# Quantum supremacy in mechanical tasks: projectiles, rockets and quantum backflow


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# Some mechanical tasks with quantum advantage

- Quantum tunnelling (1927?)

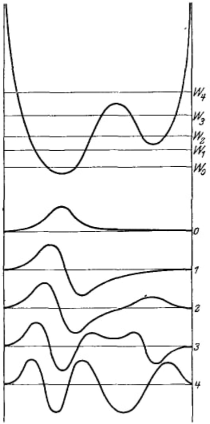


Fig. 2.

Figure: Hund, F. Zur Deutung der Molekelspektren. I. Z. Physik 40, 742–764 (1927)

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APPENDIX C. THE CURIOUS ROLE OF THE PROBABILITY CURRENT

Our first task here is to show that the ideal probability current $j(t, 0)$ (Eq. 5.1) can be appreciably negative for an appreciable part of the total time interval, even though $\psi$ itself is travelling wholly in the positive direction. It will suffice to take a wave of the type treated in Appendix A, having just two components with positive energies $E_1$ and $E_2(>E_1)$ and real amplitudes $a_1$ and $a_2$. Using (A.5) and (5.1) we find that

$$j(t,0) = \sigma\pi^{-\frac{1}{2}} \exp(-\sigma^2 t^2)[a_1{}^2 + a_2{}^2 + a_1 a_2\{(E_1/E_2)^{\frac{1}{4}} + (E_2/E_1)^{\frac{1}{4}}\} \cos(E_2 - E_1)t]. \quad \text{(C.1)}$$

It is clear that $j$ here is not always positive, and that the backflow effects can be made indefinitely large by increasing the ratio $E_2/E_1$.

Figure: Allcock, G.R. The time of arrival in quantum mechanics III. The measurement ensemble. Ann.Phys. 53, 311 (1969)

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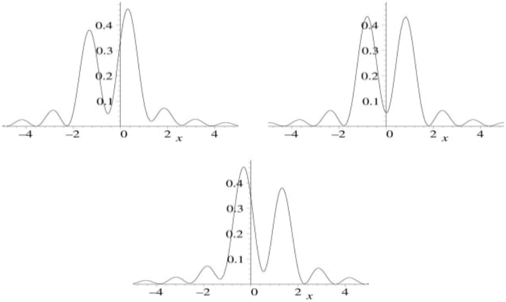


Figure 1: Evolution of a wavepacket under the free dynamics, illustrating the backflow phenomenon. From left to right, the plots show the position probability density at times $t = -0.1$, $t = 0$ and $t = 0.1$.

Figure: Eveson, S.P., Fewster, C.J. & Verch, R. Quantum Inequalities in Quantum Mechanics. Ann. Henri Poincaré 6, 1–30 (2005)

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The Bracken-Melloy constant [1]

$$c_{bm} = \sup_{\hat{\psi} \in L^2([0,\infty),dp),\Delta T} \int_0^{\Delta T} -j(0,t)dt,$$

where

$$j(x,t) := \frac{1}{m} Im[\overline{\psi}(x,t)\partial_x \psi(x,t)]$$

is the usual probability current density. Estimated to be $c_{bm} \approx 0.04$.

[1] A J Bracken and G F Melloy. Probability backflow and a new dimensionless quantum number. J. Phys. A: Math. Gen. 27 2197 (1994)

# Some mechanical tasks with quantum advantage

- Quantum tunnelling
- Quantum backflow
- Tsirelson's other problem (2006)

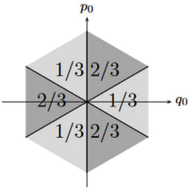


Figure: Tsirelson, B.S. How often is the coordinate of a Harmonic oscillator positive? arXiv:quant-ph/0611147

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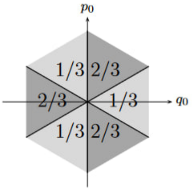


Figure: Tsirelson, B.S. How often is the coordinate of a Harmonic oscillator positive? arXiv:quant-ph/0611147

However, in quantum mechanics:

$$1 > \frac{1}{3} \sup_{\psi \in L^2(\mathbb{R})} \langle \psi | \, \Theta(X) + e^{\frac{iHT}{3}} \Theta(X) e^{-\frac{iHT}{3}} + e^{\frac{iH2T}{3}} \Theta(X) e^{-\frac{iH2T}{3}} \, | \psi \rangle \gtrsim 0.71$$

# Some mechanical tasks with quantum advantage

- Quantum tunnelling
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- Tsirelson's other problem (2006)

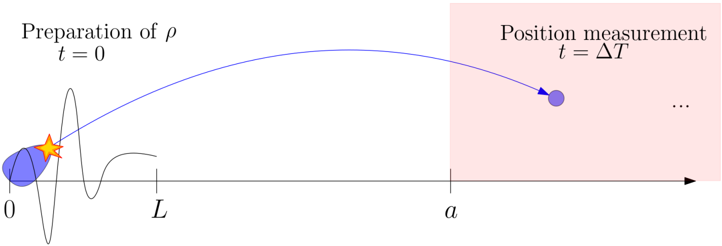
Finally picked up in [2] to perform quantumness [3] and entanglement [4] certification.

[2] Zaw, L.H., Aw, C.C., Lasmar, Z., Scarani, V. Detecting quantumness in uniform precessions. Phys.Rev.A 106, 032222 (2022)

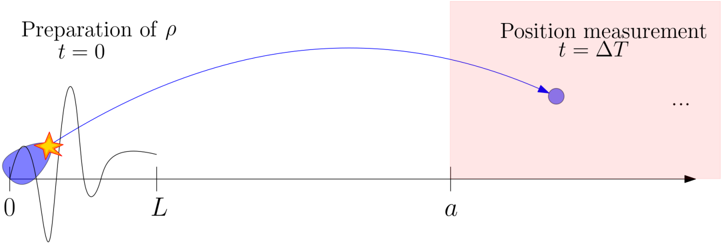
[3] Zaw, L.H. and Scarani, V. Dynamics-based quantumness certification of continuous variables with generic time-independent Hamiltonians. arXiv:2212.06017

[4] Jayachandran, P., Zaw, L.H., Scarani, V. Dynamic-based entanglement witnesses for harmonic oscillators. arXiv:2210.10357

# Quantum projectiles



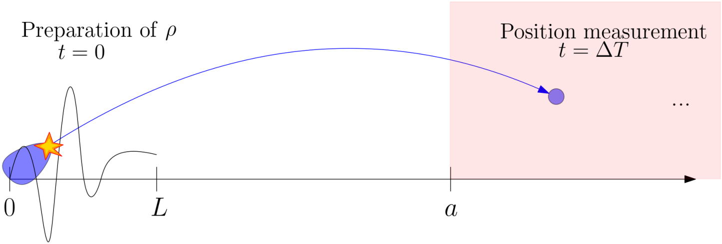
# Quantum projectiles




We compare a state $\psi \in L^2[0, L]$ with a classical particle confined to $[0, L]$ with momentum probability density $P(p) := \left|\hat{\psi}(p)\right|^2$.

$$\mathrm{Prob}_c(x(\Delta T) \geq a) \geq \mathrm{Prob}\left(p \geq \frac{(a-L)m}{\Delta T}\right)$$

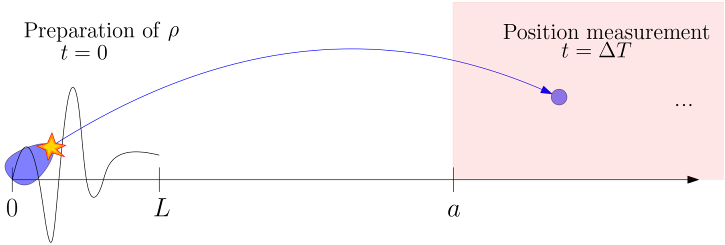
# Quantum projectiles




$$\mathrm{Prob}_c(x(\Delta T) \geq a) = \mathrm{Prob}\left(p \geq \frac{(a-L)m}{\Delta T}\right) = \int_{\frac{(a-L)m}{\Delta T}}^{\infty} \left|\hat{\psi}(p)\right|^2 dp$$

$$\mathrm{Prob}_q(x(\Delta T) \geq a) = \langle\psi| \, e^{iH\Delta T} \Theta(X-a) e^{-iH\Delta T} \, |\psi\rangle \, .$$

# Quantum projectiles




Quantity of interest:

$$\varphi := \sup_{\psi \in L^2[0,L]} \left( \langle \psi | \, e^{iH\Delta T} \Theta(X - a) e^{-iH\Delta T} \, | \psi \rangle - \int_{\frac{(a-L)m}{\Delta T}}^{\infty} \left| \hat{\psi}(p) \right|^2 dp \right) .$$

# Quantum projectiles

$$\varphi := \sup_{\psi \in L^2([0,L],dx)} \left( \langle \psi | e^{iH\Delta T} \Theta(X-a) e^{-iH\Delta T} | \psi \rangle - \int_{\frac{(a-L)m}{\Delta T}}^{\infty} \left| \hat{\psi}(p) \right|^2 dp \right)$$

$$= \sup_{\psi \in L^2([0,L],dx)} \langle \psi | \Theta\left( X + P\frac{\Delta T}{m} - a \right) - \Theta\left( P - \frac{(a-L)m}{\Delta T} \right) | \psi \rangle$$

$$= \sup_{\psi} \int_{\mathbb{R}^2} \Theta\left( x + p\frac{\Delta T}{m} - a \right) - \Theta\left( p - \frac{(a-L)m}{\Delta T} \right) W_\psi(x,p) dx dp,$$

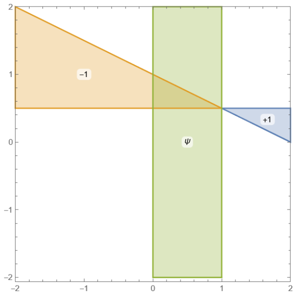
where

$$W_\psi(x,p) := \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(x - y/2) \overline{\psi(x + y/2)} e^{ipy} dy$$

is the Wigner quasi-probability distribution.

# Quantum projectiles

$$\sup_{\psi \in L^2([0,L],dx)} \int_{\mathbb{R}^2} \Theta\left(x + p\frac{\Delta T}{m} - a\right) - \Theta\left(p - \frac{(a-L)m}{\Delta T}\right) W_\psi(x,p)$$



# Quantum projectiles

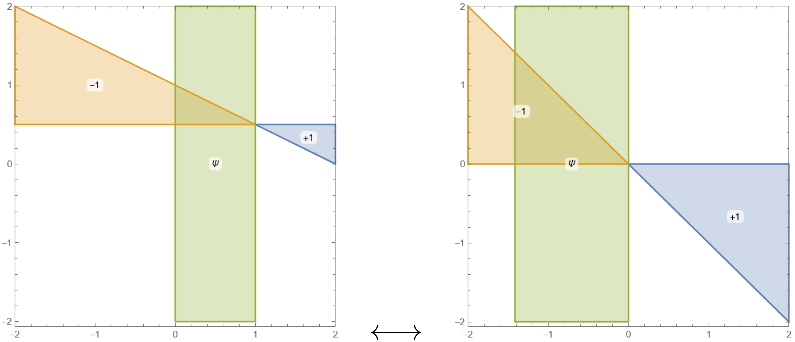



Figure: With $\Delta T = 2m$, $L = 1$, $a = 2$, we perform the transformation $X \mapsto \sqrt{2}(X-1)$, $P \mapsto \sqrt{0.5}P - \sqrt{2}$

# Quantum projectiles

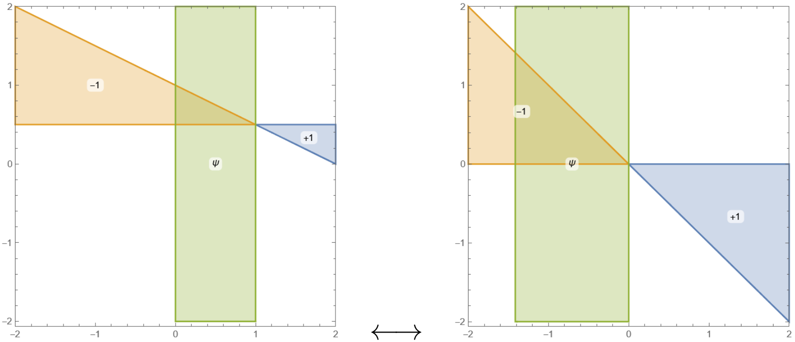
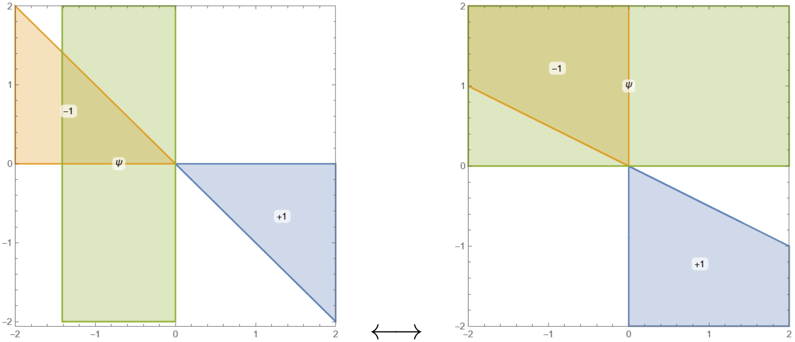



Figure: With $\Delta T = 2m$, $L = 1$, $a = 2$, we perform the transformation $X \mapsto \sqrt{2}(X-1)$, $P \mapsto \sqrt{0.5}P - \sqrt{2}$

So $\varphi$ only depends on $\alpha = \dfrac{mL^2}{\Delta T}$,

# Quantum projectiles



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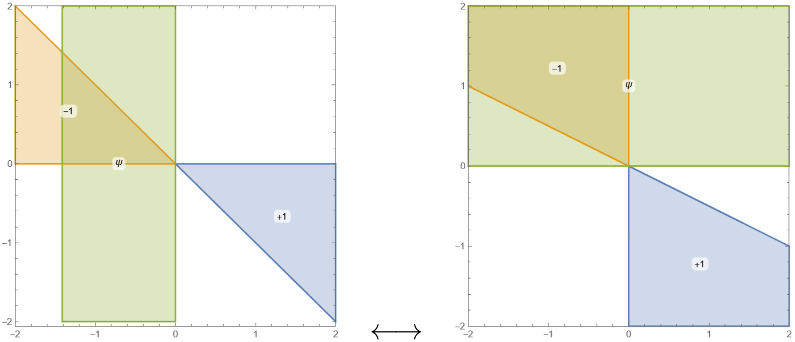



So $\varphi$ only depends on $\alpha = \dfrac{mL^2}{\Delta T}$, and is actually equal to $c_{bm}$ in the limit $\alpha \to \infty$.

# Quantum projectiles: lower bounds

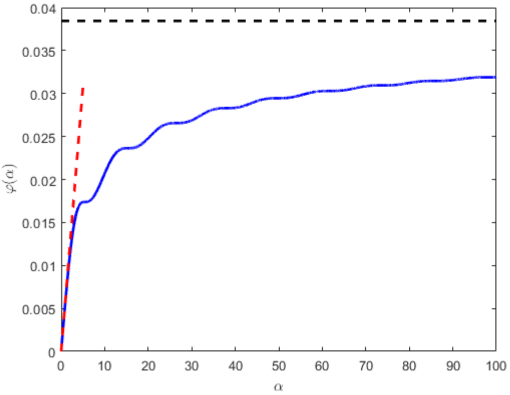
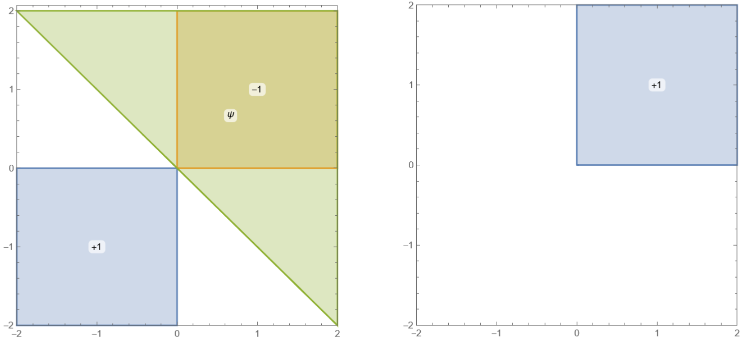


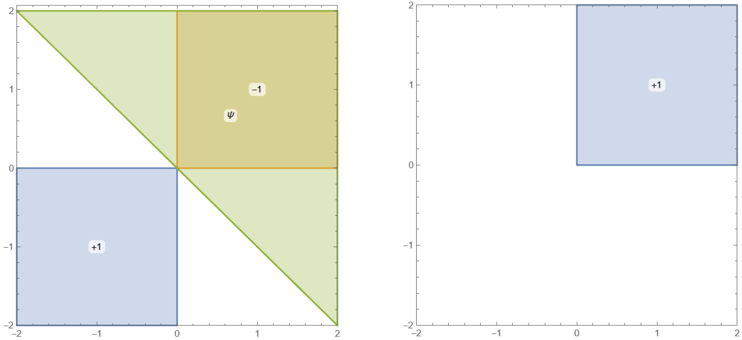

Figure: Blue, continuous: $\varphi(\alpha)$ to precision $10^{-4}$. Red, dashed: A linear approximation near 0. Black, dashed, constant: The conjectured value of $c_{bm}$.

# Quantum projectiles: upper bounds[5]




[5] R F Werner, Wigner quantisation of arrival time and oscillator phase, J. Phys. A: Math. Gen. 21 4565 (1988)

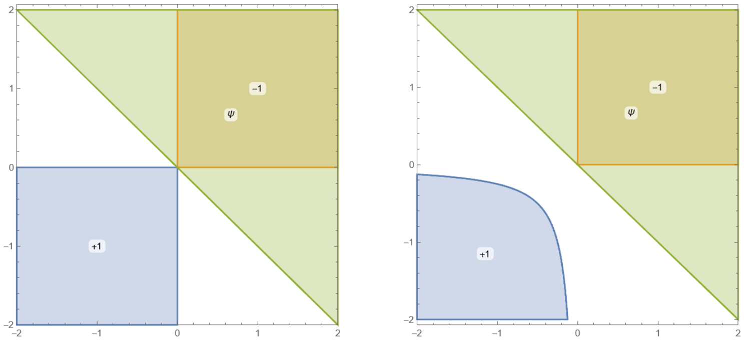
# Quantum projectiles: upper bounds[5]




Werner's operator: spectrum in $\approx [-0.1559, 1.0077]$

[5] R F Werner, Wigner quantisation of arrival time and oscillator phase, J. Phys. A: Math. Gen. 21 4565 (1988)

# Quantum projectiles: upper bounds[6]

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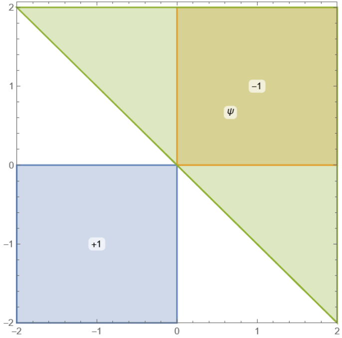
# Quantum projectiles: upper bounds[6]





Result: $c_{bm} \leq 0.08$ (Werner: $c_{bm} \leq 0.15$)

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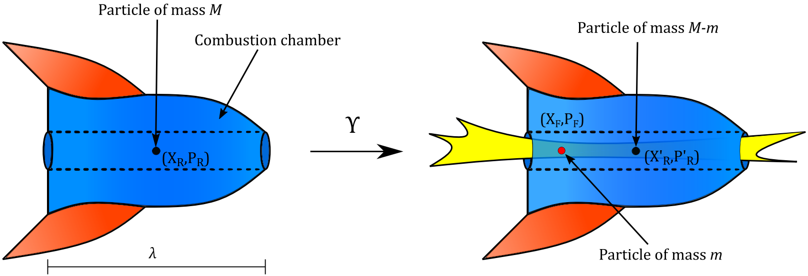
# Quantum rockets: an advantage beyond $c_{bm}$?



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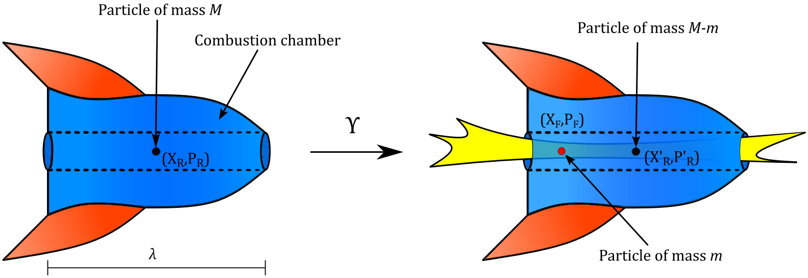


No.

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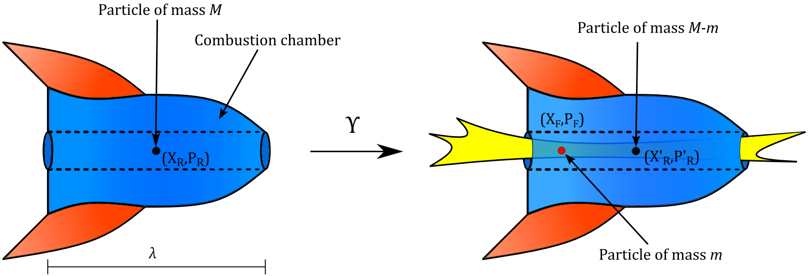


No.

If we further fix in the projectile problem the position probability density of the particle to be $P(x) := |\psi(x)|^2$, then we can go beyond $c_{bm}$ to $\geq 0.1262$

# Thank you for your attention!