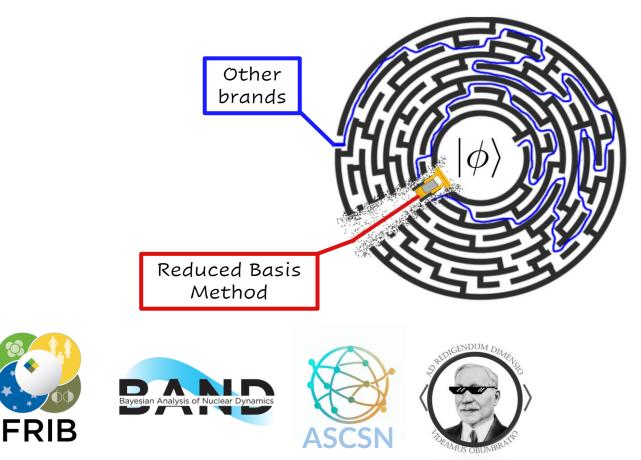
Sorry Maria, I forgot to change my title:

Dimensionality reduction for accelerating uncertainty quantification



Pablo Giuliani giulianp@frib.msu.edu

Outline

Why Uncertainty Quantification? How

The Reduced Basis Method How it works Applications and Results

Upcoming Highlights

Takeaways

Outline

Why Uncertainty Quantification? How

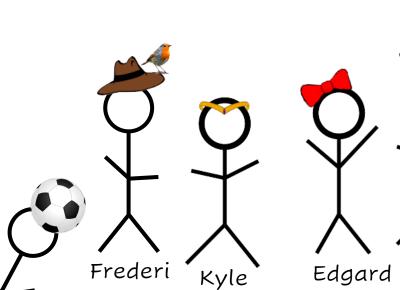
The Reduced Basis Method How it works

Applications and Results

Jorge

Upcoming Highlights

Takeaways

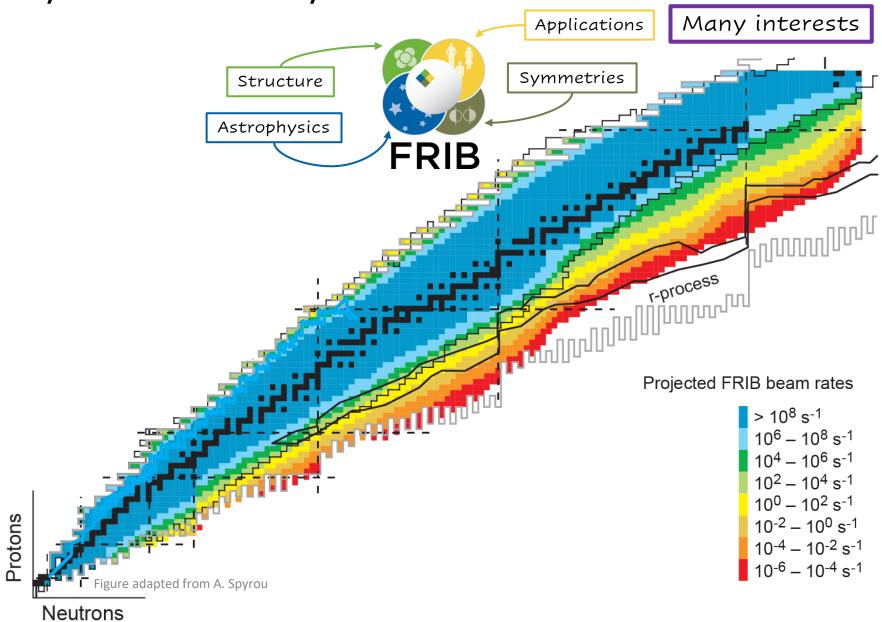


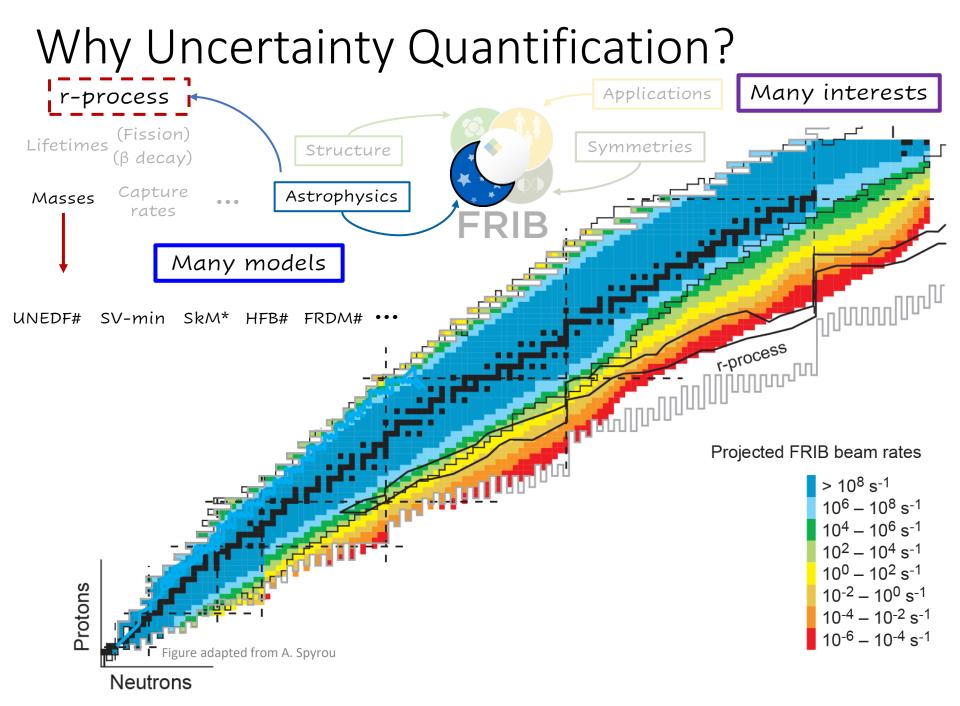


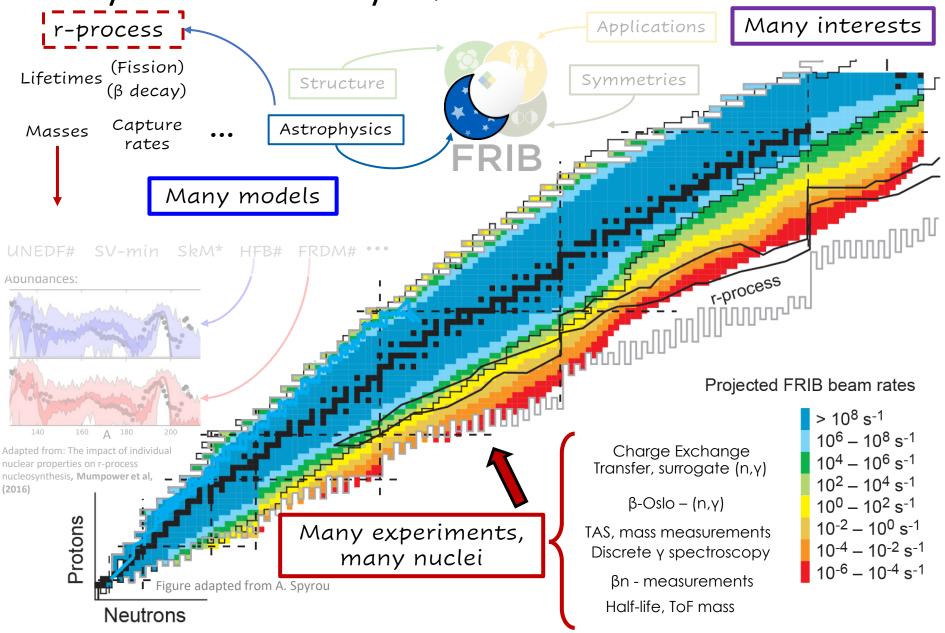


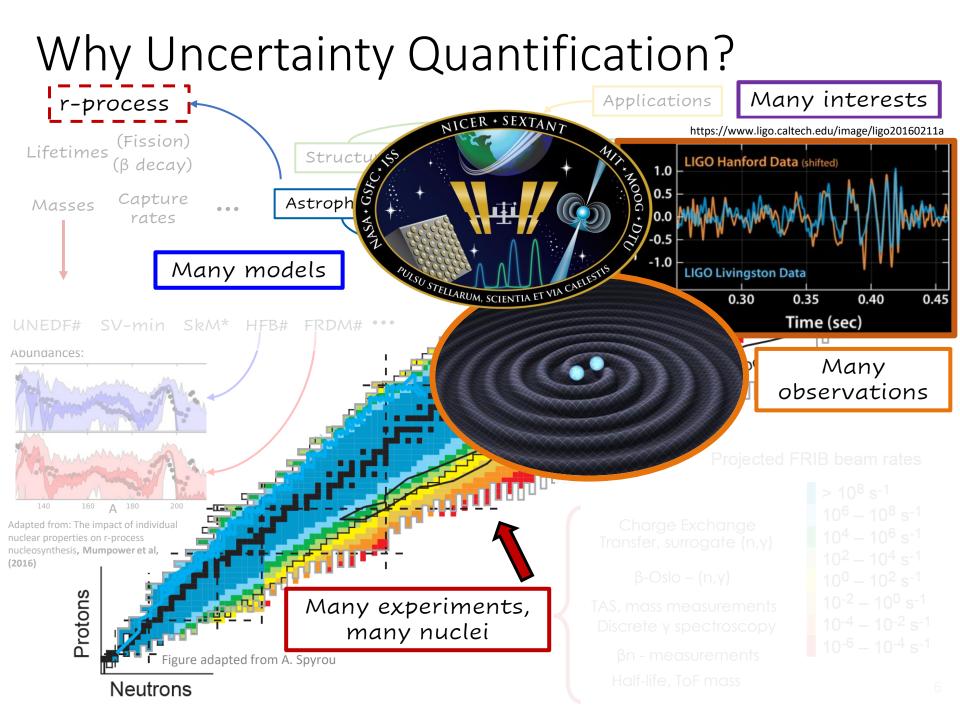
me

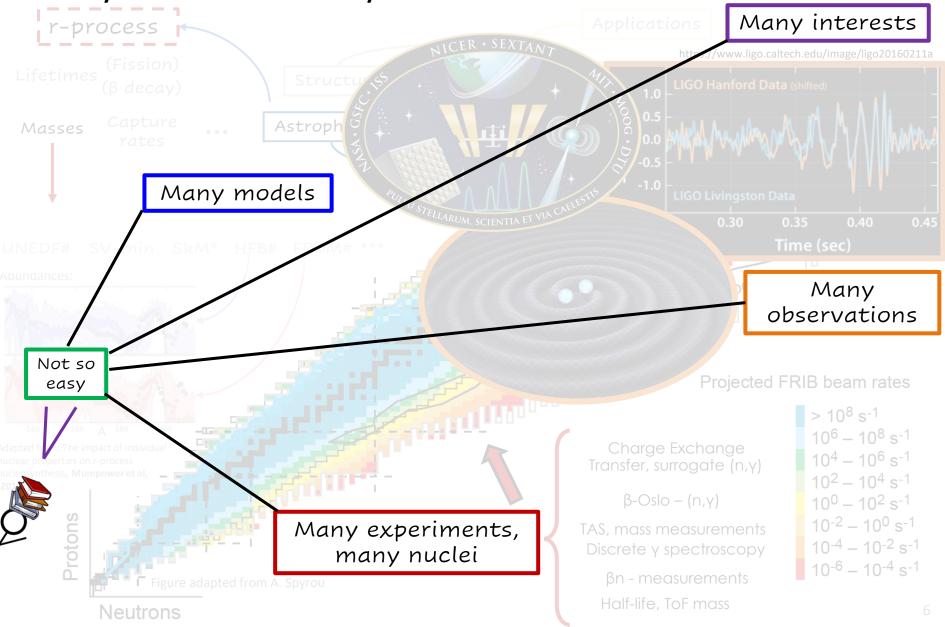
Dean







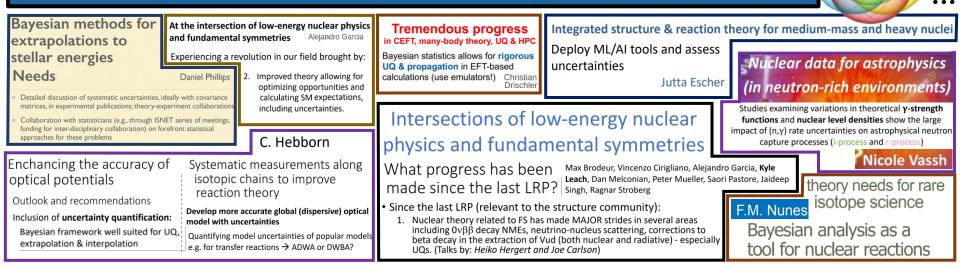




NSAC Long Range Plan Town Hall Meeting on Nuclear Structure, Reactions and Astrophysics Nov 14 – 16, 2022



NSAC Long Range Plan Town Hall Meeting on Nuclear Structure, Reactions and Astrophysics Nov 14 – 16, 2022



Computing (HPC, Quantum, AI/ML)

What are the most compelling scientific opportunities over the next decade & their potential scientific impact?

- Development of emulators, AI/ML and Bayesian methods:
 - Opens up entirely new ways to make predictions and quantify uncertainties
 - Experimental design: which measurements will help constrain/inform theoretical models (maximize the success of an experiment)

Gaute Hagen, Calvin Johnson, Michelle Kuchera Dean Lee, Pieter Maris, Kyle Wendt

Neutron Stars and Dense Matter

Since LRP2015, major breakthroughs

Quantification of uncertainties in data & models→Bayesian Analysis

Betty Tsang

5-10 year priorities for nuclear data covariances and uncertainty quantification as defined by the Nuclear Data Uncertainty Quantification Meeting D. Neudecker

Nuclear Structure and Reaction Theory

Working group: Papenbrock, Phillips, Piarulli, Potel, Schunck, Tews, Volya + Fossez, Hebborn, Koenig

· Reactions are awesome: Reactions are the best window into the structure and dynamics of nuclei, and address data needed for other fields. Full UQ and reaction-theory modeling crucial

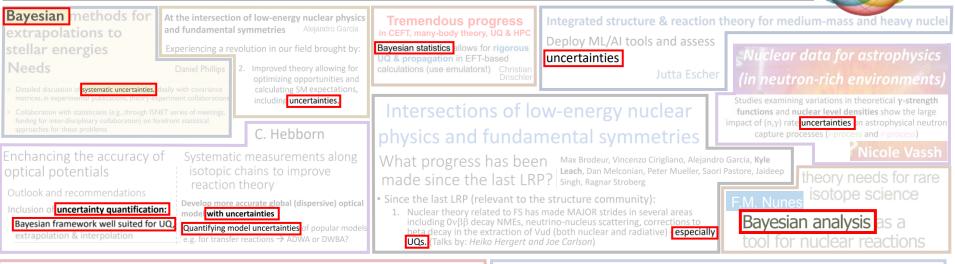
Predictive theory of nuclei and their interactions

We have entered a precision era: field moves Thomas towards quantified uncertainties

Papenbrock

Uncertainty guantification & Bayesian machine learning have advanced nuclear theory

NSAC Long Range Plan Town Hall Meeting on Nuclear Structure, Reactions and Astrophysics Nov 14 – 16, 2022



Computing (HPC, Quantum, AI/ML)

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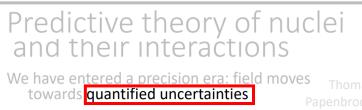
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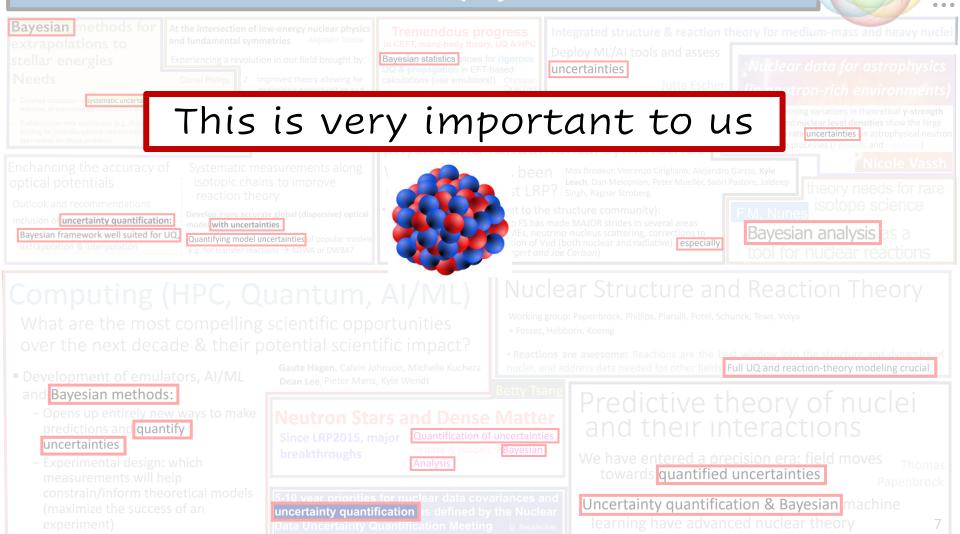
+ Fossez, Hebborn, Koenig

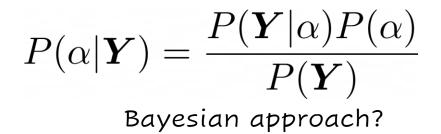
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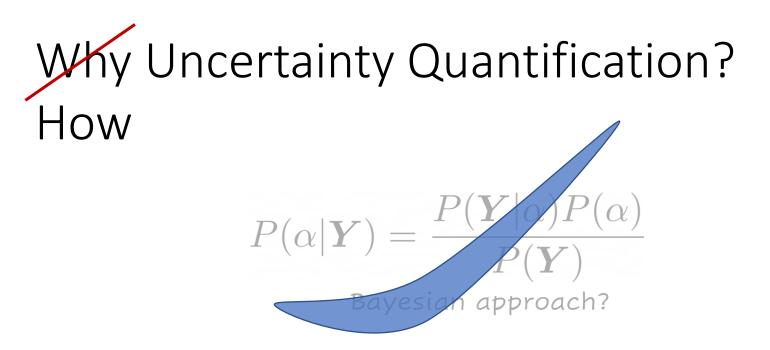


Uncertainty quantification & Bayesian machine learning have advanced nuclear theory

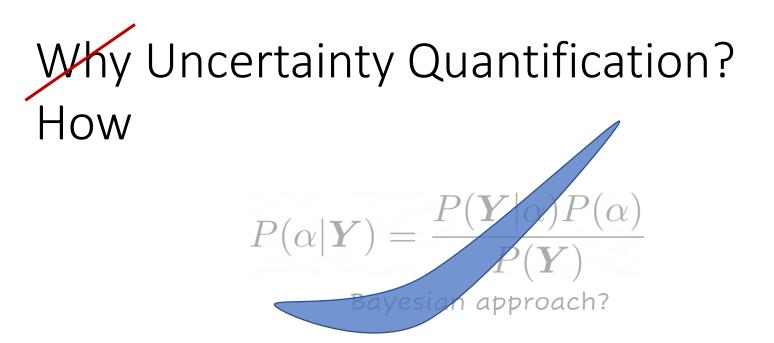
NSAC Long Range Plan Town Hall Meeting on Nuclear Structure, Reactions and Astrophysics Nov 14 – 16, 2022







The most important thing in my opinion:

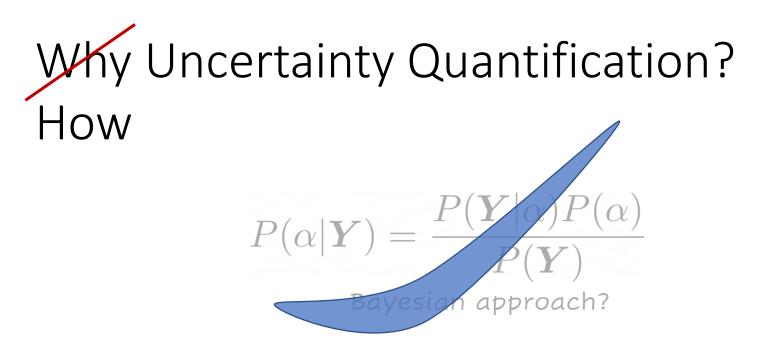


The most important thing in my opinion:



mathematics statistics computational

Work in collaboration with experts



The most important thing in my opinion:



→ Work in collaboration with experts

Eigenvector Continuation with Subspace Learning (2018)

Dillon Frame,^{1,2} Rongzheng He,^{1,2} Ilse Ipsen,³ Daniel Lee,⁴ Dean Lee,^{1,2} and Ermal Rrapaj⁵

Google Scholar

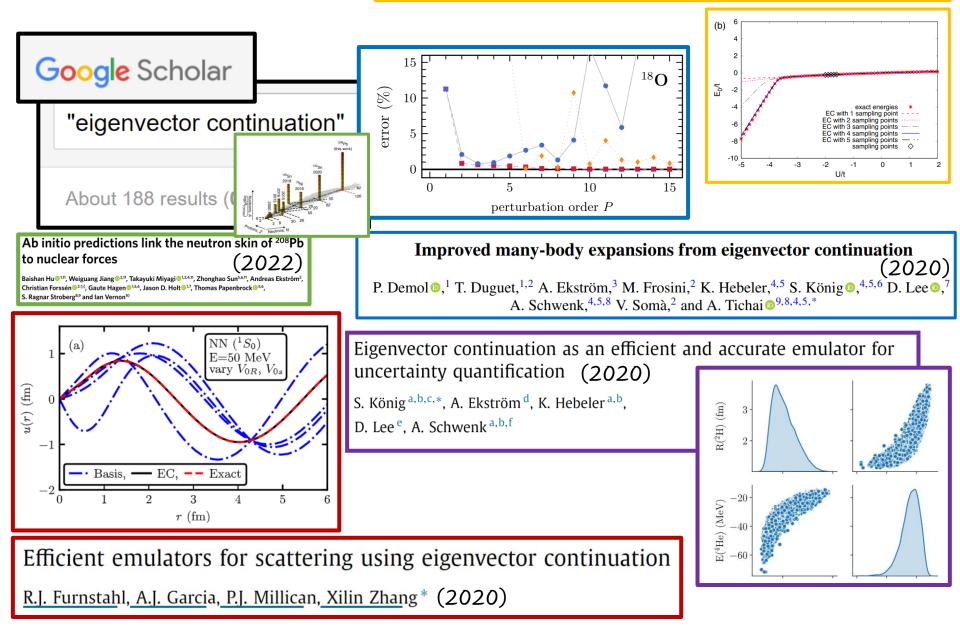
"eigenvector continuation"

About 188 results (0.08 sec)

control parameter in the Hamiltonian matrix exceeds some threshold value. In this Letter we present a new technique called eigenvector continuation that can extend the reach of these methods. The key insight is that while an eigenvector resides in a linear space with enormous dimensions, the eigenvector trajectory generated by smooth changes of the Hamiltonian matrix is well approximated by a very low-dimensional manifold. We

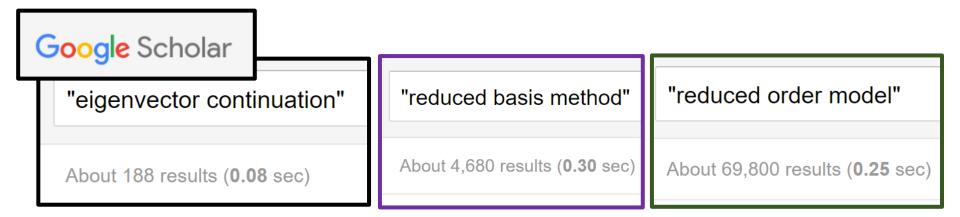
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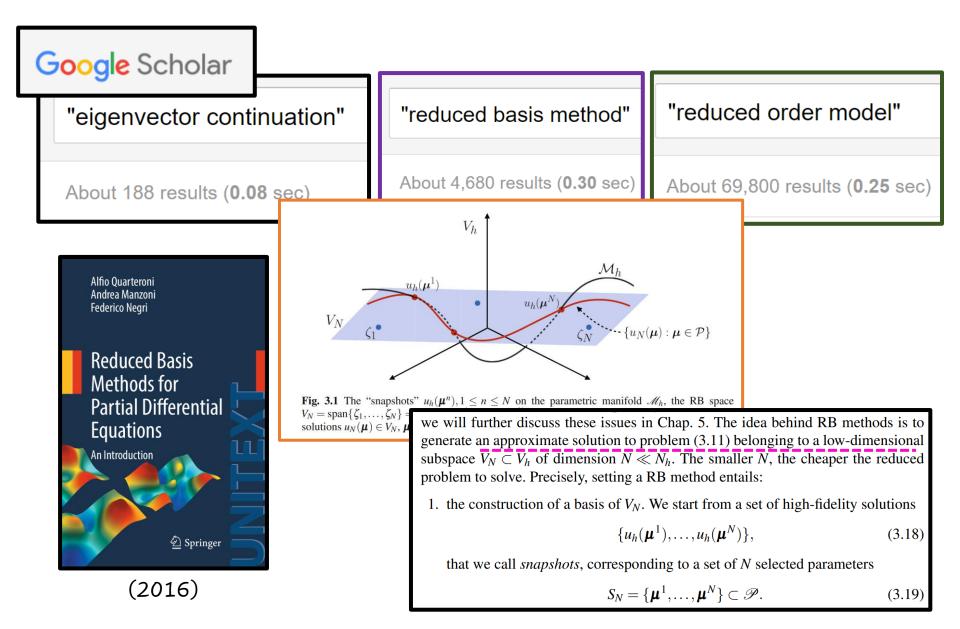
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(2016)

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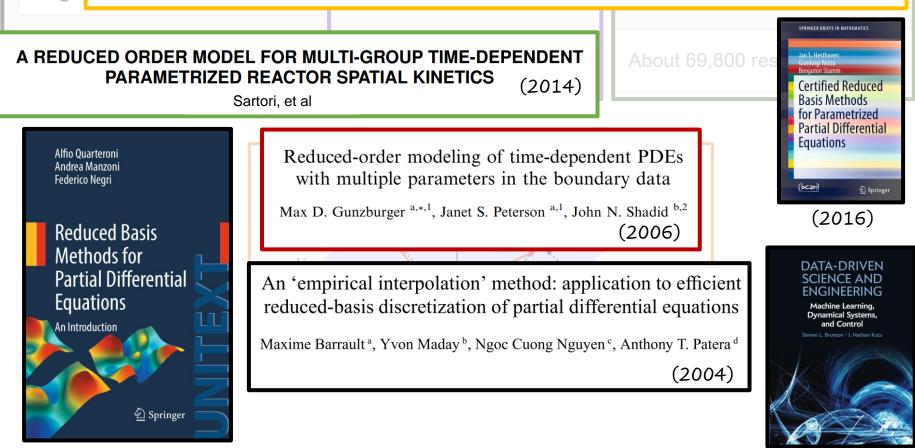
Dillon Frame,^{1,2} Rongzheng He,^{1,2} Ilse Ipsen,³ Daniel Lee,⁴ Dean Lee,^{1,2} and Ermal Rrapaj⁵

Dimensionality reduction and polynomial chaos acceleration of Bayesian inference in inverse problems

"eige Youssef M. Marzouk^{a,*}, Habib N. Najm^b

(2009)

2019



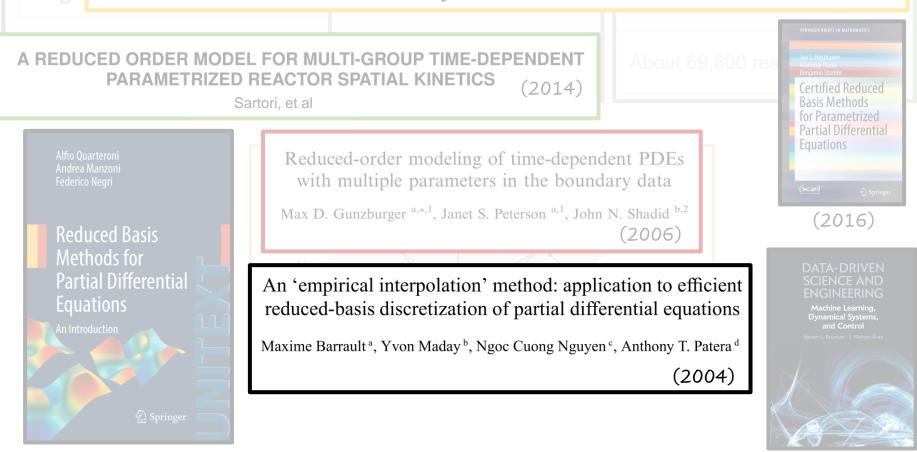
Eigenvector Continuation with Subspace Learning (2018)

Dillon Frame,^{1,2} Rongzheng He,^{1,2} Ilse Ipsen,³ Daniel Lee,⁴ Dean Lee,^{1,2} and Ermal Rrapaj

Google Dimensionality reduction and polynomial chaos acceleration of Bayesian inference in inverse problems

"eige Youssef M. Marzouk^{a,*}, Habib N. Najm^b

(2009)



(2016)

Improved many-body expansions from eigenvector continuation

(2020) P. Demol[®],¹ T. Duguet,^{1,2} A. Ekström,³ M. Frosini,² K. Hebeler,^{4,5} S. König[®],^{4,5,6} D. Lee[®],⁷ A. Schwenk,^{4,5,8} V. Somà,² and A. Tichai[®],^{9,8,4,5,*}

Dimensionality reduction and polynomial chaos acceleration of Bayesian inference in inverse problems

"eige Youssef M. Marzouk^{a,*}, Habib N. Najm^b

¹ Note that because the κ parameters do not appear linearly in the Hamiltonian, one can no longer make a single set of matrix elements calculations for all of the test parameter sets. In other contexts this might be a relevant computational disadvantage.

$$V_{1S_{0}}(r) \equiv V_{0R}e^{-\kappa_{R}r^{2}} + V_{0S}e^{-\kappa_{S}r^{2}}$$
$$V_{3S_{1}}(r) \equiv V_{0R}e^{-\kappa_{R}r^{2}} + V_{0t}e^{-\kappa_{t}r^{2}}$$



An 'empirical interpolation' method: application to efficient reduced-basis discretization of partial differential equations

Maxime Barrault^a, Yvon Maday^b, Ngoc Cuong Nguyen^c, Anthony T. Patera^d

(2004)

Efficient emulators for scattering using eigenvector continuation

<u>R.J. Furnstahl, A.J. Garcia, P.J. Millican, Xilin Zhang</u>* (2020)

The nuclear potential that we employ is additive in the d = 16 LECs, i.e., we can express the Hamiltonian as $H(\mathbf{c}) = H_0 + \sum_{i=1}^{d} c_i H_i$, where H_0 includes the kinetic energy. Any Hamiltonian with more than one interaction parameter can be written in this form, where each c_i in general may be depend nonlinearly on other parameters. Furthermore, each term H_i for i = 1, ..., 16 can be projected onto the EC subspace once and then used for an arbitrary number of emulations. Each of these corresponds to a

for Parametrized

matrix. This problem can be avoided by running an orthogonalization on the EC vectors that stabilizes the subsequent numerical steps and reveals the effective dimension of the EC subspace. Since this step leads to a unit norm matrix, it also reduces the per-sample evaluation cost at the price of additional preprocessing effort (see Appendix A).

¹, John N. Shadid ^{0,2} (2006)

SCIENCE AND

2016

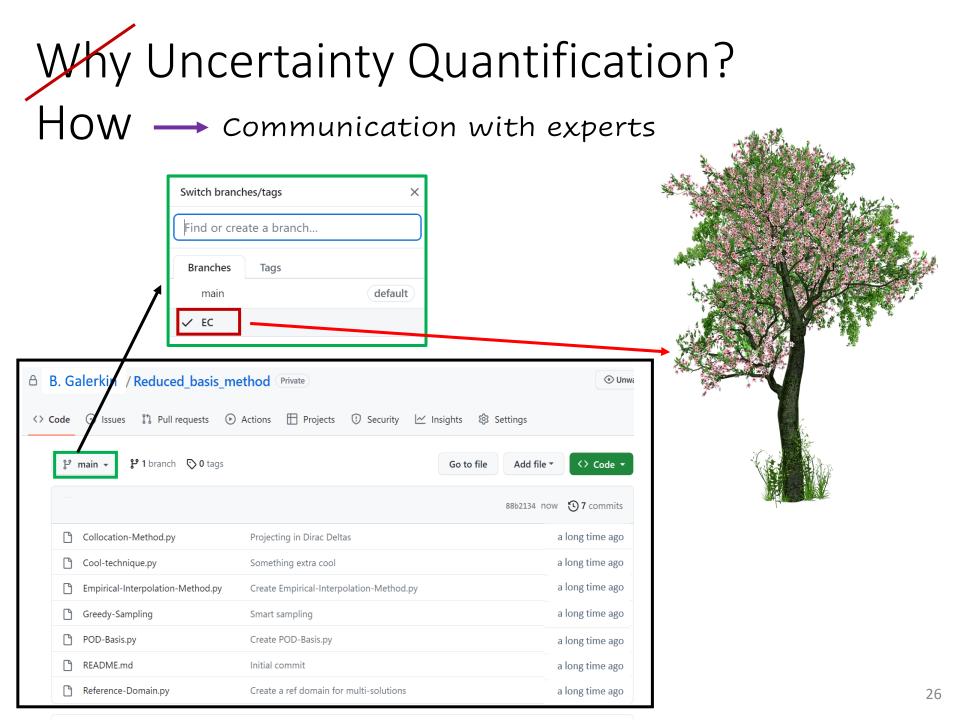
Machine Learning, Dynamical Systems, and Control

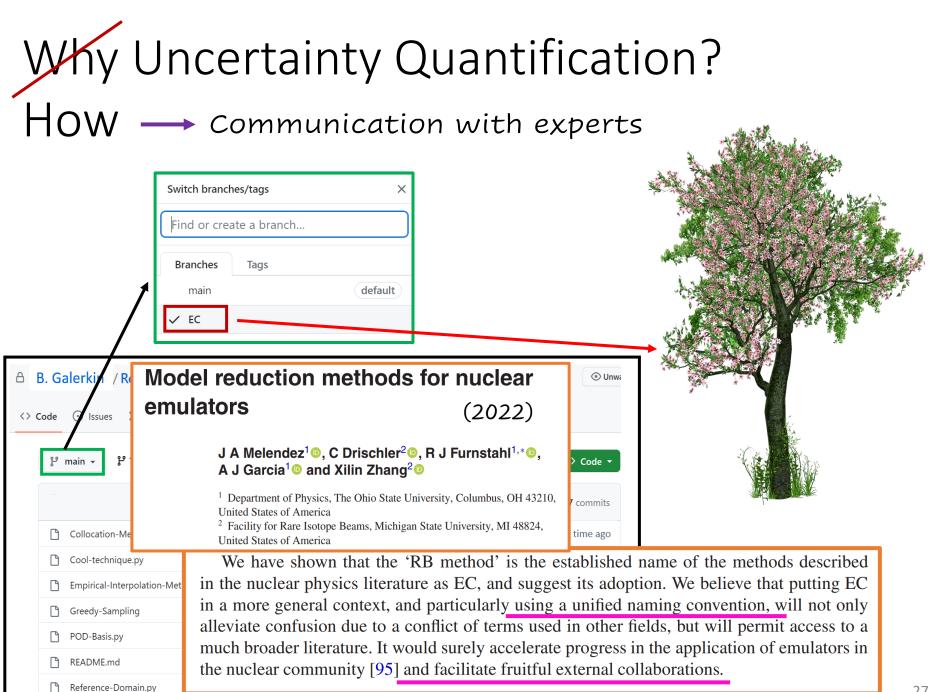
Why Uncertainty Quantification? How — communication with experts

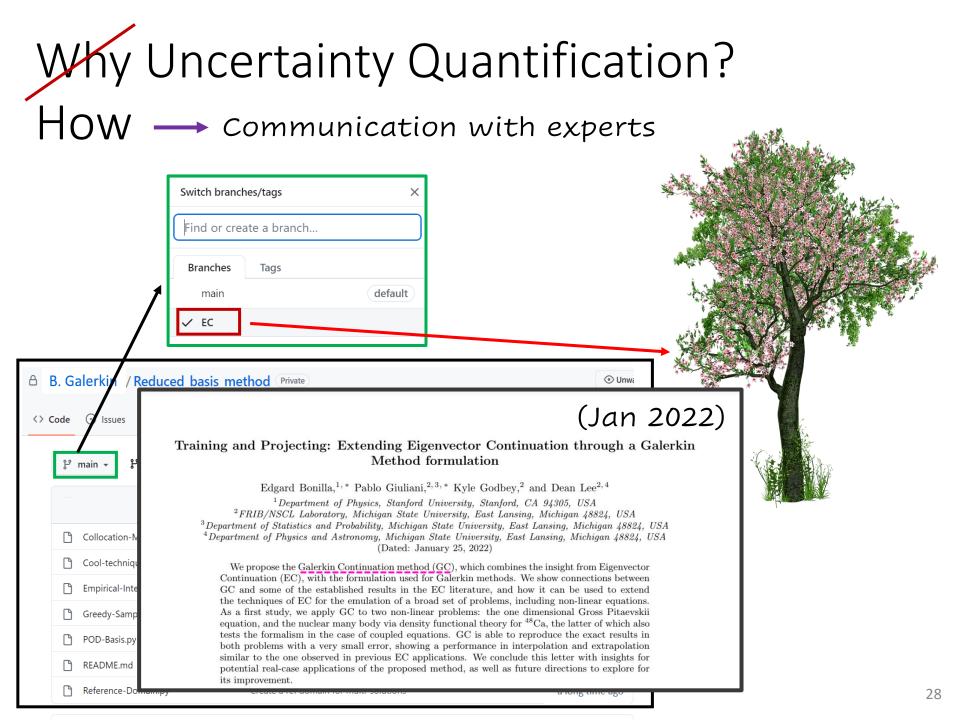
A	B. Galerkin / Reduced_basis_method Private						
<> Code 💿 Issues 👔 Pull requests 🕑 Actions 🖽 Projects 😲 Security 🗠 Insights 🕸 Settings							
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				88b2134	now 🕲 7 commits		
		Collocation-Method.py	Projecting in Dirac Deltas		a long time ago		
	<u></u>	Cool-technique.py	Something extra cool		a long time ago		
	C E	Empirical-Interpolation-Method.py	Create Empirical-Interpolation-Method.py		a long time ago		
	Greedy-Sampling Smart sampling			a long time ago			
	C F	POD-Basis.py	Create POD-Basis.py		a long time ago		
	C F	README.md	Initial commit		a long time ago		
	P F	Reference-Domain.py	Create a ref domain for multi-solutions		a long time ago		

Why Uncertainty Quantification? How — communication with experts

Γ	Switch branches/tags	×				
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1	main	default				
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A B. Galerkin / Reduced_basis_method Private <> Code Issues \$% Pull requests						
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Collocation-Method.py	Projecting in Dirac D	eltas	a long time ago			
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Empirical-Interpolation-I	Method.py Create Empirical-Inte	rpolation-Method.py	a long time ago			
Greedy-Sampling	Smart sampling		a long time ago			
POD-Basis.py	Create POD-Basis.py		a long time ago			
🗋 README.md	Initial commit		a long time ago			
Reference-Domain.py	Create a ref domain	for multi-solutions	a long time ago			







Why l	Jncertainty Quantification?
How -	> Communication with experts
	Switch branches/tags × Find or create a branch
1	Branches Tags main default V EC
🔒 B. Galerkin / Redu	aced basis method (Private)
<> Code 🖓 Issues	(Jan 2022)
१° main → १°	Training and Projecting: Extending Eigenvector Continuation through a Galerkin Method formulation
	Edgard Bonilla, ^{1,*} Pablo Giuliani, ^{2,3,*} Kyle Godbey, ² and Dean Lee ^{2,4}
Collocation-N	¹ Department of Physics, Stanford University, Stanford, CA 94305, USA ² FRIB/NSCL Laboratory, Michigan State University, East Lansing, Michigan 48824, USA ³ Department of Statistics and Probability, Michigan State University, East Lansing, Michigan 48824, USA ⁴ Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA (Dated: January 25, 2022)
🗋 Cool-techniqu	We propose the Galerkin Continuation method (GC), which combines the insight from Eigenvector Continuation (EC), with the formulation used for Galerkin methods. We show connections between
🗋 Empirical-Inte	GC and some of the established results in the EC literature, and how it can be used to extend the techniques of EC for the emulation of a broad set of problems, including non-linear equations.
🕒 Greedy-Samp	As a first study, we apply GC to two non-linear problems: the one dimensional Gross Pitaevskii equation, and the nuclear many body via density functional theory for ⁴⁸ Ca, the latter of which also
POD-Basis.py	tests the formalism in the case of coupled equations. GC is able to reproduce the exact results in both problems with a very small error, showing a performance in interpolation and extrapolation
🗋 README.md	similar to the one observed in previous EC applications. We conclude this letter with insights for potential real-case applications of the proposed method, as well as future directions to explore for its improvement.
Reference-Domen	its improvement.

Outline

Why Uncertainty Quantification? How

The Reduced Basis Method

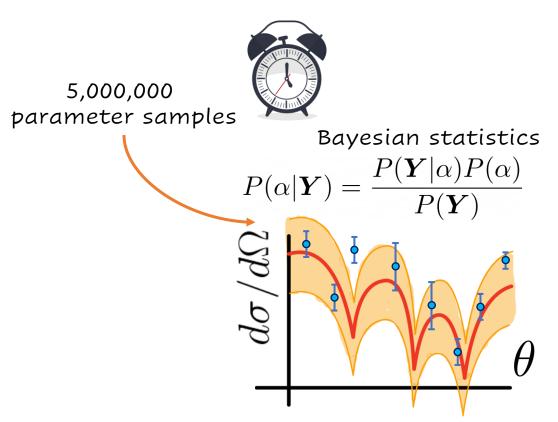
How it works

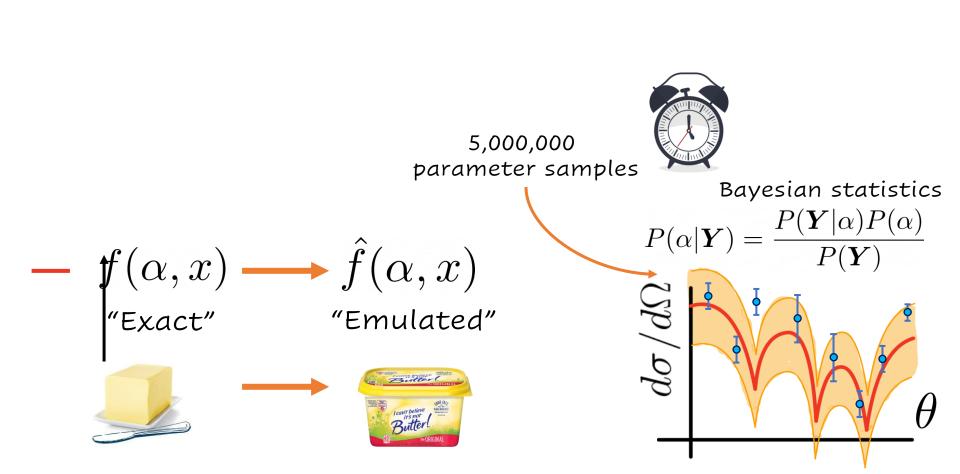
Applications and Results

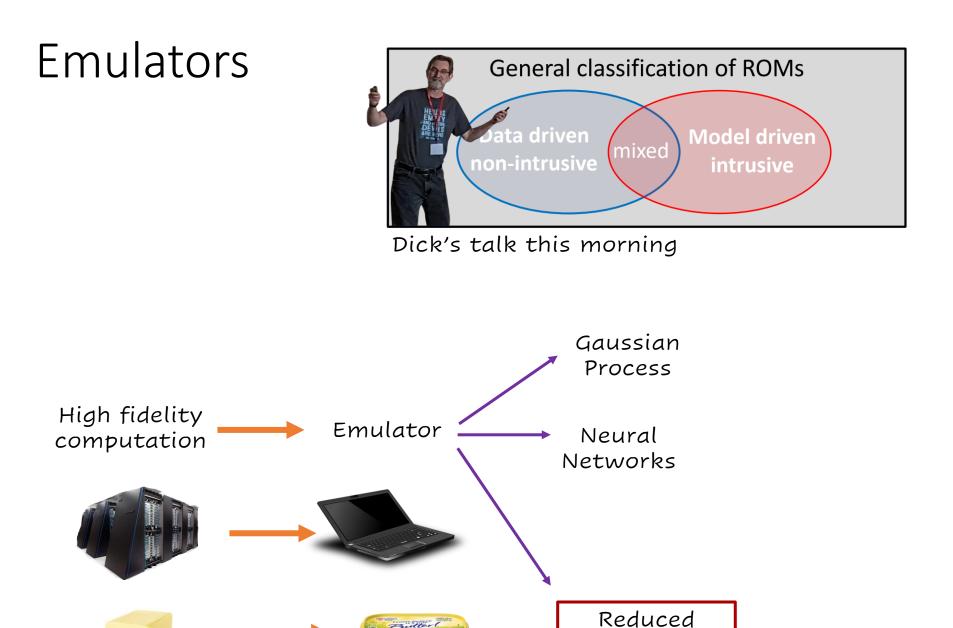
Upcoming Highlights

Takeaways

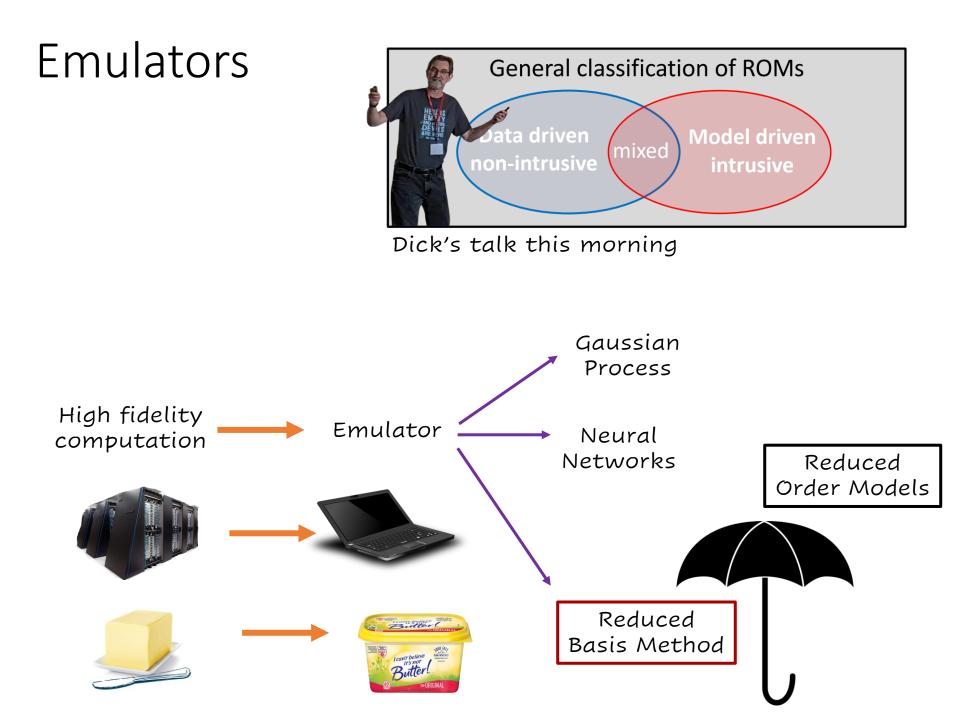
Emulators





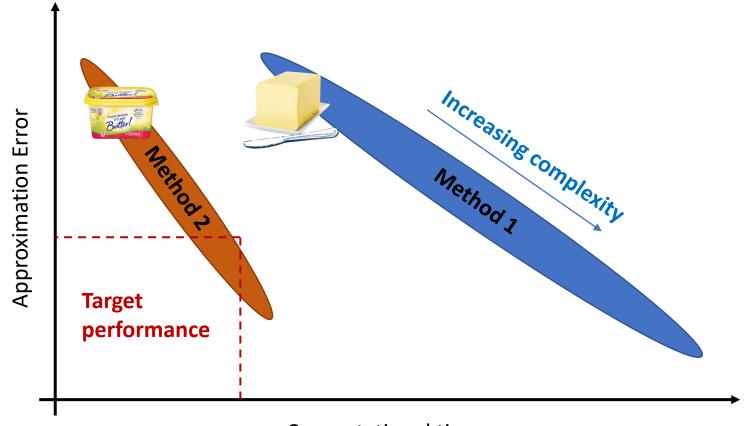


Basis Method



Computation Accuracy vs Time





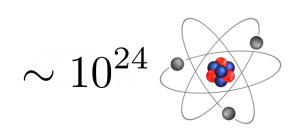
Computational time

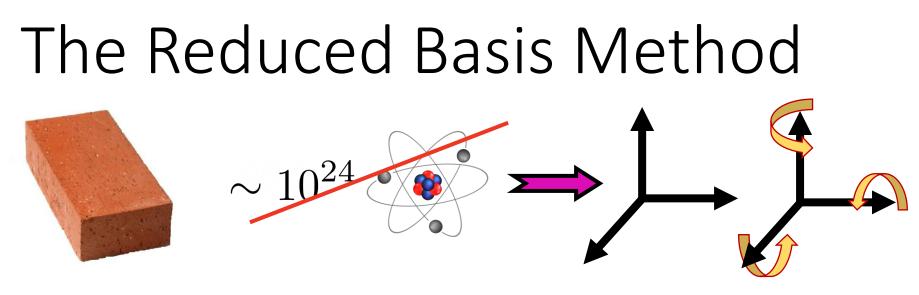
The Reduced Basis Method



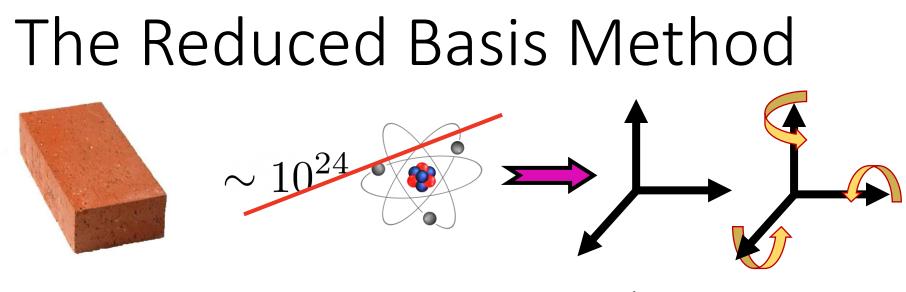
The Reduced Basis Method







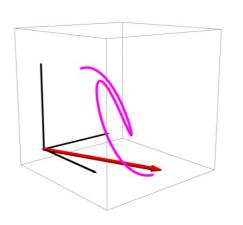
3 translations + 3 rotations

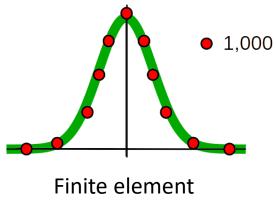


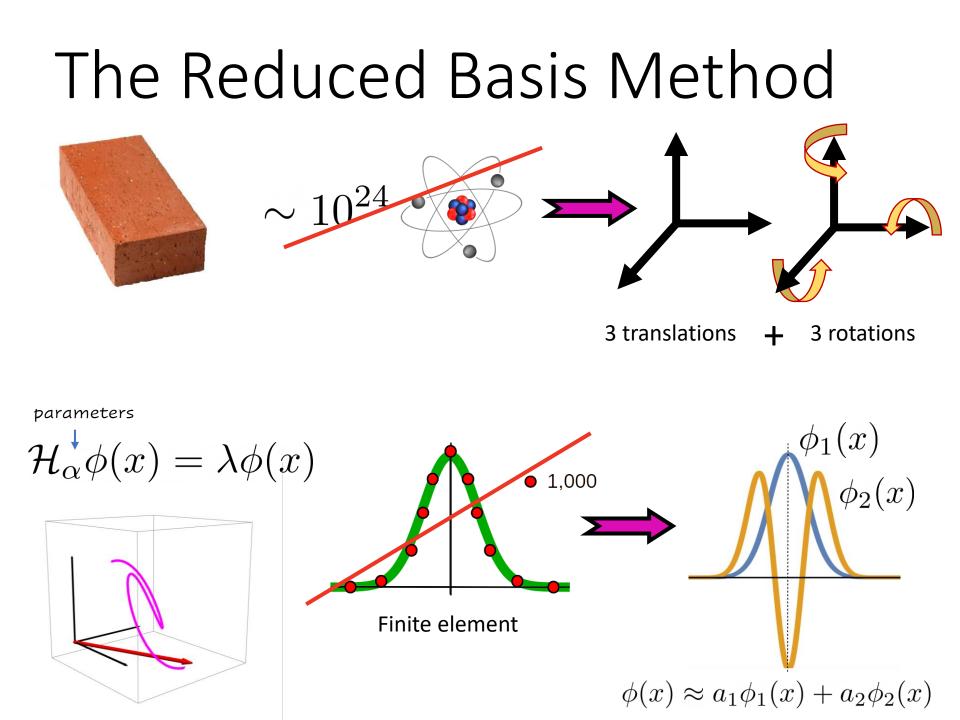
3 translations + 3 rotations

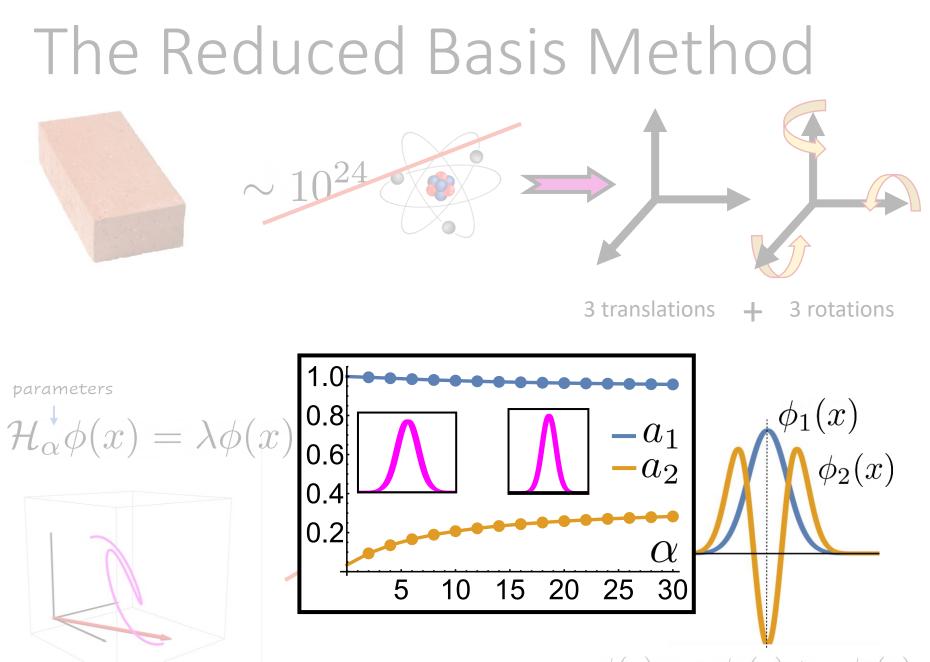
parameters

 $\mathcal{H}_{\alpha}^{\downarrow}\phi(x) = \lambda\phi(x)$

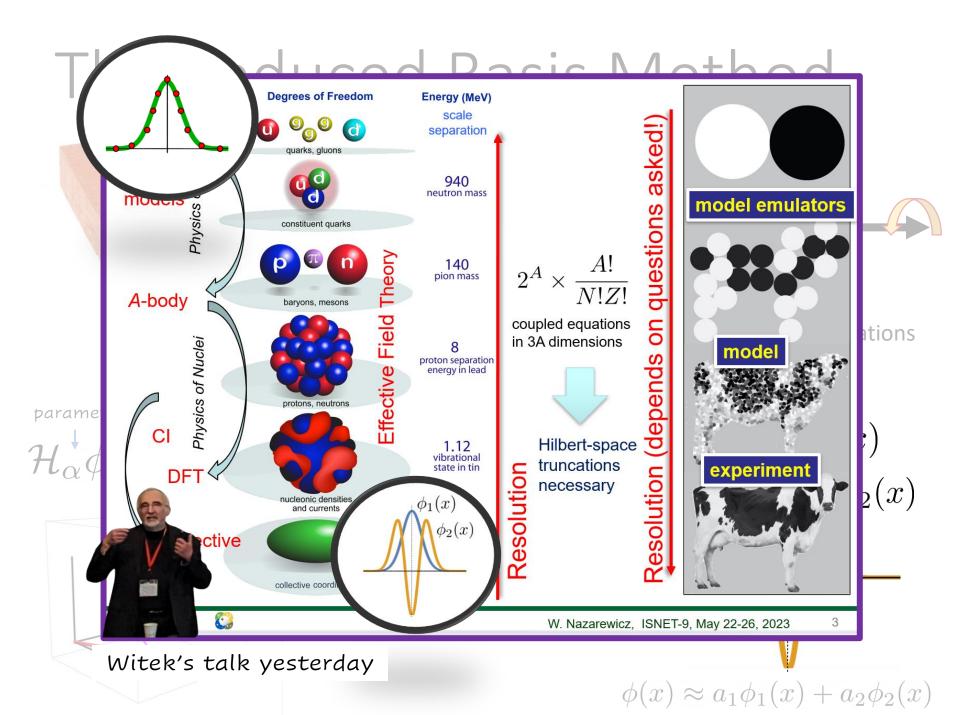


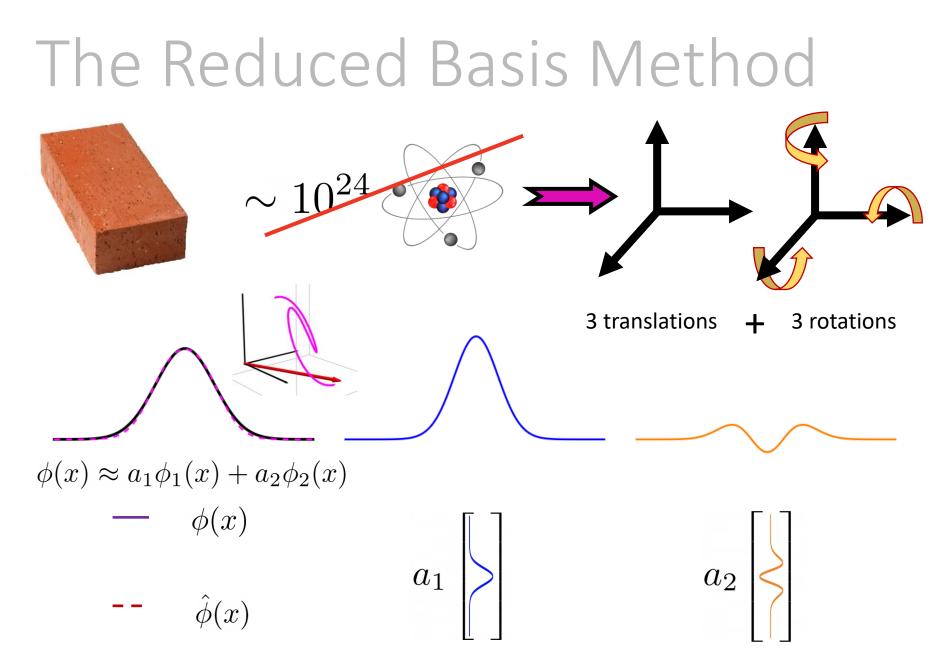






 $\phi(x) \approx a_1 \phi_1(x) + a_2 \phi_2(x)$



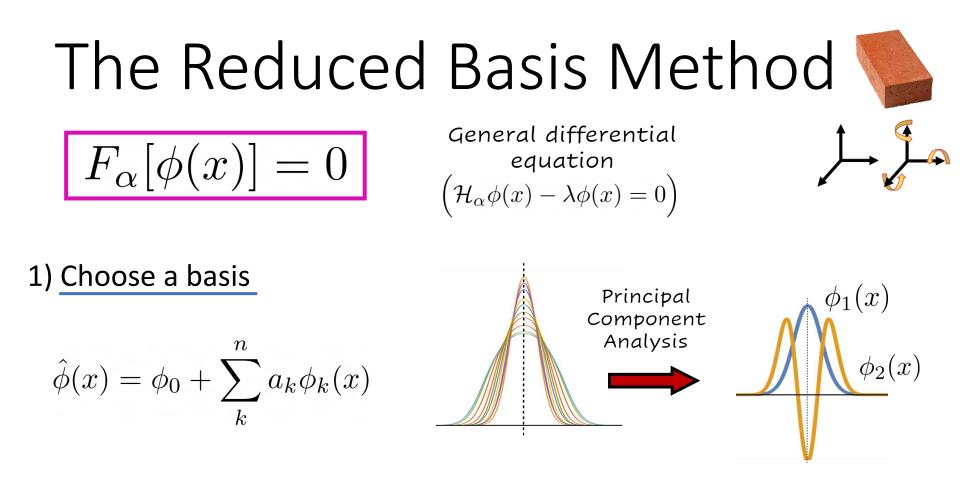


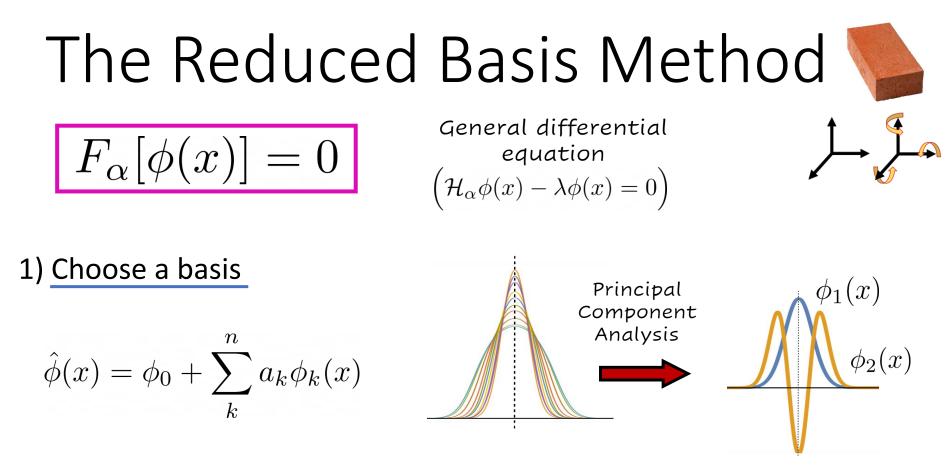
Changing the trapping strength $\,lpha$

The Reduced Basis Method

 $F_{\alpha}[\phi(x)] = 0$

General differential equation $\left(\mathcal{H}_{\alpha}\phi(x)-\lambda\phi(x)=0\right)$





2) Project onto judges

$$j = \{1, n\} \quad \langle \psi_j | F_\alpha[\hat{\phi}(x)] \rangle = 0$$

One equation per coefficient

The Reduced Basis Method

$$F_{\alpha}[\phi(x)] = 0$$

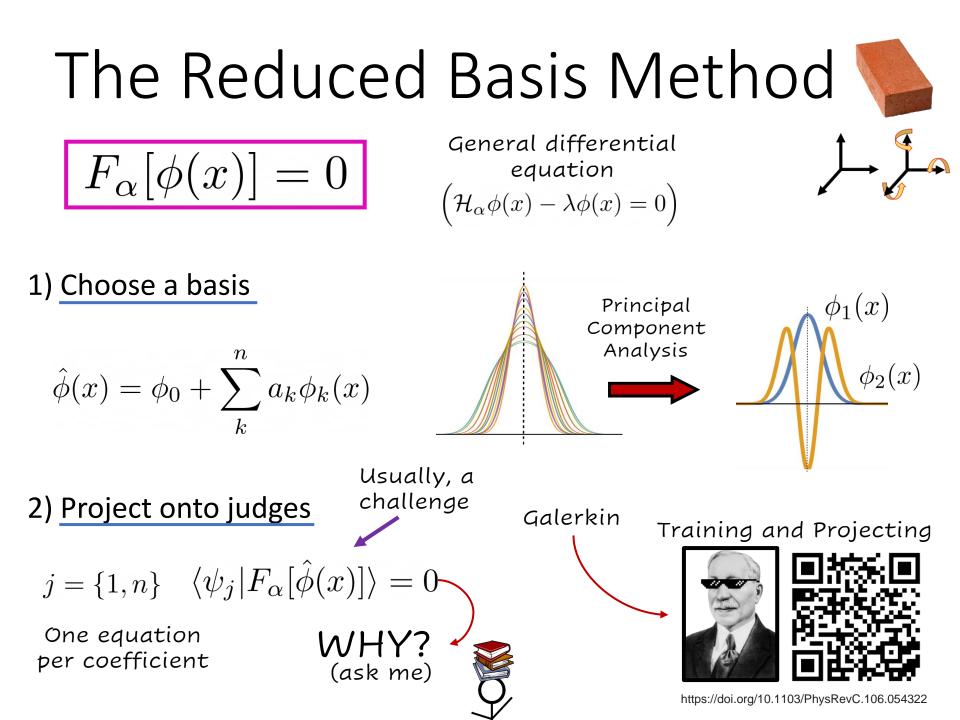
$$f_{\alpha}[\phi(x)] = 0$$

$$f_{\alpha}[\phi(x)] = 0$$

$$f_{\alpha}[\phi(x) - \lambda\phi(x) = 0]$$

$$f_{\alpha}(x) - \lambda\phi(x) = 0$$

per coefficient



https://kylegodbey.github.io/nuclear-rbm

jupyter {book}

Reduced Basis Methods in Nuclear Physics





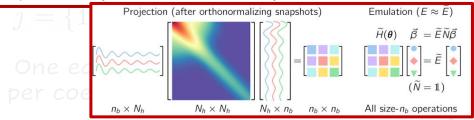
https://www.frontiersin.org/articles/10.3389/fphy.2022.1092931/full

BUQEYE Guide to Projection-Based Emulators in Nuclear Physics

C. Drischler 1,2,* , J. A. Melendez 3, R. J. Furnstahl 3, A. J. Garcia 3, and Xilin Zhang 2

¹Department of Physics and Astronomy & Institute of Nuclear and Particle Physics, Ohio University, Athens, OH 45701, USA

² Facility for Rare Isotope Beams, Michigan State University, MI 48824, USA
 ³ Department of Physics, The Ohio State University, Columbus, OH 43210, USA



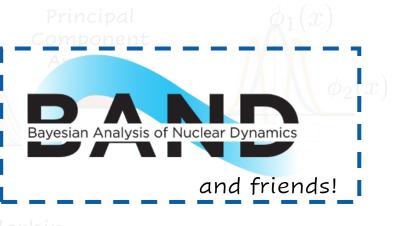
Model reduction methods for nuclear emulators

J A Melendez¹, C Drischler², R J Furnstahl^{1,*}, A J Garcia¹ and Xilin Zhang²

- ¹ Department of Physics, The Ohio State University, Colu United States of America
- ² Facility for Rare Isotope Beams, Michigan State Univer United States of America

https://doi.org/10.1088/1361-6471/ac83dd





Training and Projecting

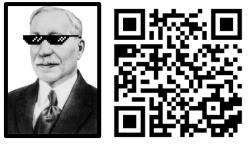






Edgard Bonilla,^{1, *} Pablo Giuliani,^{2, 3, †} Kyle Godbey,^{2, ‡} and Dean Lee^{2, 4, §}

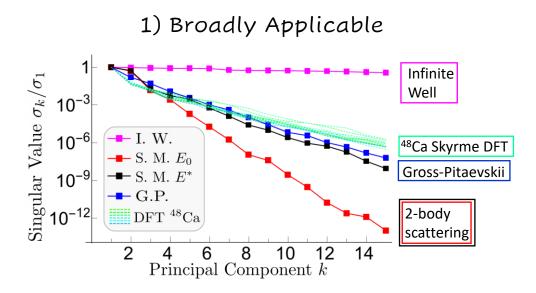
Training and Projecting





^{vy}q_{rch} ما Training and Projecting: A Reduced Basis Method Emulator for Many-Body Physics

Edgard Bonilla,^{1, *} Pablo Giuliani,^{2, 3, †} Kyle Godbey,^{2, ‡} and Dean Lee^{2, 4, §}



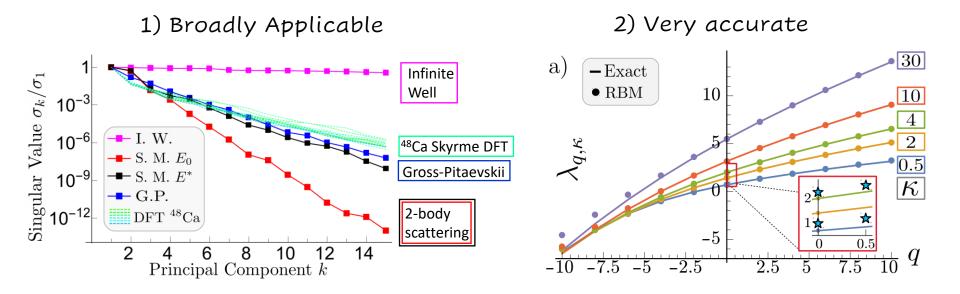
Training and Projecting



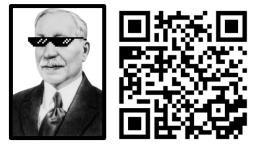


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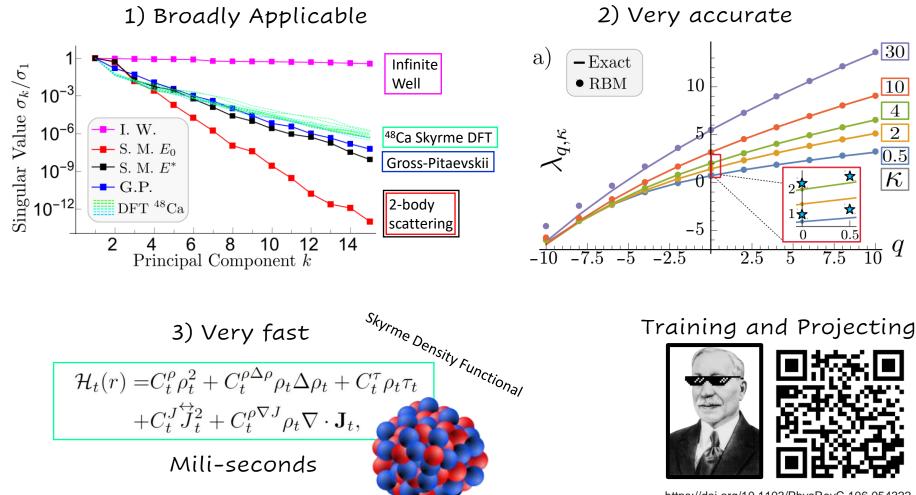
Training and Projecting





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^{(۱}۹_{۲-ch} کر Training and Projecting: A Reduced Basis Method Emulator for Many-Body Physics

Edgard Bonilla,^{1, *} Pablo Giuliani,^{2, 3, †} Kyle Godbey,^{2, ‡} and Dean Lee^{2, 4, §}

2) Very accurate 1) Broadly Applicable -Exact Infinite Singular Value σ_k/σ_1 • RBM 10^{-3} 4 2 ⁴⁸Ca Skyrme DFT Gross-Pitaevskii κ 2-body * 2 10 12 14 Principal Component kSkyrme Density Functional Training and Projecting 3) Very fast VERY non-linear $\mathcal{H}_t(r) = C_t^{\rho} \rho_t^2 + C_t^{\rho \Delta \rho} \rho_t \Delta \rho_t + C_t^{\tau} \rho_t \tau_t$ $+C_t^J \overleftrightarrow{J}_t^2 + C_t^{\rho \nabla J} \rho_t \nabla \cdot \mathbf{J}_t,$ $\rho(r)^{\alpha}$ Mili-seconds

Applications and Results



Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

Pablo Giuliani,^{1, 2, *} Kyle Godbey,^{1, †} Edgard Bonilla,^{3, ‡} Frederi Viens,^{2, 4, §} and Jorge Piekarewicz^{5, ¶}

Bayes goes fast

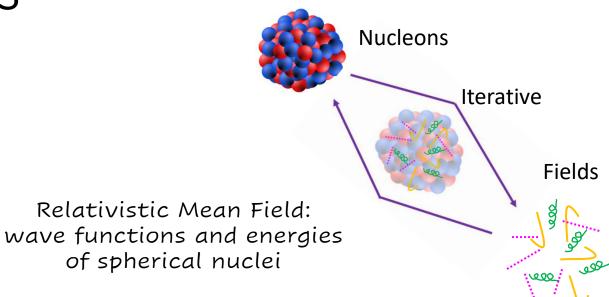


Applications and Results



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Bayes goes fast



Applications and Results



Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

Pablo Giuliani,^{1, 2, *} Kyle Godbey,^{1, †} Edgard Bonilla,^{3, ‡} Frederi Viens,^{2, 4, §} and Jorge Piekarewicz^{5, ¶}

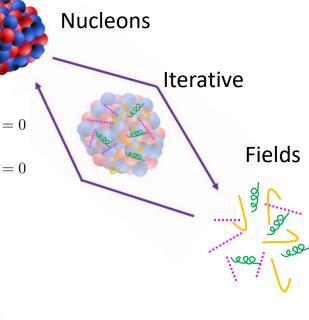
Dirac Equations

$$\left(\frac{d}{dr} + \frac{\kappa}{r}\right)g_a(r) - \left[E_a + M - \Phi_0(r) - W_0(r) \mp \frac{1}{2}B_0(r) - e\left\{\begin{array}{c}1\\0\end{array}\right\}A_0(r)\right]f_a(r) = 0$$

$$\left(\frac{d}{dr} - \frac{\kappa}{r}\right)f_a(r) + \left[E_a - M + \Phi_0(r) - W_0(r) \mp \frac{1}{2}B_0(r) - e\left\{\begin{array}{c}1\\0\end{array}\right\}A_0(r)\right]g_a(r) = 0$$

Field Equations

$$\begin{split} &\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - m_{\rm s}^2\right)\Phi_0(r) - g_{\rm s}^2\left(\frac{\kappa}{2}\Phi_0^2(r) + \frac{\lambda}{6}\Phi_0^3(r)\right) = -g_{\rm s}^2\Big(\rho_{\rm s,p}(r) + \rho_{\rm s,n}(r)\Big),\\ &\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - m_{\rm v}^2\right)W_0(r) - g_{\rm v}^2\left(\frac{\zeta}{6}W_0^3(r) + 2\Lambda_{\rm v}B_0^2(r)W_0(r)\right) = -g_{\rm v}^2\Big(\rho_{\rm v,p}(r) + \rho_{\rm v,n}(r)\Big),\\ &\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - m_{\rho}^2\right)B_0(r) - 2\Lambda_{\rm v}g_{\rho}^2W_0^2(r)B_0(r) = -\frac{g_{\rho}^2}{2}\Big(\rho_{\rm v,p}(r) - \rho_{\rm v,n}(r)\Big),\\ &\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right)A_0(r) = -e\rho_{\rm v,p}(r), \end{split}$$



Bayes goes fast



Fields

Applications and Results



Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

Pablo Giuliani,^{1, 2, *} Kyle Godbey,^{1, †} Edgard Bonilla,^{3, ‡} Frederi Viens,^{2, 4, §} and Jorge Piekarewicz^{5, ¶}

Nucleons

Dirac Equations

$$\left(\frac{d}{dr} + \frac{\kappa}{r}\right)g_a(r) - \left[E_a + M - \Phi_0(r) - W_0(r) \mp \frac{1}{2}B_0(r) - e\left\{\begin{array}{c}1\\0\end{array}\right\}A_0(r)\right]f_a(r) = 0$$

$$\left(\frac{d}{dr} - \frac{\kappa}{r}\right)f_a(r) + \left[E_a - M + \Phi_0(r) - W_0(r) \mp \frac{1}{2}B_0(r) - e\left\{\begin{array}{c}1\\0\end{array}\right\}A_0(r)\right]g_a(r) = 0$$

Field Equations

$$\begin{split} &\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - m_{\rm s}^2\right)\Phi_0(r) - g_{\rm s}^2\left(\frac{\kappa}{2}\Phi_0^2(r) + \frac{\lambda}{6}\Phi_0^3(r)\right) = -g_{\rm s}^2\left(\rho_{\rm s,p}(r) + \rho_{\rm s,n}(r)\right),\\ &\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - m_{\rm v}^2\right)W_0(r) - g_{\rm v}^2\left(\frac{\zeta}{6}W_0^3(r) + 2\Lambda_{\rm v}B_0^2(r)W_0(r)\right) = -g_{\rm v}^2\left(\rho_{\rm v,p}(r) + \rho_{\rm v,n}(r)\right),\\ &\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - m_{\rho}^2\right)B_0(r) - 2\Lambda_{\rm v}g_{\rho}^2W_0^2(r)B_0(r) = -\frac{g_{\rho}^2}{2}\left(\rho_{\rm v,p}(r) - \rho_{\rm v,n}(r)\right),\\ &\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right)A_0(r) = -e\rho_{\rm v,p}(r), \end{split}$$

Parameters lpha

Bayes goes fast

Iterative



Fields

Applications and Results



Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

Pablo Giuliani,^{1, 2, *} Kyle Godbey,^{1, †} Edgard Bonilla,^{3, ‡} Frederi Viens,^{2, 4, §} and Jorge Piekarewicz^{5, ¶}

Nucleons

Dirac Equations

$$\left(\frac{d}{dr} + \frac{\kappa}{r}\right)g_a(r) - \left[E_a + M - \Phi_0(r) - W_0(r) \mp \frac{1}{2}B_0(r) - e\left\{\begin{array}{c}1\\0\end{array}\right\}A_0(r)\right]f_a(r) = 0 \\ \left(\frac{d}{dr} - \frac{\kappa}{r}\right)f_a(r) + \left[E_a - M + \Phi_0(r) - W_0(r) \mp \frac{1}{2}B_0(r) - e\left\{\begin{array}{c}1\\0\end{array}\right\}A_0(r)\right]g_a(r) = 0$$

Field Equations

$$\begin{split} & \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - m_{\rm s}^2\right)\Phi_0(r) - g_{\rm s}^2\left(\frac{\kappa}{2}\Phi_0^2(r) + \frac{\lambda}{6}\Phi_0^3(r)\right) = -g_{\rm s}^2\left(\rho_{\rm s,p}(r) + \rho_{\rm s,n}(r)\right),\\ & \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - m_{\rm v}^2\right)W_0(r) - g_{\rm v}^2\left(\frac{\zeta}{6}W_0^3(r) + 2\Lambda_{\rm v}B_0^2(r)W_0(r)\right) = -g_{\rm v}^2\left(\rho_{\rm v,p}(r) + \rho_{\rm v,n}(r)\right),\\ & \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - m_{\rho}^2\right)B_0(r) - 2\Lambda_{\rm v}g_{\rho}^2W_0^2(r)B_0(r) = -\frac{g_{\rho}^2}{2}\left(\rho_{\rm v,p}(r) - \rho_{\rm v,n}(r)\right),\\ & \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right)A_0(r) = -e\rho_{\rm v,p}(r), \end{split}$$

Bayes goes fast

Iterative





Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

Pablo Giuliani,^{1, 2, *} Kyle Godbey,^{1, †} Edgard Bonilla,^{3, ‡} Frederi Viens,^{2, 4, §} and Jorge Piekarewicz^{5, ¶}

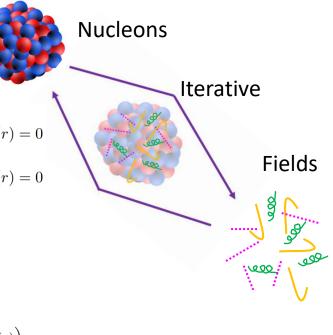
Dirac Equations

$$\left| \left(\frac{d}{dr} + \frac{\kappa}{r} \right) g_a(r) - \left[E_a + M - \Phi_0(r) - W_0(r) \mp \frac{1}{2} B_0(r) - e \left\{ \begin{array}{c} 1\\0 \end{array} \right\} A_0(r) \right] f_a(r) = 0$$

$$\left| f_{a,k}^{(j)} \right| \left(\frac{d}{dr} - \frac{\kappa}{r} \right) f_a(r) + \left[E_a - M + \Phi_0(r) - W_0(r) \mp \frac{1}{2} B_0(r) - e \left\{ \begin{array}{c} 1\\0 \end{array} \right\} A_0(r) \right] g_a(r) = 0$$

Field Equations

$$\begin{split} \left\langle \Phi_{j}(r)\right| \left(\frac{d^{2}}{dr^{2}} + \frac{2}{r}\frac{d}{dr} - m_{\rm s}^{2}\right) \Phi_{0}(r) - g_{\rm s}^{2} \left(\frac{\kappa}{2}\Phi_{0}^{2}(r) + \frac{\lambda}{6}\Phi_{0}^{3}(r)\right) &= -g_{\rm s}^{2} \left(\rho_{\rm s,p}(r) + \rho_{\rm s,n}(r)\right), \\ \left\langle W_{j}(r)\right| \left(\frac{d^{2}}{dr^{2}} + \frac{2}{r}\frac{d}{dr} - m_{\rm v}^{2}\right) W_{0}(r) - g_{\rm v}^{2} \left(\frac{\zeta}{6}W_{0}^{3}(r) + 2\Lambda_{\rm v}B_{0}^{2}(r)W_{0}(r)\right) &= -g_{\rm v}^{2} \left(\rho_{\rm v,p}(r) + \rho_{\rm v,n}(r)\right), \\ \left\langle B_{j}(r)\right| \left(\frac{d^{2}}{dr^{2}} + \frac{2}{r}\frac{d}{dr} - m_{\rho}^{2}\right) B_{0}(r) - 2\Lambda_{\rm v}g_{\rho}^{2}W_{0}^{2}(r)B_{0}(r) &= -\frac{g_{\rho}^{2}}{2} \left(\rho_{\rm v,p}(r) - \rho_{\rm v,n}(r)\right), \\ \left\langle A_{j}\left(r\right) \left(\frac{d^{2}}{dr^{2}} + \frac{2}{r}\frac{d}{dr}\right) A_{0}(r) &= -e\rho_{\rm v,p}(r), \end{split}$$



Bayes goes fast

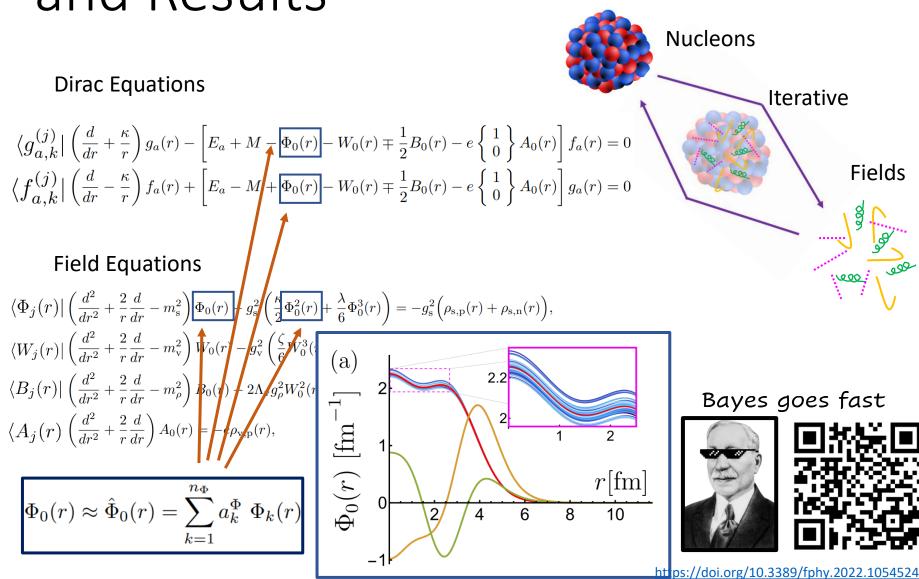


Applications and Results



Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

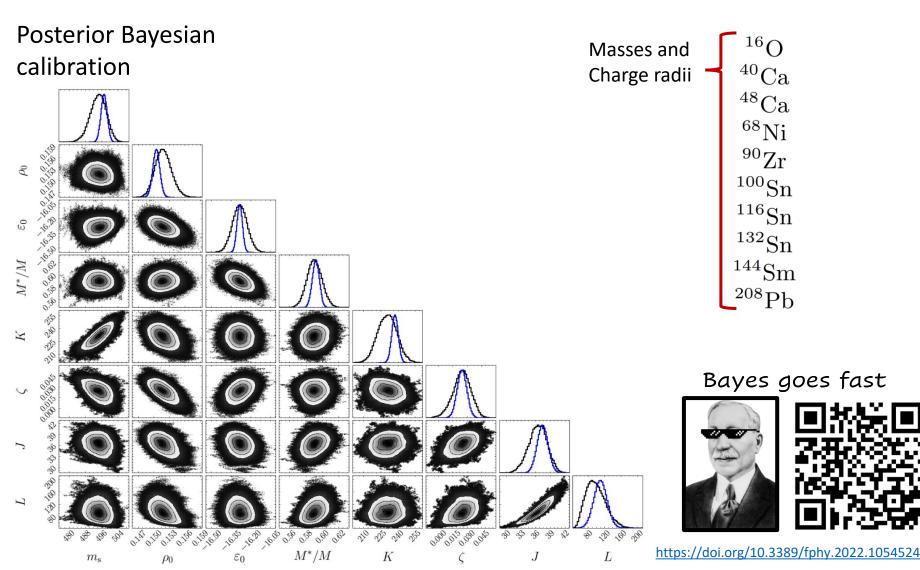
Pablo Giuliani,^{1, 2, *} Kyle Godbey,^{1, †} Edgard Bonilla,^{3, ‡} Frederi Viens,^{2, 4, §} and Jorge Piekarewicz^{5, ¶}





Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

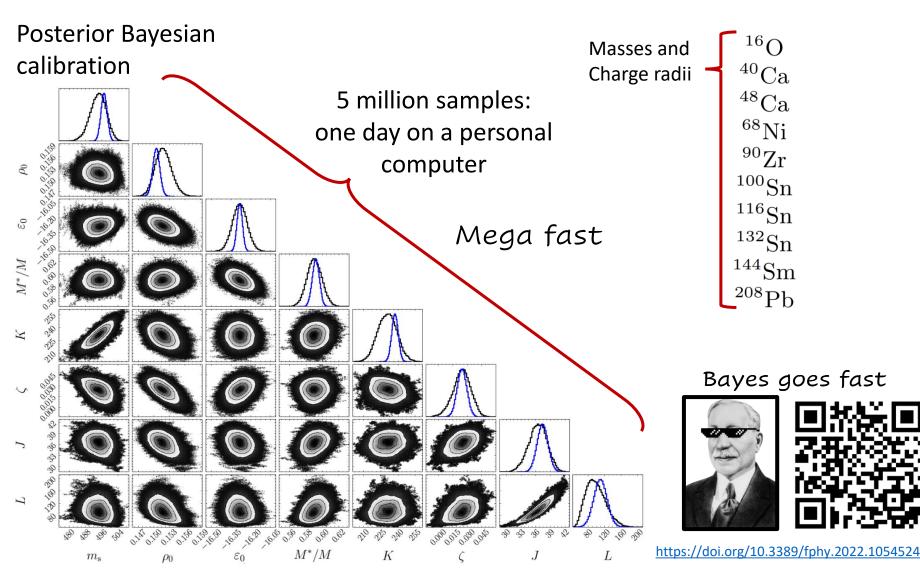
Pablo Giuliani,^{1, 2, *} Kyle Godbey,^{1, †} Edgard Bonilla,^{3, ‡} Frederi Viens,^{2, 4, §} and Jorge Piekarewicz^{5, ¶}





Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

Pablo Giuliani,^{1, 2, *} Kyle Godbey,^{1, †} Edgard Bonilla,^{3, ‡} Frederi Viens,^{2, 4, §} and Jorge Piekarewicz^{5, ¶}

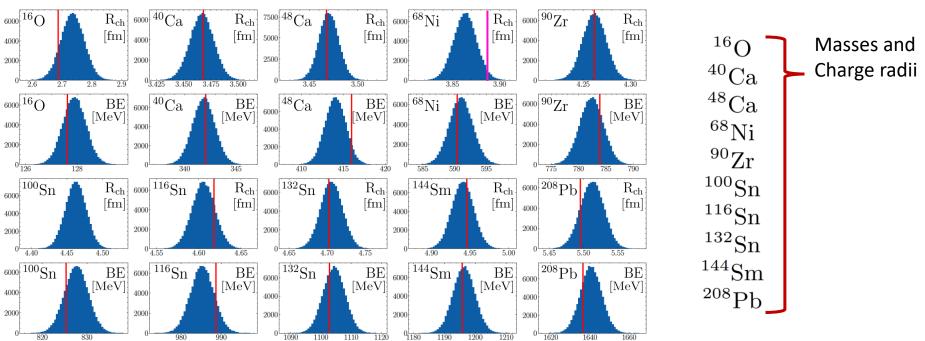


Applications and Results

2 в

Bayes goes fast: Uncertainty Quantification for a Covariant Energy Density Functional emulated by the Reduced Basis Method

Pablo Giuliani,^{1, 2, *} Kyle Godbey,^{1, †} Edgard Bonilla,^{3, ‡} Frederi Viens,^{2, 4, §} and Jorge Piekarewicz^{5, ¶}





Bayes goes fast



almost done....

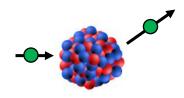
Applications 3 and Results

Presenting ROSE, a Reduced Order Scattering Emulator

D. Odell,¹, P. Giuliani,^{2,3} M. Catacora-Rios,^{2,4} M. Chan,⁵ E. Bonilla,⁶ K. Godbey,² R. J. Furnstahl,⁷ and F. M. Nunes^{2,4},

Daniel Odell





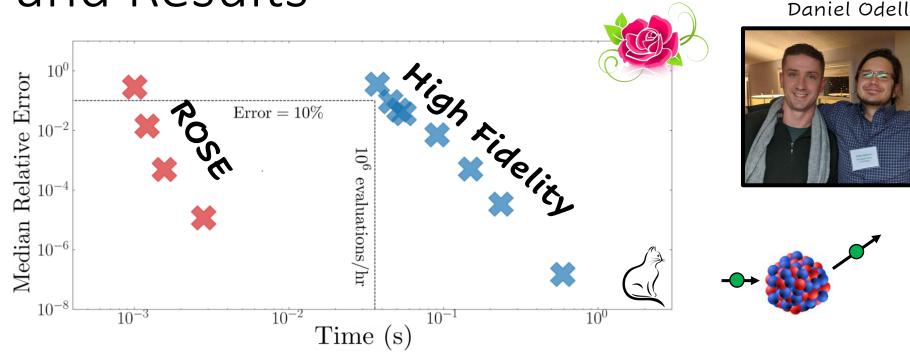
The roses

almost done....

Applications and Results

Presenting ROSE, a Reduced Order Scattering Emulator

D. Odell,^{1,6} P. Giuliani,^{2,3} M. Catacora-Rios,^{2,4} M. Chan,⁵ E. Bonilla,⁶ K. Godbey,² R. J. Furnstahl,⁷ and F. M. Nunes^{2,4,6}

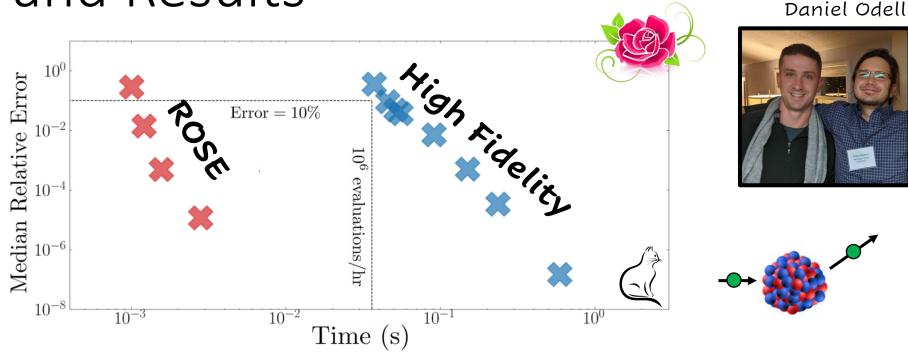


3

The roses

Presenting ROSE, a Reduced Order Scattering Emulator

D. Odell,^{1, 1} P. Giuliani,^{2,3} M. Catacora-Rios,^{2,4} M. Chan,⁵ E. Bonilla,⁶ K. Godbey,² R. J. Furnstahl,⁷ and F. M. Nunes^{2,4, 1}



3

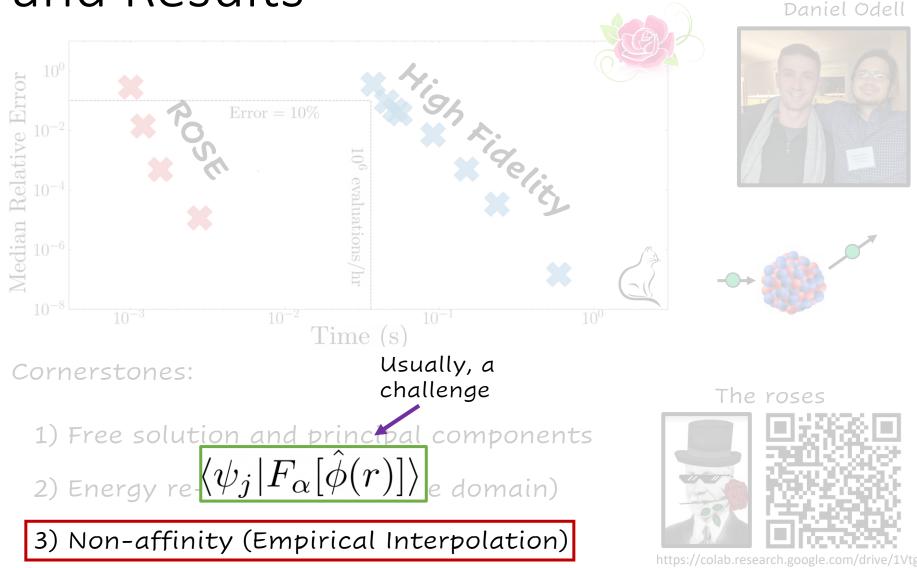
Cornerstones:

- 1) Free solution and principal components
- 2) Energy re-scaling (reference domain)
- 3) Non-affinity (Empirical Interpolation)



Presenting ROSE, a Reduced Order Scattering Emulator

D. Odell,^{1,1} P. Giuliani,^{2,3} M. Catacora-Rios,^{2,4} M. Chan,⁵ E. Bonilla,⁶ K. Godbey,² R. J. Furnstahl,⁷ and F. M. Nunes^{2,4},

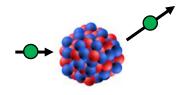


almost done....

Applications 3 and Results

Presenting ROSE, a Reduced Order Scattering Emulator

D. Odell,¹, P. Giuliani,^{2,3} M. Catacora-Rios,^{2,4} M. Chan,⁵ E. Bonilla,⁶ K. Godbey,² R. J. Furnstahl,⁷ and F. M. Nunes^{2,4},



Optical Potential

$$U(r,\alpha) = -V_v \left[1 + \exp\left(\frac{r - R_v}{a_v}\right) \right] - iW_v \left[1 + \exp\left(\frac{r - R_w}{a_w}\right) \right] - i4a_d W_d \frac{d}{dr} \left[1 + \exp\left(\frac{r - R_d}{a_d}\right) \right]$$

$$F_lpha(\phi)=igg(-rac{d^2}{dr^2}+rac{\ell(\ell+1)}{r^2}+U(r,lpha)-p^2igg)\phi(r)=0$$

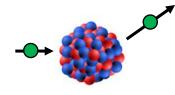


almost done....

Applications 3 and Results

Presenting ROSE, a Reduced Order Scattering Emulator

D. Odell,^{1,} P. Giuliani,^{2, 3} M. Catacora-Rios,^{2, 4} M. Chan,⁵ E. Bonilla,⁶ K. Godbey,² R. J. Furnstahl,⁷ and F. M. Nunes^{2, 4},



Optical Potential

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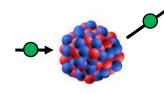
$$F_lpha(\phi)=igg(-rac{d^2}{dr^2}+rac{\ell(\ell+1)}{r^2}+U(r,lpha)-p^2igg)\phi(r)=0$$

$$\langle \psi_j | F_\alpha[\hat{\phi}(r)] \rangle = \int \psi_j(r) F_\alpha[\hat{\phi}(r)] dr = 0$$



Presenting ROSE, a Reduced Order Scattering Emulator

D. Odell,¹, P. Giuliani,^{2,3} M. Catacora-Rios,^{2,4} M. Chan,⁵ E. Bonilla,⁶ K. Godbey,² R. J. Furnstahl,⁷ and F. M. Nunes^{2,4},



Optical Potential

$$U(r,\alpha) = -V_v \Big[1 + \exp\left(\frac{r - R_v}{a_v}\right) \Big] - iW_v \Big[1 + \exp\left(\frac{r - R_w}{a_w}\right) \Big] - i4a_d W_d \frac{d}{dr} \Big[1 + \exp\left(\frac{r - R_d}{a_d}\right) \Big]$$

$$F_\alpha(\phi) = \left(-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + U(r,\alpha) - p^2 \right) \phi(r) = 0$$
Non affine problem

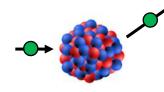
Non-affine problem

 $\langle \psi_j | F_\alpha[\hat{\phi}(r)] \rangle = \int \psi_j(r) F_\alpha[\hat{\phi}(r)] dr = 0$



Presenting ROSE, a Reduced Order Scattering Emulator

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Optical Potential

$$U(r,\alpha) = -V_v \Big[1 + \exp\left(\frac{r - R_v}{a_v}\right) \Big] - iW_v \Big[1 + \exp\left(\frac{r - R_w}{a_w}\right) \Big] - i4a_d W_d \frac{d}{dr} \Big[1 + \exp\left(\frac{r - R_d}{a_d}\right) \Big]$$
$$F_\alpha(\phi) = \left(-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + U(r,\alpha) - p^2 \right) \phi(r) = 0$$
$$Non-affine \text{ problem}$$

$$\langle \psi_j | F_\alpha[\hat{\phi}(r)] \rangle = \int \psi_j(r) F_\alpha[\hat{\phi}(r)] dr = 0$$

 $U(r, \alpha) \approx \sum_{i=1}^m b_i(\alpha) f(r)$

Empirical Interpolation Method: one work-around

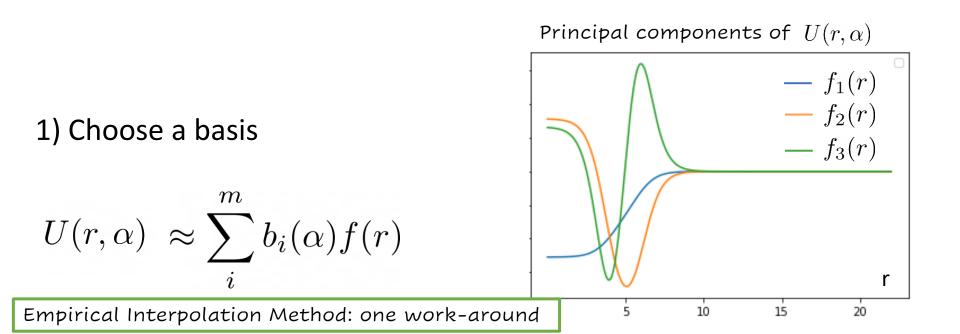


almost done....

Applications 3 and Results

Presenting ROSE, a Reduced Order Scattering Emulator

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almost done....

Applications 3 and Results

Presenting ROSE, a Reduced Order Scattering Emulator

D. Odell,^{1,†} P. Giuliani,^{2,3} M. Catacora-Rios,^{2,4} M. Chan,⁵ E. Bonilla,⁶ K. Godbey,² R. J. Furnstahl,⁷ and F. M. Nunes^{2,4,†}

$$U(r_{j}, \alpha) - \sum_{i}^{m} b_{i}(\alpha)f(r_{j}) = 0$$
Obtained by
interpolation $j = \{1, m\}$
Principal components of $U(r, \alpha)$

1) Choose a basis
$$U(r, \alpha) \approx \sum_{i}^{m} b_{i}(\alpha)f(r)$$

$$U(r, \alpha) \approx \sum_{i}^{m} b_{i}(\alpha)f(r)$$
Empirical Interpolation Method: one work-around

almost done....

Applications (and Results

2) Project onto judges

Presenting ROSE, a Reduced Order Scattering Emulator

D. Odell,^{1,1} P. Giuliani,^{2,3} M. Catacora-Rios,^{2,4} M. Chan,⁵ E. Bonilla,⁶ K. Godbey,² R. J. Furnstahl,⁷ and F. M. Nunes^{2,4},

Principal components of $U(r, \alpha)$

10

5



 $\psi_j(r) = \delta(r - r_j)$ $\langle \psi_j | F_\alpha[\hat{\phi}(r)] \rangle = F_\alpha[\hat{\phi}(r_j)]$

 $- f_1(r)$

 $- f_2(r)$

 $- f_3(r)$

 $--U(r,\alpha)$

20

15

 $U(r_j, \alpha) - \sum_{i}^{m} b_i(\alpha) f(r_j) = 0$ Obtained by $j = \{1, m\}$ interpolation

Dirac

1) Choose a basis

m $U(r, \alpha) \approx \sum b_i(\alpha) f(r)$

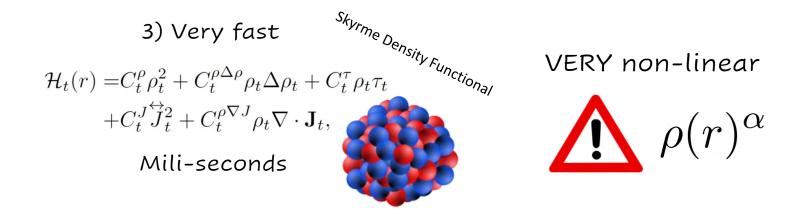
Empirical Interpolation Method: one work-around

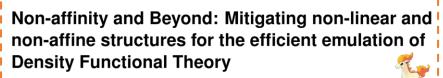


Non-affinity and Beyond: Mitigating non-linear and non-affine structures for the efficient emulation of Density Functional Theory

Kyle Godbey $^{1,\ast,+},$ Edgard Bonilla $^{2,+},$ Pablo Giuliani $^{1,3},$ and Yanlai Chen 4

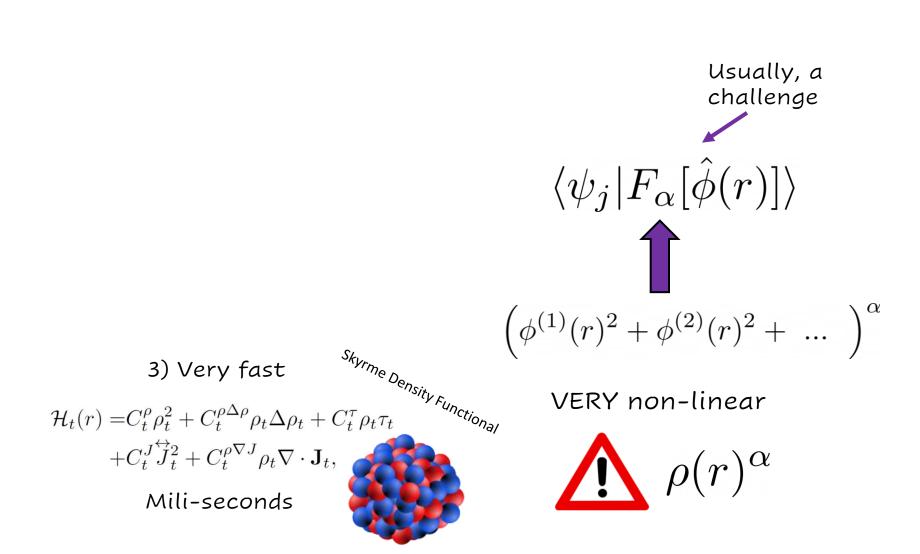
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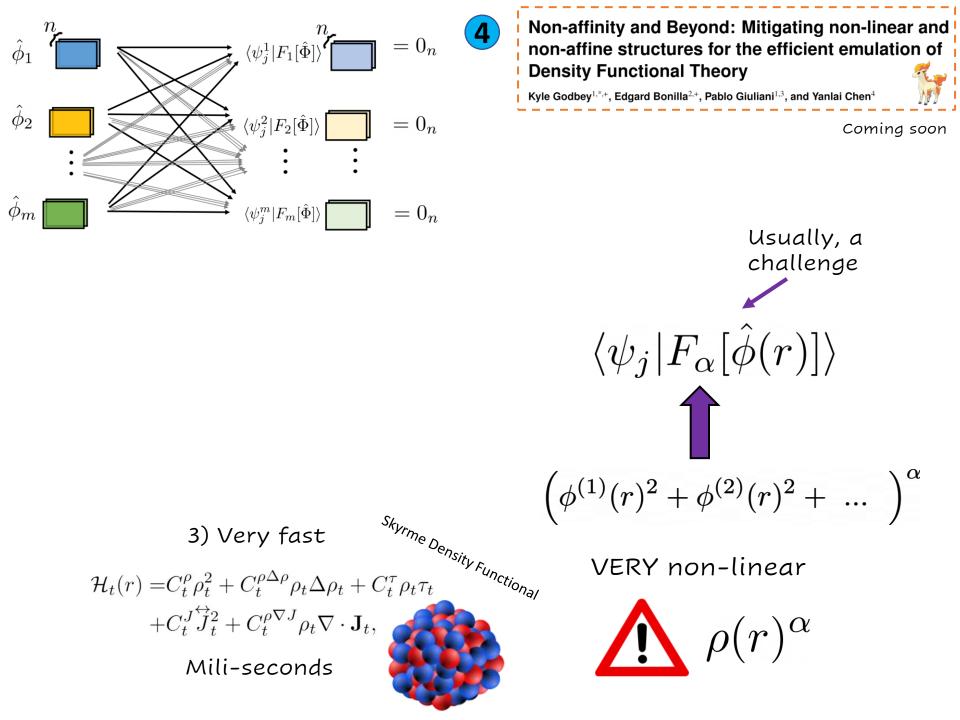


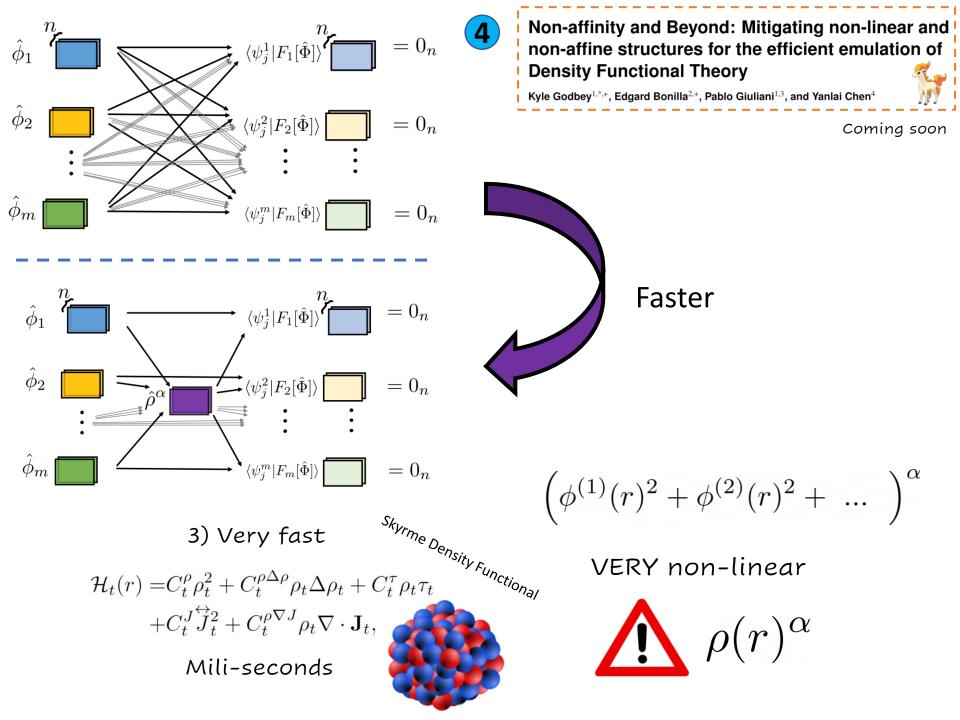


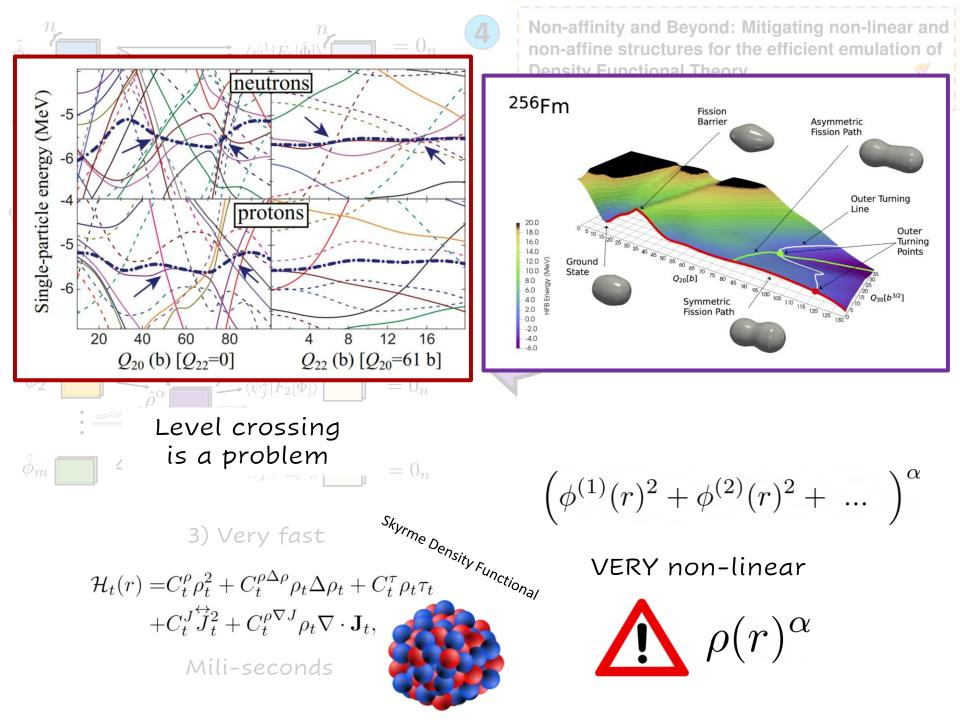
Kyle Godbey^{1,*,+}, Edgard Bonilla^{2,+}, Pablo Giuliani^{1,3}, and Yanlai Chen⁴

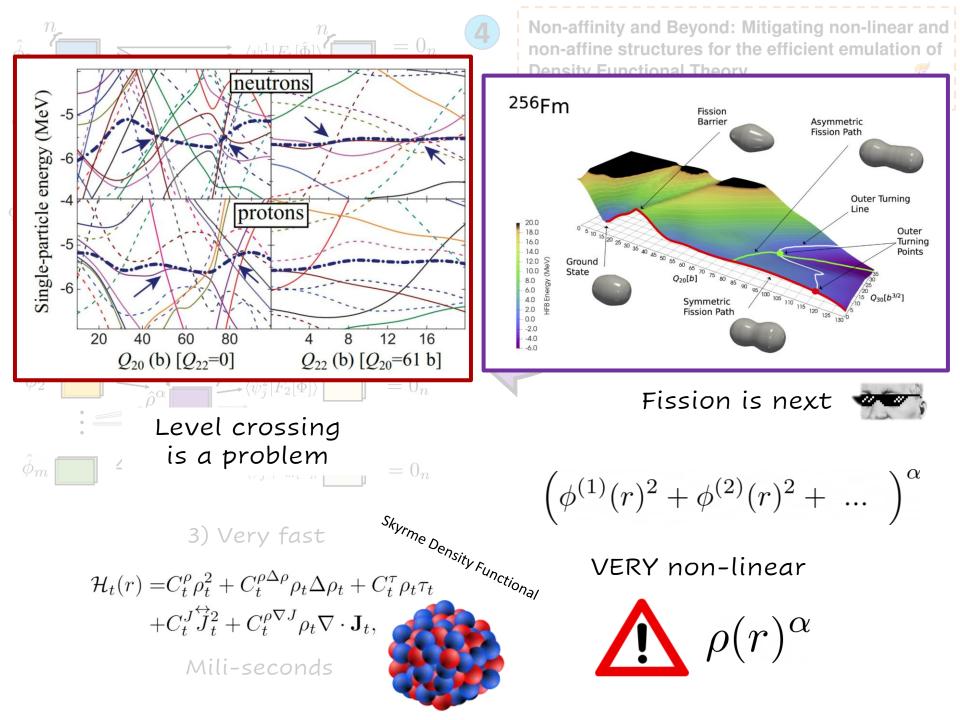
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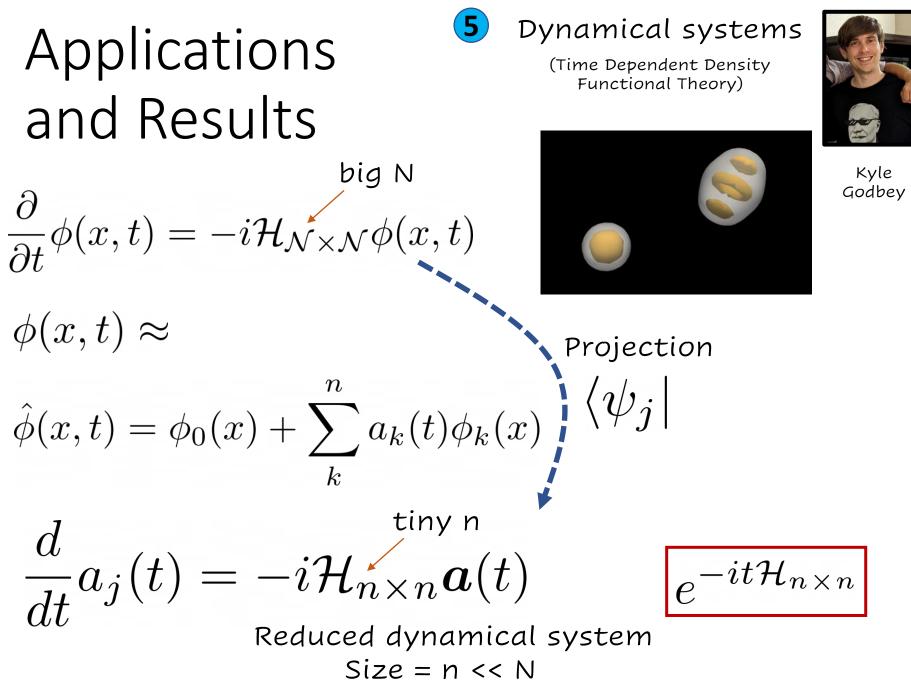








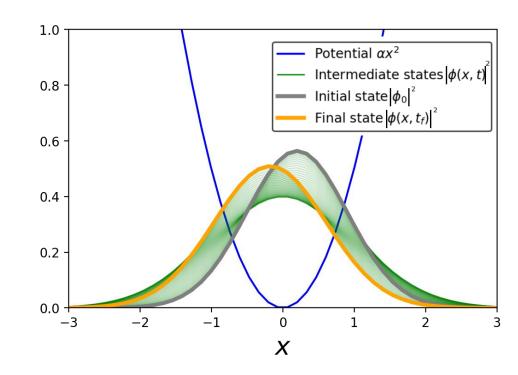




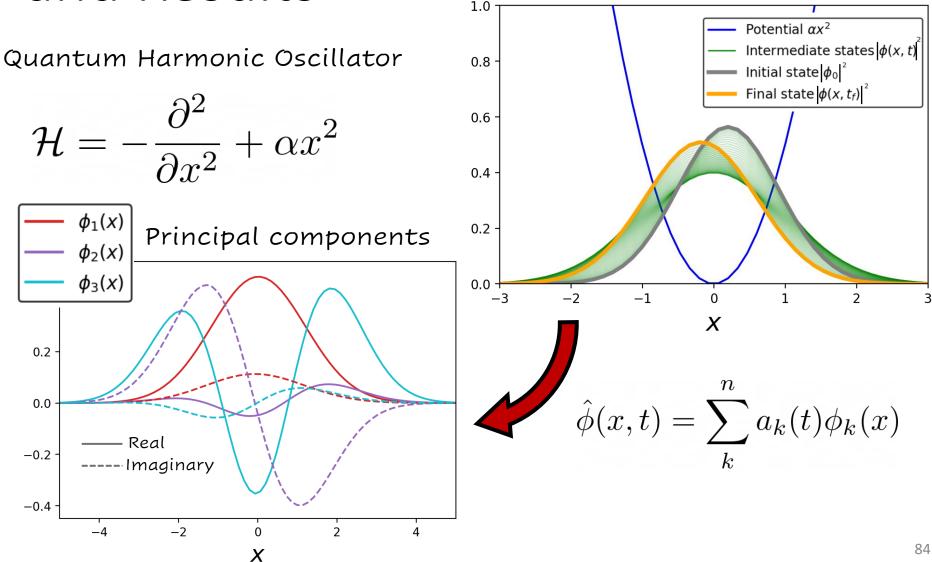
Quantum Harmonic Oscillator

$$\mathcal{H} = -\frac{\partial^2}{\partial x^2} + \alpha x^2$$



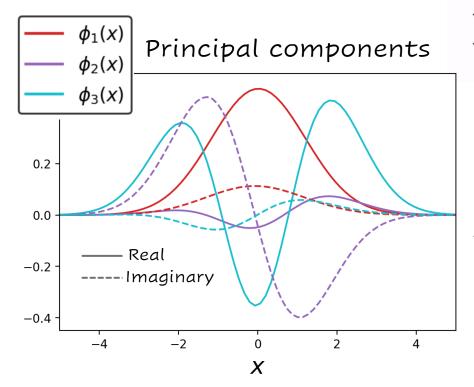




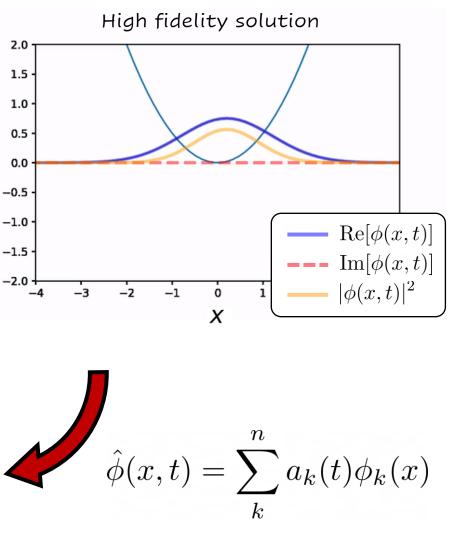


Quantum Harmonic Oscillator

$$\mathcal{H} = -\frac{\partial^2}{\partial x^2} + \alpha x^2$$

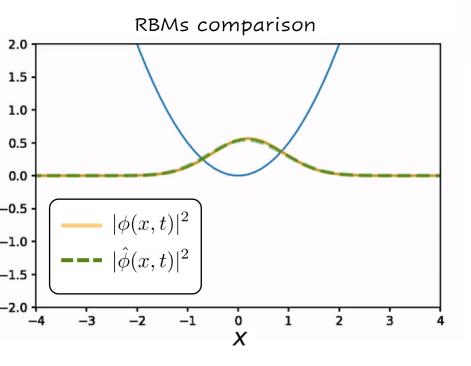




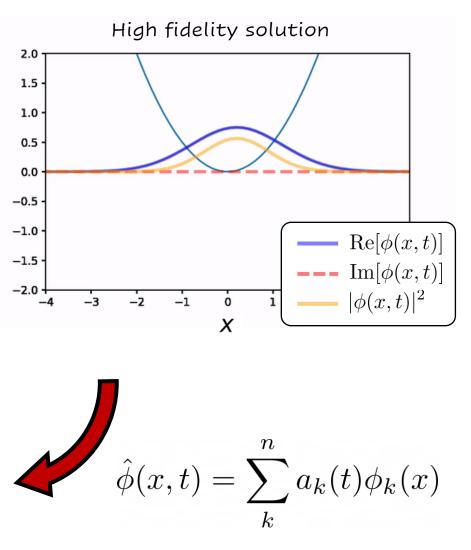


Quantum Harmonic Oscillator

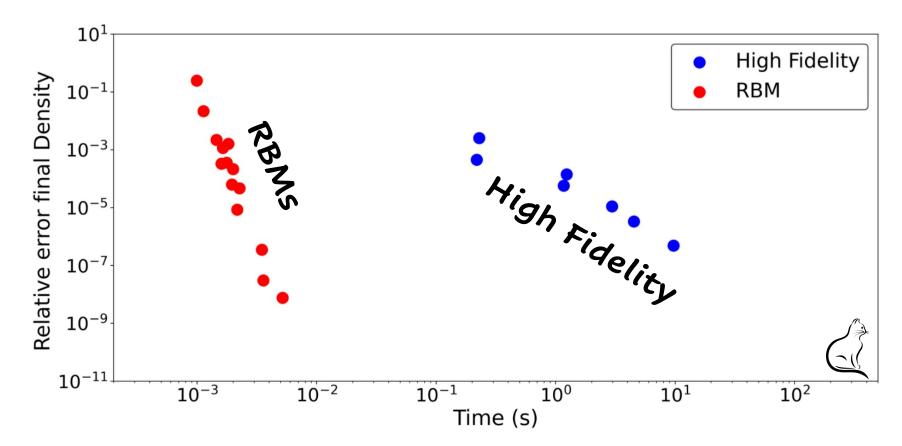
$$\mathcal{H} = -\frac{\partial^2}{\partial x^2} + \alpha x^2$$











5

Quantum Harmonic Oscillator



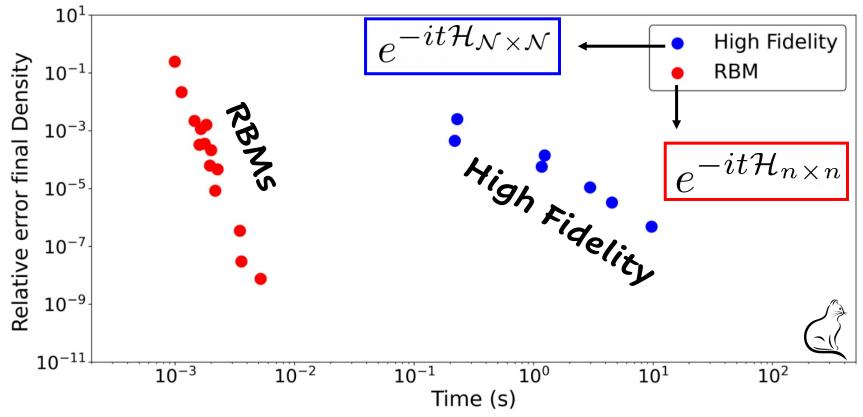
Dynamical systems

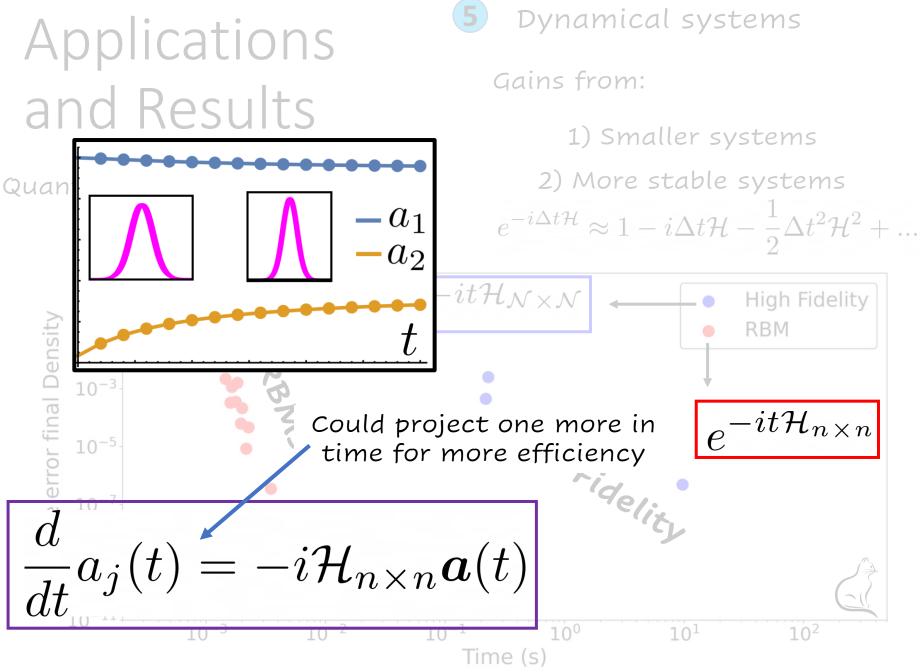
Gains from:

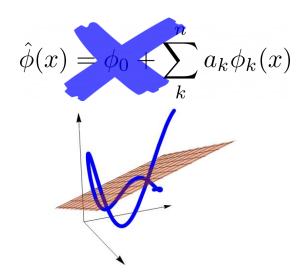
1) Smaller systems

2) More stable systems

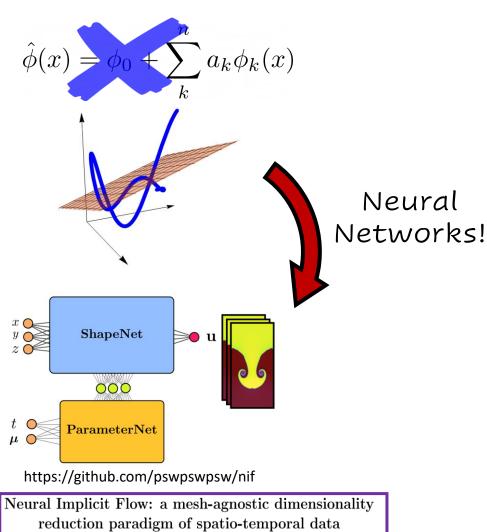






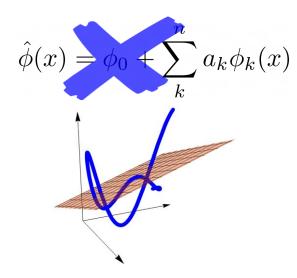


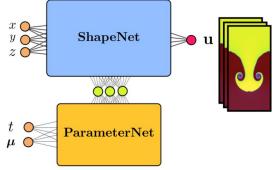




Shaowu Pan Steven L. Brunton J. Nathan Kutz





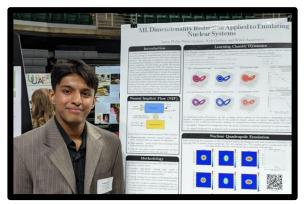


https://github.com/pswpswpsw/nif

Neural Implicit Flow: a mesh-agnostic dimensionality reduction paradigm of spatio-temporal data Shaowu Pan Steven L. Brunton J. Nathan Kutz

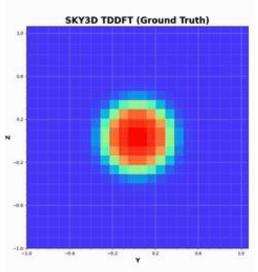


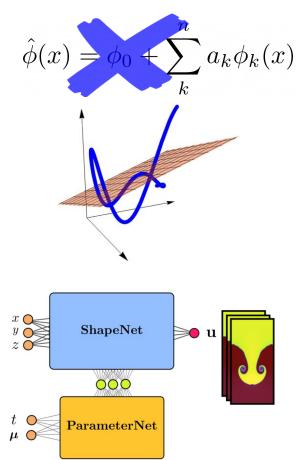
Dynamical systems



Aaron Phillip

 $m ^{40}Ca$ Quadrupole vibrations



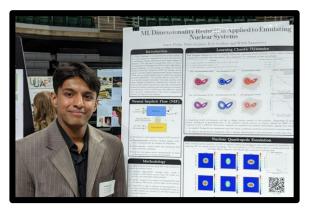


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Neural Implicit Flow: a mesh-agnostic dimensionality reduction paradigm of spatio-temporal data Shaowu Pan Steven L. Brunton J. Nathan Kutz

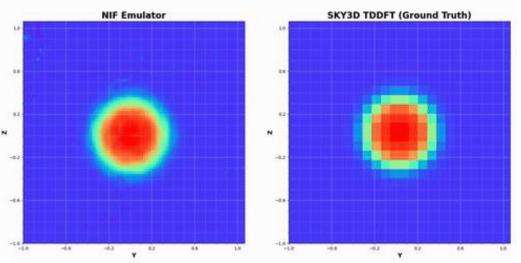


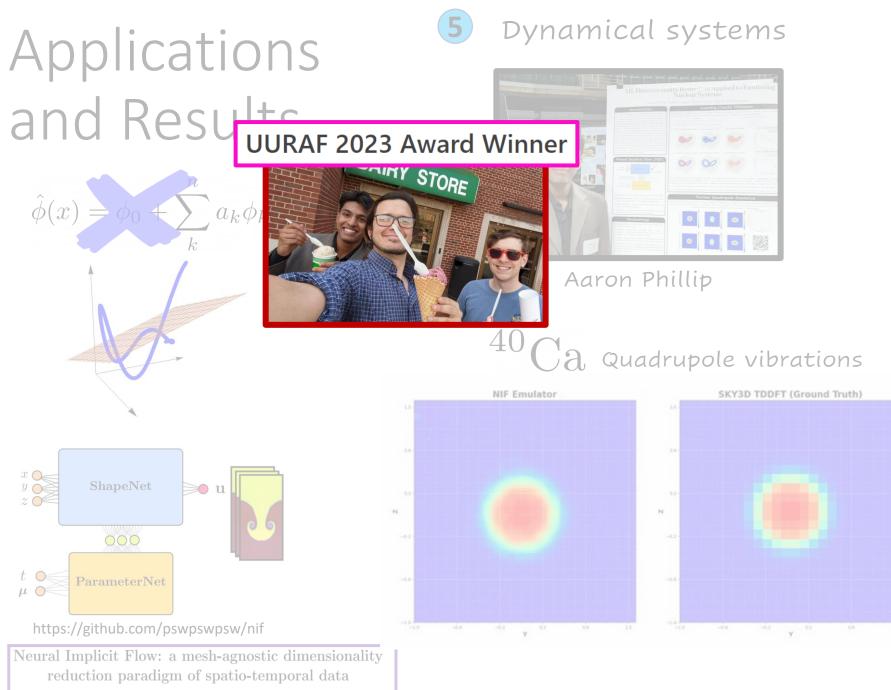
Dynamical systems



Aaron Phillip







Shaowu Pan Steven L. Brunton J. Nathan Kutz

Outline

Why Uncertainty Quantification? How

The Reduced Basis Method How it works Applications and Results

Upcoming Highlights



Takeaways



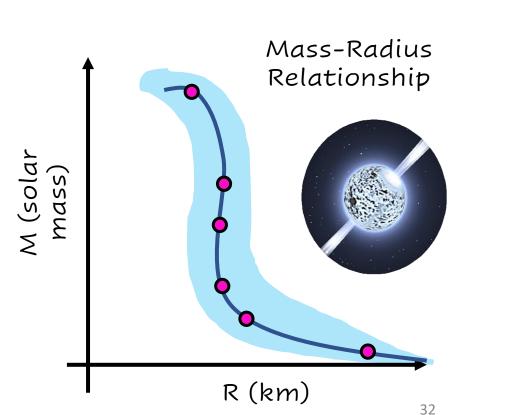
Application of reduced basis methods to compact stars

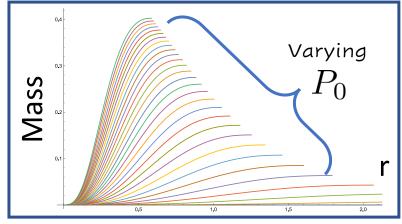
Amy Anderson,^{1, *} Pablo Giuliani,^{2, †} and J.Piekarewicz^{1, ‡}

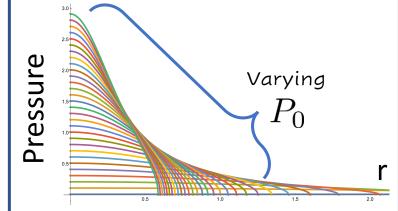
¹Department of Physics, Florida State University, Tallahassee, FL 323 ²FRIB/NSCL Laboratory, Michigan State University, East Lansing, Michiga

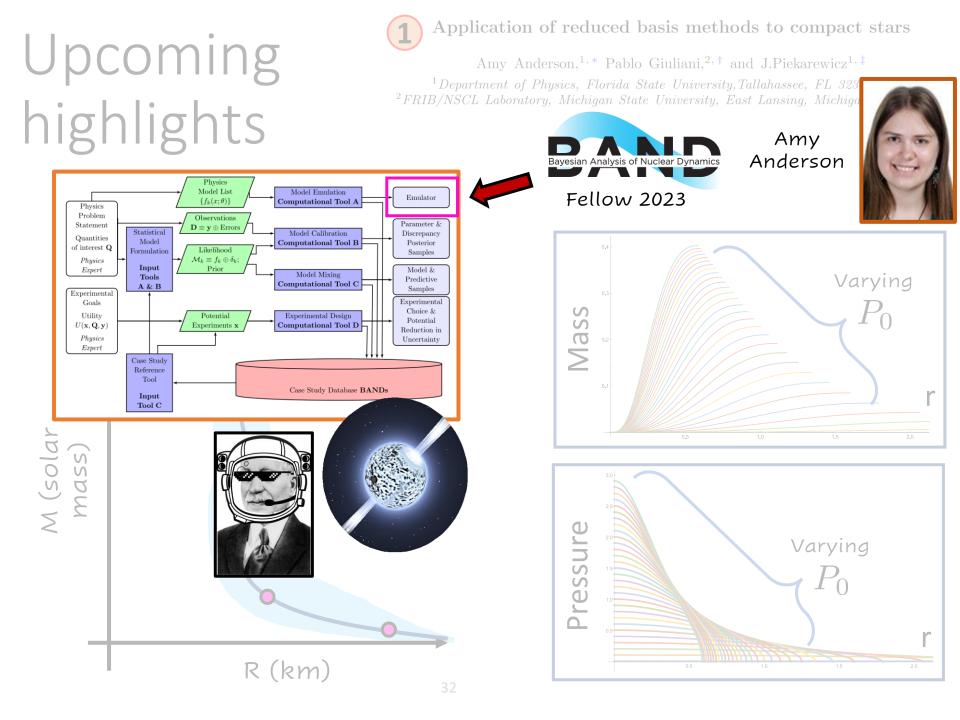
> Amy Anderson





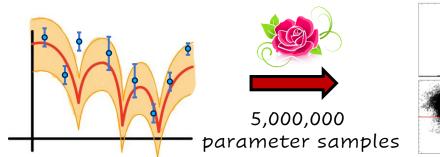


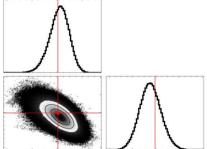






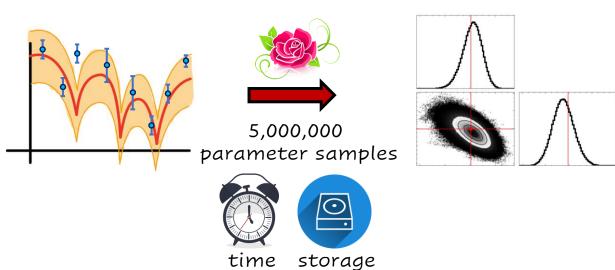
Smart posterior handling

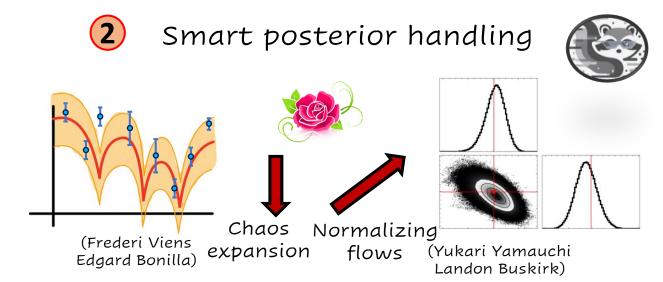






Smart posterior handling



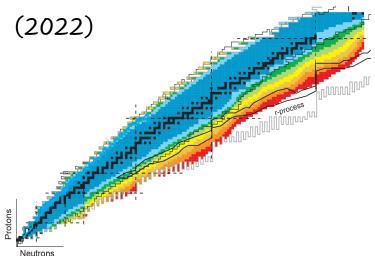


Upcoming	2 Smart posterior handling					
highlights						
Landon Buskirk	(Frederi Viens Edgard Bonilla) expansion flows (Yukari Yamauchi Landon Buskirk)					
Bayezian Mazz E	Image: second					
Compute For: Neutron + Target	Welcome to BMEX! Please input your requested nuclei on the left.					
Select Quantity: Differential Cross Section	$\frac{10^{3}}{M}$					
Select Interaction:						
Protons:	$\begin{array}{c} \mathbf{c} \\ $					
20						
Neutrons:						
20	$V_v \hspace{0.5cm} W_v \hspace{0.5cm} W_d \hspace{0.5cm} R_v \hspace{0.5cm} a_v \hspace{0.5cm} R_d \hspace{0.5cm} a_d$					



Optical potentials for the rare-isotope beam era

In regions of the nuclear chart away from stability, which represent a frontier in nuclear science over the coming decade and which will be probed at new rareisotope beam facilities worldwide, there is a targeted **need to quantify and reduce theoretical reaction model uncertainties**, especially with respect to nuclear optical potentials.







Compute For:

		VVCIC01					
Neutron + Target • Select Quantity: Differential Cross Section	defined b rather by	e where n by parame v distribu with neu	eter valu tions cor	es, but	/	$^{40}\mathrm{Ca}(n,n)$	
Select Interaction: Koning-Delaroche	Kyle Godbey				50	$\begin{array}{c} 100 \\ \theta \ (deg) \end{array}$	
20 eutrons: 20					ad	$ = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3$	

Welcome to RMFXI Please input your reque

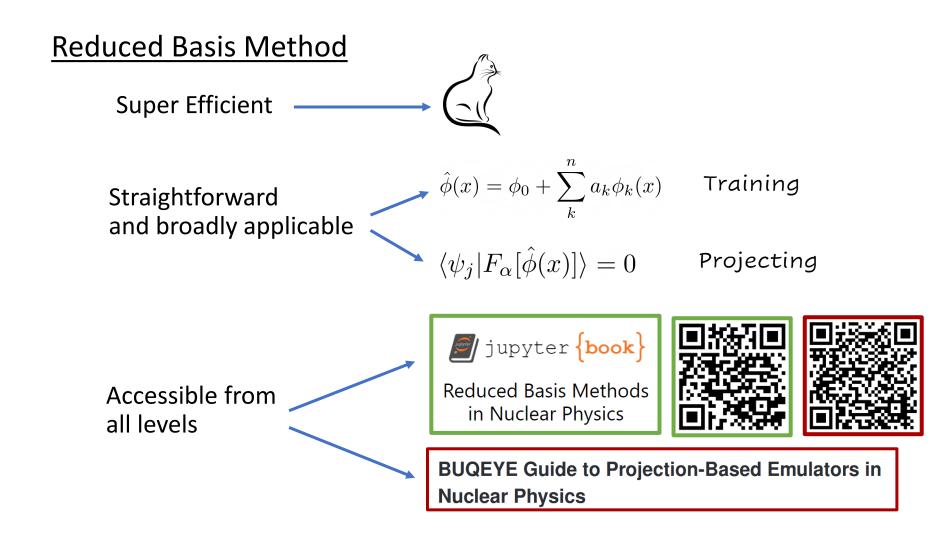
I have two



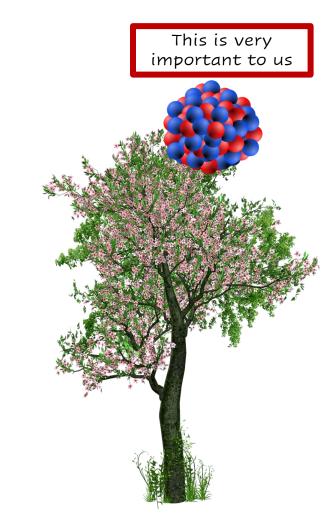
Takeaways

1) These methods are SO cool

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- 1) These methods are SO cool
- 2) UQ needs multidisciplinary efforts

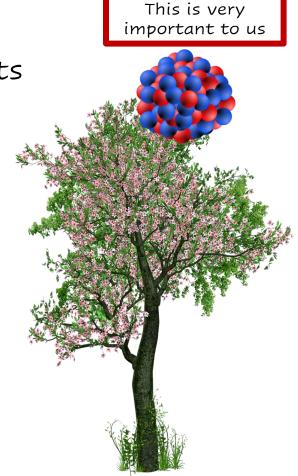




mathematics
 statistics
 computational
 experimental

- 1) These methods are SO cool
- 2) UQ needs multidisciplinary efforts

Work in collaboration with experts





mathematics
 statistics
 computational
 experimental

This is very important to us

- 1) These methods are SO cool
- 2) UQ needs multidisciplinary efforts

Work in collaboration with experts



Advanced Scientific Computing and Statistics Network



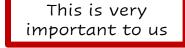
mathematics
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1) These methods are SO cool

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Work in collaboration with experts























Work in collaboration with experts ...

















... and find that the real UQ is the friends you made along the way