# Effective Field Theories for BSM Physics

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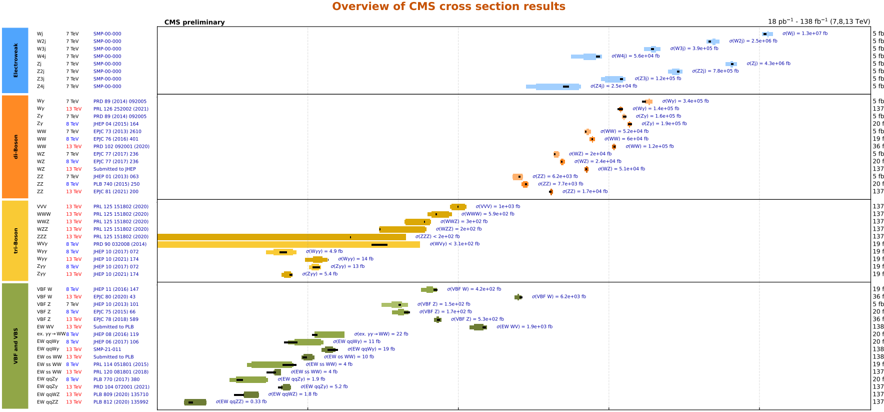
# Outline

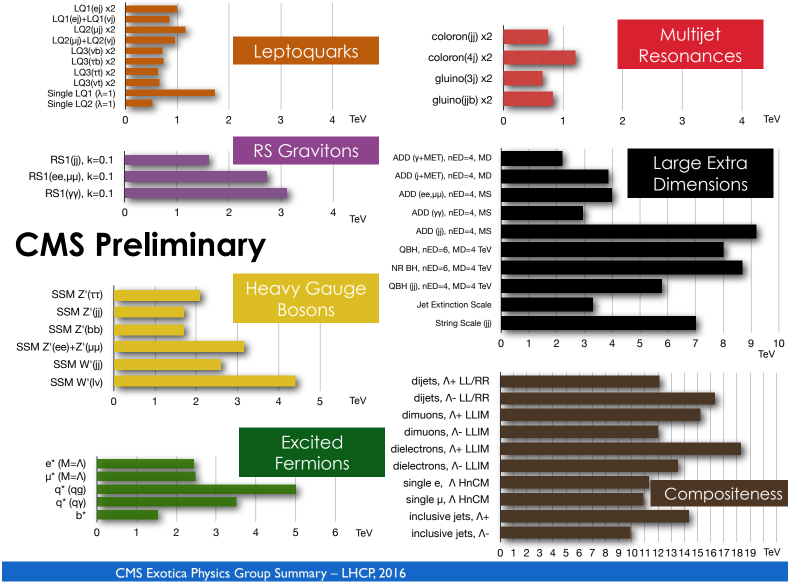
# CMS SM Summary




Measured cross sections and exclusion limits at 95% C.L.
See here for all cross section summary plots

Inner colored bars statistical uncertainty, outer narrow bars statistical+systematic uncertainty
Light colored bars: 7 TeV, Medium bars: 8 TeV, Dark bars: 13 TeV, Black bars: theory prediction

# CMS Exotics Summary



# EFTs

$\Lambda_{\rm NP}$

## The major underlying assumption of EFTs

$\Lambda_{\rm NP} \gg E$ of the scale of experiments/measurements

Weinberg 1967:

This remark is based on a "theorem", which as far as I know has never been proven, but which I cannot imagine could be wrong. The "theorem" says that although individual quantum field theories have of course a good deal of content, quantum field theory itself has no content beyond analyticity, unitarity, cluster decomposition, and symmetry. This can be put more precisely in the context of perturbation theory: if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible *S*-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. As I said, this has not been proved, but any counterexamples would be of great interest, and I do not know of any.

$E$–scale of experiments

# EFTs

$\Lambda_{\rm NP}$

## Top down EFTs:

- example: SM $\rightarrow$ Fermi theory
- Start w/ ultraviolet (UV)–model in mind (Seesaw, SUSY, etc)
- "integrate out" heavy degrees of freedom at UV scale
- model dependent relations between ops, $Q_i^{d>4}$
- $Q_i$ are only in terms of IR degrees of freedom

$E$–scale of experiments

# EFTs

$\Lambda_{\mathrm{NP}}$

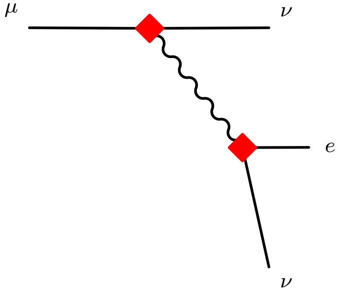
## Bottom up EFTs:

- example: Fermi theory
- Start w/ infrared (IR)–model in mind (QED, SM)
- using symmetries of model put together ops, $Q^{d>4}$
- truncate EFT at some $\mathcal{O}(1/\Lambda)$
- constrain $Q$ in experiment & infer properties of NP at $\Lambda$
- $Q$s are unrelated $\rightarrow$ model independent
- $Q$s are only in terms of IR degrees of freedom

$E$–scale of experiments

# The Fermi-theory example

## In the SM




$$\mathcal{M} \sim \frac{g_{\mathrm{W}}^2}{2} \frac{(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e)}{k^2 - M_W^2}$$

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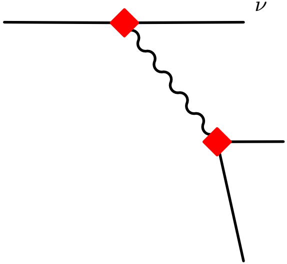


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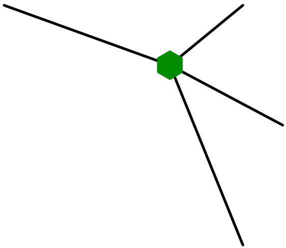
## In the Fermi theory

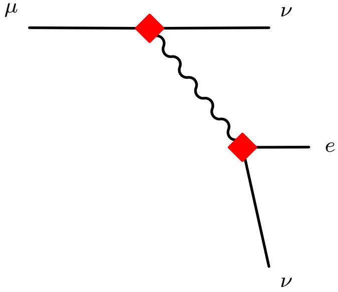



$$\frac{1}{M_W^2} (\bar{\psi}\gamma^\mu P_L \psi)^2$$

$$\mathcal{M} \sim -\frac{g_{\mathrm{W}}^2}{2M_W^2} (\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e) + \cdots$$

# The Fermi-theory example

## In the SM

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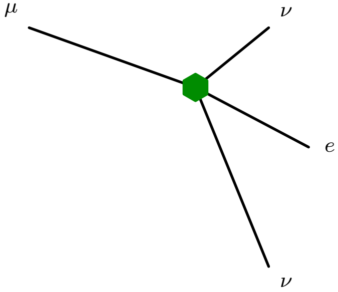


$$\mathcal{M} \sim \frac{g_{\mathrm{W}}^2}{2} \frac{(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e)}{k^2 - M_W^2}$$

$\frac{1}{M_W^2}(\bar{\psi}\gamma^\mu P_L \psi)^2$ $\qquad$ $\frac{1}{M_W^4}\partial^2(\bar{\psi}\gamma^\mu P_L \psi)^2$

$$\mathcal{M} \sim -\frac{g_{\mathrm{W}}^2}{2M_W^2}(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e) - \frac{g_{\mathrm{W}}^2 k^2}{2M_W^4}(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e) + \cdots$$

# SMEFT

In studying NP at $\Lambda_{\text{NP}} \gg v$, we employ the Standard Model EFT

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots \qquad \mathcal{L}_d = \sum_i c_i Q_i$$

The SMEFT is formed of $\mathcal{L}_{\text{SM}}$ and $Q$ of $d > 4$ respecting SM symmetries & $c_i$ embedding UV physics

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The leading operator:

$$\mathcal{L}_5 = c_{\alpha\beta}(\bar{L}^c_\alpha \tilde{H})(\tilde{H}^\dagger L_\beta) \sim v^2 \bar{\nu}_\alpha \nu_\beta$$

$$\Rightarrow m_\nu \sim v^2/\Lambda$$

# The SMEFT at dimension-six

D6 operators from SM field content ⇒ SMEFT @ D6

| Type I: $X^3$ | | Type II, III: $H^6$, $H^4D^2$ | | Type V: $\Psi^2H^3$ + h.c. | |
|---|---|---|---|---|---|
| $Q_G$ | $f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$ | $Q_H$ | $(H^\dagger H)^3$ | $Q_{eH}$ | $(H^\dagger H)(\bar{L}eH)$ |
| $Q_{\tilde{G}}$ | $f^{ABC}\tilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$ | $Q_{H\Box}$ | $(H^\dagger H)\Box(H^\dagger H)$ | $Q_{uH}$ | $(H^\dagger H)(\bar{Q}u\tilde{H})$ |
| $Q_W$ | $\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$ | $Q_{HD}$ | $(H^\dagger D^\mu H)^*(H^\dagger D^\mu H)$ | $Q_{dH}$ | $(H^\dagger H)(\bar{Q}dH)$ |
| $Q_{\tilde{W}}$ | $\epsilon^{IJK}\tilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$ | | | | |

| Type IV: $X^2\Phi^2$ | | Type VI: $\Psi^2HX$ | | Type VII: $\Psi^2H^2D$ | |
|---|---|---|---|---|---|
| $Q_{HG}$ | $(H^\dagger H)G_{\mu\nu}^A G^{A\mu\nu}$ | $Q_{eW}$ | $(\bar{L}\sigma^{\mu\nu}e)\tau^I HW_{\mu\nu}^I$ | $Q_{HL}^{(1)}$ | $(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}\gamma^\mu L)$ |
| $Q_{H\tilde{G}}$ | $(H^\dagger H)\tilde{G}_{\mu\nu}^A G^{A\mu\nu}$ | $Q_{eW}$ | $(\bar{L}\sigma^{\mu\nu}e)\tau^I HB_{\mu\nu}$ | $Q_{HL}^{(3)}$ | $(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{L}\tau^I\gamma^\mu L)$ |
| $Q_{HW}$ | $(H^\dagger H)W_{\mu\nu}^I W^{I\mu\nu}$ | $Q_{uG}$ | $(\bar{Q}\sigma^{\mu\nu}T^A u)\tilde{H}G_{\mu\nu}^A$ | $Q_{He}$ | $(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$ |
| $Q_{H\tilde{W}}$ | $(H^\dagger H)\tilde{W}_{\mu\nu}^I W^{I\mu\nu}$ | $Q_{uW}$ | $(\bar{Q}\sigma^{\mu\nu}u)\tau^I\tilde{H}W_{\mu\nu}^I$ | $Q_{HQ}^{(1)}$ | $(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$ |
| $Q_{HB}$ | $(H^\dagger H)B_{\mu\nu}B^{\mu\nu}$ | $Q_{uB}$ | $(\bar{Q}\sigma^{\mu\nu}u)\tilde{H}B_{\mu\nu}$ | $Q_{HQ}^{(3)}$ | $(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{q}\tau^I\gamma^\mu q)$ |
| $Q_{H\tilde{B}}$ | $(H^\dagger H)\tilde{B}_{\mu\nu}B^{\mu\nu}$ | $Q_{dG}$ | $(\bar{Q}\sigma^{\mu\nu}T^A d)HG_{\mu\nu}^A$ | $Q_{Hu}$ | $(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$ |
| $Q_{HWB}$ | $(H^\dagger\tau^I H)W_{\mu\nu}^I B^{\mu\nu}$ | $Q_{dW}$ | $(\bar{Q}\sigma^{\mu\nu}d)\tau^I HW_{\mu\nu}^I$ | $Q_{Hd}$ | $(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$ |
| $Q_{H\tilde{W}B}$ | $(H^\dagger\tau^I H)\tilde{W}_{\mu\nu}^I B^{\mu\nu}$ | $Q_{dB}$ | $(\bar{Q}\sigma^{\mu\nu}d)\tilde{H}B_{\mu\nu}$ | $Q_{Hud}$ | $(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu d)$ |

$$\text{Type VIII: } 5\times(\bar{L}L)(\bar{L}L)+7\times(\bar{R}R)(\bar{R}R)+8\times(\bar{L}L)(\bar{R}R)$$
$$+(\bar{L}R)(\bar{R}L)+4[(\bar{L}R)(\bar{L}R)+\text{h.c.}]=25(\bar{\Psi}\Psi)(\bar{\Psi}\Psi)$$

# SMEFT: Effective Vertices

T3: $Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$

T3: $Q_{HD} = (H^\dagger D^\mu H)^*(H^\dagger D^\mu H)$

T4: $Q_{HV} = (H^\dagger H)V^{\mu\nu}V_{\mu\nu}$

T4: $Q_{HWB} = (H^\dagger \tau^I H)W^I_{\mu\nu}B^{\mu\nu}$

T5: $Q_{\psi H} = (H^\dagger H)(\bar\Psi H\psi)$

T7: $Q^{(3)}_{HL} = (H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar L\gamma^\mu L)$

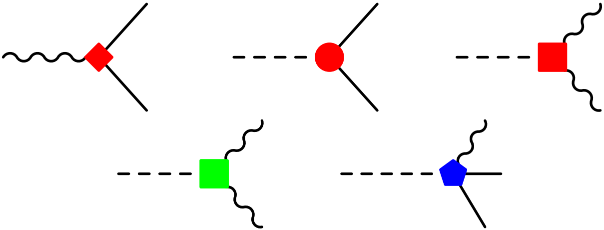
T7: $Q^{(1,3)}_{H\Psi} = (H^\dagger \overleftrightarrow{D}_\mu H)(\bar\Psi\gamma^\mu\Psi)$

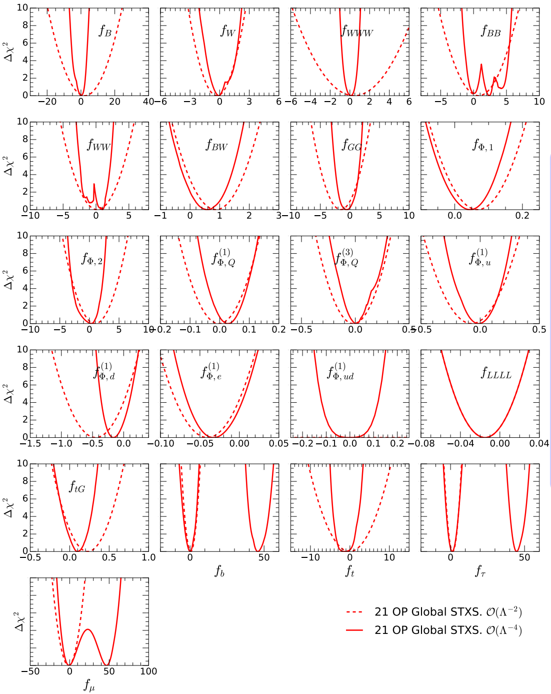
T7: $Q_{H\psi} = (H^\dagger \overleftrightarrow{D}_\mu H)(\bar\psi\gamma^\mu\psi)$

T8: $Q_{LL} = (\bar L\gamma^\mu L)(\bar L\gamma^\mu L)$

SM–like

Non-SM-like kinematic structure






Almeida, Alves, Éboli, Gonzalez-Garcia
arXiv:2108.04828

Uses:

- EWPD
- EW diboson production
- Higgs data

dashed - $\mathcal{O}\left(\frac{1}{\Lambda^2}\right)$

solid - $\sigma\left(\mathcal{O}\frac{1}{\Lambda^4}\right) \times \mathrm{BR}\left(\mathcal{O}\frac{1}{\Lambda^4}\right)$

# D6, D6$^2$, and D8

- Big impact from D6$^2$ $\sim \left(\frac{1}{\Lambda^2}\right)^2$
- LHC EFT WG, Area 1 – Truncation, validity, uncertainties
  "although they only constitute a partial set of $1/\Lambda^4$ corrections, the squares of amplitudes featuring a single dimension-six operator insertion provide a convenient proxy to estimate $1/\Lambda^4$ corrections, as they are well defined and unambiguous. They are indeed gauge invariant and can be translated exactly from one dimension-six operator basis to the other."
- Cen Zhang, SMEFTs living on the edge, arXiv:2112.11665
  "Our results indicate that the dimension-8 operators encode much more information about the UV than one would naively expect, which can be used to reverse engineer the UV physics from the SMEFT."

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# Beyond leading order in the SMEFT

At D6 in the SMEFT we have 59 operator forms, at D8 we have 895!

Two complete bases have been formulated:

- Chris Murphy, arXiv:2005.00059
- Hao-Lin Li et al., arXiv:2005.00008

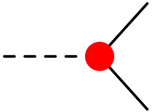
A *bit of a nightmare* to achieve, but some groups make predictions at D8 (e.g.):

- Hays et al., Assoc. Production of the Higgs, arXiv:1808.00442
- Boughezal et al., Dilepton production, arXiv:2106.05337
- Boughezal et al., Drell Yan, arXiv:2207.01703
- Asteriadis et al., Gluon fusion of Higgs, arXiv:2212.03258

But this is greatly simplified by employing the geoSMEFT methodology,
Helset et al. arXiv:2001.01453

# Higgs decay to two fermions

Imagine a Higgs is produced and we want to study its decay to two fermions:



What does a SM-theorist see?

$$i\mathcal{M} = Y_\psi \bar{u} v + \text{loops}$$

$$|\mathcal{M}|^2 \sim Y_\psi^2 (2 p_\psi \cdot p_{\bar{\psi}}) + \text{loops}$$

What does a SMEFT-theorist see?

$$i\mathcal{M} \sim \left( Y_\psi + \frac{c_{H\psi}^{(6)}}{\Lambda^2} v^2 + \frac{c_{H\psi}^{(8)}}{\Lambda^2} v^4 \right) \bar{u} v + \text{loops}$$

$$|\mathcal{M}|^2 \sim \left[ Y_\psi^2 + 2 Y_\psi \frac{c_{H\psi}^{(6)}}{\Lambda^2} v^2 + \left( \frac{c_{H\psi}^{(6)}}{\Lambda^2} v^2 \right)^2 + Y_\psi \frac{c_{H\psi}^{(8)}}{\Lambda^4} v^4 + \cdots \right] 2 p_\psi \cdot p_{\bar{\psi}} + \text{loops}$$

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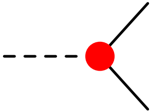

What does an experimentalist see/what does a theorist think an experimentalist sees?

$$\Gamma \sim \left|\langle\Omega|\, T\left\{h, \bar{\psi}, \psi\right\} |\Omega\rangle\right|^2 = \left|\left(-i\frac{\delta}{\delta h}\right)\left(-i\frac{\delta}{\delta\bar{\psi}}\right)\left(-i\frac{\delta}{\delta\psi}\right) Z(\phi)\right|^2_{\text{fields}\to 0}$$

This is exact. Nature doesn't care about perturbation theory.

In the loop expansion this isn't tractable.
But if we study the geometry of the SMEFT expansion, in special cases it is.

# The geoSMEFT I

Consider the $h\bar{\psi}\psi$ correlation function:

$$
\begin{aligned}
\left\langle h\bar{\psi}\psi\right\rangle &\sim \langle\Omega|\, T\{h\bar{\psi}\psi\}\,|\Omega\rangle \\
&\sim \frac{\delta^3}{\delta h\delta\bar{\psi}\delta\psi}\int\mathcal{D}(\text{fields})\exp\left[iS_{\text{SMEFT}}\right]\Big|_{\text{fields}\to 0} \\
&\sim \left\langle\frac{\delta}{\delta h}\frac{\delta^2}{\delta\bar{\psi}\delta\psi}\mathcal{L}_{\text{SMEFT}}\right\rangle
\end{aligned}
$$

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\end{aligned}
$$

Simplifying:

$$
\begin{aligned}
\frac{\delta}{\delta h}\frac{\delta^2}{\delta\bar{\psi}\delta\psi}\mathcal{L}_{\text{SMEFT}} &\Leftrightarrow \frac{\delta}{\delta h}\frac{\delta^2}{\delta\bar{\psi}\delta\psi}\Big[\underbrace{(\text{something})}_{\text{Not } W,B,\psi,\text{ bc fields}\to 0}\bar{\psi}\psi\Big] \\
&= \frac{\delta}{\delta h}\left[(\text{something})\right] \\
&\equiv \frac{\delta}{\delta h}\mathcal{Y}(H)
\end{aligned}
$$

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Simplifying:

$$\begin{aligned}\frac{\delta}{\delta h}\frac{\delta^2}{\delta\bar{\psi}\delta\psi}\mathcal{L}_{\text{SMEFT}} &\Leftrightarrow \frac{\delta}{\delta h}\frac{\delta^2}{\delta\bar{\psi}\delta\psi}\Big[\underbrace{(\text{something})}_{\text{Not } W,B,\psi,\text{ bc fields}\to 0}\bar{\psi}\psi\Big] \\ &= \frac{\delta}{\delta h}\big[(\text{something})\big] \\ &\equiv \frac{\delta}{\delta h}\mathcal{Y}(H)\end{aligned}$$

We can define $\mathcal{Y}$, a field-space connection, (an analogue to a metric in GR)
From this we can define two important (tree level) geometric quantities:

$$\bar{m} = \langle\mathcal{Y}\rangle \qquad\qquad \langle h\bar{\psi}\psi\rangle = \left\langle\frac{\delta\mathcal{Y}}{\delta h}\right\rangle$$

# The geoSMEFT II

Can define more field-space connections/geometric quantities by varying $\mathcal{L}_{\mathrm{SMEFT}}$:

$$M_{F_1,F_2,\cdots} \sim \frac{\delta}{\delta F_1}\frac{\delta}{\delta F_2}\cdots\mathcal{L}_{\mathrm{SMEFT}}\bigg|_{F\to 0}$$

with $F_i \in \{W^A_{\mu\nu}, B_{\mu\nu}, (D_\mu H), \psi, \bar{\psi}\}$ ($H$ is by definition a part of $M$ as it has a vev)

From this we can define to all-orders in $1/\Lambda$ the two-point functions:

$$g_{AB}\mathcal{W}^A_{\mu\nu}\mathcal{W}^B_{\mu\nu} \quad \Leftrightarrow \quad g_{AB} = \frac{-2g^{\mu\nu}g^{\sigma\rho}}{d^2}\frac{\delta^2\mathcal{L}}{\delta\mathcal{W}^A_{\mu\sigma}\delta\mathcal{W}^B_{\nu\rho}}$$

$$h_{IJ}(D^\mu\phi)^I(D_\mu\phi)^J \quad \Leftrightarrow \quad h_{IJ} = \frac{g^{\mu\nu}}{d}\frac{\delta^2\mathcal{L}}{\delta(D_\mu\phi)^I\delta(D_\nu\phi)^J}$$

$$Y\bar{\Psi}\psi \quad \Leftrightarrow \quad Y(\phi) = \frac{\delta\mathcal{L}}{\delta(\bar{\Psi}\psi)}$$

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And the three-point functions:

$$L^\psi_J(D^\mu\phi)^J(\bar{\psi}\Gamma_\mu\psi) \quad \Leftrightarrow \quad L^\psi_J = \frac{\delta^2\mathcal{L}}{\delta(D^\mu\phi)^J\delta(\bar{\psi}\Gamma_\mu\sigma\psi)}$$

$$d^\psi_A(\bar{\psi}\sigma^{\mu\nu}\psi)\mathcal{W}^A_{\mu\nu} \quad \Leftrightarrow \quad d^\psi_A = \frac{\delta^2\mathcal{L}}{\delta(\bar{\psi}\sigma^{\mu\nu}\psi)\delta\mathcal{W}^A_{\mu\nu}}$$

$$f_{ABC}W^{A,\mu\nu}W^B_{\nu\rho}W^{C,\rho}_\mu \quad \Leftrightarrow \quad f_{ABC} = \frac{g^{\nu\rho}g^{\sigma\alpha}g^{\beta\mu}}{3!d^3}\frac{\delta^3\mathcal{L}}{\delta\mathcal{W}^A_{\mu\nu}\delta\mathcal{W}^B_{\rho\sigma}\delta\mathcal{W}^C_{\alpha\beta}}$$

$$k^A_{IJ}(D_\mu\phi)^I(D_\nu\phi)^J W^A_{\mu\nu} \quad \Leftrightarrow \quad k^A_{IJ} = \frac{g^{\mu\rho}g^{\nu\sigma}}{2d^2}\frac{\delta^3\mathcal{L}}{\delta(D_\mu\phi)^I\delta(D_\nu\phi)^J\delta\mathcal{W}^A_{\rho\sigma}}$$

# Four point functions

In formulating the SMEFT we use the Eqs of Motion to reduce the basis of operators:

$$
\begin{aligned}
D^2 H &= Y\bar{\psi}_R\psi_L + \frac{1}{\Lambda}\cdots \\
i\not{D}\psi_L &= -Y\bar{\psi}_R H + \frac{1}{\Lambda}\cdots \\
i\not{D}\psi_R &= -Y H^\dagger\psi_L + \frac{1}{\Lambda}\cdots \\
D_\nu V^{\mu\nu} &= g_V J^\mu + \frac{1}{\Lambda}\cdots
\end{aligned}
$$

There is sufficient freedom in the EOM to guarantee that the field-space connections in the last slide are all that contribute to 2 and 3 point functions.

We cannot do this for all 4+ point functions.
Can think of this in analogy with momentum conservation ($D$s in EOM are momentum):

- 2pt: $p_1 = -p_2$, $p_1^2 = m^2$
- 3pt: $p_1 + p_2 = p_3$, $p_i \cdot p_j = \frac{1}{2}\left(m_k^2 - m_i^2 - m_j^2\right)$
- 4pt: $s + t + u = \sum_i m_i$
  We cannot fully reduce products of momenta to masses
  $\Leftrightarrow$ we cannot fully reduce derivatives in operators effecting four-point functions

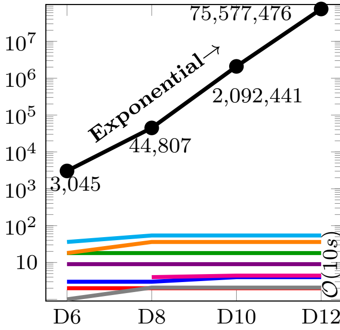
# three–point functions from geoSMEFT

| operator form | shifts: |
|---|---|
| $h_{IJ}(D\phi)^I(D\phi)^J$ | SM 3–point functions + Masses |
| $g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$ | SM triple gauge couplings + $h(\partial V)^2$ + mixing angles |
| $Y^\psi\bar{\Psi}_L\psi_R + h.c.$ | SM Yukawas+$\psi$ masses |
| $L^\psi_J(D^\mu\phi)^J(\bar{\psi}\Gamma_\mu\psi)$ | SM gauge-fermion couplings |
| $d^\psi_A W^{A,\mu\nu}(\bar{\psi}\sigma_{\mu\nu}\psi)$ | Dipoles |
| $f_{ABC}W^{A,\mu\nu}W^B_{\nu\rho}W^{C,\rho}_\mu$ | new TGCs $(\partial V)^3$ |
| $\kappa^A_{IJ}(D_\mu\phi)^I(D_\nu\phi)^J W^A_{\mu\nu}$ | new TGCs $(\partial h)^2(\partial V)$, removed from D6 in Warsaw |

# Saturation of number of operators

(This information is contained in the Hilbert Series)
(see e.g. Lehman & Martin 2015, Henning et al. 2015)

| Operator form: | Mass Dimension 6 | 8 | 10 |
|---|---|---|---|
| $h_{IJ}(D_\mu\phi)^I(D^\mu\phi)^J$ | 2 | 2 | 2 |
| $g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$ | 3 | 4 | 4 |
| $k_{IJA}(D^\mu\phi)^I(D^\nu\phi)^J W^A_{\mu\nu}$ | 0 | 3 | 4 |
| $f_{ABC}W^A_{\mu\nu}W^{B,\nu\rho}W^{C,\mu}_\rho$ | 1 | 2 | 2 |
| $Y^\psi_{pr}\bar{\Psi}_L\psi_R + h.c.$ | $2N_f^2$ | $2N_f^2$ | $2N_f^2$ |
| $d^{\psi,pr}_A\bar{\Psi}_L\sigma_{\mu\nu}\psi_R W^{\mu\nu}_A + h.c.$ | $4N_f^2$ | $6N_f^2$ | $6N_f^2$ |
| $L^{\psi_R}_{pr,J,A}(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$ | $N_f^2$ | $N_f^2$ | $N_f^2$ |
| $L^{\Psi_L}_{pr,J,A}(D^\mu\phi)^J(\bar{\Psi}_{p,L}\gamma_\mu\sigma_A\Psi_{r,L})$ | $2N_f^2$ | $4N_f^2$ | $4N_f^2$ |



# geoSMEFT on the $Z$-pole

| operator form | shifts: |
|---|---|
| $h_{IJ}(D\phi)^I(D\phi)^J$ | SM 3–point functions + Masses |
| $g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$ | SM triple gauge couplings + $h(\partial V)^2$ + mixing angles |
| $L^\psi_J(D^\mu\phi)^J(\bar{\psi}\Gamma_\mu\psi)$ | SM gauge-fermion couplings |

$$\psi = \{Q, L, u_R, d_R, e_R\}$$

$$g^{Z,\psi}_{\rm eff} = \frac{\bar{g}_Z}{2}\left[(2\, s^2_{\theta_Z} Q_\psi - \sigma_3) + \bar{v}_T \langle L^\psi_{3,4}\rangle + \sigma_3 \bar{v}_T \langle L^\psi_{3,3}\rangle\right]$$

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$$g_{\rm eff}^{Z,\psi} = \frac{\bar g_Z}{2}\left[(2\, s^2_{\theta_Z} Q_\psi - \sigma_3) + \bar v_T \langle L^\psi_{3,4}\rangle + \sigma_3 \bar v_T \langle L^\psi_{3,3}\rangle\right]$$

$$\bar m_Z^2 = \frac{\bar g_Z}{4}^2 \langle h_{33}\rangle \bar v_T^2 \qquad s^2_{\theta_Z} = f(\langle g_{AB}\rangle, g_1, g_2)$$

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$$\bar{m}_Z^2 = \frac{\bar{g}_Z{}^2}{4}\langle h_{33}\rangle \bar{v}_T^2 \qquad s^2_{\theta_Z} = f(\langle g_{AB}\rangle, g_1, g_2)$$

$$\Gamma_{Z\to\bar{\psi}\psi} = \frac{N_c^\psi}{24\pi}\bar{m}_Z |g_{\rm eff}^{Z,\psi}|^2 \left(1 - \frac{4\bar{m}_\psi^2}{\bar{m}_Z^2}\right)^{3/2}$$

# $Z$-pole pheno, arXiv:2102.02819

$$g_{\rm eff}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2\, s_{\theta_Z}^2 Q_\psi - \sigma_3) + \bar{v}_T \langle L_{3,4}^\psi \rangle + \sigma_3 \bar{v}_T \langle L_{3,3}^\psi \rangle\right]$$

$$\bar{m}_Z^2 = \frac{\bar{g}_Z}{4}^2 \langle h_{33} \rangle \bar{v}_T^2 \qquad s_{\theta_Z}^2 = f(\langle g_{AB} \rangle, g_1, g_2)$$

Number of parameters at each order (for LH fermion):

| | geoSMEFT | D6 | D(6+n) |
|---|---|---|---|
| $\bar{g}_Z$ | 1 | 3<br>$\{c_{HW}, c_{HB}, c_{HWB}\}$ | 4<br>$\{c_{HW}^{(6+n)}, c_{HW,2}^{(6+n)}, c_{HB}^{(6+n)}, c_{HWB}^{(6+n)}\}$ |
| $s_{\theta_Z}$ | 1 | 3<br>$\{c_{HW}, c_{HB}, c_{HWB}\}$ | 4<br>$\{c_{HW}^{(6+n)}, c_{HW,2}^{(6+n)}, c_{HB}^{(6+n)}, c_{HWB}^{(6+n)}\}$ |
| $\langle h_{33} \rangle$ | 1 | 1<br>$\{c_{HD}\}$ | 2<br>$\{c_{HD}^{(6+n)}, c_{HD,2}^{(6+n)}\}$ |
| $\langle L_{3,4}^\psi \rangle$ | 1 | 1<br>$\{c_{Hl}^{(3)}\}$ | 2<br>$\{c_{Hl}^{(6+n,2)}, c_{Hl}^{(6+n,3)}\}$ |
| $\langle L_{3,3}^\psi \rangle$ | 1 | 1<br>$\{c_{Hl}^{(1)}\}$ | 1<br>$\{c_{Hl}^{(6+n,1)}\}$ |
| sum: | 3+2 | 4+2 | 4+5 |

# geoSMEFT summary

1. all 3-point functions defined for both $\frac{v^2}{\Lambda^2}$ and $\frac{p^2}{\Lambda^2}$ expansions
   for 3pt only: $\{p_i \cdot p_j = \frac{1}{2}(m_k^2 - m_i^2 - m_j^2)\}$
2. $Z$-pole predictions can be derived to all orders
3. largest Higgs production xs ($hgg$) can be defined in geoSMEFT
   ✓ + D6-loop (TC, A Martin, M Trott, arXiv:2107.07470)
4. largest Higgs decay ($hbb$) + most accurately measured ($h\gamma\gamma$)
   for $h\gamma\gamma$:✓ + D6-loop (TC, A Martin, M Trott, arXiv:2107.07470)
5. Forthcoming: Higgs associated prod (w. A. Martin)
   Triple gauge scattering (w. Gonzalez-Garcia et al)
6. currently expanding to include 4-point functions
   only all orders in $\frac{v^2}{\Lambda^2}$ expansion
   for 4pt have infinitely many $\frac{p^2}{\Lambda^2}$ operators: $\{s^n, t^m, m_i^2\}$

# geoSMEFT goals/questions

1. Is it safe/conservative to do a fit to the geometric quantities in place of Wilson coeffs?
   $\rightarrow$ From an experimental perspective, a lot less open parameters in simulations

2. How do we identify blind directions in a fit that's nonlinear in the fit parameters?
   $\rightarrow$ This is also ideal for experimental studies of the SMEFT...
   $\rightarrow$ Combos can be made after using measurements + correlations

3. Write the 4-point geoSMEFT using op. forms occurring at D8
   $\rightarrow$ consistent studies to order $1/\Lambda^4$ resumming $v$ expansion.

4. Can geometric quantities be consistently defined to one-loop?

$$\langle\Omega|\, T\{\text{stuff}\}\, |\Omega\rangle \Leftrightarrow M_{F_1 F_2 \cdots} = \frac{\delta^n}{\delta F_1 \delta F_2 \cdots} \mathcal{L}_{\text{SMEFT}}$$

# geoSMEFT goals/questions

Also, can searches be optimized for the SMEFT?
→ Most SMEFT studies use SM measurements as a constraints on NP.

$$\mu_{pp\to h\bar{\psi}\psi} = \frac{\sigma(pp\to h\bar{\psi}\psi)_{\text{measured}}}{\sigma(pp\to h\bar{\psi}\psi)_{\text{SM}}}$$

But when we look at the phase space population for SMEFT processes, they are quite different:

$$\begin{aligned}\frac{\sigma(pp\to h\bar{\psi}\psi)_{\text{SMEFT}}}{\sigma(pp\to h\bar{\psi}\psi)_{\text{SM}}} &\sim 1 + 2.6\times(c^{(2)}_{HZZ}) + 0.38\times(c^{(2)}_{HZZ})^2 - 2.6\times(c_{HAZ}) + \cdots \\ c^{(2)}_{HZZ} &\rightarrow hZ_{\mu\nu}Z^{\mu\nu} \\ c_{HAZ} &\rightarrow hA_{\mu\nu}Z^{\mu\nu}\end{aligned}$$

But global fits ignore the difference in phase space population between $\mu$, a SM-like measurement, and the above SMEFT example...