IPHC Institut Pluridisciplinaire Hubert CURIEN STRASBOURG

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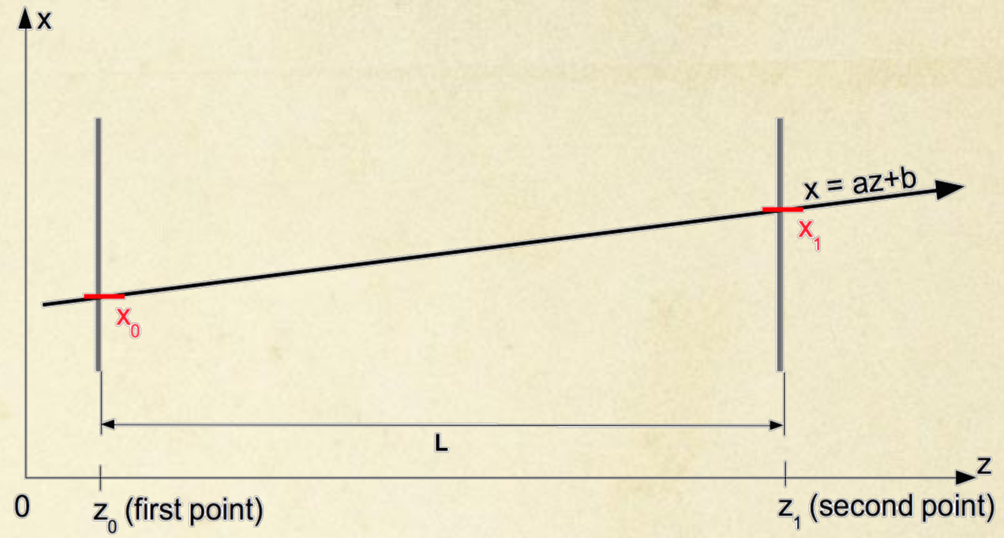
eurizon
European network
for developing new horizons for RIs

Wuppertal, July 023

Tracking 1/3

Tracking version 0.0

- **Hypothesis:**
  - Two sensors
    - <u>perfect</u> positions
    - Infinitely thin
  - 1 straight tracks
    - 2 parameters (a,b)



- **Estimation of track parameters**
  - Assuming track model is straight

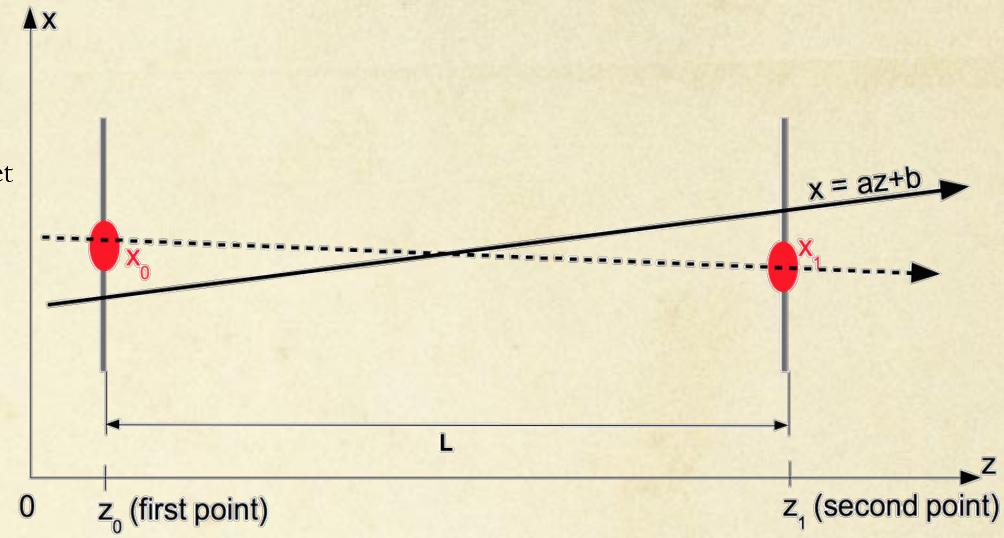
$$a = \frac{x_1 - x_0}{z_1 - z_0}, \mathrm{b} = \frac{x_0 z_1 - x_1 z_0}{z_1 - z_0}$$

  - No uncertainty !

Tracking version 1.0

Hypothesis:

- Two sensors
  - Positions with UNCERTAINTY σ_{det}
  - Infinitely thin
- 1 straight tracks
  - 2 parameters (a,b)



Estimation of track parameters

- Assuming track model is straight

$$a = \frac{x_1 - x_0}{z_1 - z_0}, \ \mathrm{b} = \frac{x_0 z_1 - x_1 z_0}{z_1 - z_0}$$

- Uncertainties from error propagation

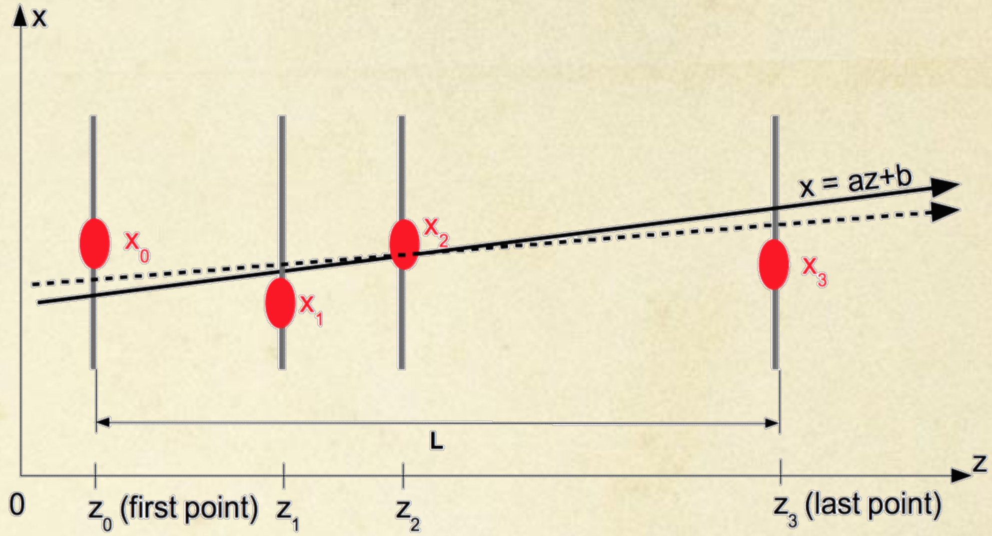
$$\sigma_a = \frac{\sqrt{2}}{z_1 - z_0}\sigma_{\mathrm{det}}, \ \sigma_b = \frac{\sqrt{z_1^2 + z_0^2}}{z_1 - z_0}\sigma_{\mathrm{det}}$$

$$\mathrm{cov}_{a,b} = -\frac{\sqrt{z_1 + z_0}}{z_1 - z_0}\sigma_{\mathrm{det}}$$

Tracking version 1.1

Hypothesis:

- More than two sensors
  - Positions with uncertainty σ_{det}
  - Infinitely thin
- 1 straight tracks
  - 2 parameters (a,b)



Estimation of track parameters

- Assuming track model is straight
  - Need FITTING PROCEDURE least square
  - Need covariance matrix of measurements (here diagonal)
- Uncertainties from error propagation
  - Detail depends on geometry

=>Both estimation & uncertainties improve

$$a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2}, \quad b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$$

$$\sigma_a^2 = \frac{S_1}{S_1 S_{z^2} - (S_z)^2}, \quad \sigma_b^2 = \frac{S_{z^2}}{S_1 S_{z^2} - (S_z)^2}$$

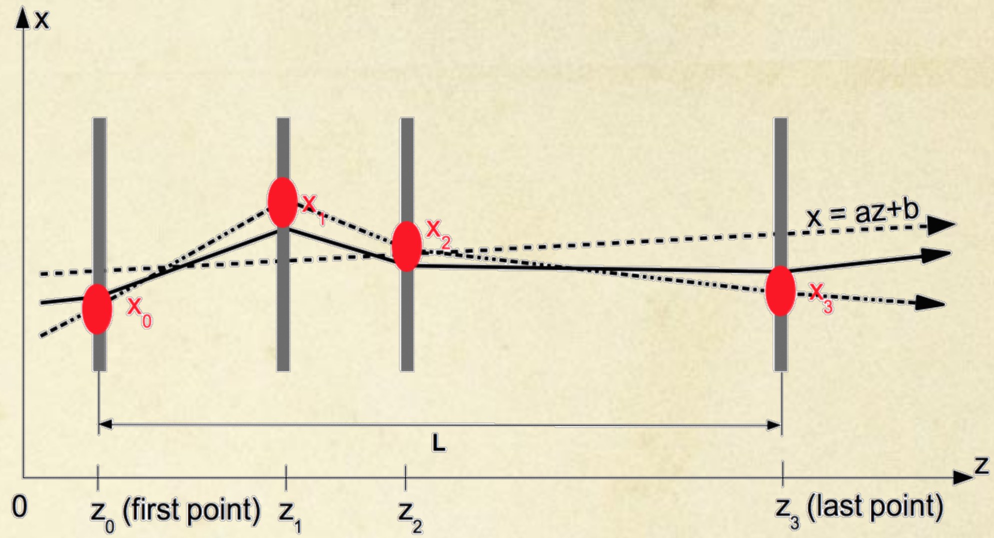
$$\mathrm{cov}_{a,b} = \frac{-S_z}{S_1 S_{z^2} - (S_z)^2}$$

$$\boxed{S_{f(x,z)} = \sum_{i=first\ point}^{last\ point} \frac{f(x_i, z_i)}{\sigma_{det}^2}}$$

Tracking version 2.0

Hypothesis:

- More than two sensors
  - Positions with uncertainty σ_{det}
  - With some THICKNESS
    → physics effects
- 1 straight tracks
  - 2 parameters (a,b)



Estimation of track parameters

- Assuming track model is straight
  - Need fitting procedure least square
  - Need covariance matrix of measurements
    physics effect → NON DIAGONAL terms
- Uncertainties from error propagation

=> Same estimators but uncertainties enlarged

$$a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2}, \quad b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$$

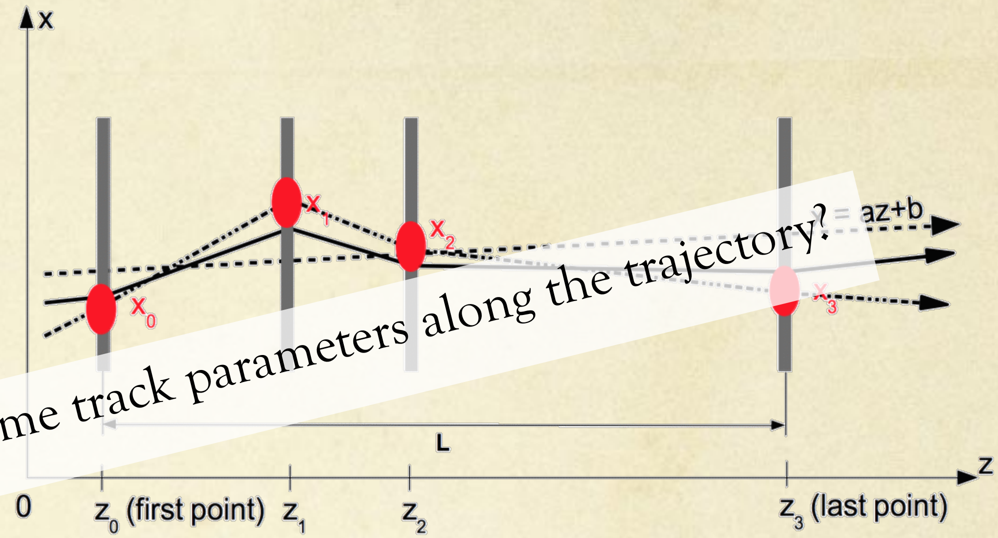
Complex covariant matrix expression

- correlation between sensors
- Depends clearly on geometry
- Various implementations possible

Tracking version 2.0

Hypothesis:

- More than two sensors
  - Positions with uncertainty σ_{det}
  - With some THICKNESS
    → physics effects
- 1 straight tracks
  - 2 parameters (a,b)



Hey, is it fine to use the same track parameters along the trajectory?

Estimation of track parameters

- Assuming track model is straight
  - Need fitting procedure least square
  - Need covariance matrix of measurements
    physics effect → NON DIAGONAL terms
- Uncertainties from error propagation

=> Same estimators but uncertainties enlarged

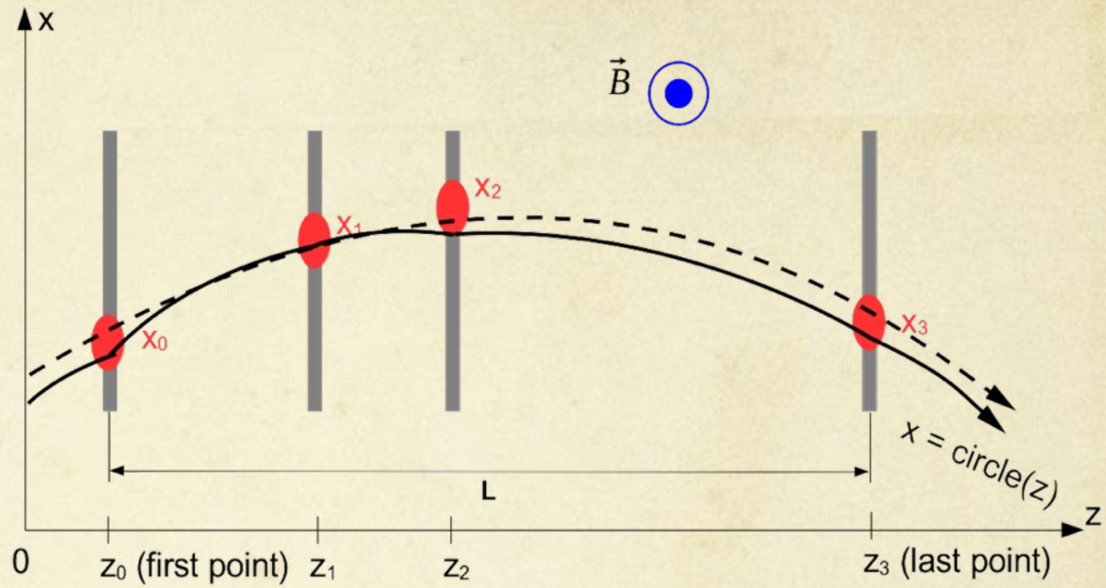
$$a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2}, \quad b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$$

Complex covariant matrix expression

- correlation between sensors
- Depends clearly on geometry
- Various implementations possible

Tracking version 2.1

- **Hypothesis:**
  - More than two sensors
    - Positions with uncertainty σ_{det}
    - With some THICKNESS
      → physics effect
  - No more straight track
    - Magnetic field → helix
    - 5 parameters → one is $\vec{p}_T$

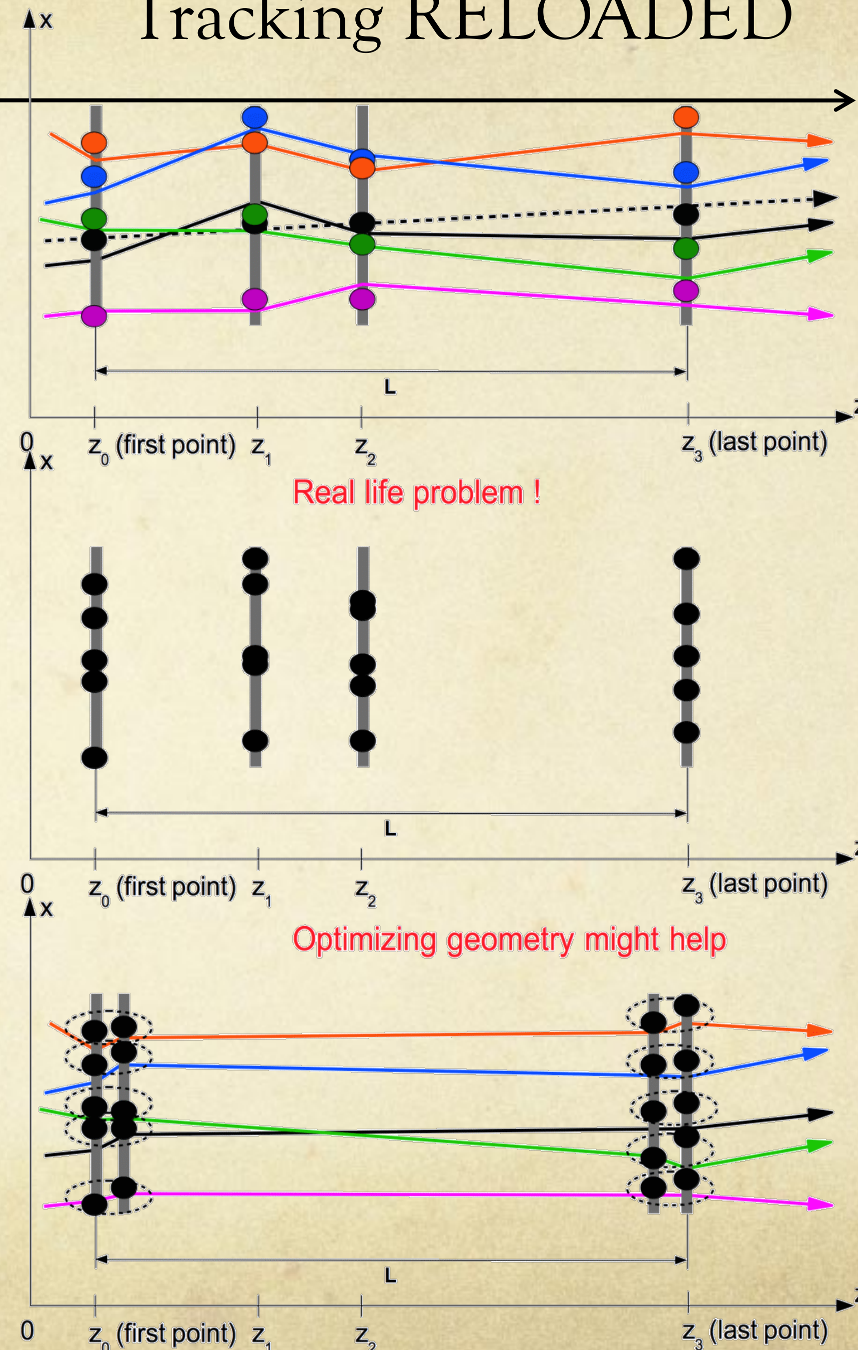


- **Estimation of track parameters**
  - As before
    - Non diagonal covariance matrix
    - Enlarged uncertainties from physics effect
  - BUT fitting more complex
    - Higher dimensions from 5 params
    - Non-linearities from model

Influence of geometry from
- Overall layout
- Interplay with physics layers
- Shape of sensing layers wrt particle incidence

Tracking RELOADED

- **Hypothesis:**
  - More than two sensors
    - Positions with uncertainty σ_{det}
    - With some thickness
  - MANY straight tracks
    - Still 2 parameters (a,b)…per track!
    - But may change along track path
- **New step = FINDING**
  - Which hits match which tracks ?
  - Strongly depends on geometry
- **Estimation of track parameters**
  - Happens after finder
  - Uncertainties involve correlation



Tracking RELOADED

Tracking needs position sensitive detectors, but …

- **Hypothesis:**
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    - Positions with uncertainty σ_{det}
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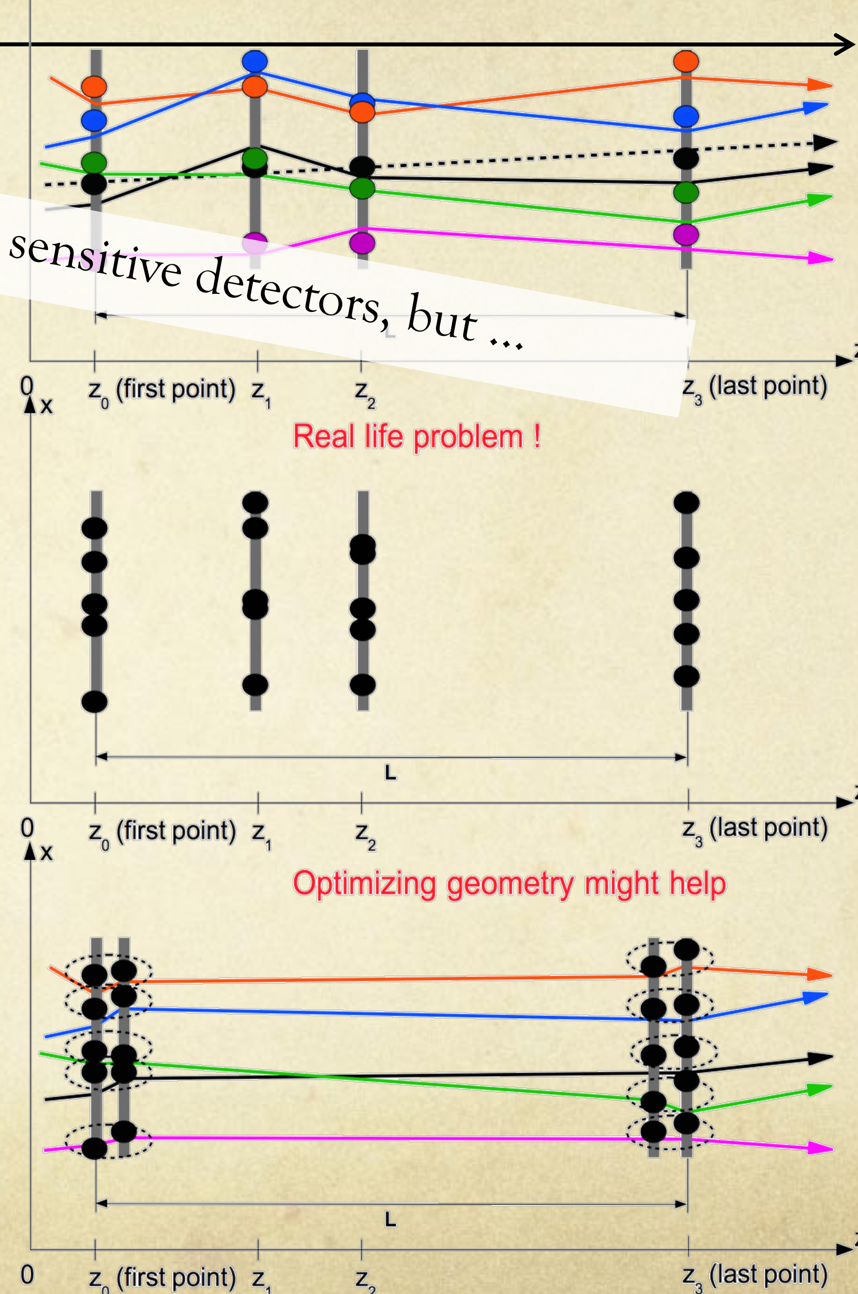


What are we talking about?

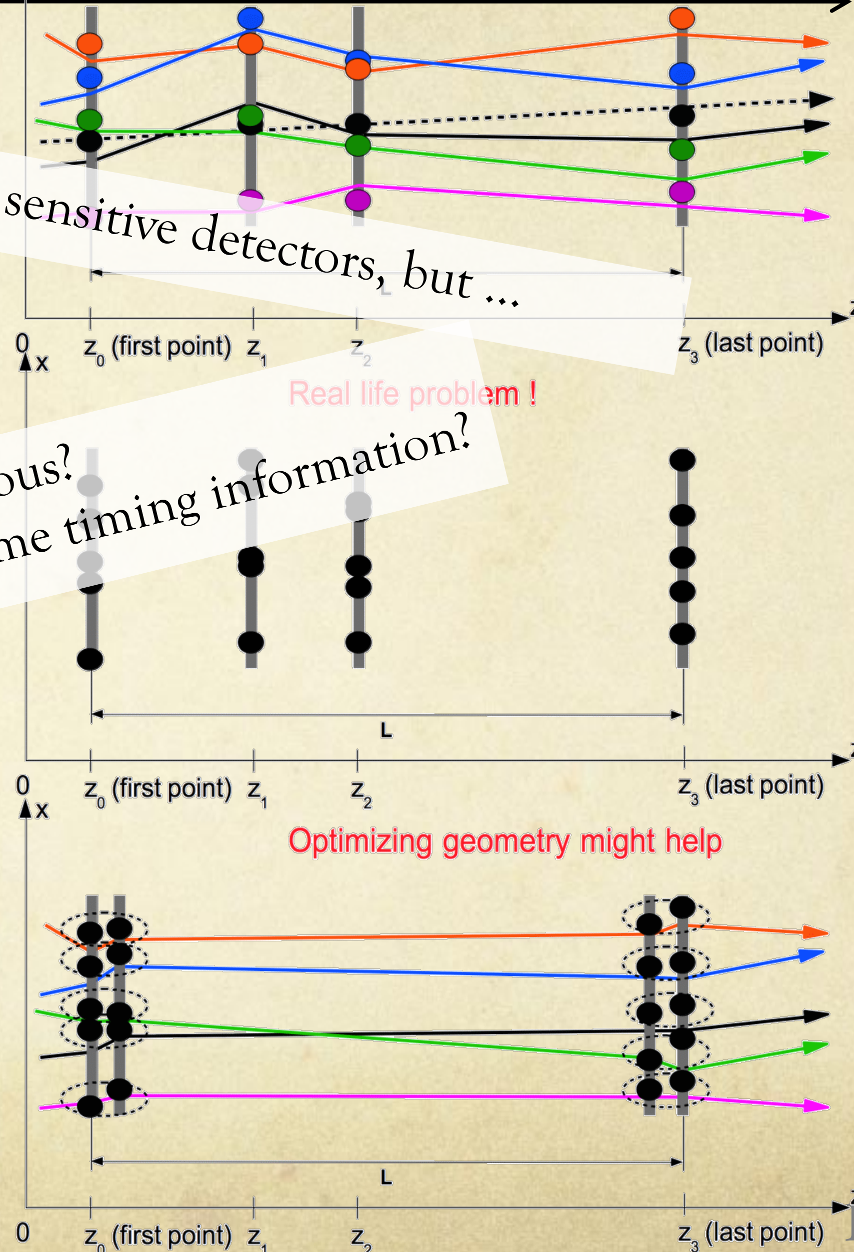
Tracking RELOADED

- **Hypothesis:**
  - More than two sensors
    - Positions with uncertainty σ_{det}
    - With some thickness
  - MANY straight tracks
    - Still 2 parameters (a,b)…per track!
    - But may change along track path
- **New step = FINDING**
  - Which hits to which tracks ?
  - Strongly depends on geometry
- **Estimation of track parameters**
  - Happens after finder
  - Uncertainties involve correlation

Tracking needs position sensitive detectors, but …

hey, are all these tracks simultaneous?

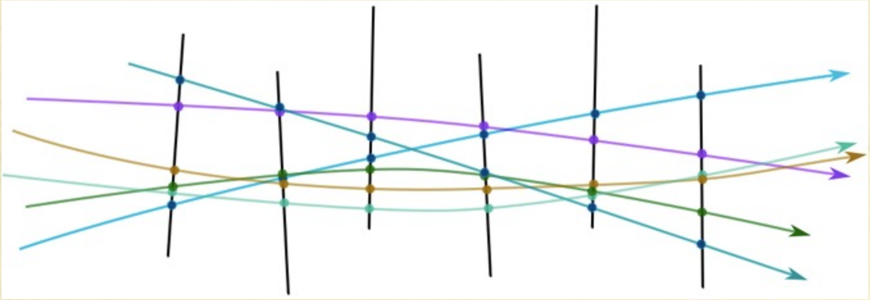
Could finding be helped with some timing information?



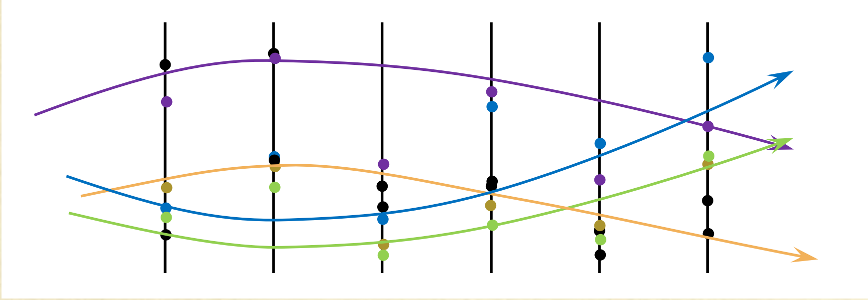
Additional real-life troubles

Alignment:

- What was mechanically constructed



- What you think you built!



Alignment procedure needed

Radiation environment

- Intensity & energy frontiers
  ➡ large radiation exposure
- Total ionizing dose
  - Possible effects starting ~ kGy
  - Worst conditions ~ GGy
- Non ionizing energy loss fluence
  - Possible effects starting ~ 10^{12} $n_{eq(1MeV)}/cm^2$
  - Worst conditions ~ 10^{18} $n_{eq(1MeV)}/cm^2$

Hardening/Monitoring needed

Temperature

- Electronics heat-up
  => performance degrades with temperature
- Radiation tolerance depends on temperature

Cooling => additional material

Lecture outline

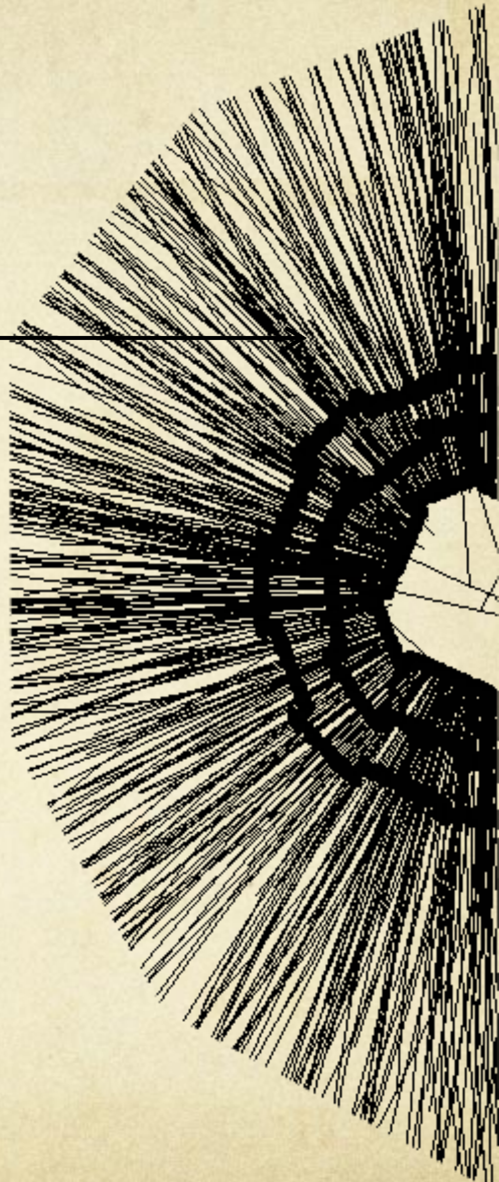
1. Basic concepts
2. Finding algorithms
3. Fitting algorithms
4. Existing tracking systems

References

first lecture

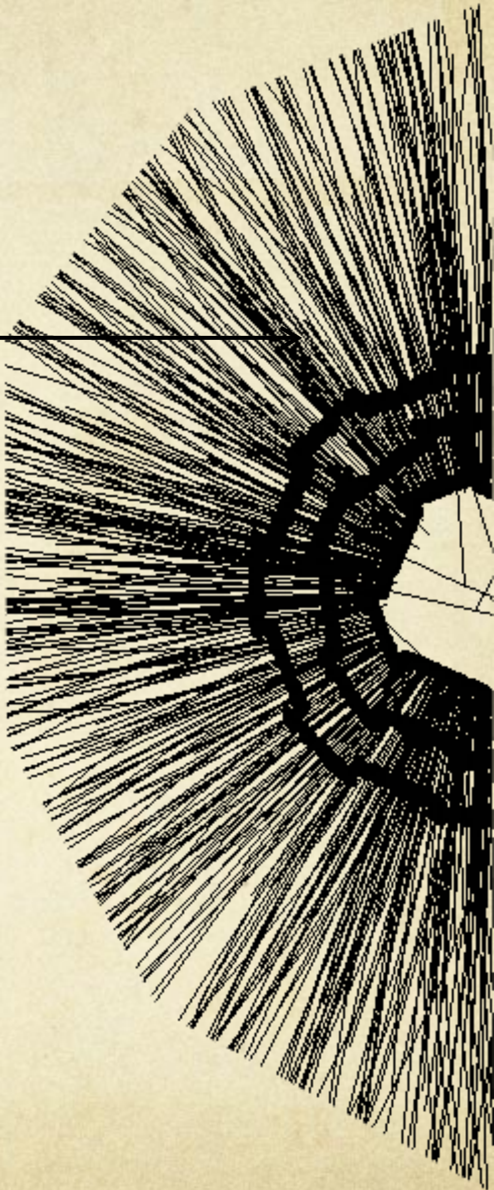
second lecture

third lecture



1. Motivations & basic concepts

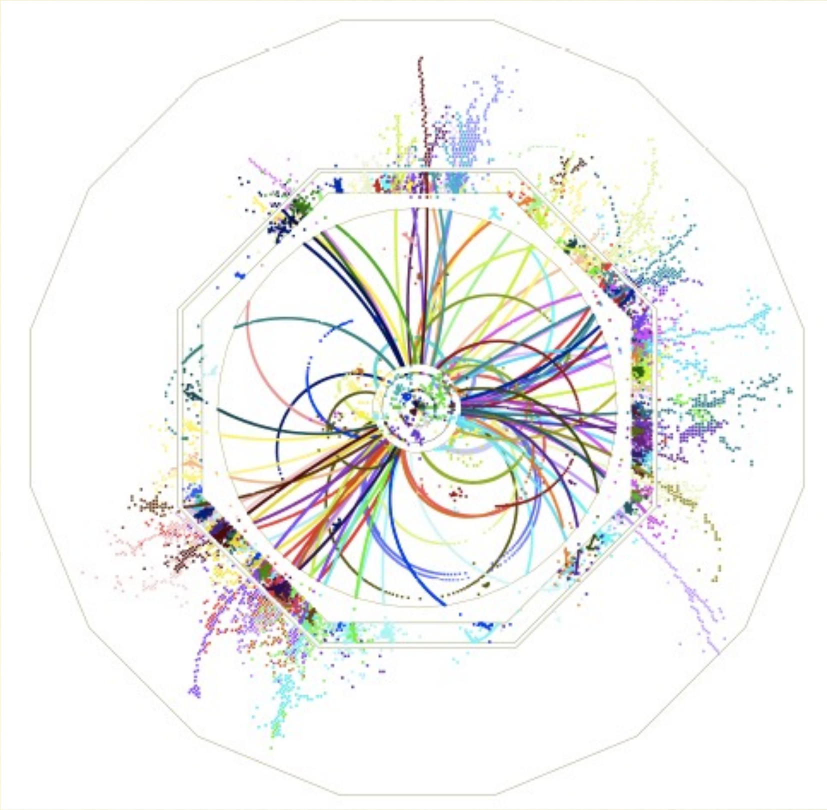
- Motivations
- Types of measurements
- The 2 main tasks
- Environmental considerations
- Figures of merit



Motivations

Understanding an event

- Individualize tracks ≃ particles
- Measure their properties
- LHC: ~1000 particles per 25 ns "event"



8 jets event (tt-bar h) @ 1 TeV ILC

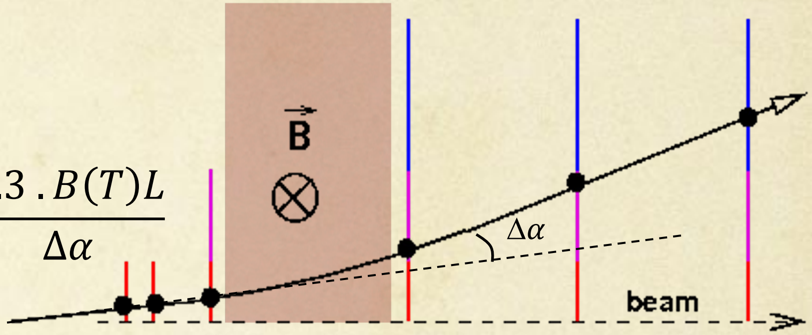
Particle/Track properties

- Momentum/charge ⇔ curvature in B field
  - Reconstruct invariant masses
  - Contribute to jet energy estimation
    ➥ particle flow algorithm (pfa)
- Origin ⇔ vertexing (connecting track)
  - Identify decays
  - Measure flight distance
- Mass ⇔ dE/dx measurement
- Time/collision ⇔ Identification through time of flight
- Energy ⇔ range measurement
  - Limited to low penetrating particle (typically ions at few 100 MeV/u)

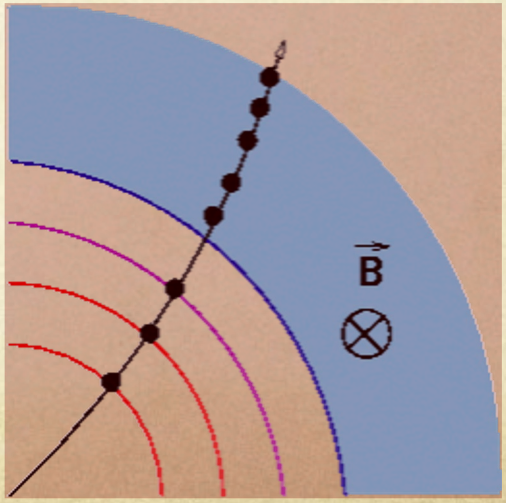
Momentum measurement

- Magnetic field curves trajectories $\frac{d\vec{p}}{dt} = q\vec{v} \times \vec{B}$
  - (r-φ) plane: circle $p_T = 0.3zBR$
  - (r-z) plane: straight line $p_z = p\cos\lambda$

  $\vec{p} = \vec{p_T} + \vec{p_z}$
  - In B=4T a 10 GeV/c particle will get a saggita of 1.5 cm @ 1m
- Fixed-target experiments
  - Dipole magnet on a restricted path segment
  - Measurement of deflection (angle variation) $\frac{p_T}{q} = \frac{0.3\,.\,B(T)L}{\Delta\alpha}$



- Collider experiment
  - Barrel-type with axial B over the whole path
  - Measurement of curvature (sagitta) $\frac{p_T(\mathrm{GeV/c})}{q} = 0.3 \cdot B(\mathrm{T}) \cdot R(\mathrm{m})$

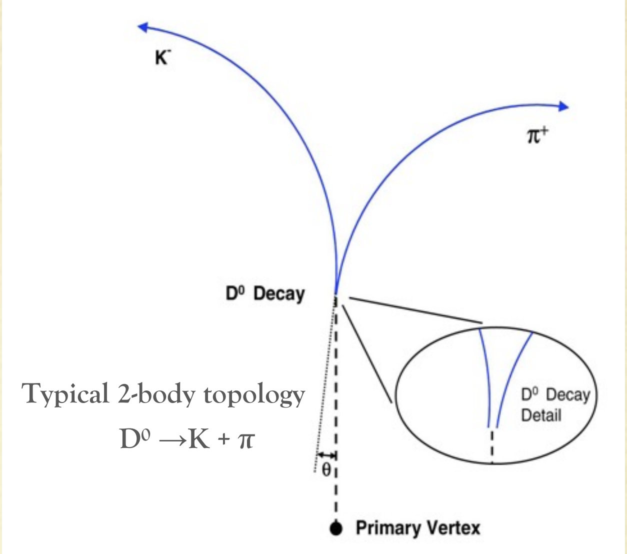


- Other arrangements
  - Toroidal B… not covered
- Two consequences
  - Position sensitive detectors needed
  - Perturbation effects on trajectories

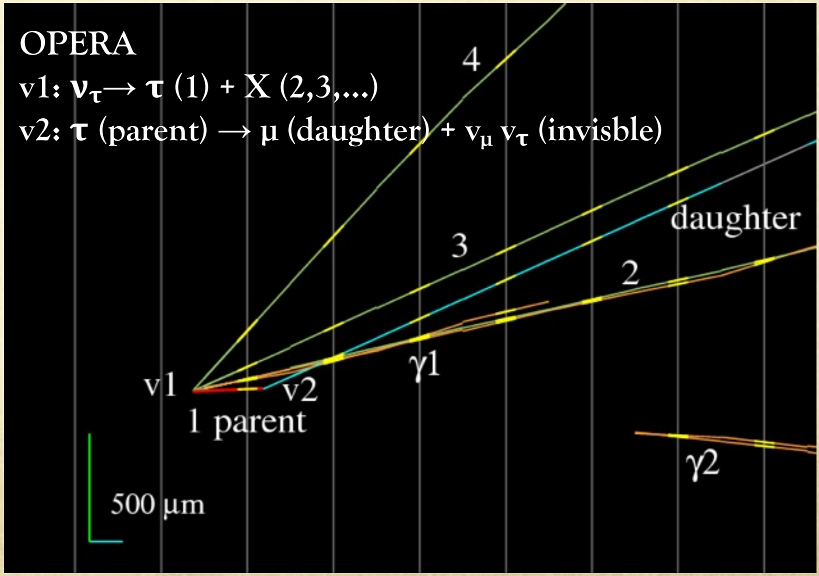
    ➡ limit precision on track parameters

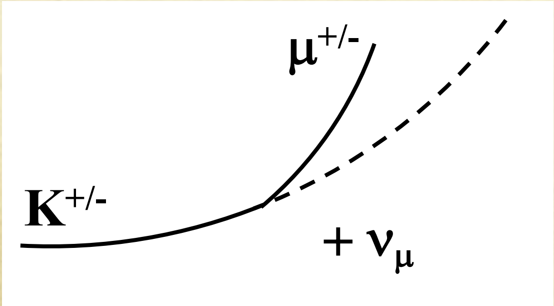
Vertex measurements 1/3

- Identifying through topology
  - Short-lived weakly decaying particles
    - Charm $c\tau \sim 120\ \mu m$
    - Beauty $c\tau \sim 470\ \mu m$
    - τ, strange (K_S,Λ)/charmed (D)/beauty (B) particles
- Exclusive reconstruction
  - Decay topology with secondary vertex
  - Exclusive = all particles in decay associated
- Inclusive "kink" reconstruction
  - Some particles are invisible (ν)



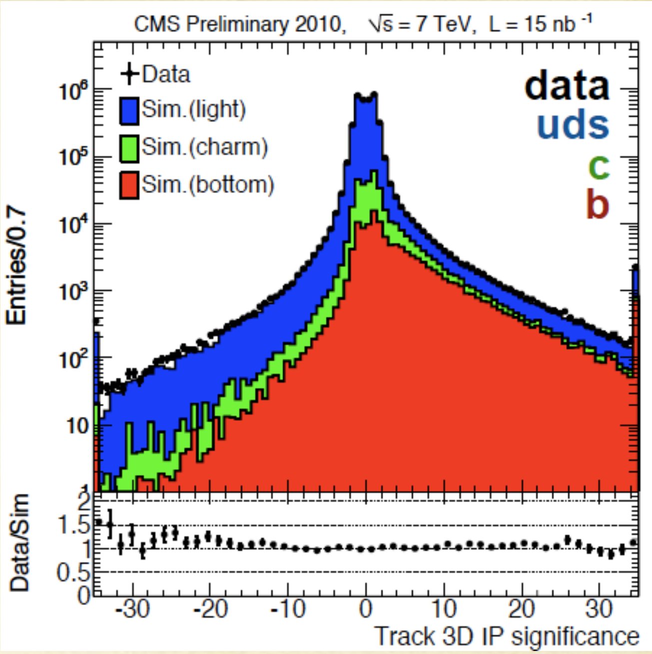
Typical 2-body topology
$D^0 \rightarrow K + \pi$





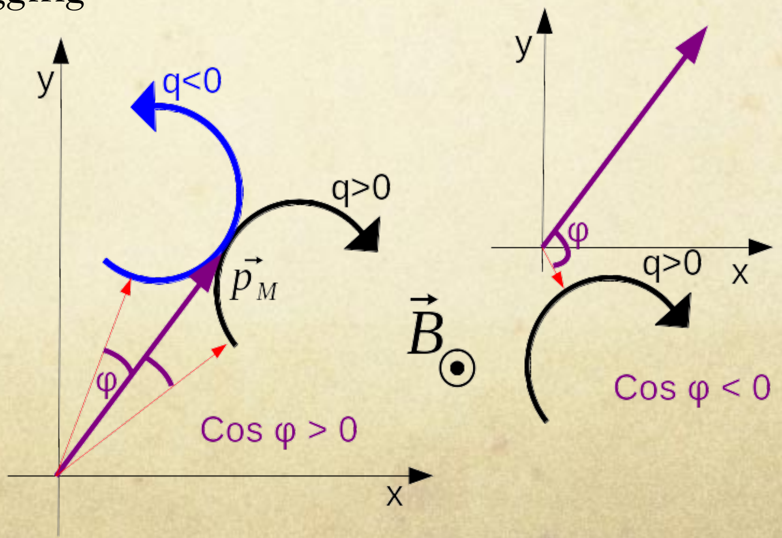
Vertex measurements 2/3

- Inclusive reconstruction
  - Selecting parts of the daughter particles = flavor tagging for high energy colliders
  - based on impact parameter (IP)
  - σ_{IP} ~ 20-100 μm and $SIGN_{IP}$ requested



- Definition of impact parameter (IP)
  - Also DCA = distance of closest approach from the trajectory to the primary vertex
  - Full 3D or 2D (transverse plane: d_ρ) +1D (beam axis: d_z)
  - Sign extremely useful for flavor-tagging

Sign defined by angle dca / jet momentum



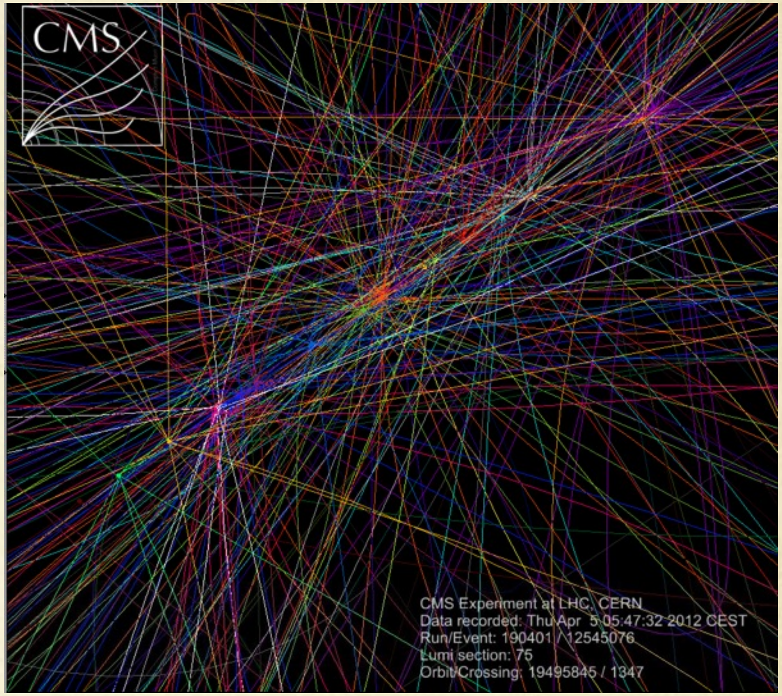
Vertex measurements 3/3

Finding the event origin

- Where did the collision did occur?

  = Primary vertex

- (life)Time dependent measurements
  - CP-asymmetries @ B factories ($\Delta z \simeq$60-120 μm)
- Case of multiple collisions / event
  - >> 10 (100) vertex @ LHC (HL-LHC)



Remarks for collider

- Usually no measurement below 1-2 cm / primary vertex
  - Due to beam-pipe maintaining vacuum
  - Vacuum operation practically easier for fixed target (eg MVD of CBM)
    BUT realized for LHCb (VErtex LOcator) at LHC
- Requires extrapolation → expect "unreducible" uncertainties

Multiple scattering - 1/5

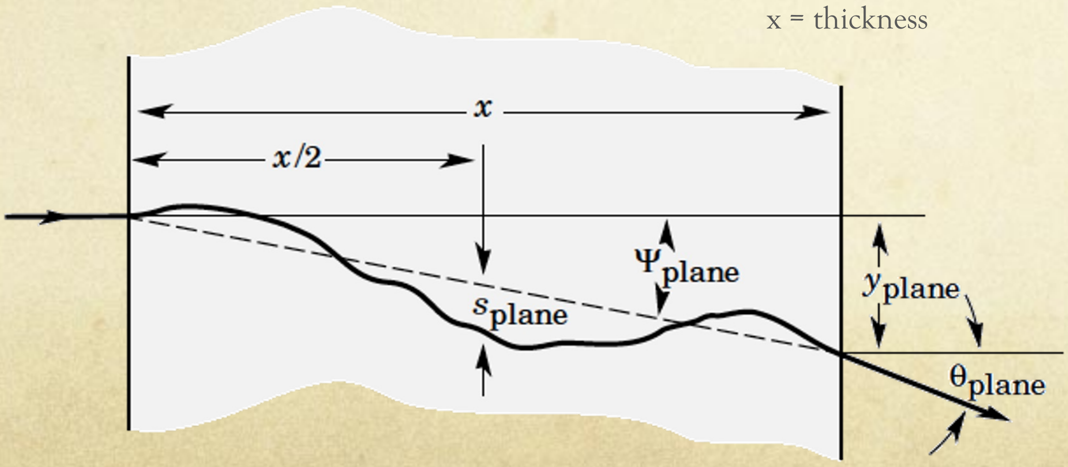
- Reminder on the physics (see other courses)
  - Coulomb scattering mostly on nuclei
  - Molière theory description as a **centered** gaussian process
    - the thinner the material, the less true ➛ large tails

- In-plane description (defined by vectors $\mathbf{p}_{in}$, $\mathbf{p}_{out}$)
  - Corresponds to (φ, $\theta=\theta_{plane}$) with $\mathbf{p}_{in} = \mathbf{p}_z$ and $p_{out}^2 = p_{out,z}^2 + p_{out,T}^2$ $\begin{cases} p_{out}\cos\theta \approx p_{out,z} \\ p_{out,T} = p_{out}\sin\theta \approx p_{out}\theta \end{cases}$

Highland formula: $\sigma_\theta = \frac{13.6\ (\mathrm{MeV/c})}{\beta p}\cdot z\cdot\sqrt{\frac{\text{thickness}}{X_0}}\cdot\left[1+0.038\ \ln\left(\frac{\text{thickness}}{X_0}\right)\right]$ $\left(\text{note}: \ \phi\in[0,2\pi]\ \text{uniform}\right)$

β, p, z = particle boost, momentum, charge

x = thickness



Xo = radiation length

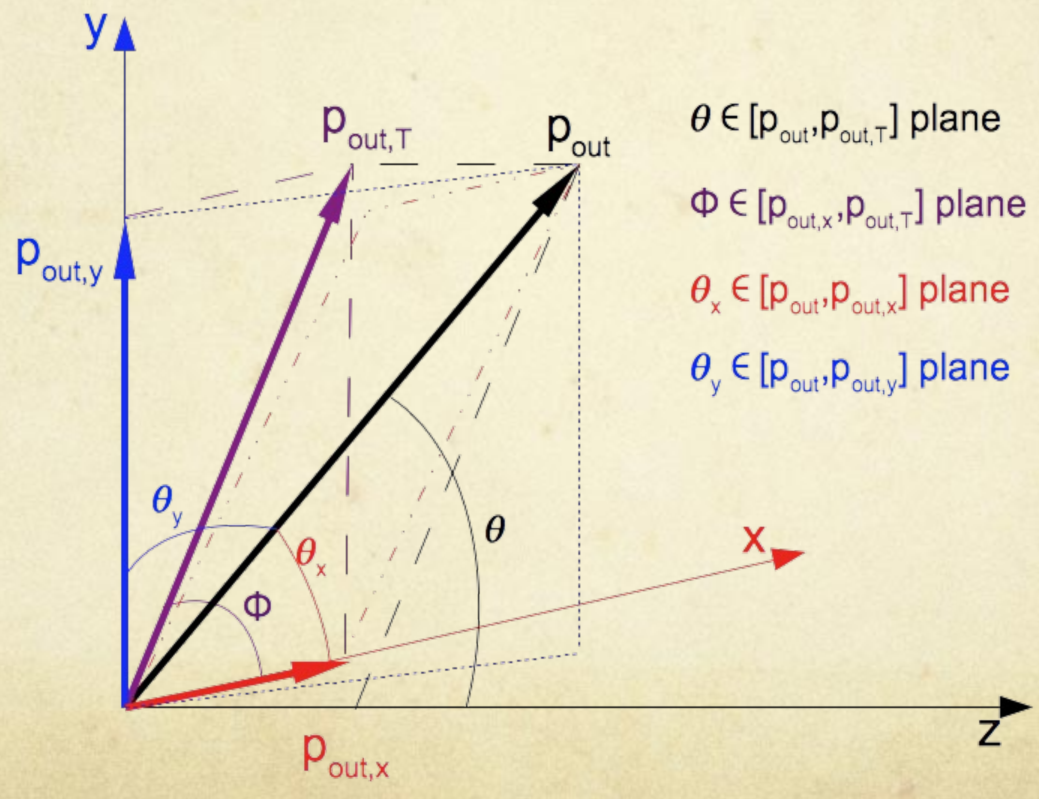
Same definition as in calorimetry

... though this is accidental

Multiple scattering – 2/5

- <u>In-space</u> description (defined by fixed x/y axes)
  - Corresponds to ($\boldsymbol{\theta}_x$,$\boldsymbol{\theta}_y$) with $p^2_{out,T} = p^2_{out,x} + p^2_{out,y}$ $\begin{cases} p_{out}\sin\theta_x \approx p_{out}\theta_x \\ p_{out}\sin\theta_y \approx p_{out}\theta_y \end{cases}$ => $\theta^2_{\text{plane}} = \theta^2_{\text{x}} + \theta^2_{\text{y}}$
  - $\boldsymbol{\theta}$x and $\boldsymbol{\theta}$y are independent gaussian processes

$$\sigma_{\theta_x} = \sigma_{\theta_y} = \frac{\sigma_{\theta_{plane}}}{\sqrt{2}}$$



$\theta \in [p_{out},p_{out,T}]$ plane

$\Phi \in [p_{out,x},p_{out,T}]$ plane

$\theta_x \in [p_{out},p_{out,x}]$ plane

$\theta_y \in [p_{out},p_{out,y}]$ plane

Multiple scattering – 3/5

Important remark when combining materials

- Total thickness $T = \Sigma T_i$, each material (i) with $X_0(i)$
- Definition of effective radiation length => $X_{0,eff} = \frac{\sum_i T_i X_0(i)}{T}$
- Consider single gaussian process $\sigma_{eff} = \sqrt{\frac{T}{X_{o,eff}}}$

  and never do variance addition (which minimize deviation) $\cancel{\sigma_{eff} = \sqrt{\sum_i \frac{T_i}{X_o(i)}}}$

Multiple scattering – 4/5

- Some common materials in trackers

material	X_0	std deviation for protons at 1 GeV/c			B=2T, p=1 GeV/c	
		thickness	Budget	σ_{hit} @ 10 cm	sagitta at 10 cm	Relative difference
Silicon	9.4 cm	100 µm	0.1 %	50 µm	760 µm	6 %
Epoxy	30 cm	1 mm	0.3 %	90 µm		12 %
NaI	2.6 cm	1 mm	3.8%	350 µm		46 %
Gas (air-CH4)	300-700 m	10 cm	$<10^{-3}$	-		

Actually very large!
/ goal $\frac{\sigma_p}{p} \approx 10^{-3}$

<u>Particle type matters!</u>

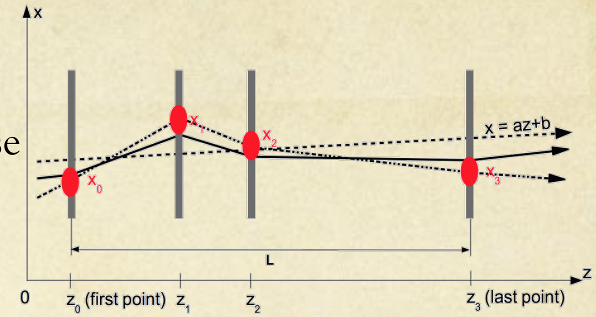
$\boldsymbol{\sigma_\theta \propto \frac{1}{\beta}}$ with depending on mass

Multiple scattering – 5/5

Impact on tracking algorithm

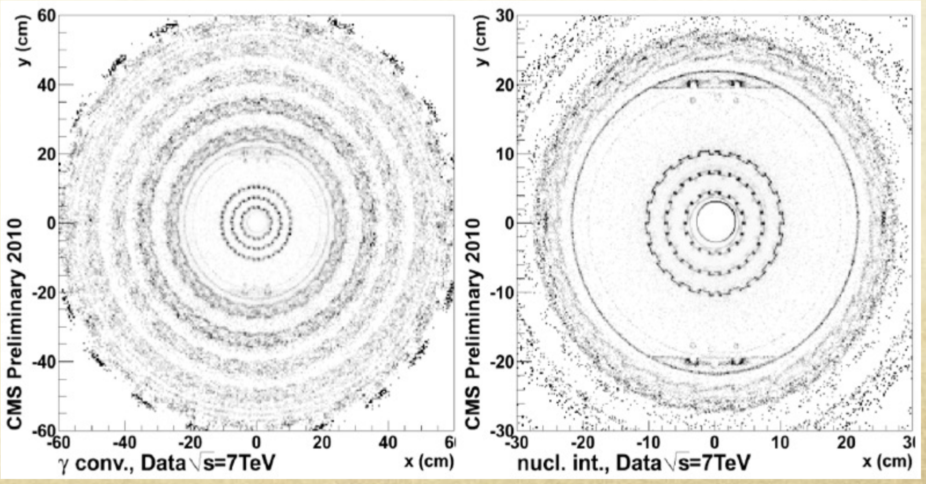
- The track parameters evolves along the track !
- May drive choice of reconstruction method
- Effect amplitude depends on particle mass hypothesis

Remember this simple case



Photon conversion

- Alternative definition of radiation length
  probability for a high-energy photon to generate a pair over a path dx: $\text{Prob} = \dfrac{dx}{\frac{9}{7}X_0}$
- $\gamma \rightarrow e^+e^-$ = conversion vertex
- Generate troubles :
  - Additional unwanted tracks
  - Decrease statistics for electromagnetic calorimeter

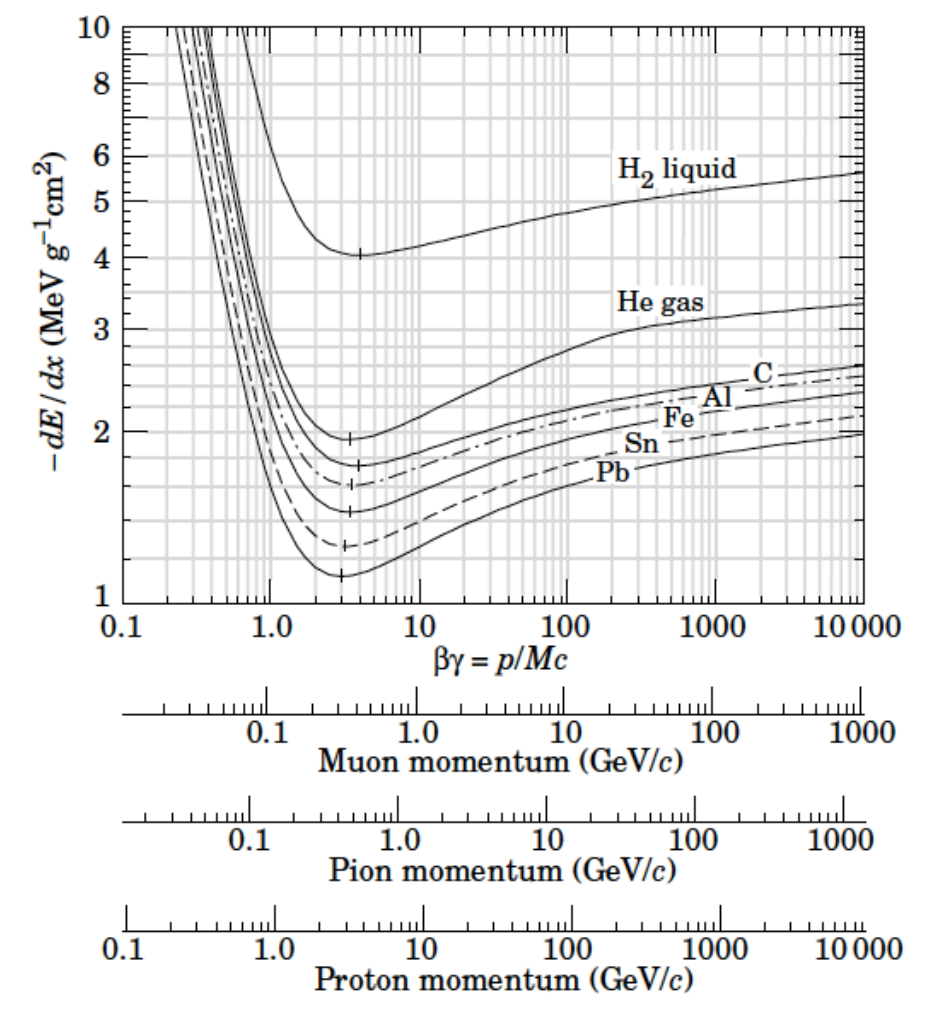


CMS “picture” of material budget through photon conversion vertices (silicon tracker only)

Energy Loss

2nd reminder on physics

- Detector measures something because there is energy loss: *dE/dx*
- Typically
  - ΔE ~100 keV over 300 μm Si
  - ΔE ~2.5 keV over 1 cm Ar



Non-negligeable impact on p

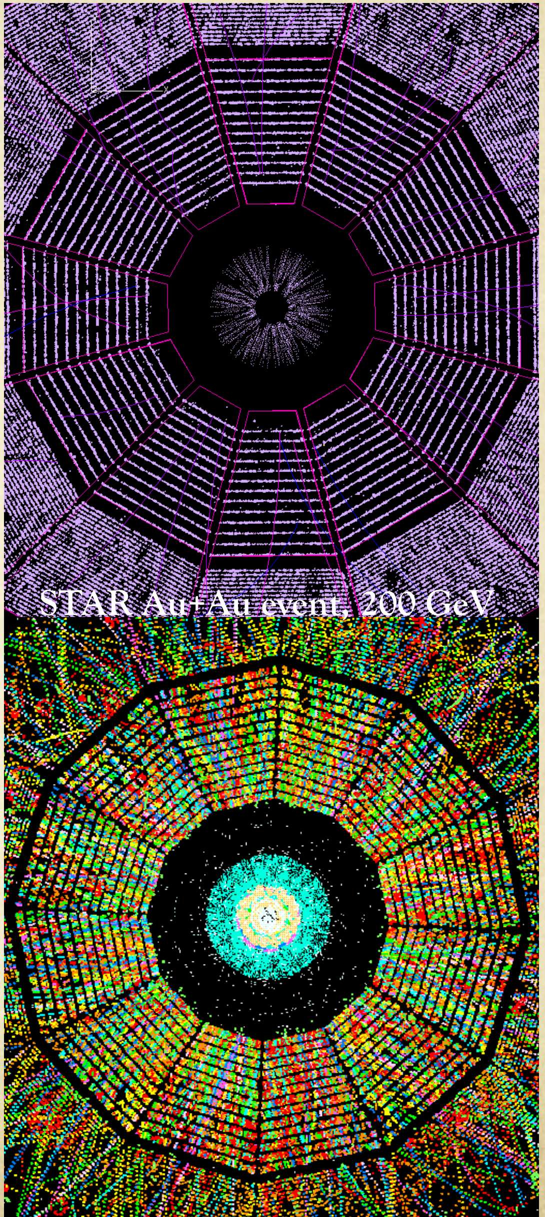
- Assuming MIP around p ≈ 1 GeV/c
- Assume 10 Si layers of 300 μm or 40cm of Ar
- $\boldsymbol{\delta}$p ~1 MeV => $\frac{\delta_p}{p} \approx 10^{-3}$
- Reminder typical goal $\frac{\sigma_p}{p} \approx 10^{-3}$

The two main tasks - 1/2

The collider paradigm

- Basic inputs from detectors
  - Succession of 2D or 3D points (or track segments)

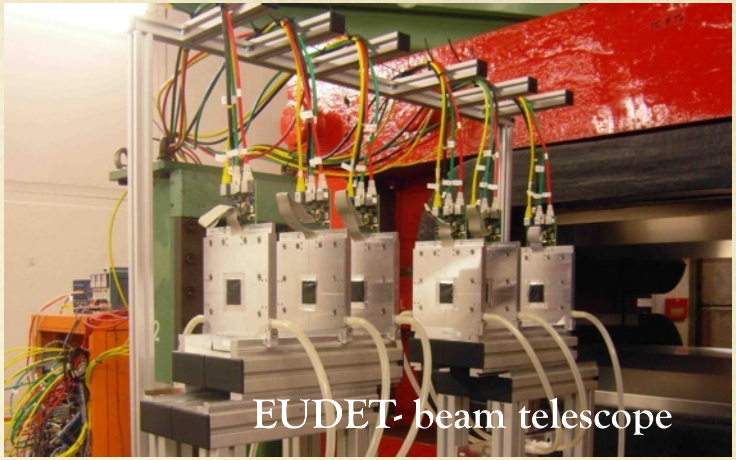
    => Who's who ?
- 2 steps process
  - Step 1: track identification = **finding** = pattern recognition
    - Associating a set of points to a track
  - Step 2: track **fitting**
    - Estimating trajectory parameters ➛ momentum
- Both steps require
  - Track model (signal, background)
  - Knowledge of measurement uncertainties (position resolution)
  - Knowledge of materials traversed (Eloss, mult. scattering)
- Vertexing needs same 2 steps
  - Identifying tracks belonging to same vertex
  - Estimating vertex properties (position + 4-vector)



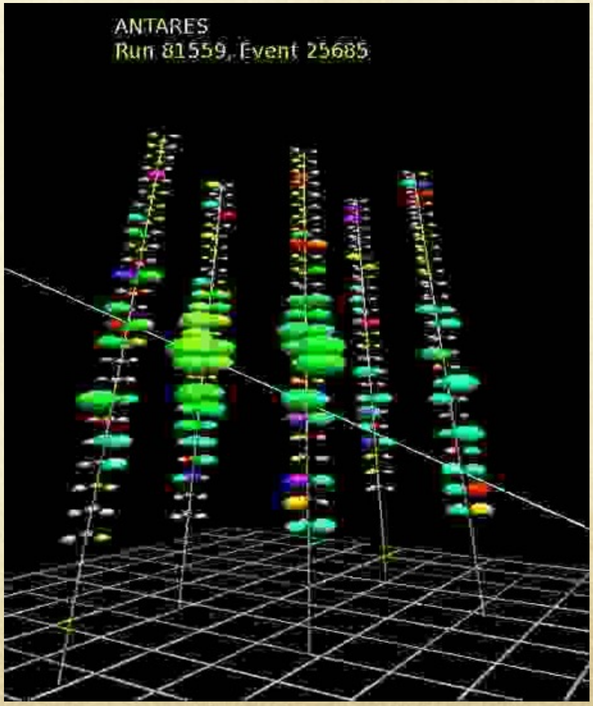
The two main tasks - 2/2

The Telescope mode

- Beam test
  - Single (or a few) particle at a time
    - Sole nuisances = noise and material budget
  - Trigger from beam
    - Often synchronous
  - Goal = extrapolate particle position on detector under test



- The astroparticle way
  - Similar to telescope mode but estimate incoming direction
  - No synchronous timing
  - Ex: deep-water ν telescopes



=> For 2 last cases: mostly a fitting problem
  - Usually with straight track model

Environmental conditions – 1/2

- Life in a real experiment is tough (*for detectors of course, students are welcome!*)
  - Chasing small cross-sections ➞ large luminosity and/or energy
  - Short interval between beam crossing (LHC: 25 ns)
  - Pile-up of events (HL-LHC >100 collisions / crossing)
  - Large amount of particles (could be > 10^8 part/cm^2/s)

    ⇒ background, radiation

    ⇒ Finding more complicated!

    ⇒ Requirements on detectors:
    - Fast timing
    - High granularity
  - Vacuum could be required (space, very low momentum particles (CBM, LHCb))

- Radiation tolerance
  - Two types of energy loss
    - Ionizing (generate charges): dose in Gy = 100 Rad
    - Non-ionizing (generate defects in solid): fluence in n_{eq}(1MeV)/cm^2
  - The innermost the detection layer, the harder the radiation (radius^2 effect)
  - Examples for most inner layers:
    - LHC: 10^{15} to $<10^{17}$ n_{eq}(1MeV)/cm^2 with 50 to 1 MGy
    - ILC: $<10^{12}$ n_{eq}(1MeV)/cm^2 with 5 kGy

Environmental conditions – 2/2

- Timing consideration
  - Integration time drives occupancy level (important for finding algorithm)
    - Currently range in the ns (CMS, ATLAS) to μs (ALICE) levels
  - Time resolution offers time-stamping of tracks
    - Tracks in one "acquisition event" could be associated to their proper collision, event if several have piled-up
    - O(100) ps required
  - Time ultra-resolution for particle identification with Time Of Flight method
    - O(10) ps required

- Heat concerns
  - Spatial resolution → segmentation→ many channels
    Readout speed → power dissipation/channel
  - Efficient cooling techniques exist BUT
    add material budget and may not work everywhere (space)

  } Hot cocktail!

- Summary

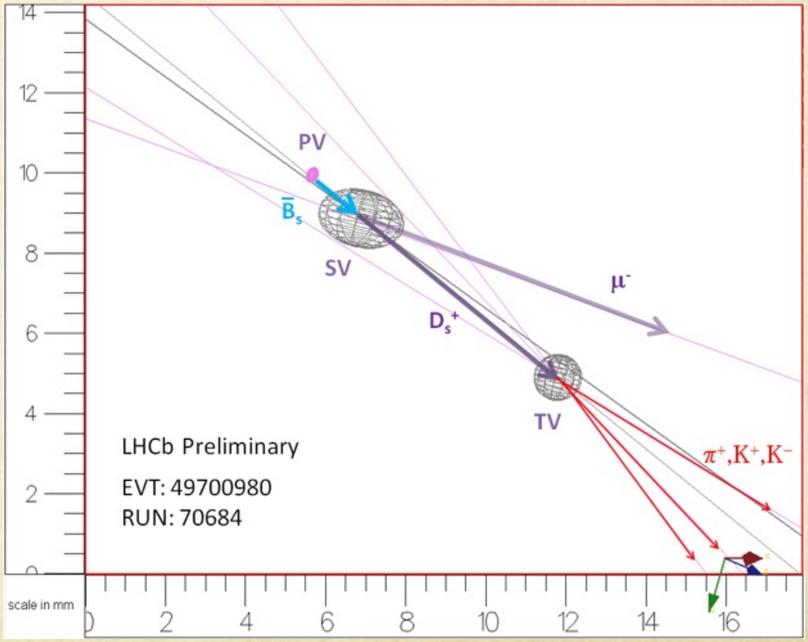
  → Tracker technology driven by environmental conditions: hadron colliders (LHC)

  → Tracker technology driven by physics performances: lepton colliders (B factories, ILC),
  heavy-ion colliders (RHIC, LHC)

  → Of course, some intermediate cases: superB factories, CLIC

Figures of Merit

- For detection layer
  - Detection efficiency
    - Mostly driven by Signal/Noise
    - <u>Note:</u> Noise = signal fluctuation ⊕ readout (electronic) noise
  - Intrinsic spatial resolution
    - Driven by segmentation (not only)
    - Useful tracking domain σ< 1mm
  - Linearity and resolution on dE/dx for PID
  - Material budget
  - "Speed" (time resolution, hit rate)
    - Driven by signal collection & electronics
  - Radiation tolerance
- For detection systems (multi-layers)
  - Track finding efficiency & purity
  - Two-track resolution
    - Ability to distinguish two nearby trajectories
    - Mostly governed by signal spread / segments
  - Momentum resolution $\frac{\sigma(p)}{p}$
  - Impact parameter resolution
    - Sometimes called "distance of closest approach" to a vertex



Figures of Merit: initial estimates

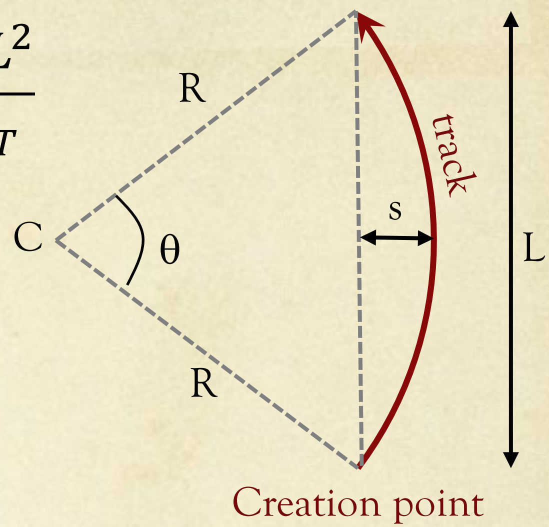
Momentum resolution

- Based on sagitta (s) measurement in collider geometry
- L = lever arm of measurements
- R = curvature radius $p_T/0.3B \gg L$

Units: T, GeV/c, m

$$s \approx \frac{L^2}{8R} = 0.038\,\frac{BL^2}{p_T}$$

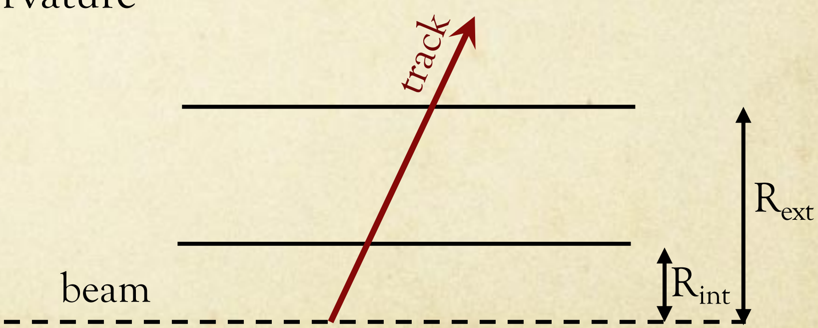
$$\frac{\sigma_{p_T}}{p_T} = \frac{\sigma_s}{s}$$



Impact parameter resolution

- Based on two layers measurements
- assume track straight over small distance: $R_{ext} \ll$ curvature
- Each layer with spatial resolution: σ_{int}, σ_{ext}
- Material budget → σ_θ
- Telescope equation:

$$\sigma_{IP} \propto \frac{\sqrt{R_{ext}^2\sigma_{int}^2 + R_{int}^2\sigma_{ext}^2}}{R_{ext} - R_{int}} \oplus \frac{R_{int}\sigma_{\vartheta(ms)}}{p\sin^{3/2}(\theta)}$$



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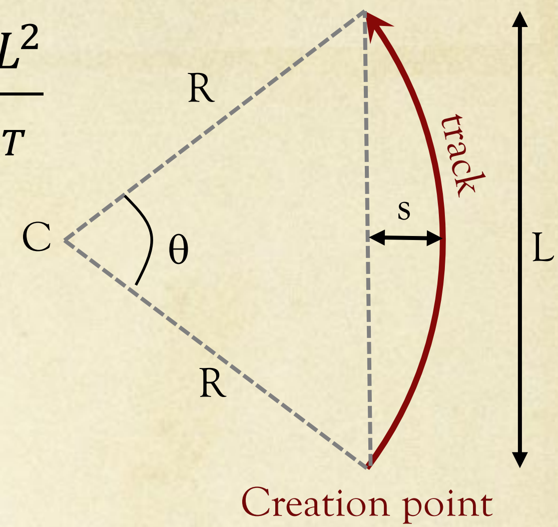

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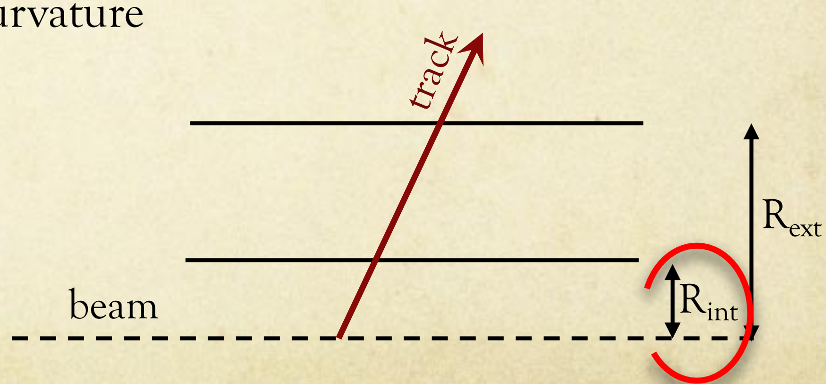


Simulating your favourite techno.

Of course GEANT 4 is there for you (https://geant4.web.cern.ch)

- But specialized for radiation-matter interaction & description of large geometries
- detail detector response to a given energy loss ? => specialized simulator

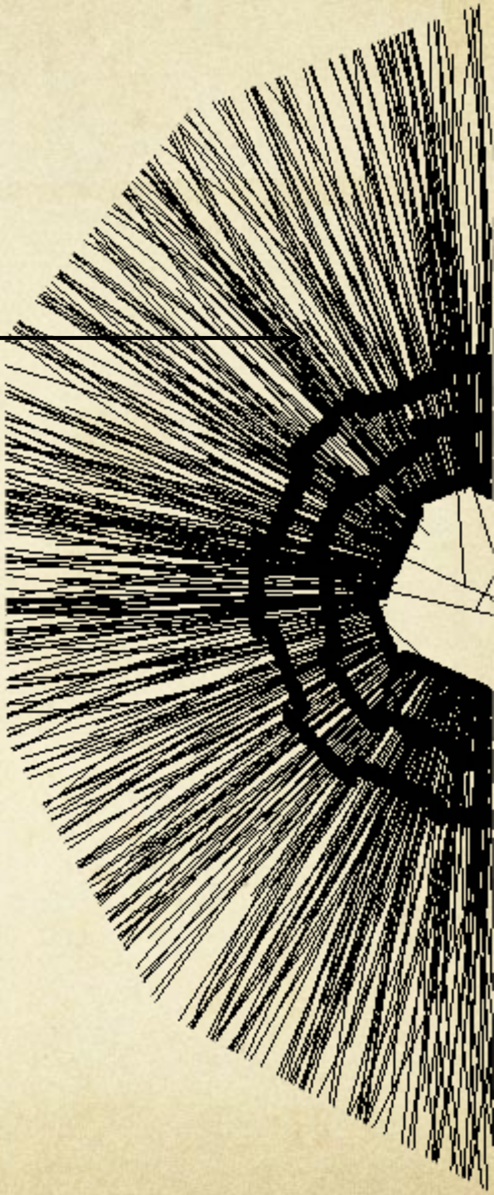
Some specialized tools

- Semiconductors:
  - Allpix2 (https://cern.ch/allpix-squared)
- Gas:
  - Garfield++ (http://cern.ch/garfieldpp)

- Solve or implement electric field in your drift volume
- Drift charges from ionization
- Simulate avalanche in gas
- Model electric signal

2. Finding algorithms

- Local method
- Global method
- Methods based on machine learning



FINDING : 2 strategies

Global methods

- Transform the coordinate space into pattern space
  - "pattern" = parameters used in track model
- Identify the "best" solutions in the new phase space
- Use all points at a time
  - No history effect
- Well adapted to evenly distributed points with same accuracy

Local methods

- Start with a track seed = restricted set of points
  - Could require good accuracy from the beginning
- Then extrapolate to next layer-point
  - And so on…iterative procedure
- "Wrong" solutions discarded at each iteration
- Possibly sensitive to "starting point"
- Well adapted to redundant information

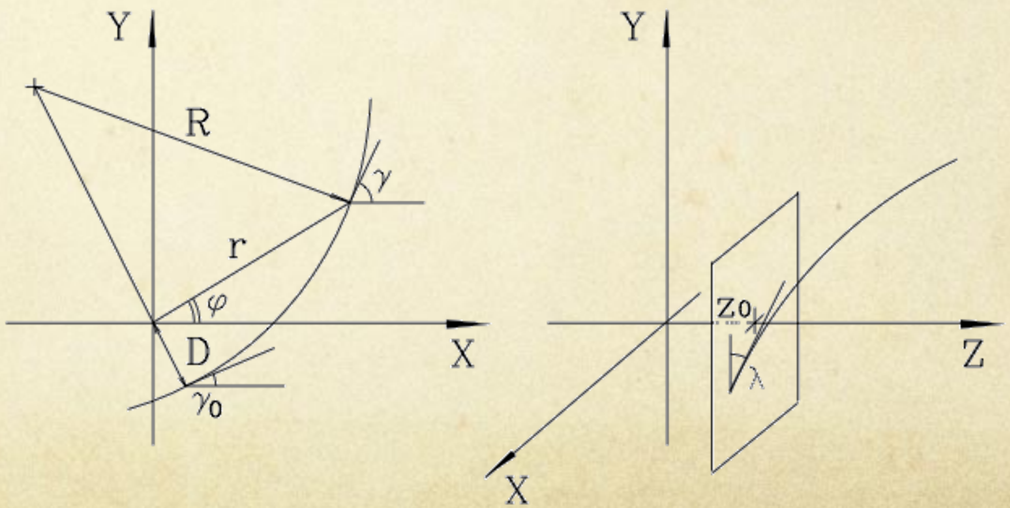
FINDING drives
tracking efficiency
fake track rate

Track model

- A simple example
  - Straight line in 2D: model is x = a*z + b
  - Track parameters (a,b); N measurements x_i at z_i (i=1..N)
- A more complex example
  - Helix in 3D with magnetic field
  - Track parameters (γ_0, z_0, D, $tan\lambda$, C=R)
  - Measurements/point (r, φ, z)

$$\varphi(r) = \gamma_0 + asin\frac{C\,r\,(1+CD)D/r}{1+2CD}$$

$$z(r) = z_0 + \frac{\tan\lambda}{C}\,asin\left(C\sqrt{\frac{r^2-D^2}{1+2CD}}\right)$$



- **Generalization**
  - Parameters: P-vector **p**
  - Measurements: N-vector **c**
  - Model: function f ($\mathcal{R}^P \rightarrow \mathcal{R}^N$)

f(**p**) = **c** ↔ propagation

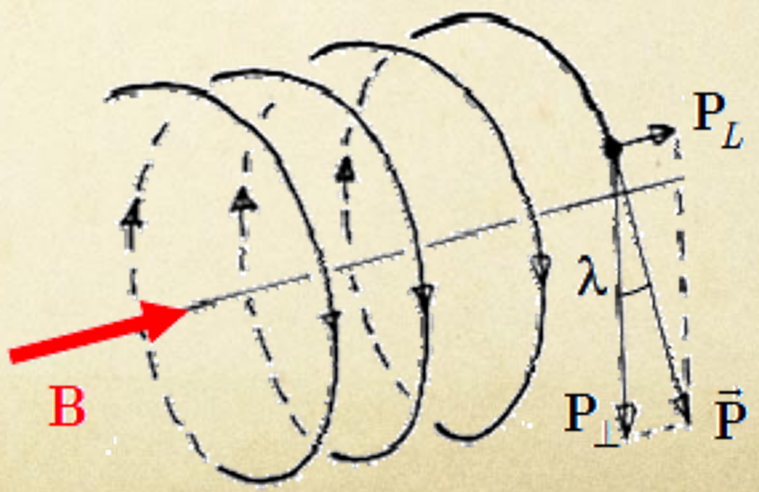
Helix model

- Another view of the helix
  - s = track length
  - h = rotation direction
  - λ= dip angle
  - Pivot point (s=0):
    - position (x_0, y_0, z_0)
    - orientation φ_0

$$x(s) = x_o + R\left[\cos\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \cos\Phi_o\right]$$

$$y(s) = y_o + R\left[\sin\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_o\right]$$

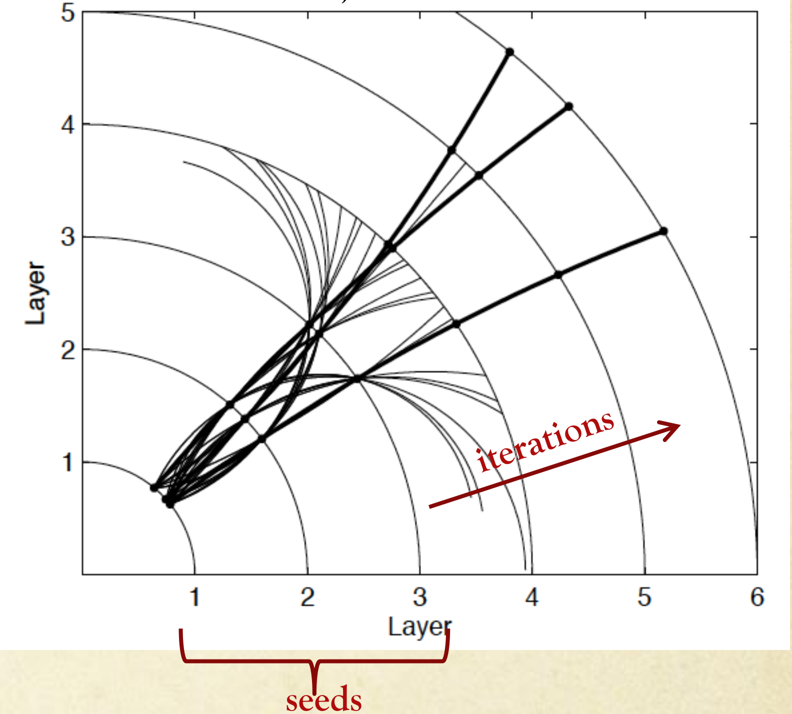
$$z(s) = z_o + s\sin\lambda$$



Local method 1/3

- Track seed = initial segment
  - Made of few (2 to 4) points
    - One point could be the expected primary vtx
  - Allows to initialize parameter for track model
  - Choose most precise layers first
    - usually inner layers
  - But if high hit density
    - Start farther from primary interaction @ lowest density
    - Limit mixing points from different tracks

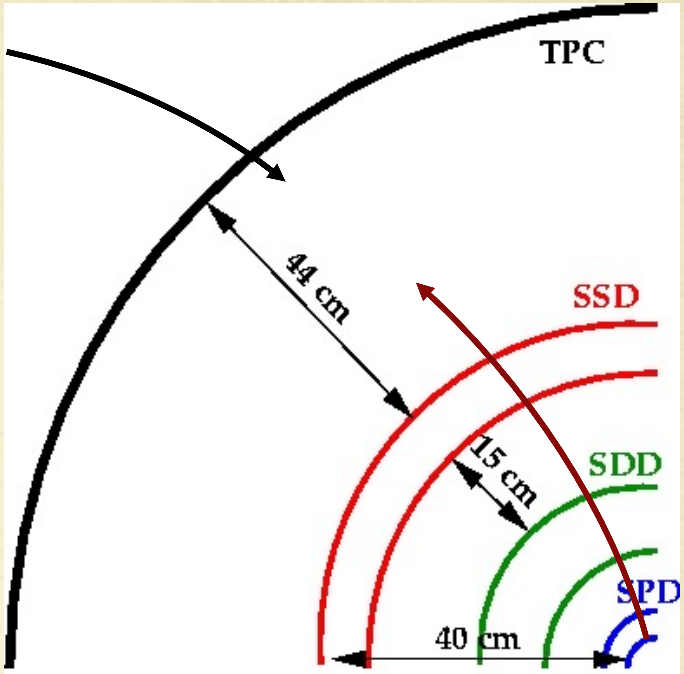
Frühwirth, Strandlie 2009



- Extrapolation step
  - Out or inward (=toward primary vtx) onto the next layer
  - Not necessarily very precise, especially **only local model** needed
    - Extrapolation uncertainty ≲ layer point uncertainty
    - Computation speed important
  - Match (associate) nearest point on the new layer
    - Might skip the layer if point missing
    - Might reject a point: if worst track-fit or if fits better with another track

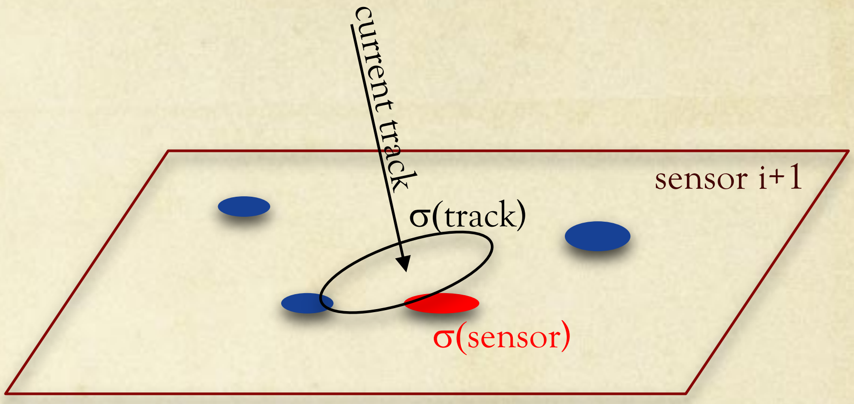
Local method 2/3

- Variant with track segments
  - First build “tracklets” on natural segments
    - Sub-detectors, or subparts with same resolution
  - Then match segments together
  - Typical application:
    - Segments large tracker (TPC) with vertex detector (Si) ➛ layers dedicated to matching



- (Variant with Kalman filter ⟶ See later)

Local method 3/3



- Figure of merit
  - σ_{eff} = σ(sensor) ⊕ σ(track extrapolation)
    = effective spatial resolution
  - ρ = background hit density
  - Probability to match correct hit
    => decided on distance hit to track-extrapolation ($\chi 2$ test)

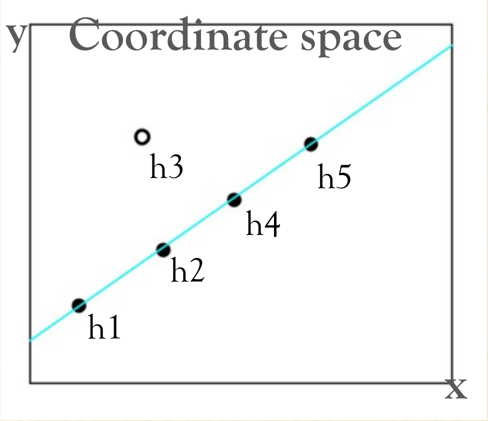
$$\text{Proba} = \frac{1}{1 + 2\pi\sigma_{eff,z}\times\sigma_{eff,\varphi}\times\rho_{bkg}}$$

$$\boxed{\sigma_{eff,\phi} \times \sigma_{eff,z} \times \rho_{bckgrnd}}$$

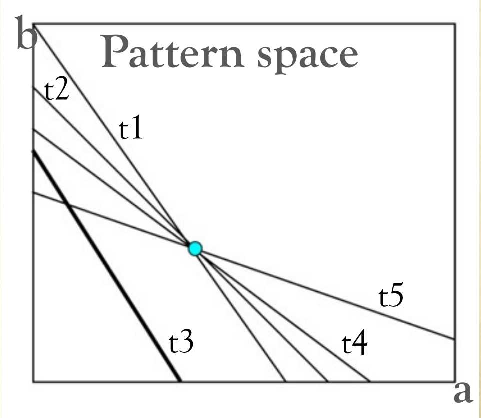
- Best suited to
  - Accommodate diverse extrapolation precision at each layer
    - Multi-layer system with non-equidistant & non-equivalent resolution layers
  - Easy to include timing information (just sum position & time χ^2)

Global methods 1/2

- Brute force = combinatorial way
  - Consider all possible combination of points to make a track
  - Keep only those compatible with model
  - Usually too time consuming…



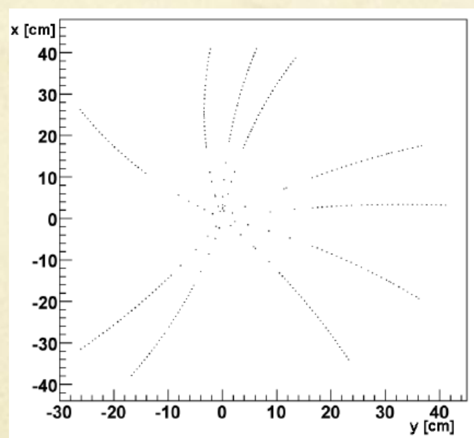
- Hough transform
  - Example straight track:
    - Coord. space y = a*x + b ⇔ pattern space b = y - x*a
    - Each point (y,x) defines a line in pattern space
    - All lines, from points belonging to same straight-track, cross at same point (a,b)
    - In practice: discretize pattern space and search for maximum
  - Applicable to circle finder
    - needs two parameters as well (r,φ of center) if track is assumed to originate from (0,0)
  - More difficult for more than 2 parameters…

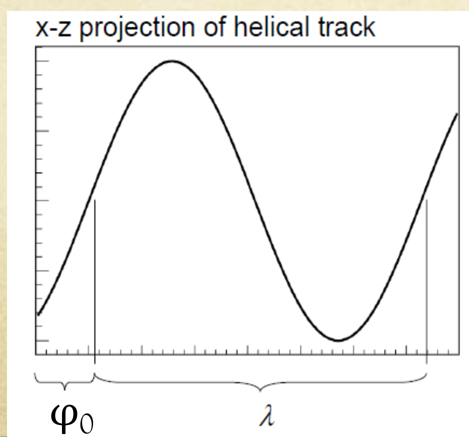


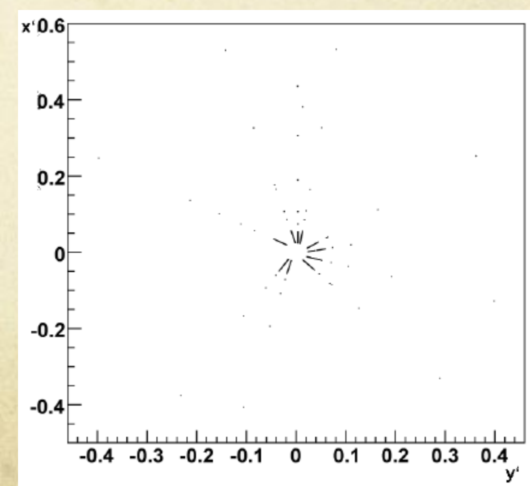
Global methods 2/2

Conformal mapping for helix

- (x_0,y_0,z_0) a (pivot) point on the helix with (a,b) the center of the projected circle of radius r
  - $(x\text{-}a)^2 + (y\text{-}b)^2 = r^2$
- Transforming to $x' = \frac{x-x_0}{r^2}, y' = \frac{y-y_0}{r^2}$ leads to $y' = -\frac{a}{b}x' + \frac{1}{2b}$ i.e. a line!
  - So all measured points (x,y) in circles are aligned in (x',y') plane
- Use Hough transform (x',y') → (r,θ) so that $r = x' \cos\theta + y' \sin\theta$
  - To find the lines corresponding to true circles with $\boxed{a = r\cos\theta \text{ and } b = r\sin\theta}$
- Repeat for different z_0
  - New Hough transforms
  - λ = dip angle
  - φ_0 = orientation of pivot point







Global methods 2/2

Figure of merit

- Search precision in pattern space depends on bin-size in the pattern space
- Such bin-size ~ uncertainty on the measurements = σ(sensor) ⊕ σ(multiple scatt.)

$$\boxed{\sigma_{eff,\phi} \times \sigma_{eff,z} \times \rho_{bckgrnd}}$$

Best suited for

- Homogenous set of measurements
- Typically large gas volume
  or multi equidistant equivalent layers