## Sorry Maria, I forgot to change my

 title:Dimensionality reduction for accelerating uncertainty quantification




Pablo Giuliani

## Outline

Why
Uncertainty Quantification?
How

The Reduced Basis Method
How it works
Applications and Results
Upcoming Highlights

Takeaways

## Outline

Why
Uncertainty Quantification?
How

The Reduced Basis Method
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## Why Uncertainty Quantification?



## Why Uncertainty Quantification?

r-process
Lifetimes (Fission)
( decay)


Many models

UNEDF\# SV-min $S k M^{*}$ HFB\# FRDM\# •••

Projected FRIB beam rates

$$
\begin{aligned}
& >10^{8} \mathrm{~s}^{-1} \\
& 10^{6}-10^{8} \mathrm{~s}^{-1} \\
& 10^{4}-10^{6} \mathrm{~s}^{-1} \\
& 10^{2}-10^{4} \mathrm{~s}^{-1} \\
& 10^{0}-10^{2} \mathrm{~s}^{-1} \\
& 10^{-2}-10^{0} \mathrm{~s}^{-1} \\
& 10^{-4}-10^{-2} \mathrm{~s}^{-1} \\
& 10^{-6}-10^{-4} \mathrm{~s}^{-1}
\end{aligned}
$$

## Why Uncertainty Quantification?



## Why Uncertainty Quantification?



## Why Uncertainty Quantification?

## Why Uncertainty Quantification?

NSAC Long Range Plan Town Hall Meeting on Nuclear Structure, Reactions and Astrophysics

## Why Uncertainty Quantification?

## NSAC Long Range Plan Town Hall Meeting on Nuclear Structure, Reactions and Astrophysics <br> Nov 14 - 16, 2022

Bayesian methods for extrapolations to stellar energies Needs

At the intersection of low-energy nuclear physics and fundamental symmetries Alejandro Garcia
Experiencing a revolution in our field brought by:
Daniel Phillips

- Detailed discussion of systematic uncertainties, ideally with covariance Detailed discussion of systematic uncertainties, deally with covariance
matrices, in experimental publications; theory-experiment collaborations Collaboration with statisticians (e.g., through ISNET series of meetings, funding for inter-disciplinary collaboration) on forefront statistical approaches for these problems

2. Im
Improved theory allowing for optimizing opportunities and calculating SM expectations, including uncertainties.

Enchancing the accuracy of optical potentials

Outlook and recommendations Inclusion of uncertainty quantification: Bayesian framework well suited for UQ, extrapolation \& interpolation

Systematic measurements along isotopic chains to improve reaction theory

Develop more accurate global (dispersive) optical model with uncertainties
Quantifying model uncertainties of popular model e.g. for transfer reactions $\rightarrow$ ADWA or DWBA?

Tremendous progress in CEFT, many-body theory, UQ \& HPC Bayesian statistics allows for rigorous UQ \& propagation in EFT-based calculations (use emulators!) Christian

Integrated structure \& reaction theory for medium-mass and heavy nuclei Deploy ML/AI tools and assess uncertainties

Jutta Escher
${ }_{5}^{5} N u$ clear data for astrophysics (in neutron-rich environments)

## Intersections of low-energy nuclear

 physics and fundamental symmetriesMax Brodeur, Vincenzo Cirigliano, Alejandro Garcia, Kyle Leach, Dan Melconian, Peter Mueller, Saori Pastore, Jaideep What progress has been made since the last LRP? Singh, Ragnar Stroberg
Since the last LRP (relevant to the structure community):

1. Nuclear theory related to FS has made MAJOR strides in several areas F.M. Nunes including $0 v \beta \beta$ decay NMEs, neutrino-nucleus scattering, corrections to Bayesian analysis as a beta decay in the extraction of Vud (both nuclear and radiative) - especially UQs. (Talks by: Heiko Hergert and Joe Carlson)

Studies examining variations in theoretical $\boldsymbol{\gamma}$-strength functions and nuclear level densities show the large impact of $(n, \gamma)$ rate uncertainties on astrophysical neutron capture processes (i-process and

## Computing (HPC, Quantum, AI/ML)

What are the most compelling scientific opportunities over the next decade \& their potential scientific impact?

- Development of emulators, $\mathrm{Al} / \mathrm{ML}$ and Bayesian methods:
- Opens up entirely new ways to make predictions and quantify uncertainties
- Experimental design: which measurements will help constrain/inform theoretical models (maximize the success of an experiment)

Gaute Hagen, Calvin Johnson, Michelle Kuchera Dean Lee, Pieter Maris, Kyle Wendt

## Nuclear Structure and Reaction Theory

Working group: Papenbrock, Phillips, Piarulli, Potel, Schunck, Tews, Volya

+ Fossez, Hebborn, Koenig
- Reactions are awesome: Reactions are the best window into the structure and dynamics of nuclei, and address data needed for other fields. Full UQ and reaction-theory modeling crucial


## Betty Tsang

## Neutron Stars and Dense Matter

Since LRP2015, major Quantification of uncertainties breakthroughs

5-10 year priorities for nuclear data covariances and uncertainty quantification as defined by the Nuclear Data Uncertainty Quantification Meeting

## Predictive theory of nuclei and their interactions

We have entered a precision era: field moves Thomas Papenbrock

Uncertainty quantification \& Bayesian machine learning have advanced nuclear theory

## Why Uncertainty Quantification?

## NSAC Long Range Plan Town Hall Meeting on Nuclear Structure, Reactions and Astrophysics <br> Nov 14 - 16, 2022

Bayesian


uncertainty quantification:

node with uncertainties Quantifying model uncertainties



Bayesian analysis


What are the most compelling scientific opportunities over the next decade \& their potential scientific impact?

- Development of emulators, $\mathrm{Al} / \mathrm{ML}$


## Bayesian methods:

> uncertainties
> quantify

## Experimental

 measurements will help constrain/inform theoretical models (maximize the success of an experiment) Data Uncertainty Quantification Meeting

## Nuclear Structure and Reaction Theory

 + Fossez, Hebborn, Koenig
$\qquad$

```
Predictive theory of nuclei
and their interactions
```


towards quantified uncertainties
Uncertainty quantification \& Bayesian machine

## Why Uncertainty Quantification?

## NSAC Long Range Plan Town Hall Meeting on Nuclear Structure, Reactions and Astrophysics <br> Nov 14 - 16, 2022

## This is very important to us

$\qquad$
uncertainty quantification:


## Bayesian methods:

predictions an quantify

uncertainty quantification
quantified uncertainties
Uncertainty quantification \& Bayesian

## Why Uncertainty Quantification?

How

$$
P(\alpha \mid \boldsymbol{Y})=\frac{P(\boldsymbol{Y} \mid \alpha) P(\alpha)}{P(\boldsymbol{Y})}
$$

Bayesian approach?

Why Uncertainty Quantification? How


The most important thing in my opinion:

Why Uncertainty Quantification? How


The most important thing in my opinion:

mathematics
statistics
computational

Work in collaboration with experts

## Why Uncertainty Quantification?

 How

The most important thing in my opinion:

## Communication


mathematics
$\rightarrow$ Work in collaboration with experts

## Communication

## Google Scholar

"eigenvector continuation"

About 188 results ( 0.08 sec )
control parameter in the Hamiltonian matrix exceeds some threshold value. In this Letter we present a new technique called eigenvector continuation that can extend the reach of these methods. The key insight is that while an eigenvector resides in a linear space with enormous dimensions, the eigenvector trajectory generated by smooth changes of the Hamiltonian matrix is well approximated by a very low-dimensional manifold. We

## Communication

Eigenvector Continuation with Subspace Learning (2018)
Dillon Frame, ${ }^{1,2}$ Rongzheng He, ${ }^{1,2}$ Ilse Ipsen, ${ }^{3}$ Daniel Lee, ${ }^{4}$ Dean Lee,,${ }^{1,2}$ and Ermal Rrapaj ${ }^{5}$


Ab initio predictions link the neutron skin of ${ }^{208} \mathrm{~Pb}$ to nuclear forces
Baishan Hu $\odot^{י 1 n}$, Weizuang Jiang $\oplus^{2 n}$, Takayuki Miyagi $\oplus^{134 n}$, Zhonghao Sun ${ }^{5.6 n}$, Andreas Ekström ${ }^{2}$ Christian Forssén $\oplus^{2 \boxtimes,}$, Gaute Hagen $\oplus^{15.56}$, Jason D. Holt $\oplus^{י}$, Thomas Papenbrock $\oplus^{\text {5.6. }}$, S. Ragnar Stroberg ${ }^{\text {s, },}$ and lan Vernon ${ }^{10}$


Efficient emulators for scattering using eigenvector continuation
Eigenvector continuation as an efficient and accurate emulator for uncertainty quantification (2020)
S. König ${ }^{\text {a,b,c,* }}$, A. Ekström ${ }^{\text {d }}$, K. Hebeler ${ }^{\text {a,b }}$,
D. Lee ${ }^{\mathrm{e}}$, A. Schwenk ${ }^{\mathrm{a}, \mathrm{b}, \mathrm{f}}$

## Improved many-body expansions from eigenvector continuation

(2020)
P. Demol $\odot,{ }^{1}$ T. Duguet, ${ }^{1,2}$ A. Ekström, ${ }^{3}$ M. Frosini, ${ }^{2}$ K. Hebeler, ${ }^{4,5}$ S. König $\odot,{ }^{4,5,6}$ D. Lee $\oplus,{ }^{7}$ A. Schwenk, ${ }^{4,5,8}$ V. Somà, ${ }^{2}$ and A. Tichai $\odot^{9,8,4,5,{ }^{*}}$
R.J. Furnstahl, A.J. Garcia, P.J. Millican, Xilin Zhang* (2020)

## Communication

Eigenvector Continuation with Subspace Learning (2018)
Dillon Frame, ${ }^{1,2}$ Rongzheng He, ${ }^{1,2}$ Ilse Ipsen, ${ }^{3}$ Daniel Lee, ${ }^{4}$ Dean Lee, ${ }^{1,2}$ and Ermal Rrapaj ${ }^{5}$


## Communication

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## Dimensionality reduction and polynomial chaos acceleration of Bayesian inference in inverse problems

Youssef M. Marzouk ${ }^{\text {a,* }}$, Habib N. Najm ${ }^{\text {b }}$

## A REDUCED ORDER MODEL FOR MULTI-GROUP TIME-DEPENDENT PARAMETRIZED REACTOR SPATIAL KINETICS <br> (2014)



Sartori, et al


Reduced-order modeling of time-dependent PDEs with multiple parameters in the boundary data

Max D. Gunzburger ${ }^{\text {a, }, 1}$, Janet S. Peterson ${ }^{\text {a,1 }}$, John N. Shadid ${ }^{\text {b,2 }}$
(2006)

An 'empirical interpolation' method: application to efficient reduced-basis discretization of partial differential equations

Maxime Barrault ${ }^{\text {a }}$, Yvon Maday ${ }^{\mathrm{b}}$, Ngoc Cuong Nguyen ${ }^{\mathrm{c}}$, Anthony T. Patera ${ }^{\text {d }}$ (2004)

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| Certified Reduced |
| :--- |
| Basi Methods |
| for Prametrized |
| Partial Diffierential |
| Equations |
| (20en) |
| (2016) |

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'eige

## Youssef M. Marzouk ${ }^{\text {a,* }}$, Habib N. Najm ${ }^{\text {b }}$

1 Note that because the $\kappa$ parameters do not appear linearly in the Hamiltonian, one can no longer make a single set of matrix elements calculations for all of the test parameter sets. In other contexts this might be a relevant computational disadvantage.

$$
\begin{aligned}
& V_{1_{S_{0}}}(r) \equiv V_{0 R} e^{-\kappa_{R} r^{2}}+V_{0 s} e^{-\kappa_{s} r^{2}} \\
& V_{{ }_{3} S_{1}}(r) \equiv V_{0 R} e^{-\kappa_{R} r^{2}}+V_{0 t} e^{-\kappa_{t} r^{2}}
\end{aligned}
$$

modeling of time-d
parameters in the

The nuclear potential that we employ is additive in the $d=$ 16 LECs, ie., we can express the Hamiltonian as $H(\mathbf{c})=H_{0}+$ $\sum_{i=1}^{d} c_{i} H_{i}$, where $H_{0}$ includes the kinetic energy. Any Hamiltosian with more than one interaction parameter can be written in this form, where each $c_{i}$ in general may be depend nonlinearly on other parameters. Furthermore, each term $H_{i}$ for $i=1, \ldots, 16$ can be projected onto the EC subspace once and then used for an arbitrary number of emulations. Each of these corresponds to a
matrix. This problem can be avoided by running an orthogonalization on the EC vectors that stabilizes the subsequent humerical steps and reveals the effective dimension of the EC subspace. Since this step leads to a unit norm matrix, it also reduces the per-sample evaluation cost at the price of additional preprocessing effort (see Appendix A).


## 

An 'empirical interpolation' method: application to efficient reduced-basis discretization of partial differential equations

Maxime Barrault ${ }^{\text {a }}$, Yvon Maday ${ }^{\text {b }}$, Ngoc Cong Nguyen ${ }^{\text {c }}$, Anthony T. Patera ${ }^{\text {d }}$

## Why Uncertainty Quantification? How $\longrightarrow$ communication with experts



## Why Uncertainty Quantification?

How $\longrightarrow$ communication with experts


## Why Uncertainty Quantification?

HOW $\longrightarrow$ communication with experts
[] Collocation-Method.py
[ Cool-technique.py
[] Empirical-Interpolation-Method.py
[ Greedy-Sampling
(1) POD-Basis.py
(1) README.md
[ Reference-Domain.py

Projecting in Dirac Deltas
Something extra cool
Create Empirical-Interpolation-Method.py
Smart sampling
Create POD-Basis.py
Initial commit
Create a ref domain for multi-solutions
a long time ago
a long time ago
a long time ago
a long time ago
a long time ago
a long time ago
a long time ago


# Why Uncertainty Quantification? <br> HOW $\longrightarrow$ communication with experts 



## Why Uncertainty Quantification?

HOW $\longrightarrow$ Communication with experts


## Why Uncertainty Quantification?

 How $\longrightarrow$ communication with experts

## Outline

Uncertainty Quantification?
How
The Reduced Basis Method
How it works
Applications and Results

## Upcoming Highlights

Takeaways

## Emulators




## Emulators



Dick's talk this morning


## Emulators



Dick's talk this morning


## Computation Accuracy vs Time



## The Reduced Basis Method

## The Reduced Basis Method



## The Reduced Basis Method



## The Reduced Basis Method



## parameters

$\mathcal{H}_{\alpha}^{\downarrow} \phi(x)=\lambda \phi(x)$


Finite element

## The Reduced Basis Method



## parameters

$\mathcal{H}_{\alpha}^{\downarrow} \phi(x)=\lambda \phi(x)$


## The Reduced Basis Method




## The Reduced Basis Method



Changing the trapping strength $\alpha$

## The Reduced Basis Method

$$
F_{\alpha}[\phi(x)]=0
$$

General differential equation
$\left(\mathcal{H}_{\alpha} \phi(x)-\lambda \phi(x)=0\right)$

## The Reduced Basis Method

$$
F_{\alpha}[\phi(x)]=0
$$

General differential equation

$$
\left(\mathcal{H}_{\alpha} \phi(x)-\lambda \phi(x)=0\right)
$$



1) Choose a basis

$$
\hat{\phi}(x)=\phi_{0}+\sum_{k}^{n} a_{k} \phi_{k}(x)
$$



## The Reduced Basis Method

$$
F_{\alpha}[\phi(x)]=0
$$

General differential equation

$$
\left(\mathcal{H}_{\alpha} \phi(x)-\lambda \phi(x)=0\right)
$$



1) Choose a basis

$$
\hat{\phi}(x)=\phi_{0}+\sum_{k}^{n} a_{k} \phi_{k}(x)
$$


2) Project onto judges
$j=\{1, n\} \quad\left\langle\psi_{j} \mid F_{\alpha}[\hat{\phi}(x)]\right\rangle=0$
One equation per coefficient

## The Reduced Basis Method

$$
F_{\alpha}[\phi(x)]=0
$$

General differential equation

$$
\left(\mathcal{H}_{\alpha} \phi(x)-\lambda \phi(x)=0\right)
$$



1) Choose a basis

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\hat{\phi}(x)=\phi_{0}+\sum_{k}^{n} a_{k} \phi_{k}(x)
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usually, a
2) Project onto judges
 challenge
$j=\{1, n\} \quad\left\langle\psi_{j} \mid F_{\alpha}[\hat{\phi}(x)]\right\rangle=0$
One equation per coefficient

## The Reduced Basis Method

$$
F_{\alpha}[\phi(x)]=0
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General differential equation

$$
\left(\mathcal{H}_{\alpha} \phi(x)-\lambda \phi(x)=0\right)
$$



1) Choose a basis

$$
\hat{\phi}(x)=\phi_{0}+\sum_{k}^{n} a_{k} \phi_{k}(x)
$$

Usually, a
2) Project onto judges
$j=\{1, n\} \quad\left\langle\psi_{j} \mid F_{\alpha}[\hat{\phi}(x)]\right\rangle=0$
One equation per coefficient
challenge

Galerkin


Training and Projecting




J A Melendez ${ }^{1} \oplus$, C Drischler ${ }^{2} \oplus$, R J Furnstahl ${ }^{1, *}{ }^{\oplus}$, A J Garcia ${ }^{1}{ }^{\text {© }}$ and Xilin Zhang ${ }^{2}{ }^{\text {© }}$
${ }^{1}$ Department of Physics, The Ohio State University, Colv United States of America
${ }^{2}$ Facility for Rare Isotope Beams, Michigan State Univer United States of America


Training and Projecting


[^0](1) Training and Projecting: A Reduced Basis ${ }^{\text {arh }}{ }_{2022}$ Method Emulator for Many-Body Physics

Edgard Bonilla, ${ }^{1, *}$ Pablo Giuliani, ${ }^{2,3, \dagger}$ Kyle Godbey, ${ }^{2, \ddagger}$ and Dean Lee ${ }^{2,4, \S}$

Training and Projecting


# Applications and Results 

(1) Training and Projecting: A Reduced Basis ${ }^{\text {arh }}{ }_{2022}$ Method Emulator for Many-Body Physics

Edgard Bonilla, ${ }^{1, *}$ Pablo Giuliani, ${ }^{2,3, \dagger}$ Kyle Godbey, ${ }^{2, \ddagger}$ and Dean Lee ${ }^{2,4, \S}$

1) Broadly Applicable


Training and Projecting


# Applications and Results 

 Method Emulator for Many-Body PhysicsEdgard Bonilla,,$^{1, *}$ Pablo Giuliani, ${ }^{2,3, \dagger}$ Kyle Godbey, ${ }^{2, \ddagger}$ and Dean Lee ${ }^{2,4, \S}$


# Applications and Results 

(1) Training and Projecting: A Reduced Basis ${ }^{\text {(1) }}{ }_{2022}$

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1) Broadly Applicable




Training and Projecting


# Applications and Results 

## 1) Broadly Applicable


2) Very accurate



Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method


# Applications and Results 

Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

Fields


# Applications and Results 

$$
\begin{aligned}
& \text { Dirac Equations } \\
& \qquad \begin{array}{l}
\left(\frac{d}{d r}+\frac{\kappa}{r}\right) g_{a}(r)-\left[E_{a}+M-\Phi_{0}(r)-W_{0}(r) \mp \frac{1}{2} B_{0}(r)-e\left\{\begin{array}{l}
1 \\
0
\end{array}\right\} A_{0}(r)\right] f_{a}(r)=0 \\
\left(\frac{d}{d r}-\frac{\kappa}{r}\right) f_{a}(r)+\left[E_{a}-M+\Phi_{0}(r)-W_{0}(r) \mp \frac{1}{2} B_{0}(r)-e\left\{\begin{array}{l}
1 \\
0
\end{array}\right\} A_{0}(r)\right] g_{a}(r)=0
\end{array}
\end{aligned}
$$

## Nucleons

Field Equations

$$
\begin{aligned}
& \left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\mathrm{s}}^{2}\right) \Phi_{0}(r)-g_{\mathrm{s}}^{2}\left(\frac{\kappa}{2} \Phi_{0}^{2}(r)+\frac{\lambda}{6} \Phi_{0}^{3}(r)\right)=-g_{\mathrm{s}}^{2}\left(\rho_{\mathrm{s}, \mathrm{p}}(r)+\rho_{\mathrm{s}, \mathrm{n}}(r)\right), \\
& \left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\mathrm{v}}^{2}\right) W_{0}(r)-g_{\mathrm{v}}^{2}\left(\frac{\zeta}{6} W_{0}^{3}(r)+2 \Lambda_{\mathrm{v}} B_{0}^{2}(r) W_{0}(r)\right)=-g_{\mathrm{v}}^{2}\left(\rho_{\mathrm{v}, \mathrm{p}}(r)+\rho_{\mathrm{v}, \mathrm{n}}(r)\right), \\
& \left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\rho}^{2}\right) B_{0}(r)-2 \Lambda_{\mathrm{v}} g_{\rho}^{2} W_{0}^{2}(r) B_{0}(r)=-\frac{g_{\rho}^{2}}{2}\left(\rho_{\mathrm{v}, \mathrm{p}}(r)-\rho_{\mathrm{v}, \mathrm{n}}(r)\right), \\
& \left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}\right) A_{0}(r)=-e \rho_{\mathrm{v}, \mathrm{p}}(r),
\end{aligned}
$$

Bayes goes fast



# Applications and Results 

$$
\begin{aligned}
& \text { Dirac Equations } \\
& \qquad\left(\frac{d}{d r}+\frac{\kappa}{r}\right) g_{a}(r)-\left[E_{a}+M-\Phi_{0}(r)-W_{0}(r) \mp \frac{1}{2} B_{0}(r)-e\left\{\begin{array}{l}
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& \left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\rho}^{2}\right) B_{0}(r)-2 \Lambda_{\mathrm{v}} g_{\rho}^{2} W_{0}^{2}(r) B_{0}(r)=-\frac{g_{\rho}^{2}}{2}\left(\rho_{\mathrm{v}, \mathrm{p}}(r)-\rho_{\mathrm{v}, \mathrm{n}}(r)\right), \\
& \left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}\right) A_{0}(r)=-e \rho_{\mathrm{v}, \mathrm{p}}(r), \\
& \text { Parameters } \boldsymbol{\alpha}
\end{aligned}
$$

Bayes goes fast


# Applications and Results 

# (2) <br> Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method 

## Dirac Equations

$$
\begin{aligned}
& \left(\frac{d}{d r}+\frac{\kappa}{r}\right) g_{a}(r)-\left[E_{a}+M-\Phi_{0}(r)-W_{0}(r) \mp \frac{1}{2} B_{0}(r)-e\left\{\begin{array}{l}
1 \\
0
\end{array}\right\} A_{0}(r)\right] f_{a}(r)=0 \\
& \left(\frac{d}{d r}-\frac{\kappa}{r}\right) f_{a}(r)+\left[E_{a}-M+\Phi_{0}(r)-W_{0}(r) \mp \frac{1}{2} B_{0}(r)-e\left\{\begin{array}{l}
1 \\
0
\end{array}\right\} A_{0}(r)\right] g_{a}(r)=0
\end{aligned}
$$



$$
\begin{aligned}
& \left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\mathrm{s}}^{2}\right) \Phi_{0}(r)-g_{\mathrm{s}}^{2}\left(\frac{\kappa}{2} \Phi_{0}^{2}(r)+\frac{\lambda}{6} \Phi_{0}^{3}(r)\right)=-g_{\mathrm{s}}^{2}\left(\rho_{\mathrm{s}, \mathrm{p}}(r)+\rho_{\mathrm{s}, \mathrm{n}}(r)\right), \\
& \left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\mathrm{v}}^{2}\right) W_{0}(r)-g_{\mathrm{v}}^{2}\left(\frac{\zeta}{6} W_{0}^{3}(r)+2 \Lambda_{\mathrm{v}} B_{0}^{2}(r) W_{0}(r)\right)=-g_{\mathrm{v}}^{2}\left(\rho_{\mathrm{v}, \mathrm{p}}(r)+\rho_{\mathrm{v}, \mathrm{n}}(r)\right), \\
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& \left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}\right) A_{0}(r)=-e \rho_{\mathrm{v}, \mathrm{p}}(r),
\end{aligned}
$$

Bayes goes fast


# Applications and Results 

# 2 Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method 

$$
\begin{aligned}
& \text { Dirac Equations } \\
& \left\langle g_{a, k}^{(j)}\right|\left(\frac{d}{d r}+\frac{\kappa}{r}\right) g_{a}(r)-\left[E_{a}+M-\Phi_{0}(r)-W_{0}(r) \mp \frac{1}{2} B_{0}(r)-e\left\{\begin{array}{l}
1 \\
0
\end{array}\right\} A_{0}(r)\right] f_{a}(r)=0 \\
& \left\langle f_{a, k}^{(j)}\right|\left(\frac{d}{d r}-\frac{\kappa}{r}\right) f_{a}(r)+\left[E_{a}-M+\Phi_{0}(r)-W_{0}(r) \mp \frac{1}{2} B_{0}(r)-e\left\{\begin{array}{l}
1 \\
0
\end{array}\right\} A_{0}(r)\right] g_{a}(r)=0
\end{aligned}
$$



Fields


## Field Equations

$$
\begin{aligned}
& \left\langle\Phi_{j}(r)\right|\left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\mathrm{s}}^{2}\right) \Phi_{0}(r)-g_{\mathrm{s}}^{2}\left(\frac{\kappa}{2} \Phi_{0}^{2}(r)+\frac{\lambda}{6} \Phi_{0}^{3}(r)\right)=-g_{\mathrm{s}}^{2}\left(\rho_{\mathrm{s}, \mathrm{p}}(r)+\rho_{\mathrm{s}, \mathrm{n}}(r)\right) \\
& \left\langle W_{j}(r)\right|\left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\mathrm{v}}^{2}\right) W_{0}(r)-g_{\mathrm{v}}^{2}\left(\frac{\zeta}{6} W_{0}^{3}(r)+2 \Lambda_{\mathrm{v}} B_{0}^{2}(r) W_{0}(r)\right)=-g_{\mathrm{v}}^{2}\left(\rho_{\mathrm{v}, \mathrm{p}}(r)+\rho_{\mathrm{v}, \mathrm{n}}(r)\right), \\
& \left\langle B_{j}(r)\right|\left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\rho}^{2}\right) B_{0}(r)-2 \Lambda_{\mathrm{v}} g_{\rho}^{2} W_{0}^{2}(r) B_{0}(r)=-\frac{g_{\rho}^{2}}{2}\left(\rho_{\mathrm{v}, \mathrm{p}}(r)-\rho_{\mathrm{v}, \mathrm{n}}(r)\right), \\
& \left\langle A_{j}(r)\left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}\right) A_{0}(r)=-e \rho_{\mathrm{v}, \mathrm{p}}(r)\right.
\end{aligned}
$$

Bayes goes fast


# Applications and Results 

# (2) <br> Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method 

## Dirac Equations

$$
\left.\left\langle g_{a, k}^{(j)}\right|\left(\frac{d}{d r}+\frac{\kappa}{r}\right) g_{a}(r)-\left[E_{a}+M-\Phi_{0}(r)\right]-W_{0}(r) \mp \frac{1}{2} B_{0}(r)-e\left\{\begin{array}{l}
1 \\
0
\end{array}\right\} A_{0}(r)\right] f_{a}(r)=0
$$

$$
\left\langle f_{a, k}^{(j)}\right|\left(\frac{d}{d r}-\frac{\kappa}{r}\right) f_{a}(r)+\left[E_{a}-M++\Phi_{0}(r)-W_{0}(r) \mp \frac{1}{2} B_{0}(r)-e\left\{\begin{array}{l}
1 \\
0
\end{array}\right\} A_{0}(r)\right] g_{a}(r)=0
$$

Field Equations

$$
\left\langle\Phi_{j}(r)\right|\left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\mathrm{s}}^{2}\right) \Phi_{0}(r)-g_{\mathrm{s}}^{2}\left(\frac{1}{2} \Phi_{0}^{2}(r)+\frac{\lambda}{6} \Phi_{0}^{3}(r)\right)=-g_{\mathrm{s}}^{2}\left(\rho_{\mathrm{s}, \mathrm{p}}(r)+\rho_{\mathrm{s}, \mathrm{n}}(r)\right),
$$

$$
\left\langle W_{j}(r)\right|\left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\mathrm{v}}^{2}\right) W_{0}(r)-g_{\mathrm{v}}^{2}\left(\frac{\zeta}{6} W_{0}^{3}(\underset{(\mathrm{a})}{ }\right.
$$

$$
\left\langle B_{j}(r)\right|\left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}-m_{\rho}^{2}\right) B_{0}(\eta)-2 \Lambda / g_{\rho}^{2} W_{0}^{2}(r
$$

$$
\left\langle A_{j}(r)\left(\frac{d^{2}}{d r^{2}}+\frac{2}{r} \frac{d}{d r}\right) A_{0}(r)=-\rho_{y} \rho_{p}(r)\right.
$$

$$
\Phi_{0}(r) \approx \hat{\Phi}_{0}(r)=\sum_{k=1}^{n_{\Phi}} a_{k}^{\Phi} \Phi_{k}(r)
$$



# Applications and Results 

(2) Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

## Posterior Bayesian calibration


Masses and
Charge radii $\left\{\begin{array}{l}{ }^{16} \mathrm{O} \\ { }^{40} \mathrm{Ca} \\ { }^{48} \mathrm{Ca} \\ { }^{68} \mathrm{Ni} \\ { }^{90} \mathrm{Zr} \\ { }^{100} \mathrm{Sn} \\ { }^{116} \mathrm{Sn} \\ { }^{132} \mathrm{Sn} \\ { }^{144} \mathrm{Sm} \\ { }^{208} \mathrm{~Pb}\end{array}\right.$

Bayes goes fast

https://doi.org/10.3389/fphy.2022.1054524

# Applications and Results 



# Applications and Results 




Bayes goes fast


Presenting ROSE, a Reduced Order Scattering Emulator
 Bonilla, ${ }^{6}$ K. Godbey, ${ }^{2}$ R. J. Furnstahl, ${ }^{7}$ and F. M. Nunes ${ }^{2}, 4$, 团


The roses

https://colab.research.google.com/drive/1Vtg 11apJy0o4D2MloDz1D0WbxbxlwW8H

 Bonilla, ${ }^{6}$ K. Godbey, ${ }^{2}$ R. J. Furnstahl, ${ }^{7}$ and F. M. Nunes ${ }^{2}, 4$, 团

Daniel Odell


The roses

https://colab.research.google.com/drive/1Vtg 11apJy0o4D2MloDz1DOWbxbxlwW8H


The roses

1) Free solution and principal components
2) Energy re-scaling (reference domain)
3) Non-affinity (Empirical Interpolation)

https://colab.research.google.com/drive/1Vtg 11apJy0o4D2MloDz1D0WbxbxlwW8H

# Applications and Results 

Usually, a challenge 1) Free solution and princtal components

3) Non-affinity (Empirical Interpolation)


The roses


## Applications and Results

Presenting ROSE, a Reduced Order Scattering Emulator
D. Odell, ${ }^{1, \text {, }}$. P. Giuliani, ${ }^{2,3}$ M. Catacora-Rios, ${ }^{2,4}$ M. Chan, ${ }^{5}$ E. Bonilla, ${ }^{6}$ K. Godbey, ${ }^{2}$ R. J. Furnstahl, ${ }^{7}$ and F. M. Nunes ${ }^{2,4,}$,

## Optical Potential



$$
\begin{aligned}
& U(r, \alpha)=-V_{v}\left[1+\exp \left(\frac{r-R_{v}}{a_{v}}\right)\right]-i W_{v}\left[1+\exp \left(\frac{r-R_{w}}{a_{w}}\right)\right]-i 4 a_{d} W_{d} \frac{d}{d r}\left[1+\exp \left(\frac{r-R_{d}}{a_{d}}\right)\right] \\
& F_{\alpha}(\phi)=\left(-\frac{d^{2}}{d r^{2}}+\frac{\ell(\ell+1)}{r^{2}}+U(r, \alpha)-p^{2}\right) \phi(r)=0
\end{aligned}
$$

The roses

https://colab.research.google.com/drive/1Vtg 11apJy0o4D2MloDz1D0WbxbxlwW8H

## Applications and Results

D. Odell, ${ }^{1,}{ }^{1}$ O P. Giuliani, ${ }^{2,3}$ M. Catacora-Rios, ${ }^{2,4}$ M. Chan, ${ }^{5}$ E. Bonilla, ${ }^{6}$ K. Godbey, ${ }^{2}$ R. J. Furnstahl, ${ }^{7}$ and F. M. Nunes ${ }^{2}$, 4, ,

## Optical Potential



$$
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& F_{\alpha}(\phi)=\left(-\frac{d^{2}}{d r^{2}}+\frac{\ell(\ell+1)}{r^{2}}+U(r, \alpha)-p^{2}\right) \phi(r)=0
\end{aligned}
$$

$$
\left\langle\psi_{j} \mid F_{\alpha}[\hat{\phi}(r)]\right\rangle=\int \psi_{j}(r) F_{\alpha}[\hat{\phi}(r)] d r=0
$$

The roses

https://colab.research.google.com/drive/1Vtg 11apJy0o4D2MloDz1D0WbxbxlwW8H

## Applications and Results

Presenting ROSE, a Reduced Order Scattering Emulator
 Bonilla, ${ }^{6}$ K. Godbey, ${ }^{2}$ R. J. Furnstahl, ${ }^{7}$ and F. M. Nunes ${ }^{2,4,}$, ${ }^{5}$

## Optical Potential

$$
\begin{aligned}
& U(r, \alpha)=-V_{v}\left[1+\exp \left(\frac{r-\mid R_{v}}{a_{v}}\right)\right]-i W_{v}\left[1+\exp \left(\frac{r-\mid R_{w}}{a_{w}}\right)\right]-i 4 a_{d} W_{d} \frac{d}{d r}\left[1+\exp \left(\frac{r-\left|R_{d}\right|}{a_{d}}\right)\right] \\
& \underbrace{F_{\alpha}(\phi)=\left(-\frac{d^{2}}{d r^{2}}+\frac{\ell(\ell+1)}{r^{2}}+U(r, \alpha)-p^{2}\right) \phi(r)=0} \text { Non-affine problem }
\end{aligned}
$$

$$
\left\langle\psi_{j} \mid F_{\alpha}[\hat{\phi}(r)]\right\rangle=\int \psi_{j}(r) F_{\alpha}[\hat{\phi}(r)] d r=0
$$

The roses

https://colab.research.google.com/drive/1Vtg 11apJy0o4D2MloDz1D0WbxbxlwW8H

## Applications and Results

Presenting ROSE, a Reduced Order Scattering Emulator
D. Odell, ${ }^{1, \text { 团 P. Giuliani, }}{ }^{2,3}$ M. Catacora-Rios, ${ }^{2,4}$ M. Chan, ${ }^{5}$ E. Bonilla, ${ }^{6}$ K. Godbey, ${ }^{2}$ R. J. Furnstahl, ${ }^{7}$ and F. M. Nunes ${ }^{2}, 4$, 团

## Optical Potential

$$
\begin{aligned}
& U(r, \alpha)=-V_{v}\left[1+\exp \left(\frac{r-R_{v}}{a_{v}}\right)\right]-i W_{v}\left[1+\exp \left(\frac{r-\sqrt{R_{w}}}{a_{w}}\right)\right]-i 4 a_{d} W_{d} \frac{d}{d r}\left[1+\exp \left(\frac{r-\sqrt{R_{d}}}{a_{d}}\right)\right] \\
& \underbrace{F_{\alpha}(\phi)} \underbrace{\left(-\frac{d^{2}}{d r^{2}}+\frac{\ell(\ell+1)}{r^{2}}+U(r, \alpha)-p^{2}\right) \phi(r)=0} \text { Non-affine problem }
\end{aligned}
$$

$$
\begin{aligned}
& \left\langle\psi_{j} \mid F_{\alpha}[\hat{\phi}(r)]\right\rangle=\int \psi_{j}(r) F_{\alpha}[\hat{\phi}(r)] d r=0 \\
& U(r, \alpha) \approx \sum_{i}^{m} b_{i}(\alpha) f(r)
\end{aligned}
$$

The roses

https://colab.research.google.com/drive/1Vtg 11apJy0o4D2MloDz1DOWbxbxlwW8H

## Applications and Results

Presenting ROSE, a Reduced Order Scattering Emulator
 Bonilla, ${ }^{6}$ K. Godbey, ${ }^{2}$ R. J. Furnstahl, ${ }^{7}$ and F. M. Nunes ${ }^{2,4,}{ }^{\text {P }}$

1) Choose a basis
$U(r, \alpha) \approx \sum_{i}^{m} b_{i}(\alpha) f(r)$

Principal components of $U(r, \alpha)$


# Applications and Results 

$$
U\left(r_{j}, \alpha\right)-\sum_{i}^{m} b_{i}(\alpha) f\left(r_{j}\right)=0
$$

Obtained by $\quad j=\{1, m\}$ interpolation

1) Choose a basis
$U(r, \alpha) \approx \sum_{i}^{m} b_{i}(\alpha) f(r)$

# Applications and Results 

D. Odell, , , 团 P. Giuliani, ${ }^{2,3}$ M. Catacora-Rios, ${ }^{2,4}$ M. Chan, ${ }^{5}$ E. Bonilla, ${ }^{6}$ K. Godbey, ${ }^{2}$ R. J. Furnstahl, ${ }^{7}$ and F. M. Nunes ${ }^{2}, 4$, 用
2) Project onto judges

$$
U\left(r_{j}, \alpha\right)-\sum_{i}^{m} b_{i}(\alpha) f\left(r_{j}\right)=0
$$



$$
\begin{gathered}
\psi_{j}(r)=\delta\left(r-r_{j}\right) \\
\left\langle\psi_{j} \mid F_{\alpha}[\hat{\phi}(r)]\right\rangle=F_{\alpha}\left[\hat{\phi}\left(r_{j}\right)\right]
\end{gathered}
$$

Dirac
Obtained by $j=\{1, m\}$ interpolation

1) Choose a basis

$$
U(r, \alpha) \approx \sum_{i}^{m} b_{i}(\alpha) f(r)
$$

# Applications and Results 

Non-affinity and Beyond: Mitigating non-linear and non-affine structures for the efficient emulation of Density Functional Theory


# Applications and Results 

Non-affinity and Beyond: Mitigating non-linear and non-affine structures for the efficient emulation of Density Functional Theory

Kyle Godbey ${ }^{1, *+,}$, Edgard Bonilla ${ }^{2,+,}$, Pablo Giuliani ${ }^{1.3}$, and Yanlai Chen ${ }^{4}$




Non-affinity and Beyond: Mitigating non-linear and non-affine structures for the efficient emulation of Density Functional Theory

```
Kyle Godbey }\mp@subsup{}{}{1,*+,},\mathrm{ Edgard Bonilla (2,+,}\mathrm{ , Pablo Giuliani }\mp@subsup{}{}{1.3}\mathrm{ , and Yanlai Chen }\mp@subsup{}{}{4
```

Coming soon challenge

$$
\left.\left\langle\psi_{j}\right| F_{F}|\hat{\phi}(r)\rangle\right\rangle
$$

3) Very fast

$$
\begin{gathered}
\mathcal{H}_{t}(r)=C_{t}^{\rho} \rho_{t}^{2}+C_{t}^{\rho \Delta \rho} \rho_{t} \Delta \rho_{t}+C \\
+C_{t}^{J} \stackrel{\leftrightarrow}{J}_{t}^{2}+C_{t}^{\rho \nabla J} \rho_{t} \nabla \cdot \mathbf{J}_{t} \\
\text { Mili-seconds }
\end{gathered}
$$

VERY non-linear

$$
\leqq \rho(r)^{\alpha}
$$




## Dencity Functinnal Theory



## Level crossing

 is a problem3) Very fast

$$
\left(\phi^{(1)}(r)^{2}+\phi^{(2)}(r)^{2}+\ldots\right)^{\alpha}
$$

VERY non-linear

$$
+C_{t}^{J \overleftrightarrow{J}_{t}^{2}}+C_{t}^{\rho \nabla J} \rho_{t} \nabla \cdot \mathbf{J}_{t},
$$

$\triangleq \rho(r)^{\alpha}$


Level crossing is a problem

$$
\begin{aligned}
\mathcal{H}_{t}(r) & =C_{t}^{\rho} \rho_{t}^{2}+C_{t}^{\rho \Delta \rho} \rho_{t} \Delta \rho_{t}+C \\
& +C_{t}^{J} \stackrel{\leftrightarrow}{J}_{t}^{2}+C_{t}^{\rho \nabla J} \rho_{t} \nabla \cdot \mathbf{J}_{t}
\end{aligned}
$$

$$
\left(\phi^{(1)}(r)^{2}+\phi^{(2)}(r)^{2}+\ldots\right)^{\alpha}
$$

VERY non-linear
§ $\rho(r)^{\alpha}$

## Applications

 and Results(5) Dynamical systems
(Time Dependent Density Functional Theory)


Kyle Godbey
$\frac{\partial}{\partial t} \phi(x, t)=-i \mathcal{H}_{\mathcal{N} \times \mathcal{N}} \phi(x, t)$
$\phi(x, t) \approx$
$\hat{\phi}(x, t)=\phi_{0}(x)+\sum_{k}^{n} a_{k}(t) \phi_{k}(x)\left\langle\psi_{j}\right|$
$\frac{d}{d t} a_{j}(t)=-i \mathcal{H}_{n \times n}^{\text {Reduced dynamical system }} \underset{\substack{\text { tiny } n \\ \text { Size }=n \ll N}}{\text { a }(t)}$

$$
e^{-i t \mathcal{H}_{n \times n}}
$$

$$
\text { Size }=n \ll N
$$

## Applications and Results

Quantum Harmonic Oscillator

$$
\mathcal{H}=-\frac{\partial^{2}}{\partial x^{2}}+\alpha x^{2}
$$



## Applications and Results

Quantum Harmonic Oscillator

$$
\mathcal{H}=-\frac{\partial^{2}}{\partial x^{2}}+\alpha x^{2}
$$



## Applications and Results

Quantum Harmonic Oscillator

(5) Dynamical systems

High fidelity solution


## Applications and Results

Quantum Harmonic Oscillator

$$
\mathcal{H}=-\frac{\partial^{2}}{\partial x^{2}}+\alpha x^{2}
$$

RBMs comparison

(5) Dynamical systems

High fidelity solution



## Applications and Results

Quantum Harmonic Oscillator


## Applications and Results

Quantum Harmonic Oscillator
(5) Dynamical systems Gains from:

1) Smaller systems
2) More stable systems

$$
e^{-i \Delta t \mathcal{H}} \approx 1-i \Delta t \mathcal{H}-\frac{1}{2} \Delta t^{2} \mathcal{H}^{2}+\ldots
$$



Applications and Results


Could project one more in time for more efficiency

$$
e^{-i t \mathcal{H}_{n \times n}}
$$

$\frac{d}{d t} a_{j}(t)=-i \mathcal{H}_{n \times n} \boldsymbol{a}(t)$

## Applications and Results



## Applications and Results



[^1] reduction paradigm of spatio-temporal data

## Applications and Results



Neural Implicit Flow: a mesh-agnostic dimensionality reduction paradigm of spatio-temporal data


Aaron Phillip

## 40 Ca quadrupole vibrations

## Applications and Results


(5) Dynamical systems


Aaron Phillip


# Applications <br> <br> and Resultr 

 <br> <br> and Resultr}
(5) Dynamical systems

UURAF 2023 Award Winner




Aaron Phillip


https://github.com/pswpswpsw/nif

## Outline

Why

## Uncertainty Quantification?

How

## The Reduced Basis Method

How it works
Applications and Results

Upcoming Highlights

Takeaways


# Upcoming highlights 

Application of reduced basis methods to compact stars
Amy Anderson, ${ }^{1, *}$ Pablo Giuliani, ${ }^{2, \dagger}$ and J.Piekarewicz ${ }^{1, \ddagger}$
${ }^{1}$ Department of Physics, Florida State University, Tallahassee, FL 323
${ }^{2}$ FRIB/NSCL Laboratory, Michigan State University, East Lansing, Michiga
Amy
Anderson


# Upcoming highlights 



Amy Anderson, ${ }^{1, *}$ Pablo Giuliani, ${ }^{2, \dagger}$ and J.Piekarewicz ${ }^{1, \ddagger}$
${ }^{1}$ Department of Physics, Florida State University, Tallahassee, FL 325 ${ }^{2}$ FRIB/NSCL Laboratory, Michigan State University, East Lansing, Michig



## Upcoming highlights

(2) Smart posterior handling


## Upcoming highlights

(2) Smart posterior handling


## Upcoming highlights

(2) Smart posterior handling



Chaos Normalizing
(Frederi Viens expansion flows
(Yukari Yamauchi

## Upcoming

(2) Smart posterior handling highlights


Landon
Buskirk

(Frederi Viens Edgard Bonilla)


Chaos Normalizing
 expansion flows


Compute For:


Select Quantity:
Differential Cross Section

Select Interaction:
Koning-Delaroche

## Protons:

20

Neutrons:
20

Welcome to BMEX! Please input your requested nuclei on the left.


## Upcoming

(2) Smart posterior handling highlights


Landon Buskirk

(Frederi Viens Edgard Bonilla)


Chaos Normalizing expansion flows (Yukari Yamauchi Landon Buskirk)

## Paysrion flage Firplores



Compute For:


Select Quantity:
Differential Cross Section -

Welcome to BMEX! Please input vour requested nuclei on the left.
"A future where models are not defined by parameter values, but rather by distributions constantly updated with new data"


## Optical potentials for the rare-isotope beam era

In regions of the nuclear chart away from stability, which represent a frontier in nuclear science over the coming decade and which will be probed at new rareisotope beam facilities worldwide, there is a targeted need to quantify and reduce theoretical reaction model uncertainties, especially with respect to nuclear optical potentials.


## Bayspian may Firplorsr




Select Quantity:
Differential Cross Section -

Select Interaction:

| Koning-Delaroche |
| :--- |

"A future where models are not defined by parameter values, but rather by distributions constantly


## Takeaways

## Takeaways

1) These methods are SO cool

## Takeaways

1) These methods are SO cool

Reduced Basis Method


Straightforward
and broadly applicable $\begin{array}{ll}\hat{\phi}(x)=\phi_{0}+\sum_{k}^{n} a_{k} \phi_{k}(x) & \text { Training } \\ \left\langle\psi_{j} \mid F_{\alpha}[\hat{\phi}(x)]\right\rangle=0 & \text { Projecting }\end{array}$


## Takeaways

1) These methods are SO cool
2) UQ needs multidisciplinary efforts


## Takeaways

1) These methods are SO cool
2) UQ needs multidisciplinary efforts

## Takeaways

1) These methods are SO cool
2) UQ needs multidisciplinary efforts

Work in collaboration with experts


Advanced Scientific Computing and Statistics Network

## Takeaways

1) These methods are SO cool
2) UQ needs multidisciplinary efforts


## Takeaways



Work in collaboration with experts ...

... and find that the real UQ is the friends you made along the way


[^0]:    https://doi.org/10.1103/PhysRevC.106.054322

[^1]:    Neural Implicit Flow: a mesh-agnostic dimensionality

