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European network for developing new horizons for RIs

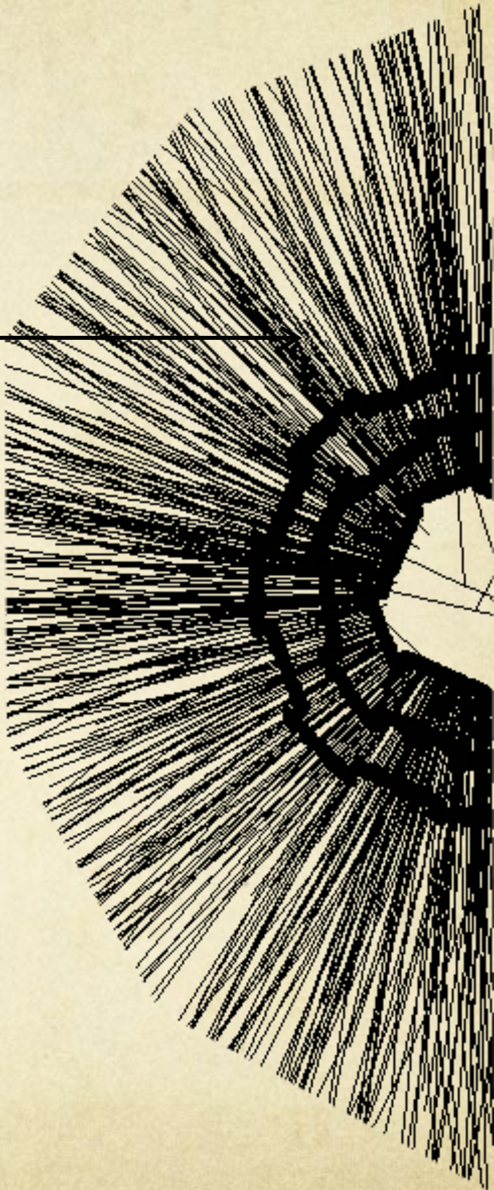
Wuppertal, July 023

Tracking 2/3

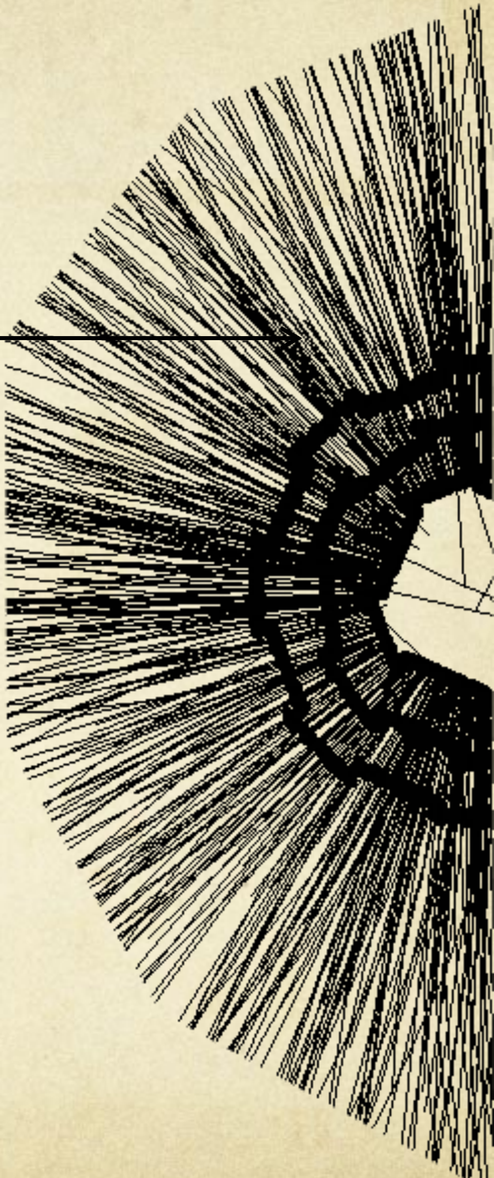
Lecture outline

1. Basic concepts — first lecture
2. Finding algorithms — second lecture
3. Fitting algorithms — third lecture
4. Existing tracking systems — third lecture

References



2. Finding algorithms



- Local method
- Global method
- Methods based on machine learning

FINDING : 2 strategies

Global methods

- Transform the coordinate space into pattern space
  - "pattern" = parameters used in track model
- Identify the "best" solutions in the new phase space
- Use all points at a time
  - No history effect
- Well adapted to evenly distributed points with same accuracy

Local methods

- Start with a track seed = restricted set of points
  - Could require good accuracy from the beginning
- Then extrapolate to next layer-point
  - And so on…iterative procedure
- "Wrong" solutions discarded at each iteration
- Possibly sensitive to "starting point"
- Well adapted to redundant information

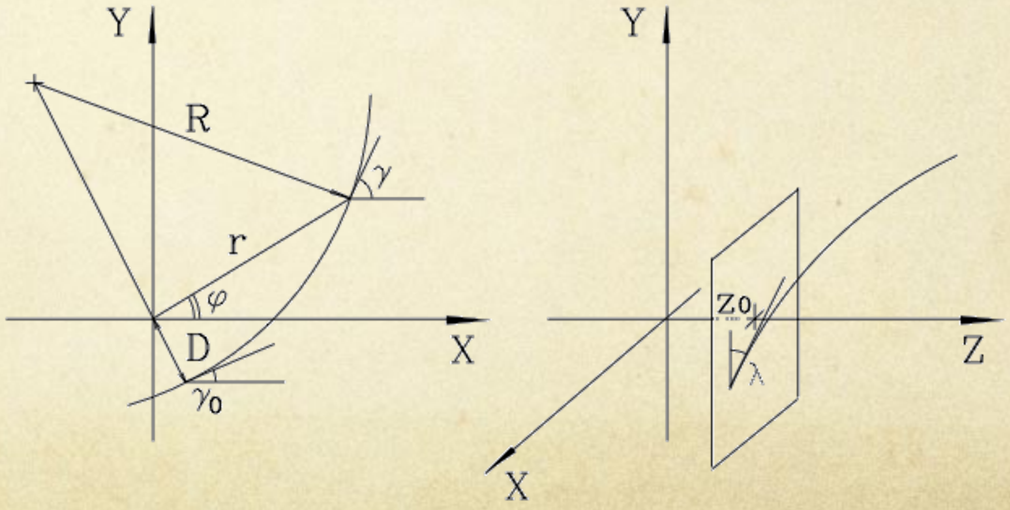
FINDING drives
tracking efficiency
fake track rate

Track model

- A simple example
  - Straight line in 2D: model is x = a*z + b
  - Track parameters (a,b); N measurements x_i at z_i (i=1..N)
- A more complex example
  - Helix in 3D with magnetic field
  - Track parameters (γ_0, z_0, D, $tan\lambda$, C=R)
  - Measurements/point (r, φ, z)

$$\varphi(r) = \gamma_0 + asin\frac{C\,r\,(1+CD)D/r}{1+2CD}$$

$$z(r) = z_0 + \frac{\tan\lambda}{C}\,asin\left(C\sqrt{\frac{r^2-D^2}{1+2CD}}\right)$$



- **Generalization**
  - Parameters: P-vector **p**
  - Measurements: N-vector **c**
  - Model: function f ($\mathcal{R}^P \rightarrow \mathcal{R}^N$)

f(**p**) = **c** ↔ propagation

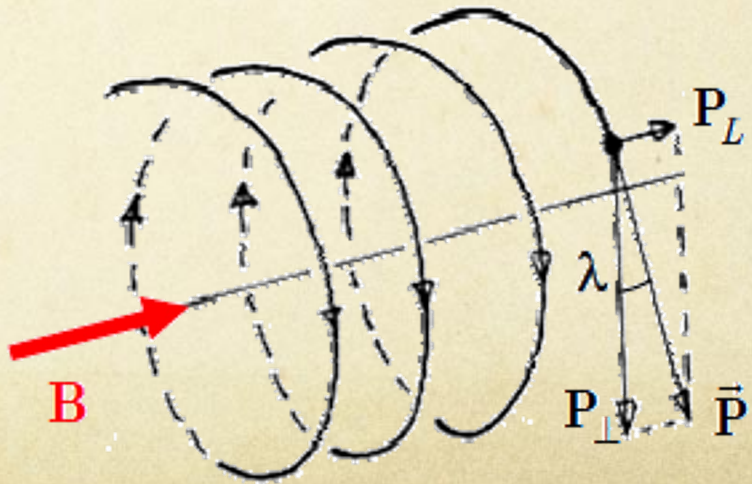
Helix model

- Another view of the helix
  - s = track length
  - h = rotation direction
  - λ= dip angle
  - Pivot point (s=0):
    - position (x_0, y_0, z_0)
    - orientation φ_0

$$x(s) = x_o + R\left[\cos\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \cos\Phi_o\right]$$

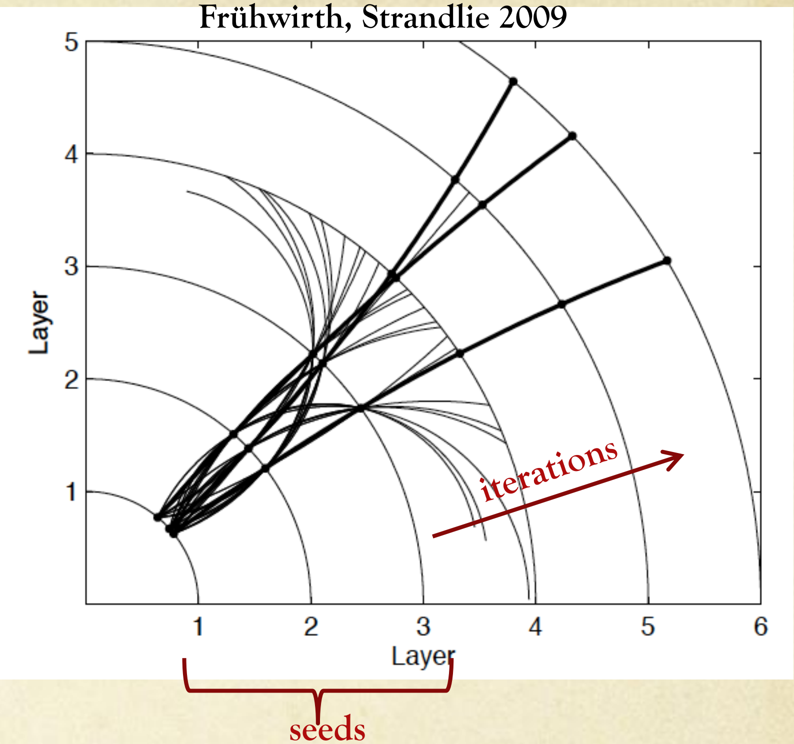
$$y(s) = y_o + R\left[\sin\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_o\right]$$

$$z(s) = z_o + s\sin\lambda$$



Local method 1/3

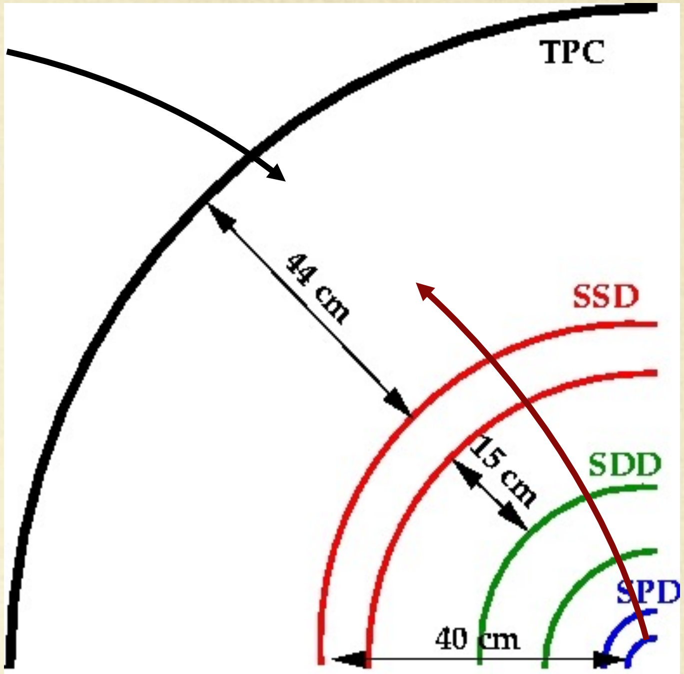
- **Track seed = initial segment**
  - Made of few (2 to 4) points
    - One point could be the expected primary vtx
  - Allows to initialize parameter for track model
  - Choose most precise layers first
    - usually inner layers
  - But if high hit density
    - Start farther from primary interaction @ lowest density
    - Limit mixing points from different tracks



- **Extrapolation step**
  - Out or inward (=toward primary vtx) onto the next layer
  - Not necessarily very precise, especially **only local model** needed
    - Extrapolation uncertainty ≲ layer point uncertainty
    - Computation speed important
  - Match (associate) nearest point on the new layer
    - Might skip the layer if point missing
    - Might reject a point: if worst track-fit or if fits better with another track

Local method 2/3

- Variant with track segments
  - First build "tracklets" on natural segments
    - Sub-detectors, or subparts with same resolution
  - Then match segments together
  - Typical application:
    - Segments large tracker (TPC) with vertex detector (Si)
      ➛ layers dedicated to matching
- (Variant with Kalman filter ⟶ See later)



2. Finding algorithms:

Local method 3/3

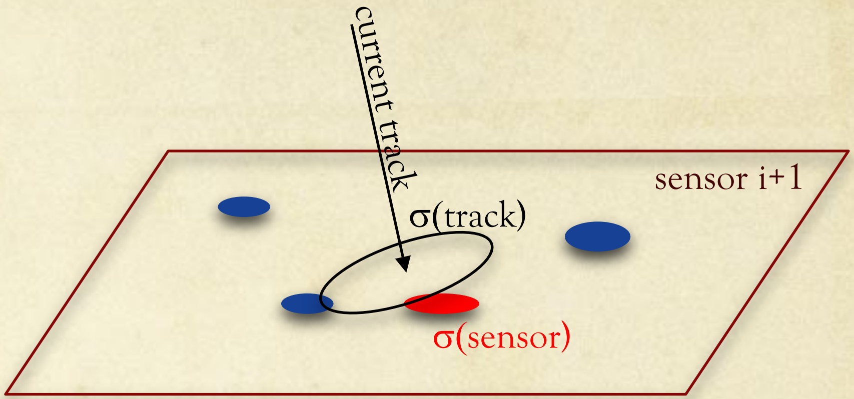


Figure of merit

- σ_{eff} = σ(sensor) ⊕ σ(track extrapolation)
  = effective spatial resolution
- ρ = background hit density
- Probability to match correct hit
  => decided on distance hit to track-extrapolation (χ2 test)

$$\text{Proba} = \frac{1}{1 + 2\pi\sigma_{eff,z}\times\sigma_{eff,\varphi}\times\rho_{bkg}}$$

$$\sigma_{eff,\phi}\times\sigma_{eff,z}\times\rho_{bckgrnd}$$

Best suited to

- Accommodate diverse extrapolation precision at each layer
  - Multi-layer system with non-equidistant & non-equivalent resolution layers
- Easy to include timing information (just sum position & time χ^2)

Interlude

Occupancy

= segmentation area (pitch²) x $\rho_{bckgrnd}$

- Knowing that $\sigma_{det} = \frac{\text{pitch}}{k}$ with $\sqrt{12} = 3.46$

  we got $\sigma_r \times \sigma_\varphi \times \rho_{bkgdrnd} = \frac{\text{pitch}}{k_r} \times \frac{\text{pitch}}{k_\varphi} \times \rho_{bkgdrnd} = \frac{\text{occupancy}}{k_r \times k_\varphi} < \frac{\text{occupancy}}{10}$

Back to probability to match correct hit

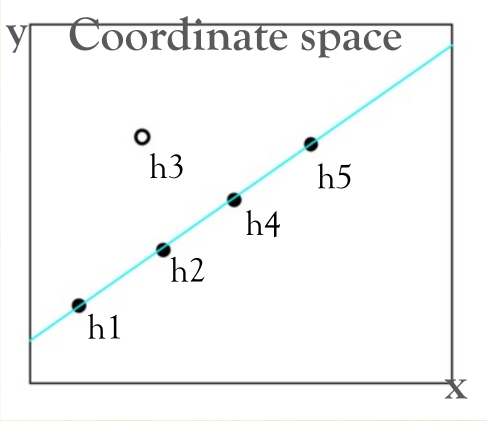
$$\text{Proba} = \frac{1}{1 + 2\pi\sigma_{eff,z} \times \sigma_{eff,\varphi} \times \rho_{bkg}} > \frac{1}{1 + 2\pi\ \text{occupancy}}$$

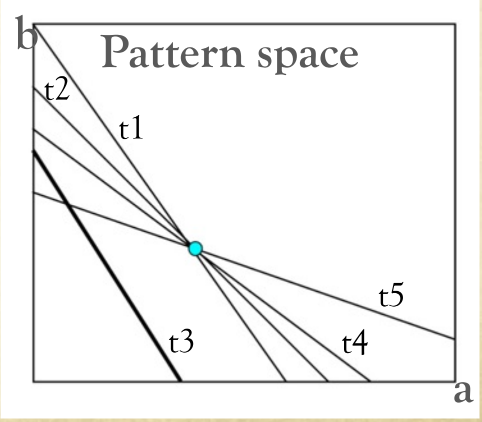
occupancy	$\frac{1}{1+2\pi\ \text{occupancy}}$
0.1 %	99.4%
1 %	94.1%
5%	76.1%

Global methods 1/2

- Brute force = combinatorial way
  - Consider all possible combination of points to make a track
  - Keep only those compatible with model
  - Usually too time consuming…

- Hough transform
  - Example straight track:
    - Coord. space y = a*x + b ⇔ pattern space b = y - x*a
    - Each point (y,x) defines a line in pattern space
    - All lines, from points belonging to same straight-track, cross at same point (a,b)
    - In practice: discretize pattern space and search for maximum
  - Applicable to circle finder
    - needs two parameters as well (r,φ of center) if track is assumed to originate from (0,0)
  - More difficult for more than 2 parameters…

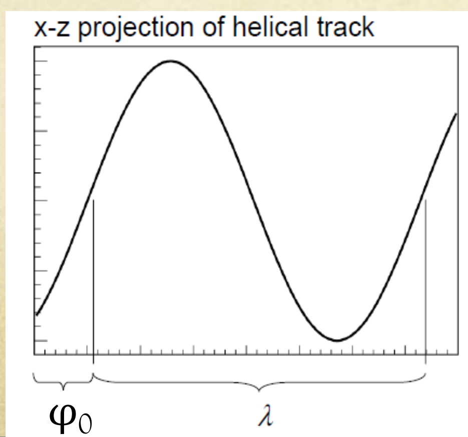


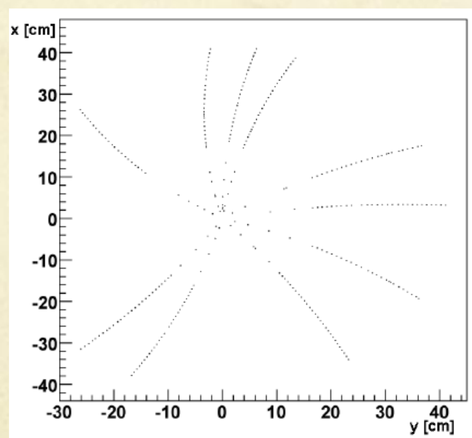


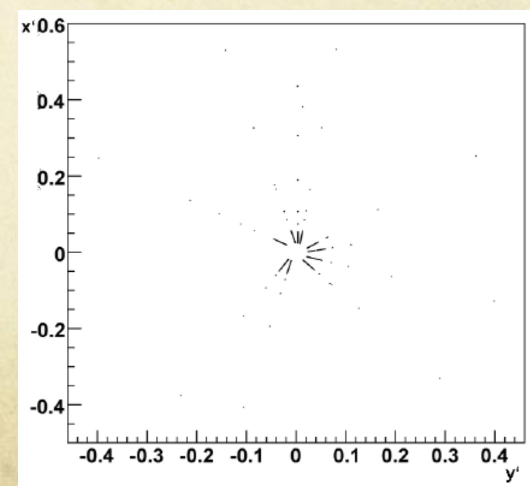
Global methods 2/2

Conformal mapping for helix

- (x_0,y_0,z_0) a (pivot) point on the helix with (a,b) the center of the projected circle of radius r
  - $(x-a)^2 + (y-b)^2 = r^2$
- Transforming to $x' = \frac{x-x_0}{r^2}, y' = \frac{y-y_0}{r^2}$ leads to $y' = -\frac{a}{b}x' + \frac{1}{2b}$ i.e. a line!
  - So all measured points (x,y) in circles are aligned in (x',y') plane
- Use Hough transform (x',y') → (r,θ) so that $r = x' \cos\theta + y' \sin\theta$
  - To find the lines corresponding to true circles with $a = r\cos\theta$ and $b = r\sin\theta$
- Repeat for different z_0
  - New Hough transforms
  - λ = dip angle
  - φ_0 = orientation of pivot point







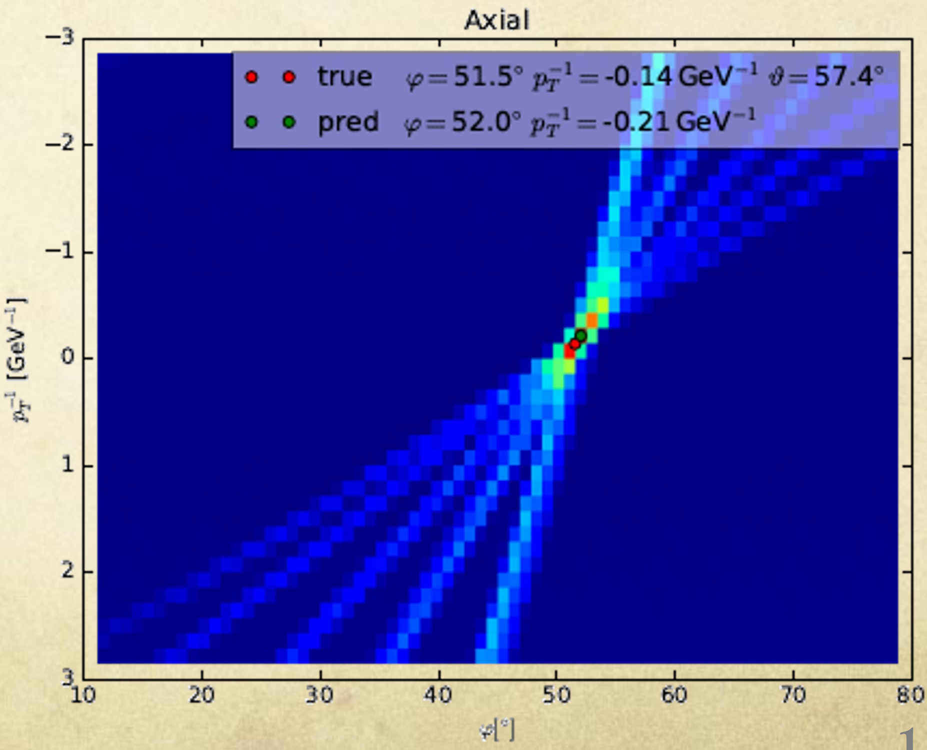
Illustrations from PANDA track finding

Global methods 2/2

- Figure of merit
  - Search precision in pattern space depends on bin-size in the pattern space
  - Such bin-size ~ uncertainty on the measurements = σ(sensor) ⊕ σ(multiple scatt.)

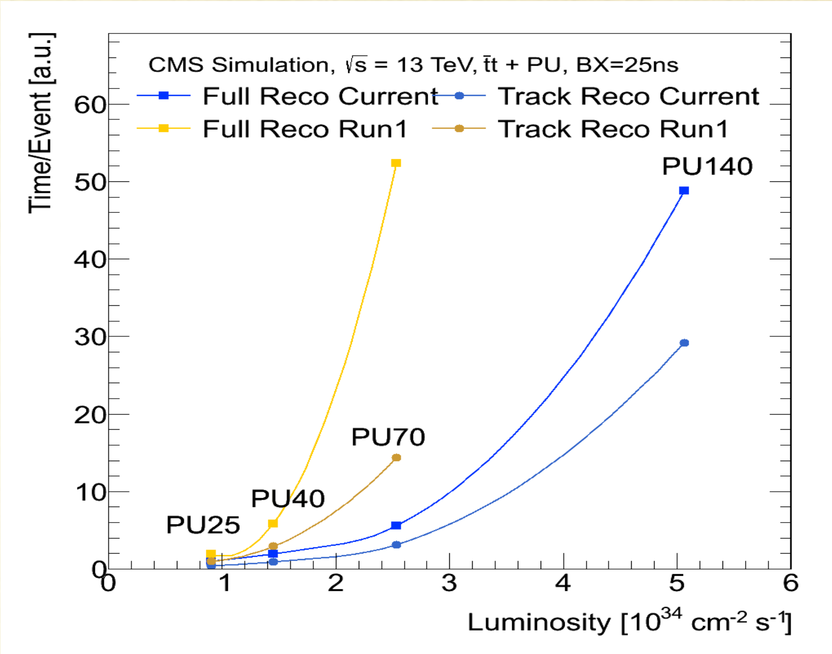
$$\sigma_{eff,\phi} \times \sigma_{eff,z} \times \rho_{bckgrnd}$$

- Best suited for
  - Homogenous set of measurements
  - Typically large gas volume
    or multi equidistant equivalent layers



Adaptive (machine learning) methods

- Shall we do better?
  - Higher track/vertex density => lower efficient for classical methods + Processing intensive
  - Allows for many options and best choice



- Adaptive features
  - Dynamic change of track parameters during finding/fitting
  - Measurements are weighted / uncertainties
    - Allows to take into account many info
  - Many hypothesis are handled simultaneously
    - But their number decrease with iterations (annealing like behavior)
  - Non-linearity
  - Effective with respect to processing time

- Examples
  - Neural network (NN), Elastic nets, Gaussian-sum filters, Deterministic annealing, Cellular automaton, convolutional NN, graph NN

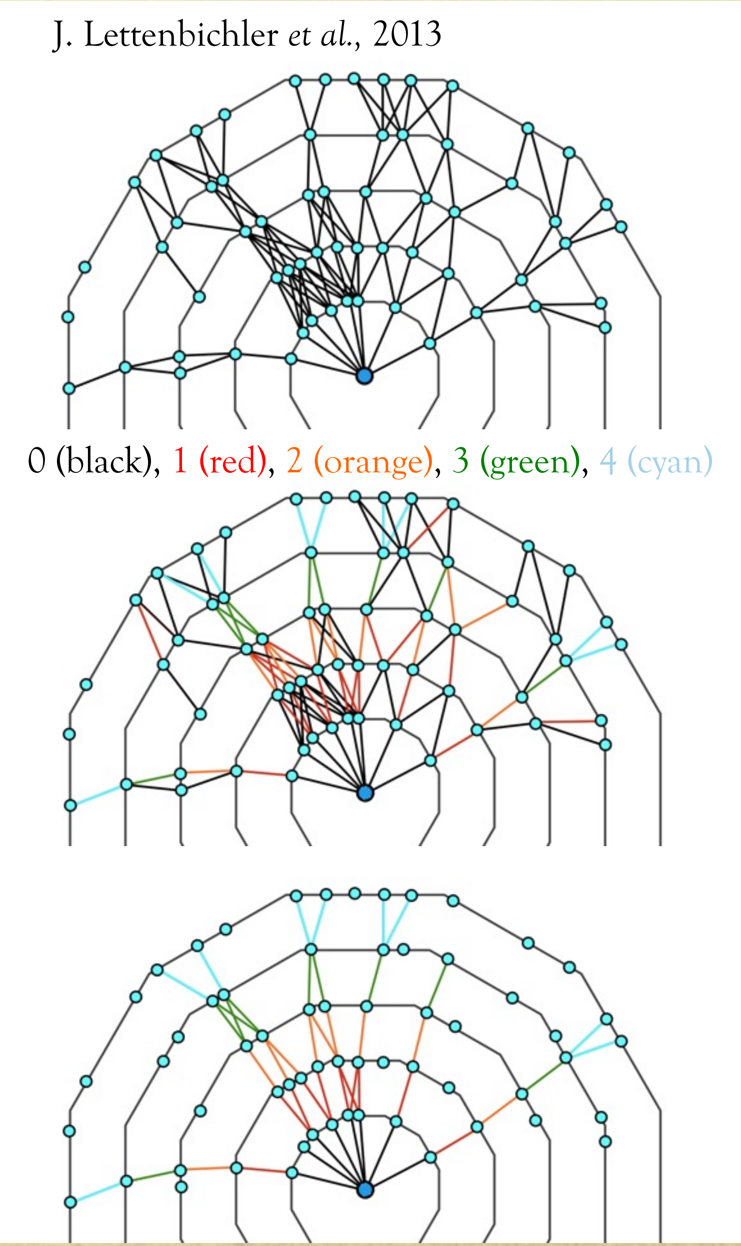
Cellular automaton

Concepts

- Initialization
  - Build all 'possible' cell (= segment of 2 points)
  - With rule(s) like:
    2 points belonging to same detector 'sector'
- Iterative step
  - Associate neighbour cells (inner-wise here =>)
    - following rule(s) like:
      the two cells math the track model
  - Rise "state" with associated cells
  - Kill lowest state cells
    - need a cut on the minimal accepted state

Usage

- For full tracking or seeding



Graph neural network (GNN)

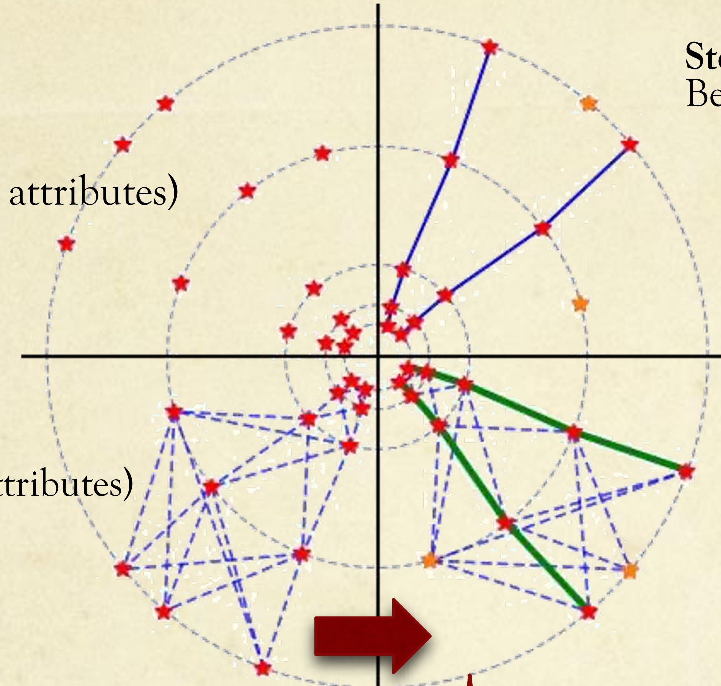
- Concepts

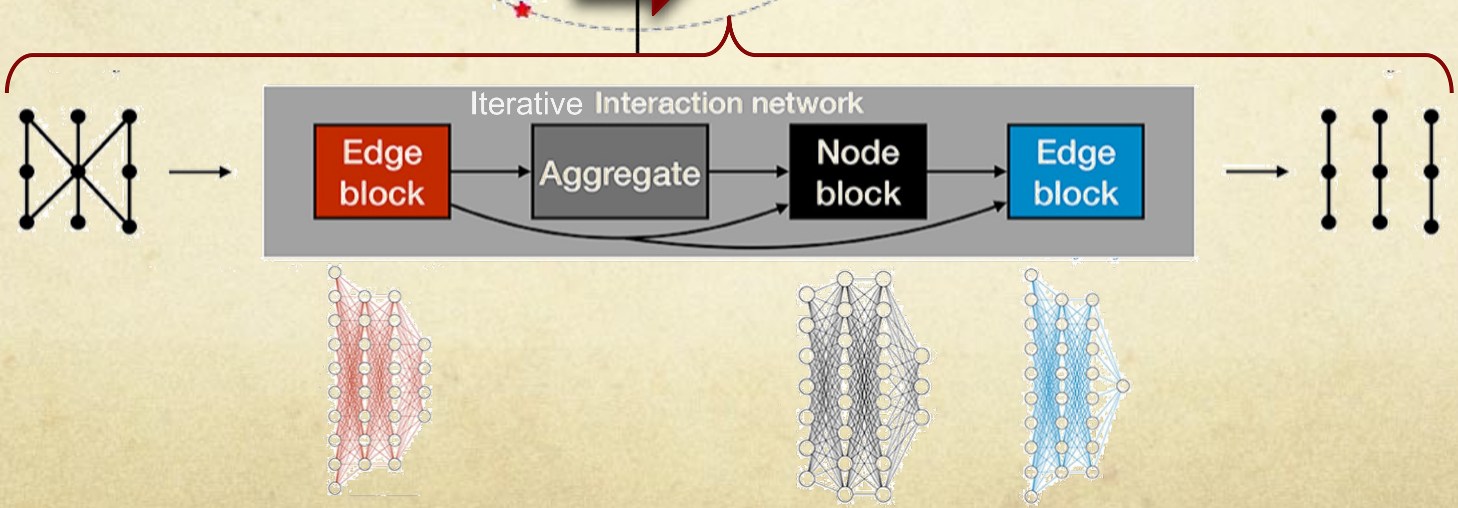
Step 1:
One hit =
one node (with attributes)

Step 2:
Two nodes =
one edge (with attributes)

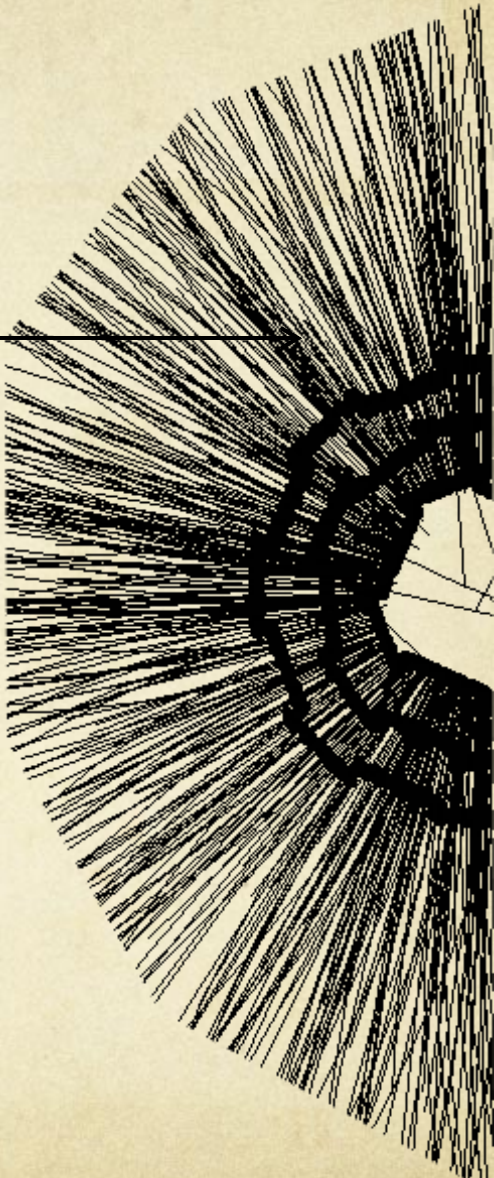
Step 3:
GNN updates extended edges

Step 4:
Best edges = tracks





3. Fitting algorithms



- Least square method (global)
- Kalman filter (local)
- Alignment

FITTING

- Why do we need to fit?
  - Measurement error
  - Multiple scattering error

- Global fit
  - Assume knowledge of:
    - all track points
    - full correlation matrix
      → difficult if $\sigma_{\text{mult. scatt.}} \gtrsim \sigma_{\text{meas.}}$
  - Least square method

- Iterative (local) fit
  - Iterative process:
    - points included in the fit one by one
    - could be merged with finder step
  - Kalman filter

FITTING drives track extrapolation & momentum res.

Nb of measured points to start?

The rule

- For the fit: nb of constraints > nb of free parameters in the track model

Measurements

- 1 point in 2D = 1 constraint ($x \leftrightarrow y$) or ($r \leftrightarrow \phi$)
- 1 point in 3D = 2 constraints ($x \leftrightarrow z$ & $y \leftrightarrow z$)

Models

- Straight track in 2D = 2 parameters
  - 1 coordinate @ origin (z=0) , 1 slope
- Straight track in 3D = 4 parameters
  - 2 coordinates @ origin, 2 slopes
- Circle in 2D = 3 parameters
  - 2 coordinates for center, 1 radius
- Helix in 3D = 5 parameters
  - 3 coordinates for center, 1 radius, 1 dip angle

Minimal #points needed

- $\Leftarrow$ 2 points in 2D (straight track in 2D)
- $\Leftarrow$ 2 points in 3D (straight track in 3D)
- $\Leftarrow$ 3 points in 2D (circle in 2D)
- $\Leftarrow$ 3 points in 3D (helix in 3D)

Least Square Method (LSM)

Linear model hypothesis

- P track parameters **p**, with N measurements **c**

$$\vec{c} = \vec{c}_s + A(\vec{p} - \vec{p}_s) + \vec{\varepsilon}$$

- $\mathbf{p}_s$ = known starting point (pivot), **A** = track model NxP matrix,
  ε = error vector corresponding to **V** = covariance NxN matrix

"N measurements" means:
- K points (or layers)
- D coordinates at each point
- N = KxD

Sum of squares:

$$\sum \frac{(\text{model} - \text{measure})^2}{\text{uncertainty}^2} \Rightarrow S(\vec{p}) = \left(\vec{c}_s + A(\vec{p} - \vec{p}_s) - \vec{c}\right)^T V^{-1} \left(\vec{c}_s + A(\vec{p} - \vec{p}_s) - \vec{c}\right)$$

Best estimator (minimizing variance)

$$\frac{\mathrm{d}S}{\mathrm{d}\underline{\vec{p}}}(\underline{\vec{p}}) = 0 \Rightarrow \underline{\vec{p}} = \vec{p}_s + \left(A^T V^{-1} A\right)^{-1} A^T V^{-1} \left(\vec{c} - \vec{c}_s\right)$$

- Variance (= uncertainty) of the estimator:

$$\underline{V_{\vec{p}}} = \left(A^T V^{-1} A\right)^{-1}$$

- Estimator p follows a χ^2 law with N-P degrees of freedom

Problem ⇔ inversion of a PxP matrix ($A^T V^{-1} A$)

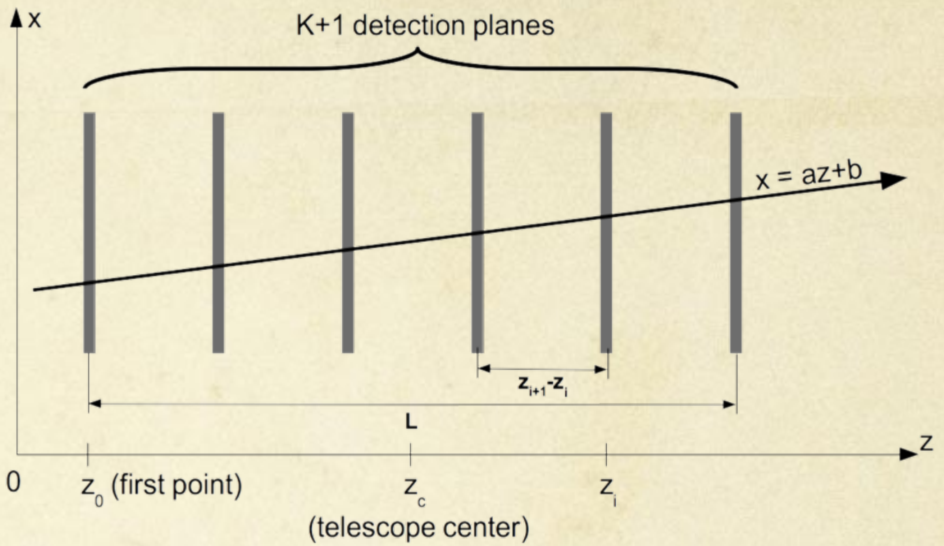
- But real difficulty could be computing **V** (NxN matrix)
  <= layer correlations if multiple scattering non-negligible if $\sigma_{\text{mult. scatt.}} \gtrsim \sigma_{\text{meas}}$

Generic tool for fitting:
https://genfit.sourceforge.net

LSM on straight tracks

Straight line model

- 2D case → D=2 coordinates (z,x)
- 2 parameters: a = slobe, b = intercept at z=0



General case

- K+1 detection planes (i=0…k)
  - located at z_i
  - Spatial resolution $\boldsymbol{\sigma}_i$
- Useful definitions

$$S_1 = \sum_{i=0}^{K} \frac{1}{\sigma_i^2} \;,\; S_z = \sum_{i=0}^{K} \frac{z_i}{\sigma_i^2} \;,\; S_{xz} = \sum_{i=0}^{K} \frac{x_i z_i}{\sigma_i^2} \;,\; S_{z^2} = \sum_{i=0}^{K} \frac{z_i^2}{\sigma_i^2}$$

- Solutions $a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2}$, $b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$

- Uncertainties

$$\sigma_a^2 = \frac{S_1}{S_1 S_{z^2} - (S_z)^2} \;,\; \sigma_b^2 = \frac{S_{z^2}}{S_1 S_{z^2} - (S_z)^2}$$

! correlation $\mathrm{cov}_{a,b} = \frac{-S_z}{S_1 S_{z^2} - (S_z)^2}$

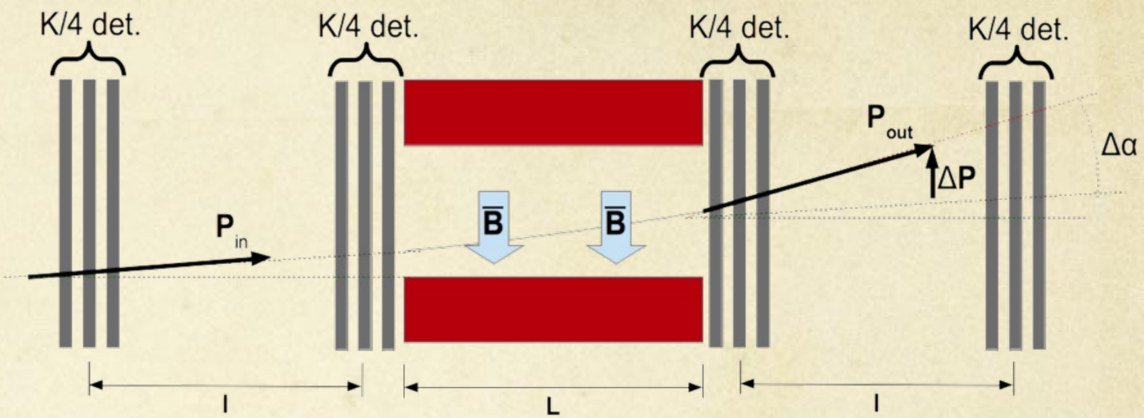
Case of uniformly distributed (K+1) planes

- $z_{i+1} - z_i = L/K$ et $\boldsymbol{\sigma}_i = \boldsymbol{\sigma}$ $\forall i$
- $S_z = 0$ → a,b uncorrelated

$$\sigma_a^2 = \frac{12K}{(K+2)L^2} \frac{\sigma^2}{K+1} \;,\; \sigma_b^2 = \left(1 + 12 \frac{K}{K+2} \frac{z_c^2}{L^2}\right) \frac{\sigma^2}{K+1}$$

- Uncertainties :
  - $\boldsymbol{\sigma}_a$ and $\boldsymbol{\sigma}_b$ improve with $1/\sqrt{(K+1)}$
  - $\boldsymbol{\sigma}_a$ and $\boldsymbol{\sigma}_b$ improve with 1/L
  - $\boldsymbol{\sigma}_b$ improve with z_c

LSM on fixed target geometry



Hypothesis

- K detectors,
  each with σ single point accuracy
- Uniform field over L from dipole
  - Trajectory: $\Delta\alpha = \left|\frac{0.3qBL}{p}\right|$
  - Bending: $\Delta p = p\,\Delta\alpha$
- Geometrical arrangement optimized for resolution
  - Angular determination on input and output angle: $\sigma_\alpha^2 = \frac{16\,\sigma^2}{K\,l^2}$

Without multiple scattering

- Uncertainty on momentum
- Note proportionality to p and to $\frac{1}{BL}$

$$\frac{\sigma_p}{p} = \frac{8}{0.3q}\frac{1}{BL}\frac{\sigma}{l\sqrt{K}}p$$

Multiple scattering contribution

- Bring **additive** term proportional to K

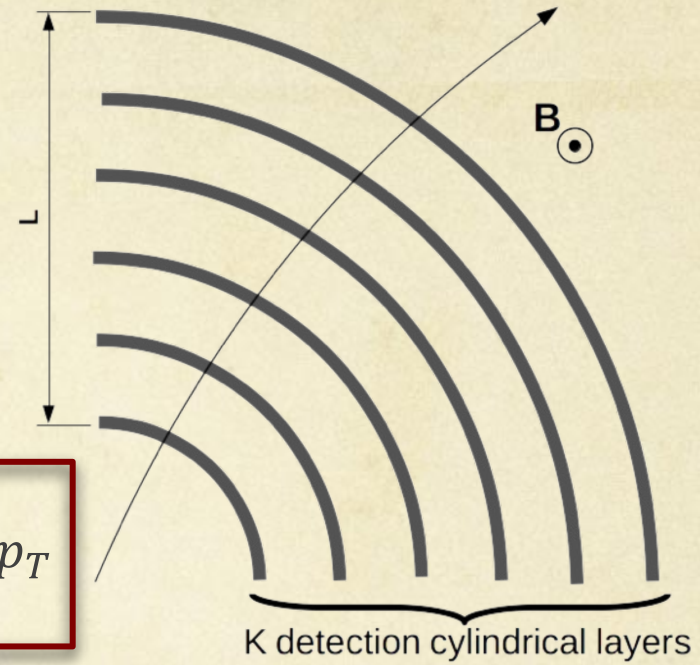
and $\sigma_\theta = \frac{13.6\ (\text{MeV/c})}{\beta p}\sqrt{\frac{\text{thickness}}{X_0}}$

$$\frac{\sigma_p}{p}(ms) = A_K \frac{13.6\ (\text{MeV/c})}{\beta}\sqrt{\frac{\text{total thickness}}{X_0}}$$

=> Constant with p!

A_K = factor depending on geometrical arrangement

LSM on collider geometry



Hypothesis

- K detectors uniformly distributed each with **σ** single point accuracy
- Uniform field over path length L

Without multiple scattering

- Uncertainty on transverse momentum (Glückstern formula)

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sqrt{720}}{0.3q} \frac{1}{BL^2} \frac{\sigma}{\sqrt{K+6}} p_T$$

- Works well with large K > 20

=> Proportional to p and to $\frac{1}{BL^2}$!

Multiple scattering contribution

- Brings additive contribution

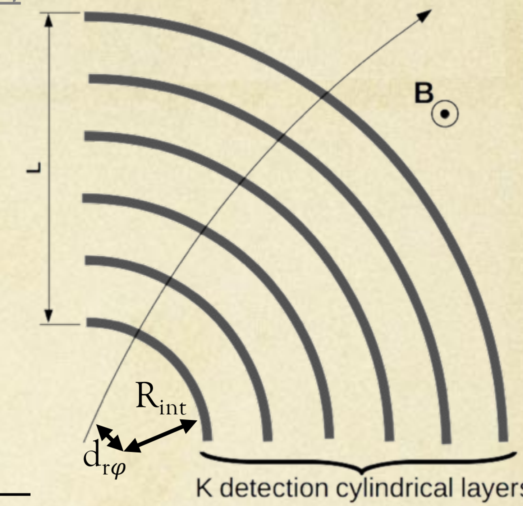
$$\frac{\sigma_{p_T}}{p_T} = \frac{1.43}{0.3q} \frac{1}{BL} \sqrt{\frac{13.6\ (\mathrm{MeV/c})}{\beta} \frac{\text{total thickness}}{X_0}}$$

*Numerical factors $\sqrt{\frac{720}{K+6}}$ and 1.43 can be refined
⟶ see https://arxiv.org/abs/1805.12014

=> Constant with p!
=> Depends on K through total thickness!

LSM for impact parameter ($d_{r\varphi}$)

⟶ see https://arxiv.org/abs/1805.12014



Hypothesis

- K detectors uniformly distributed over L
- each with σ single point accuracy
- First layer is close to PV / L => $r = R_{int}/L < 1$

Without multiple scattering

$$\sigma_{d_{r\varphi}} = \frac{\sqrt{3}\sigma}{\sqrt{K+4}}\sqrt{1+8r+28\,r^2}$$

Multiple scattering contribution

- Brings additive contribution proportional to $\sigma_\theta = \frac{13.6\ (\mathrm{MeV/c})}{\beta p_T}\sqrt{\frac{\mathrm{thickness}}{X_0}}$

$$\sigma_{d_{r\varphi}}(ms) = r\sigma_\theta\sqrt{1+\frac{1}{2}r+\frac{K-1}{4}r^2}$$

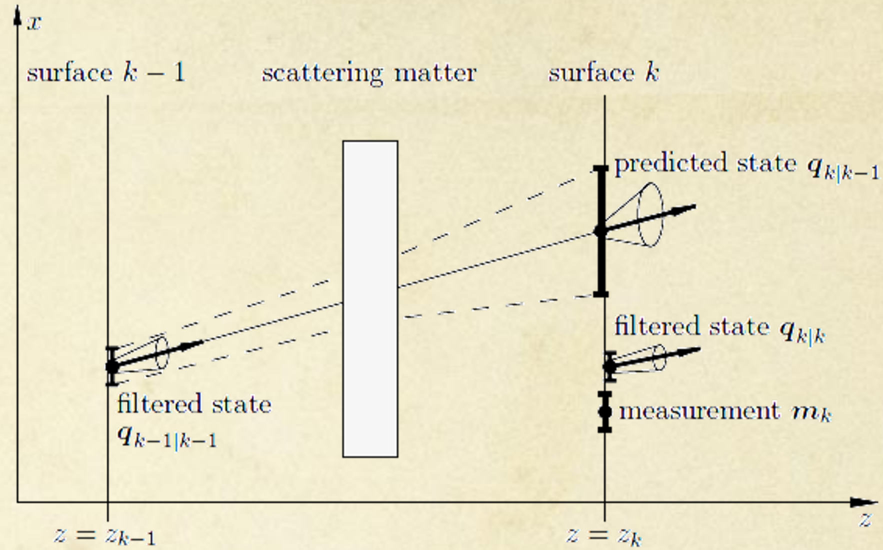
=> Proportional to r and K!

Key points

- Minimising R_{int}/L ⟷ getting close & keeping lever arm
- Multiple scattering destroys statistical gain of K>2

Kalman filter 1/2

- Dimensions
  - P parameters for track model
  - D "coordinates" measured at each point (usually D<P)
  - K measurement points (# total measures: N = KxD)



- Starting point
  - Initial set of parameters: first measurements
  - With large uncertainties if unknowns

- Iterative method
  - Propagate to next layer = prediction
    - Using the system equation $\vec{p}_k = G\,\vec{p}_{k-1} + \vec{\omega}_k$
    - G = PxP matrix, ω = perturbation associated with covariance PxP matrix V_ω
    - Update the covariance matrix with additional uncertainties (ex: material budget between layers) $V_{k|k-1} = V_{k-1} + V_{\omega_k}$
  - Add new point to update parameters and covariance, using the measure equation $\vec{m}_k = H\,\vec{p}_k + \vec{\varepsilon}_k$
    - H=DxP matrix, ε= measure error associated with diagonal covariance DxD matrix V_m
    - Weighted means of prediction and measurement using variance ⟺χ^2 fit
  - Iterate…

$$\vec{p}_k = \left(V_{k|k-1}^{-1}\vec{p}_{k|k-1} + H^T V_{m_k}^{-1}\vec{m}_k\right)\cdot\left(V_{k|k-1}^{-1} + H^T V_{m_k}^{-1} H\right)^{-1}$$

Kalman filter 2/2

Forward and backward filters

- Forward estimate of $\boldsymbol{p}_k$: from 1➛k-1 measurements
- Backward estimate of $\boldsymbol{p}_k$: from k+1➛K measurements
- Independent estimates ➞combination with weighted mean = smoother step

Computation complexity

- only PxP, DxP or DxD matrices computation (≪NxN)

Mixing with finder

- After propagation step: local finder
- Some points can be discarded if considered as outliers in the fit (use χ^2 value)

Include exogenous measurements

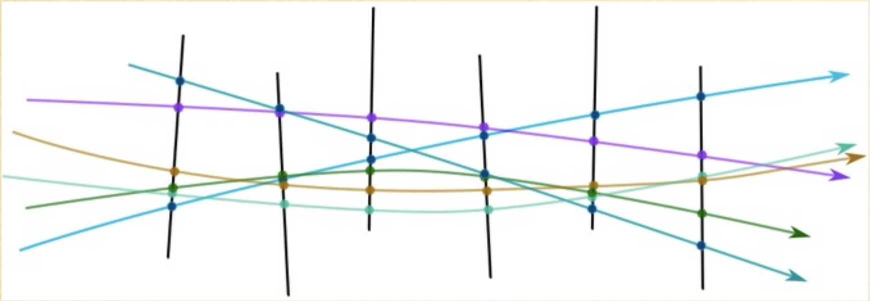
- Like dE/dx, correlated to momentum
- Additional measurement equation $\vec{m}'_k = H' \vec{p}_k + \vec{\varepsilon}'_k$

$$\vec{p}_k = \left(V_{k|k-1}^{-1} \vec{p}_{k|k-1} + H^T V_{m_k}^{-1} \vec{m}_k + H'^T V_{m'_k}^{-1} \vec{m}'_k \right) \cdot \left(V_{k|k-1}^{-1} + H^T V_{m_k}^{-1} H + H'^T V_{m'_k}^{-1} H' \right)^{-1}$$

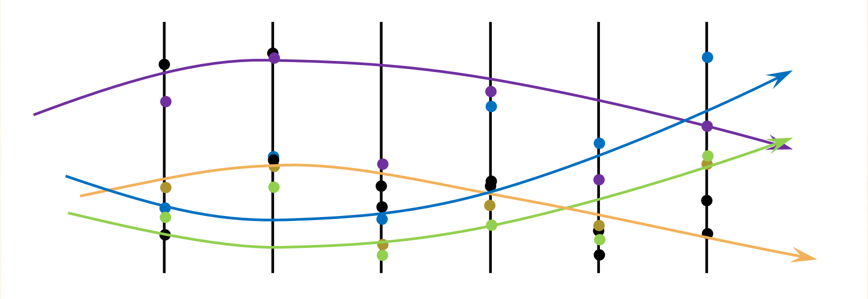
Need for Alignment

- Let's come back to one initial & implicit hypothesis
  - "We know were the point are located."
  - True to the extent we know were the detector is!
  - BUT, mechanical instability (magnetic field, temperature, air flow…) and also drift speed variation (temperature, pressure, field inhomogeneity…) limit our knowledge
  - Periodic determination of positions and deformations needed = alignment

True tracks & True detector positions



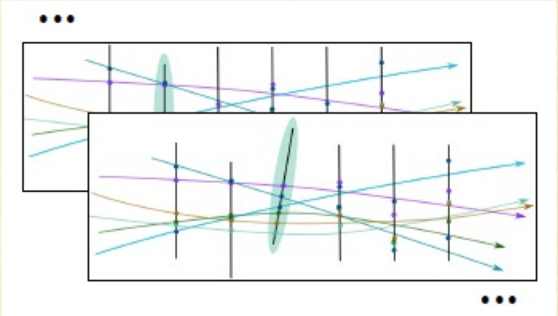
Initial assumption for detector positions & tracks built from these assumptions

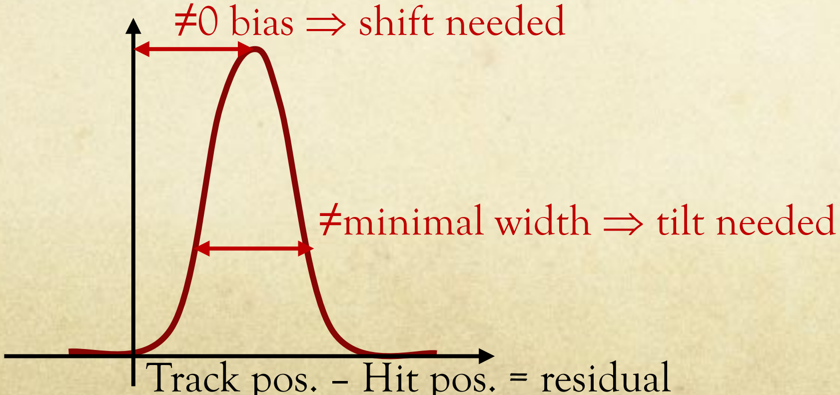


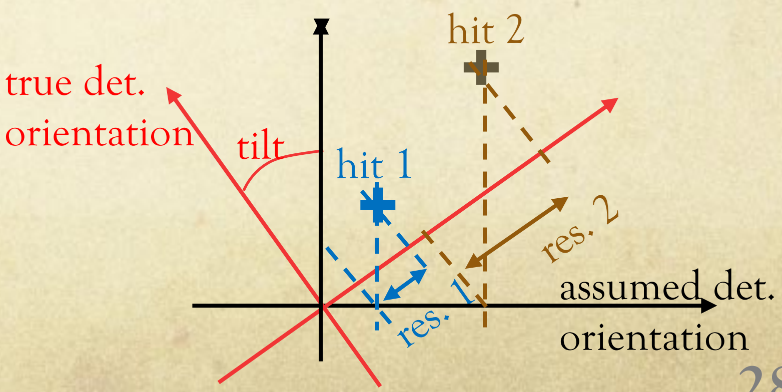
Note hit position relative to detector <u>are the same</u>
tracks reconstructed are not even close to reality… and this assuming hits can be properly associated together!

Alignment strategy 1/2

- **Alignment parameters**
  - Track model depends on additional “free” parameters, i.e. the sensor positions
- **Methods to find the relative position of individual sensors**
  - Global alignment:
    - Fit the new params. to minimize the overall χ^2 of a set of tracks
    - Beware: many parameters could be involved (few 10^3 can easily be reached) → Millepede algo.
  - Local alignment:
    - Use tracks reconstructed with reference detectors
    - Align other detectors by minimizing the “residual” (track-hit distance) width







Alignment strategy 2/2

- In both methods (global or local alignment)
  - Use a set of well know tracks and tracking-"friendly" environment to avoid bias
    - Muons (very traversing) and no magnetic field
    - Low multiplicity events
- Global deformations also possible
  - Invisible through single track χ^2 investigation
  - affect overall positions & momentum
  - Corrected through observing
    - Mass peak positions
    - Systematic differences at various track angles or detector positions