Concept and modeling of RD scan for FCC-ee

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PART I

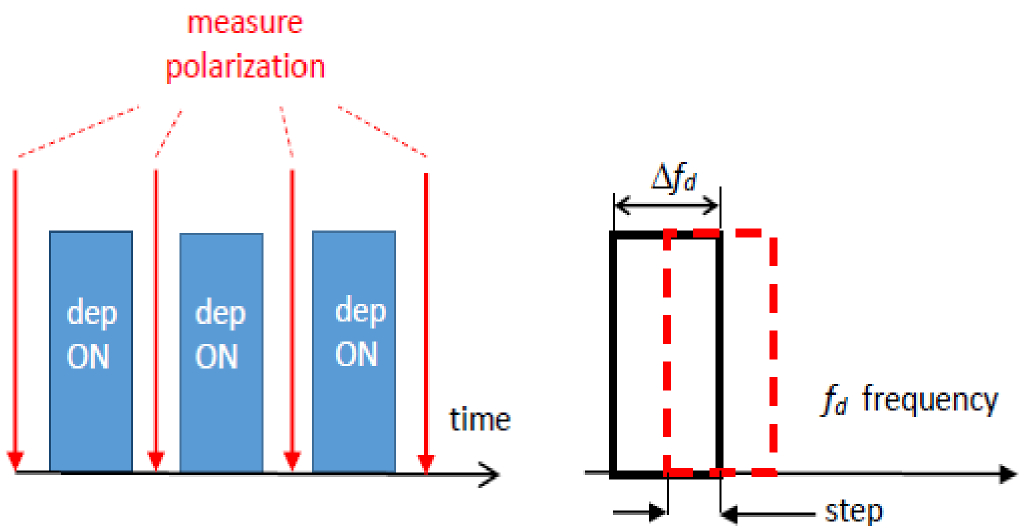
- Basic parameters of continuous RD scanning
- Analytical model of depolarization process
- Conceptual parameters of FCC-ee depolarizer
- Idea of “counter-scanning” on numerical examples including energy drift

PART II

- Monte-Carlo simulation
- RD accuracy in our concept of scanning

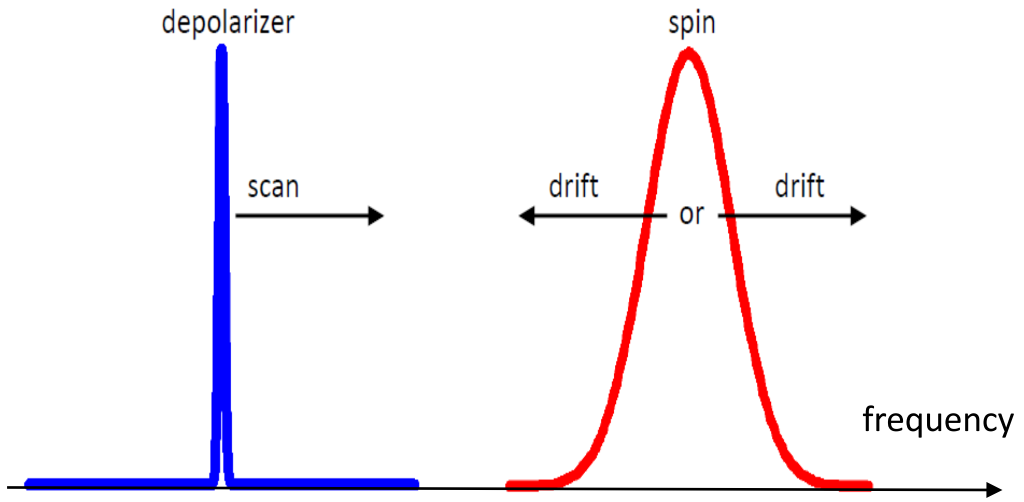
HOW TO MAKE RD SCAN

Discrete scanning with sweep of depolarizer frequency



This was being done in first RD experiments, and, for example, so was done on LEP …

Continuous scanning with narrow line of depolarizer



At VEPP-4M, in experiments with resolution of 10^{-9} in terms of depolarization frequency …

In our experience, continuous monotonic scanning is convenient and efficient for any ratio of depolarizer and spin spectral linewidths. At FCC-ee, to achieve better than 10^{-6} accuracy, depolarizer line must be narrower than spin line.

BASIC PARAMETERS

Accuracy of determination of single-moment value of spin frequency by RD depends on:

- spin line width
- depolarizer (kicker) line width
- rate of scanning
- rate of energy drift
- spin harmonic amplitude (strength of depolarizer)
- statistical error in polarization measurement

LINE WIDTH IN SPIN AND DEPOLARIZER SPECTRA

$\varepsilon_\nu \sim \nu \langle H''(\sigma_{x\beta}^2 + \sigma_{x\gamma}^2) \rangle$ broadening of spin line due to sextupoles [turn^{-1}]

$\varepsilon_{diff} \sim \frac{\sigma_\nu}{\nu_\gamma} \frac{\lambda_\gamma}{2\pi}$ broadening of spin line due to radiative diffusion of spin phase [turn^{-1}]

V. Blinov, E. Levichev, S. Nikitin and I.Nikolaev. Eur. Phys. J. Plus (2022) 137:717

	E GeV	f_0 kHz	σ_ν spin tune spread due to energy spread [turn^{-1}]	ν_γ synchrotron tune [turn^{-1}]	σ_ν / ν_γ modulation index	$\lambda_\gamma / 2\pi$ radiation decrement [rad^{-1}]	ε_ν due to non-linearity [turn^{-1}]	ε_{diff} due to radiative diffusion [turn^{-1}]	$\frac{\sqrt{\varepsilon_\nu^2 + \varepsilon_{diff}^2}}{\nu}$	Spin line half- width [keV]
VEPP-4M	1.85 4.73	820	0.0015 0.0098	~0.01 0.015	~0.015 ≈0.7	1.8e-6 3.0e-5	~4e-6 ~1e-4	2.7e-7 2.1e-5	≈1e-6 ≈1e-5	≈2 ≈40
LEP	45.6	11	0.061	0.083	0.73	4.7e-4	-	3.4e-4	~3e-6	~140
FCC-ee	45.6 80	3	0.039 0.120	0.025 0.051	1.56 2.37	1.25e-4 6.8e-4	~7.3e-5 -	2e-4 1.6e-3	≈2.3e-6 ~8.8e-6	≈108 ~705

Frequency resolution of the FCC-ee depolarizer synthesizer should be not worse than 10^{-4} Hz

$\delta f_d \sim \sqrt{df_d/dt}$ [Hz] broadening of depolarizer line when scanning rate of df_d/dt

$\varrho = \frac{\Delta f_d}{\Delta E} = \nu \frac{f_0}{E} = 0.007$ [Hz/keV] ratio of frequency interval to energy interval ($\nu = \gamma a$)

$df_d/dt \ll (100\,\varrho)^2 \approx 0.5$ [Hz/s] needed rate to provide depolarizer linewidth much smaller than that of spin

If necessary, depolarizer line can be expanded in artificial way using synthesizer, maintaining rate of scanning

ANALYTICAL MODEL OF MONOTONIC SCANNING

Spin spectrum	Depolarizer spectrum	Type
$g(p) = \frac{1}{2p_0}[H(p+p_0) - H(p-p_0)]$	$q(p) = \frac{1}{2p_{00}}[H(p+p_{00}) - H(p-p_{00})]$	Rectangle
$g(p) = \frac{1}{\sqrt{2\pi}p_0}\exp\left(-\frac{p^2}{2p_0^2}\right)$	$q(p) = \frac{1}{\sqrt{2\pi}p_{00}}\exp\left(-\frac{p^2}{2p_{00}^2}\right)$	Gaussian

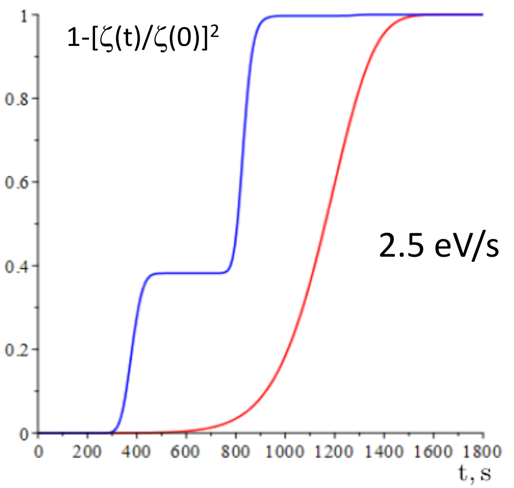
- Quantity p is detuning in relative to center of spin spectrum written in energy units
- Ratio of half-widths of spectral lines recommended for scanning: p_0 (spin) $\geq p_{00}$ (depolarizer)
- Convolution of spectral distribution functions: $G(p) = (g * q)(p)$
- Time parameter used is $\tau_d \sim \frac{\pi \Delta f_{spin}}{|w_k|^2 \omega_0^2}$, depolarization time for case when depolarizer narrow line is inside spin spectral band of width $\Delta f_{spin} = 2p_0\,\rho$ [Hz] ; $\omega_0 = 2\pi f_0$; $|w_k|$ harmonic amplitude
- At detuning rate $\dot{p} = dp/dt$, polarization ζ decreases according to

$$\frac{d\zeta}{dp}\frac{dp}{dt} \approx -\zeta\frac{\pi p_0}{\tau_d}G(p(t)) \text{ so } \zeta(p(t)) \approx \zeta_0 \exp\left[-\pi p_0\left(\frac{dp}{dt}\tau_d\right)^{-1}\int_{-\infty}^{p} G\,dp'\right]$$

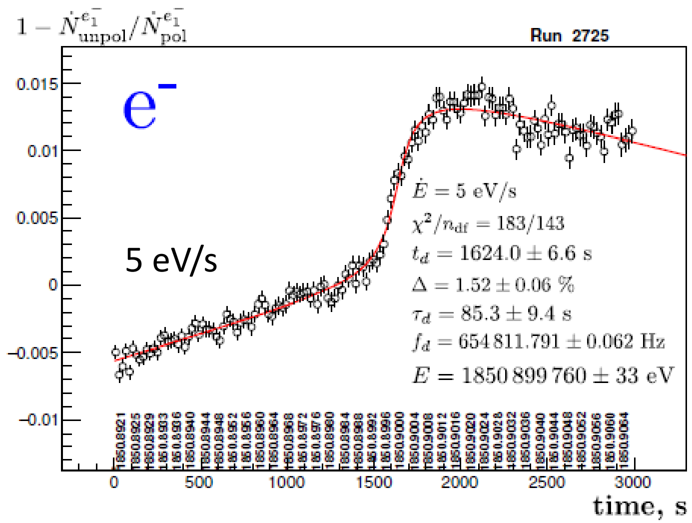
- Accounting for both scan rate and energy drift: $p(t) = p(0) + \int_0^t (\dot{p}_{dep} + \dot{p}_{drift})dt$

FINE SCANNING: ANALYTICAL MODEL VS EXRERIMENTS AT VEPP-4M

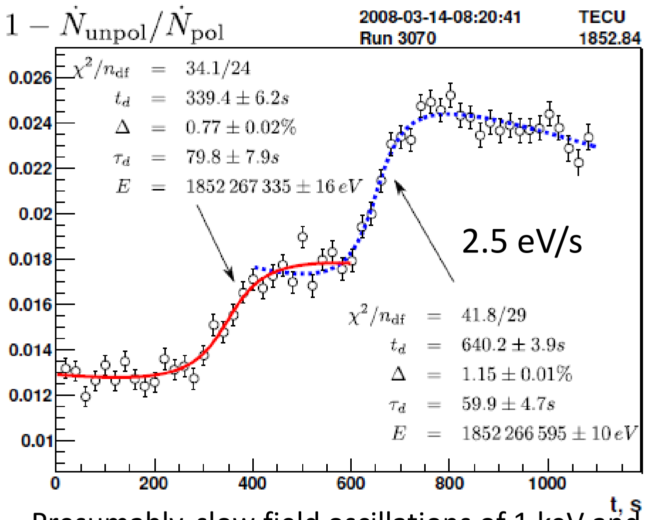
Simulation model taking into account relative spin linewidth estimated as $5\cdot10^{-7}$ at 1.85 GeV VEPP-4M (due to sextupoles) is in agreement with results of Fine-Scanning (high resolution) experiments



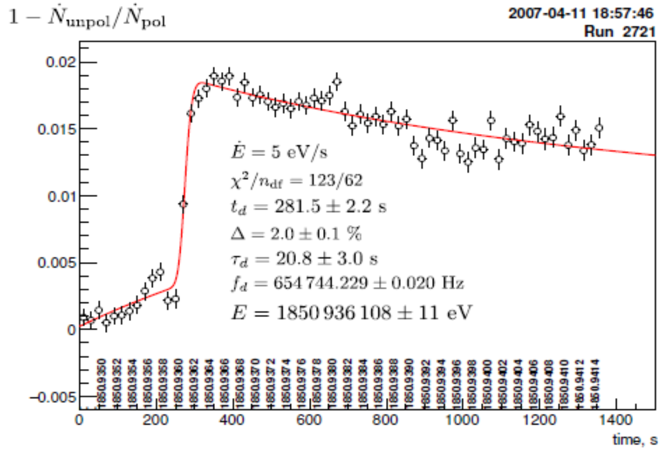
Model. Spin linewidth 1 keV; $|w_k|=10^{-7}$.
Blue: energy oscillations of 2 keV and 500 s



The typical experimental result on FS ($|w_k|\sim5\cdot10^{-8}$)



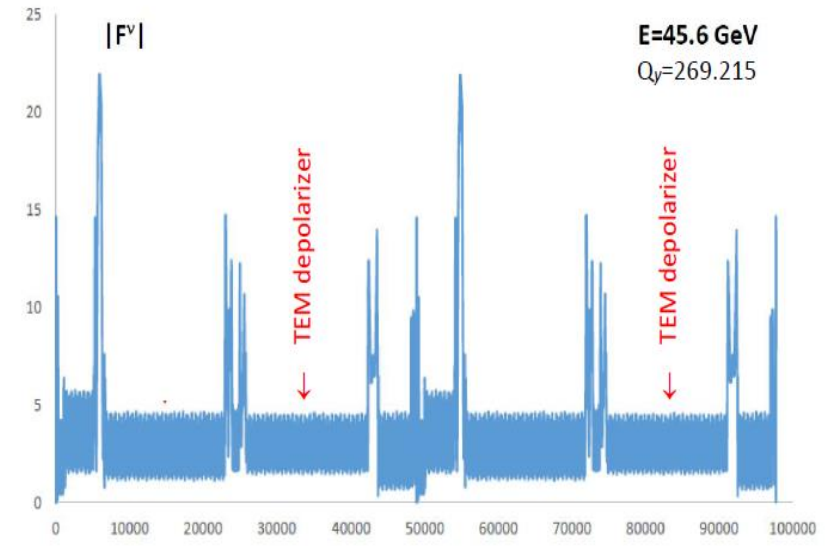
Presumably, slow field oscillations of 1 keV and a period of a few hundred seconds.

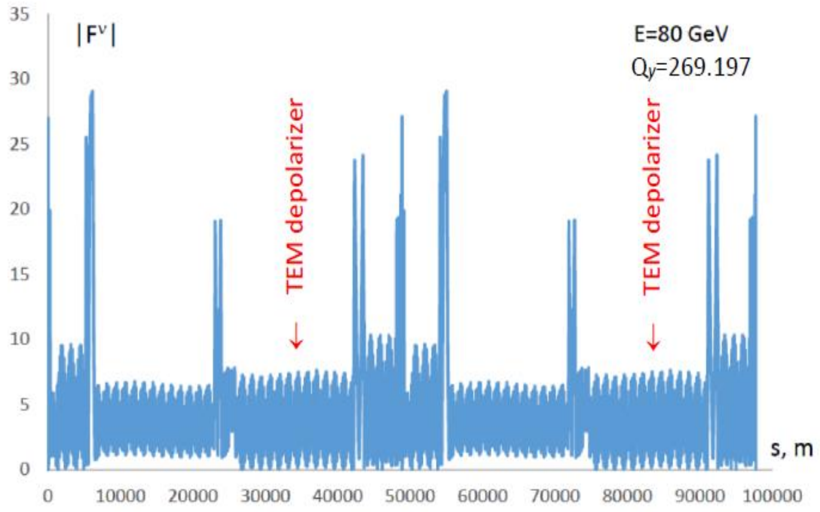


The rare case of a sharp jump, presumably, due to oncoming energy drift. Depolarization frequency resolution is found as the ratio of the fit error of 1.96 eV to the beam energy of 1852 MeV and is approximately 10^{-9}.
Note that 10^{-9} is not the accuracy of determining the absolute value of the spin frequency, but the resolution when finding the depolarization frequency! This is suitable for testing the CPT invariance by comparing e+ and e- spin frequencies with the relevant accuracy.

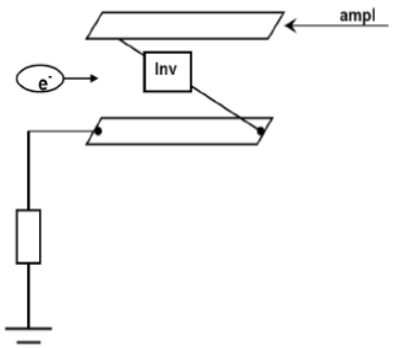
CONCEPT OF DEPOLARIZER FOR FCC-ee

Spin response factor vs azimuth at Z pole and WW threshold





Strip-line kicker



$$|w_k| = \frac{\nu \hat{B}_{\perp} l_d}{2\pi B\rho}|F^{\nu}| = |F^{\nu}|\frac{\nu\varphi_{\perp}}{2\pi}$$

$\tau_d = \frac{\pi\Delta f}{\omega_0^2|w_k|^2}$, characteristic depolarization time for selected case, when scanning is not performed, and after turning on depolarizer, its line completely turns out to be and remains inside spin spectral band Δf

($\omega_0 = 2\pi f_0$, angular revolution frequency)

Conceptual parameters

Beam energy	$E = 45.6$ GeV
spin harmonic amplitude	$\lvert w_k\rvert \propto \frac{\nu U l_d \lvert F^{\nu}\rvert}{E \cdot d}$
strip-line length	$l_d = 1$ m;
verticalgap between plates	$d = 20$ mm
amplitude of signal from amplifier	= 100 V
amplitude of voltage between plates	$U = 200$ V
spin response factor	$\lvert F^{\nu}\rvert = 5$
deflection angle in one passage	$\varphi_{\perp} = 2.2 \cdot 10^{-7}$
spin rotation in one passage`	$\nu\varphi_{\perp} = 2.3 \cdot 10^{-5}$
spin harmonic amplitude	$\lvert w_k\rvert = 1.8 \cdot 10^{-5}$
spin spectral band	$\Delta f = 1.4$ Hz
characteristic relaxation time	$\tau_d \approx 38$ second

The conceptual data shown are for scaling. For example, with $l_d \approx 3$ m, $|w_k|^2$ is 10 times larger. Our conditions are for complete or partial depolarization (without adiabatic spin flip). This is the case of fast uncorrelated crossings in spectral band. We studied it in the Fine-Scanning experiments at VEPP-4:

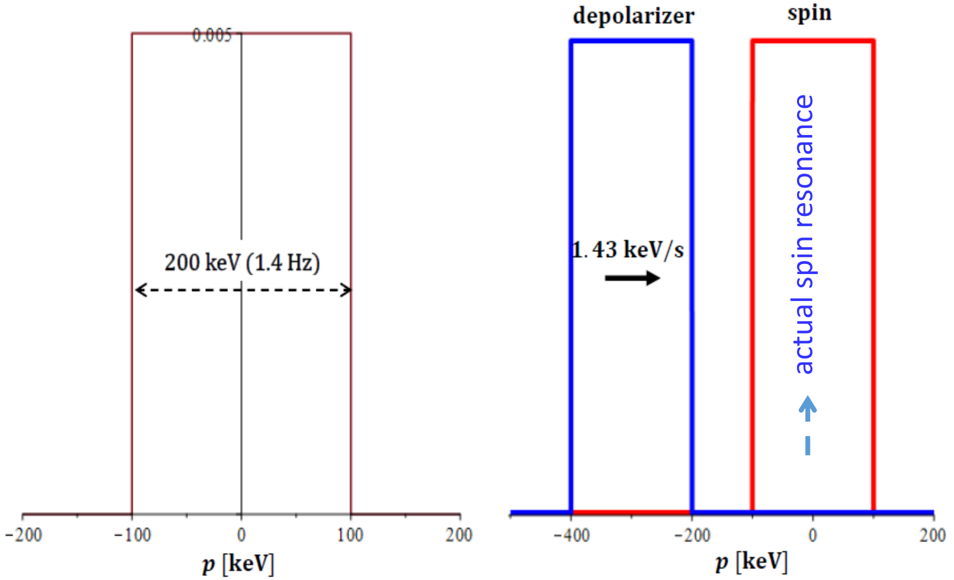
$\nu^2 \geq \tau_p(\Delta f)^3/f_0^2$, "uncorrelatedness" ($\tau_p$= S-T time);

$(\Delta f/f_0)^2 \gg |w_k|^2$, "rapidity" of resonance crossing.

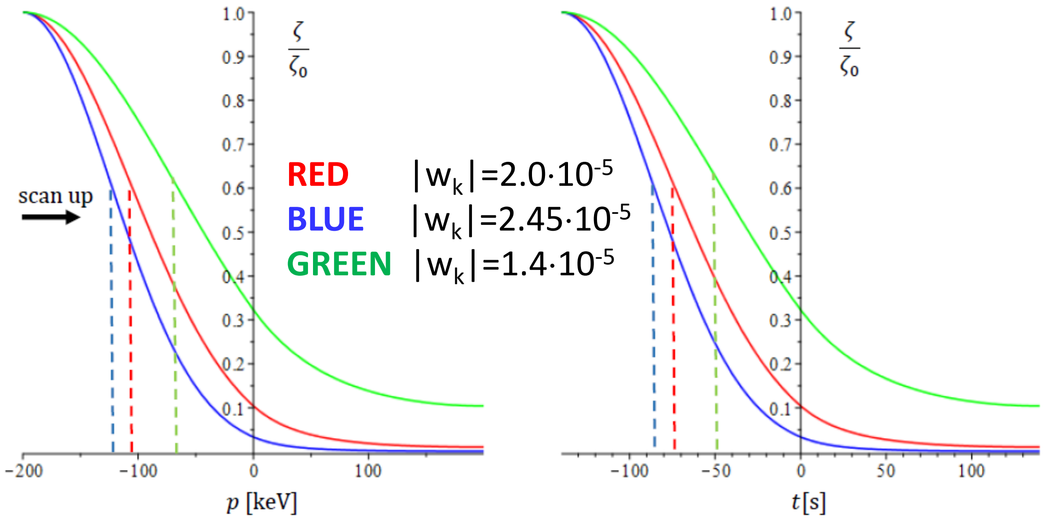
The concept we consider requires two independent selective depolarizers. They can be placed, for example, as shown in the graphs with |Fᵛ|. To implement a spin-flip for energy calibration (Ivan Koop), the spin harmonic amplitude is required by more than an order of magnitude greater than in the variant intended only for beam depolarization.. Therefore, the "soft" variant could easily fit into some spin-flip version.

EXAMPLE OF MODELING MONOTONIC SCANNING WITH SINGLE DEPOLARIZER

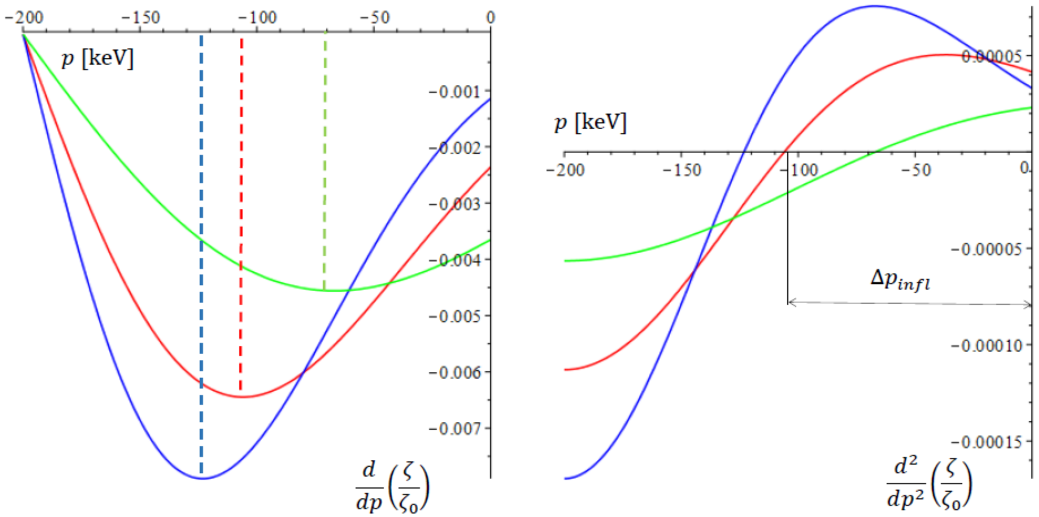
Spectra in "Equal linewidths" mode



Depolarization diagrams on energy and time scales



Inflection points. Can they help?

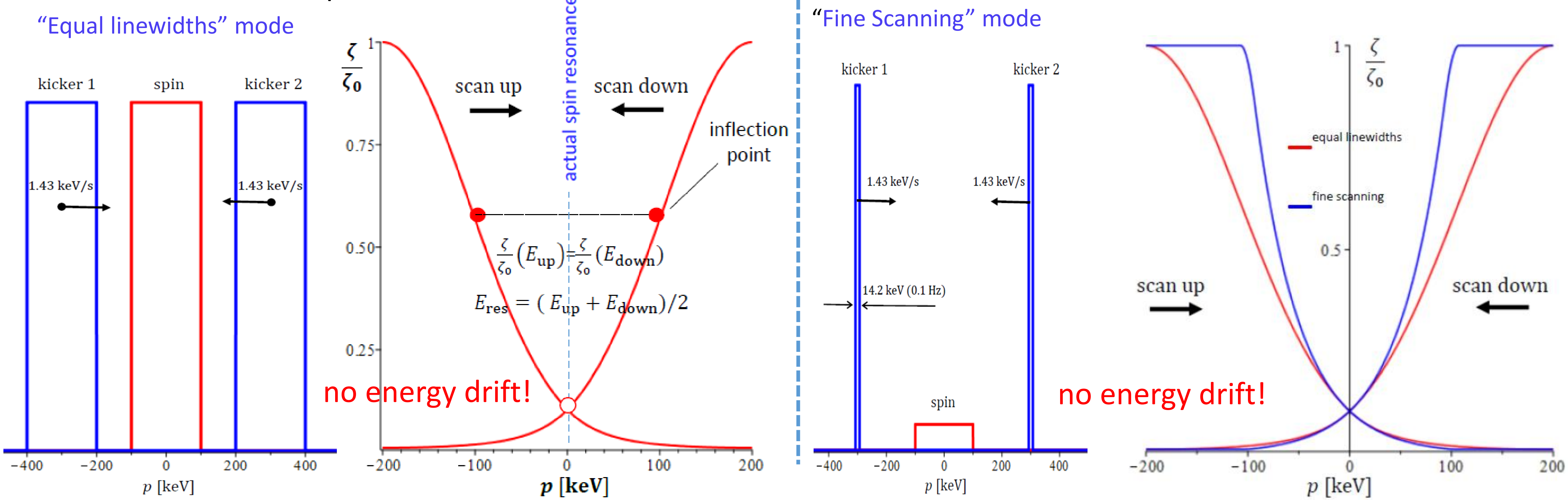


Questions

How to determine **true resonance**? Depending on width of spin line and strength of depolarizer, characteristic (**inflection**) point on diagrams can be at different distances from true resonance. Is it possible to determine **energy drift** parameters using RD **in model-independent way**?

IDEA OF SIMULTANEOUS INDEPENDENT SCANNING ON TWO BUNCHES: "COUNTER SCANNING"

Simultaneous use of two selective depolarizers, acting independently on two pilot bunches. Generally, frequency scanning in mutually opposite directions with respect to spin resonance. Model-independent way of determining true resonance, especially, if energy drift is negligible. Ability to determine rate and direction of drift and reduce systematic error related to width of spectral lines



If when scanning the energy drift does not manifest itself significantly, then "up" and "down" diagrams are mirror symmetric. Presumably, normalized fits of the polarimeter data related to these diagrams makes it possible to find average position of spin resonance with error less than spin linewidth. Monte Carlo simulation is used to explore this possibility.

EXAMPLE WITH COUNTER-SCANNING UNDER EXTREME ENERGY DRIFT

For the most complete demonstration, we consider the large drift rate that CERN estimates is possible due to tidal effects. We hope that an energy stabilization system (for example, based on BPM data-RF frequency feedback) will significantly reduce their impact.

Extreme energy drift (~45 MeV/h)

Drift V_d=12.5 keV/s (up); Dep.1 V_1=+14 keV/s; Dep.2 V_2=-14 keV/s

GREEN → and GREY → : $V_1 - V_d$ =+ 1.5 keV/s, $DE_1(t=0) = -400$ keV

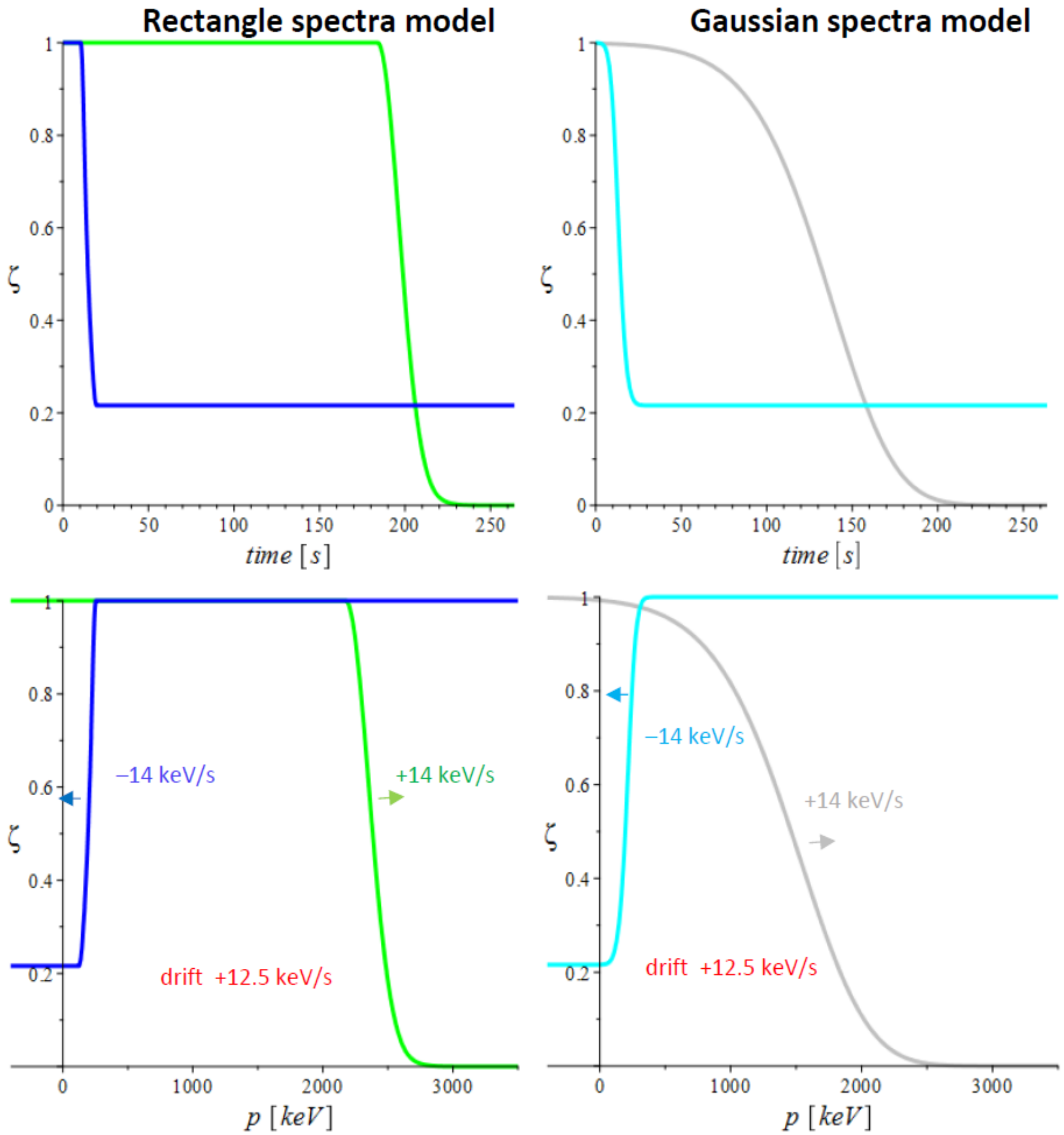
BLUE ← and CYAN ← : $V_2 - V_d = -26.5$ keV/s, $DE_2(t=0) = +400$ keV

Depolarizer harmonic amplitude: $|w_k| = 4 \cdot 10^{-5}$

Rectangle spectra model: spin linewidth = 2*100 keV, kicker linewidth= 2*24 keV= 48 keV

Gaussian spectra model: sigma of spin = 100 keV, sigma of kicker = 24 keV

Note:
Too fast scanning towards the drift leads to incomplete depolarization. On this basis, one can determine the direction of energy drift.



COUNTER-SCANNING UNDER MODERATE ENERGY DRIFT

Moderate rate of energy drift

Drift V_d=1.3 keV/s (up); Dep.1 V_1=+1.43 keV/s, Dep.2 V_2=–1.43 keV/s

GREEN → and GREY →: $V_1 - V_d$ =+ 0.13 keV/s, DE_1(t=0) = –200 keV

BLUE ← and CYAN ← : $V_2 - V_d$ = –2.73 keV/s, DE_2(t=0) = +400 keV

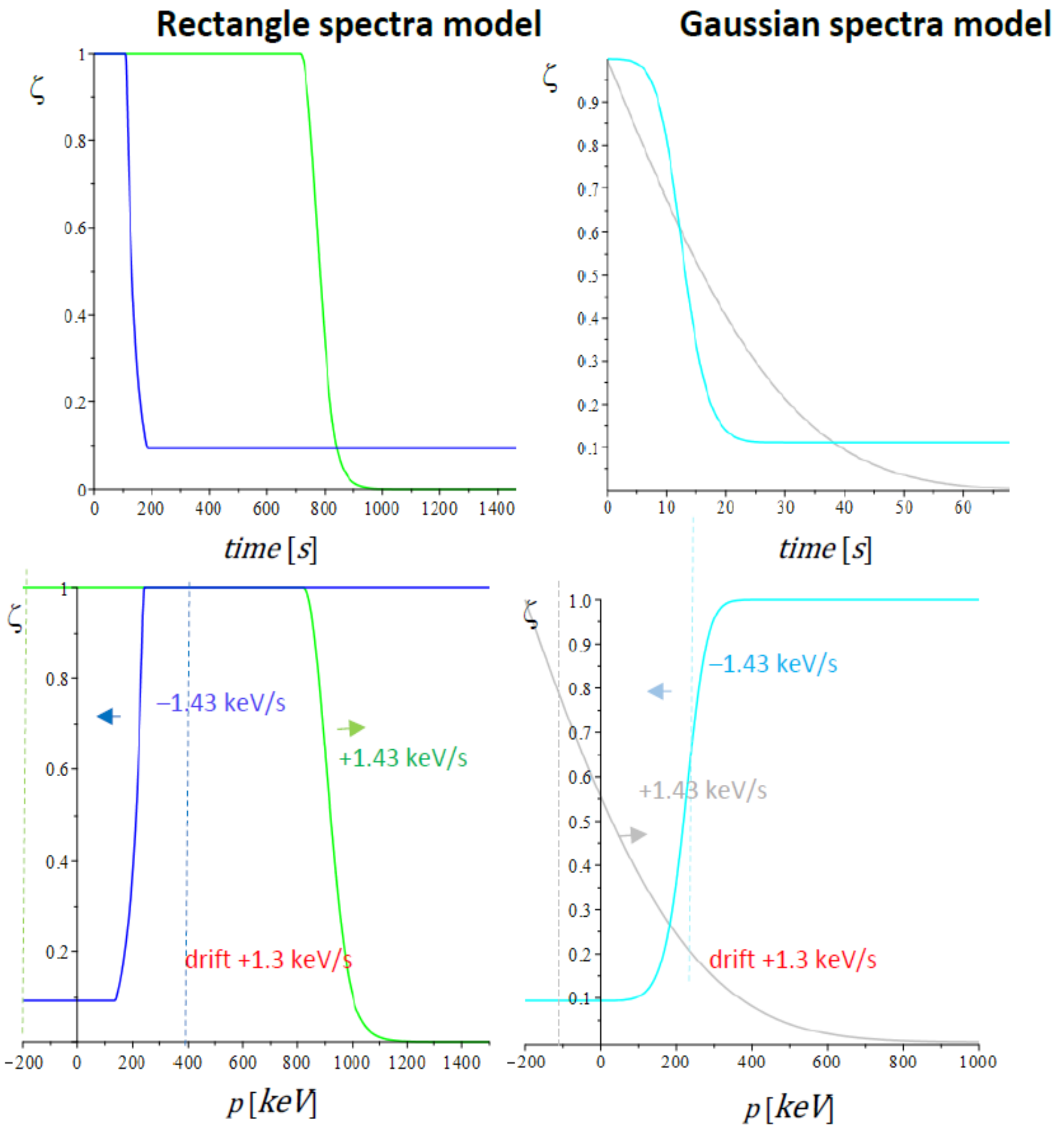
Depolarizer harmonic amplitude: $|w_k| = 2\cdot 10^{-5}$

Rectangle spectra model: spin linewidth = 2*100 keV, kicker linewidth = 2*7.1 keV

Gaussian spectra model : sigma of spin = 100 keV, sigma of kicker =7.1 keV

Note:
In comparison with the example for extreme drift, in this case, there is a notable difference in the diagrams between the two models of the shape of the spectra.

In the general case, counter-scanning is a decrease in the detuning of each of the two depolarizers that started from opposite sides of the resonance, which changes its position. This is also possible when their frequencies are scanned in the same direction, but at different speeds: one depolarizer catches up with the resonance while the resonance itself catches up with the second depolarizer.



Fitting the measured diagrams with curves obtained in the spectrum shape model that gives the smallest error, would give the most accurate information about both the drift and the position of the resonance at certain points in time.

The possibilities of Counter-Scanning for this are explored using Monte Carlo simulations with accounting the possible errors in polarization measurement.

The Monte-Carlo approach and its results are presented below, in the next part of the report.

END OF PART I

Depolarization model conditions

- Time of spin rotation under depolarizer field on big angles (π) is much smaller then synchrotron relaxation time.
- Single resonance crossing rotate spin on small angle ($\pi|w|^2\omega_0/\dot{\epsilon} \ll 1$). Thus according to Froissart-Stora:

$$\Delta\zeta = \zeta\left(2e^{-\frac{\pi|w|^2\omega_0}{2\dot{\epsilon}}} - 2\right) \approx -\frac{\pi|w|^2\omega_0}{\dot{\epsilon}}\zeta, \tag{1}$$

ω_0 is the revolution frequency (rad/s); $\dot{\epsilon}$ is the resonance crossing speed; w — harmonic amplitude.

Gaussian spin and depolarizer lines

$$d\zeta = -\zeta\pi\omega_0 \int |w(\epsilon,\epsilon_d(t))|^2 \frac{f(\epsilon,\dot{\epsilon},\epsilon_s)|\dot{\epsilon}_d-\dot{\epsilon}|dt}{|\dot{\epsilon}_d-\dot{\epsilon}|} d\epsilon d\dot{\epsilon} \qquad (2)$$

$$\frac{d\zeta}{dt} = -\zeta\pi\omega_0 \int |w(\epsilon,\epsilon_d(t))|^2 f(\epsilon,\epsilon_s(t)) d\epsilon \qquad (3)$$

In case of gaussian distribution of the depolarizer and spin spectra density and linear depolarizaer and spin drift:

$$|w(\epsilon,\epsilon_d)|^2 = |w|^2 \frac{e^{-(\epsilon-\epsilon_d)^2/2\sigma_d^2}}{\sqrt{2\pi}\sigma_d}, \qquad f(\epsilon,\epsilon_s) = \frac{e^{-(\epsilon-\epsilon_s)^2/2\sigma_s^2}}{\sqrt{2\pi}\sigma_s}$$

$$\epsilon_d(t) = \epsilon_d(t_0) + (t-t_0)\dot{\epsilon}_d, \qquad \epsilon_s(t) = \epsilon_s(t_0) + (t-t_0)\dot{\epsilon}_s$$

$$\sigma^2 = \sigma_{d0}^2 + |\dot{\epsilon}_d|/\omega_0 + \sigma_{s0}^2 + |\dot{\epsilon}_s|/\omega_0$$

$$\zeta(t) = \zeta(t_0)\exp\left\{ -\frac{\pi\omega_0|w|^2}{2(\dot{\epsilon}_d-\dot{\epsilon}_s)} \operatorname{erf}\left(\frac{\epsilon_d(t_0)-\epsilon_s(t_0)+(\dot{\epsilon}_d-\dot{\epsilon}_s)(t-t_0)}{\sqrt{2}\sigma} \right)\Bigg|_{t_0}^{t} \right\}$$

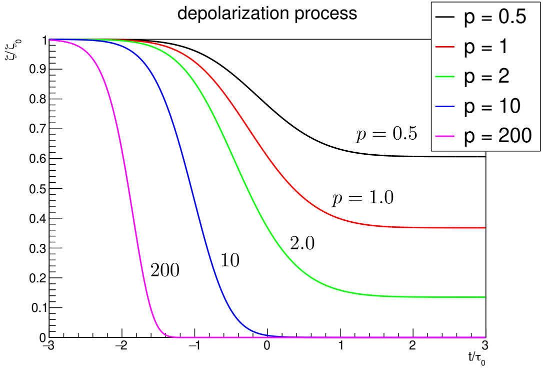
- Depolarizer strength (dimensionless parameter):

$$p = \frac{\pi\omega_0|w|^2}{|\dot{\epsilon}_d - \dot{\epsilon}_s|}$$

- Spin crossing time (seconds)

$$\tau_0 = \frac{\sqrt{2}\sigma}{|\dot{\epsilon}_d - \dot{\epsilon}_s|}$$

$$\zeta(t) = \zeta_0 \exp\left\{-\frac{p}{2}\left[1 + \text{erf}\left(\frac{t}{\tau_0}\right)\right]\right\}$$

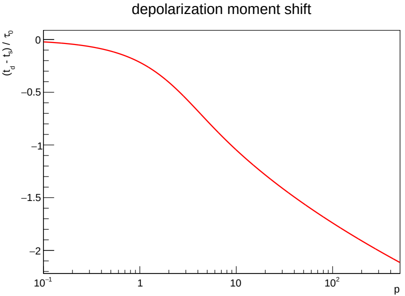


Depolarization moment and depolarization time bias

Half of the total polarization change defines depolarization moment:

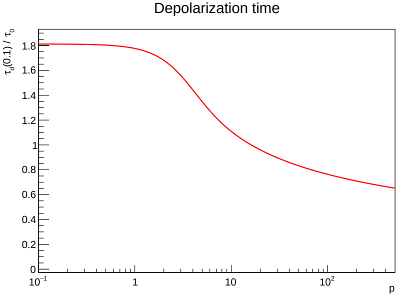
$$t_d = -\tau_0 \, \mathrm{erf}^{-1} \left(1 + \frac{2}{p} \ln \left\{ \frac{1}{2} (1 + e^{-p}) \right\} \right)$$

$\approx \; 0.5\sqrt{2}\sigma \; \approx \; 70$ keV depolarization moment shift!



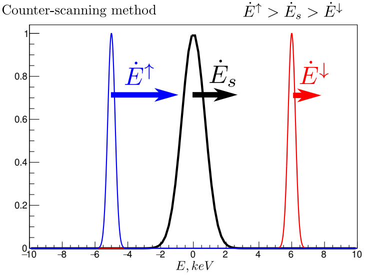
From 90% to 10% of the total polarization change defines depolarization time:

$$\tau_d \approx \frac{2\sigma}{|\dot{\epsilon}_d - \dot{\epsilon}_s|}$$



Counter-scanning method

- Model dependent depolarization moment shift requires depolarization independent depolarization of two bunches with counter scanning.
- In absence of energy drift determination and averaging of the moment of half polarization changes would give true energy value.
- But in case of energy drift one need to apply some model and use joint fit of counter scanning.
- This allow one to determine energy and drift speed at some time point.



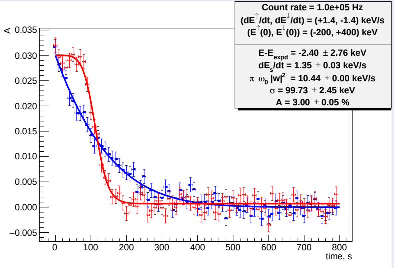
Simulation. Conditions

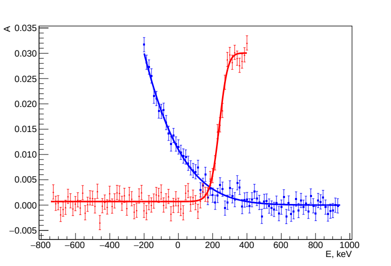
- Spin width $\sigma = 100$ keV
- Number of electrons in pilot bunch: $\sim 10^{10}$
- Laser power 100 W (disk laser).
- Counting rate $\dot{N} \sim 10^5$ Hz.
- Polarization effect (asymmetry) $0.3 \times 10\% = 0.03$
- Total measurement time $t_{max} = 1800$ s
- Count time $T = 10$ s
- Harmonic amplitude $|w| = 7.6 \cdot 10^{-6}$ and depolarizer strength:

$$p = \frac{2\pi^2 |w|^2 \cdot 3000\text{Hz} \cdot 440648\text{keV}}{\dot{E}(= 1\ \text{keV/s})} = 1.5$$

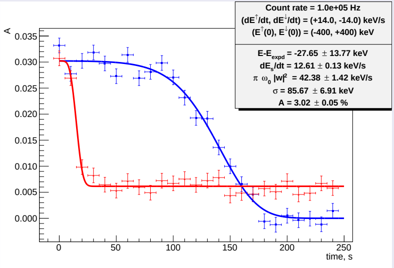
Comparison with Nikitin's calculation (different drift speed and harm. amplitudes)

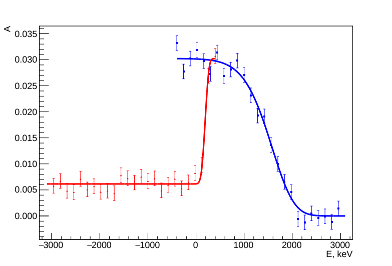
$\dot{E}_s = 1.3$ keV/s, $E_d = \pm 1.43$ keV/s $|w| = 2 \cdot 10^{-5}$





$\dot{E}_s = 12.5$ keV/s, $E_d = \pm 14$ keV/s, $|w| = 4 \cdot 10^{-5}$

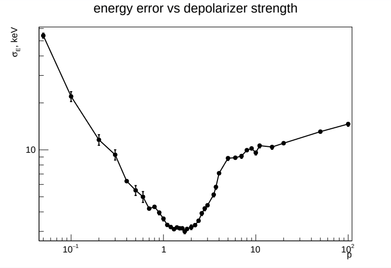




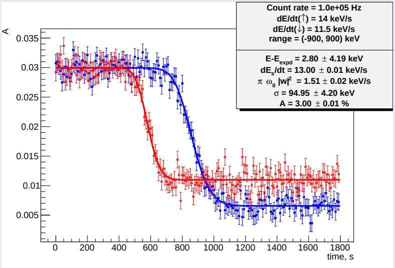
Simulation result: $\sigma_E \approx 3$ keV, $\dot{E}_s = 13 \pm 0.01$

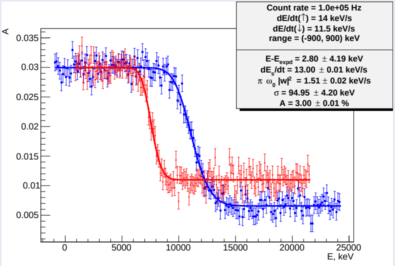
$$\sigma_E \sim \frac{|\dot{E}_d - \dot{E}_s|}{\sqrt{\dot{N}}} \frac{(T^2 + \tau_d^2)^{1/4}}{\zeta_0 (1 - e^{-p})}$$

Optimal depolarizer strength $p = 1.5 \div 2$



$p = 1.5$, $\dot{E}^{\uparrow\downarrow} = 14, 11.5$ keV/s, $\dot{E}_s = 13$ keV/s





Summary

- Accuracy is determined by spin line width σ, counting rate $\dot{N}$ and relative scan speed.
- Scan speed is limited by uncertainty of the energy and total measurement time t_{max}
- Optimal depolarizer strength $p = 1.5 \div 2$ ($|w| = 7.6 \times 10^{-6}$)
- In order to take into account earlier depolarization and energy drift during slow scanning we need to apply counter-scanning method with joint fit by gaussian model.
- Still need to understand systematics for different models of spin and depolarizer lines shape.
- At best case statistical accuracy is about 3 keV for 100 kHz counting rate, 100 keV spin width and 1800 seconds of total scan time.