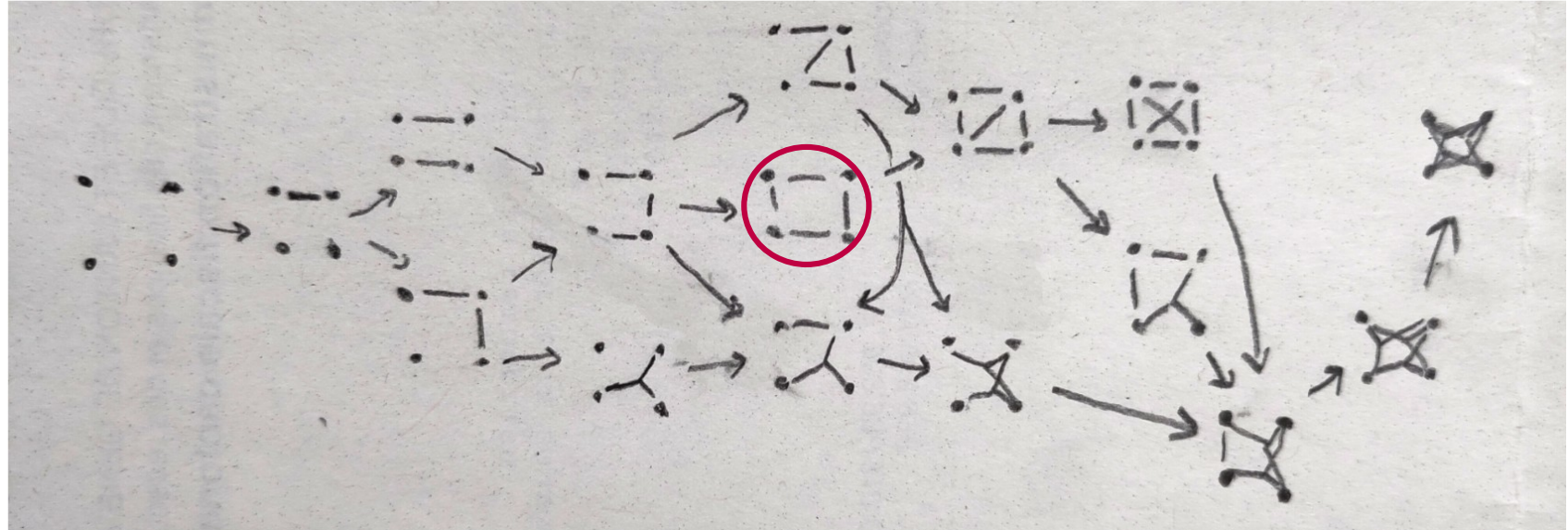
Qu



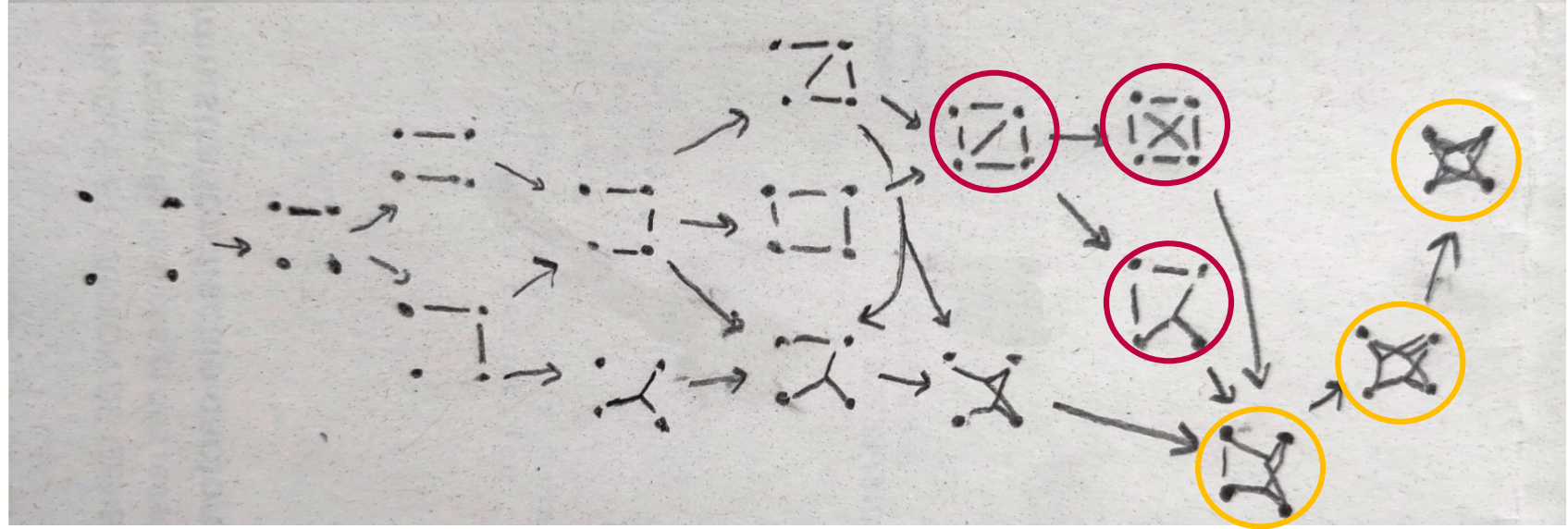
Cl



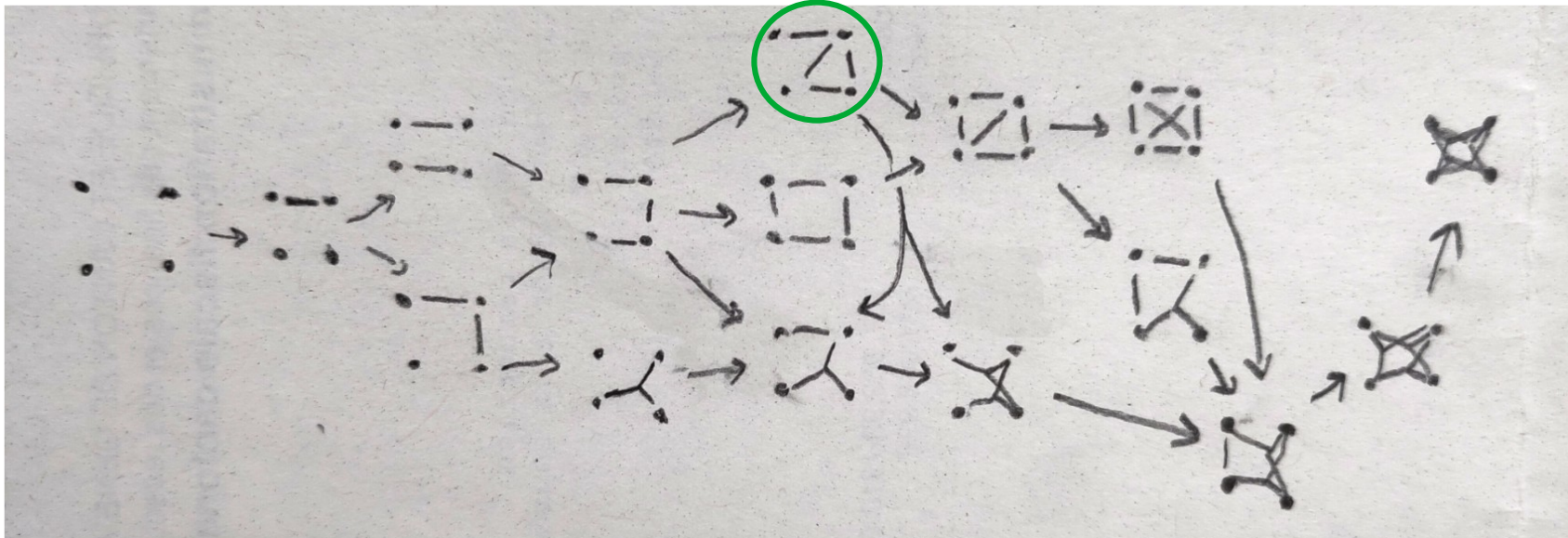
Qu



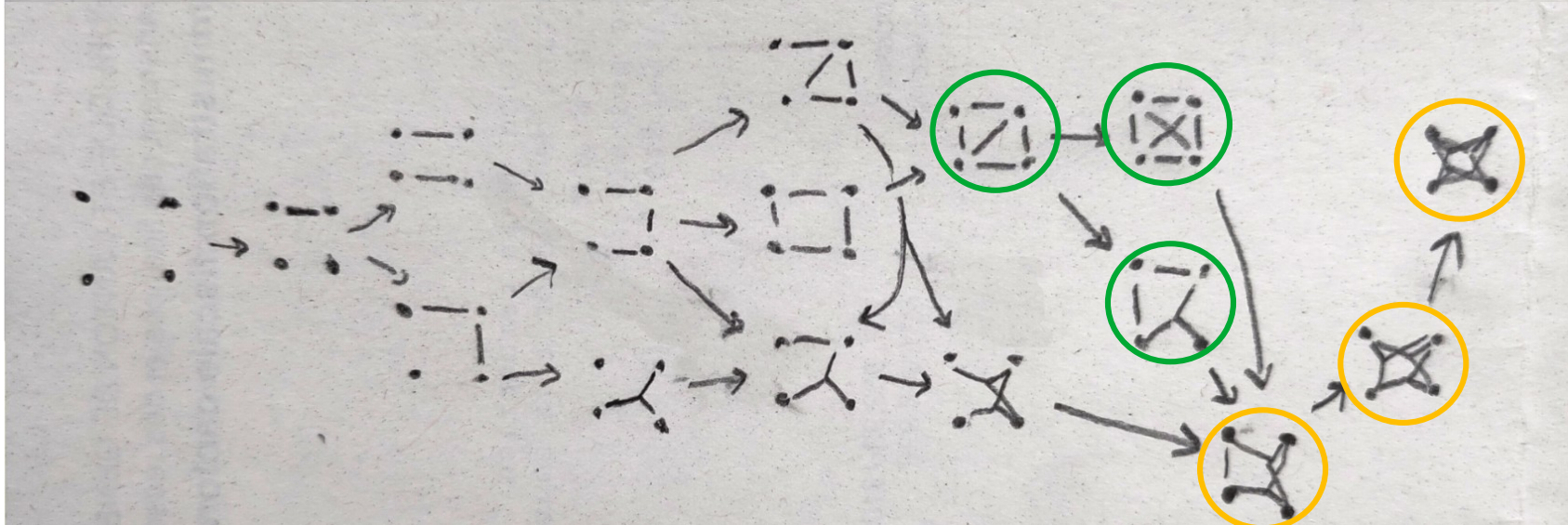
Cl



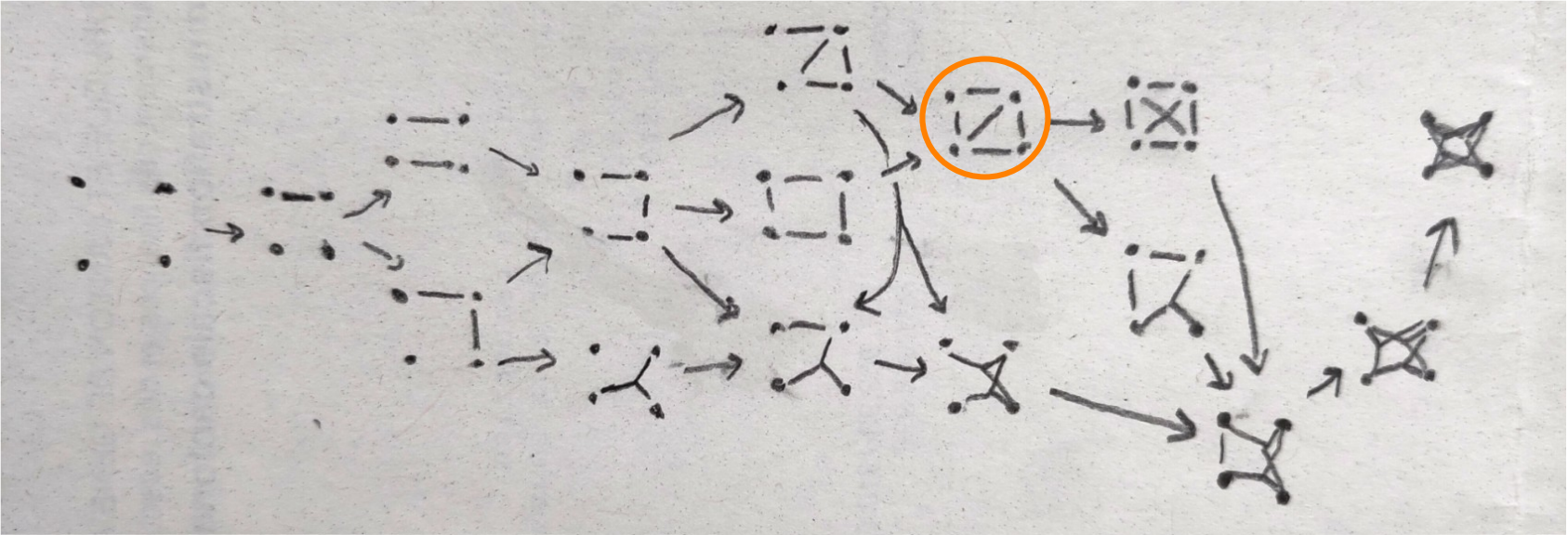
Qu



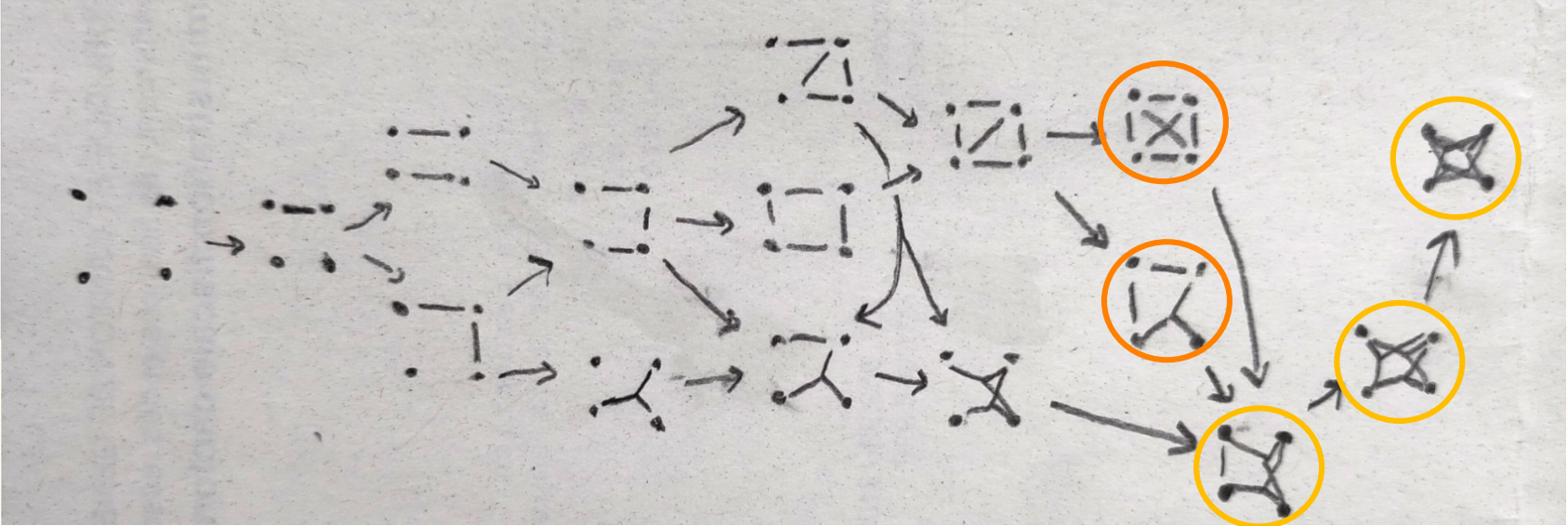
Cl



Qu



Cl



Generic criteria

Constructing whole preorder structure difficult.

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Generic criteria:

- existence of a **loop** in qu. network
 - “no double common source” cl. network
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Very **typical** in large random networks

Generic criteria

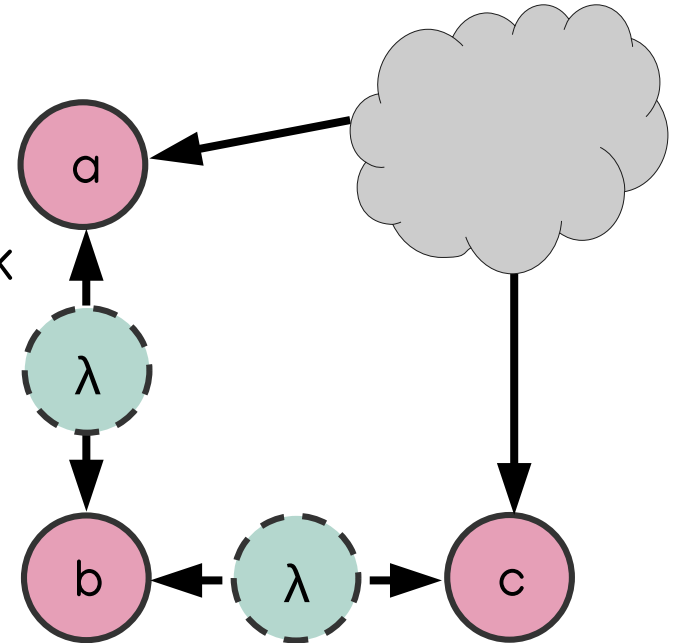
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Generic criteria

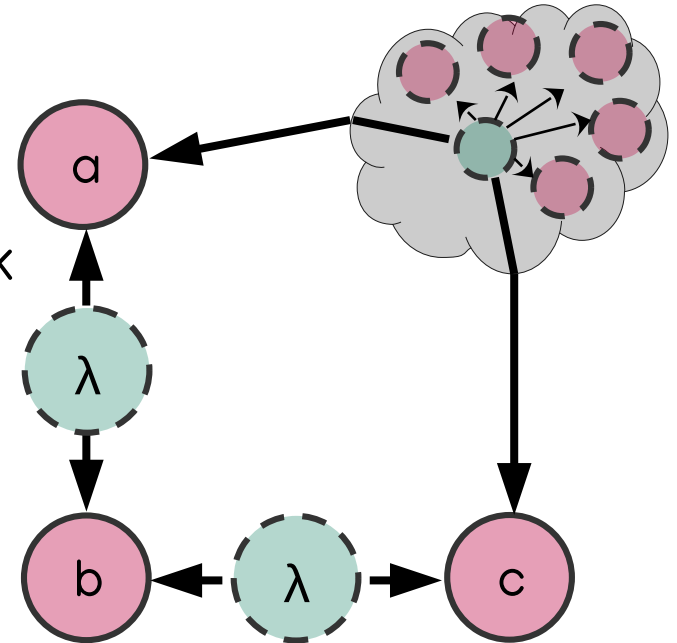
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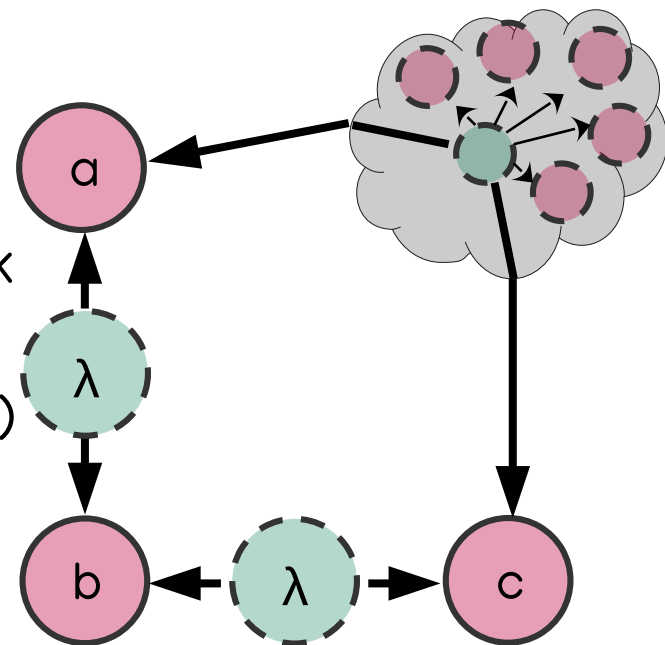
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Very **typical** in large random networks

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local information of network **structure** + $p(abcd\dots z)$

→ nonlocality

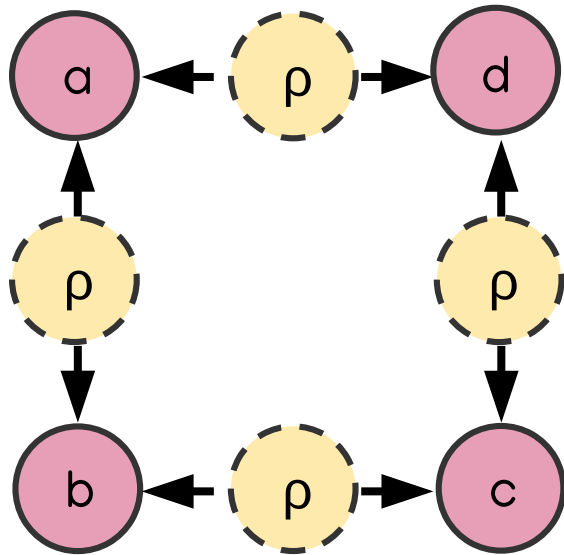


Towards applications?

- Features:
 - Proven **randomness**
 - **Robustness** to network topology

Towards applications?

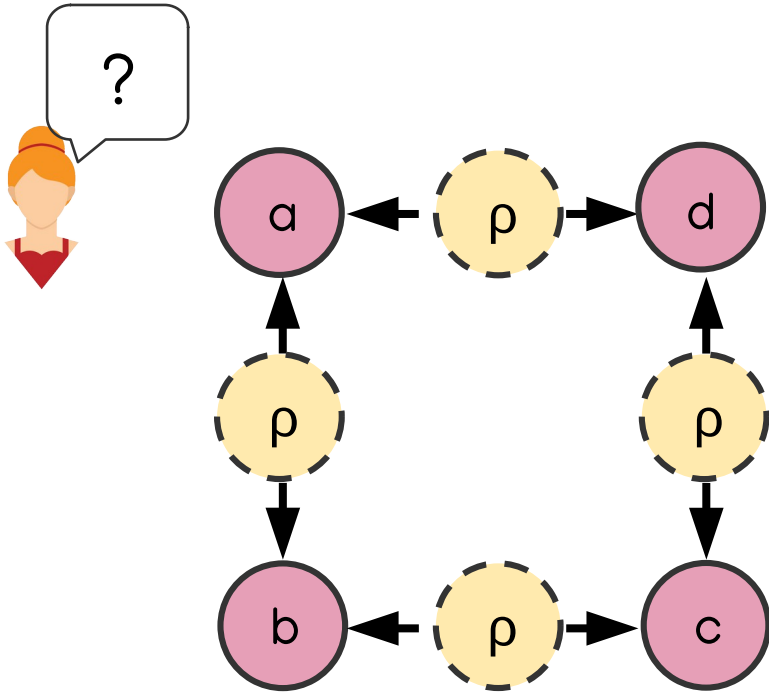
Toy example: **Trusted** quantum **sources** & each party promises to do some measurements



Towards applications?

Based **only on $p(abcd)$** , can an external party answer the question:

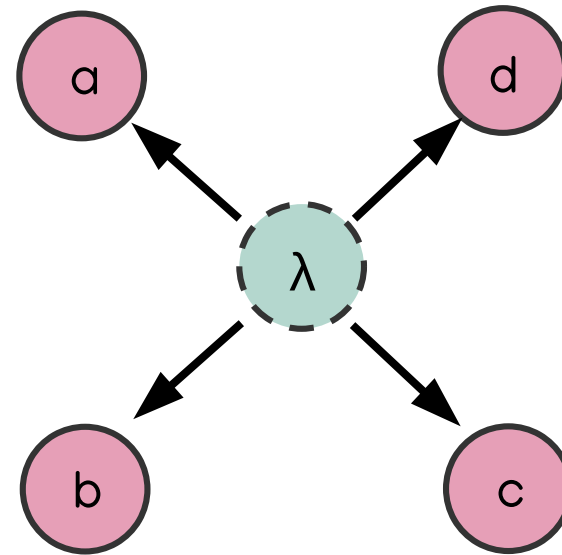
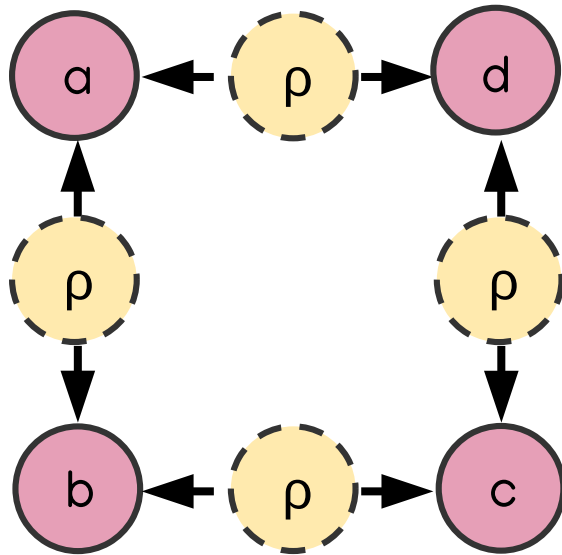
Does there exist randomness in network's outcome?



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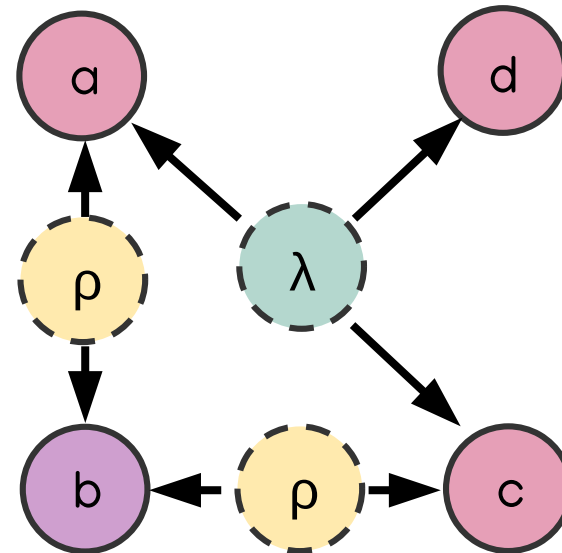
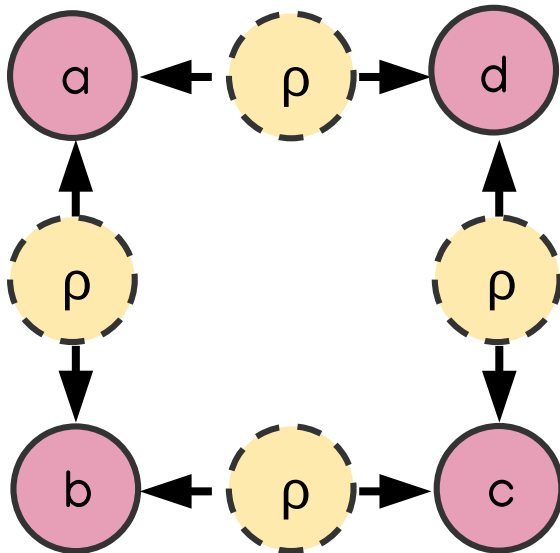
No!!

Towards applications?

Based **only on $p(abcd)$** , can an external party answer the question:

Does there exist randomness in network's outcome?

Assuming that **at least one party** is trustworthy:



Yes!!

Towards applications?

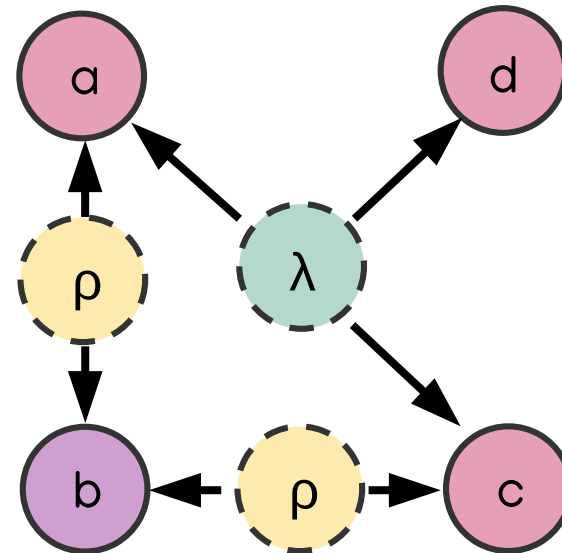
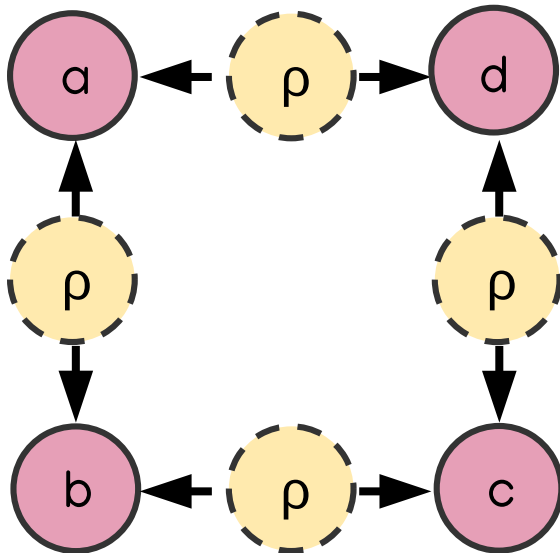
Based **only on $p(abcd)$** , can an external party answer the question:

Does there exist randomness in network's outcome?

Assuming that **at least one party** is trustworthy:



Moving towards **topologically robust quantum steering**

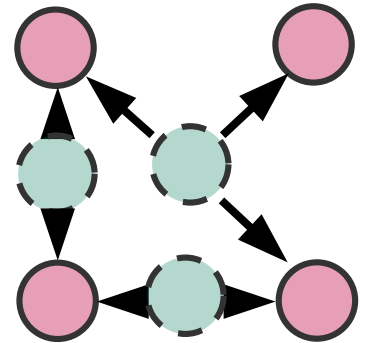
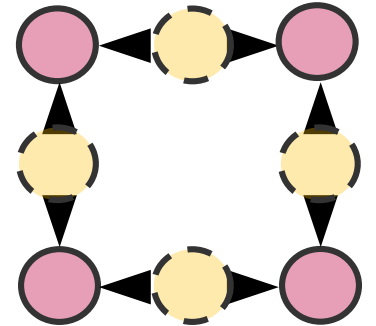


Yes!!

Summary

1) Topological robust Bell nonlocality

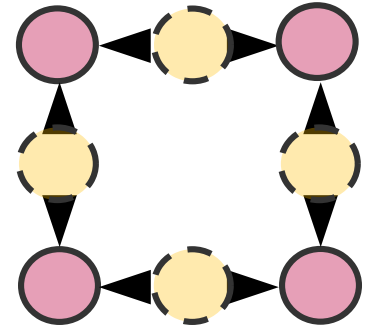
- Motivation: only sure of network structure **locally**
- Sometimes having **quantum correlations** is better than having a **stronger classical network**



Summary

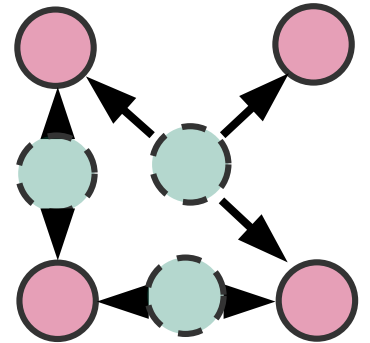
1) Topological robust Bell nonlocality

- Motivation: only sure of network structure **locally**
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2) Towards **applications**

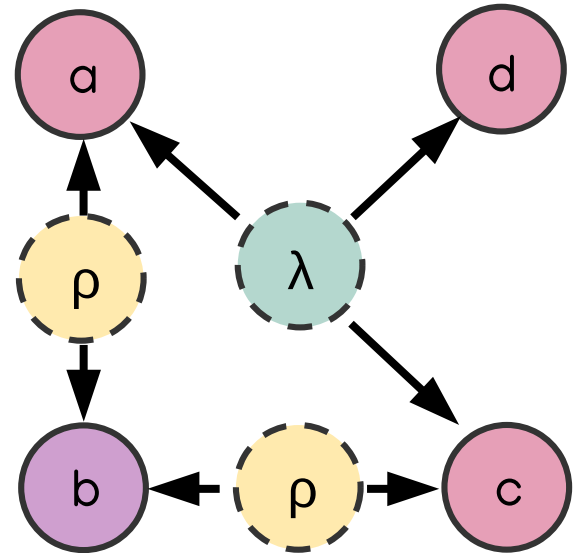
- **Randomness** without inputs, only network assumptions
- Network assumptions → **trust**/knowledge of network
- Topologically robust **steering**



Open questions

1) Foundational

- Complete characterization of topological robustness preorder
- What changes when considering **inputs** for parties?



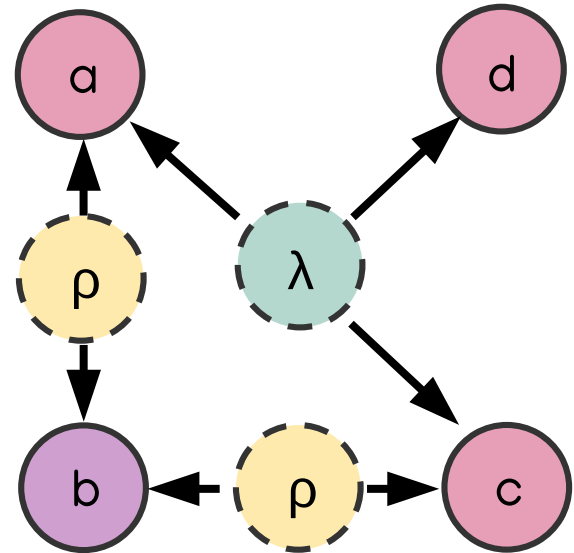
Open questions

1) Foundational

- Complete characterization of topological robustness **preorder**
- What changes when considering **inputs** for parties?

2) Towards **applications**

- Sensible **assumptions** to make?
- **Noise** robustness not there yet... **post-selection**?
- What **protocols** could we have?
 - Public randomness, secret sharing, ... ?



Contact

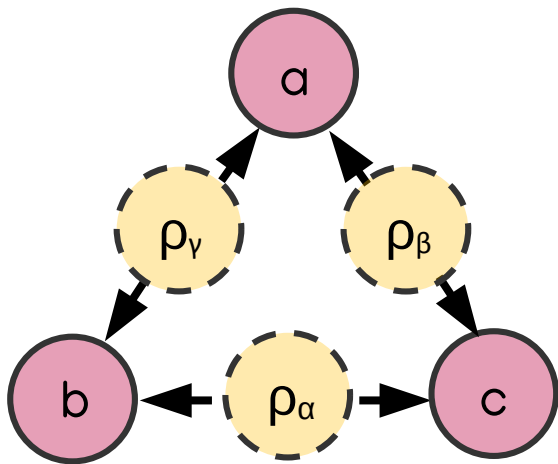
Topologically robust network nonlocality
arXiv (summer 2023)

Let's discuss [here](#) or [online](#)

tamas.krivachy@gmail.com

 @KrivachyTamas

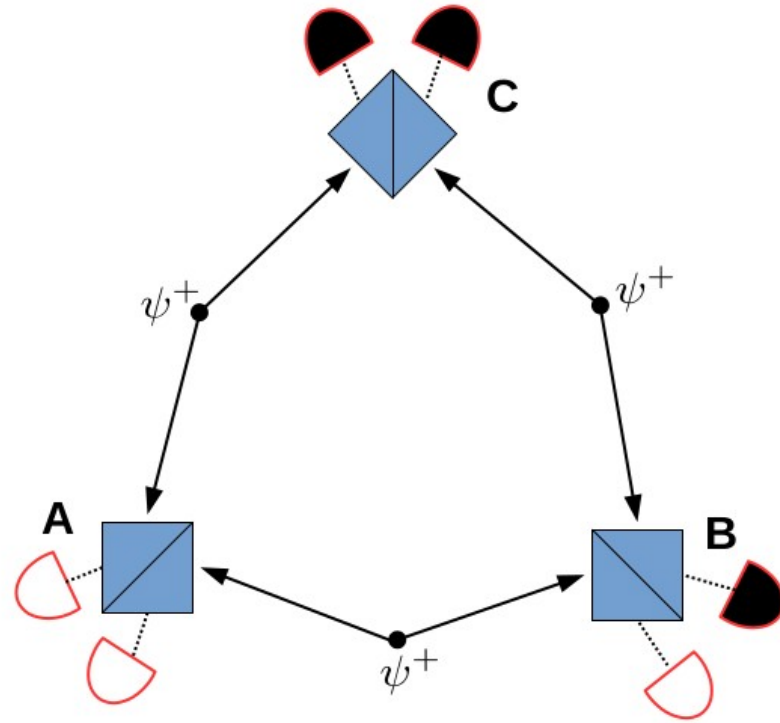
Back-up slides



$$|\psi_\gamma\rangle_{A_\gamma B_\gamma} = |\psi_\alpha\rangle_{B_\alpha C_\alpha} = |\psi_\beta\rangle_{C_\beta A_\beta} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

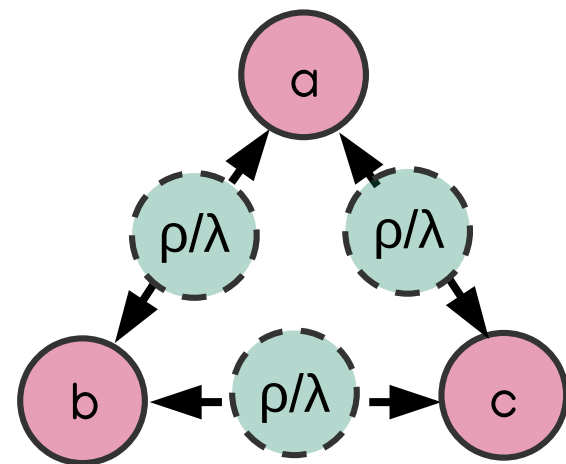
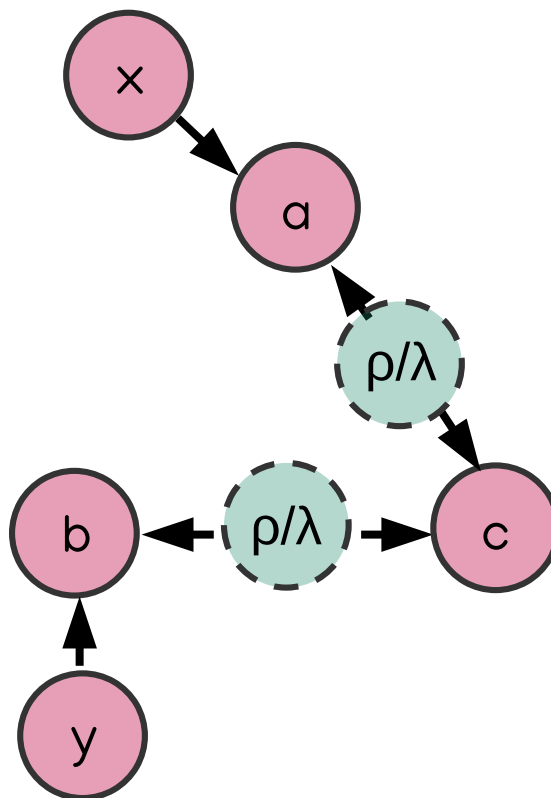
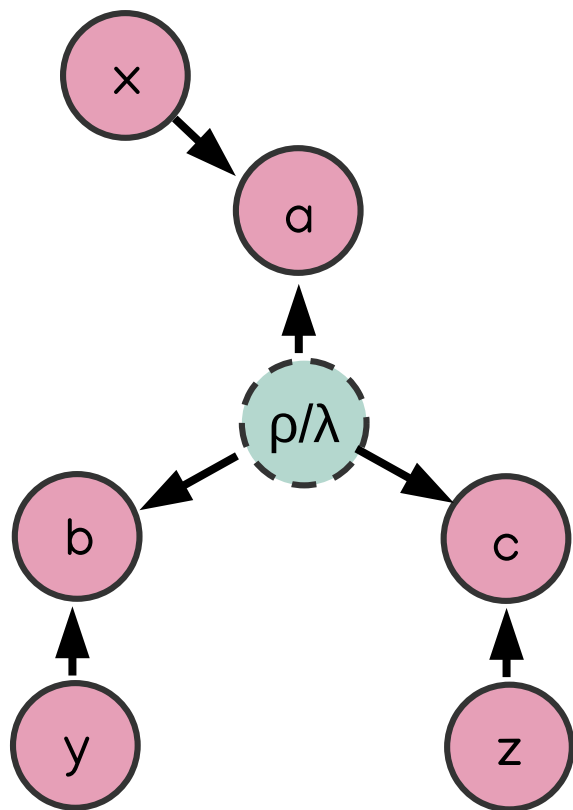
$$|\uparrow\rangle = |01\rangle, \quad |\chi_0\rangle = u|00\rangle + v|11\rangle,$$

$$|\downarrow\rangle = |10\rangle, \quad |\chi_1\rangle = v|00\rangle - u|11\rangle,$$



[Abiuso, TK et al., PRR 4 (1), L012041 (2022)]

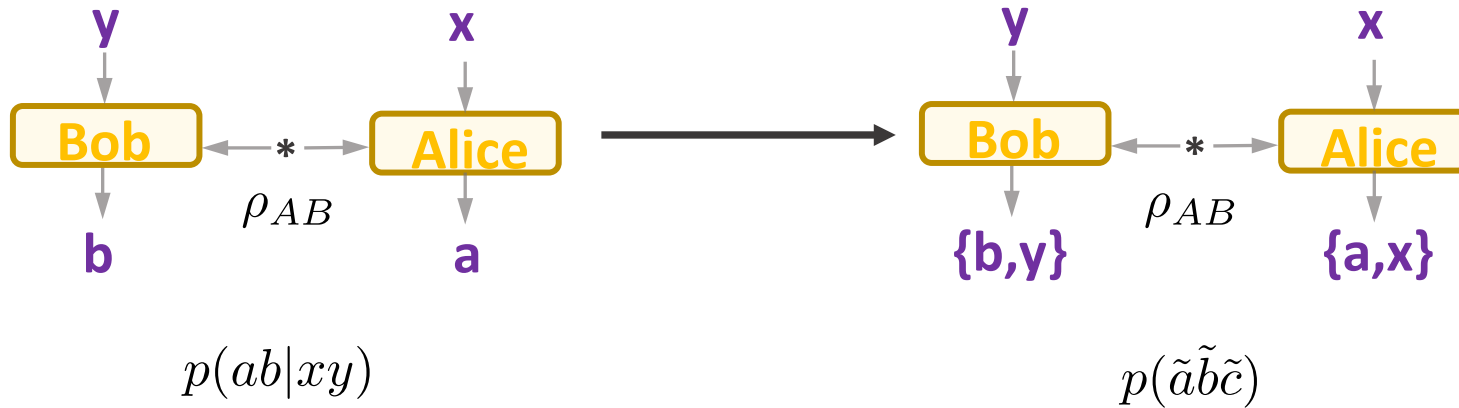
Bell nonlocality on networks



Nonlocality in networks

CHSH

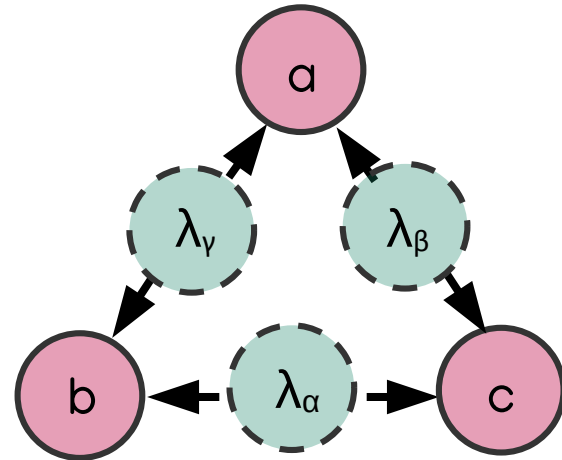
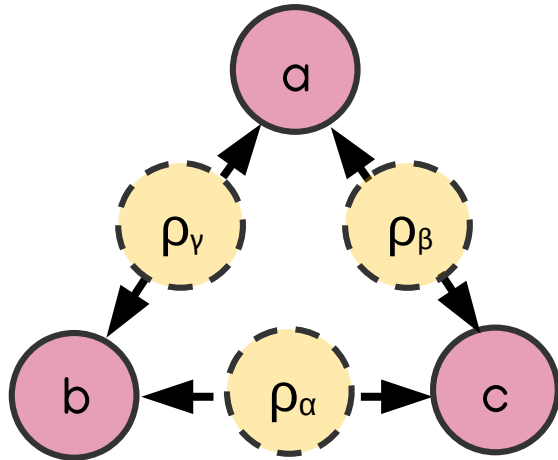
$$\rho(v) = v|\psi^-\rangle\langle\psi^-| + (1-v)\frac{\mathbb{I}}{4}$$
$$A_0 = X, B_0 = (X + Z)/2$$
$$A_1 = Z, B_1 = (X - Z)/2$$



How?

In [Sekatski et al., arXiv:2209.09921 (2022)], they prove a **partial self-test**, namely given $p(abc)$ from [Renou et al., PRL 123, 14041 (2019)], they show

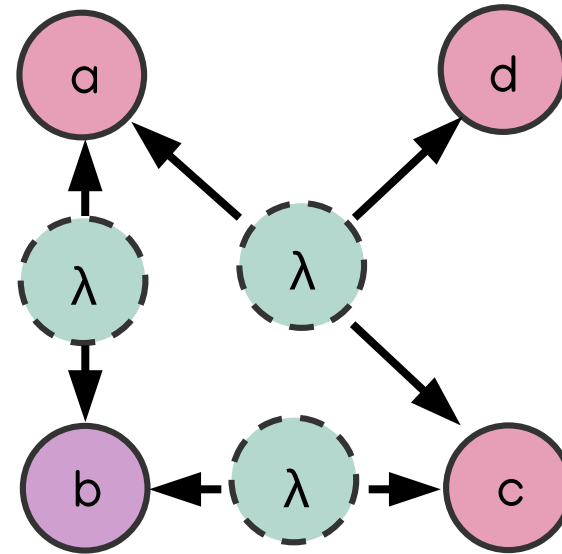
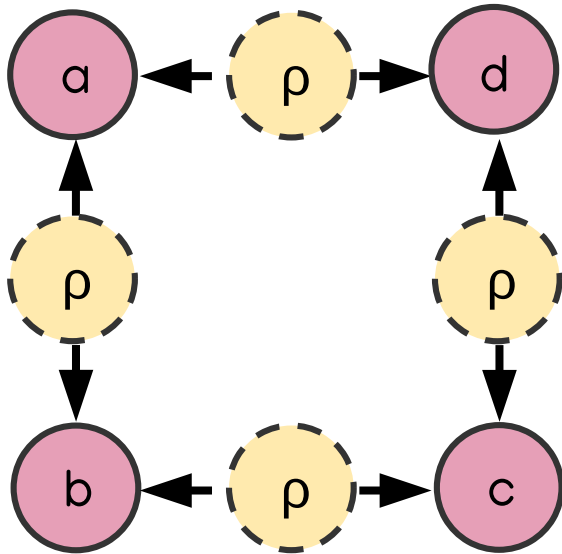
- all parties must share **entangled** states,
 - all parties must conduct **entangling measurements**,
 - there must be some randomness in outputs of **each** party,
- given that sources are independent



How?

Adapt [Sekatski et al., arXiv:2209.09921 (2022)] for the following networks.

- neighbors of honest parties must share entangled states,
- neighbors of honest parties must conduct entangling measurements,
- there must be some randomness in outputs of these parties...



Quantum Correlations

1964, John Bell



Goal: if

even, even \rightarrow HH or TT (correlated)

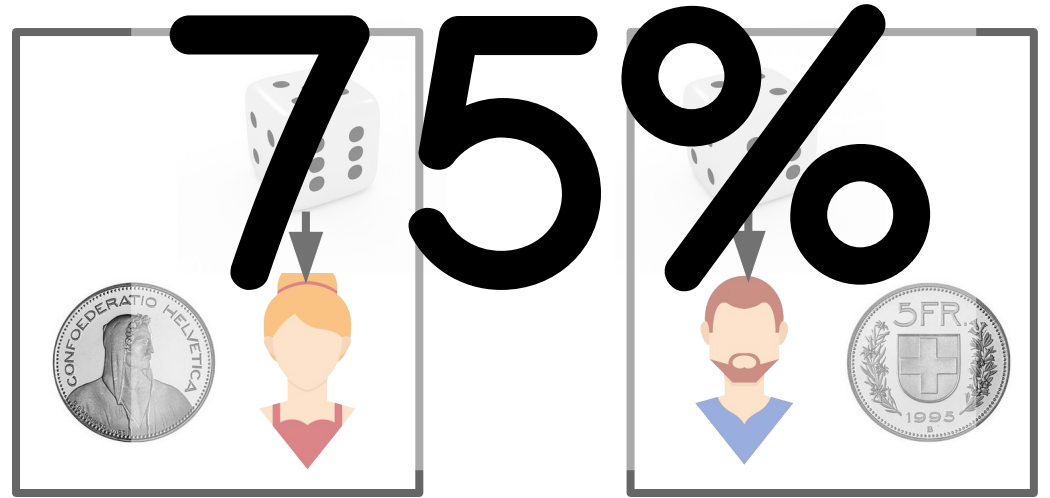
odd, even \rightarrow HH or TT (correlated)

even, odd \rightarrow HH or TT (correlated)

odd, odd \rightarrow **HT** or **TH** (anticorrelated)

Quantum Correlations

1964, John Bell



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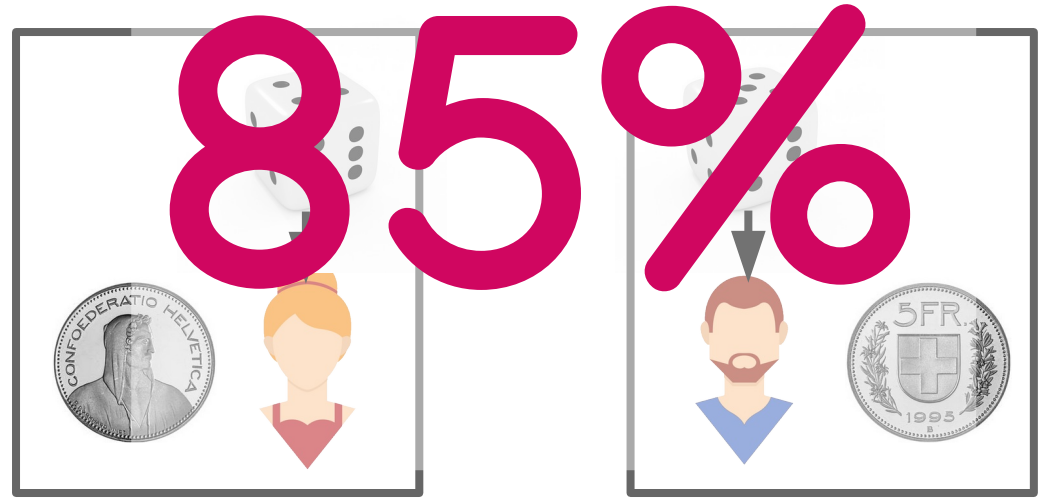
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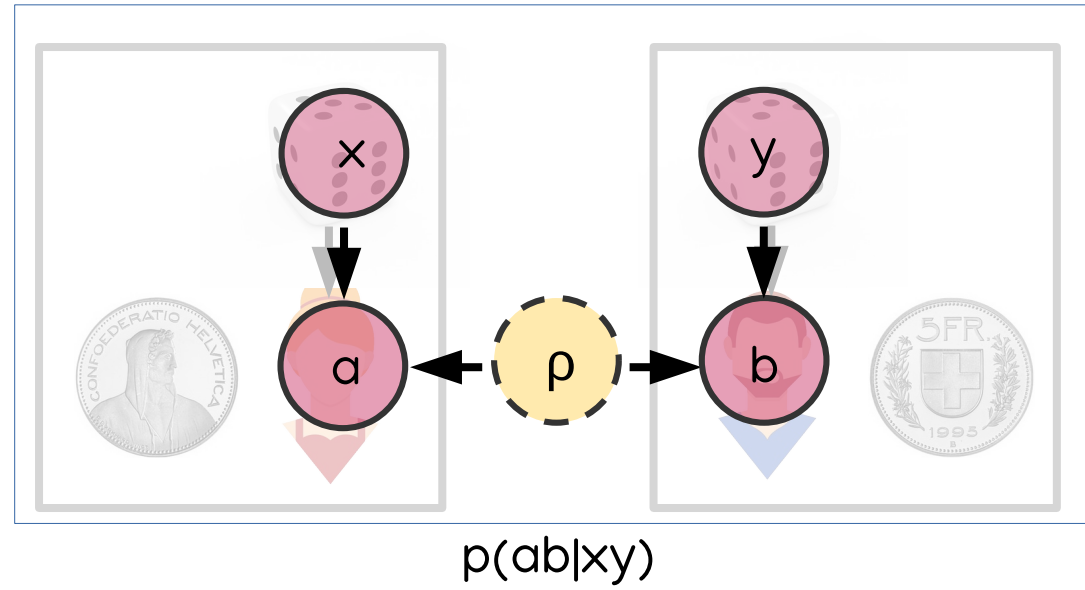
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Quantum Correlations

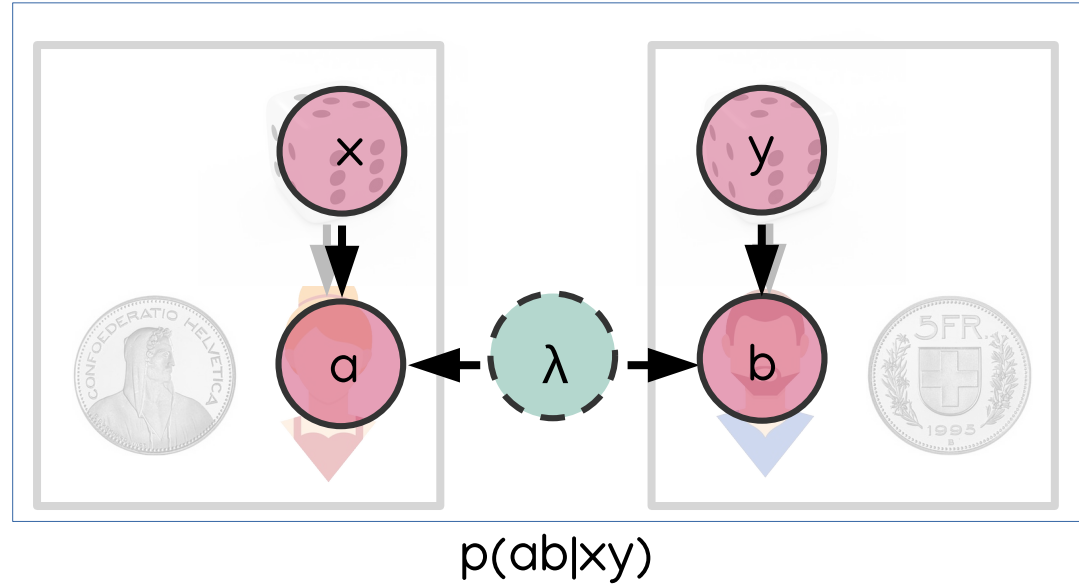
1964, John Bell



Quantum Correlations

1964, John Bell

- Use causal model

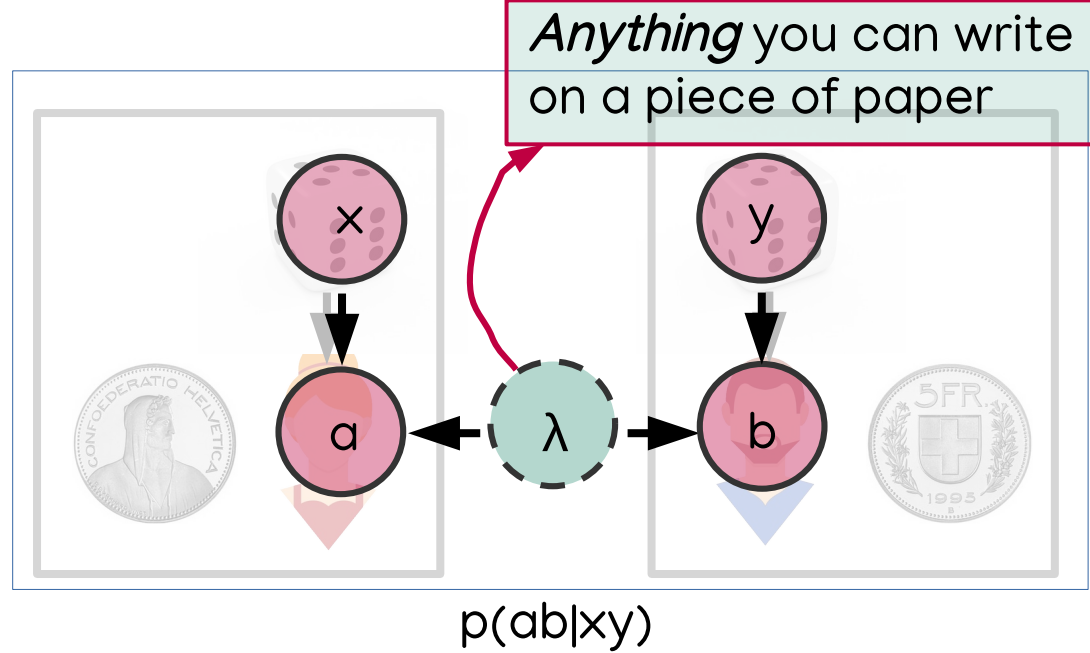


Q: Given **observed correlations** $p(ab|xy)$, does there exist a **classical causal** explanation? **Answer:** sometimes **NO!**

Quantum Correlations

1964, John Bell

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- **No** classical explanation

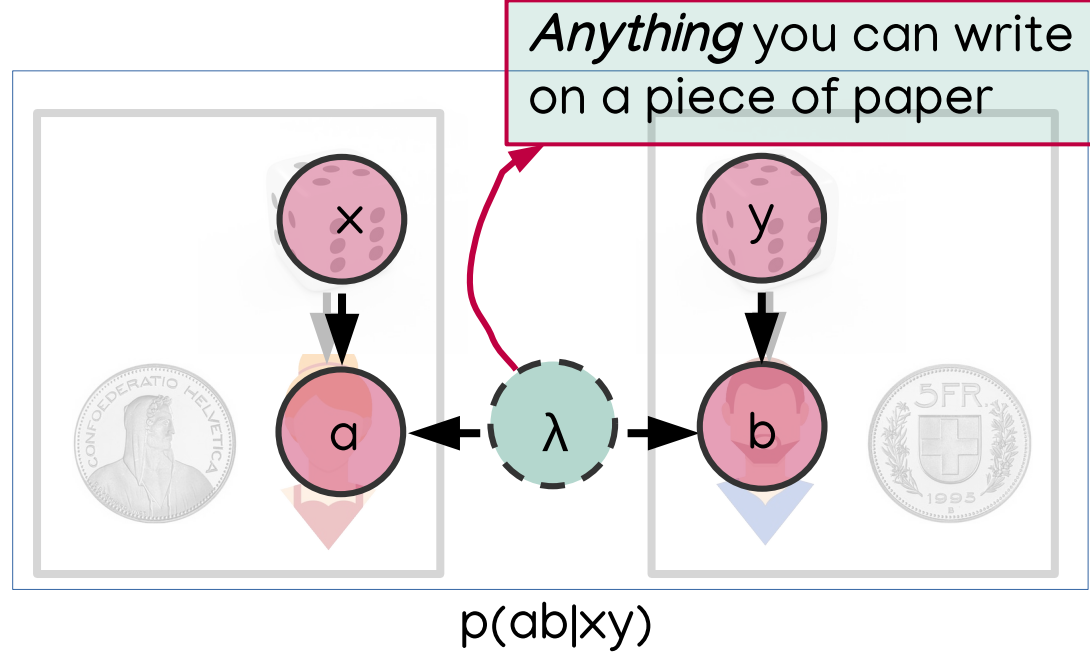


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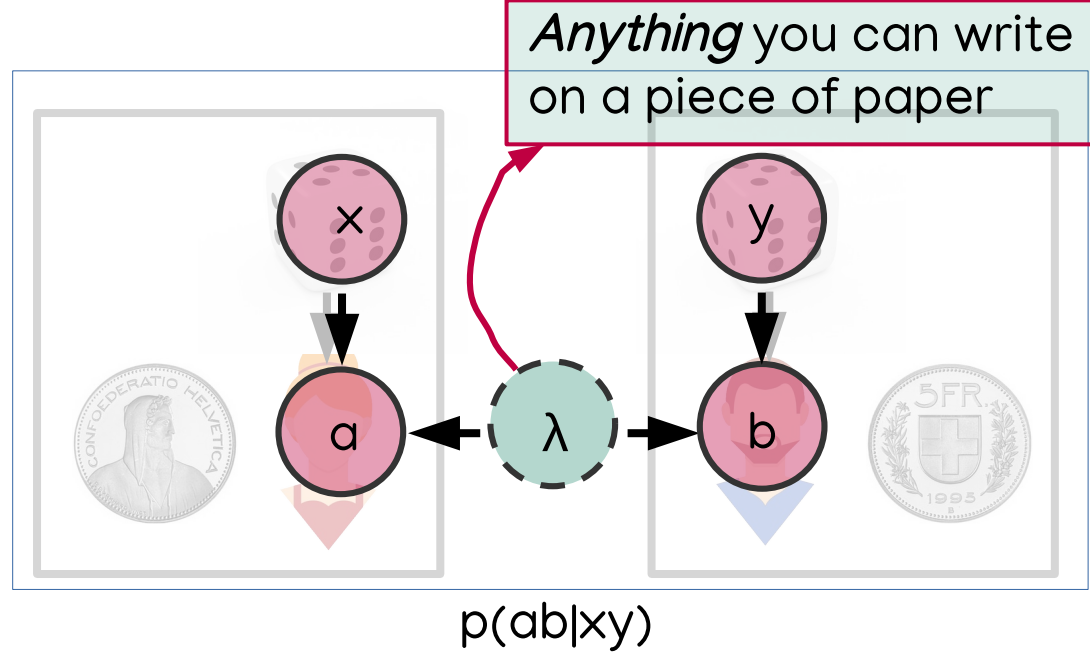


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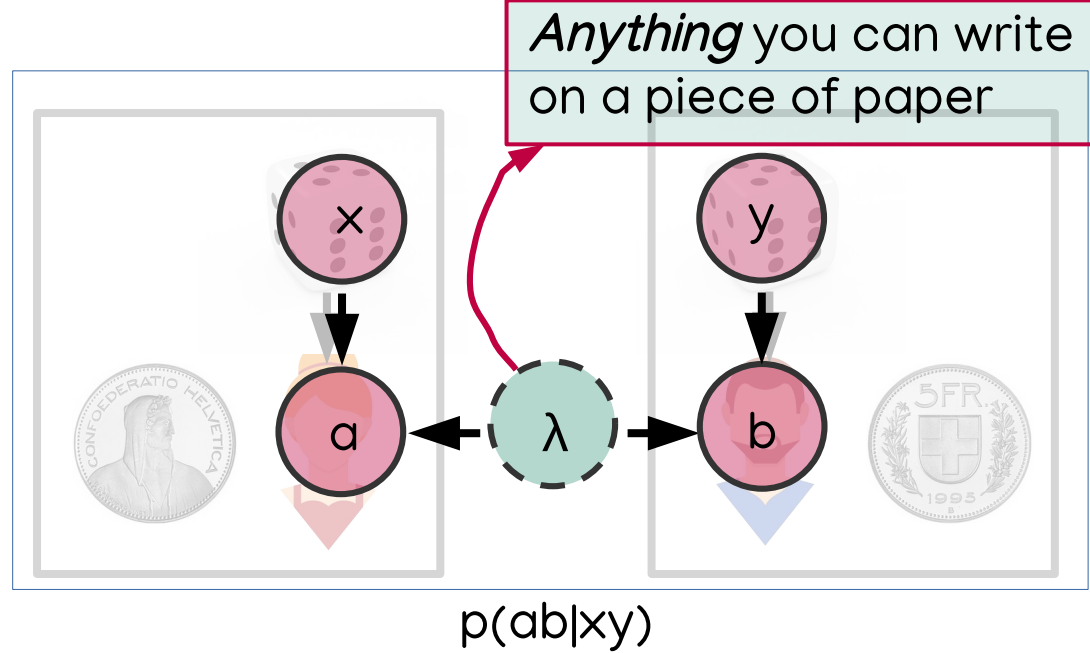


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Quantum cryptography, computing, ...

