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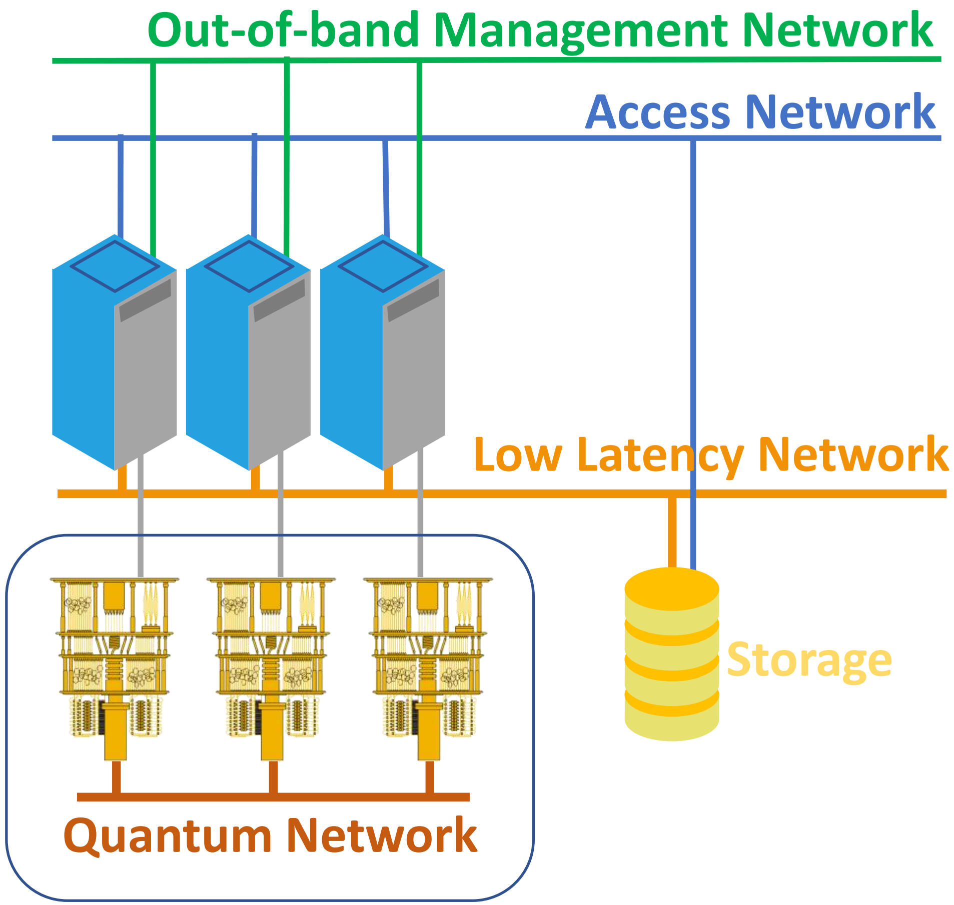
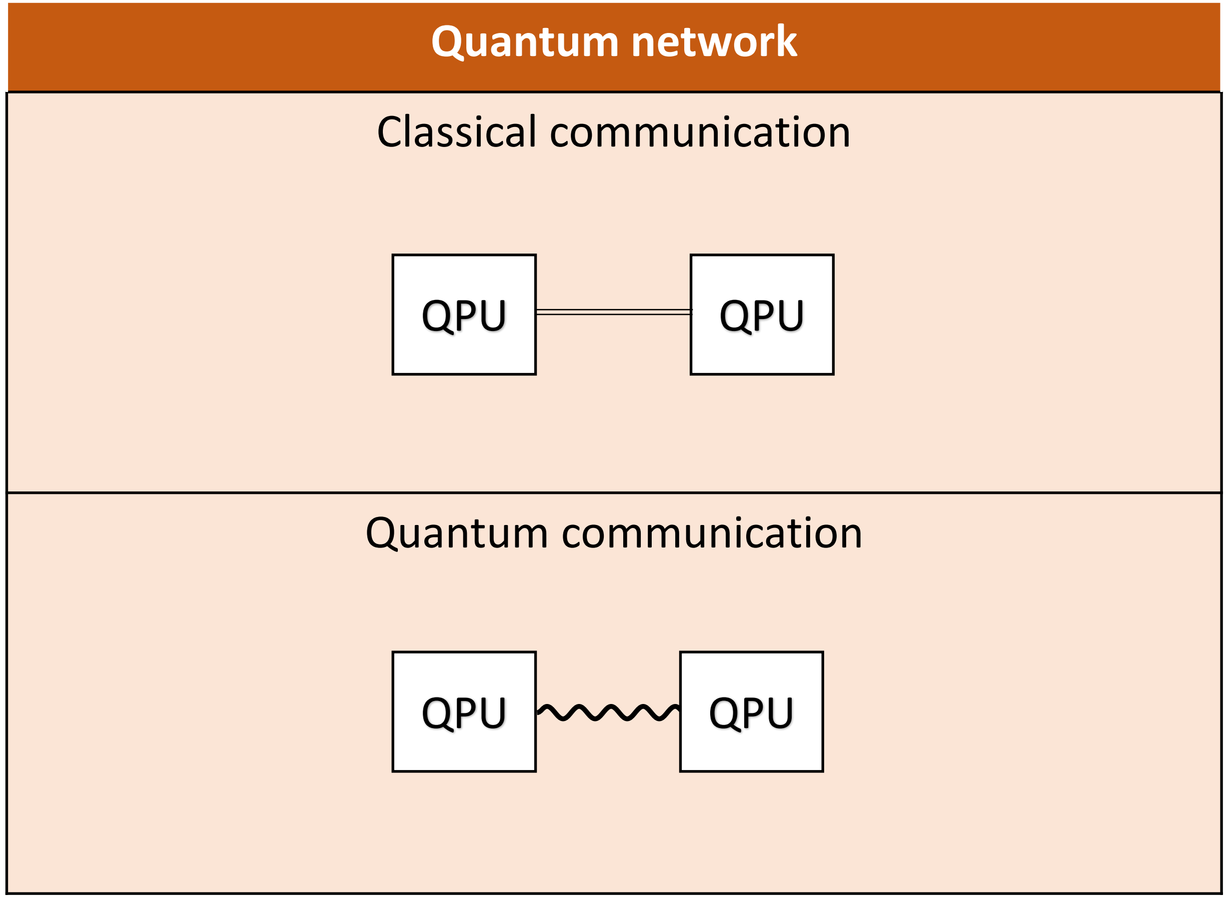
Parallelizing quantum algorithms with classical communication

PARALLEL QUANTUM COMPUTING





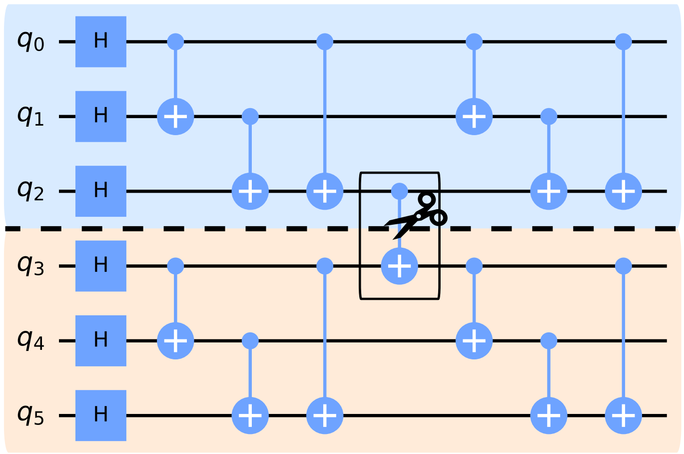
MOTIVATION

- Explore the limits of algorithm parallelization with classical communication. (Piveteau and Sutter, 2022)
- Develop tools determining whether is a good option to parallelize a given algorithm
- Use a framework that allows a smooth transition between the scenarios of classical communication and quantum communication.

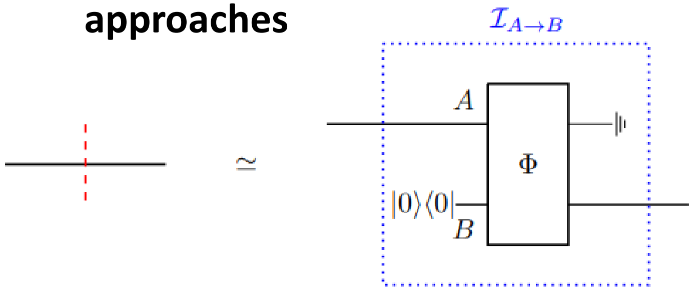
CIRCUIT CUTTING

Circuit cutting techniques

- Wire cutting (Peng, 2020)
- Gate cutting (Mitarai, 2021)



Relation between both approaches



(Brenner et al., 2023)

Sampling overhead grows ***exponentially*** *with the number of cuts.*

DECOMPOSING A TWO QUBIT GATE

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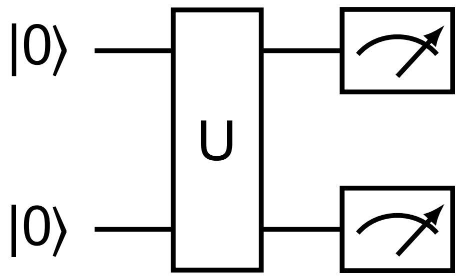


→

Quantum operation ξ_U

A linear map between density operators.

$$\rho_i \xrightarrow{\xi_U} \rho_f$$

Quantum opetations correspond to CPT maps →

Choi matrix representation CP map

$$C_\xi \in \mathcal{M}_{d^2\times d^2}\ s.t.\ C_\xi \geq 0$$

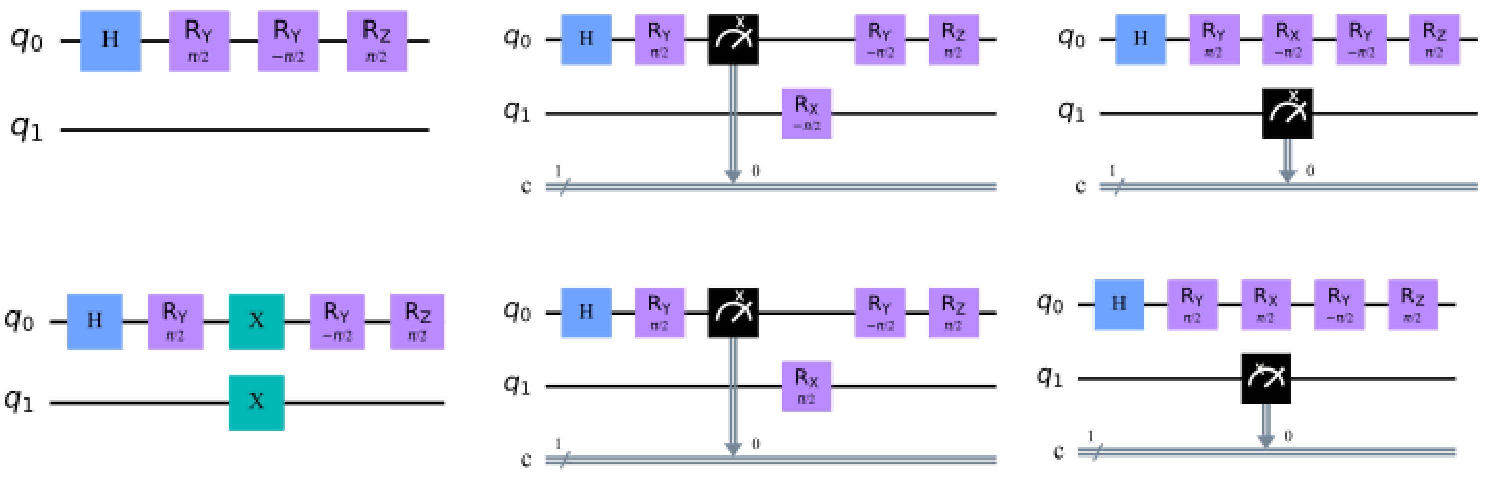
$$\xi(\rho) = \mathrm{Tr}_B\left(C_\xi(\mathbb{I}_d \otimes \rho)\right)$$

Quasiprobability decomposition for gate cutting

$$\{q_i \in \mathbb{R}, C_{\xi_{A_i}\otimes\xi_{B_i}}\} \rightarrow C_{\xi_U} = \sum_{i=0}^{m-1} q_i C_{\xi_{A_i}\otimes\xi_{B_i}}$$

EXAMPLE: CNOT GATE

$$C_{CNOT} = \frac{1}{2} C_{I \otimes I} + \frac{1}{2} C_{\sigma_x \otimes \sigma_x} + \frac{1}{2} C_{R_x\left(\frac{\pi}{2}\right) \otimes (\Pi_x^+ - \Pi_x^-)}$$
$$+ \frac{1}{2} C_{(\Pi_x^+ - \Pi_x^-) \otimes R_x\left(\frac{\pi}{2}\right)} - \frac{1}{2} C_{R_x\left(\frac{\pi}{2}\right) \otimes (\Pi_x^+ - \Pi_x^-)} - \frac{1}{2} C_{(\Pi_x^+ - \Pi_x^-) \otimes R_x\left(\frac{\pi}{2}\right)}$$



Cost of quasiporbalistic simulation? ⟶ *Sampling overhead* $\kappa^2 = \left(\sum |q_i|\right)^2$

$\kappa_{CNOT}^2 = 3^2$ *Optimal?*

OPTIMAL DECOMPOSITION

Problem. For a given C_{ξ_U}, find $\left\{C_{\xi_{A_i}\otimes\xi_{B_i}}\right\}_{i\in\mathbb{Z}_m}$ and $\{q_i\}_{i\in\mathbb{Z}_m}$ such that $C_{\xi_U} = \sum_{i=0}^{m-1} q_i C_{\xi_{A_i}\otimes\xi_{B_i}}$ minimal $\sum|q_i|$

Definition. $\gamma_U \coloneqq \min\sum|q_i|$

$$\gamma_U = 2RoE\left(C_{\xi_U}/d\right) + 1$$

$$RoE\left(C_{\xi_U}/d\right) = \left(\sum_i \alpha_i\right)^2 - 1$$

$\{\alpha_i\}_{i\in\mathbb{Z}_{2^d}}$ Schmidt coefficients

$$\gamma_U = 2\left(\sum_i \alpha_i\right)^2 - 1$$

(Piveteau and Sutter, 2022)

Robustness of entanglement (RoE)

- Entanglement monotone

$$RoE(\rho_{AB}) \coloneqq \min_{\sigma_{AB}\in SEP} R(\rho_{AB}||\sigma_{AB})$$

$$R(\rho_{AB}||\sigma_{AB}) \coloneqq \min_{t>0}\left\{t: \frac{\rho_{AB} + t\sigma_{AB}}{1+t} \in SEP\right\}$$

(Vidal and Tarranch, 1999)

Examples
$\gamma_{CNOT} = 3$
$\gamma_{iSWAP} = 7$
$\gamma_{SWAP} = 7$
$\gamma_{QFT_2} = 7$

SIMULATING MULTIPLE GATES

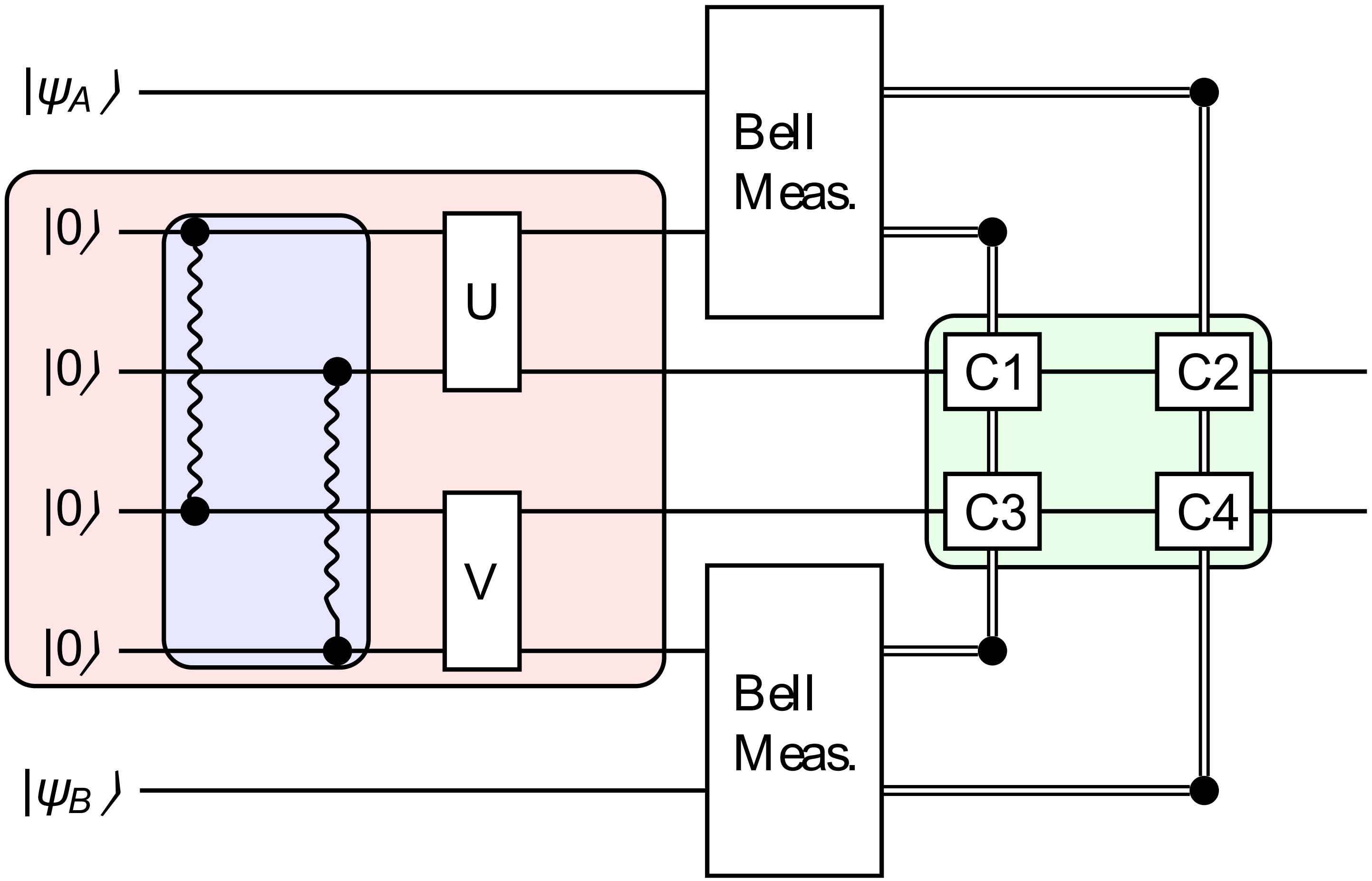
	γ	**Effective γ per gate for $n \to \infty$**
$CNOT^{\otimes n}$	3^n	3
$Bell^{\otimes n}$	$2^{n+1} - 1$	$\sqrt[n]{\lim_{n\to\infty} (2^{n+1} - 1)} = 2$

$Winning\ strategy$: $generate\ all\ entangled\ pairs\ at\ once\ and\ distribute\ them\ with\ gate\ teleportation$

Ingredients:

- LOCC scheme to implement gate teleportation
- Quasiprobability decomposition algorithm to simulate n Bell pairs

LOCC scheme I



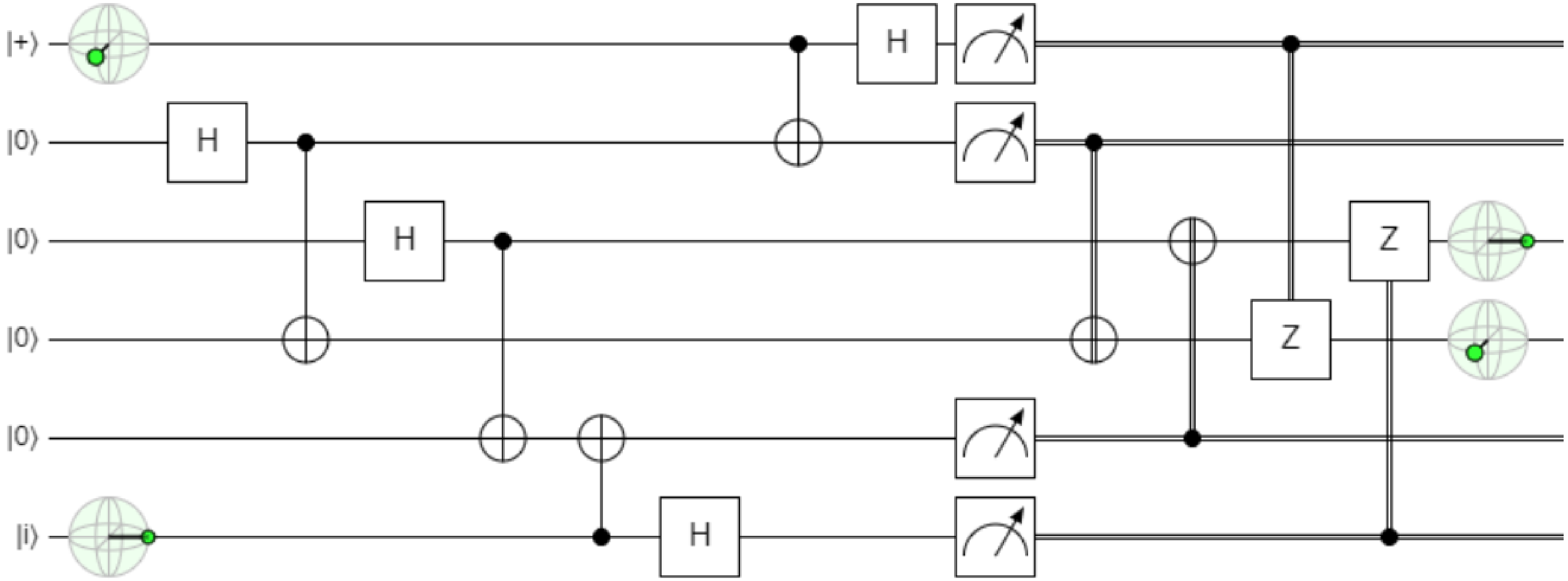
Entanglement factory (Bell pairs generation)

Choi state generation

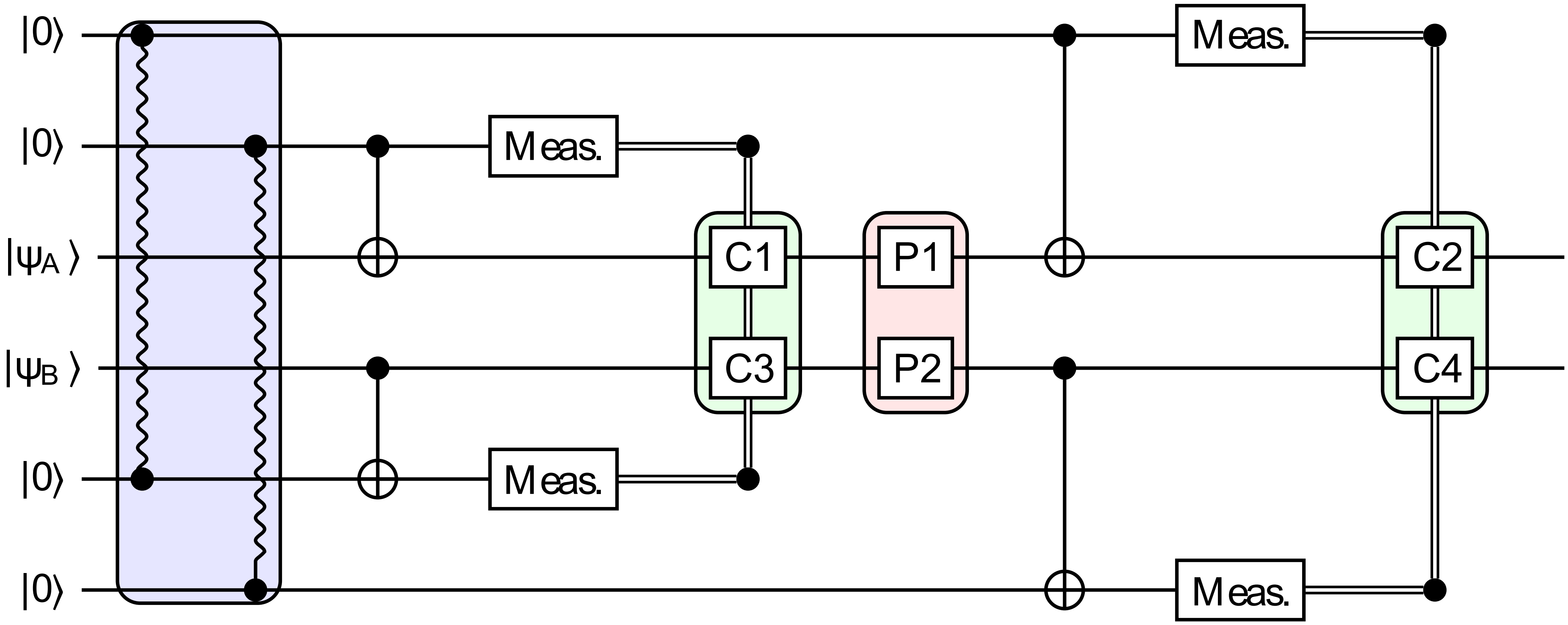
Correction gates

Only valid for Clifford gates!!

Example: SWAP gate



LOCC scheme II



ALGORITHM FOR N BELL PAIR GENERATION

Objective

$$\rho_{Bell^{\otimes n}} = \frac{1}{2^n}\sum_{i,j}^{2^n} |i\rangle|i\rangle\langle j|\langle j|$$

Quasiprobabilistic decomposition

$\rho_{Bell^{\otimes n}} = 2^n\rho^+ - (2^n - 1)\rho^-$ with $\rho^+, \rho^- \in SEP$

$$\rho^- = \frac{1}{2^n - 1}\sum_{i \neq j} |i\rangle|j\rangle\langle j|\langle j|$$

(Vidal and Tarranch, 1999)

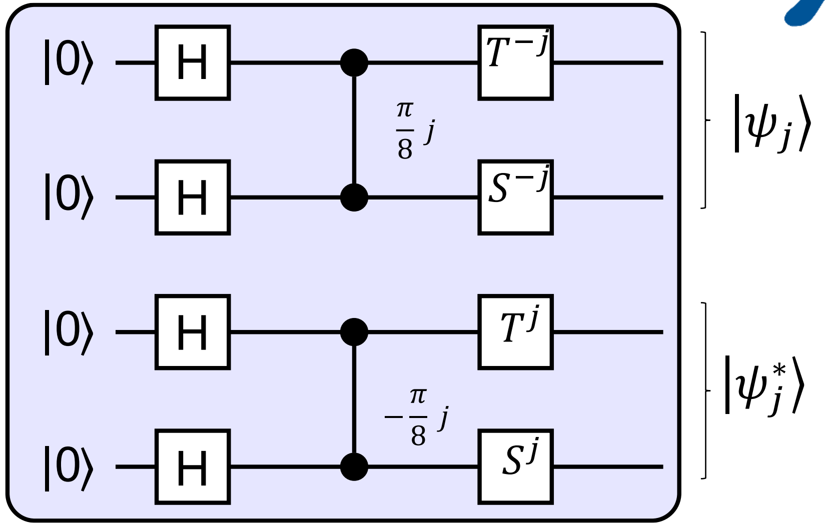
$$\rho^+ = \sum_{j=0}^{2^{2^n}-1} |\psi_j\rangle\langle\psi_j| \otimes |\psi_j^*\rangle\langle\psi_j^*|$$

$$|\psi_j\rangle = \left(\sum_{k=0}^{2^n-1} e^{\left(\frac{2^k-1}{2^k}j\right)\pi i} |k\rangle\right)$$

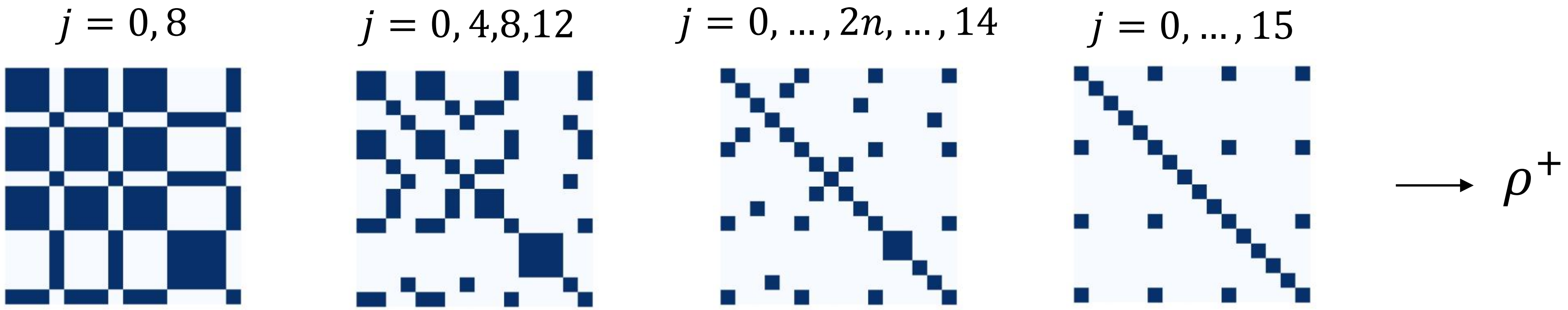
EXAMPLE: 2 BELL PAIR

$$\{|\psi_j\rangle\}_{0,\dots,15}$$

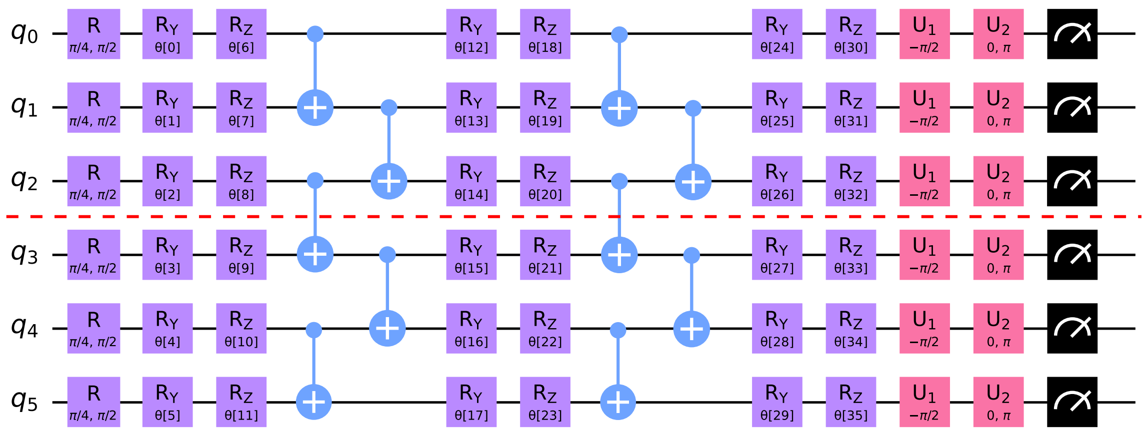
$$|\psi_j\rangle = \left(\sum_{k=0}^{3} e^{\left(\frac{2^k-1}{2^k}j\right)\pi i}|k\rangle\right)$$



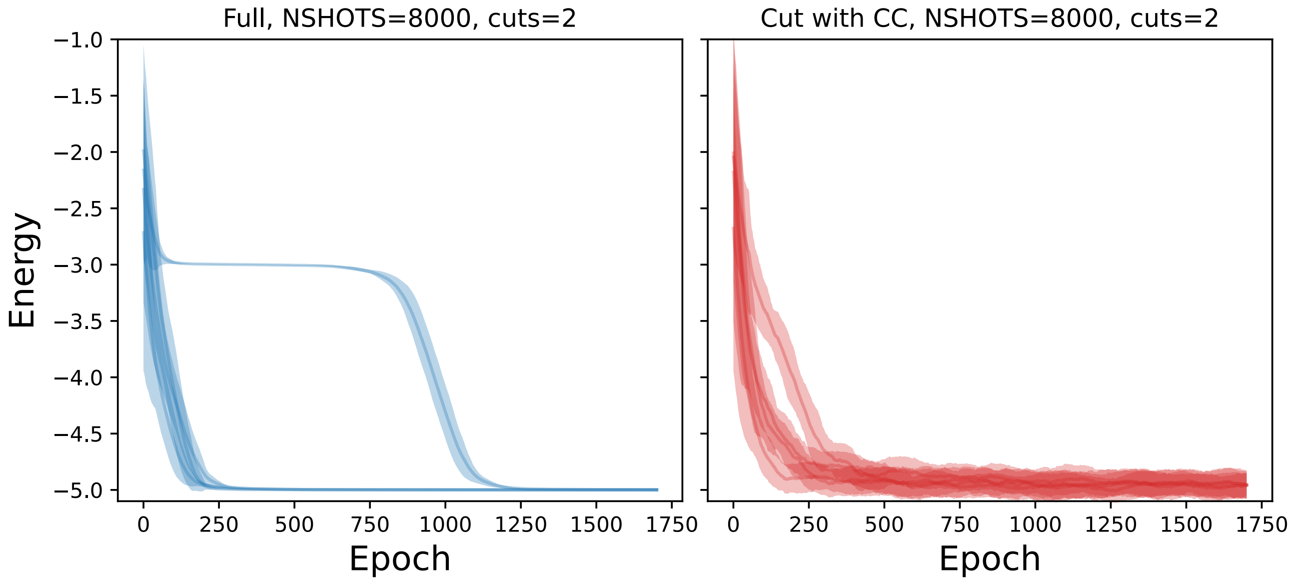
$$\sum_{j=0} |\psi_j\rangle\langle\psi_j| \otimes |\psi_j^*\rangle\langle\psi_j^*|$$



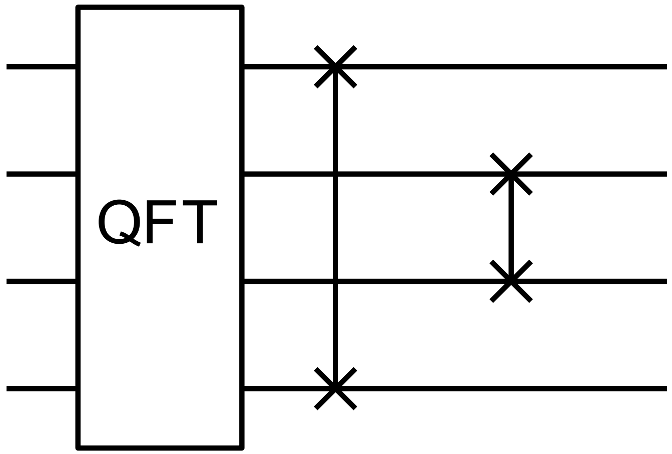
APPLICATION: 1D-ISING VQE



+ SPSA optimizer



No SWAP QFT



γ_U		
Qubits	QFT	QFT^*
2	7	2.414
4	31	3.027
6	126	3.206
2n (for high n)	$2^{n+1} - 1$	≈3.264

Thank you!

Funding:

Despliegue de una infraestructura basada en tecnologías cuánticas de la información que permita impulsar la I+D+i en Galicia.

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PROGRAMA OPERATIVO FEDER 2014-2020

Una manera de hacer Europa