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Efficient quantum simulation of jet evolution in a medium

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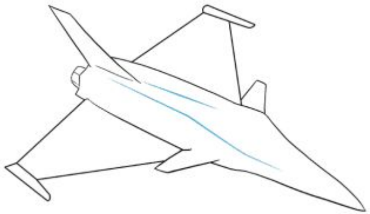
Based on 2104.04661, 2208.06750, 23XX.XXXX

In collaboration with
João Barata, Meijian Li, Xiaojian Du, Carlos Salgado

What is jet quenching?

Slide from M Li's talk at Qiskit Fall Fest at USC (2022)

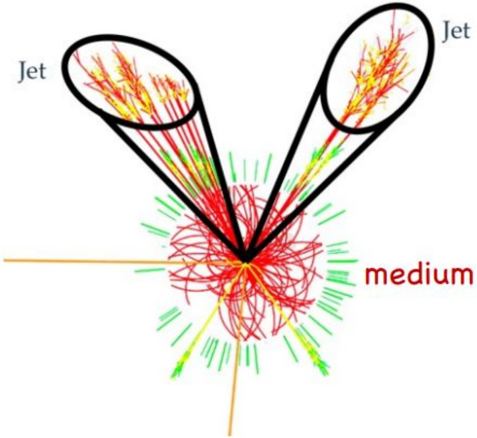


"Jet"
a rapid stream

"Quenching"
a rapid cooling process

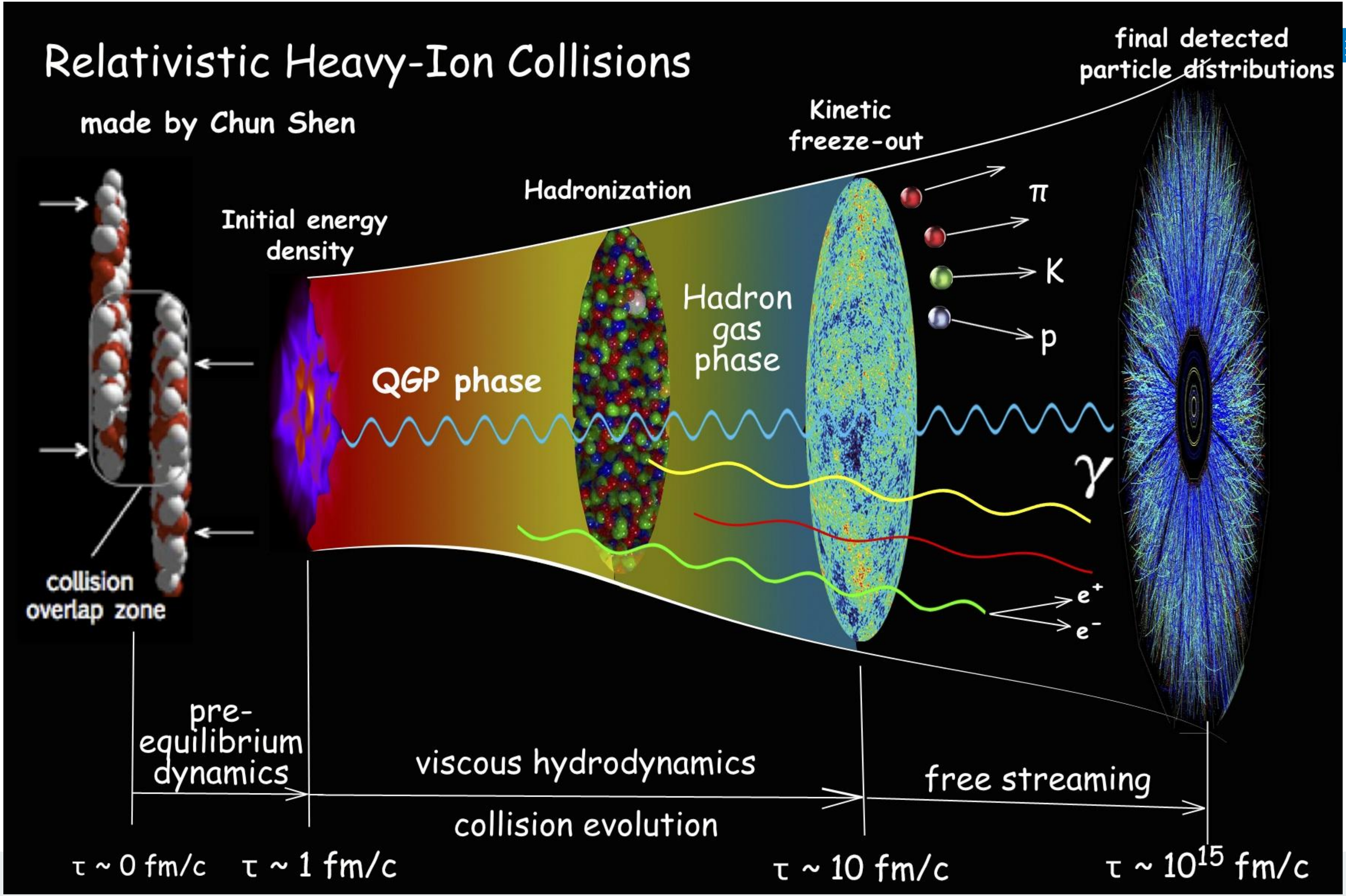
What is jet quenching?

Slide from M Li's talk at Qiskit Fall Fest at USC (2022)



In heavy ion collisions, a jet is a cone-shaped beam of energetic particles.

When propagating through the hot medium, it loses energy due to **jet-medium** interaction, a phenomenon known as “jet quenching”.



Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected particle distributions
Kinetic freeze-out
Hadronization
Initial energy density
Hadron gas phase
QGP phase
π
K
p
γ
e+
e−
collision overlap zone
pre-equilibrium dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

Outline

1. Light-front Hamiltonian and physical setups
2. Quantum simulation algorithm
3. Results

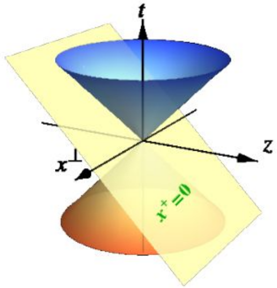
From LF Hamiltonian formalism to quantum simulation

Classical light-front Hamiltonian formalism

Scattering in Time-Dependent Basis Light-Front Quantization, PRD 88 (2013) 065014

Ultrarelativistic quark-nucleus scattering, PRD 101 (2020) 7, 076016

Scattering and gluon emission in a color field, PRD 104 (2021) 5, 056014



front form

$$x^+ \triangleq x^0 + x^3$$

Quantum simulation

Single-particle digitization strategy for quantum computation of a ϕ4 scalar field theory, PRA 103 (2021) 4, 042410

A quantum strategy to compute the jet quenching parameter, Eur.Phys.J.C 81 (2021) 10, 862

Quantum simulation of nuclear inelastic scattering, PRA 104 (2021) 1, 012611

Medium induced jet broadening in a quantum computer, PRD 106 (2022) 7, 074013

Quantum simulation of jet evolution in a medium (soon)

QCD Lagrangian

We start with the QCD lagrangian, with an external field

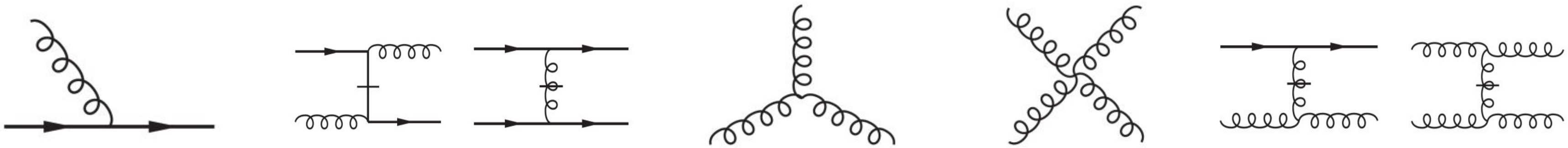
Li, Lappi, Zhao, PRD104.056014 (2021)

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}{}_{a} F^{a}_{\mu\nu} + \overline{\Psi}(i\gamma^{\mu} D_{\mu} - m_q)\Psi$$

$$D^{\mu} \equiv \partial_{\mu} + ig(A^{\mu} + \mathcal{A}^{\mu})$$

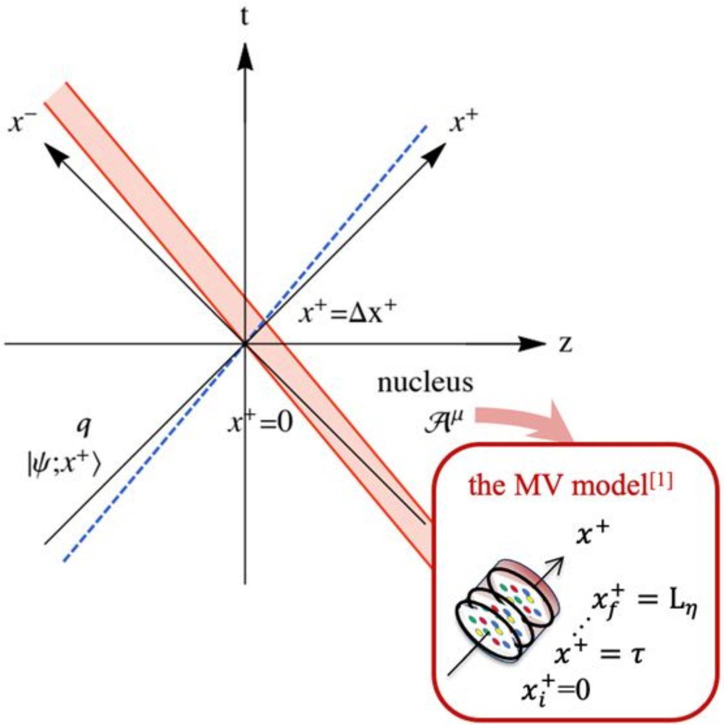
The light-front Hamiltonian is obtained by the canonical light-front quantization via the standard Legendre transformation,

Brodsky, Pauli, Pinsky, Phys.Rept.301: 299-486 (1998)



Physical set up, I

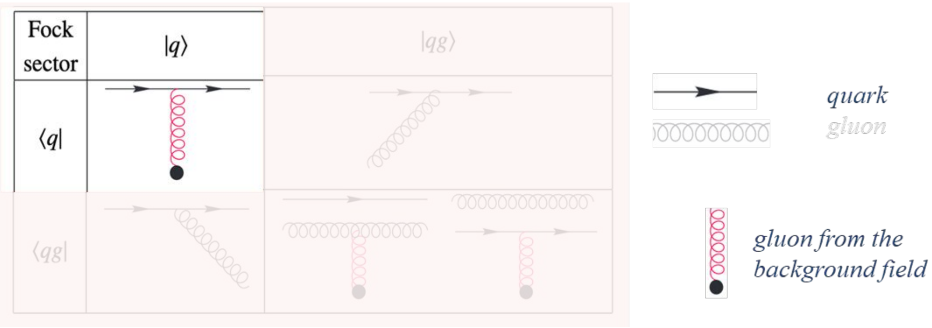
Li, Zhao, Maris, Chen, Li, Tuchin, Vary PRD101.076016 (2020)
Barata, Du, Li, WQ, Salgado, PRD106,074013 (2022)



High-energy quark moving close to the light cone scattering on a dense nucleus medium

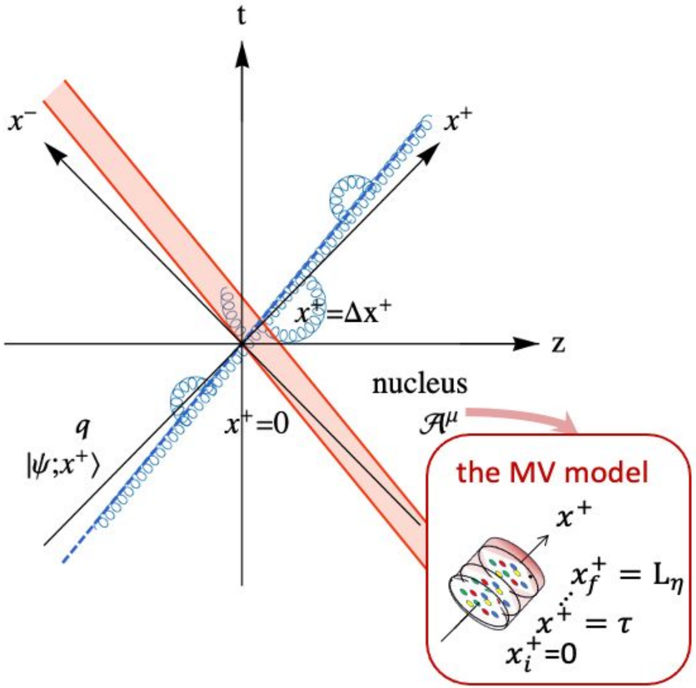
The light-front Hamiltonian in the $|q\rangle$ Fock sector:

$$P^-(x^+) = P^-_{\mathrm{KE}} + V_A(x^+)$$



Physical set up, II

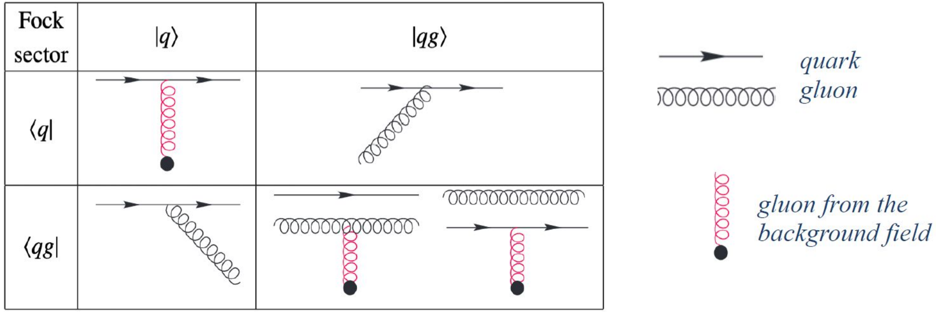
Li, Lappi, Zhao, PRD104.056014 (2021)
Barata, Du, Li, WQ, Salgado (to appear)



High-energy quark moving close to the light cone scattering on a dense nucleus medium

The light-front Hamiltonian in the $|q\rangle + |qg\rangle$ Fock sector:

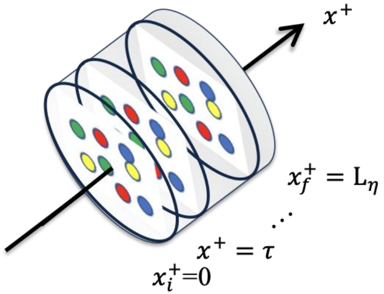
$$P^-(x^+) = P^-_{\mathrm{KE}} + V(x^+) = P^-_{\mathrm{KE}} + \left\{V_{qg} + V_{\mathcal{A}}(x^+)\right\}$$



The medium and evolution

The stochastic background field uses the McLerran-Venugopalan (MV) model

McLerran, Venugopalan, PRD49, 2233; PRD49, 3352; PRD50, 2225 (1994)



$$\langle\langle \rho_a(\vec{x}_\perp, x^+)\rho_b(\vec{y}_\perp, y^+)\rangle\rangle = g^2\tilde{\mu}^2\delta_{ab}\delta^2(\vec{x}_\perp - \vec{y}_\perp)\delta(x^+ - y^+)$$

$$(m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(\vec{x}_\perp, x^+) = \rho_a(\vec{x}_\perp, x^+)$$

$$Q_s^2 = C_F\frac{(g^2\tilde{\mu})^2 L_\eta}{2\pi}$$

saturation scales

Light-front time evolution of the probe, decomposed as sequence of unitary operators

$$|\psi_{L_\eta}\rangle = U(L_\eta;0)\,|\psi_0\rangle \equiv \mathcal{T}_+ e^{-i\int_0^{L_\eta} \mathrm{d}x^+\, P^-(x^+)}\,|\psi_0\rangle$$

$$U(L_\eta;0) = \prod_{k=1}^{N_t} U(x_k^+; x_{k-1}^+)$$

Quantum simulation algorithm

Wiesner, 9603028 (1996); Zalka, 9603026 (1996)

1. **Define problem Hamiltonian**
2. **Encode Hamiltonian onto basis**
3. Prepare initial states
4. Evolution
5. Measurement protocol

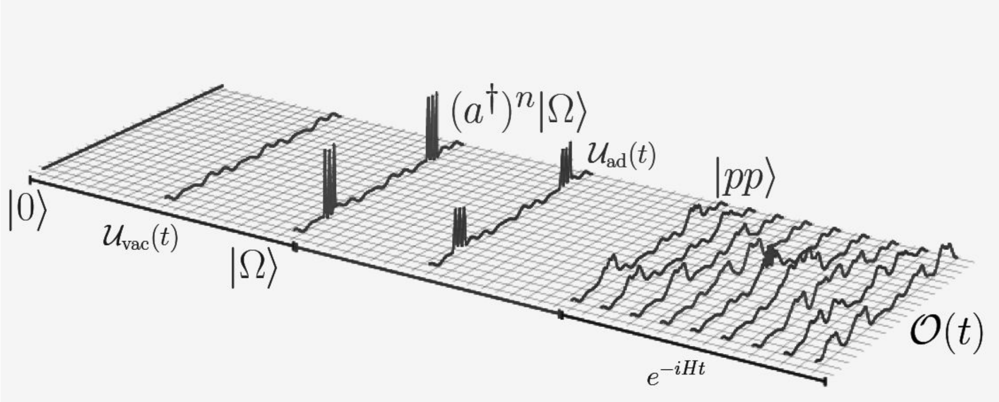


Image from Lamm's talk at Fermilab (2021)

Basis encoding

Barata, Mueller, Tarasov, Venugopalan, PRA103, 4, 042410 (2021)

The general QCD quantum state in single-particle basis:

$$|\psi\rangle = |q\rangle \cdots |q\rangle \otimes |g\rangle \cdots |g\rangle \quad \sim \prod_{i=1}^{m} \Big(|e_{g_i}\rangle \otimes |g_i\rangle \Big) \otimes \prod_{i=1}^{n} \Big(|e_{q_i}\rangle \otimes |q_i\rangle \Big)$$

In our cases of $|q\rangle + |qg\rangle$ and $|q\rangle$ Fock spaces:

$$|\psi\rangle = |\zeta\rangle \otimes \underbrace{\Big(|g_x\rangle\, |g_y\rangle\, |c_g\rangle \Big)}_{|g\rangle} \otimes \underbrace{\Big(|q_x\rangle\, |q_y\rangle\, |c_q\rangle \Big)}_{|q\rangle}$$

$$|\zeta\rangle = |0\rangle \Leftrightarrow \text{Fock } |q\rangle,\ k_g^+ = 0, k_q^+ = K$$

$$|\zeta\rangle = |1\rangle \Leftrightarrow \text{Fock } |qg\rangle, k_g^+ = 1, k_q^+ = K - 1$$

$$\cdots$$

$$N_{\text{tot}} \sim \lceil K \rceil N_\perp^4 \rightarrow n_Q \sim 4 \log N_\perp + \log \lceil K \rceil$$

$$|\psi\rangle = \underbrace{\Big(|q_x\rangle\, |q_y\rangle\, |c_q\rangle \Big)}_{|q\rangle}$$

$$N_{\text{tot}} \sim N_\perp^2 \rightarrow n_Q \sim 2 \log N_\perp$$

Transverse & Longitudinal Lattice

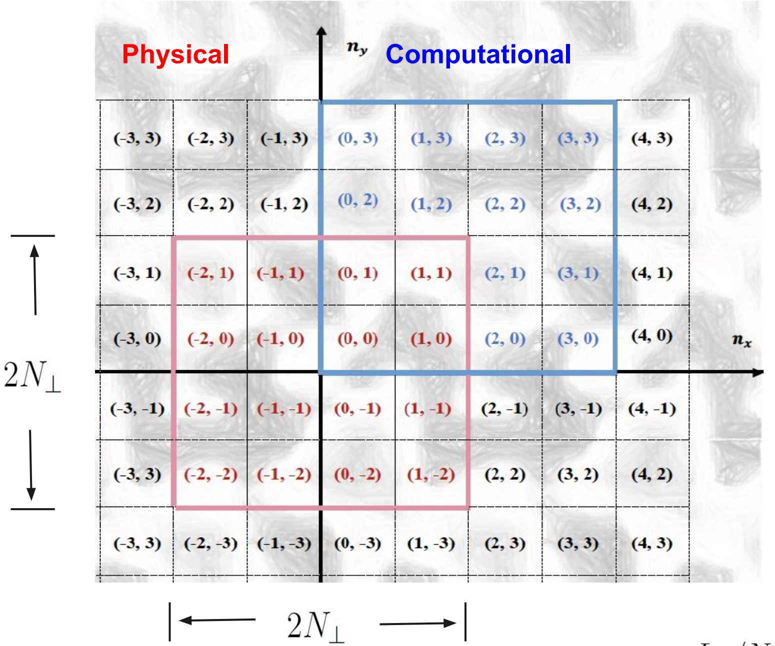
- Transverse lattice is periodical, position & momentum related via quantum fourier transform (FT)

$$(n_x, n_y) \Longleftrightarrow (n_x + i2N_\perp, n_y + j2N_\perp)$$

$$\vec{r}_\perp = (n_x, n_y)a_\perp \qquad \vec{p}_\perp = (k_x, k_y)b_\perp$$

- Longitudinal direction is periodical (anti-periodical) for bosons (fermions)

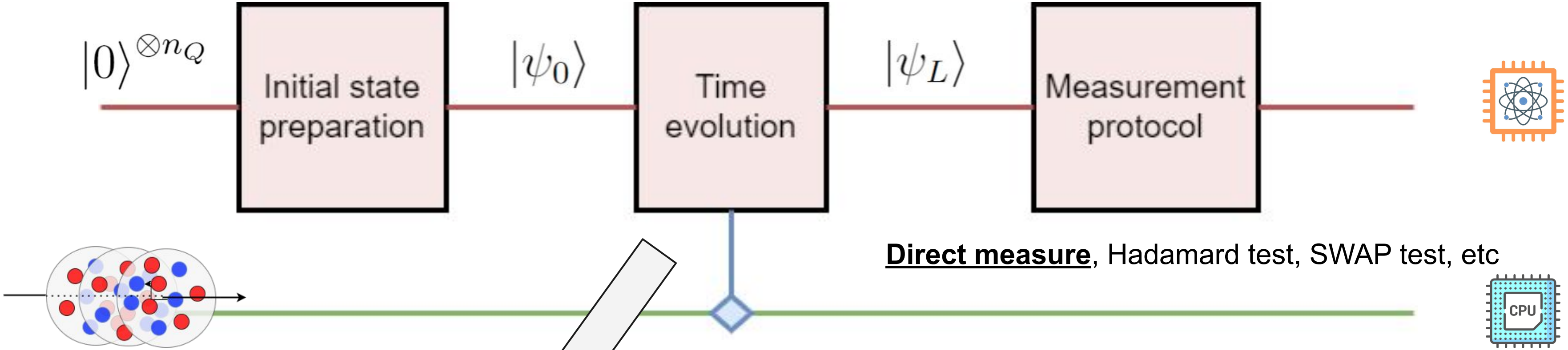
$$p_l^+ = \frac{2\pi}{L}k_l^+, \quad k_q^+ = \frac{1}{2}, \frac{3}{2}, \cdots, \quad k_g^+ = 1, 2, 3, \cdots$$



$$a_\perp = L_\perp/N_\perp$$
$$b_\perp = \pi/L_\perp$$

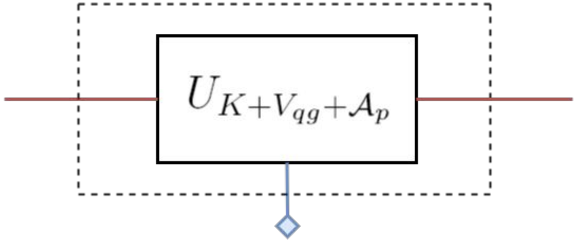
Momentum-space simulation

Amplitude level



Direct measure, Hadamard test, SWAP test, etc

many **time steps**

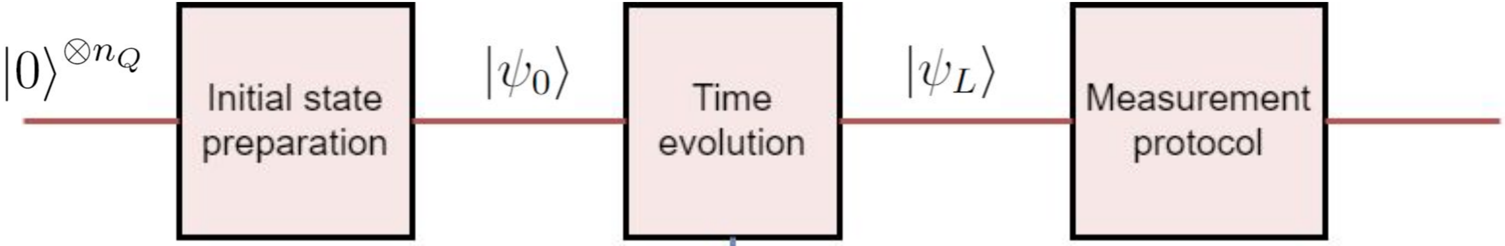


$$U_{\mathcal{A}_p}(x^+, \delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iV_{\mathcal{A}_p}\delta x^+)\,|\boldsymbol{p}\rangle$$
$$U_K(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iP^-_{\mathrm{KE}}\delta x^+)\,|\boldsymbol{p}\rangle$$
$$U_{V_{qg}}(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iV_{qg}\delta x^+)\,|\boldsymbol{p}\rangle$$

Ham => pauli strings => quantum gates

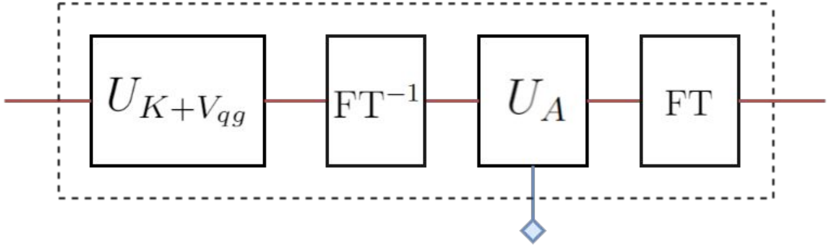
Mixed-space simulation

Amplitude level



Direct measure, Hadamard test, SWAP test, etc

many **time steps**



FT allows convenient simulation in the respective basis, esp when Hamiltonians are extremely sparse

$$U_A(x^+, \delta x^+)\,|\boldsymbol{r}\rangle = \exp(-iV_A\delta x^+)\,|\boldsymbol{r}\rangle$$
$$U_K(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iP^-_{\mathrm{KE}}\delta x^+)\,|\boldsymbol{p}\rangle$$
$$U_{V_{qg}}(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iV_{qg}\delta x^+)\,|\boldsymbol{p}\rangle$$

Sparse Ham => pauli strings => quantum gates

Simulation strategies

	Momentum-space simulation	Mixed-space simulation
	$U(x_k^+ + \delta x; x_k^+) = U_{K+V_{qg}+V_{\mathcal{A}_p}}(\delta x^+, x_k^+) \equiv \exp\left\{ -i\delta x^+ \left[\tilde{K} + \tilde{V}_{qg} + \tilde{V}_{\mathcal{A}_p}(x_k^+)\right] \right\},$	$U(x_k^+ + \delta x; x_k^+) \approx U_{K+V_{qg}}(\delta x^+) U_{V_{\mathcal{A}}}(\delta x^+, x_k^+) \equiv \exp\left\{ -i\delta x^+ \left[\tilde{K} + \tilde{V}_{qg}\right] \right\} \exp\left\{ -i\delta x^+ \left[\tilde{V}_{\mathcal{A}}(x_k^+)\right] \right\}$
Pros	Easy to trotter (no need of basis transform)	Need quantum Fourier Transforms (qFT)
Cons	Harder to scale to larger problem size; Expensive to evaluate Pauli terms	Sparse Hamiltonian; better scaling at larger problem size

Simulation parameters

- Our initial state for the jet is a quark state with zero transverse momenta: $(p_x, p_y) = (0, 0)$
- Duration of static medium: $L_\eta = 50\,\mathrm{GeV}^{-1} \approx 10\,\mathrm{fm}$
- 5 stochastic fields are used for configuration average $\langle\langle\langle \hat{O} \rangle\rangle\rangle = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \langle \hat{O} \rangle_i$
- Computational lattices:

  |q> + |qg> Transverse: $N_\perp = 1$ Longitudinal: $K = 3.5$

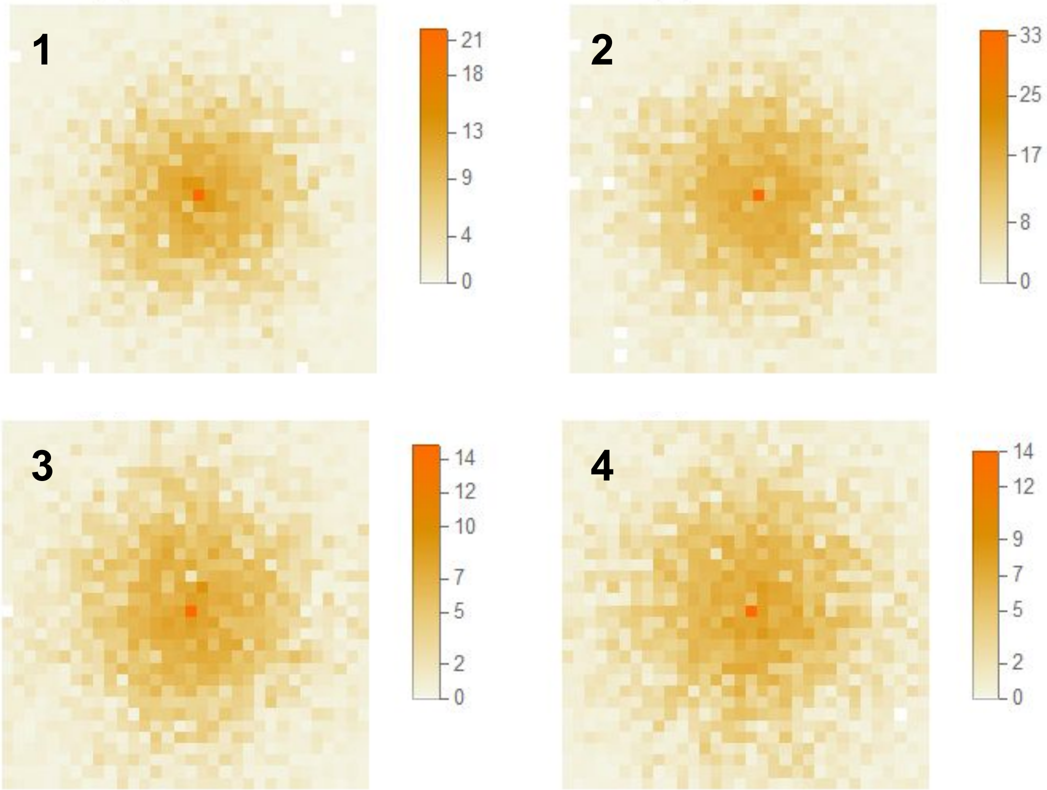
  |q> Transverse: $N_\perp = 16$
- Non-abelian SU(2) color
- Non-flip case for spin $(\lambda_Q, \lambda_q, \lambda_g) = (\uparrow, \uparrow, \uparrow)$
- Sufficient time steps in trotterization and medium layers in background medium

We present our preliminary results of jet evolution in vacuum and in medium, using IBM Qiskit quantum simulators

Momentum broadening

Fock |q> only

Barata, Du, Li, WQ, Salgado, PRD106,074013 (2022)



Increasing
saturation scales / time

transverse lattice: 32 X 32

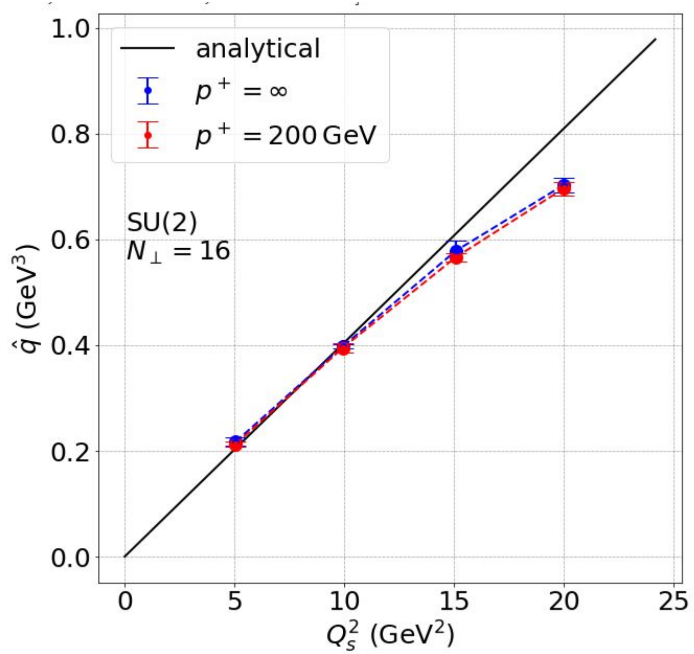
819200 shots, 11 qubits

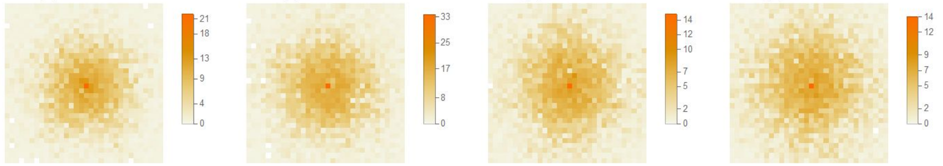
Momentum broadening

Fock |q> only

Barata, Du, Li, WQ, Salgado, PRD106,074013 (2022)

quenching parameter





Analytical (eikonal)

$$\hat{q} = \frac{g^4}{4\pi} C_F \tilde{\mu}^2 \left\{ \log\left(1 + \frac{\frac{\pi^2}{a_\perp^2}}{m_g^2}\right) - \frac{1}{1 + \frac{a_\perp^2 m_g^2}{\pi^2}} \right\} \sim Q_s^2 / L_\eta$$

Simulation

$$\hat{q} = \frac{\Delta \langle p_\perp^2 (x^+) \rangle}{\Delta x^+} = \langle \boldsymbol{p}_\perp^2 \rangle / L_\eta$$

819200 shots, 11 qubits

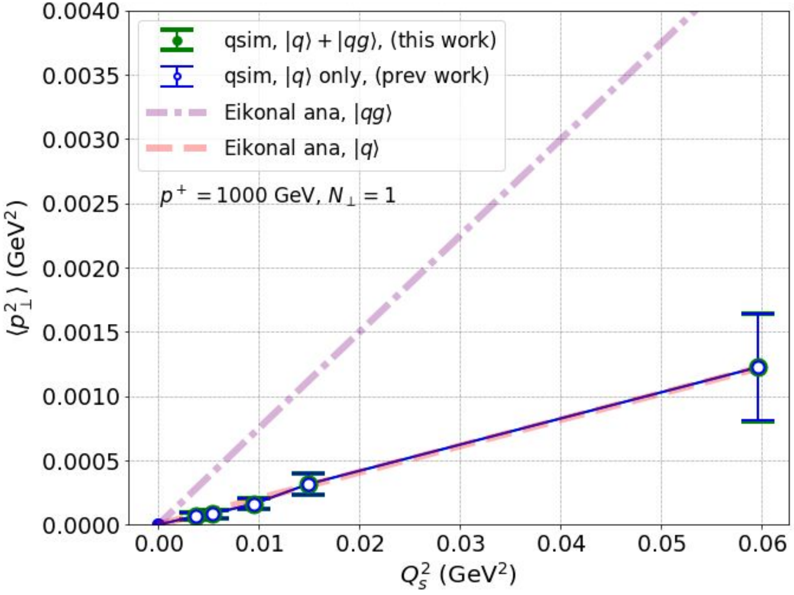
Momentum broadening

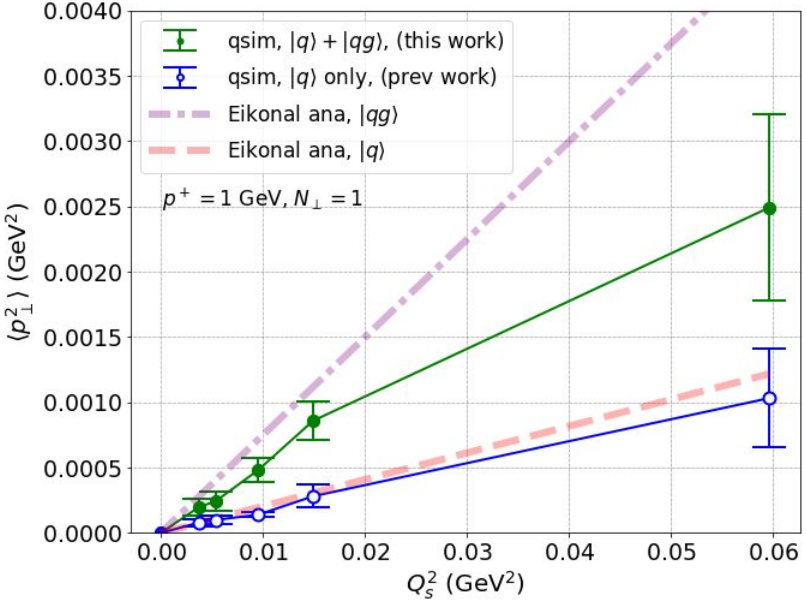
Fock |q> + |qg>

Barata, Du, Li, WQ, Salgado (to appear)

$$\langle p_\perp^2 \rangle = \mathcal{P}_{|q\rangle} \langle p_\perp^2 \rangle_{|q\rangle} + \mathcal{P}_{|qg\rangle} \langle p_\perp^2 \rangle_{|qg\rangle}$$

$$\hat{q}_{Eik}(x = a_\perp m_g/\pi, N_\perp = 1)\big|_{\text{on lattice}} = C_F g^4 \tilde{\mu}^2 \frac{1}{(2\pi)^2} \left[\frac{2}{(x^2+1)^2} + \frac{2}{(x^2+2)^2} \right] \sim Q_s^2/L_\eta$$

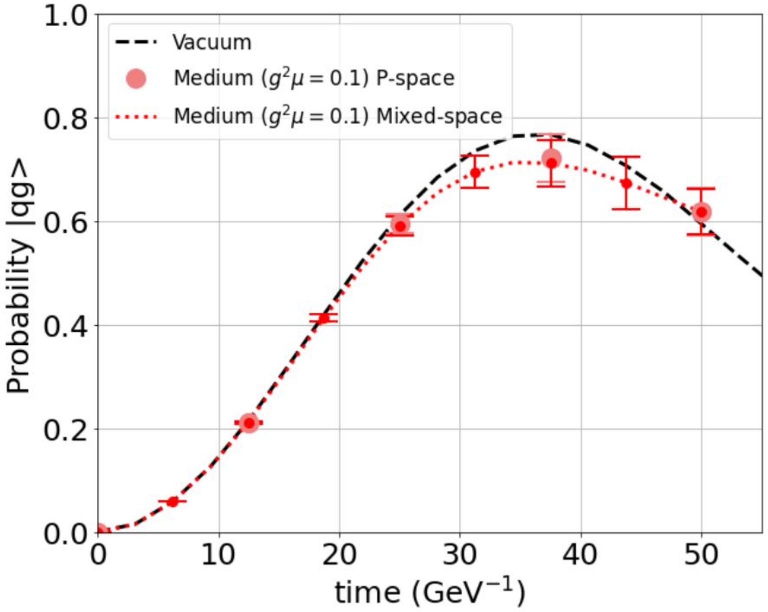


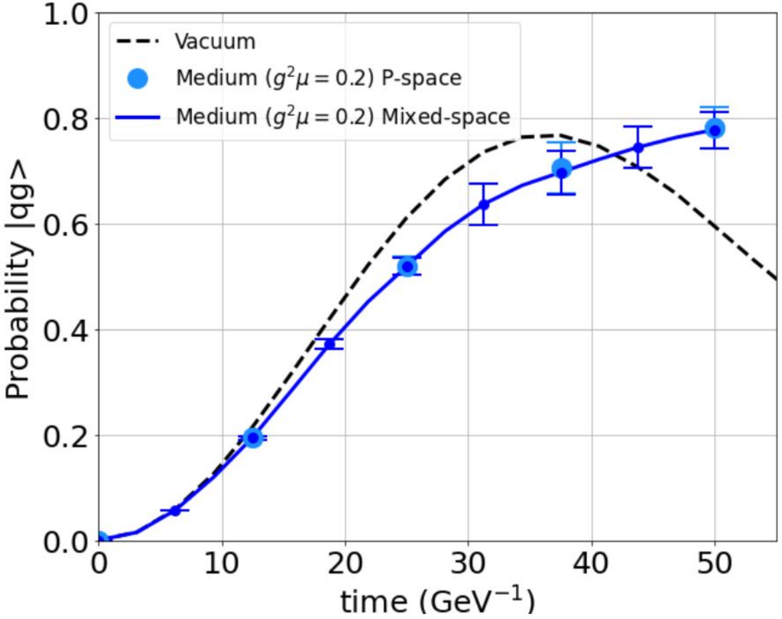


819200 shots, 9 qubits, Ndx=16, Neta=16

Quantum jet evolution

Barata, Du, Li, WQ, Salgado (to appear)



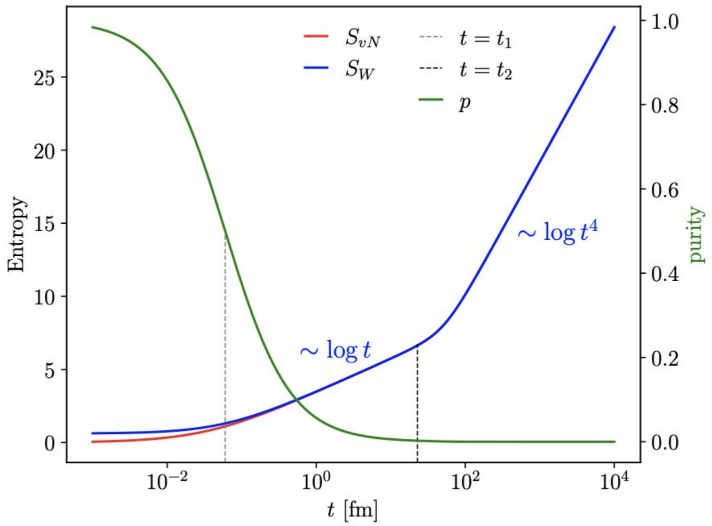


819200 shots, 9 qubits, Ndx=16, Neta=16

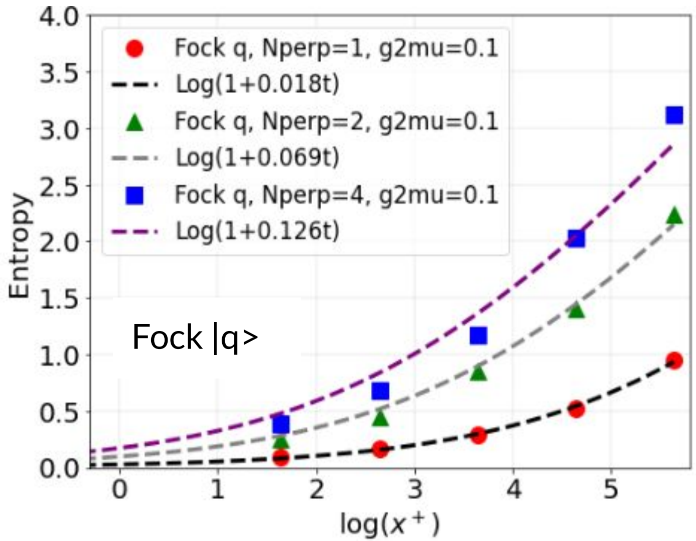
Entropy evolution of the quark jet

Barata, Du, Li, WQ, Salgado (to appear)

$$\rho_{\mathrm{N_m}} = (1/N_m)\sum_{i=1}^{N_m} \rho_{\mathrm{i}} \qquad \langle S\rangle = -\mathrm{Tr}(\rho_{\mathrm{N_m}} \log \rho_{\mathrm{N_m}})$$



Barata, Blazizot, Mehtar-Tani, 2305.10476 (2023)



819200 shots, 9 qubits, Ndx=16, Neta=16

Summary and outlook

- We study multi-particle jet evolution in a medium within two Fock sectors |q> + |qg> using light-front QCD Hamiltonian approach on quantum simulator.
- Despite of having a small model space, we can study jet evolution using quantum simulators, in agreement with analytical result and classical diagonalization.
- Quantum simulation can be effective in reducing problem complexity faced in classical simulation. Our formalism can be extended to higher Fock sectors, various QCD mediums, different physical processes.

Thank you for your attention!

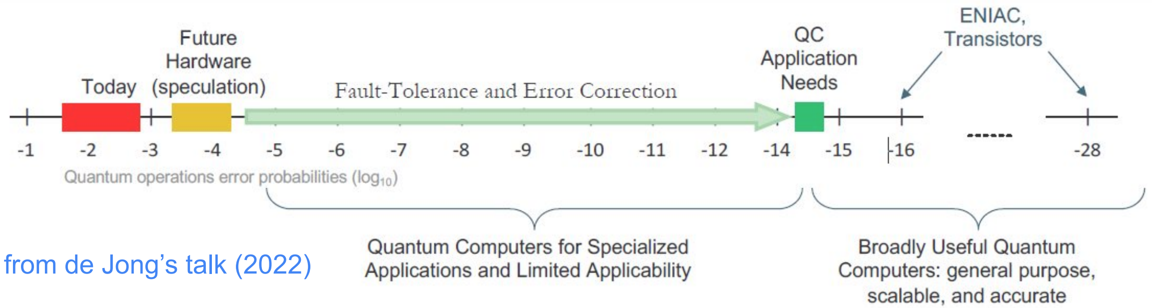
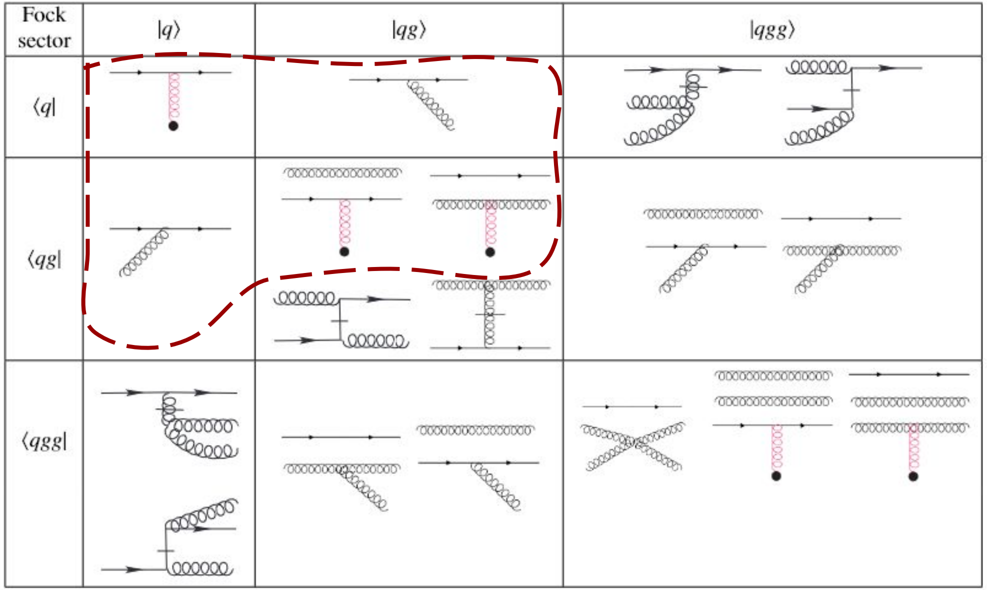


Image from de Jong's talk (2022)

Future work



McLerran, 0812.4989 (2008)