

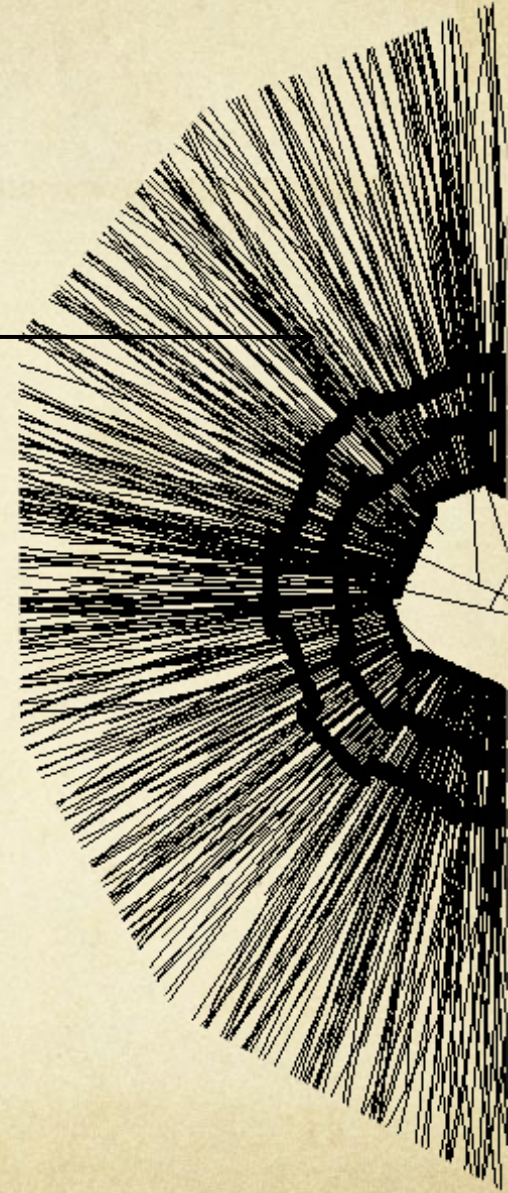


# Tracking 3/3

# 3. Fitting algorithms

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- Least square method (global)
- Kalman filter (local)
- Alignment



#### ○ Why do we need to fit?

- Measurement error
- Multiple scattering error

#### ○ Global fit

- Assume knowledge of:
  - all track points
  - full correlation matrix
    - difficult if  $\sigma_{\text{mult. scatt.}} \gtrsim \sigma_{\text{meas.}}$
- Least square method

#### ○ Iterative (local) fit

- Iterative process:
  - points included in the fit one by one
  - could be merged with finder step
- Kalman filter

**FITTING drives  
track extrapolation  
& momentum res.**

## ○ The rule

- For the fit: nb of constraints  $>$  nb of free parameters in the track model

## ○ Measurements

- 1 point in 2D = 1 constraint ( $x \leftrightarrow y$ ) or ( $r \leftrightarrow \phi$ )
- 1 point in 3D = 2 constraints ( $x \leftrightarrow z$  &  $y \leftrightarrow z$ )

## ○ Models

- Straight track in 2D = 2 parameters
  - 1 coordinate @ origin ( $z=0$ ), 1 slope
- Straight track in 3D = 4 parameters
  - 2 coordinates @ origin, 2 slopes
- Circle in 2D = 3 parameters
  - 2 coordinates for center, 1 radius
- Helix in 3D = 5 parameters
  - 3 coordinates for center, 1 radius, 1 dip angle

## ○ Minimal #points needed

← 2 points in 2D

← 2 points in 3D

← 3 points in 2D

← 3 points in 3D

### ○ Linear model hypothesis

- P track parameters  $\mathbf{p}$ , with N measurements  $\mathbf{c}$

$$\vec{c} = \vec{c}_s + A(\vec{p} - \vec{p}_s) + \vec{\varepsilon}$$

- $\mathbf{p}_s$  = known starting point (pivot),  $\mathbf{A}$  = track model NxP matrix,  $\boldsymbol{\varepsilon}$  = error vector corresponding to  $\mathbf{V}$  = covariance NxN matrix

“N measurements” means:

- K points (or layers)
- D coordinates at each point
- $N = K \times D$

### ○ Sum of squares:

$$\sum \frac{(\text{model} - \text{measure})^2}{\text{uncertainty}^2} \quad \longrightarrow \quad S(\vec{p}) = (\vec{c}_s + A(\vec{p} - \vec{p}_s) - \vec{c})^T V^{-1} (\vec{c}_s + A(\vec{p} - \vec{p}_s) - \vec{c})$$

### ○ Best estimator (minimizing variance)

$$\frac{dS}{d\vec{p}}(\vec{p}) = 0 \quad \longrightarrow \quad \underline{\vec{p}} = \vec{p}_s + (A^T V^{-1} A)^{-1} A^T V^{-1} (\vec{c} - \vec{c}_s)$$

- Variance (= uncertainty) of the estimator:

$$\underline{V}_{\vec{p}} = (A^T V^{-1} A)^{-1}$$

- Estimator p follows a  $\chi^2$  law with N-P degrees of freedom

### ○ Problem $\Leftrightarrow$ inversion of a PxP matrix ( $A^T V^{-1} A$ )

- But real difficulty could be computing  $\mathbf{V}$  (NxN matrix)

<= layer correlations if multiple scattering non-negligible if  $\sigma_{\text{mult. scatt.}} \gtrsim \sigma_{\text{meas}}$

Generic tool for fitting:  
<https://genfit.sourceforge.net>

### 3. Fitting algorithms:

## LSM on straight tracks

#### ○ Straight line model

- 2D case → D=2 coordinates (z,x)
- 2 parameters: a = slope, b = intercept at z=0

#### ○ General case

- K+1 detection planes (i=0...k)
  - located at  $z_i$
  - Spatial resolution  $\sigma_i$

- Useful definitions

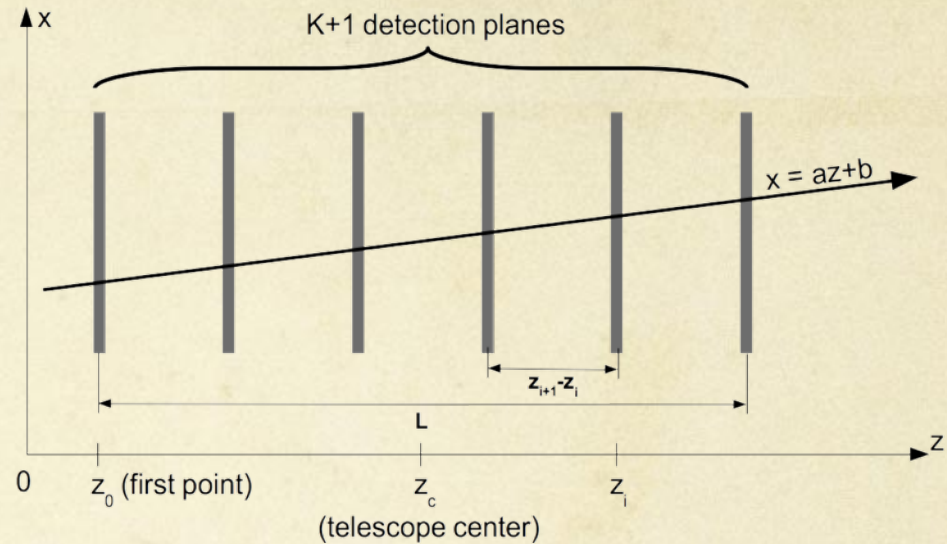
$$S_1 = \sum_{i=0}^K \frac{1}{\sigma_i^2}, \quad S_z = \sum_{i=0}^K \frac{z_i}{\sigma_i^2}, \quad S_{xz} = \sum_{i=0}^K \frac{x_i z_i}{\sigma_i^2}, \quad S_{z^2} = \sum_{i=0}^K \frac{z_i^2}{\sigma_i^2}$$

- Solutions  $a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2}, \quad b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$

- Uncertainties

$$\sigma_a^2 = \frac{S_1}{S_1 S_{z^2} - (S_z)^2}, \quad \sigma_b^2 = \frac{S_{z^2}}{S_1 S_{z^2} - (S_z)^2}$$

**! correlation**  $\text{cov}_{a,b} = \frac{-S_z}{S_1 S_{z^2} - (S_z)^2}$



#### ○ Case of uniformly distributed (K+1) planes

- $z_{i+1} - z_i = L/K$  et  $\sigma_i = \sigma \quad \forall i$

- $S_z = 0$  → a, b uncorrelated

$$\sigma_a^2 = \frac{12K}{(K+2)L^2} \frac{\sigma^2}{K+1}, \quad \sigma_b^2 = \left( 1 + 12 \frac{K}{K+2} \frac{z_c^2}{L^2} \right) \frac{\sigma^2}{K+1}$$

- Uncertainties :

- $\sigma_a$  and  $\sigma_b$  improve with  $1/\sqrt{K+1}$
- $\sigma_a$  and  $\sigma_b$  improve with  $1/L$
- $\sigma_b$  improve with  $z_c$

### 3. Fitting algorithms:

## LSM on fixed target geometry

#### ○ Hypothesis

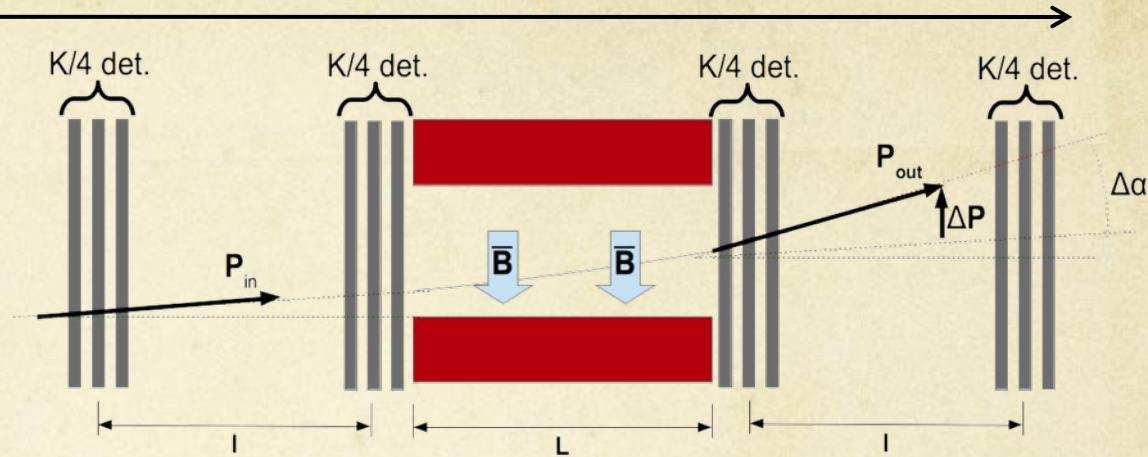
- K detectors, each with  $\sigma$  single point accuracy
- Uniform field over L from dipole

- Trajectory:  $\Delta\alpha = \left| \frac{0.3qBL}{p} \right|$

- Bending:  $\Delta p = p \Delta\alpha$

- Geometrical arrangement optimized for resolution

- Angular determination on input and output angle:  $\sigma_\alpha^2 = \frac{16 \sigma^2}{K l^2}$



#### ○ Without multiple scattering

- Uncertainty on momentum

- Note proportionality to  $p$  and to  $\frac{1}{BL}$



$$\frac{\sigma_p}{p} = \frac{8}{0.3q} \frac{1}{BL} \frac{\sigma}{l\sqrt{K}} p$$

#### ○ Multiple scattering contribution

- Bring **additive** term proportional to  $K$

$$\text{and } \sigma_\theta = \frac{13.6 \text{ (MeV/c)}}{\beta p} \sqrt{\frac{\text{thickness}}{X_0}}$$



$$\frac{\sigma_p}{p} (ms) = A_K \frac{13.6 \text{ (MeV/c)}}{\beta} \sqrt{\frac{\text{total thickness}}{X_0}}$$

=> Constant with  $p$ !

$A_K$  = factor depending on geometrical arrangement

### 3. Fitting algorithms:

## LSM on collider geometry

#### ○ Hypothesis

- K detectors uniformly distributed each with  $\sigma$  single point accuracy
- Uniform field over path length L

#### ○ Without multiple scattering

- Uncertainty on transverse momentum (Glückstern formula)

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sqrt{720}}{0.3q} \frac{1}{BL^2} \frac{\sigma}{\sqrt{K+6}} p_T$$

- Works well with large  $K > 20$

=> Proportional to p and to  $\frac{1}{BL^2}$ !

#### ○ Multiple scattering contribution

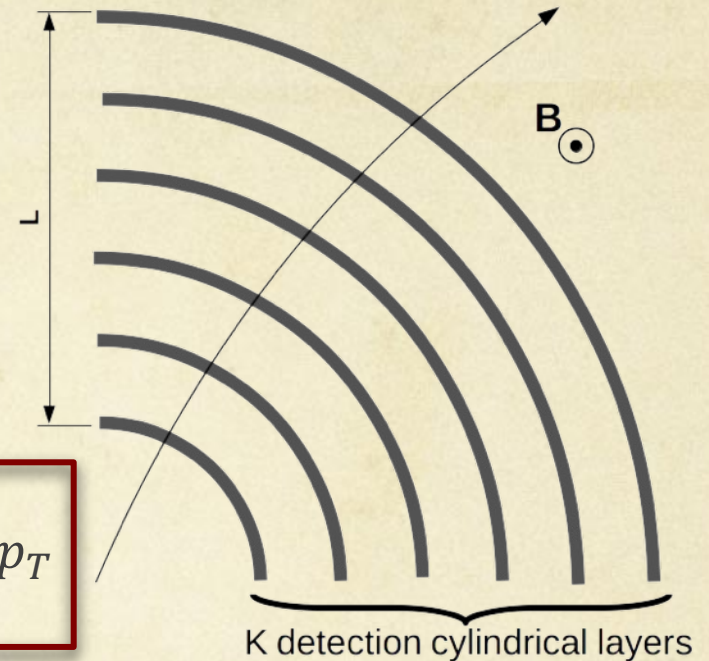
- Brings additive contribution

$$\frac{\sigma_{p_T}}{p_T} = \frac{1.43}{0.3q} \frac{1}{BL} \sqrt{\frac{13.6 \text{ (MeV/c) total thickness}}{\beta X_0}}$$

\*Numerical factors  $\sqrt{\frac{720}{K+6}}$  and 1.43 can be refined  
→ see <https://arxiv.org/abs/1805.12014>

=> Constant with p!

=> Depends on K through total thickness!





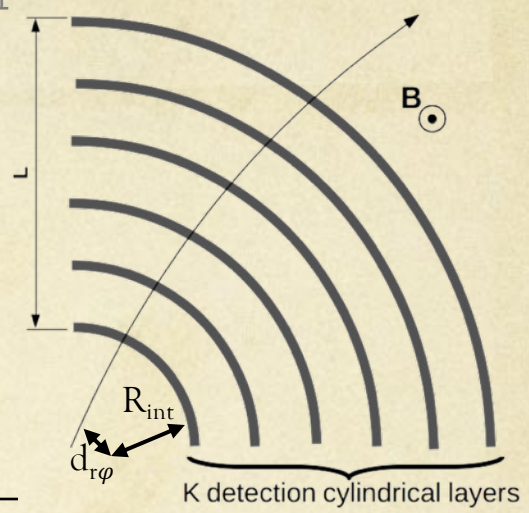
### 3. Fitting algorithms:

## LSM for impact parameter ( $d_{r\varphi}$ )

→ see <https://arxiv.org/abs/1805.12014>

#### ○ Hypothesis

- K detectors uniformly distributed over L
- each with  $\sigma$  single point accuracy
- First layer is close to PV / L =>  $r = R_{int}/L < 1$



#### ○ Without multiple scattering $\sigma_{d_{r\varphi}} = \frac{\sqrt{3}\sigma}{\sqrt{K+4}} \sqrt{1+8r+28r^2}$

#### ○ Multiple scattering contribution

- Brings additive contribution proportional to  $\sigma_\theta = \frac{13.6 \text{ (MeV/c)}}{\beta p_T} \sqrt{\frac{\text{thickness}}{X_0}}$

$$\sigma_{d_{r\varphi}}(ms) = r\sigma_\theta \sqrt{1 + \frac{1}{2}r + \frac{K-1}{4}r^2} \text{ cv}$$

=> Proportional to r and K!

#### ○ Key points

- Minimising  $R_{int}/L \leftrightarrow$  getting close & keeping lever arm
- Multiple scattering destroys statistical gain of  $K>2$

○ Final parametrisation

$$\sigma_{d_{r\varphi}} = a \oplus \frac{b}{p_T \sin^2 \theta}$$

$a \propto \sigma$   
 $b \propto \sigma_\theta$

### 3. Fitting algorithms:

# Kalman filter 1/2

#### ○ Dimensions

- P parameters for track model
- D “coordinates” measured at each point (usually  $D < P$ )
- K measurement points (# total measures:  $N = K \times D$ )

#### ○ Starting point

- Initial set of parameters: first measurements
- With large uncertainties if unknowns

#### ○ Iterative method

- Propagate to next layer = prediction

- Using the **system equation**  $\vec{p}_k = G \vec{p}_{k-1} + \vec{\omega}_k$
- $G = P \times P$  matrix,  $\omega$  = perturbation associated with covariance  $P \times P$  matrix  $V_\omega$

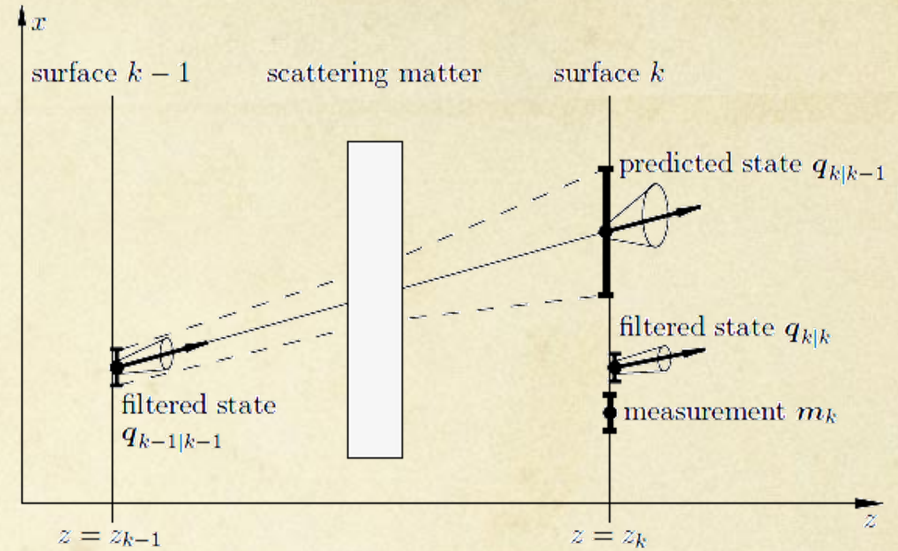
- Update the covariance matrix with additional uncertainties  $V_{k|k-1} = V_{k-1} + V_{\omega_k}$   
(ex: material budget between layers)

- Add new point to update parameters and covariance, using the **measure equation**  $\vec{m}_k = H \vec{p}_k + \vec{\epsilon}_k$

- $H = D \times P$  matrix,  $\epsilon$  = measure error associated with **diagonal** covariance  $D \times D$  matrix  $V_m$
- Weighted means of prediction and measurement using variance  $\Leftrightarrow \chi^2$  fit

- Iterate...

$$\vec{p}_k = \left( V_{k|k-1}^{-1} \vec{p}_{k|k-1} + H^T V_{m_k}^{-1} \vec{m}_k \right) \cdot \left( V_{k|k-1}^{-1} + H^T V_{m_k}^{-1} H \right)^{-1}$$





### ○ Forward and backward filters

- Forward estimate of  $\vec{p}_k$ : from 1 → k-1 measurements
- Backward estimate of  $\vec{p}_k$ : from k+1 → K measurements
- Independent estimates → combination with weighted mean = smoother step

### ○ Computation complexity

- only PxP, DxP or DxD matrices computation ( $\ll N \times N$ )

### ○ Mixing with finder

- After propagation step: local finder
- Some points can be discarded if considered as outliers in the fit (use  $\chi^2$  value)

### ○ Include exogenous measurements

- Like dE/dx, correlated to momentum
- Additional measurement equation

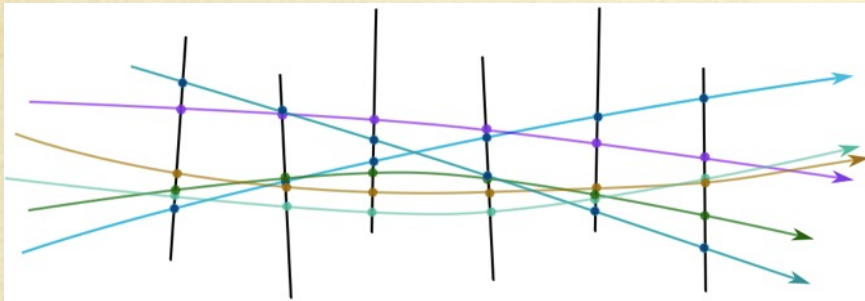
$$\vec{m}'_k = H' \vec{p}_k + \vec{\epsilon}'_k$$

$$\vec{p}_k = \left( V_{k|k-1}^{-1} \vec{p}_{k|k-1} + H^T V_{m_k}^{-1} \vec{m}_k + H'^T V_{m'_k}^{-1} \vec{m}'_k \right) \cdot \left( V_{k|k-1}^{-1} + H^T V_{m_k}^{-1} H + H'^T V_{m'_k}^{-1} H' \right)^{-1}$$

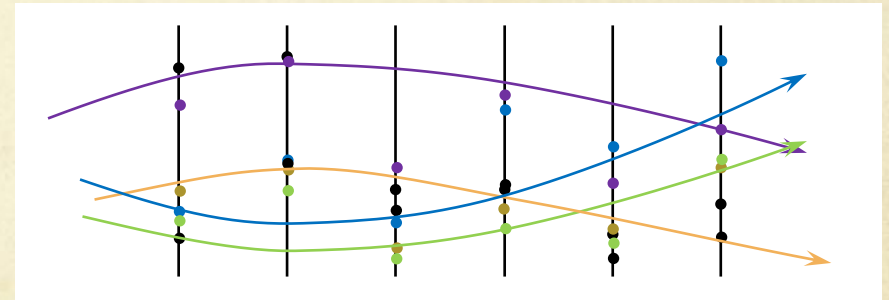
## ○ Let's come back to one initial & implicit hypothesis

- “We know where the points are located.”
- True to the extent we know where the detector is!
- BUT, mechanical instability (magnetic field, temperature, air flow...) and also drift speed variation (temperature, pressure, field inhomogeneity...) limit our knowledge
- Periodic determination of positions and deformations needed = alignment

True tracks & True detector positions



Initial assumption for detector positions & tracks built from these assumptions



Note hit position relative to detector are the same  
tracks reconstructed are not even close to reality...  
and this assuming hits can be properly associated  
together!

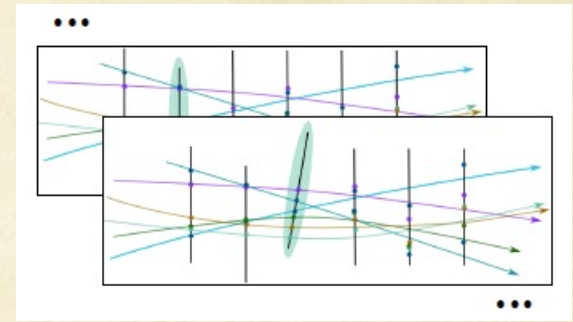
### ○ Alignment parameters

- Track model depends on additional “free” parameters, i.e. the sensor positions

### ○ Methods to find the relative position of individual sensors

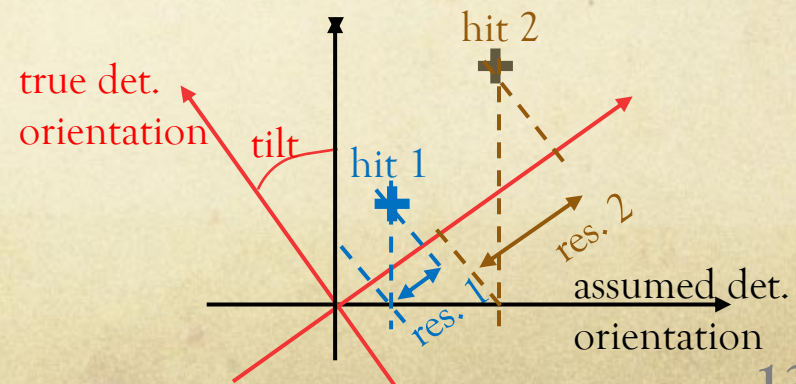
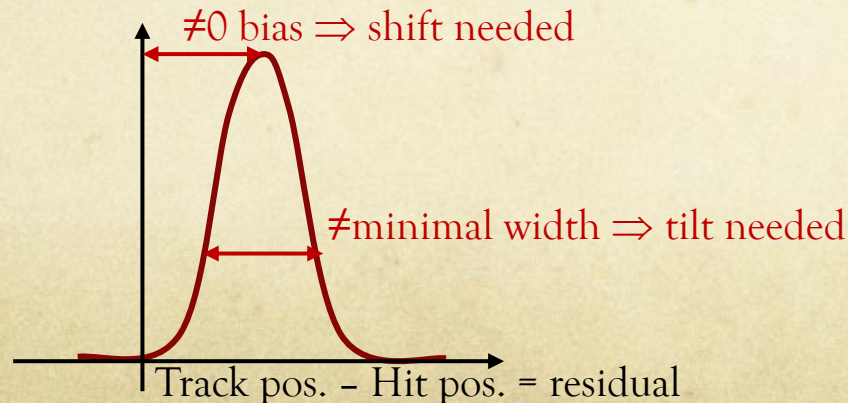
#### → Global alignment:

- Fit the new params. to minimize the overall  $\chi^2$  of a set of tracks
- Beware: many parameters could be involved (few  $10^3$  can easily be reached) → Millepede algo.



#### → Local alignment:

- Use tracks reconstructed with reference detectors
- Align other detectors by minimizing the “residual” (track-hit distance) width

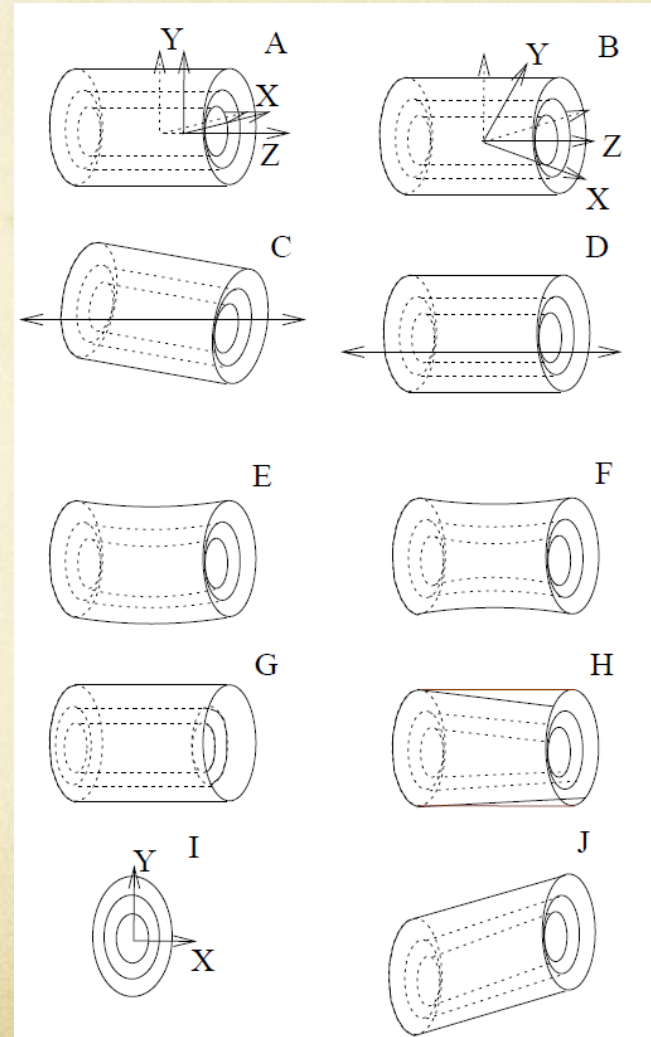


### ○ In both methods (global or local alignment)

- Use a set of well know tracks and tracking-”friendly” environment to avoid bias
  - Muons (very traversing) and no magnetic field
  - Low multiplicity events

### ○ Global deformations also possible

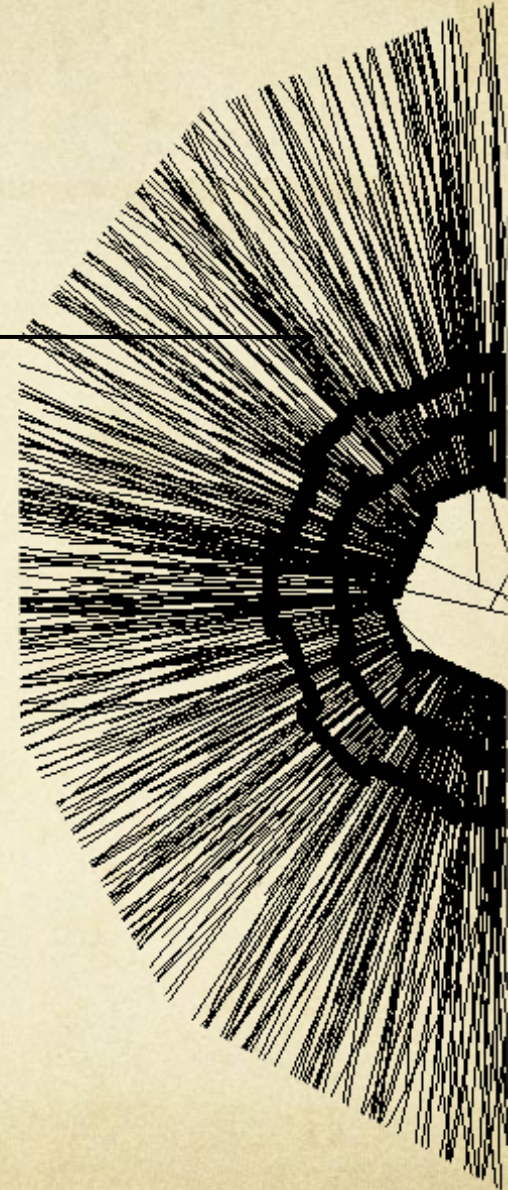
- Invisible through single track  $\chi^2$  investigation
- affect overall positions & momentum
- Corrected through observing
  - Mass peak positions
  - Systematic differences at various track angles or detector positions



# 4. Deconstructing some tracking systems

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- Collider setups:
  - CBM, LHCb
  - ALICE, Belle II, CMS
- Telescope setups:
  - AMS, ANTARES, OPERA
- Something totally(?) different
  - AGATA



## 4. Some tracking systems:

# Lessons from figures of merit

### ○ Excellent track finder requires

- Granular sensors in space & time
- Layer close to each other

### ○ Excellent momentum resolution requires

- Many measurement points
- Measurement over large lever arm
- Low material budget  
(depending on momentum range)

### ○ Excellent vertexing requires

- Very granular sensors in space
- Short & long distances
- Low material budget

Can a single algorithm all track search?

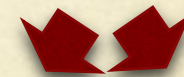


NO  
real tracking is done with many steps

Can we get all that with a single sub-detector?



NO  
(though they are similarities)



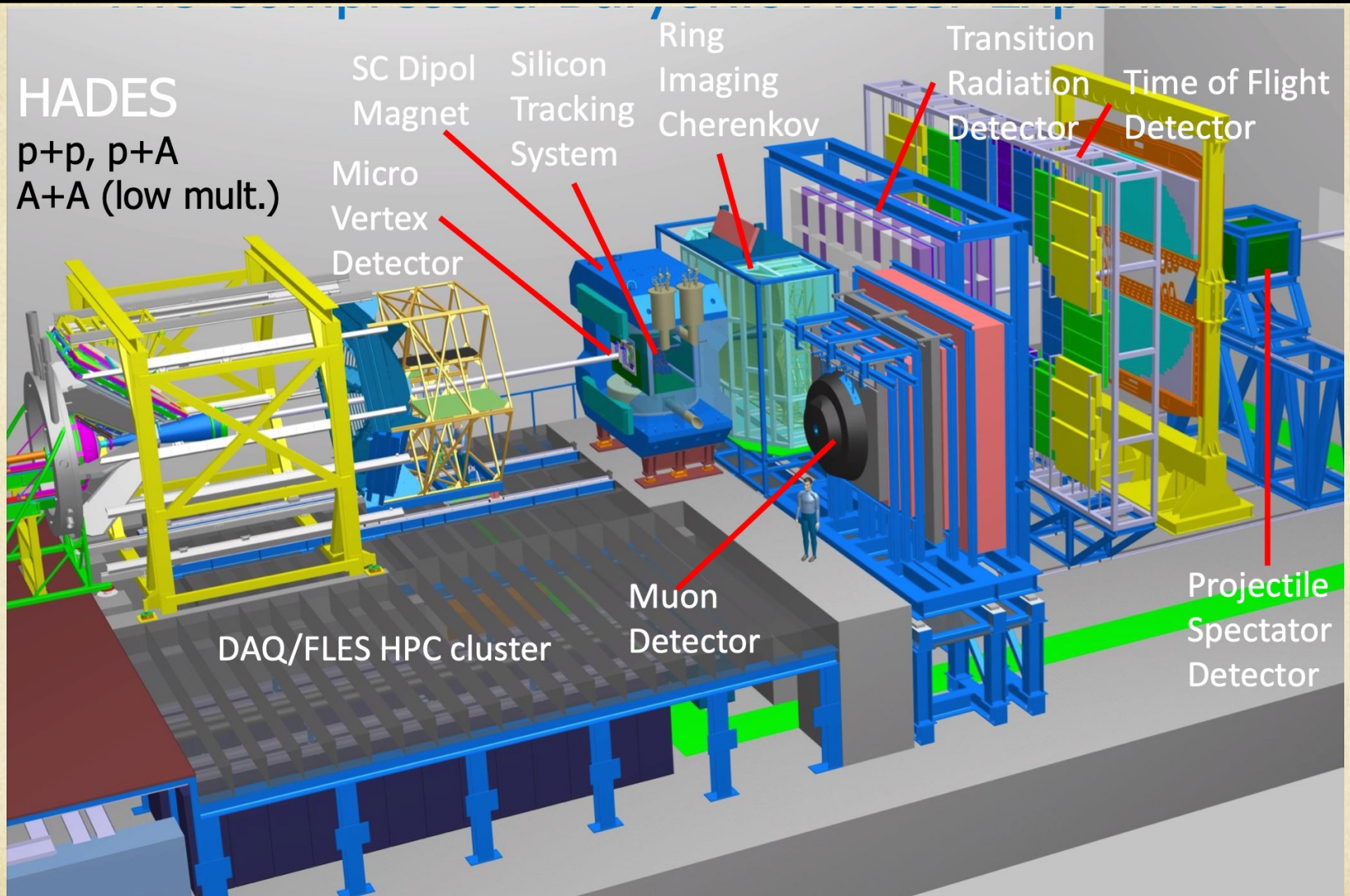
Trackers

Vertex locators



#### 4. Some tracking systems:

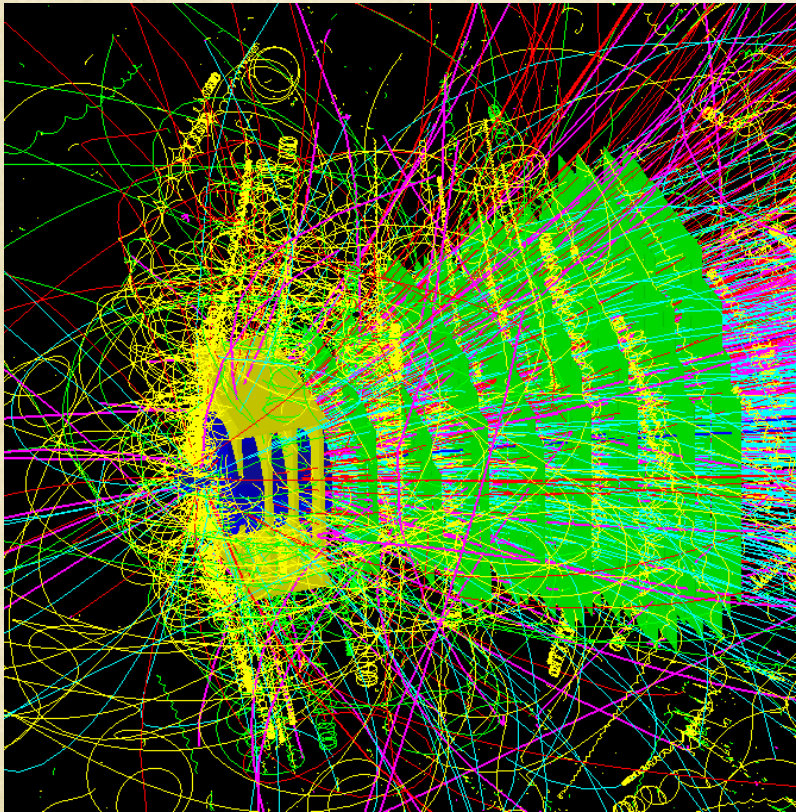
## CBM @ FAIR/SIS setup



## 4. Some tracking systems:

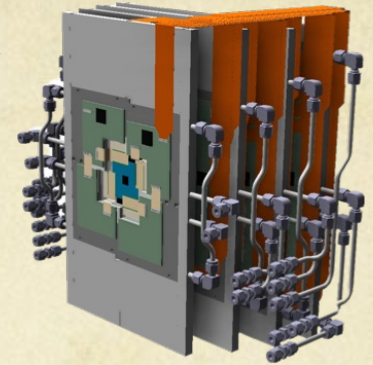
# CBM @ FAIR/SIS setup

- $10^5$ - $10^7$  Au+Au/sec,  $10^9$  p+Au/sec

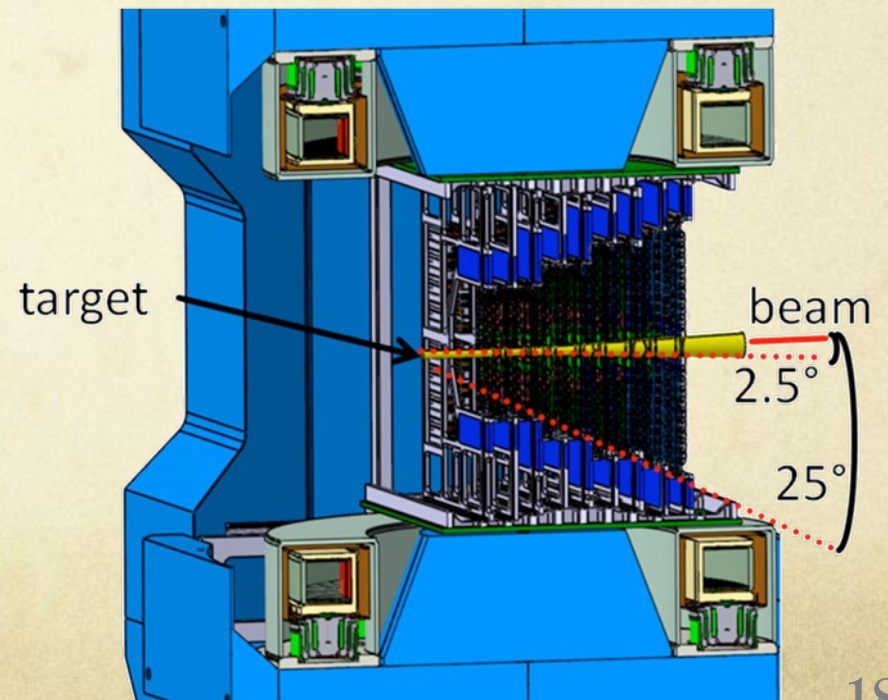


Simulated Au+Au collision at 8 A GeV

Micro Vertex Detector  
(MAPS)



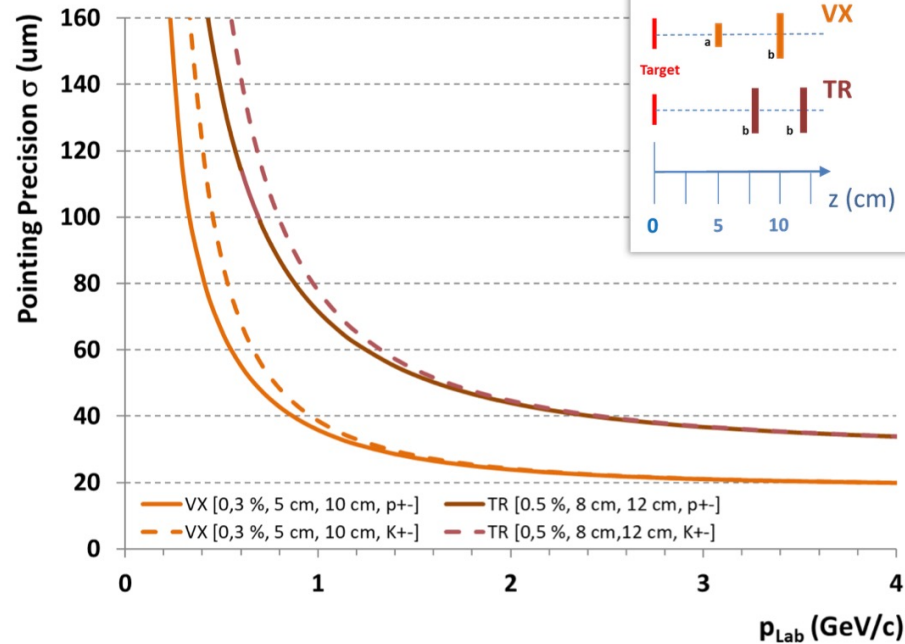
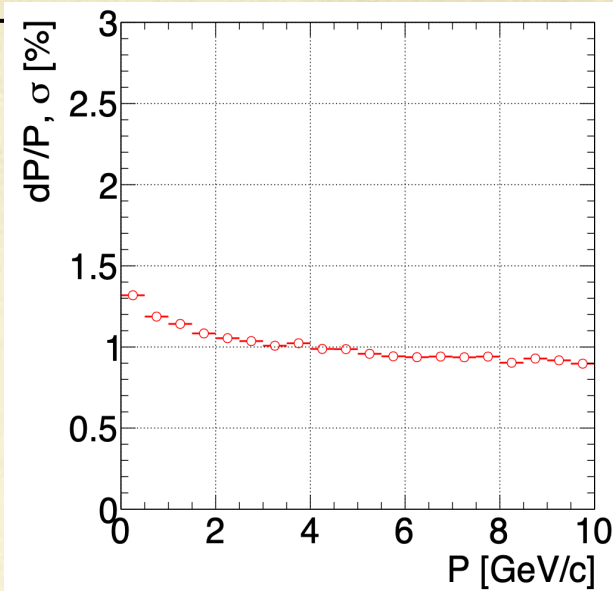
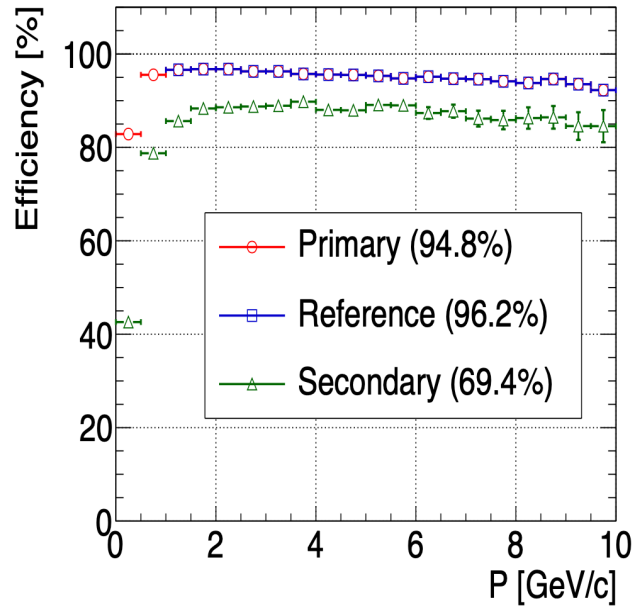
Silicon Tracking System  
(double-sided strip sensors)



## 4. Some tracking systems:

Track finder: cellular automaton  
 ~700 tracks/event

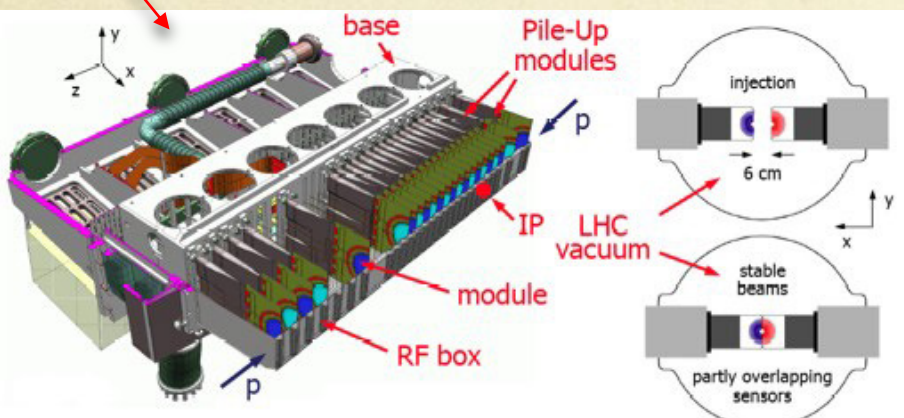
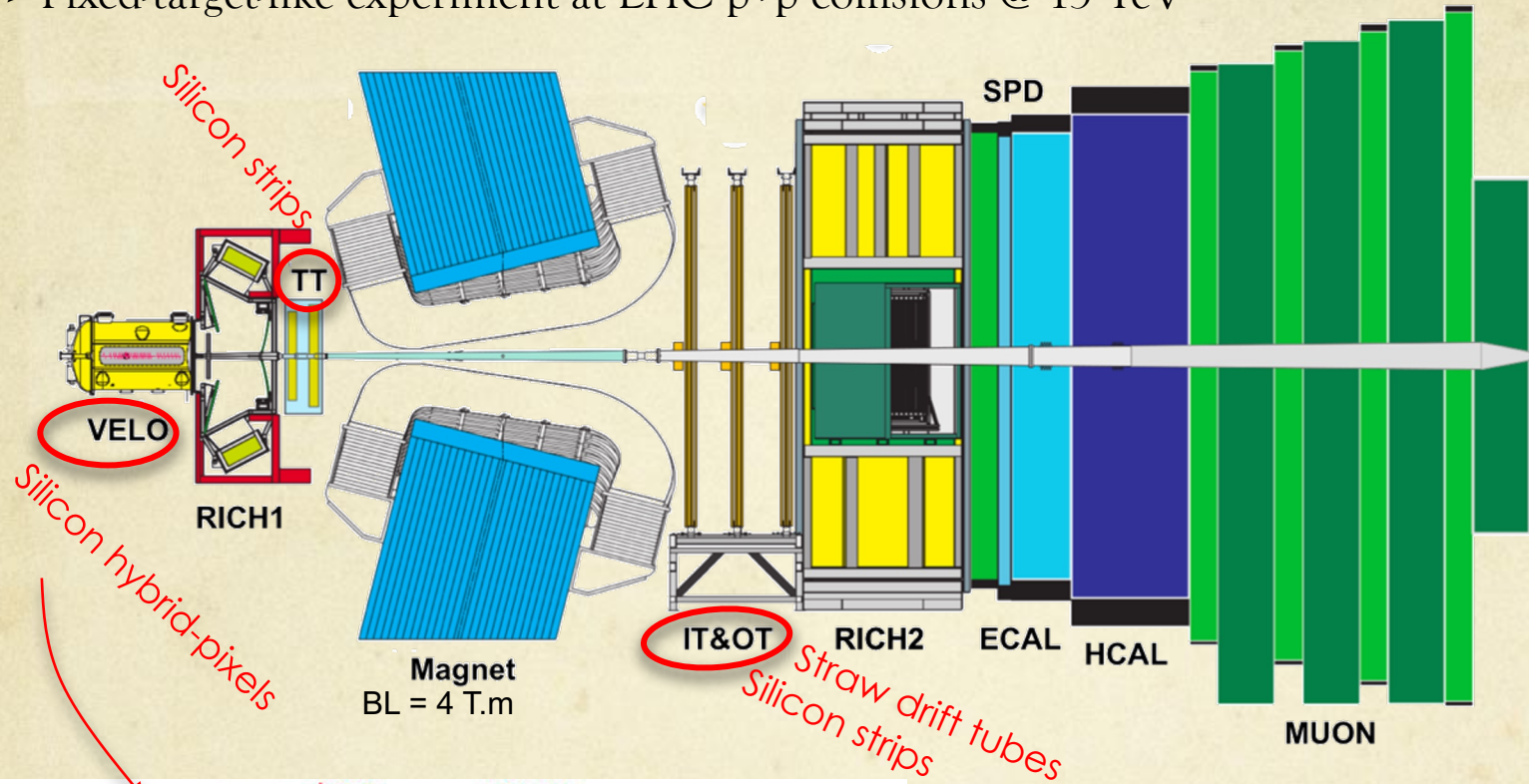
# CBM @ FAIR/SIS performance



# 4. Some tracking systems:

# LHCb setup

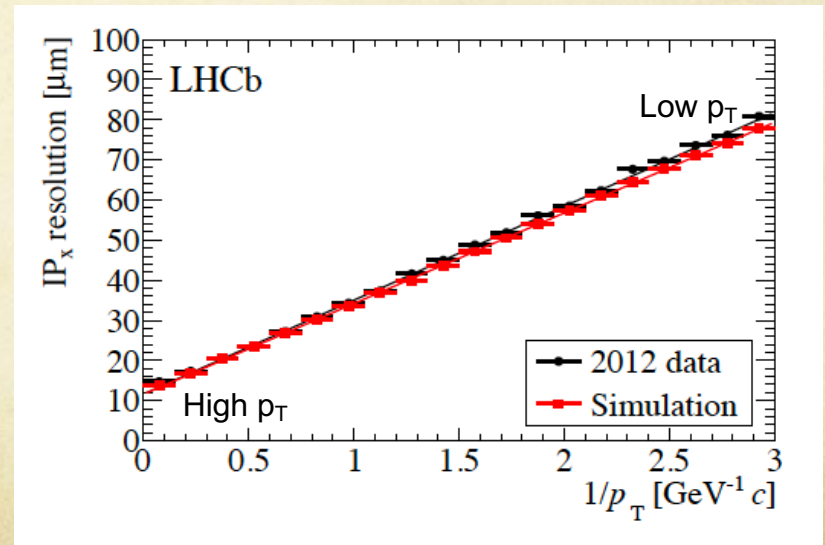
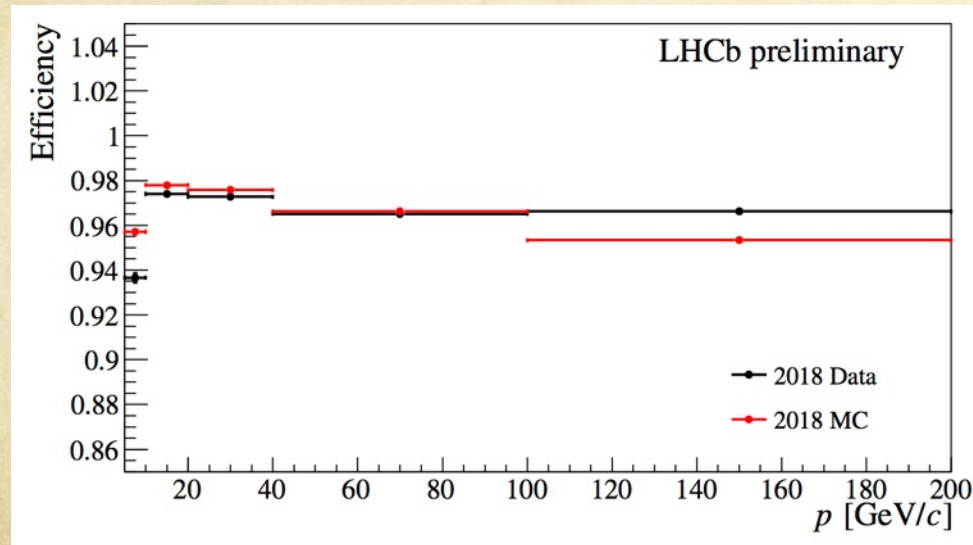
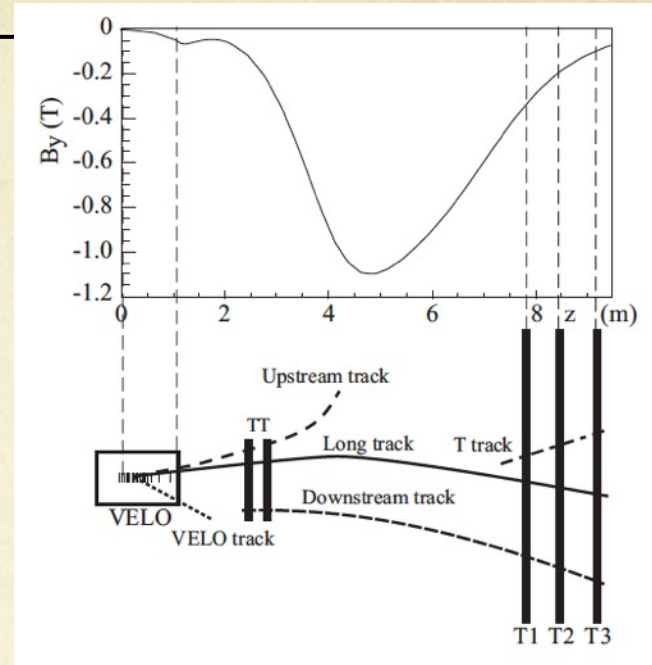
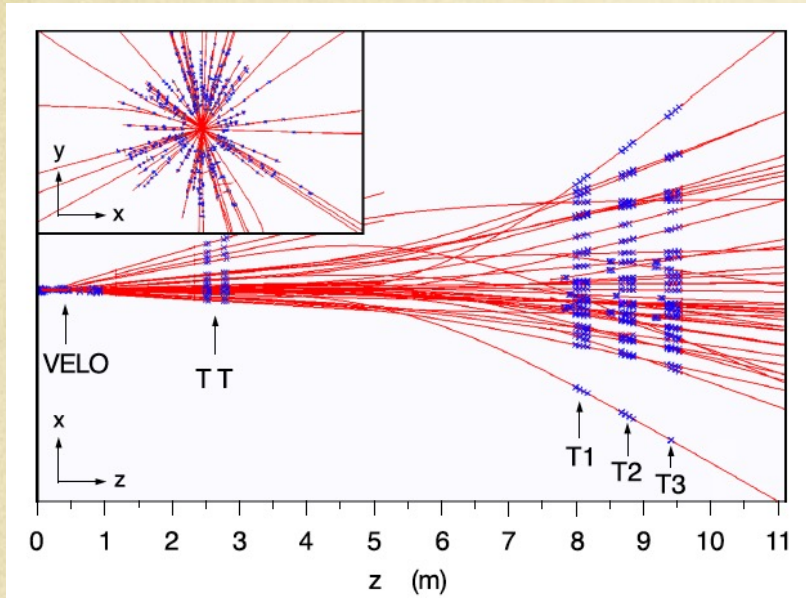
→ Fixed-target-like experiment at LHC p+p collisions @ 13 TeV



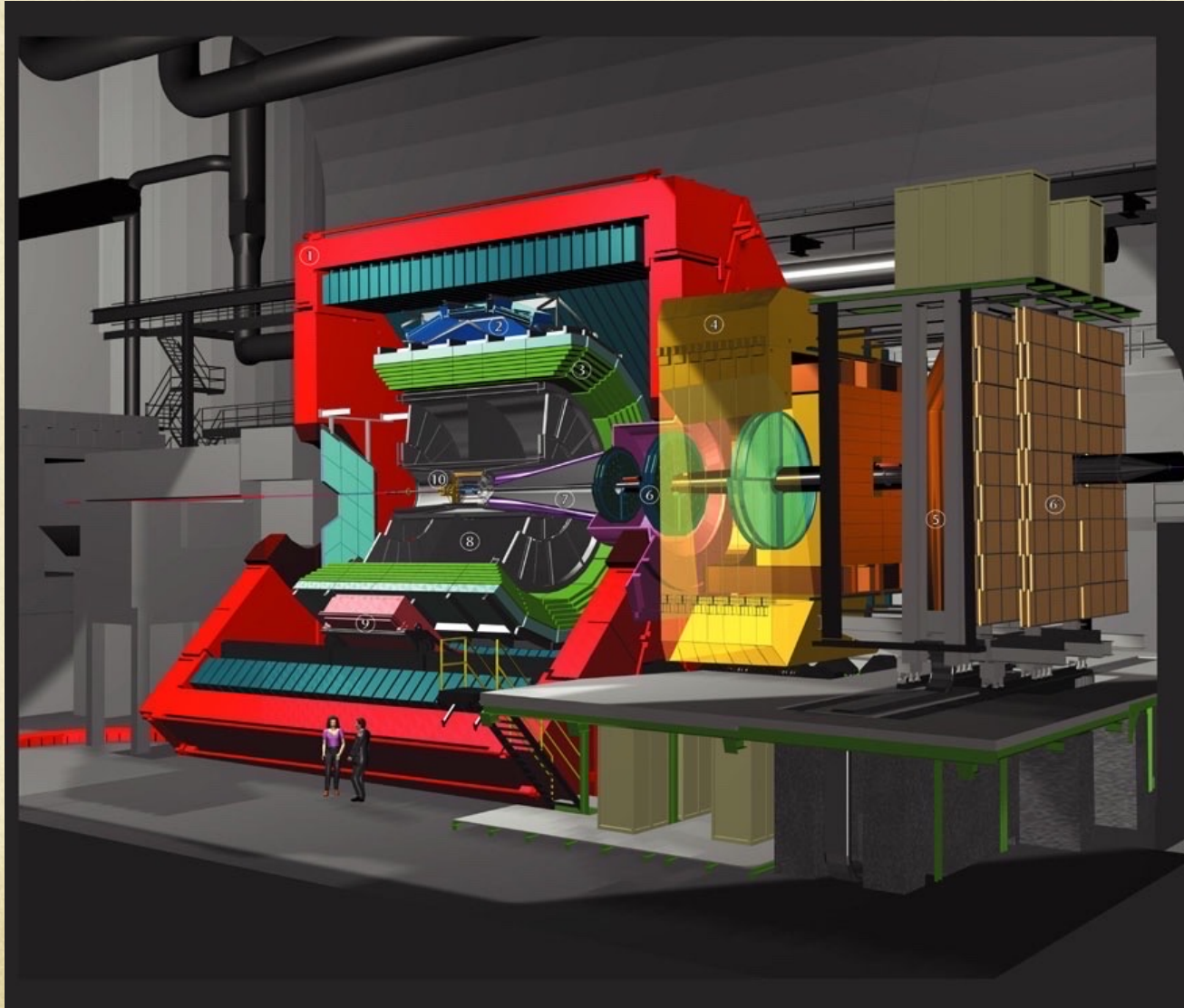
- Collision rate 40 MHz
- Trigger rates (hw & sw)  
1 MHz / 100 KHz / 12 KHz

#### 4. Some tracking systems:


# LHCb performance



A+A, A+p, p+p collisions



- 1• L3 MAGNET
- 2• HMPID
- 3• TOF
- 4• DIPOLE MAGNET
- 5• MUON FILTER
- 6• TRACKING CHAMBERS
- 6• TRIGGER CHAMBERS
- 7• ABSORBER
- 8• TPC
- 9• PHOS
- 10• ITS



← Largest TPC

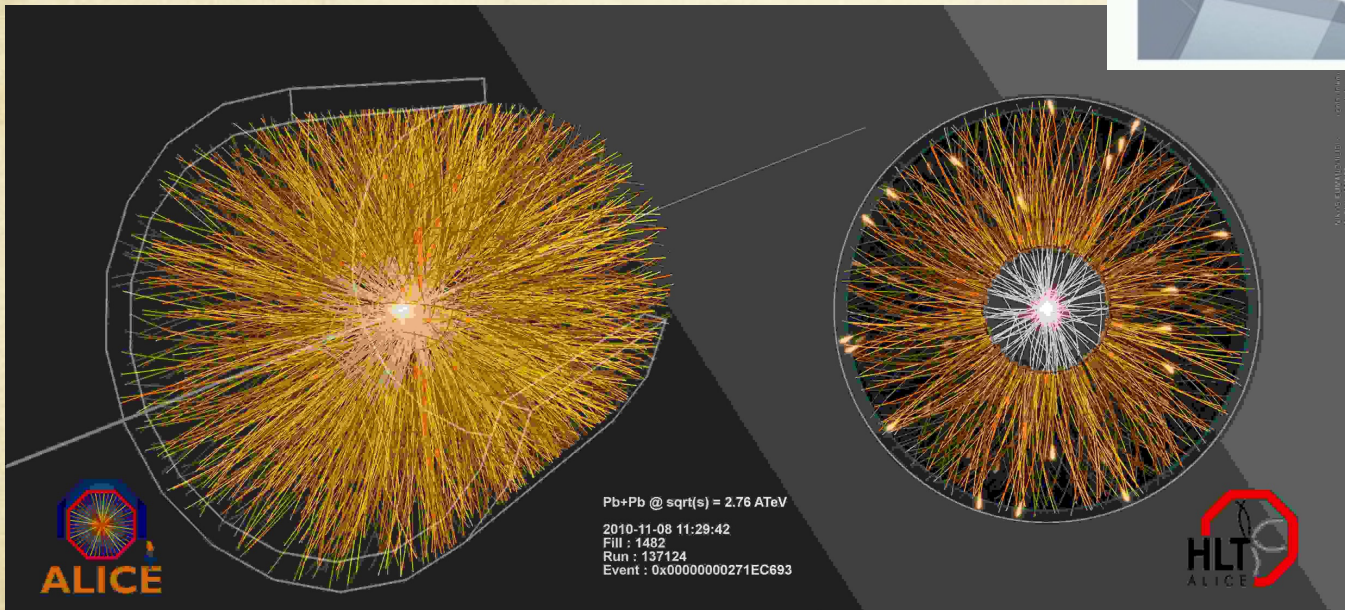
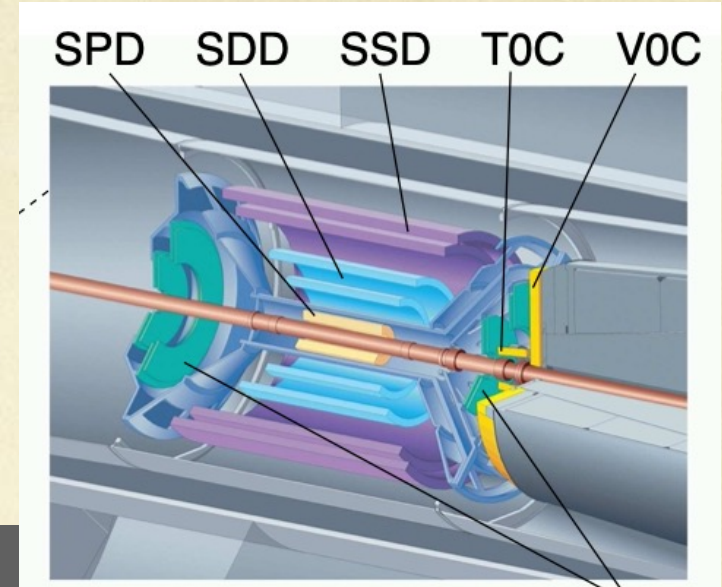
← Since 2022,  
largest MAPS  
area (10 m<sup>2</sup>,  
13 Gpixels)

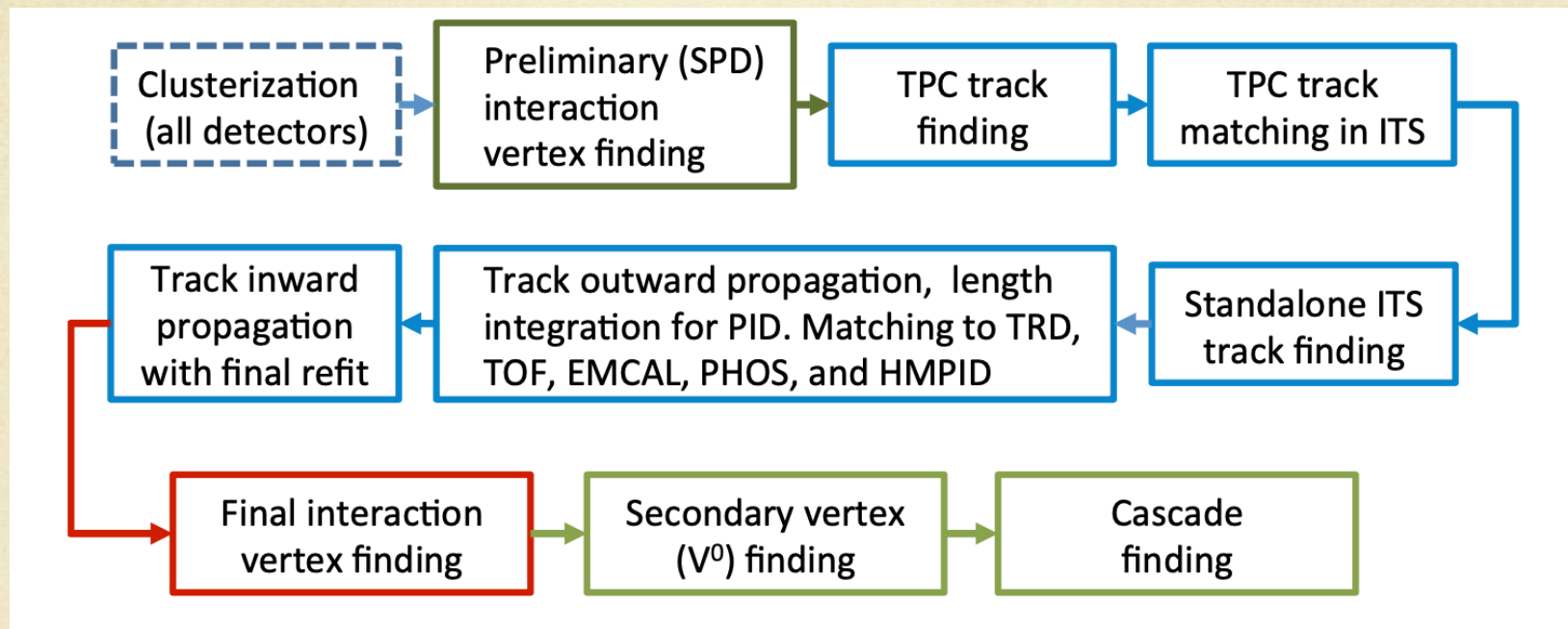
## 4. Some tracking systems:

# ALICE setup

### ○ Inside the large TPC, the Inner Tracking System

- Radius < 43 cm
- Hybrid pixels: SPD
- Silicon drift detector SDD
- Double-sided strips SSD

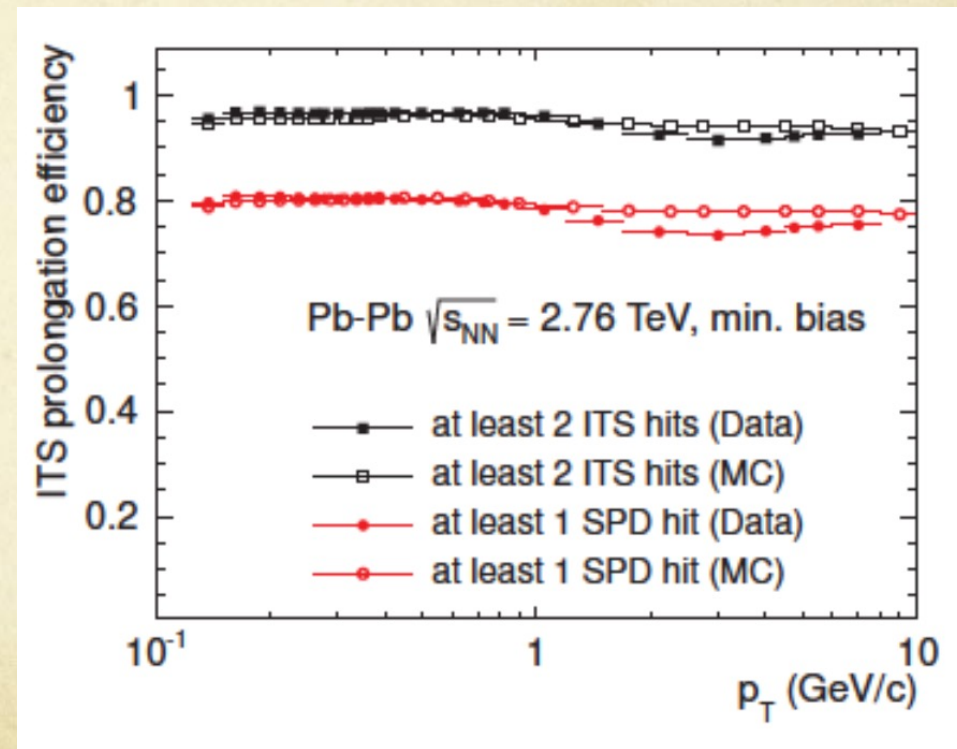
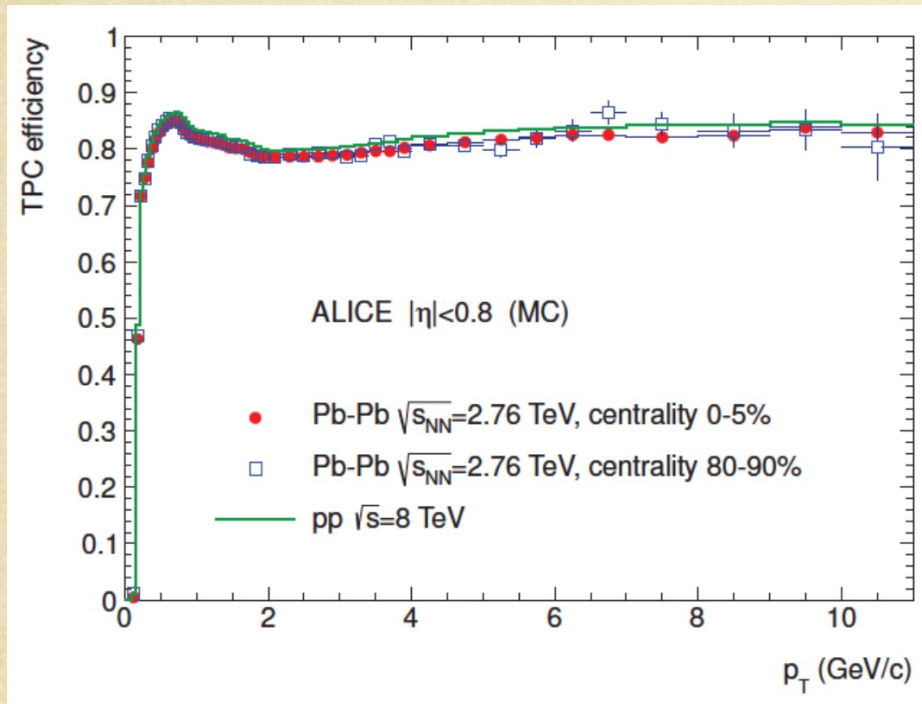






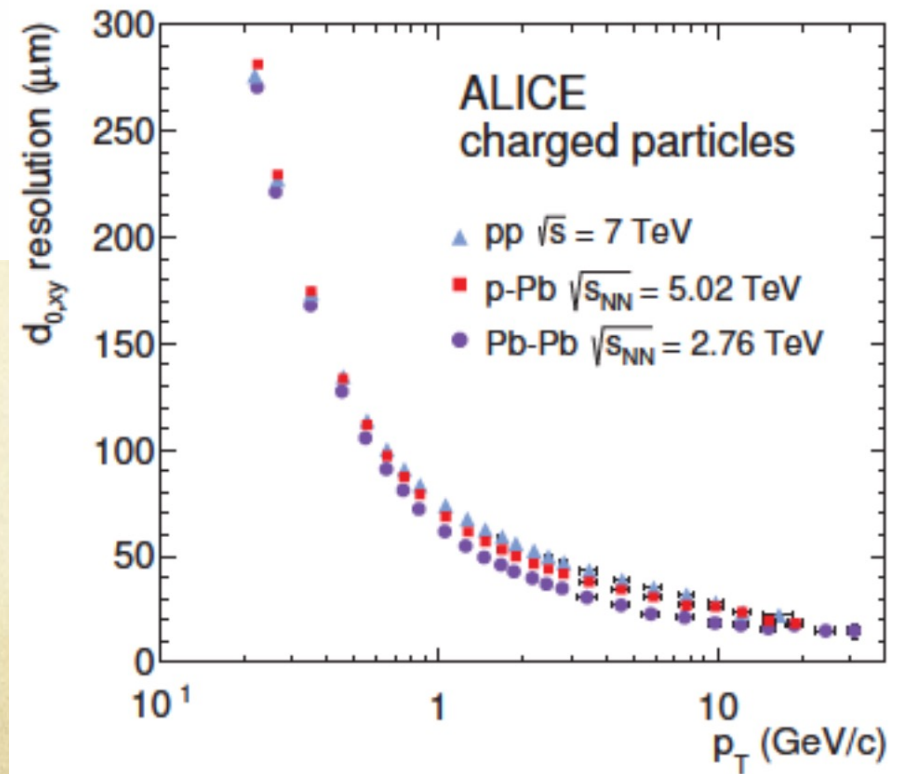
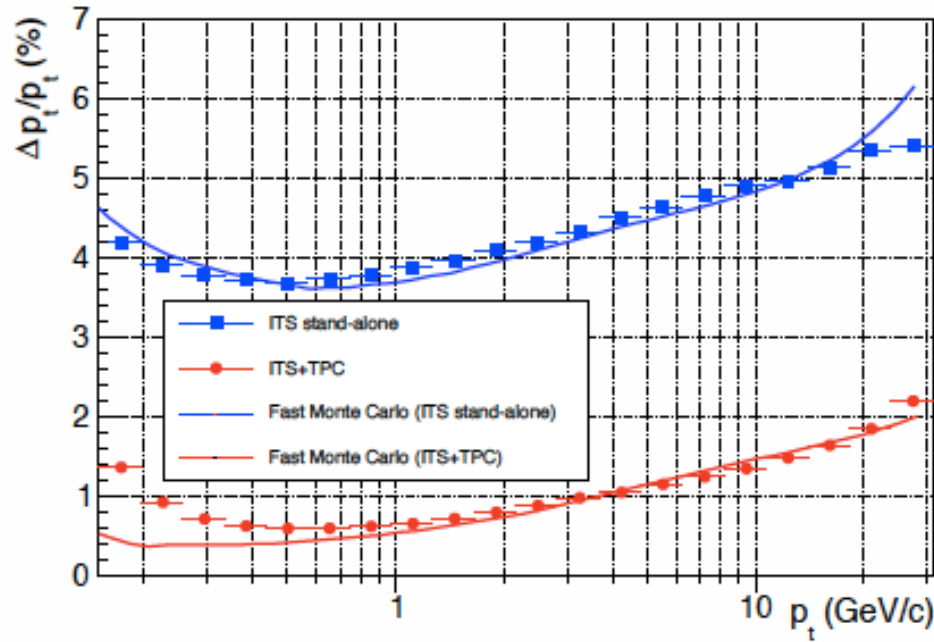
## 4. Some tracking systems:

# ALICE performance



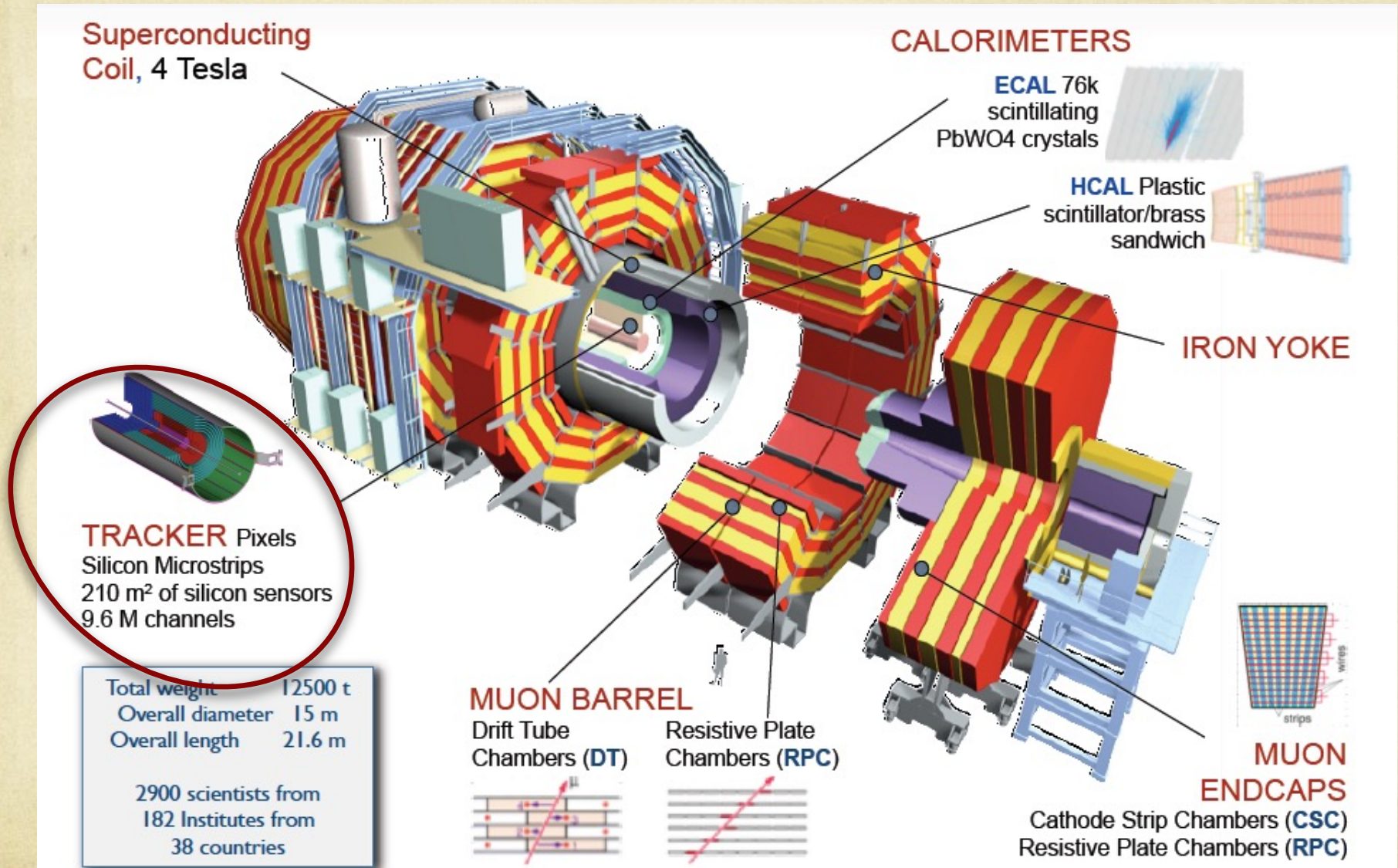
## 4. Some tracking systems:

# ALICE performance



# 4. Some tracking systems:

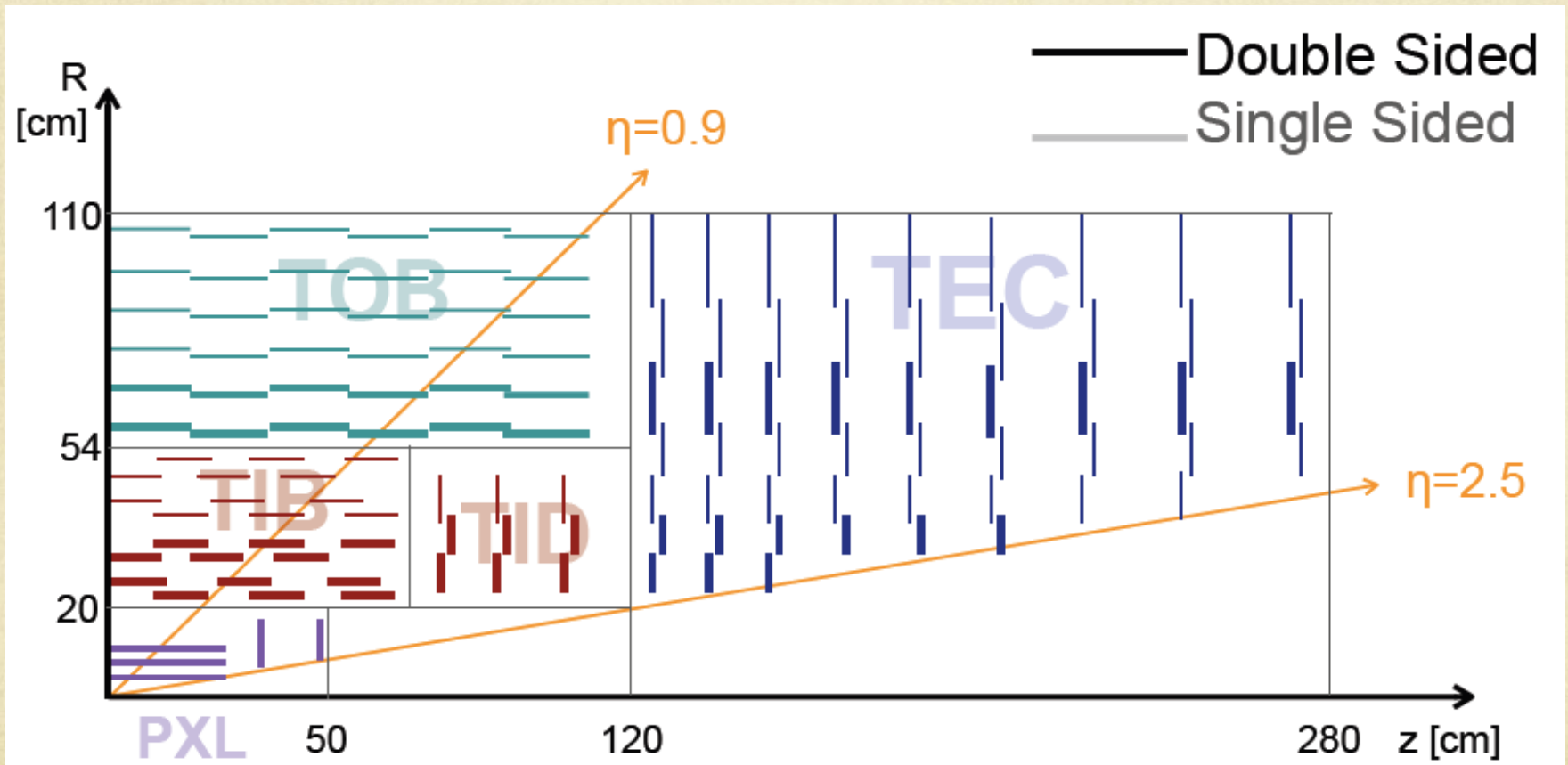
# CMS @ LHC setup



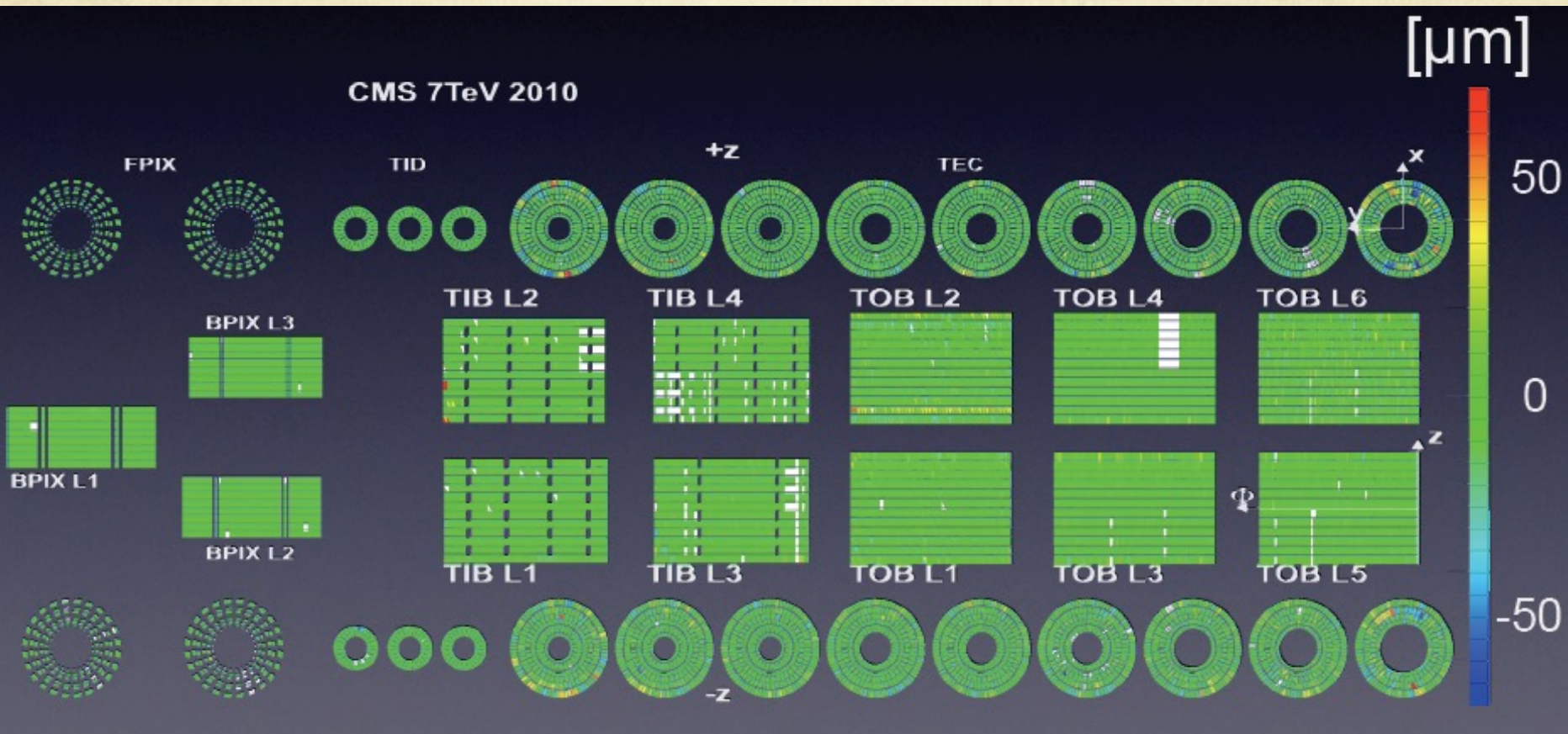
## 4. Some tracking systems:

CMS setup

### ○ The trackerS



○ Alignment residual width



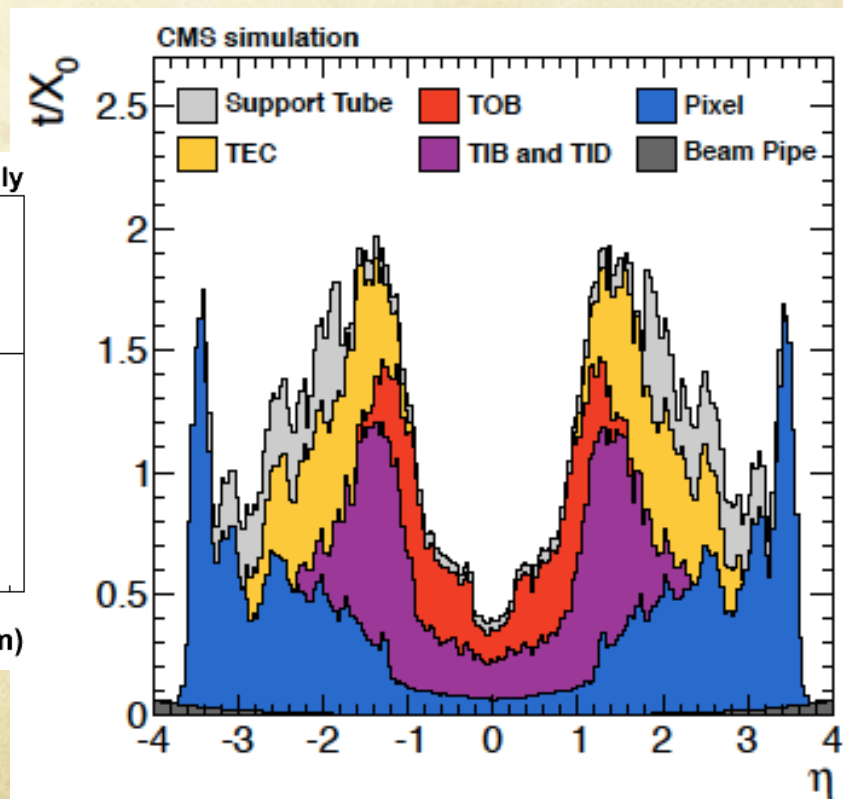
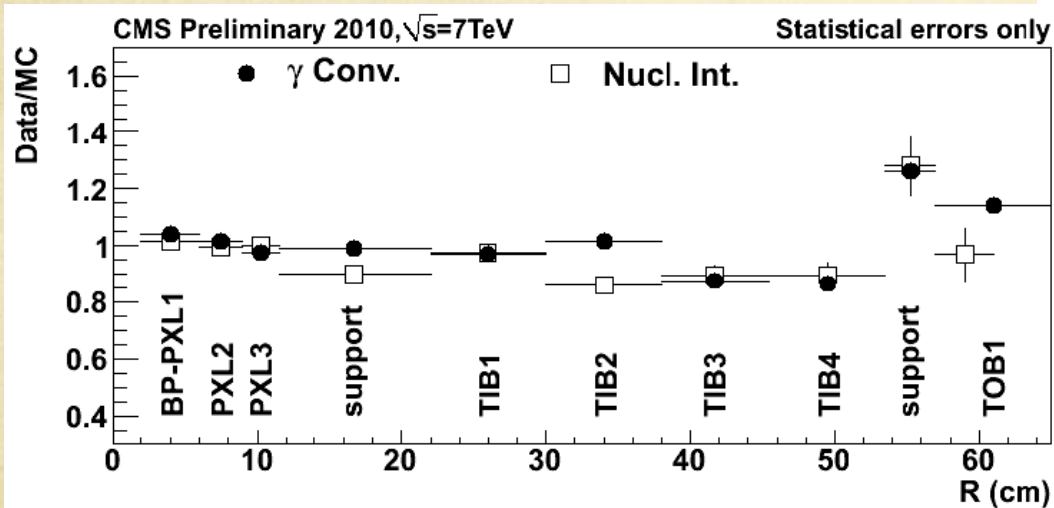
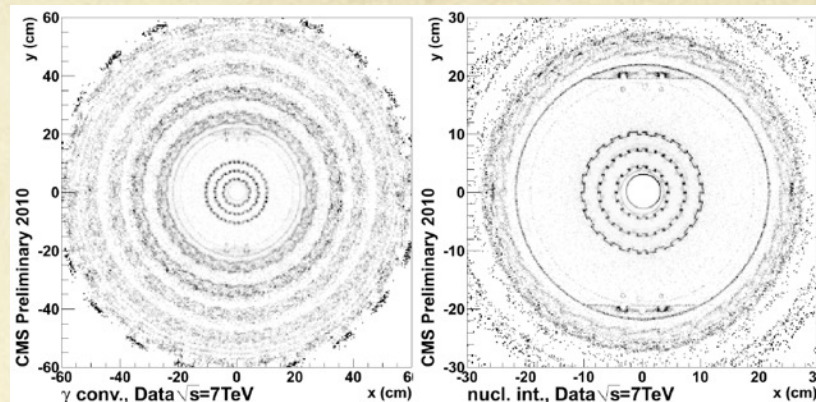
## 4. Some tracking systems:

### ○ Taking a picture of the material budget

→ Using secondary vertices from  $\gamma \rightarrow e^+e^-$

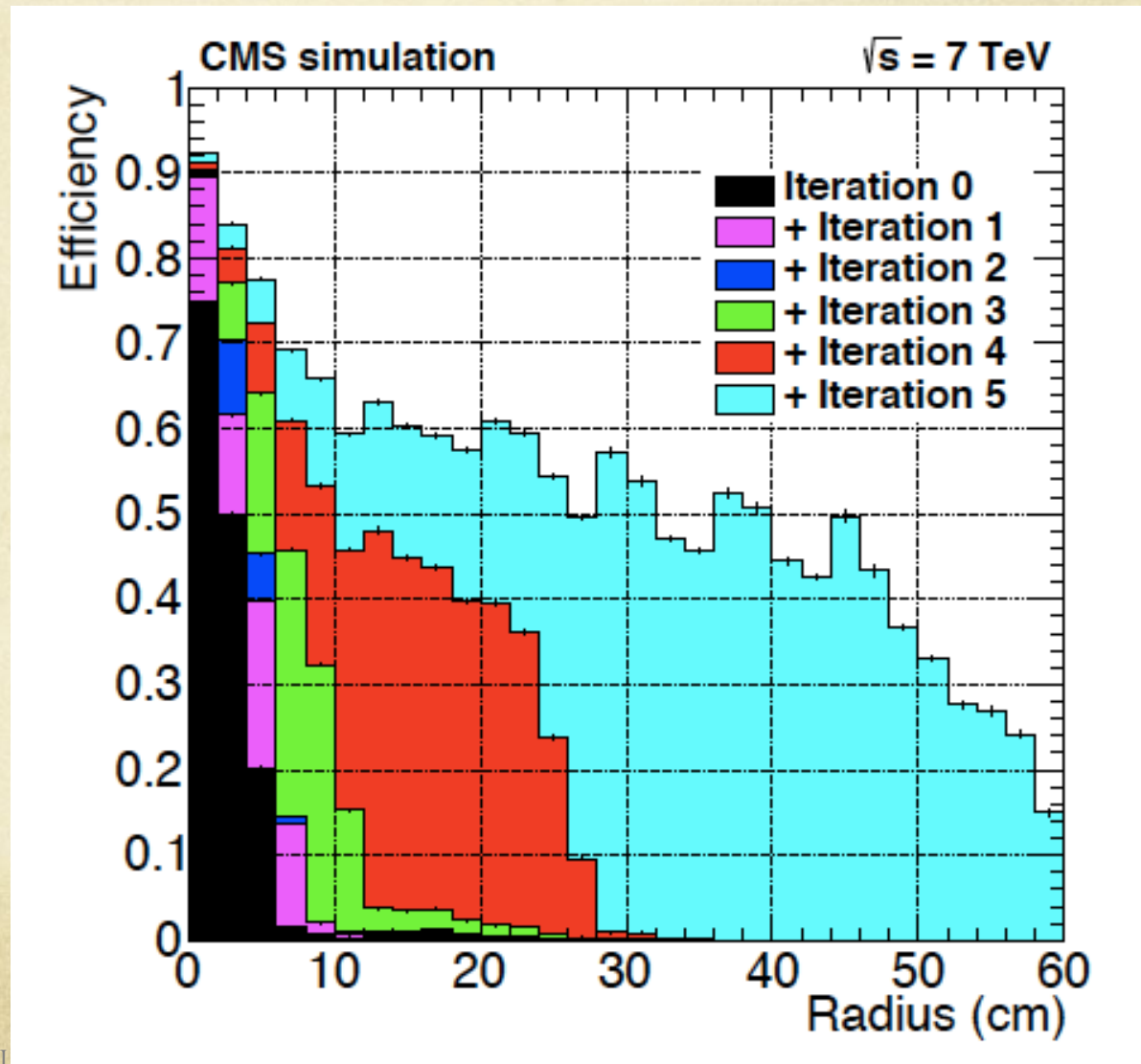
### ○ Measuring it by data/simulation comparison

## CMS material budget





- Tracking algorithm = multi-iteration process

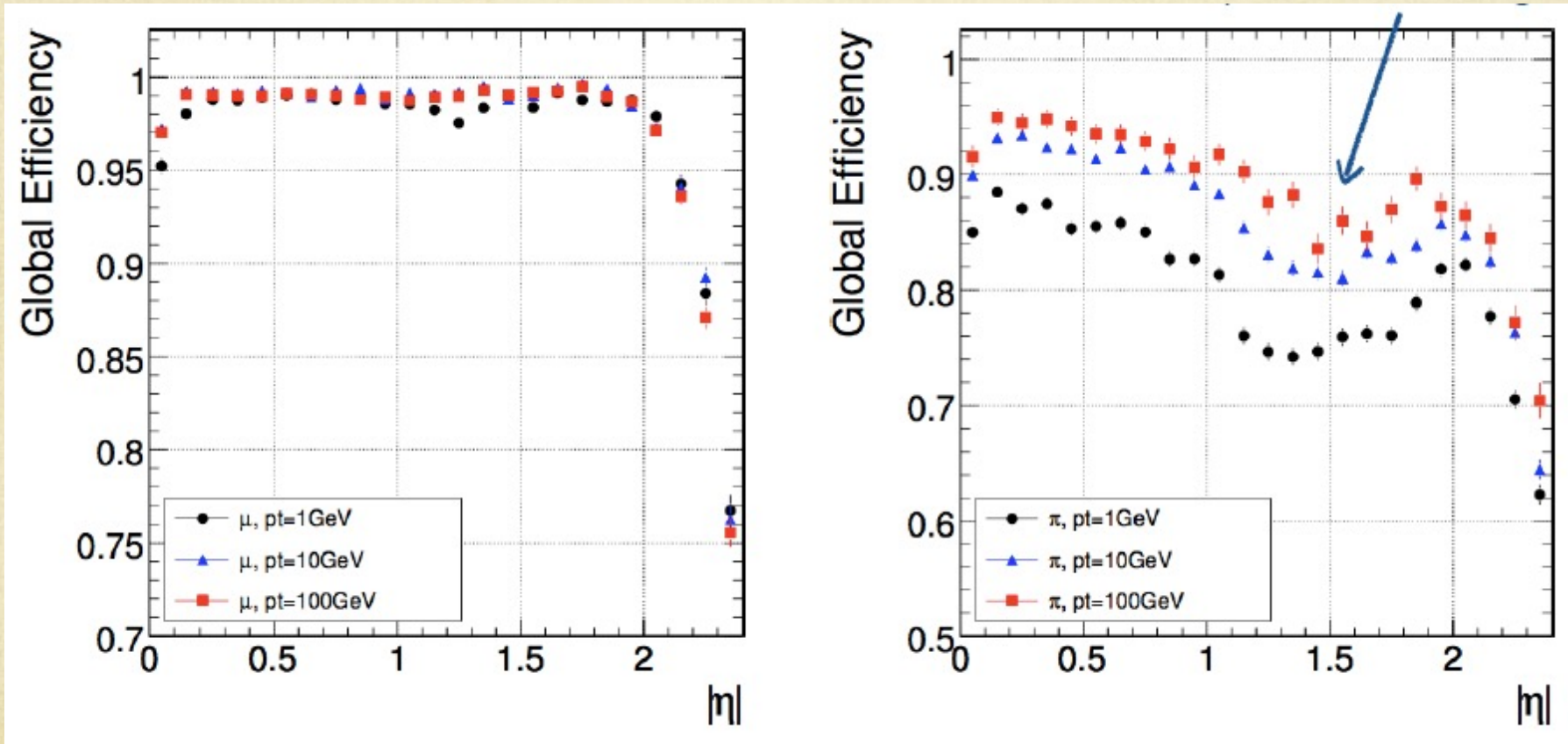


#### 4. Some tracking systems:

# CMS performance



### ○ Tracking efficiency

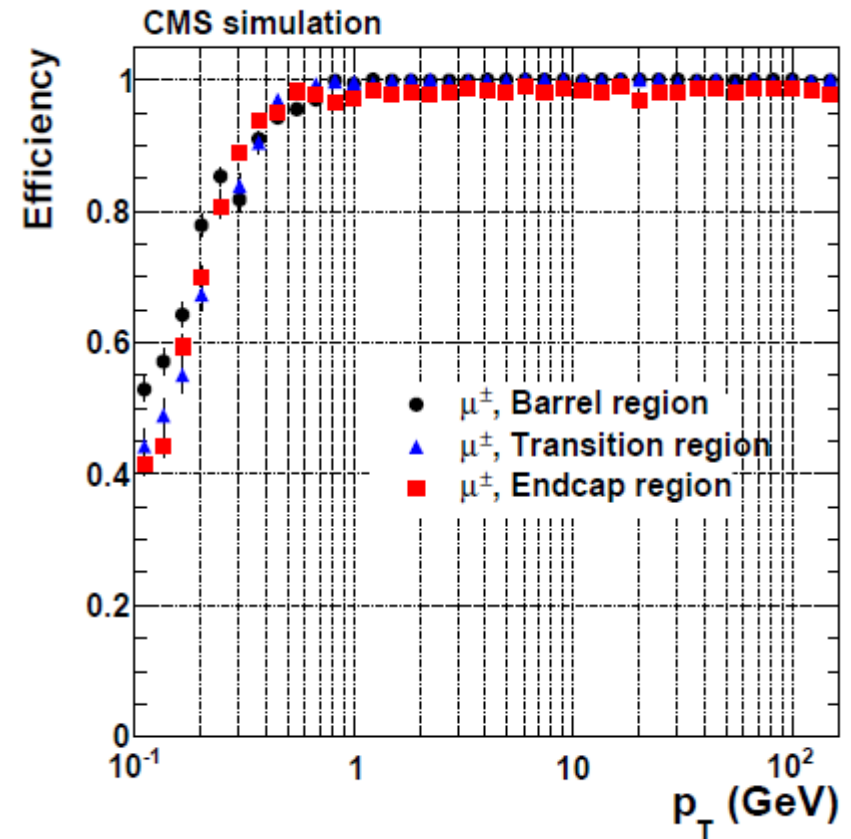
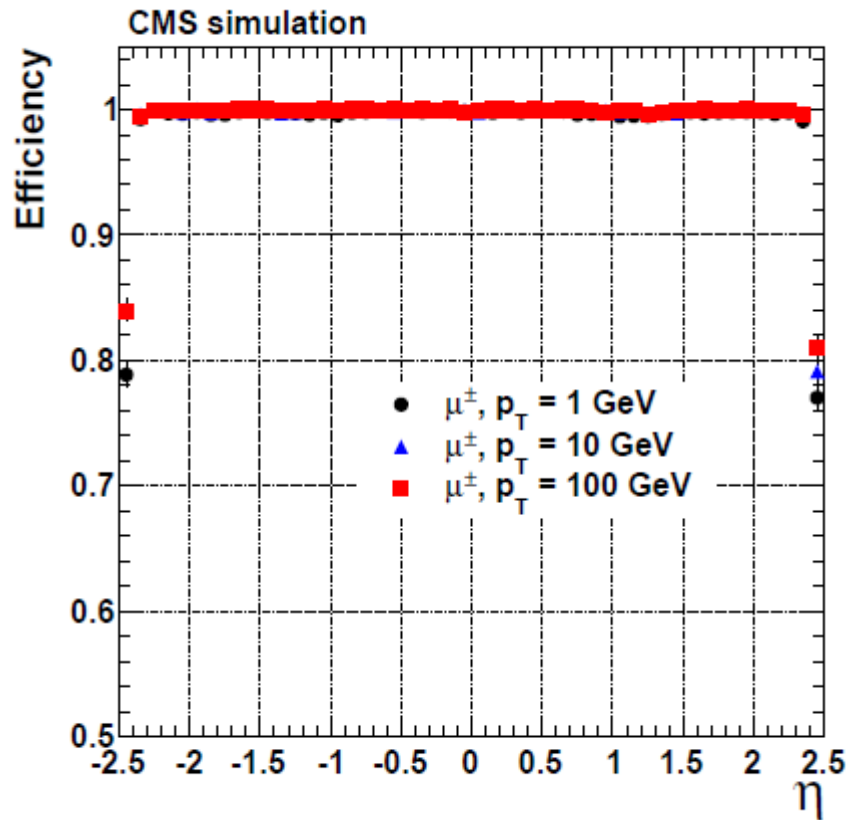






### ○ Tracking efficiency

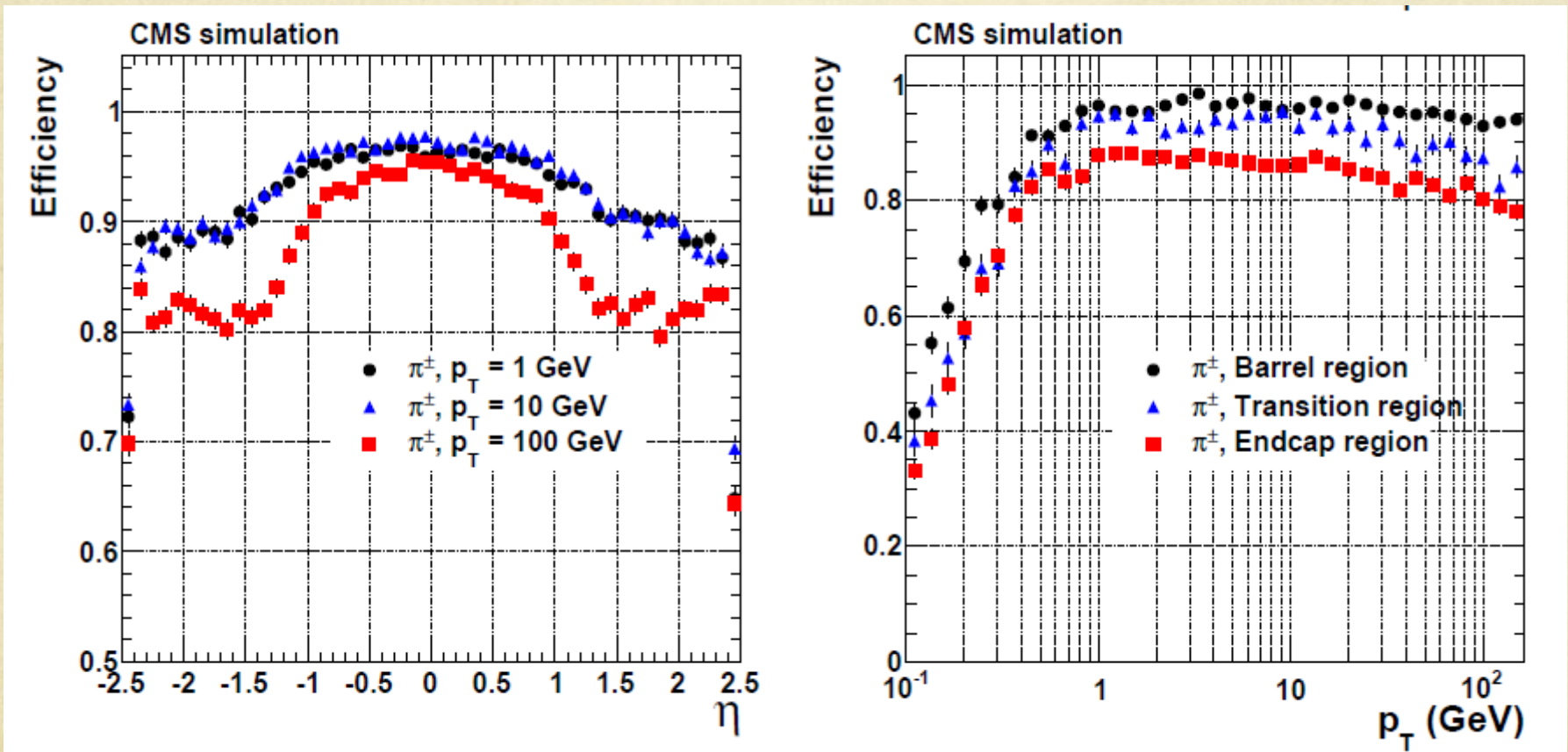
→ Single, isolated muons





## ○ Tracking efficiency

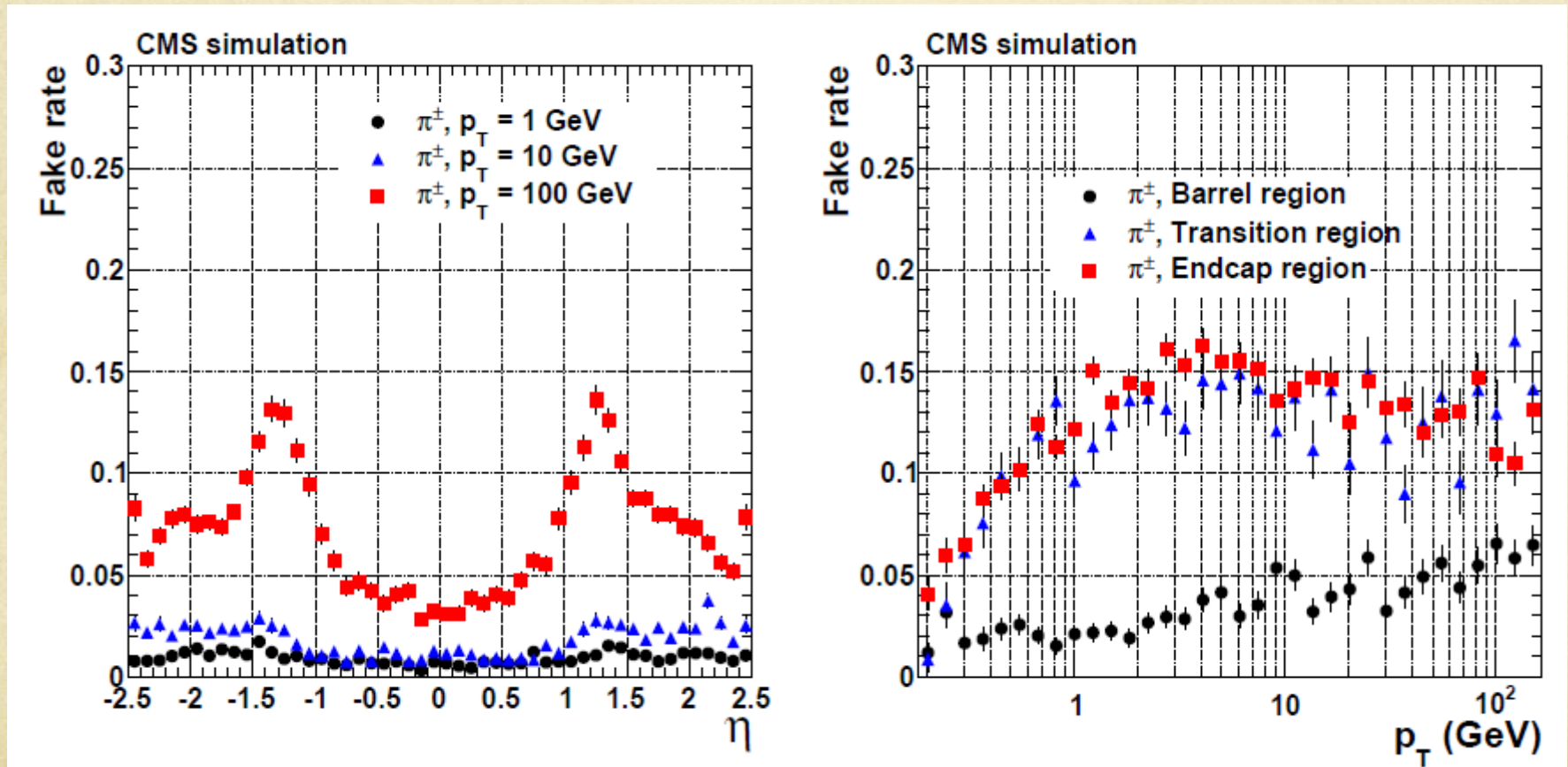
→ All pions





## ○ Tracking purity

→ All pions

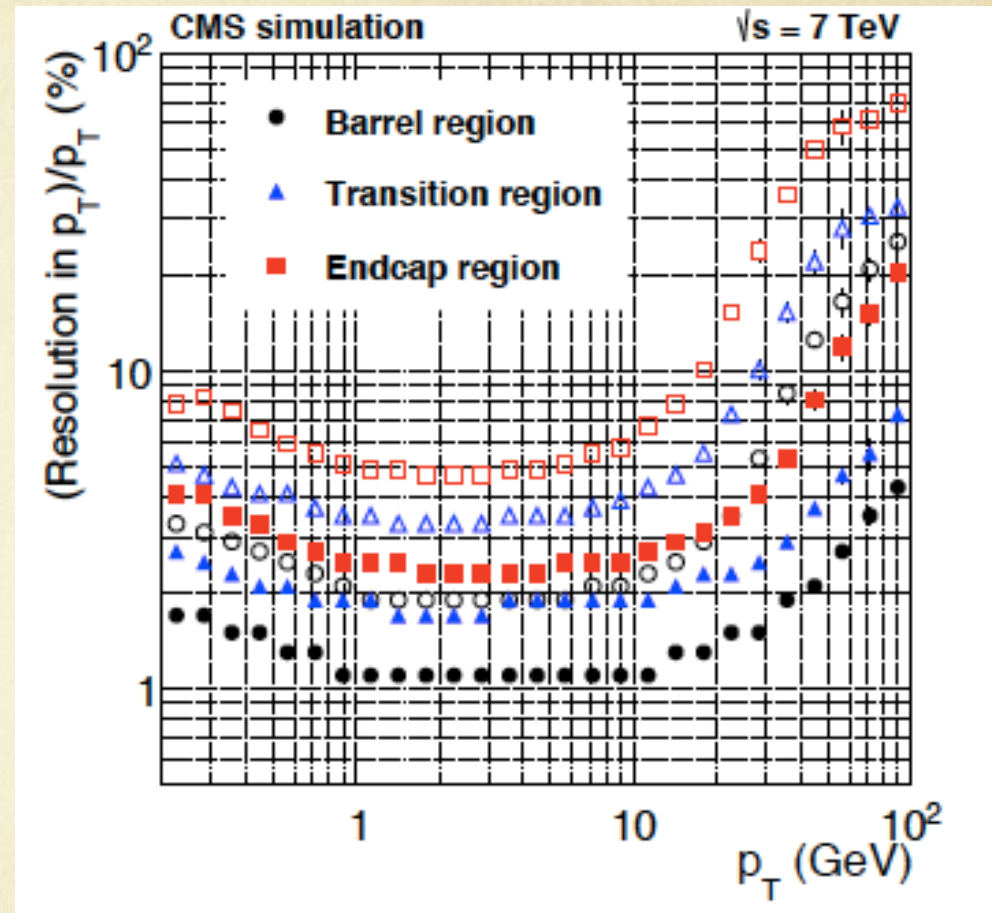
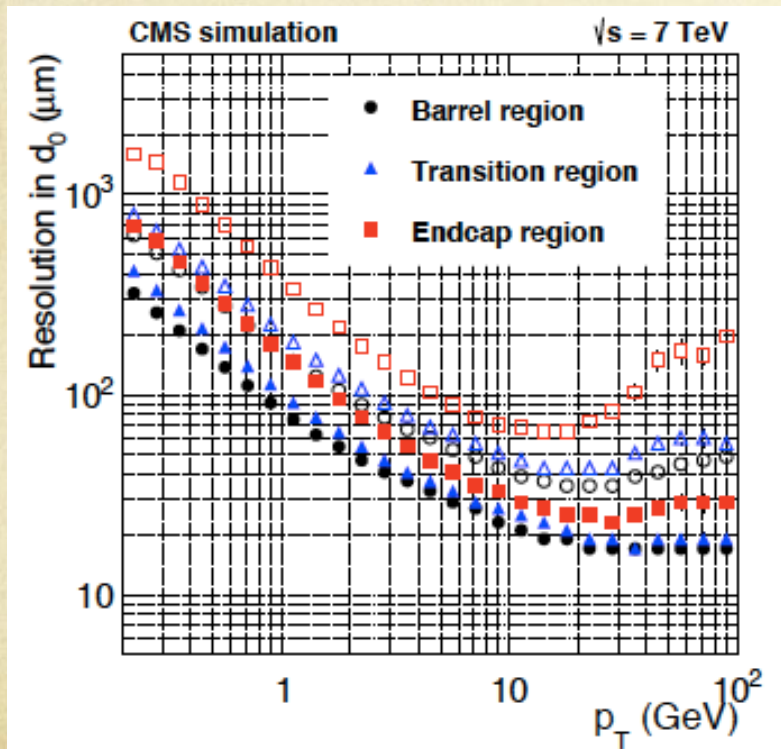


## 4. Some tracking systems:

# CMS performance

### ○ Tracking resolution

$d_0$  = transverse impact parameter

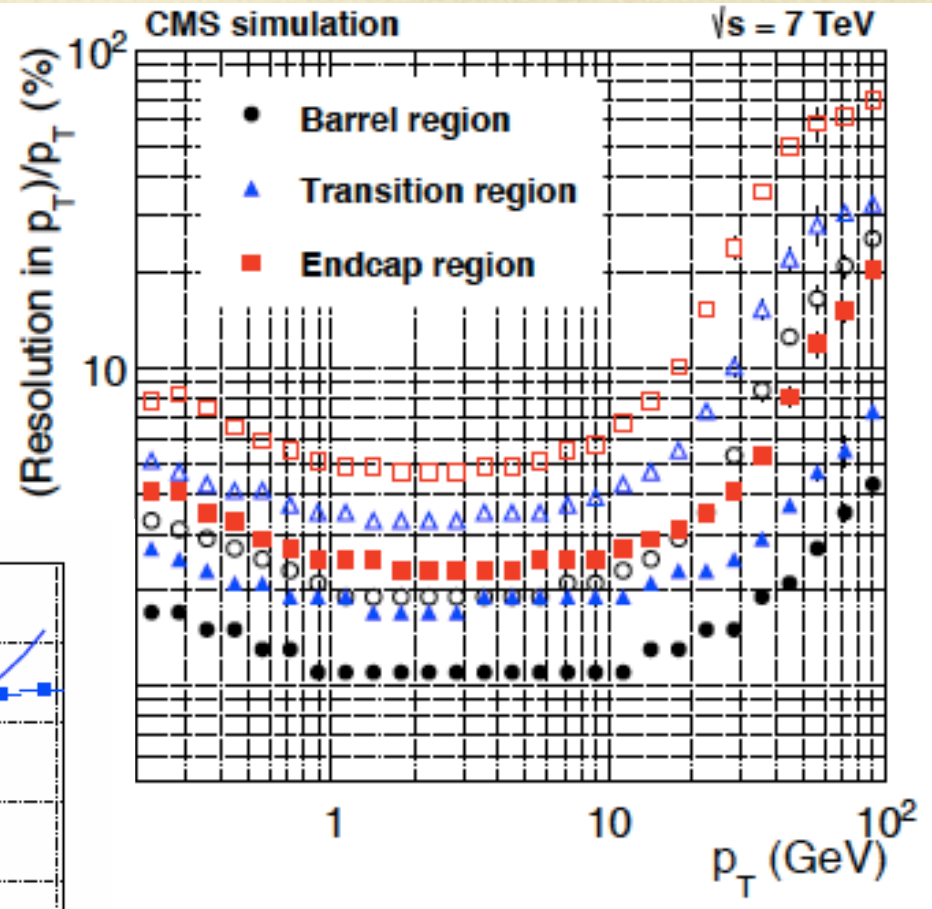
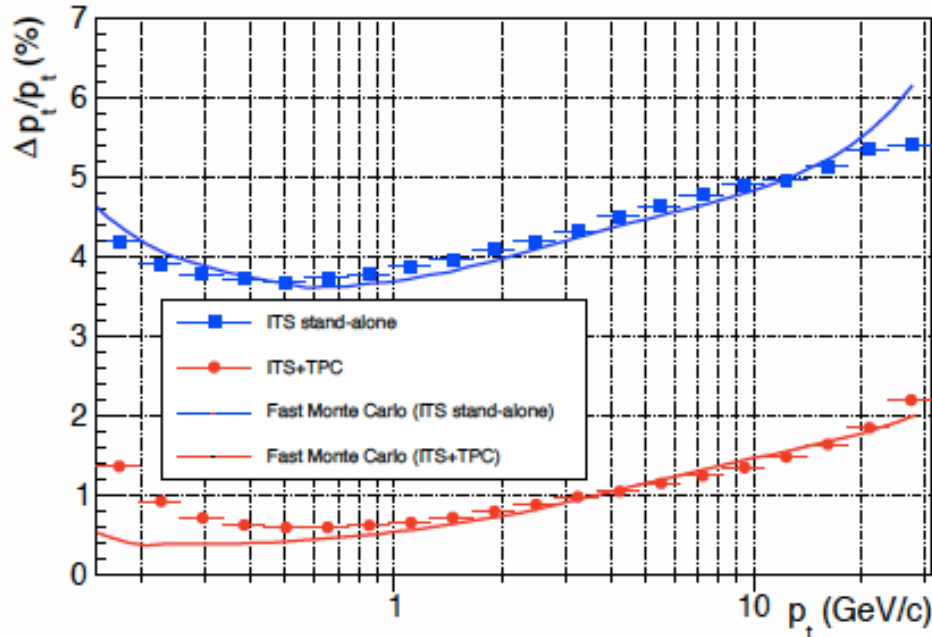


# 4. Some tracking systems:

# CMS performance

## ○ Tracking resolution

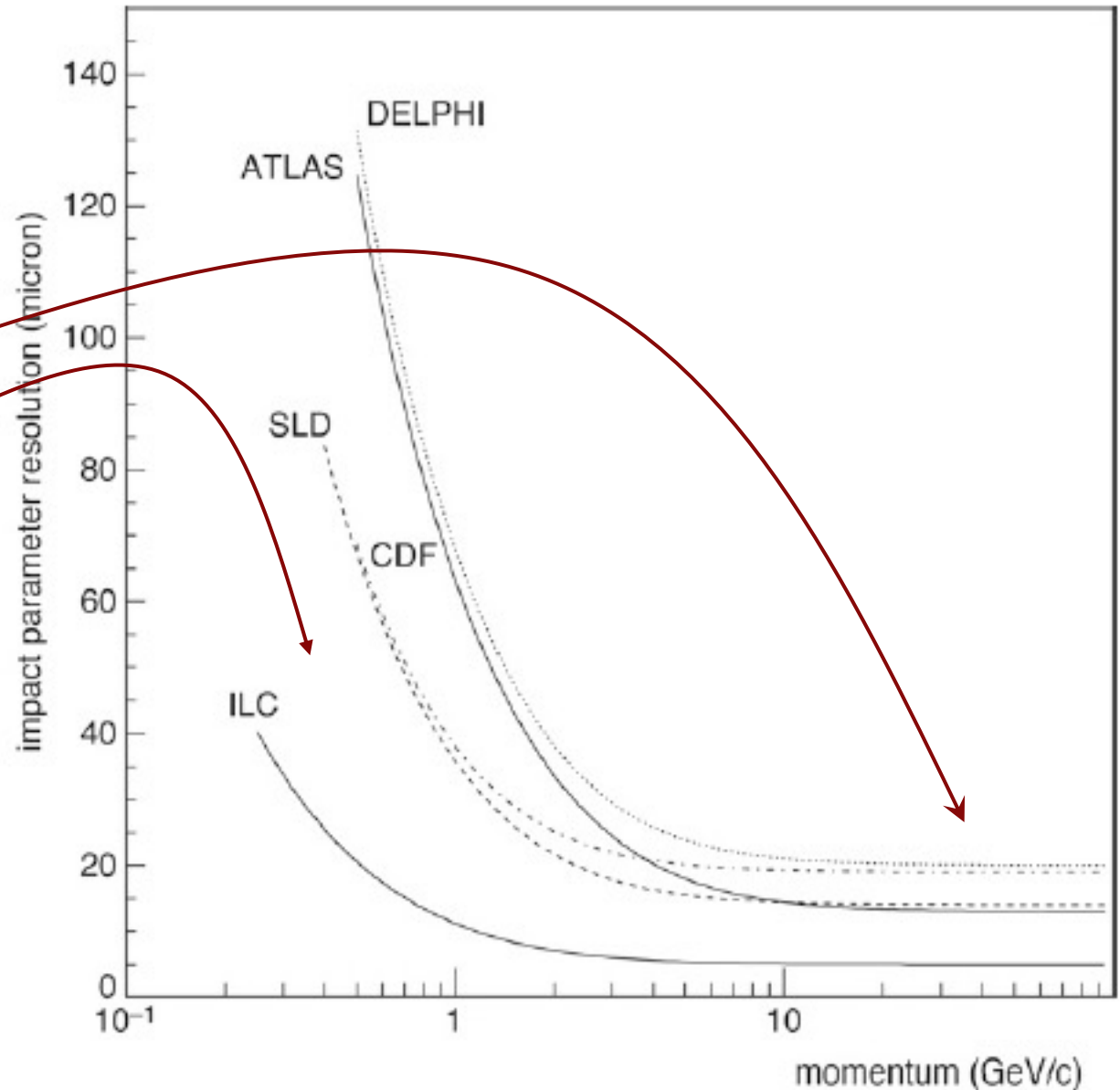
ALICE figure



#### 4. Some tracking systems:

## Impact parameter resolution

$$\sigma_{d_{r\phi}} = a \oplus \frac{b}{p_T \sin^2 \theta}$$

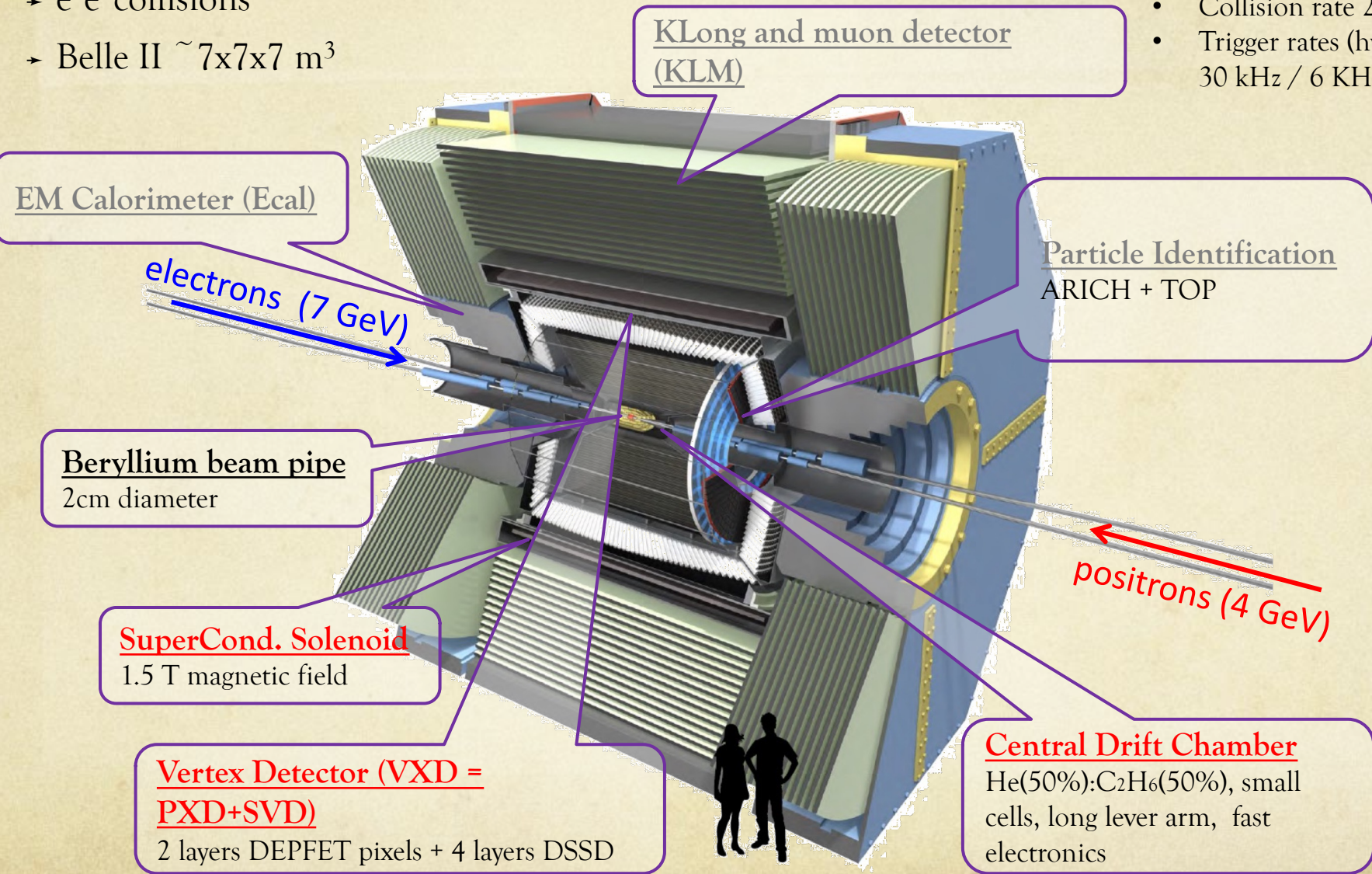


# 4. Some tracking systems:

# Belle II @ SuperKekB setup

- $e^+e^-$  collisions
- Belle II  $\sim 7 \times 7 \times 7 \text{ m}^3$

- Collision rate 250 MHz
- Trigger rates (hw & sw) 30 kHz / 6 KHz



EM Calorimeter (Ecal)

KLong and muon detector (KLM)

Particle Identification  
ARICH + TOP

Beryllium beam pipe  
2cm diameter

SuperCond. Solenoid  
1.5 T magnetic field

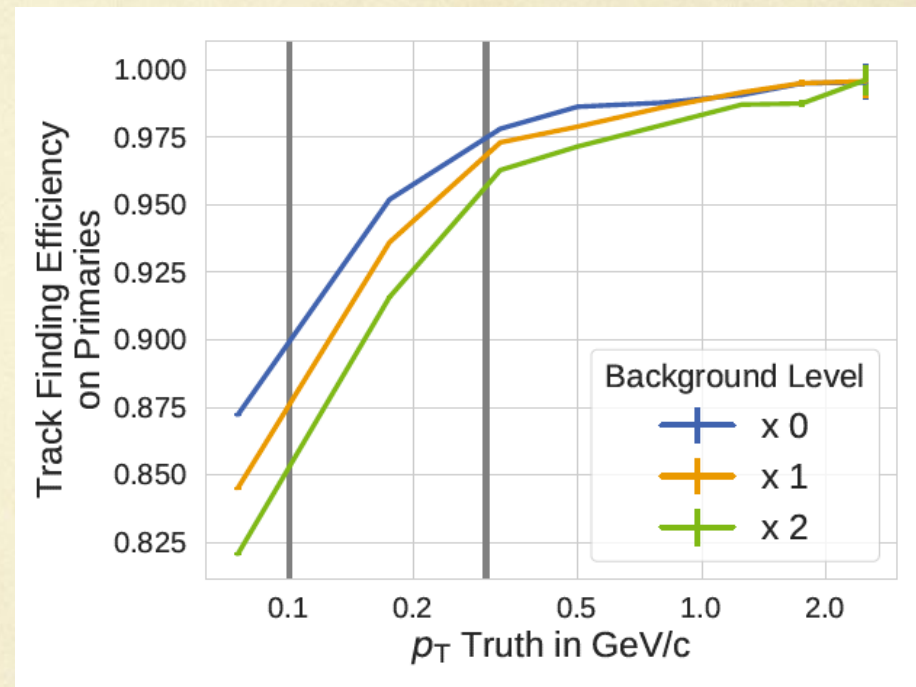
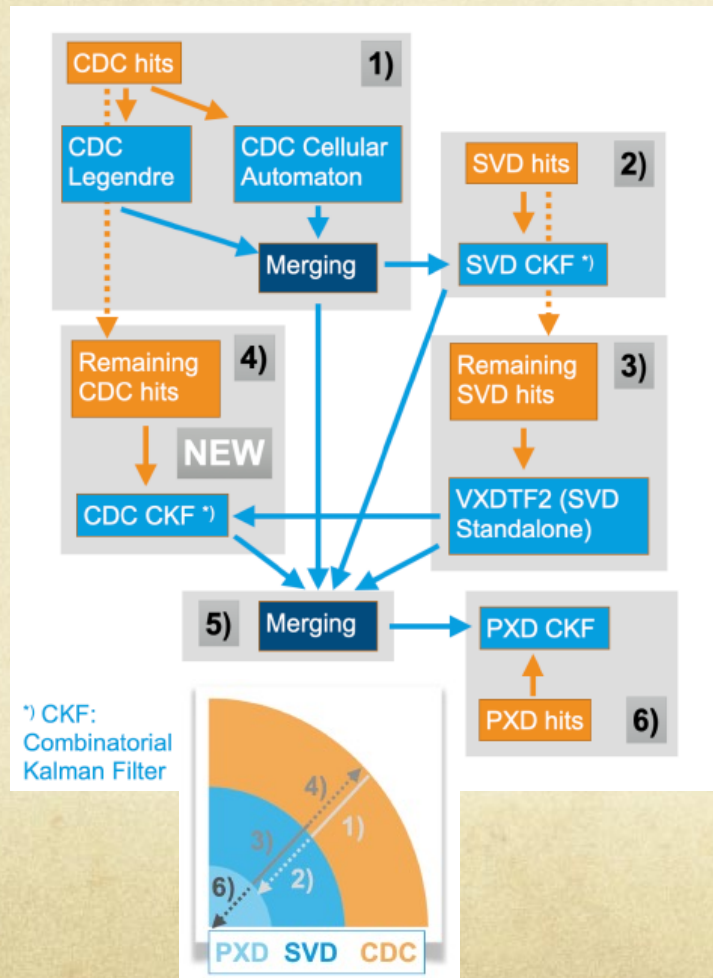
Vertex Detector (VXD = PXD+SVD)  
2 layers DEPFET pixels + 4 layers DSSD

Central Drift Chamber  
He(50%):C<sub>2</sub>H<sub>6</sub>(50%), small cells, long lever arm, fast electronics

## 4. Some tracking systems:

# Belle II algorithm

- Elementary collisions produce few tracks ( $\sim 10/\text{evts}$ )
- Beam induced background largely dominates occupancies.  $\Rightarrow$  Occupancies for Lumi  $> 10^{35} \text{ cm}^{-2} \cdot \text{s}^{-1}$   
 PXD:  $\sim 1\%$ , SVD: 1-3 %  
 at peak luminosities  $> 10^{35} \text{ cm}^{-2} \cdot \text{s}^{-1}$



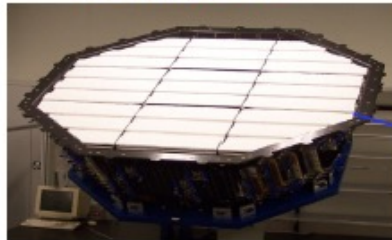


# AMS: A TeV precision, multipurpose particle physics spectrometer in space.

Particles and nuclei are defined by their charge ( $Z$ ) and energy ( $E \sim P$ )

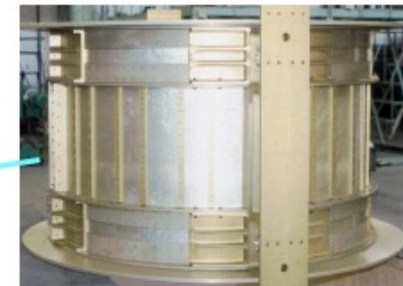
**TRD**  
Identify  $e^+$ ,  $e^-$

**TOF**  
 $Z, E$



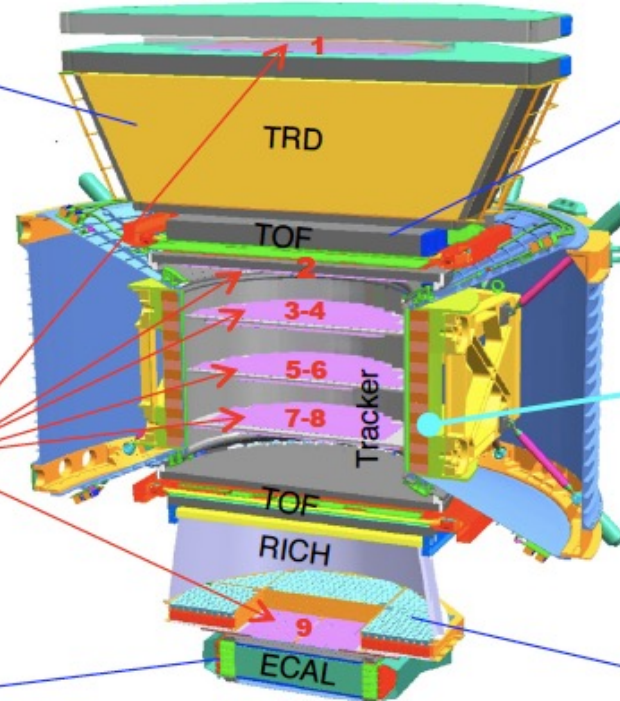
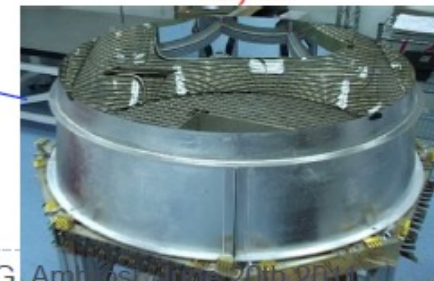
**Silicon Tracker**  
 $Z, P$

**Magnet**  
 $\pm Z$



**ECAL**  
 $E$  of  $e^+$ ,  $e^-$ ,  $\gamma$

**RICH**  
 $Z, E$



$Z, P$  are measured independently by the Tracker, RICH, TOF and ECAL

G. Ambrosi, June 20th 2011

# 4. Some tracking systems:

Silicon strips

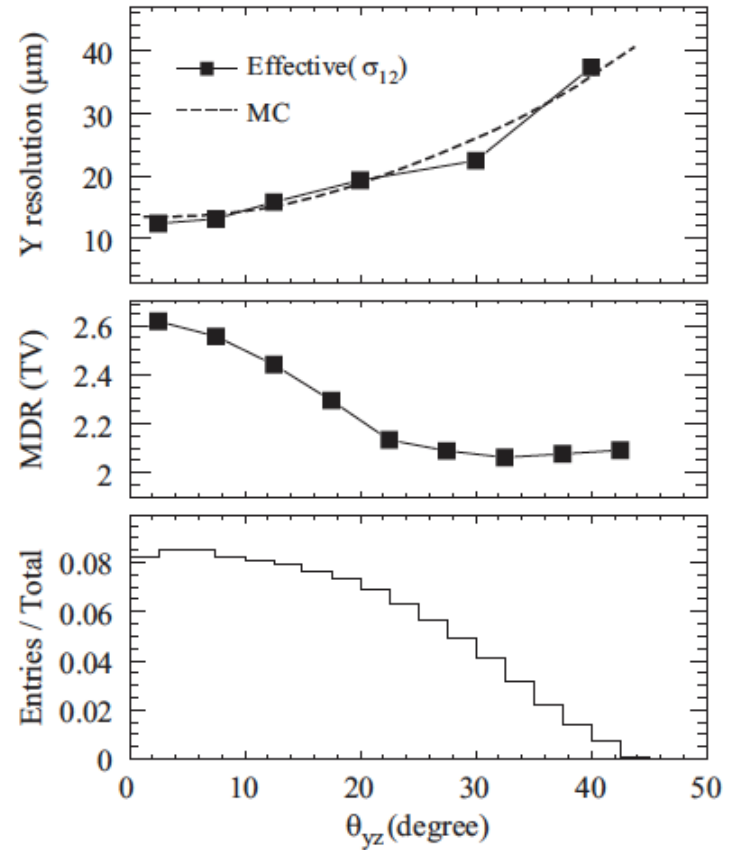
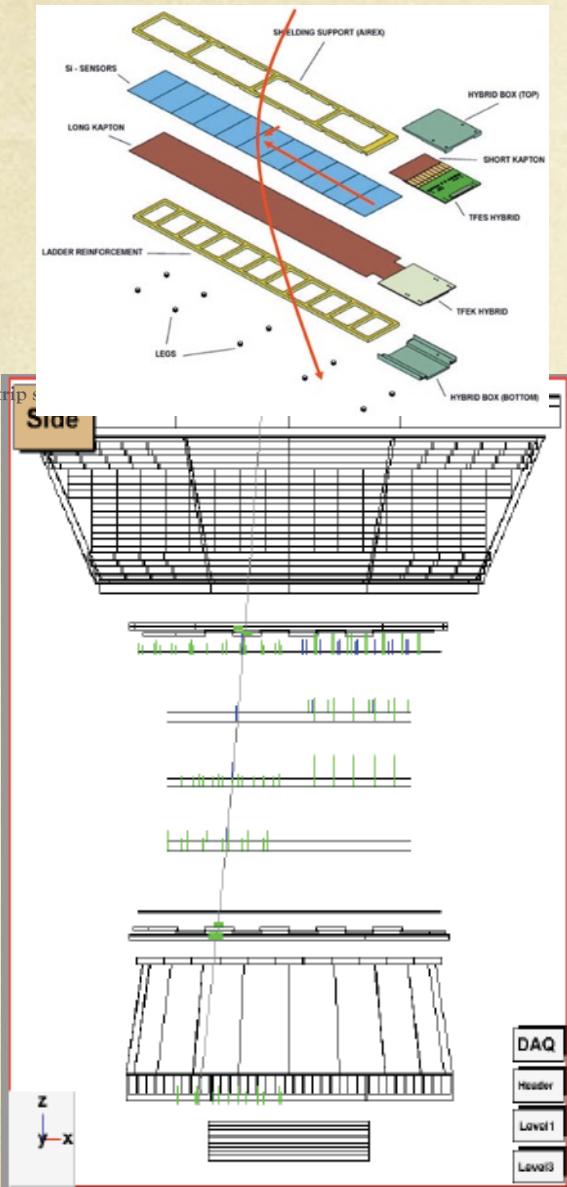
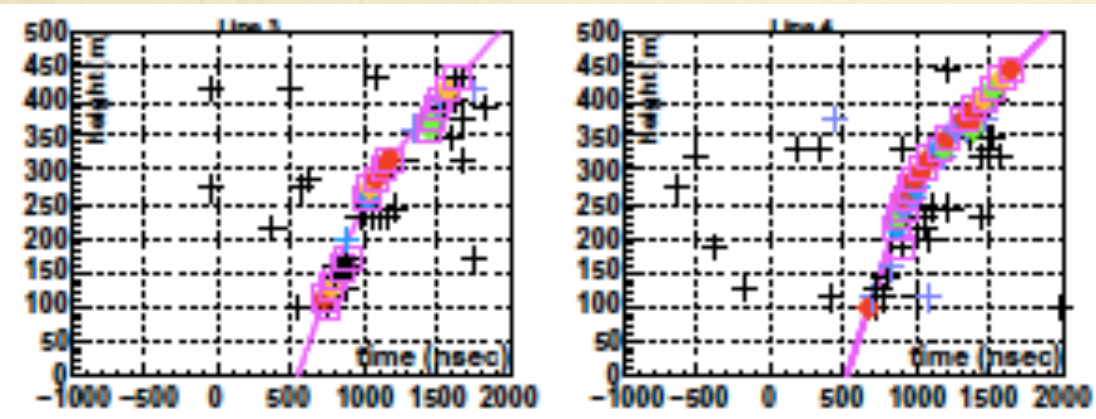
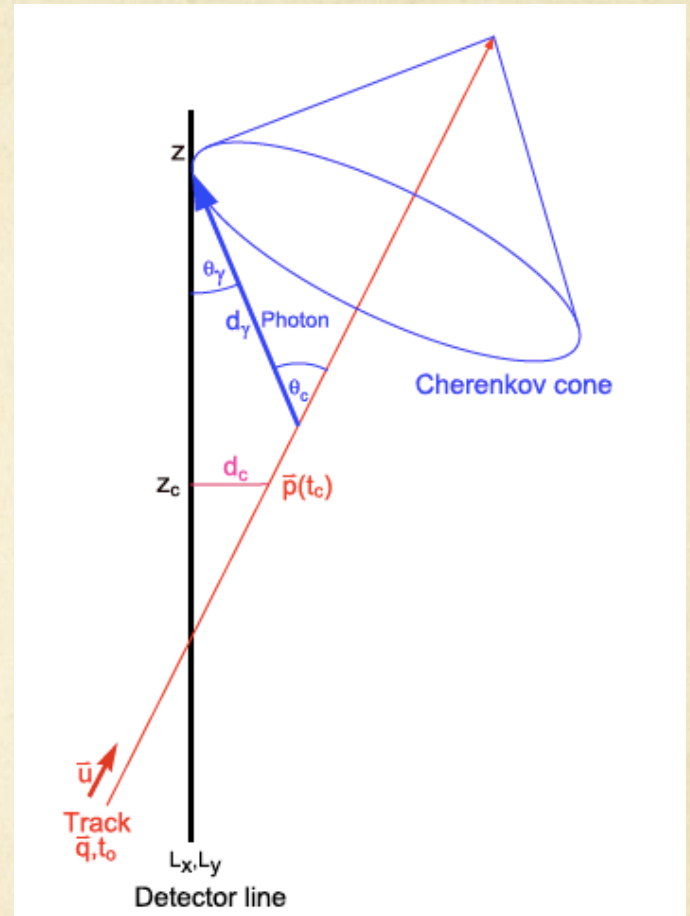
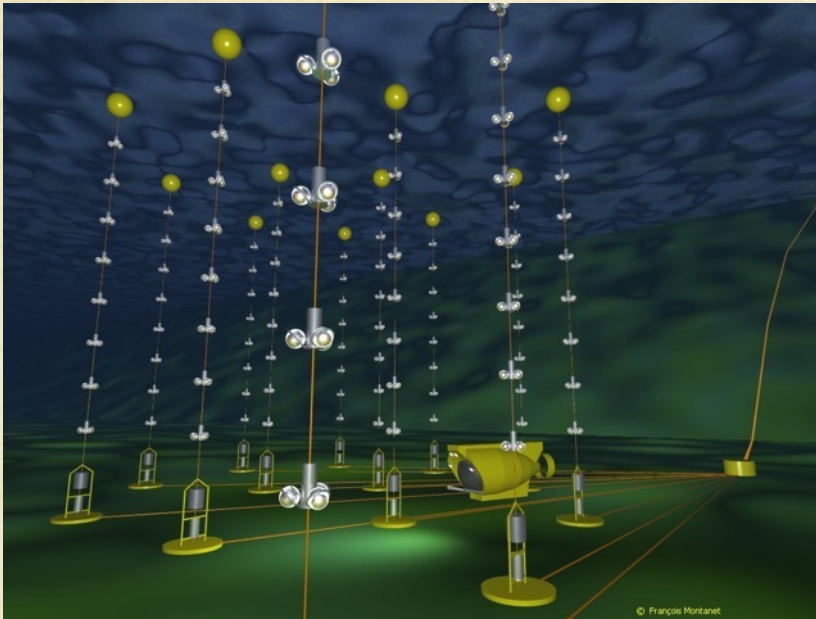


Fig. 5. The effective position resolution (weighted average of two Gaussian widths) in the y-coordinate for different inclination angles (top), the Maximum Detectable Rigidity (MDR, 100% rigidity measurement error) as a function of the inclination angle estimated for 1TV proton incidence with the simulation (middle), and the inclination angle distribution in the geometric acceptance of the tracker (bottom).

# 4. Some tracking systems:

# ANTARES

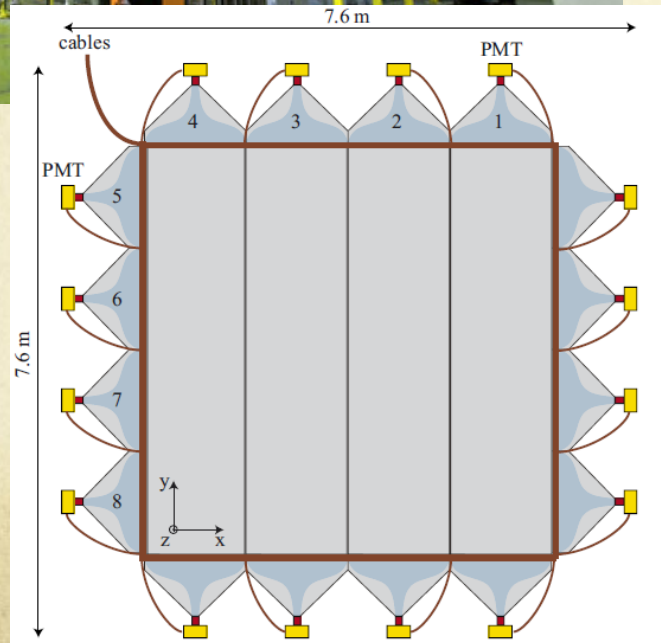
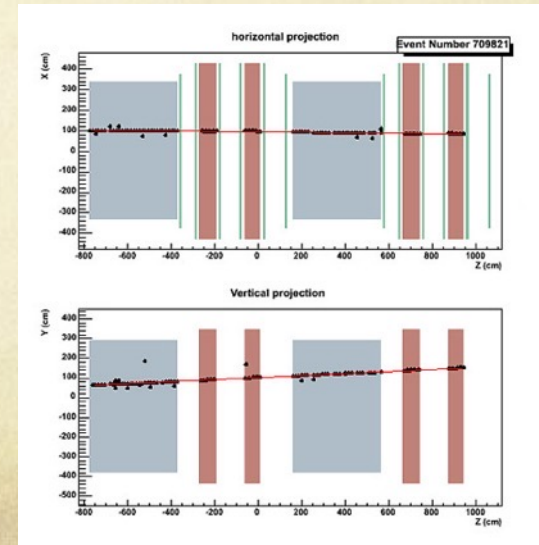
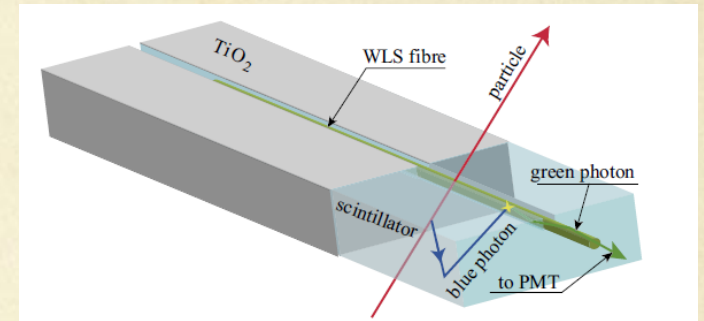


## 4. Some tracking systems:

# OPERA

Detecting  $\nu$  created 700 km away

Target Tracker with scintillator strips:  
 1 strip = 6.86m long,  
 10.6mm thick, 26.3mm wide



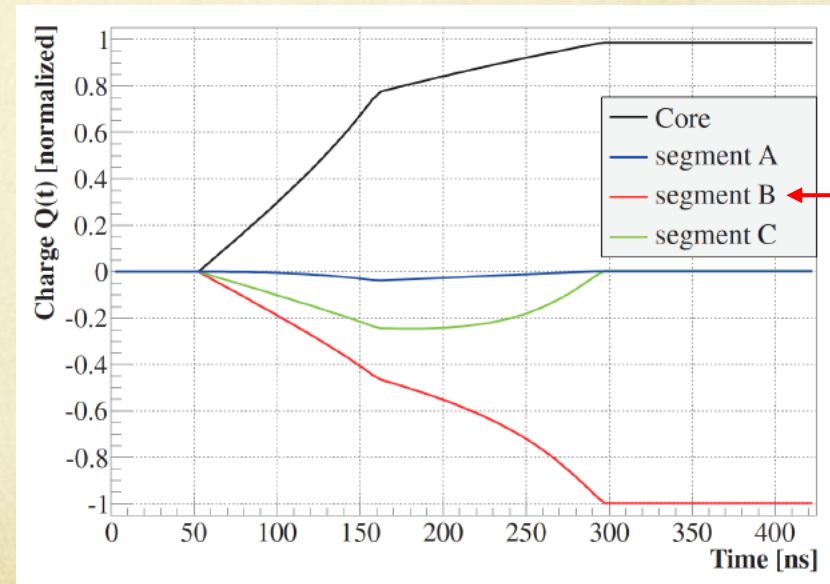
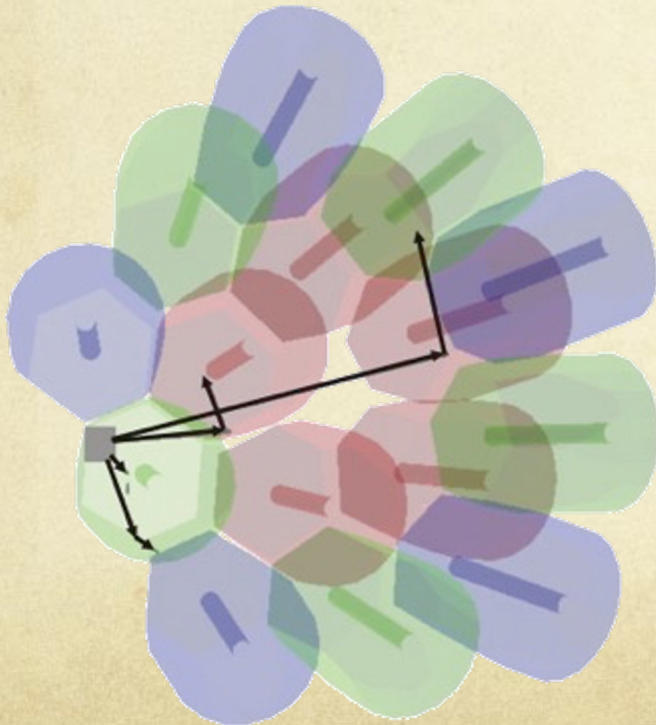
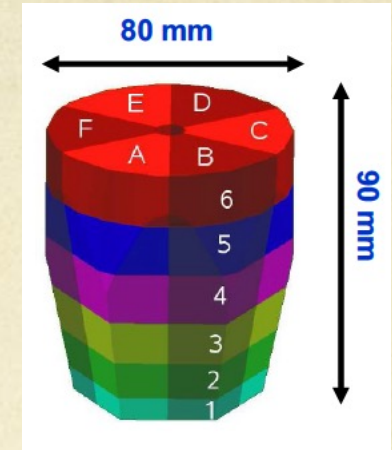
## 4. Some tracking systems:

# AGATA



### ○ Advanced GAMMA Tracking Array

- European gamma-ray spectrometer → nuclear structure studies
- Photon energy: 150 keV → 10 MeV
- Physical effect = multiple Compton scattering
- Sensors = segmented germanium



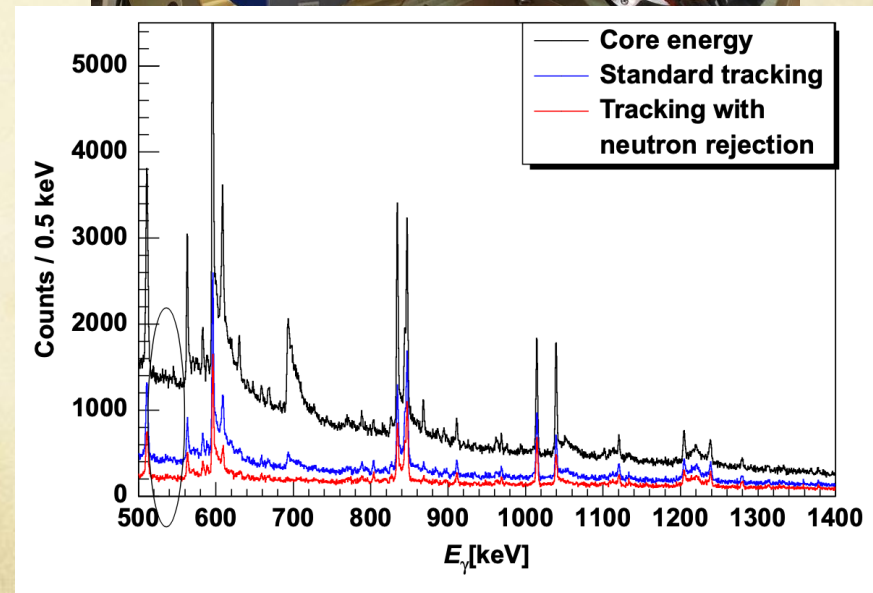
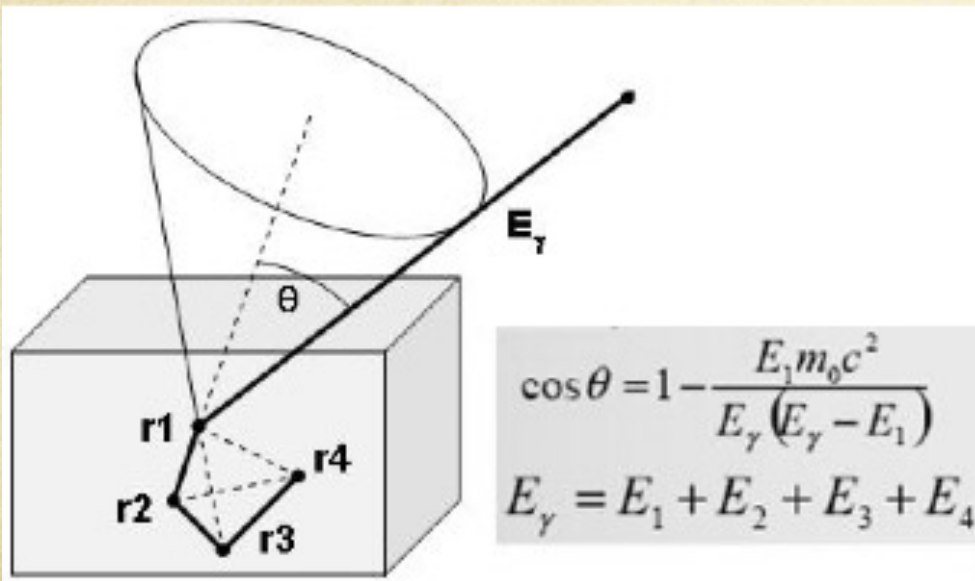
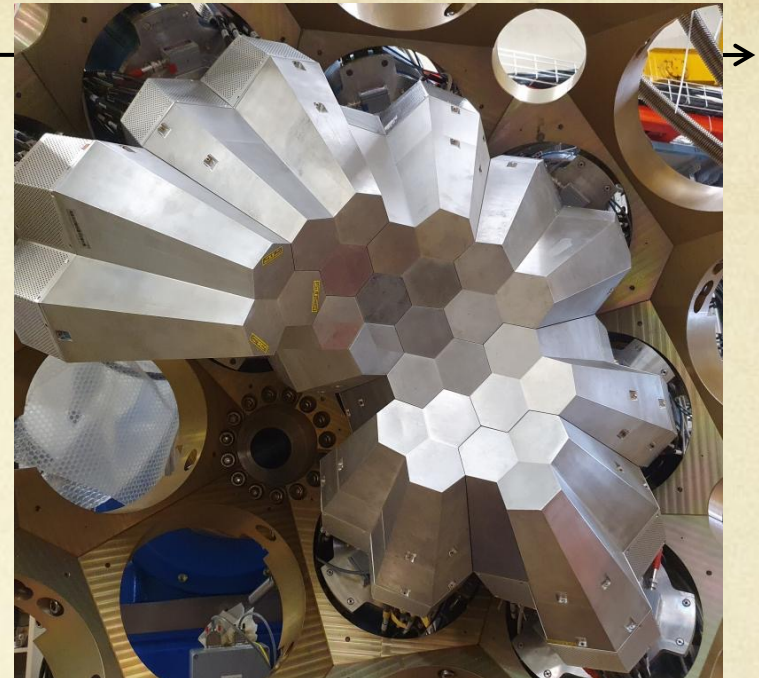
Segment hit by  $\gamma$

## 4. Some tracking systems:

### ○ Tracking algorithm

- Various methods: standard iterative to adaptive
- 2 steps (similar to charged particle tracking)
  - Grouping energy deposits
  - Ordering of energy => Initial  $E_\gamma$

AGATA

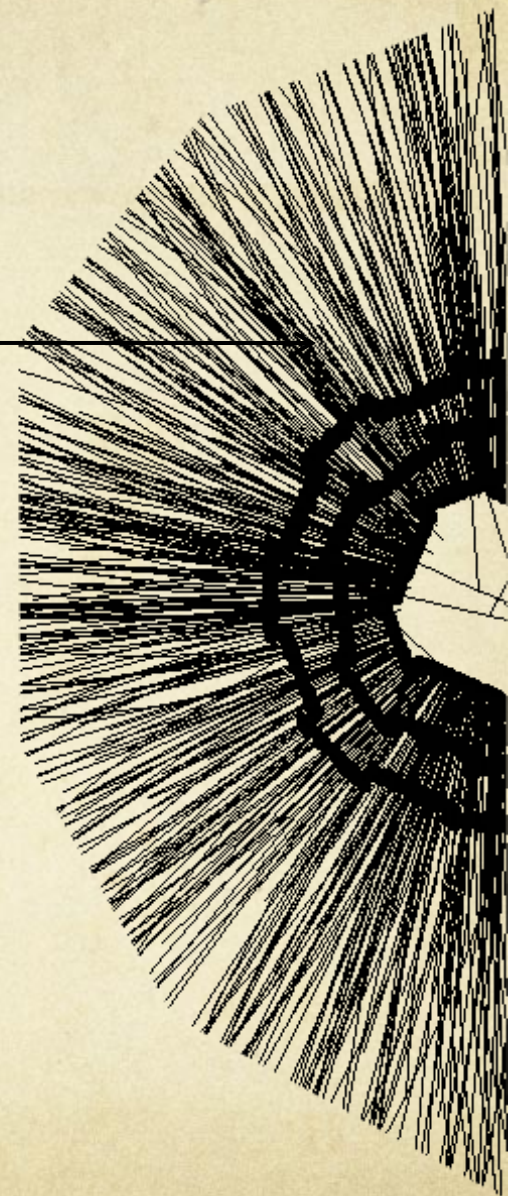


- Fundamental characteristics of any tracking & vertexing device:
  - (efficiency), granularity, material budget, power dissipation, “timing”, radiation tolerance
  - All those figures are correlated!
- Reconstruction algorithms
  - Enormous boost (variety and performances) in the last 10 years
  - Each tracking system has its optimal method: usually requires several algorithms
- Development trend
  - Always higher hit rates call for more data reduction
  - Tracking info in trigger → high quality online tracking/vertexing

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