Applications of novel chiral interactions to quantum Monte Carlo methods and astrophysical data analysis

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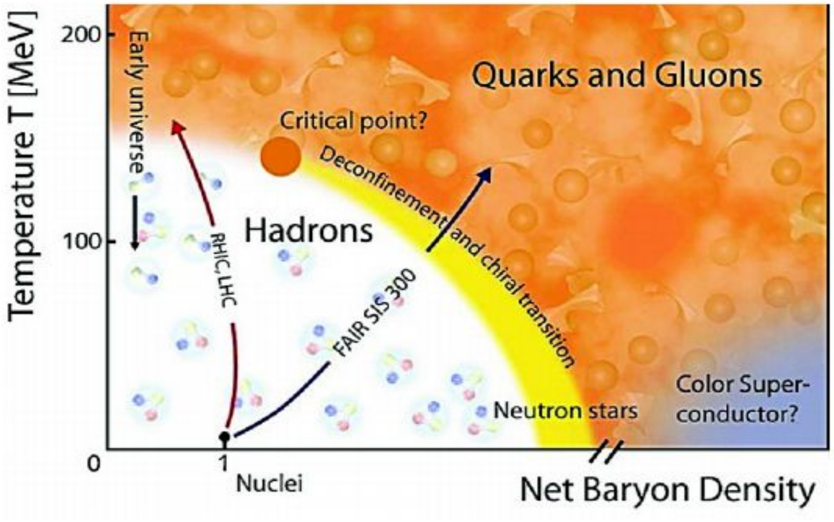
The Basic Question



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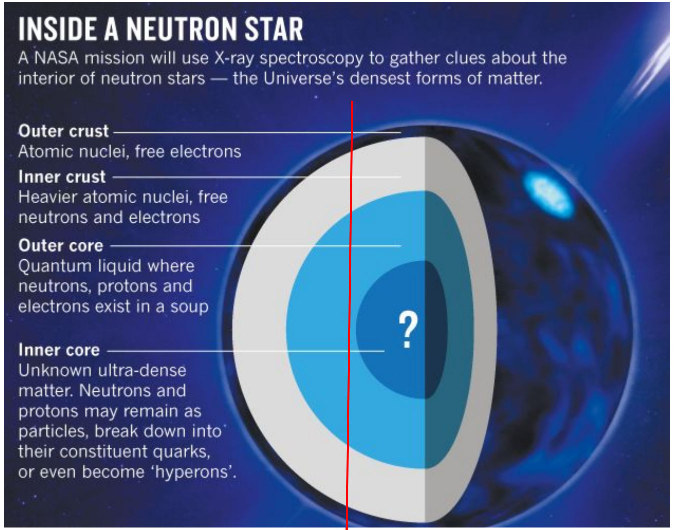
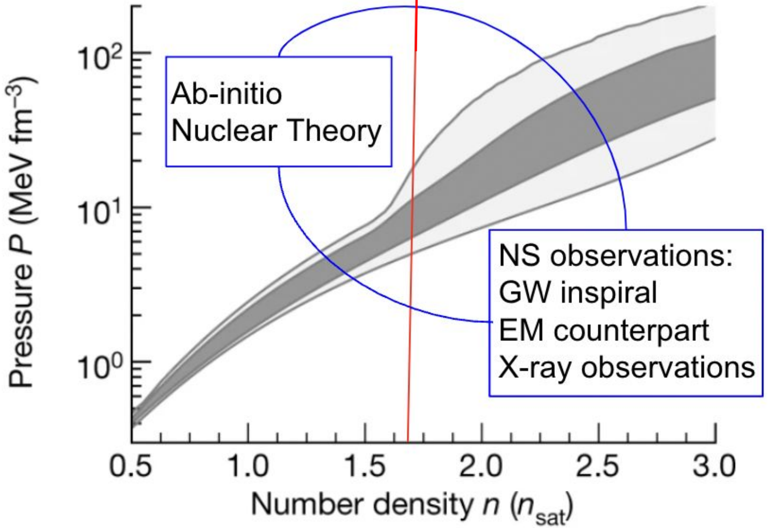
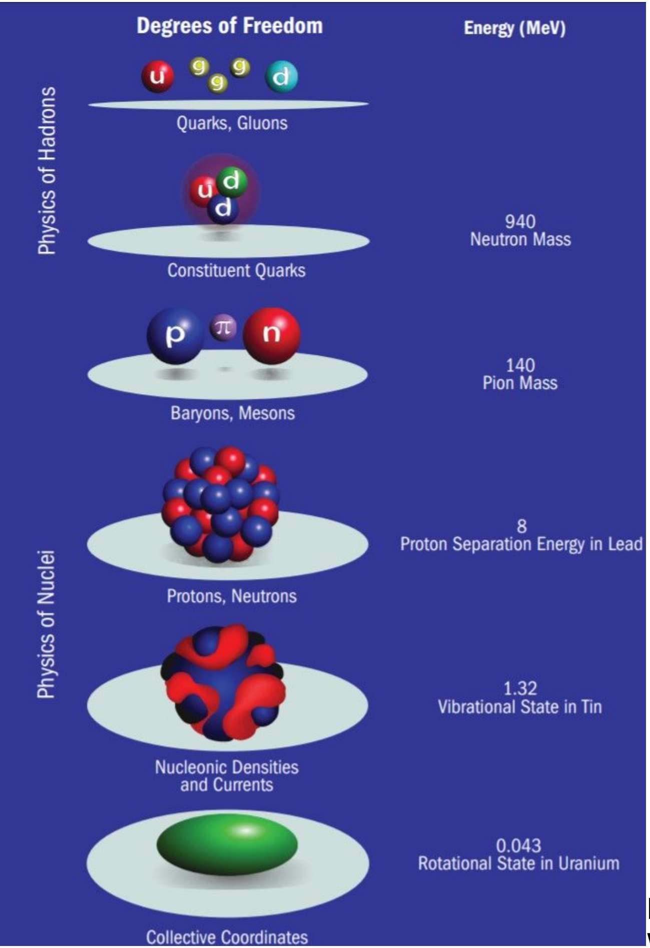
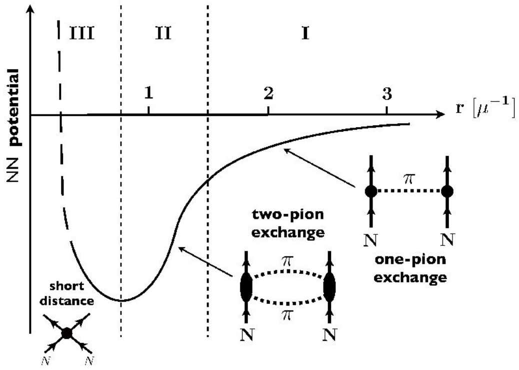




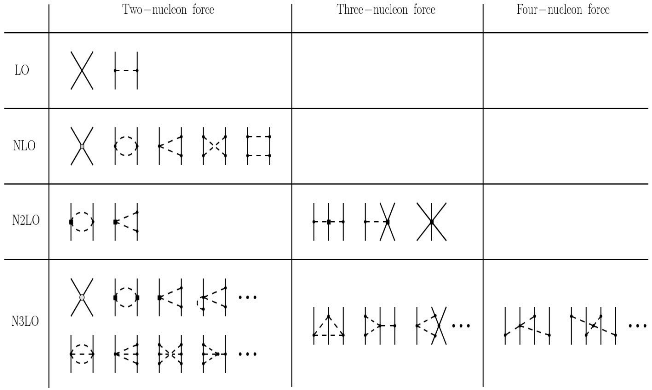
- What is the nature of the densest matter in the universe?
- How and when do new states of dense matter appear?
- How do neutrons and protons interact under extreme conditions of density and isospin asymmetry?

Nuclear Effective Field Theory





Holt et al., 10.1016/j.ppnp.2013.08.001



Bertsch et al., United States: N. p., 2007. Web.

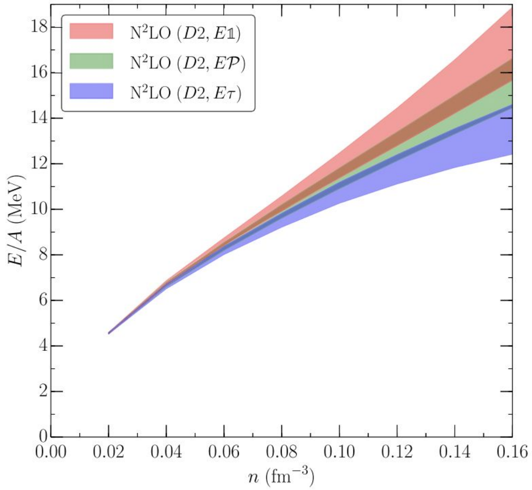
- The long-range component of the NN interaction is mediated by the pion which is constrained directly by the symmetries of QCD.
- Heavy mesons (ρ,ω) are 'integrated out' and are replaced with contact interactions.
- The Lagrangian is an expansion $Q = p/\Lambda_b$, where Λ_b is the EFT breakdown scale (~600 MeV) and p is of the order of the pion mass (~140 MeV).
- The Low Energy Couplings (LECs) are fit to NN scattering data.

Local chiral interactions at N²LO: EFT at large cutoffs

- Quantum Monte Carlo (QMC) methods are among the most accurate many-body methods to solve nuclear systems, but they require local interactions as input.

$$\lim_{\tau\to\infty} e^{-H\tau} |\Psi_T\rangle \to |\Psi_0\rangle$$

where H does not include derivative operators



Lynn et al., PRL 116 (2016) 6, 062501

- We also need to regulate the interaction using local regulators:

$$V_{\text{long},\pi}(r) \to V_{\text{long},\pi}(r) f_{\text{long}}(r) \qquad \delta(r) \to f_{\text{short}}(r)$$

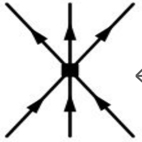
where $f_{\text{long}}(r) = \left(1 - \exp\left(-\left(\frac{r}{R_0}\right)^{n_1}\right)\right)^{n_2}$ and $f_{\text{short}}(r) \sim \exp\left(-\left(\frac{r}{R_0}\right)^{n}\right)$

- The cutoff R_0 (Λ_c in momentum space) induces a scale in the EFT: $Q' = \frac{p}{\Lambda_c}$

Taking $\Lambda_c \gg \Lambda_B (500 - 600\text{MeV})$ should therefore improve the chiral expansion, i.e. decrease regulator artifacts and EFT truncation uncertainties.

Local chiral interactions at N²LO: EFT at large cutoffs

- Local interactions violate the Fierz rearrangement freedom. This can have a drastic effect in the 3-body sector



$$V_{E\tau} = \frac{c_E}{\Lambda_\chi F_\pi^4} \sum_{i<j<k} \sum_{\text{cyc}} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k \delta_{R_{3N}}(\mathbf{r}_{kj}) \delta_{R_{3N}}(\mathbf{r}_{ij}),$$

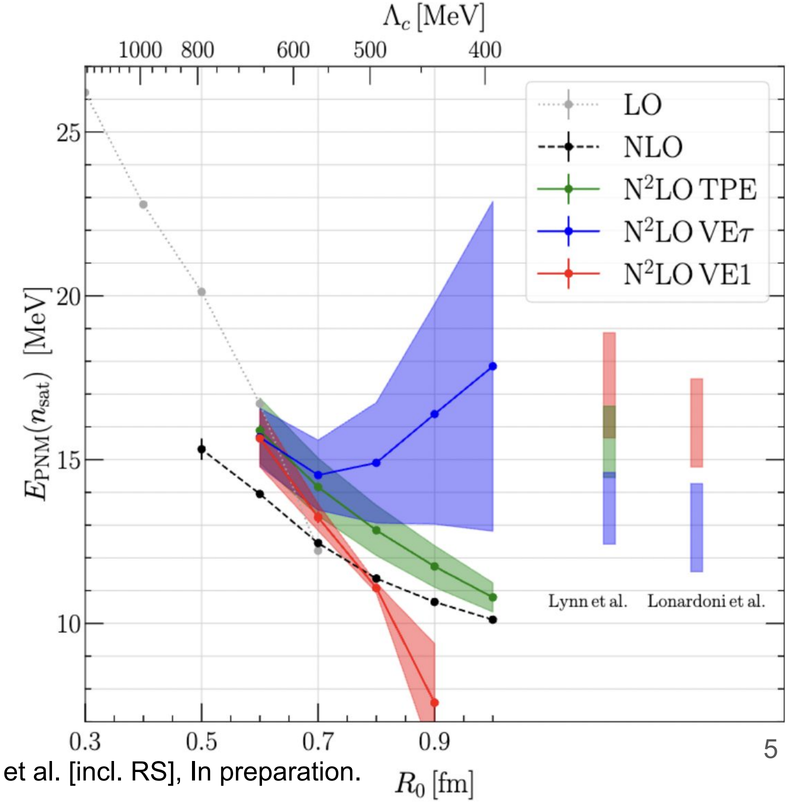
$$V_{E\mathbb{1}} = \frac{c_E}{\Lambda_\chi F_\pi^4} \sum_{i<j<k} \sum_{\text{cyc}} \delta_{R_{3N}}(\mathbf{r}_{kj}) \delta_{R_{3N}}(\mathbf{r}_{ij}),$$

$$V_{E\mathcal{P}} = \frac{c_E}{\Lambda_\chi F_\pi^4} \sum_{i<j<k} \sum_{\text{cyc}} \mathcal{P} \delta_{R_{3N}}(\mathbf{r}_{kj}) \delta_{R_{3N}}(\mathbf{r}_{ij}).$$

- Different operator structures are possible for the same 3N force, V_E. This ambiguity should vanish in the limit of infinite cutoff.

- QMC calculations of neutron matter for a wide range of cutoffs seem to confirm this, greatly improving the applicability of local chiral interactions to many-body systems.

QMC calculation of pure neutron matter



I. Tews et al. [incl. RS], In preparation.

Maximally local NN chiral interactions at N^{3}LO

- As the next step, we aim to construct chiral interactions at N^{3}LO that can be used for QMC calculations. However, the short-range part of the NN interaction contains pieces that are inevitably non-local.

$$H^{\text{N3LO}} = \underbrace{H^{\text{local}}}_{\text{17 operators}} + \underbrace{H^{\text{non-local}}}_{\text{4 operators}}$$

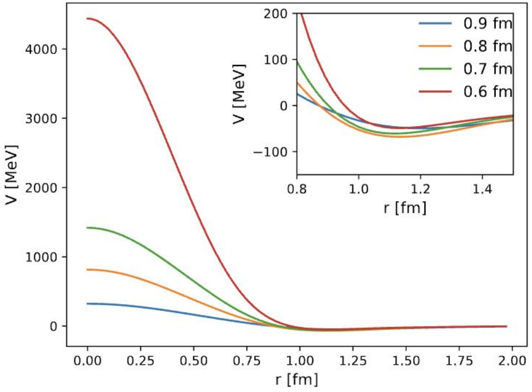
Our idea is to compute H^{local} exactly in QMC and treat $H^{\text{non-local}}$ perturbatively, $\Delta E = \langle \psi_0 | H^{\text{non-local}} | \psi_0 \rangle$

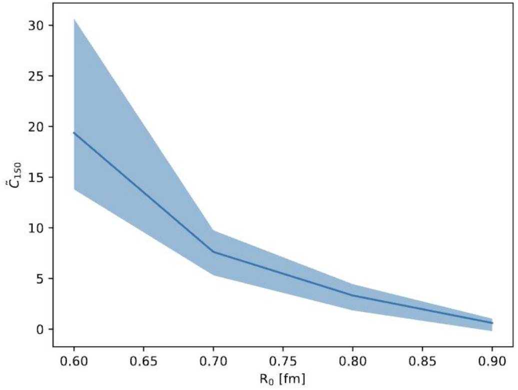
- All non-local and pion-exchange pieces are consistently included up to N^{3}LO.

See also: Piarulli et al., PRC (2015) 2, 024003
Saha et al., PRC (2023) 3, 034002

- Once again, we explore a wide range of cutoffs in the range ~440 MeV to ~660 MeV

- Going one order higher in the chiral expansion, as well as exploring high-cutoff interactions, is expected to decrease uncertainties by factor 2-3.





RS et al., In preparation

Maximally local NN chiral interactions at N^{3}LO: A Bayesian analysis

- The NN interaction is calibrated to scattering data using the method of Bayesian inference. This allows us to incorporate EFT truncation uncertainties using order-by-order calculations.

- We perform a MCMC sampling of the posterior distribution function

$$P = \frac{\mathcal{L} \times \Pi}{Z}, \qquad \mathcal{L} \propto \prod_i \exp\left\{ -\frac{1}{2}\left(\frac{X_i^{\rm exp} - X_i^{\rm theo}}{\sigma_i}\right)^2 \right\},$$

$$\sigma^2 = \sigma^2_{\rm exp} + \sigma^2_{\rm theo};$$

EFT Truncation error:

$$X^{\rm LO}(p) = c_0(p),$$

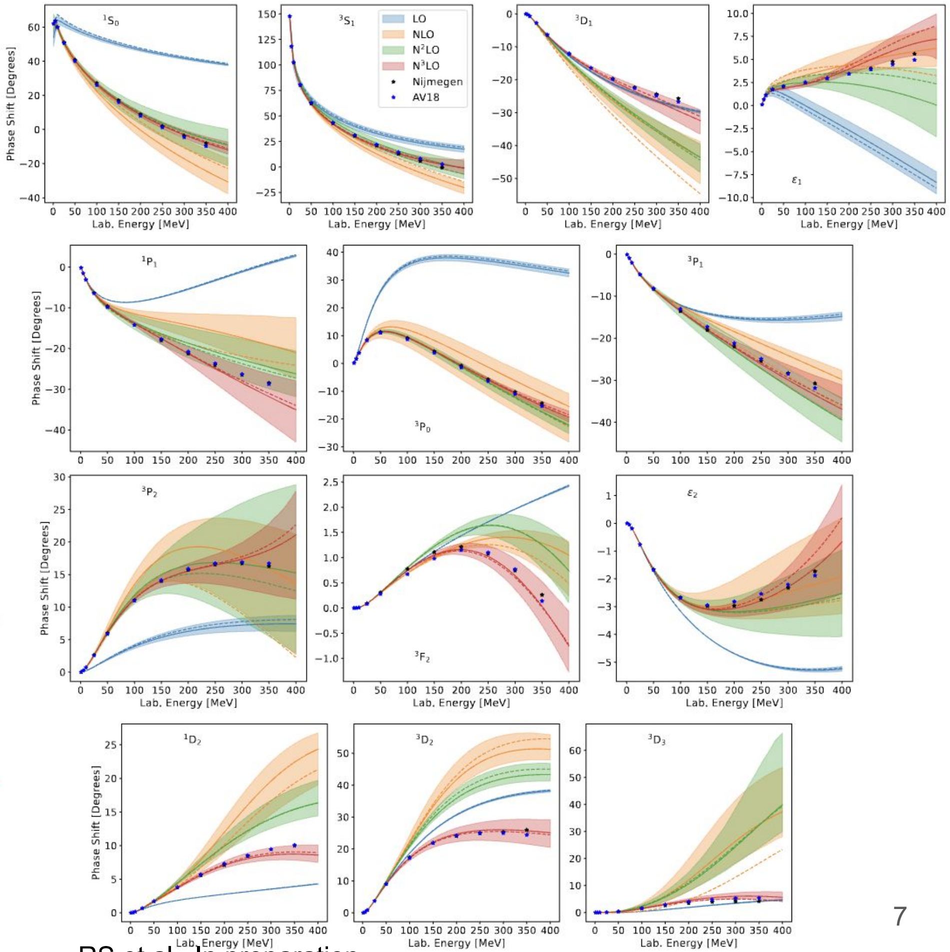
$$X^{\rm NLO}(p) = \sum_{n=0}^{2} c_n(p) Q^n,$$

$$X^{{\rm N}^j{\rm LO}}(p) = \sum_{n=0}^{j+1} c_n(p) Q^n,$$

$$\Delta X^{{\rm N}^j{\rm LO}} = Q^{j+2} \max(|c_0|, |c_1|, \ldots, |c_{j+1}|),$$

$$\text{where } Q = \frac{\max(m_\pi, p)}{\min(\Lambda_c, \Lambda_B)} \text{ and } \Lambda_B = 600 \text{ MeV}$$

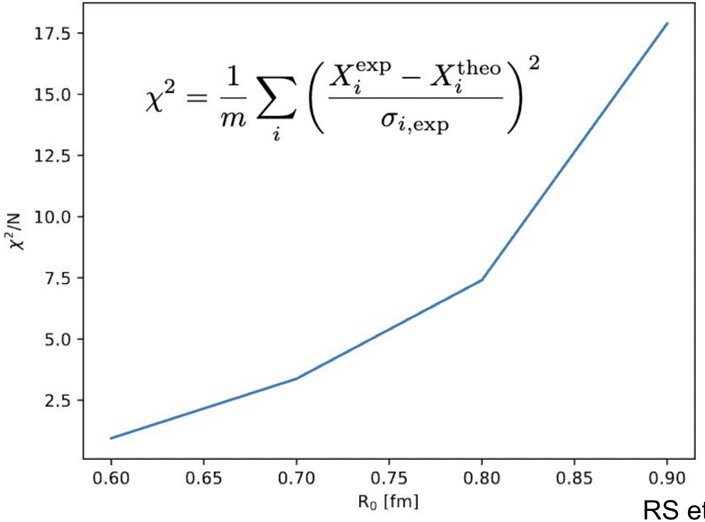
Epelbaum et al., EPJA 51 (2015) 5, 53
See also: Wesolowski et al., JPG 46 (2019) 4, 045102

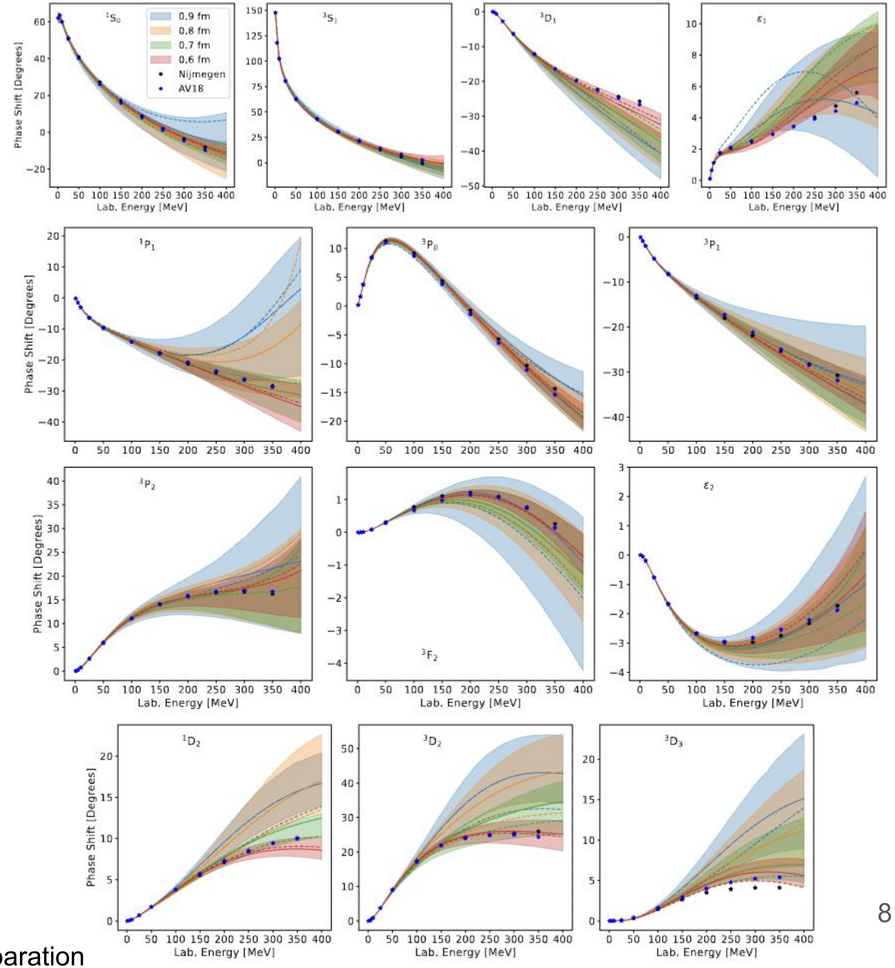


RS et al., In preparation

Maximally local NN chiral interactions at N^{3}LO: A Bayesian analysis

- We see that harder-core interactions exhibit significantly lower EFT truncation uncertainties.
- The more non-perturbative potentials are also better at reproducing NN scattering data.
- By comparing with a simple least-squares fit, we see the importance of modeling truncation errors via Bayesian methods.



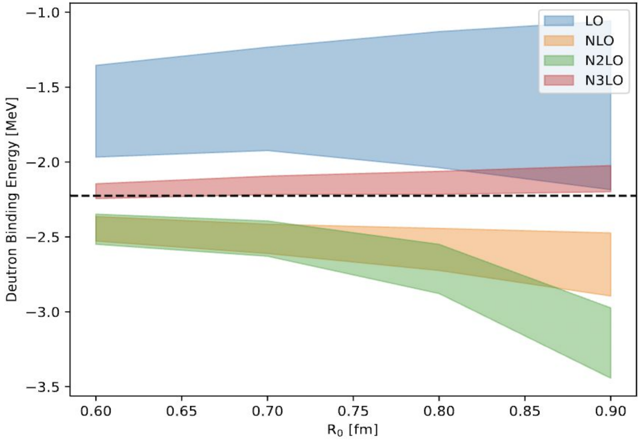


RS et al., In preparation

From the deuteron to neutron stars

- Our best interaction (Λ_c~ 660 MeV) is in excellent agreement with the deuteron binding energy at the 68% confidence level.

- Our next step would be to incorporate (parameter free) 3N forces at N^3LO. This would allow for a fully consistent N^3LO calculation of few and many body nuclear systems using QMC.



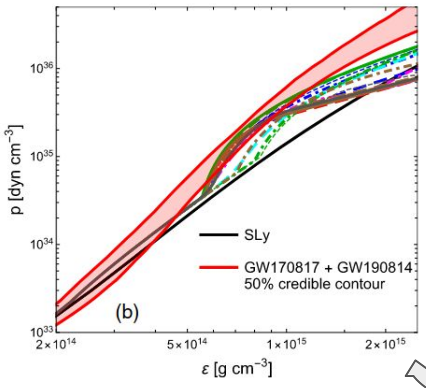
RS et al., In preparation

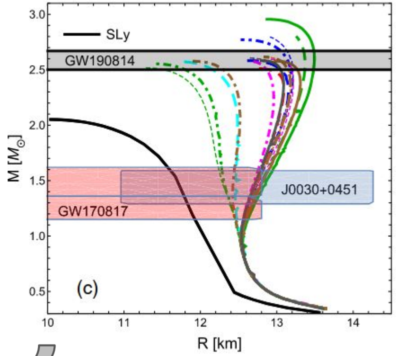
In particular, we are interested in the Equation of State of neutron stars

The Equation of State

$$G^{\mu\nu} = \kappa T^{\mu\nu}$$

Neutron star observables

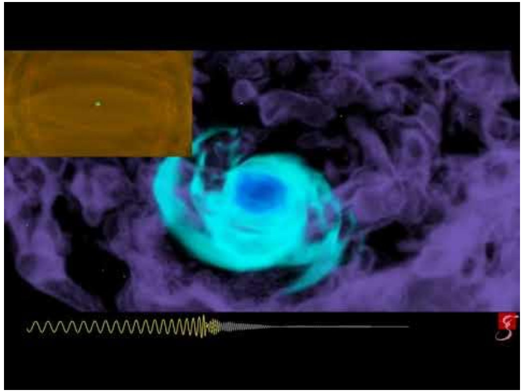




Bayesian Inference methods

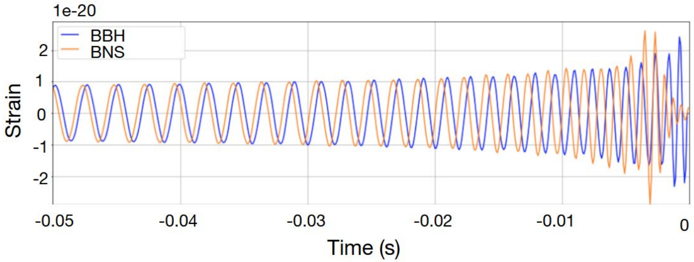
EOS constraints from Gravitational Wave constraints

In 17th August, 2017 the first GW emission from NS merger was detected !



(Max Planck Institute for Gravitational Physics)

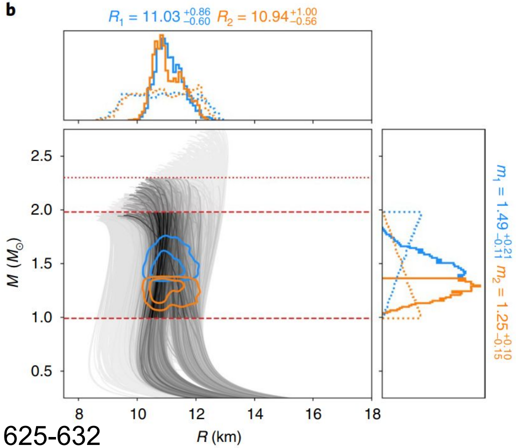
Virgo GW detector



The tidal deformability: $\Lambda = \frac{2}{3} k_2 \left(\frac{R}{M} \right)^5$

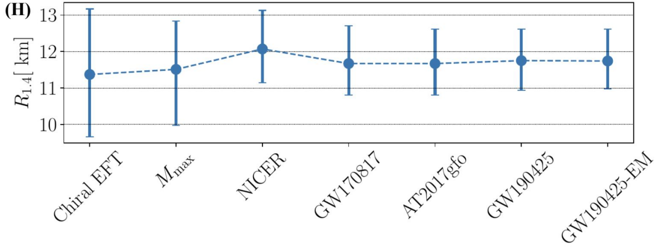
During the merger, NSs deform under the gravitational field of their partner.

This 'Tidal Deformability' leaves an imprint in the GW signal that can be measured. This leads to strong constraints on the radius.



Capano et al. Nature Astron. 4 (2020) 6, 625-632

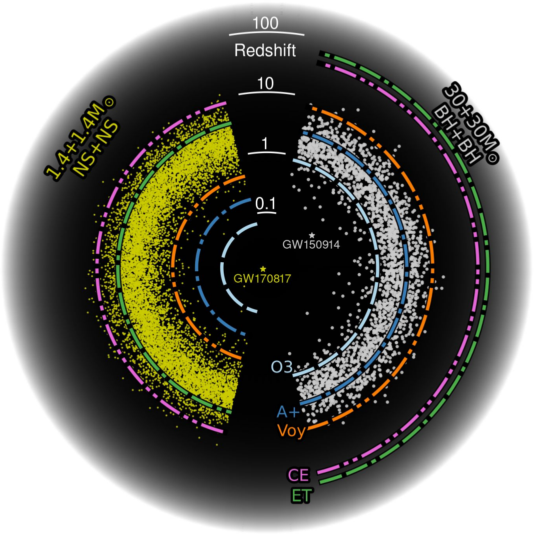
$$R_{1.4} = 11.75^{+0.86}_{-0.81} \text{ km}$$



Dietrich et al., Science 370 (2020) 6523, 1450-1453

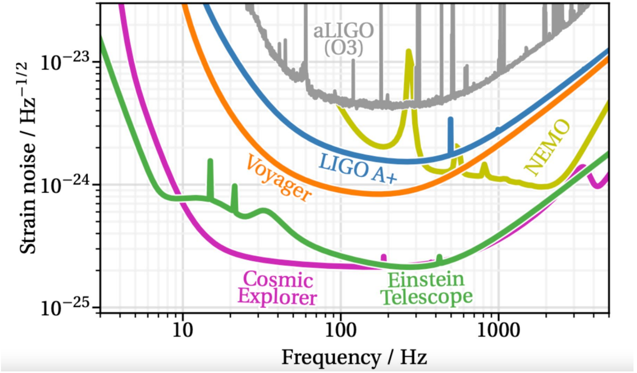
The future of GW astronomy: Precision measurement of the EOS

- Third generation GW detectors (10x more sensitive) are expected to come online by mid 2030s.
- A network of 2 Cosmic Explorers and 1 Einstein Telescope will be able to see all neutron star mergers in the universe throughout cosmic time!
- This will lead to ~300 detections of neutron star mergers with signal-to-noise ratio greater than 100, leading to unprecedented, high-precision measurement of the EOS.



<u>What do we need from the nuclear physics community?</u>

1. Improved models of nuclear interactions that lead to smaller theoretical uncertainties.
2. **Efficient emulators** for the fast determination of many body observables (the EOS). This is required for a full propagation of nuclear uncertainties via Bayesian methods.

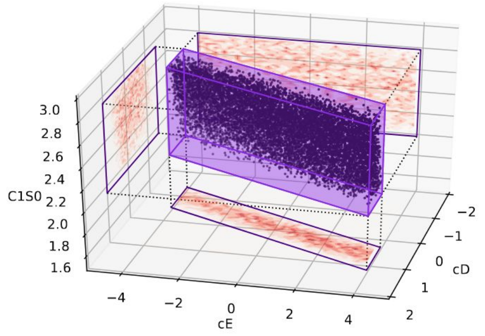


Credits: https://cosmicexplorer.org/

Emulators for coupled-cluster calculations of the neutron star EOS:

- Recently, emulators for the nuclear matter EOS have been developed based on the subspace-projected coupled-cluster method. The emulator employs a nonlocal Δ-full EFT interaction at N^2LO.

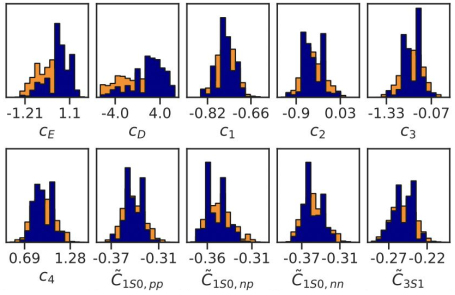
History matching to identify non-implausible domains for the LECs



See Christian Forssén's talk at 12:00 PM.

Bayesian analysis using sampling/importance resampling

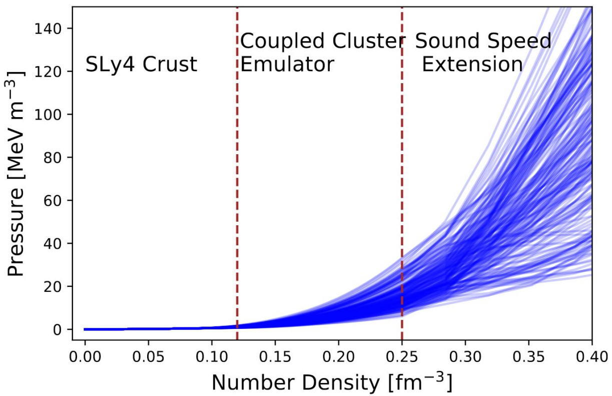
See Weiguang Jiang's talk at 1:30 PM.



W.G. Jiang et al., 2212.13216

Each sample can be converted to a NS EOS:

- The prediction of the emulator, for a given LEC parameterization, determines the NS EOS up to ~1.5 n_{sat}.
- For every such low-density sample, we construct a large number (~100) of high density extensions in an agnostic manner by sampling the sound-speed c^2_s (n) ~ U[0,1].



An Injection study of a year's worth of 3G GW detections

- We inject GW signals into a network of 3 next gen. GW detectors using the APR EOS (AV18 + UIX).

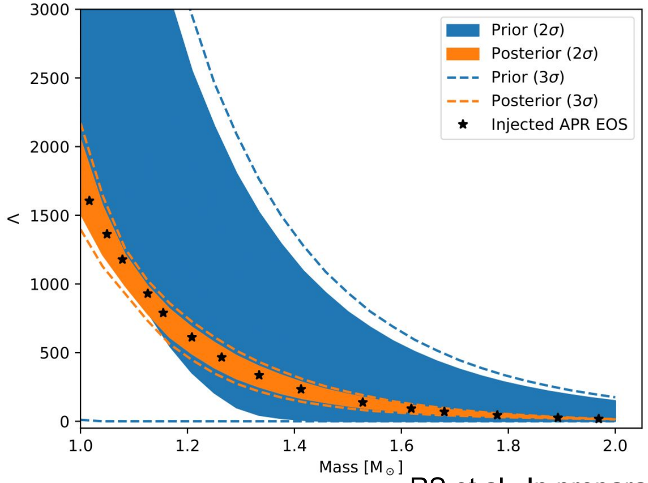
- The GW signals are analysed using ~ 10^4 samples calibrated to A=2-4 observables. This constitutes our prior. Each sample is given a GW likelihood using Bayesian statistics. This results in posteriors on all EOS parameters

EOS input

$$P(\boldsymbol{d}|\boldsymbol{\theta}) \propto \prod_i \int P(d_i|\mathcal{M}, q, \tilde{\Lambda})\, \pi(\mathcal{M}, q, \tilde{\Lambda}|\boldsymbol{\theta})\, d\mathcal{M}\, dq\, d\tilde{\Lambda}.$$

GW likelihood of EOS sample $\boldsymbol{\theta}$

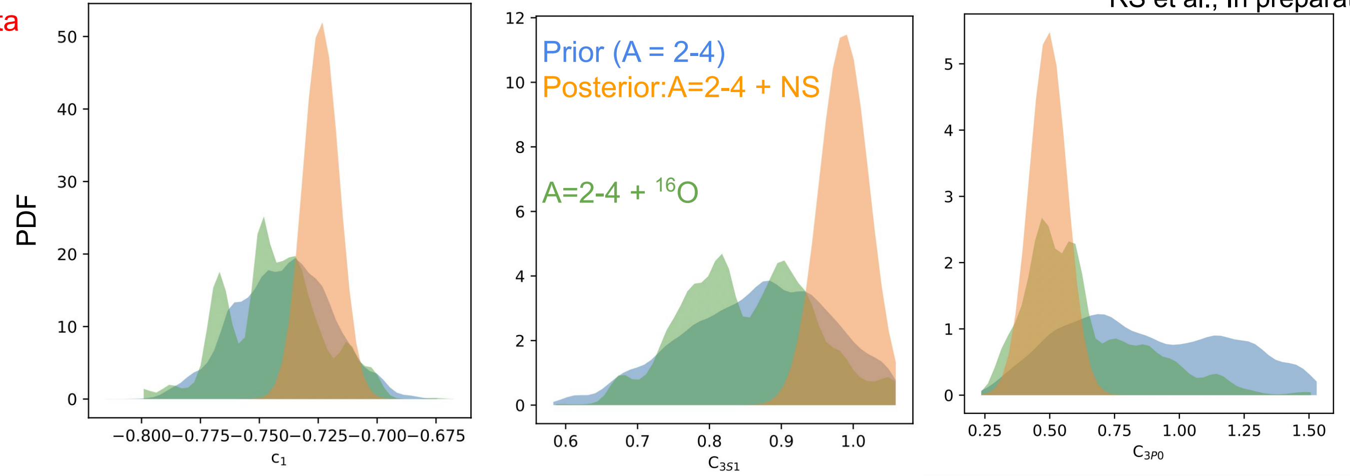
GW data



RS et al., In preparation

- Marginalizing over the high-density EOS, leads to posterior over the LECs.

- In the future, we will be able to calibrate subleading 3N, 4N and possibly YN couplings to NSs.

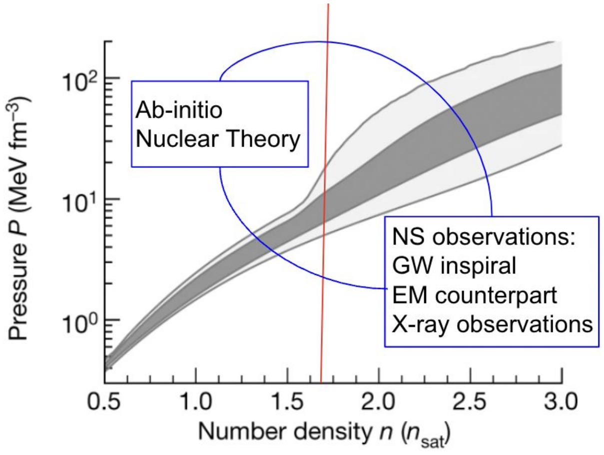


Conclusions and outlook

Ab-initio nuclear theory:

- We have developed novel (maximally) local chiral interactions at N^2LO (NN+3N) and N^3LO (NN) for quantum monte carlo calculations. Our work includes exploring a wide range of cutoffs and Bayesian uncertainty quantification.

- The next step would be to include 3N forces N^3LO, allowing for the complete QMC calculations of few and many body nuclear systems.



Applications to astrophysics:

- We have shown how efficient emulators can help in the analysis of astrophysical observations of NSs. The 'golden age' of GW astronomy will allow us to measure the nuclear interaction and calibrate subleading 3N, 4N and possibly YN LECs to astrophysical data.

- Development of fast and efficient emulators for different many-body methods (such as QMC) is crucial for this research direction.

Acknowledgements

I am very grateful to my collaborators and mentors:

Nuclear physicists:
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Thank You!