Progress of Muon Collider Lattice Design

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With special thanks to K. Oide, D. Schulte and R. Tomas

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Outline

- 10TeV Muon Collider
  - Final Focusing Scheme
  - Chromatic Correction Scheme
- Tracing Studies
- Discussion

10TeV Muon Collider

TABLE I. 10 TeV center of mass energy muon collider.

Parameters	Symbol	Unit	10TeV com mc
Particle energy	E	GeV	5000
Particle momentum	P_0	GeV c^{-1}	5000
Luminosity per IP	$\mathcal{L}$	10^{34} cm^{-2} s^{-1}	20
Bunch population	N_p	10^{12}	1.8
Transverse normalized rms emittance	$\varepsilon_{nx} = \varepsilon_{ny}$	μm	25
Longitudinal emittance ($4\pi\ \sigma_E\ \sigma_T$)	ε_l	eVs	0.314
Rms bunch length	σ_z	mm	1.5
Relative rms energy spread	δ	%	0.1
Beta function at IP	$\beta_x^\star = \beta_y^\star$	mm	1.5
Beam power with 5 Hz repetition rate	P_{beam}	MW	7.2

10TeV Muon Collider - In a nutshell

1.5mm β*

=> ~500Km βs in the Final Focusing (FF) scheme (also large δ=0.1%).

=> Enormous chromatic aberrations at the optical functions (described by Montague functions).

=> Necessity for a Chromatic Correction (CC) scheme right after the FF.

=> Need for strong sextupolar kick (beta values, dispersion, sextuple strength) that degrade the DA due to not exactly 0 or π phase advance between the kicks.

=> The CC generate large positive momentum compaction factor (α_p) and should be controlled (keep the bunch length short) in the arcs among other parameters.

Muon decay (short lifetime τ_0~2.2μs or τ_{5TeV}~0.1s)

=> The resulting neutrinos even from a short straight piece of collider generate a narrow "radiation cone" that is an issue at the location, where they reach the earth surface

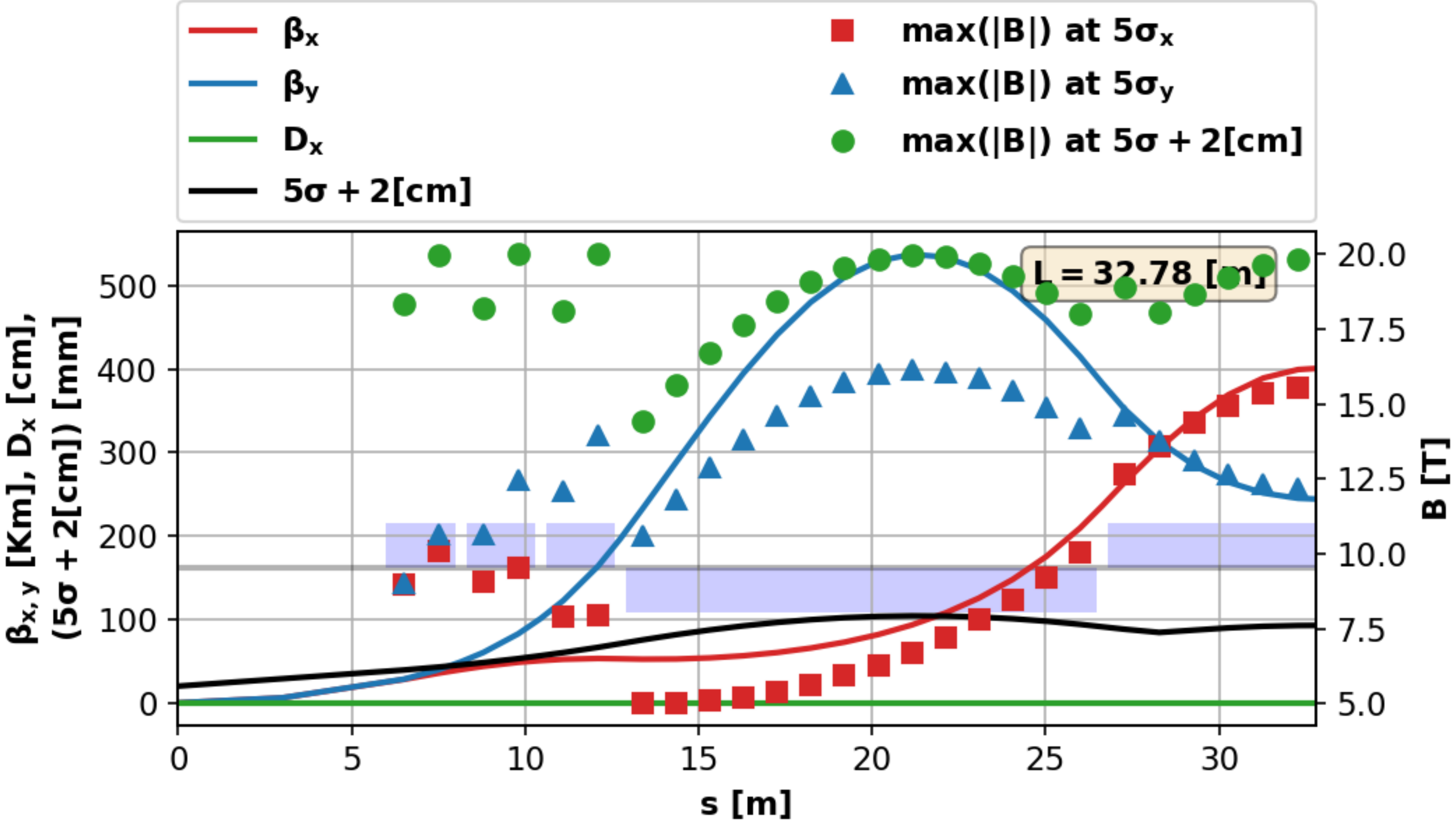
=> The planned shape of the collider is like a race track (2 straight sections for IPs)

=> Extensive use of dipoles and combined function magnets

10TeV Muon Collider - Final Focusing Scheme

- $L^* = 6m$ and a triplet is used for the Final Focusing (FF).
- The maximum allowed magnetic field at the FF scheme is assumed to be the 20T.
- Due to the fast increase of the β functions right after the IP, the first magnet is splitted in three shorter ones with different gradient, reducing that way the length of the FF scheme.
- The first focusing magnets can be used to control the beta ratio (β_x/β_y) at the end of the FF scheme while the last two elements are used for the point to parallel matching ($\alpha_{x,y} = 0$ at the FF triplet end).

10TeV Muon Collider - Final Focusing Scheme



$$B\rho = 16678.205\ [Tm]$$

$$Aperture = 2(5\sigma + 0.002)\ [m]$$

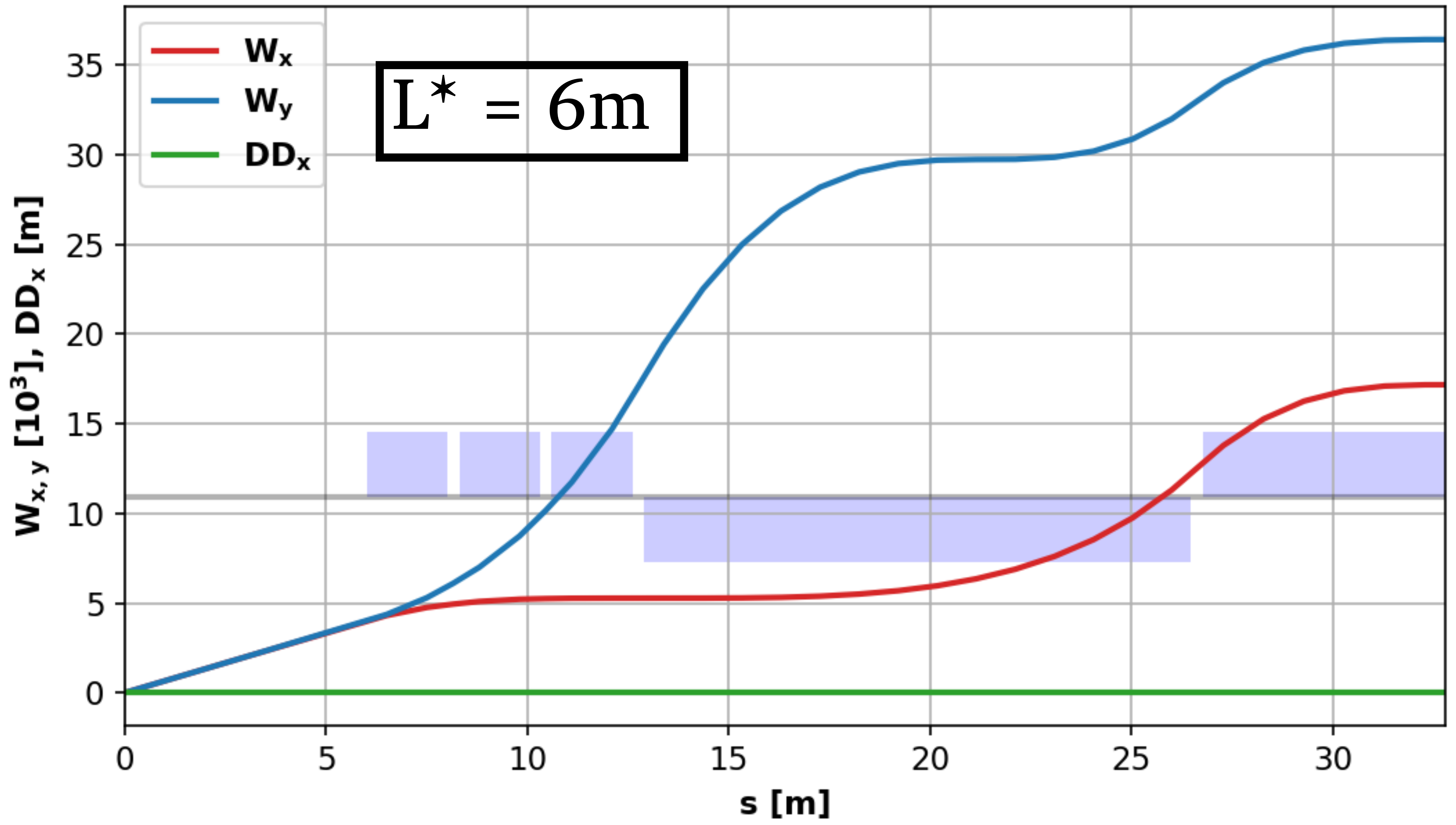
$$\sigma_j = \sqrt{\frac{\beta_j \varepsilon_{nj}}{\beta_r \gamma_r} + (D_j \delta_p)^2}\ [m]\ with\ j = x, y$$

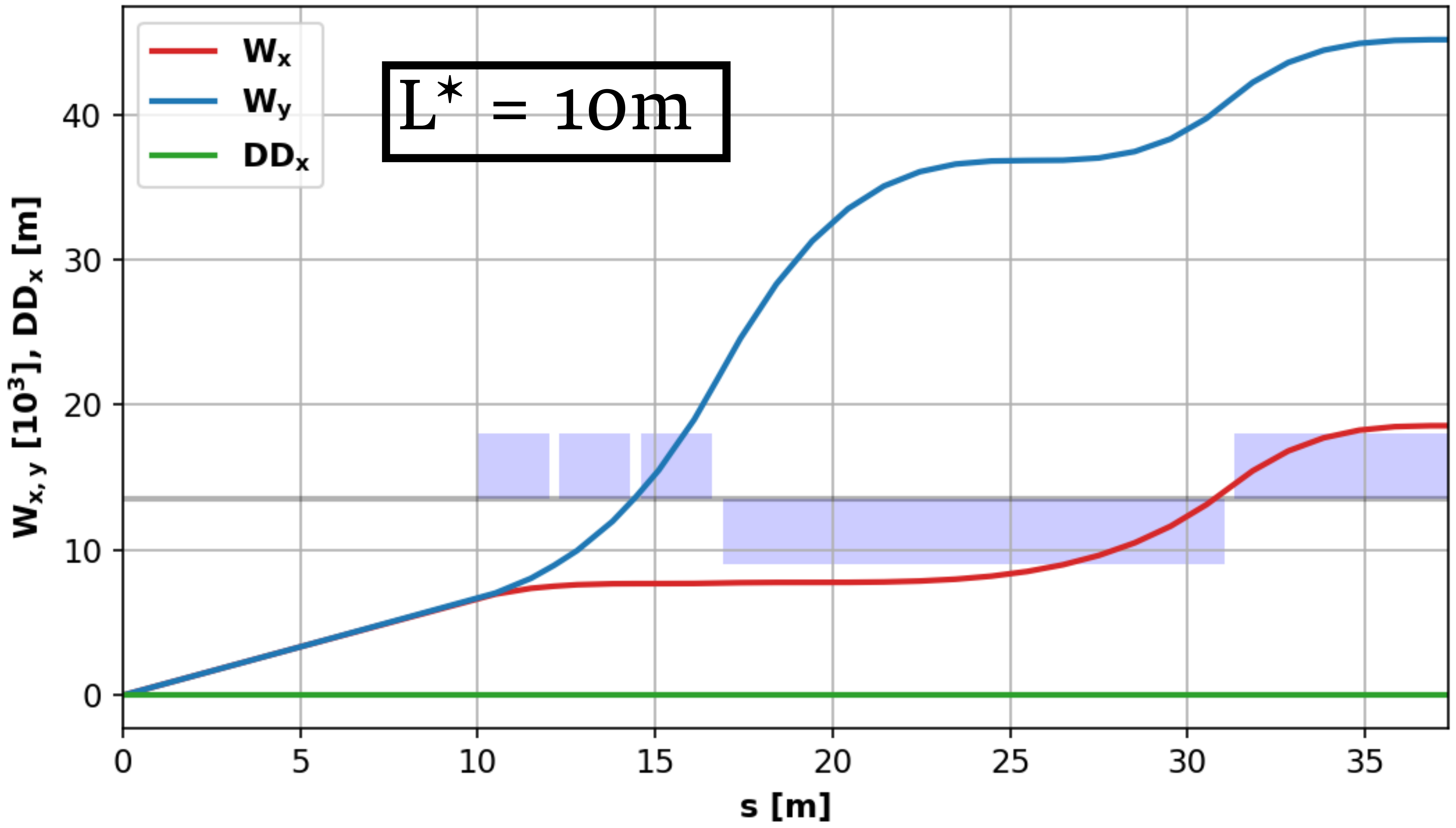
$$\sigma = max(\sigma_x, \sigma_y)\ [m]$$

10TeV Muon Collider - Final Focusing Scheme

- Due to strong focusing quadrupoles (β^*=1.5mm), the Montague chromatic functions ($W_{x,y}$) that describe the optics perturbation for off-momentum particles w.r.t on-momentum one become very large.

- Together with the large momentum spread ($\delta=10^{-3}$), these W values indicate enormous chromatic effects that should be compensated in order to avoid performance degradation.





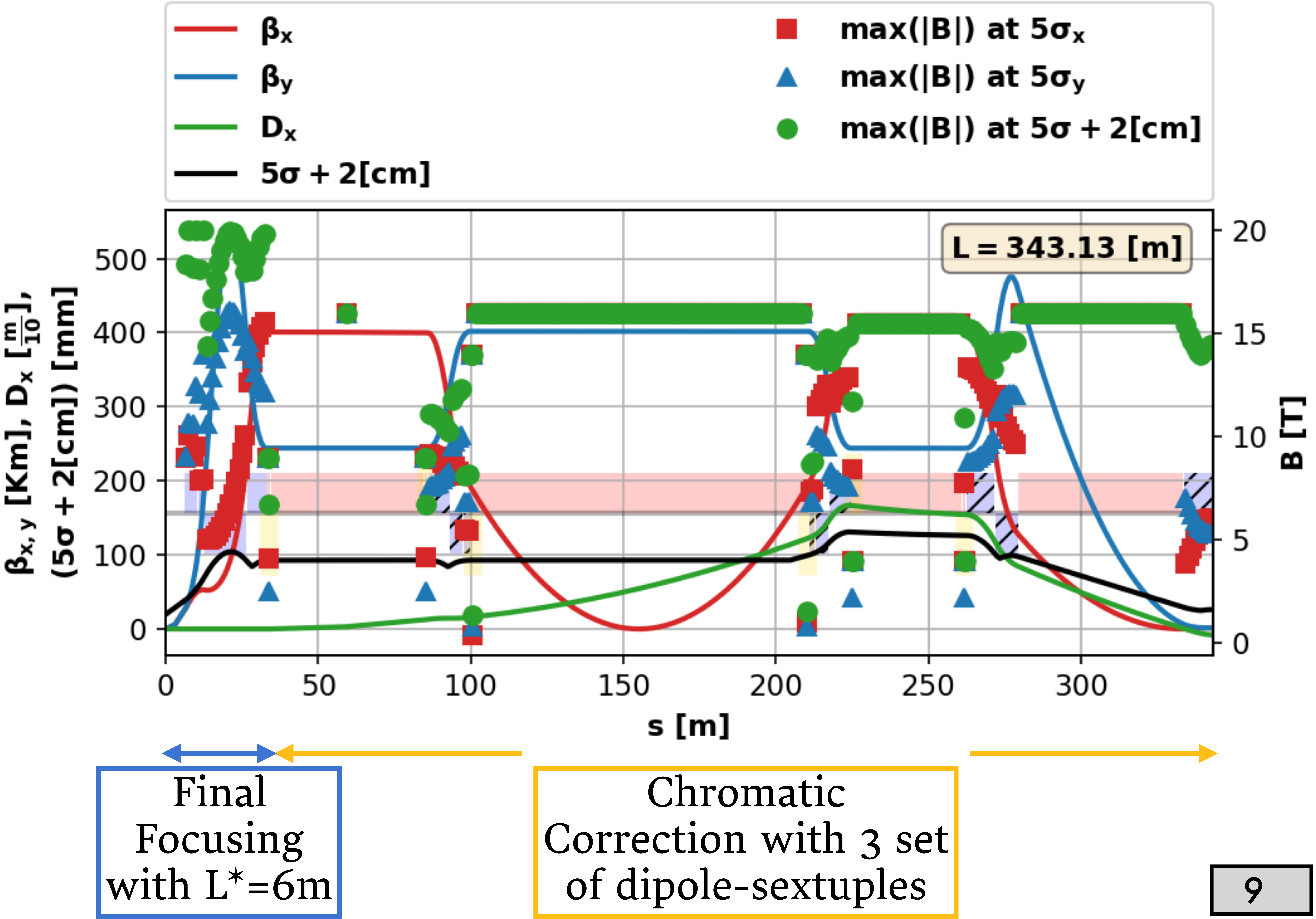
10TeV Muon Collider - Chromatic Correction Scheme

- In order to address the chromatic phenomena before entering the arcs, the Chromatic Correction (CC) schemes is designed and placed right after the FF quads.
- The maximum allowed magnetic field is assumed to be the 16T.
- The CC scheme include 3 sets (doublets) of combined function dipole-sextupole magnets and each set is placed at positions with large β_q, where q=x or y, for the correction of the W_q at the end of CC scheme and the correction of the DD_x at the IPs.
- Each set include a pair of dipole-sextupole magnets with opposite polarity (k_2,$-k_2$) when are separated by an identity like transformation and with the same k_2 when are separated by -I transform at x plane for the compensation of the RDTs excited by the sextupolar component.
- The D_{px} is also controlled in the CC scheme (by generating a π phase advance jump at the x plane) facilitating the matching between the CC and arc optics.

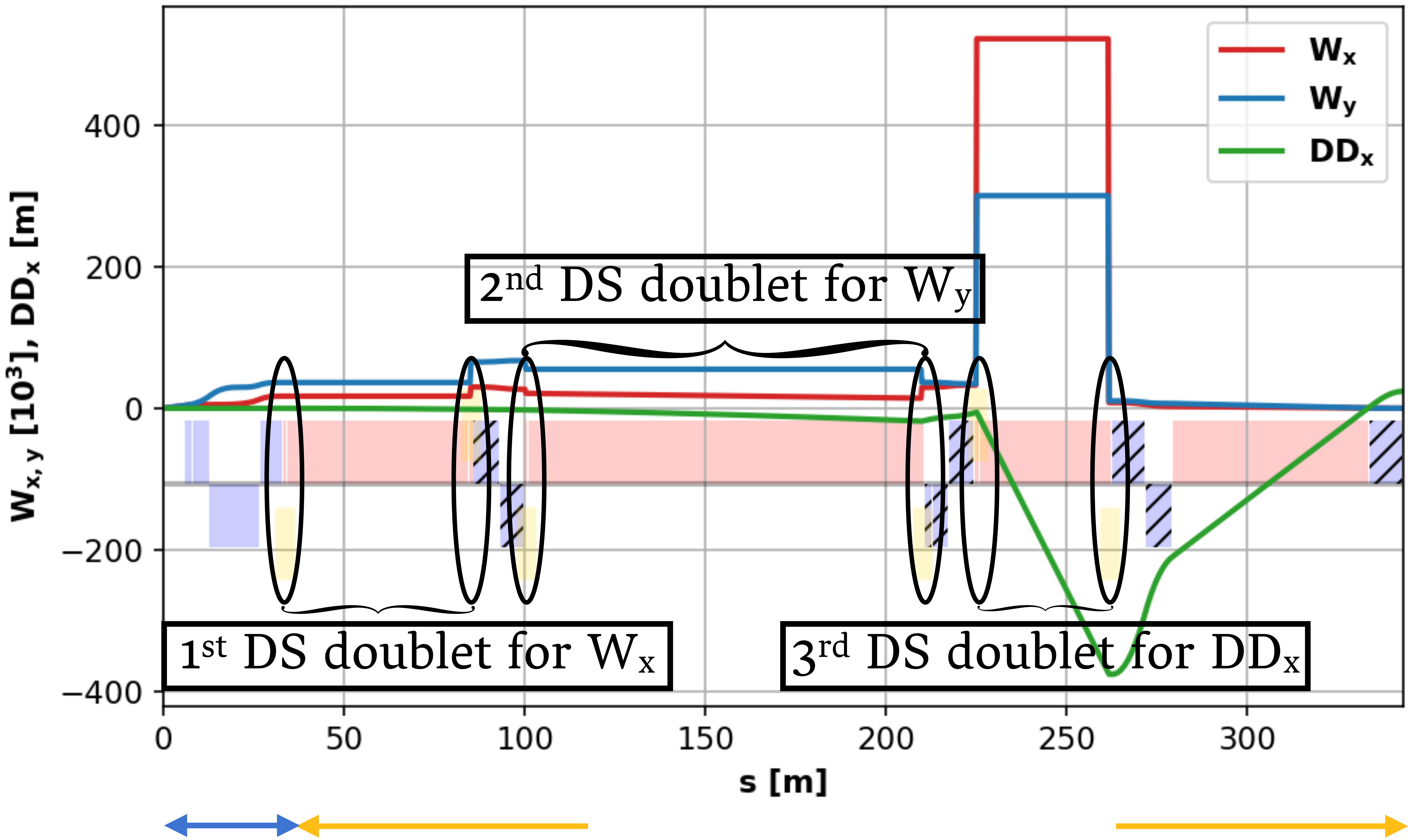
10TeV Muon Collider - Chromatic Correction Scheme

Colour code for lattice elements:

- Red dipoles
- Blue quadrupoles
- Hashed blue dipole-quadrupoles
- Red + Gold dipole-sextupoles (all 1m long)



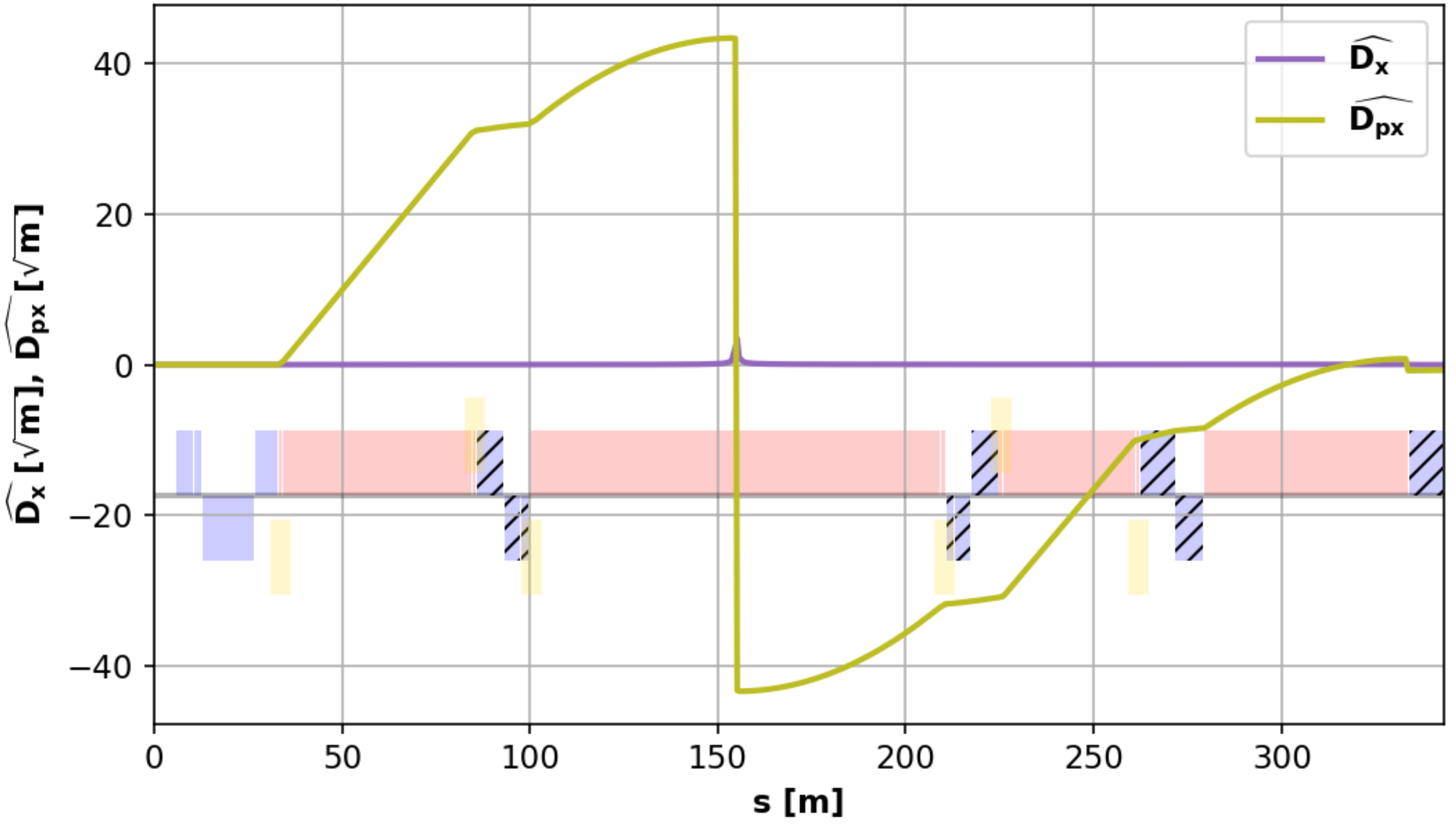
10TeV Muon Collider - Chromatic Correction Scheme



Final Focusing with L*=6m

Chromatic Correction with 3 set of dipole-sextuples

10TeV Muon Collider - Chromatic Correction Scheme

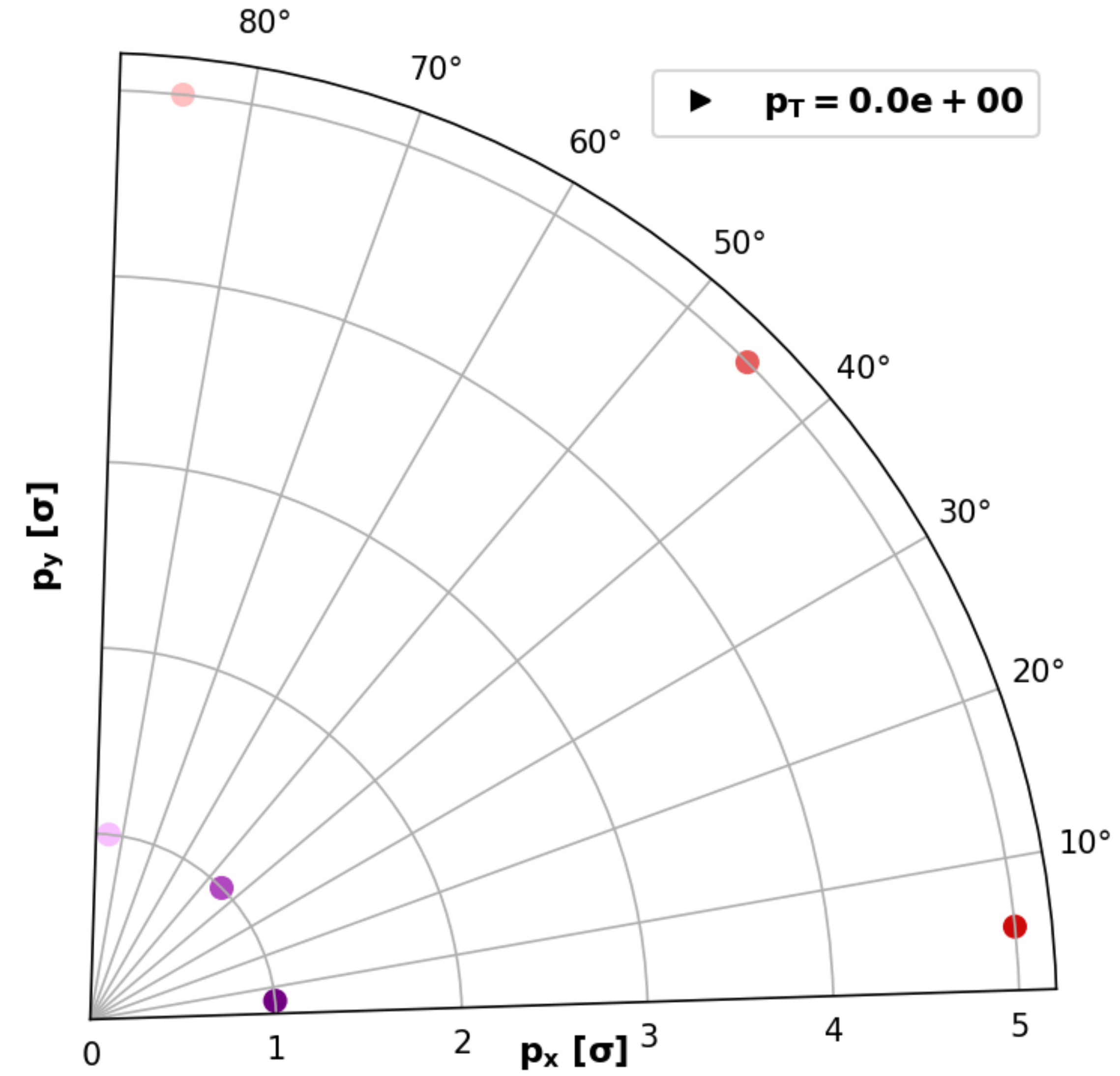


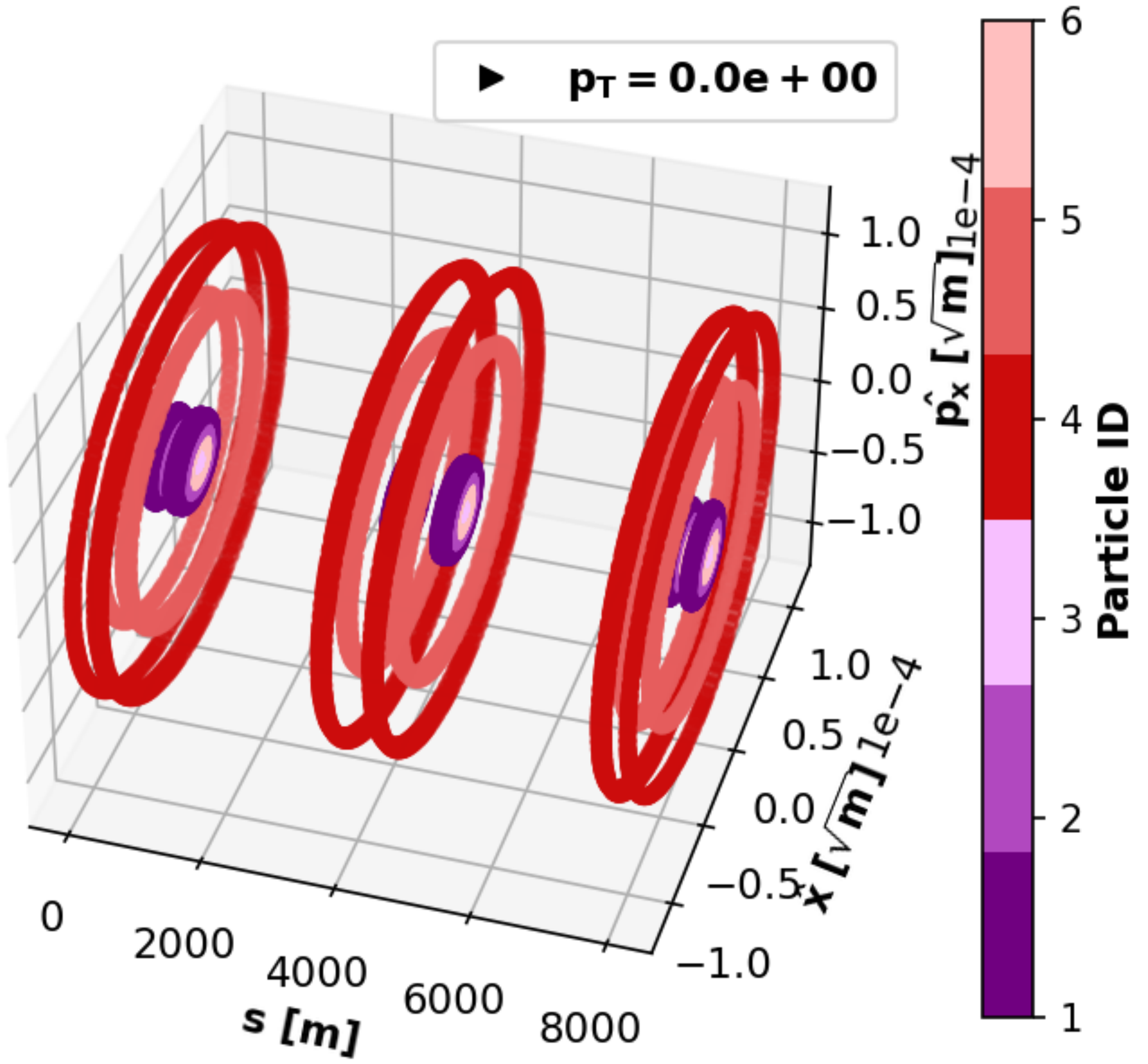
Final Focusing with L*=6m

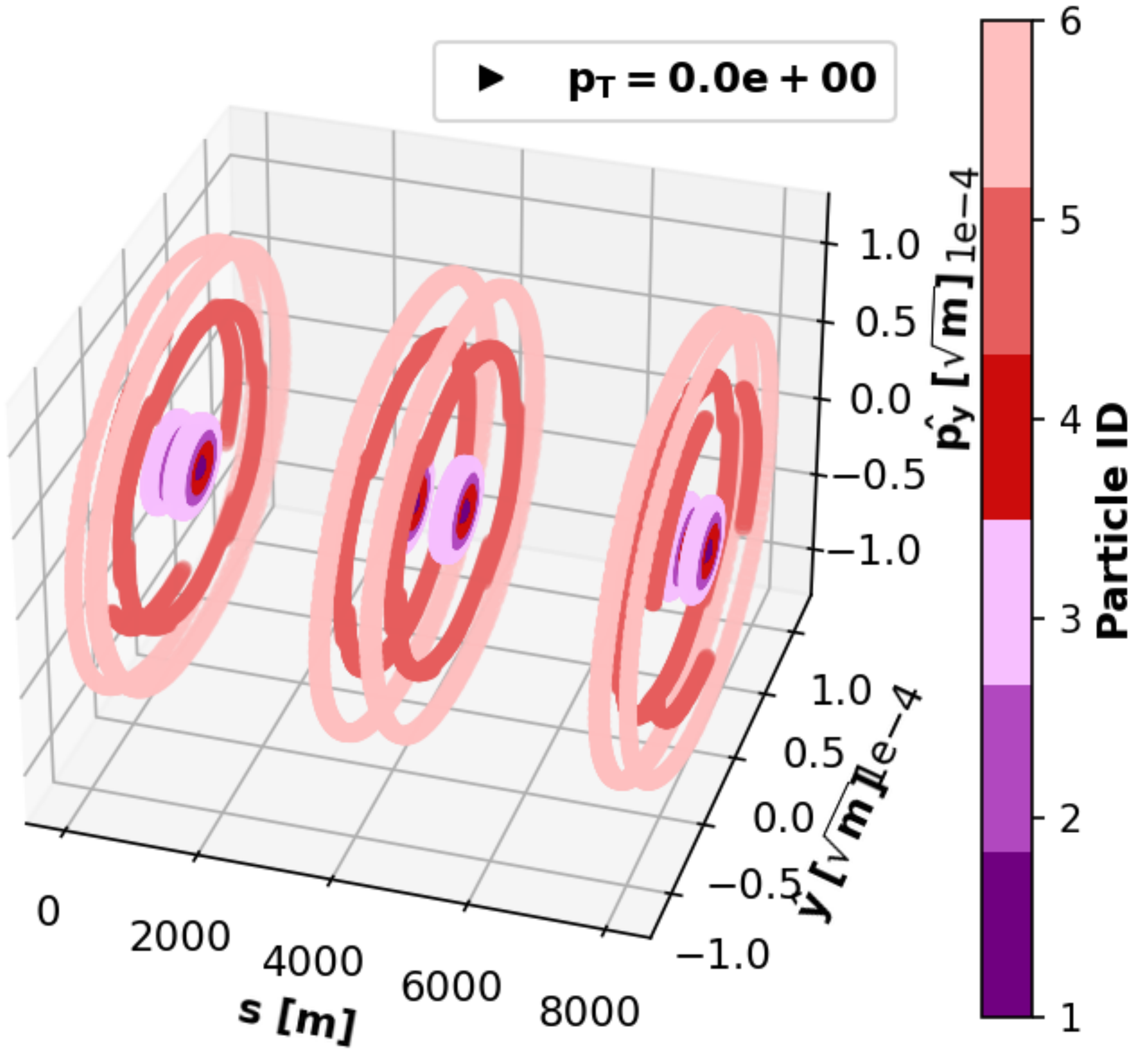
Chromatic Correction with 3 set of dipole-sextuples

10TeV Muon Collider - Tracking Studies

- The linear lattice guaranty stable motion for long time (nonlinear elements switched off).

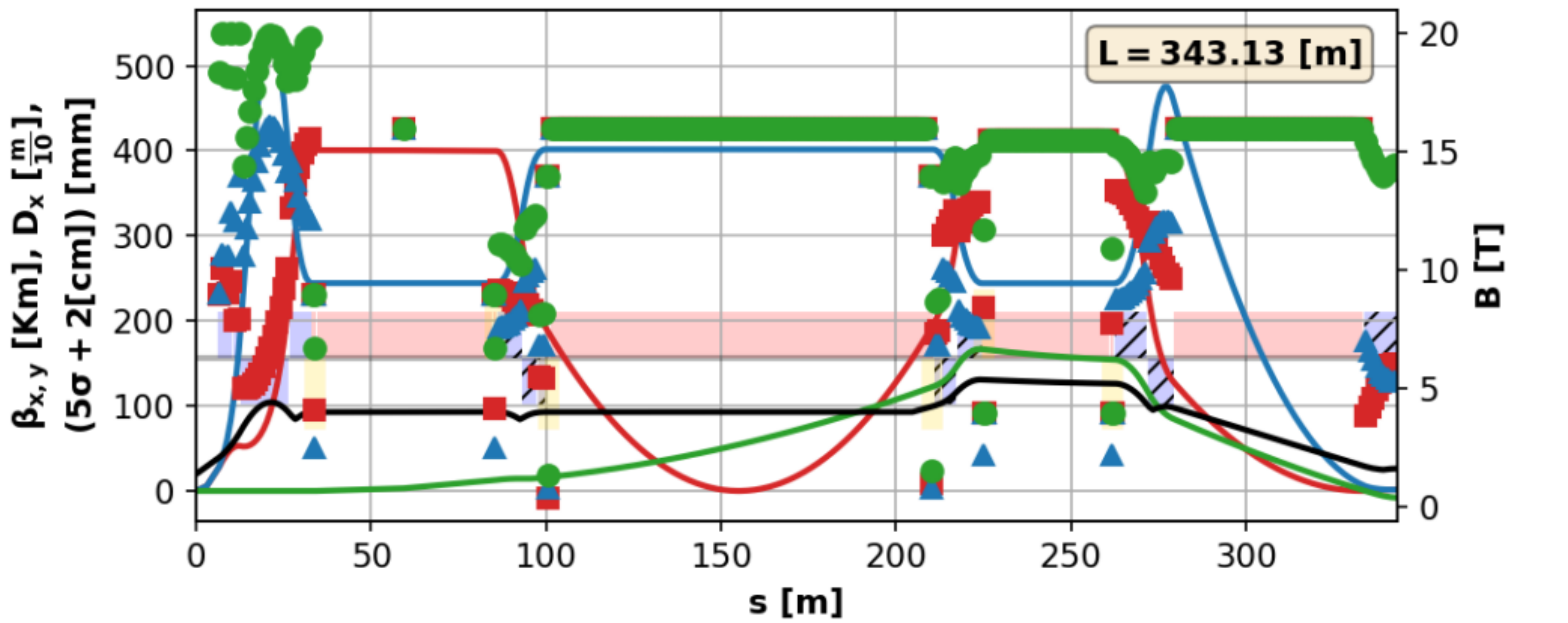






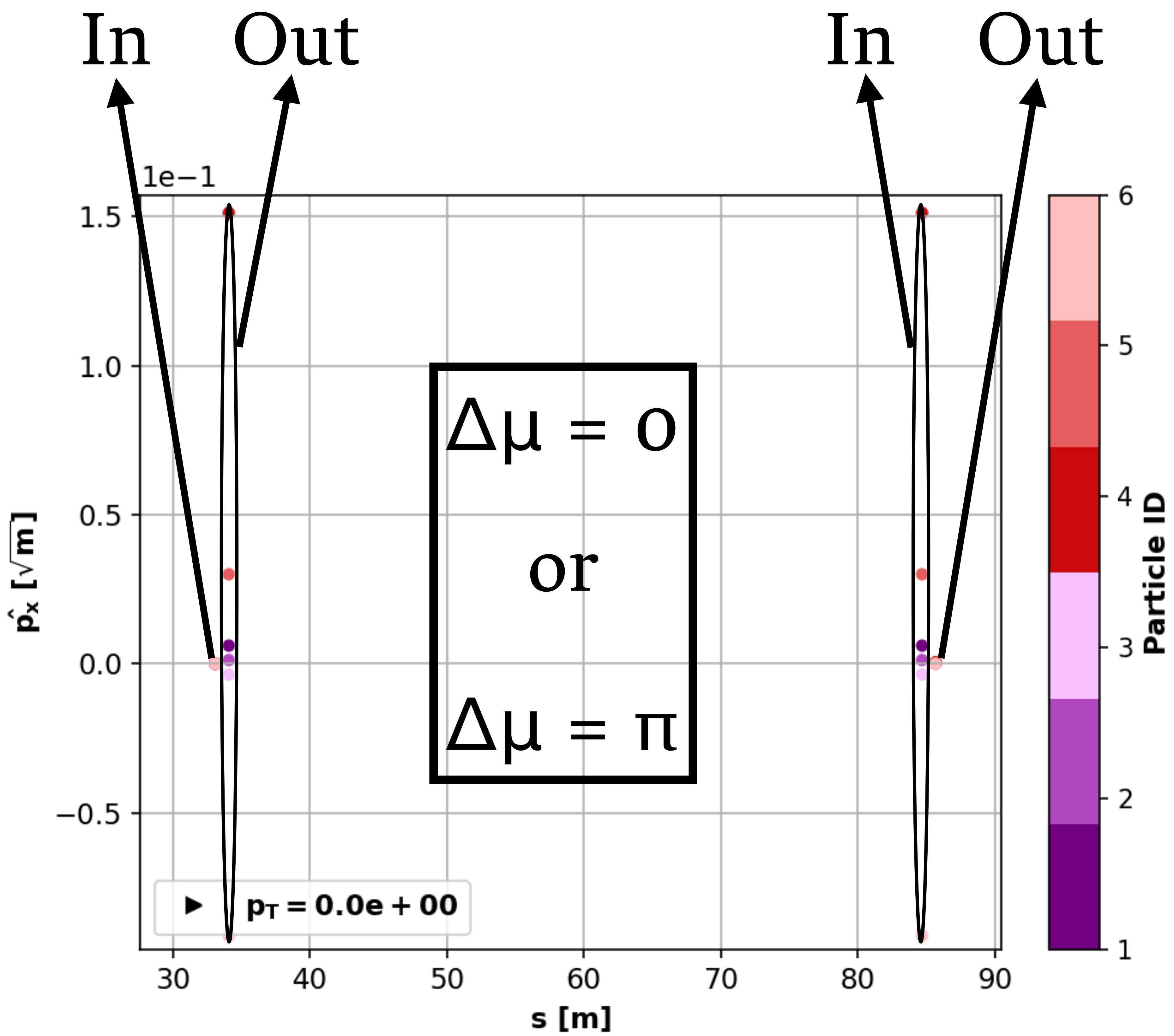
10TeV Muon Collider - Tracking Studies

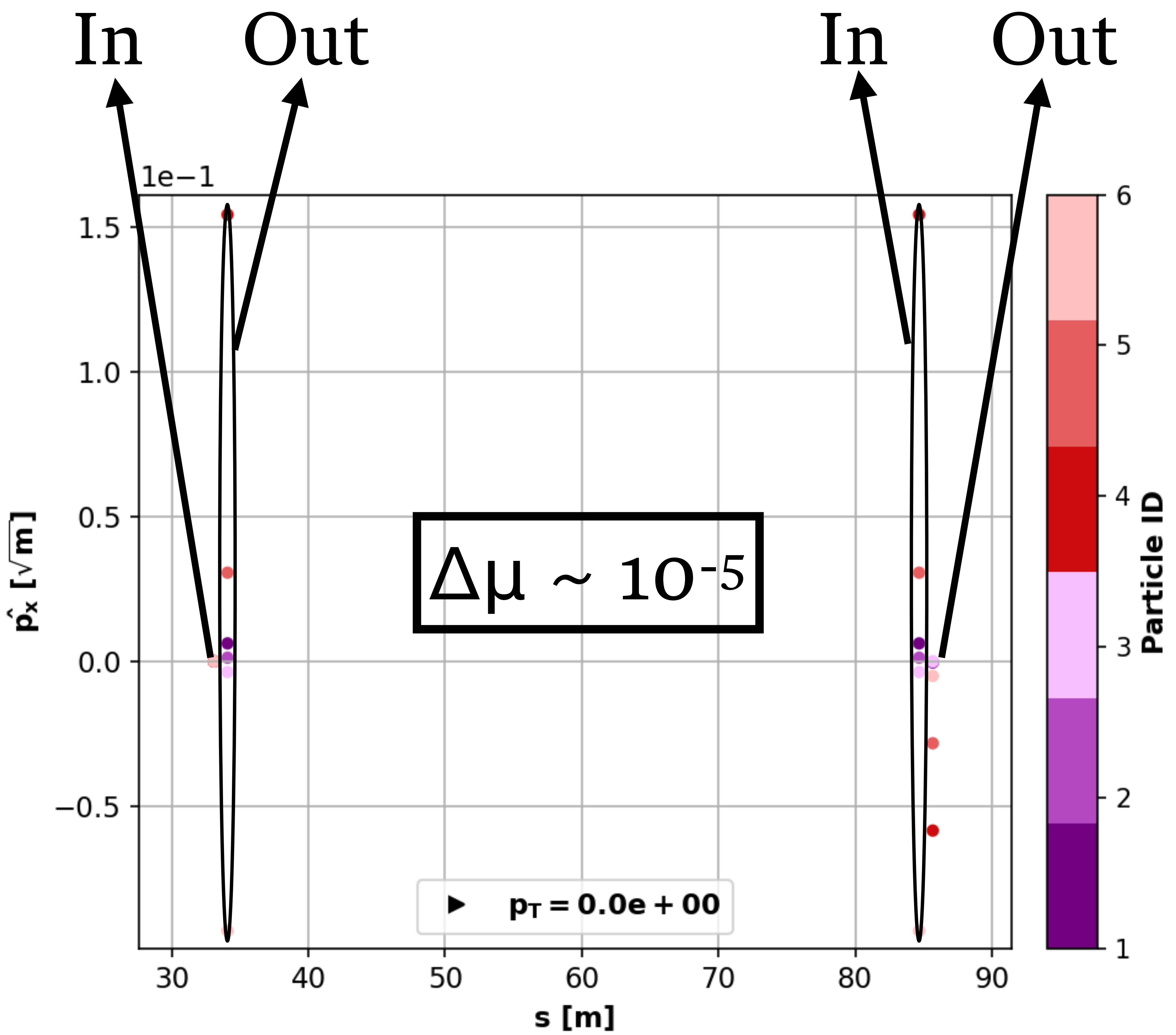
- Due to very large beta values at the IRs, small variations of the phase advance (~10^{-5}) can be detrimental for the particle dynamics (beam lifetime) thus, the compensation of sextupolar aberrations are quite demanding.



- Very large ratio between kicks from X-poles to beam divergence.

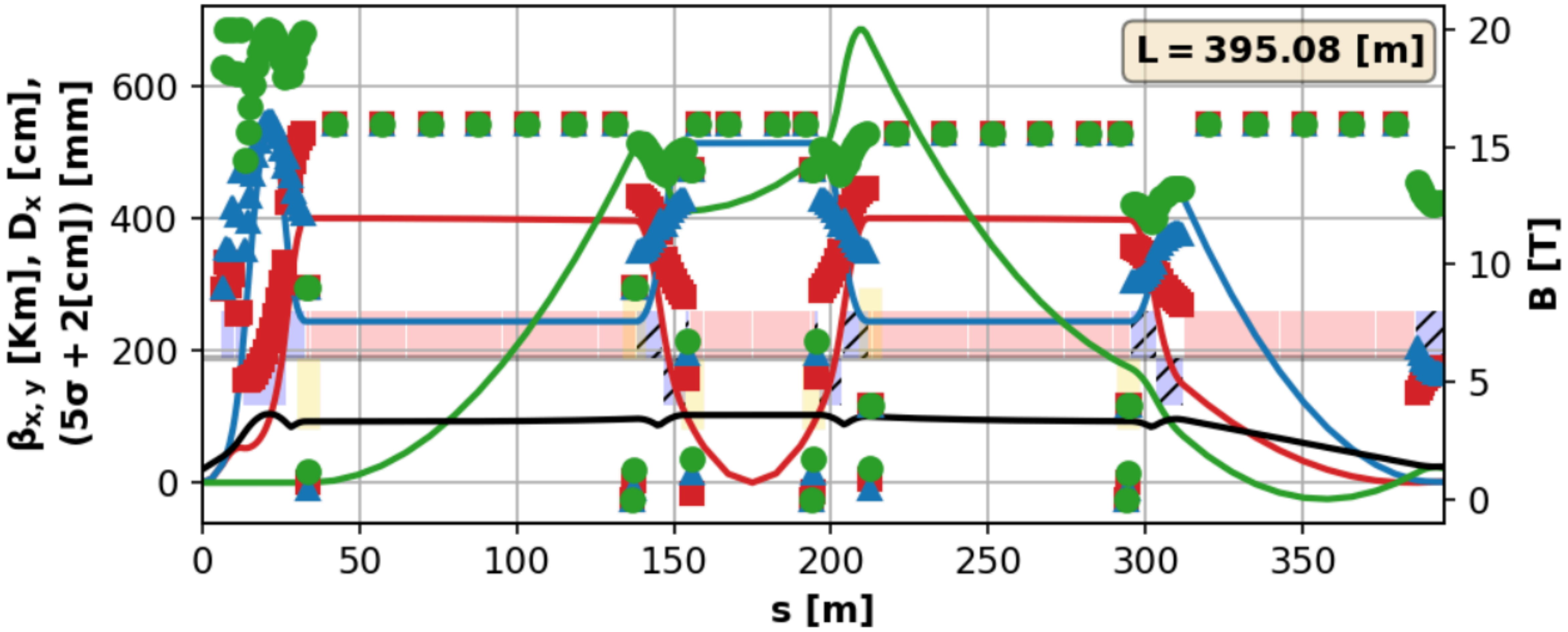
- This ratio can be decreased with smaller βs and/or longer CC section (increase of dispersion).

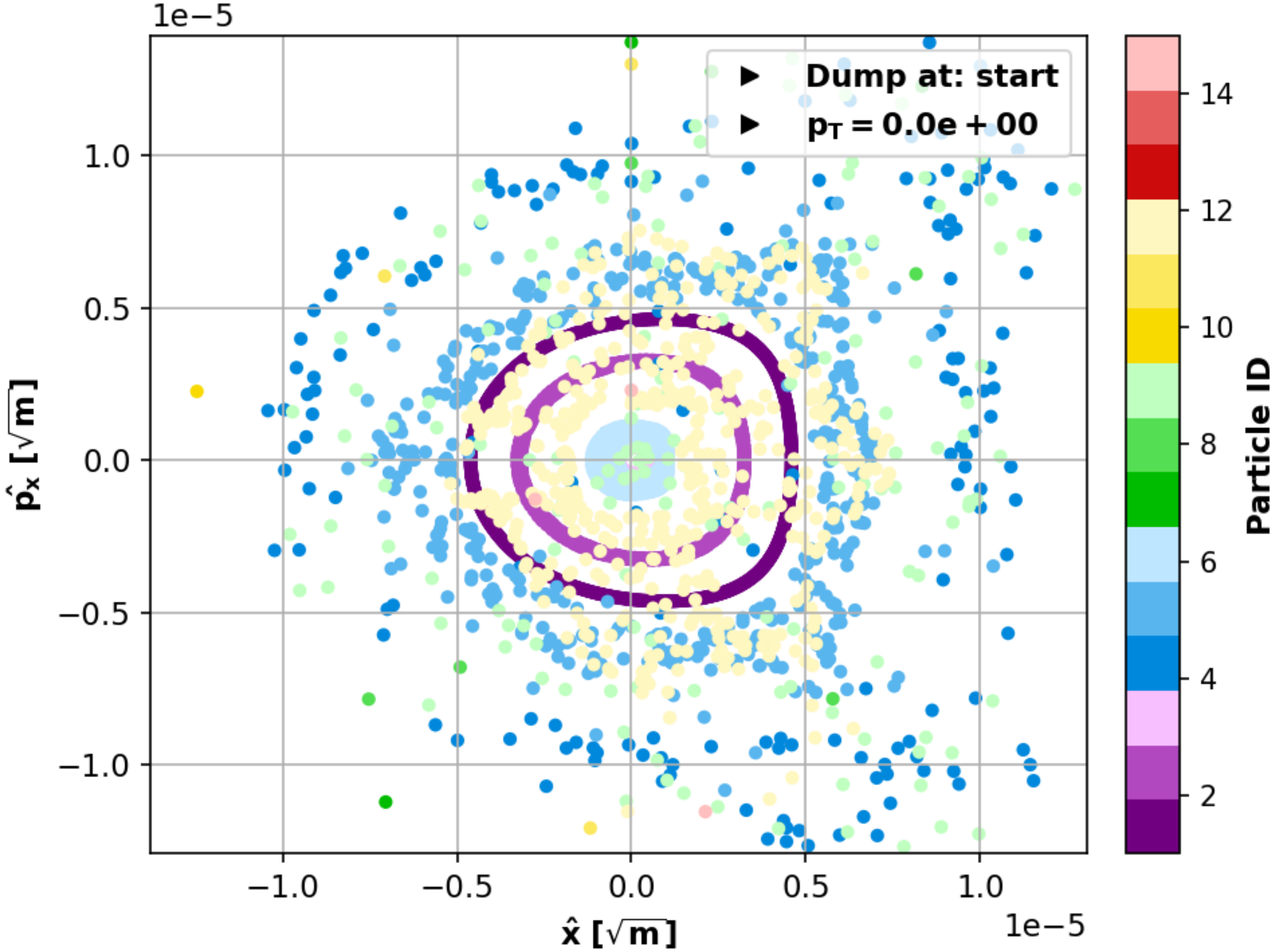


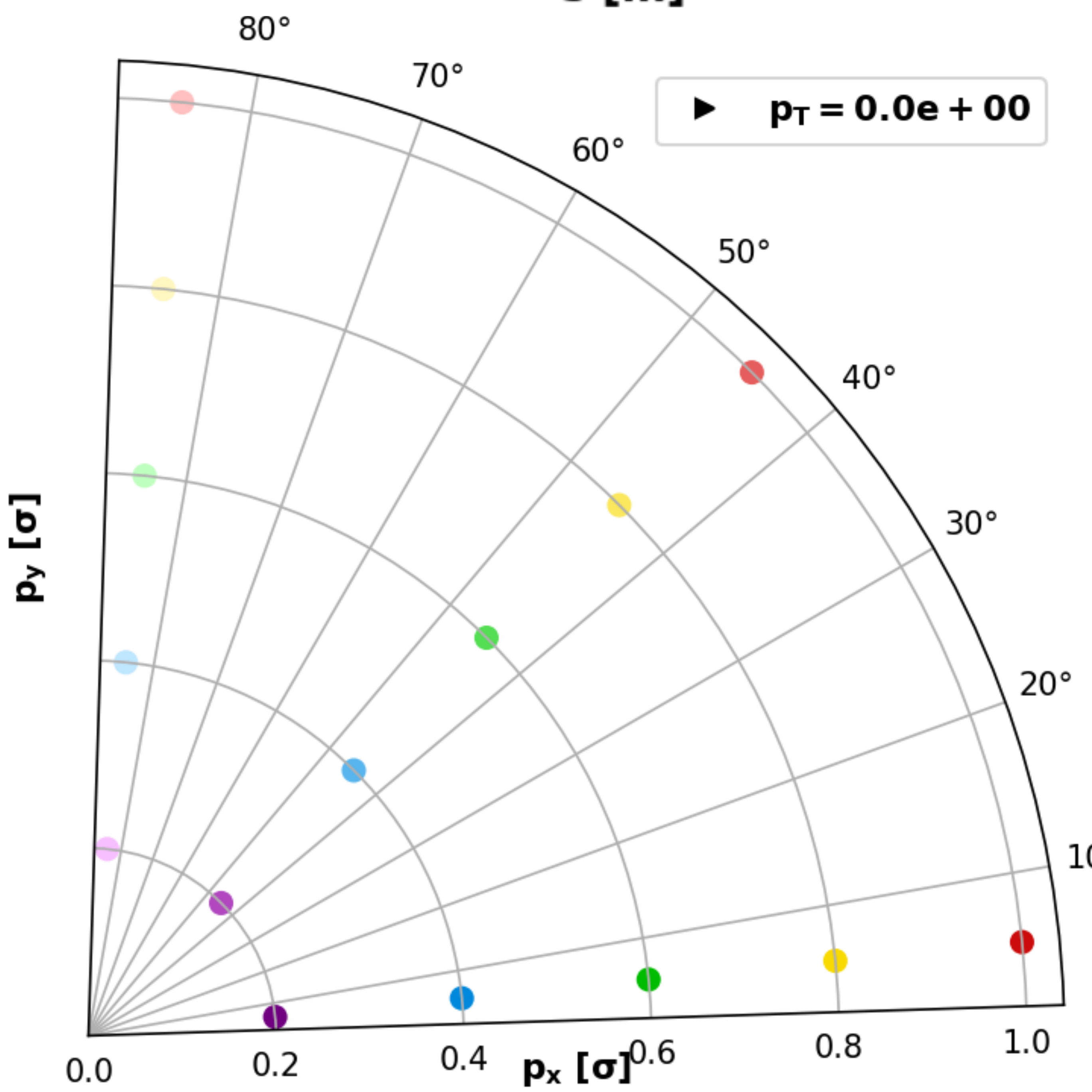


10TeV Muon Collider - Tracking Studies

- Alternative design of the CC with 1m long dipole-sextuple magnets with sextupolar components weaker than 2T (increase of dispersion with the addition of dipoles between sextuples of all sets).

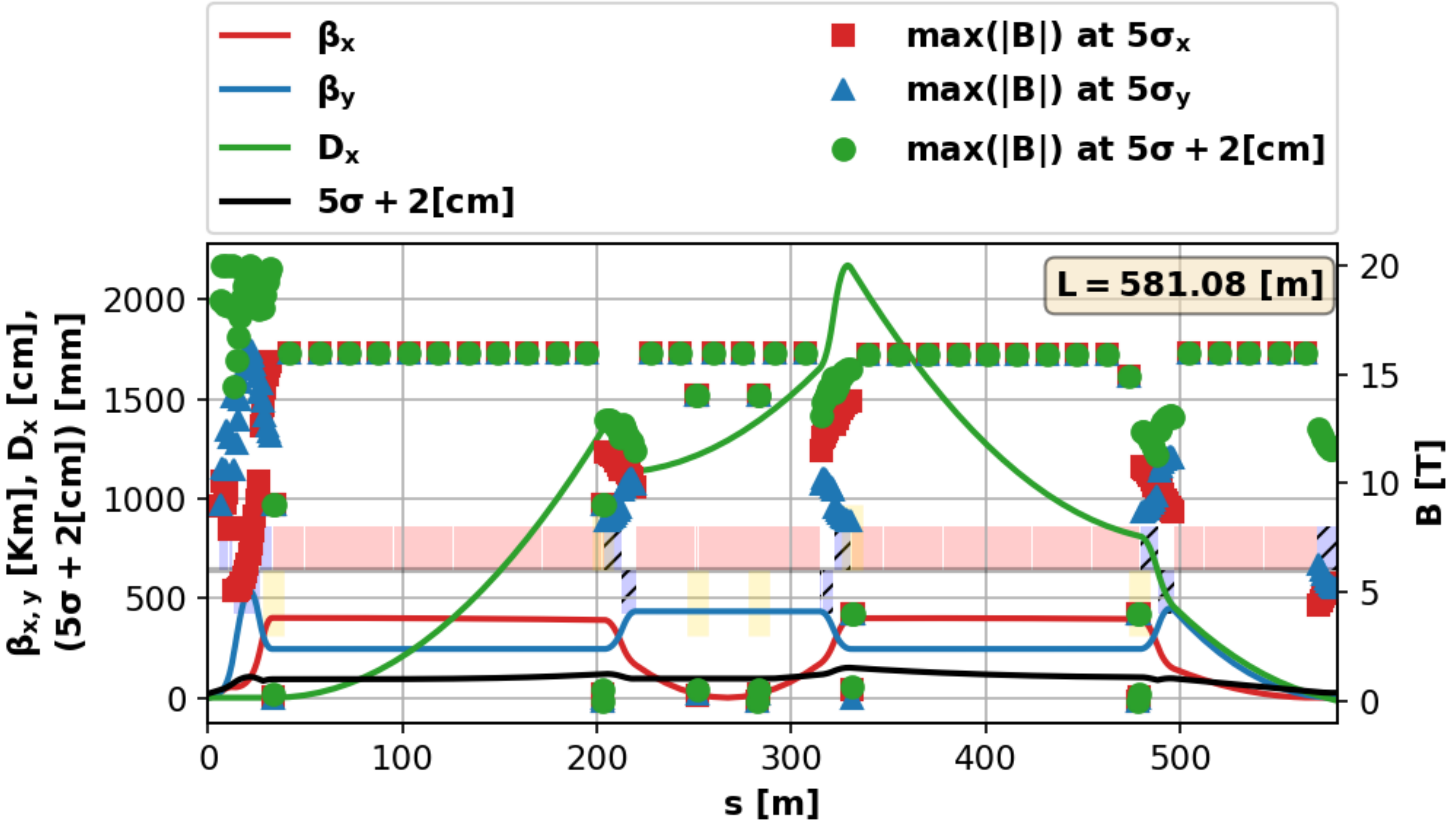


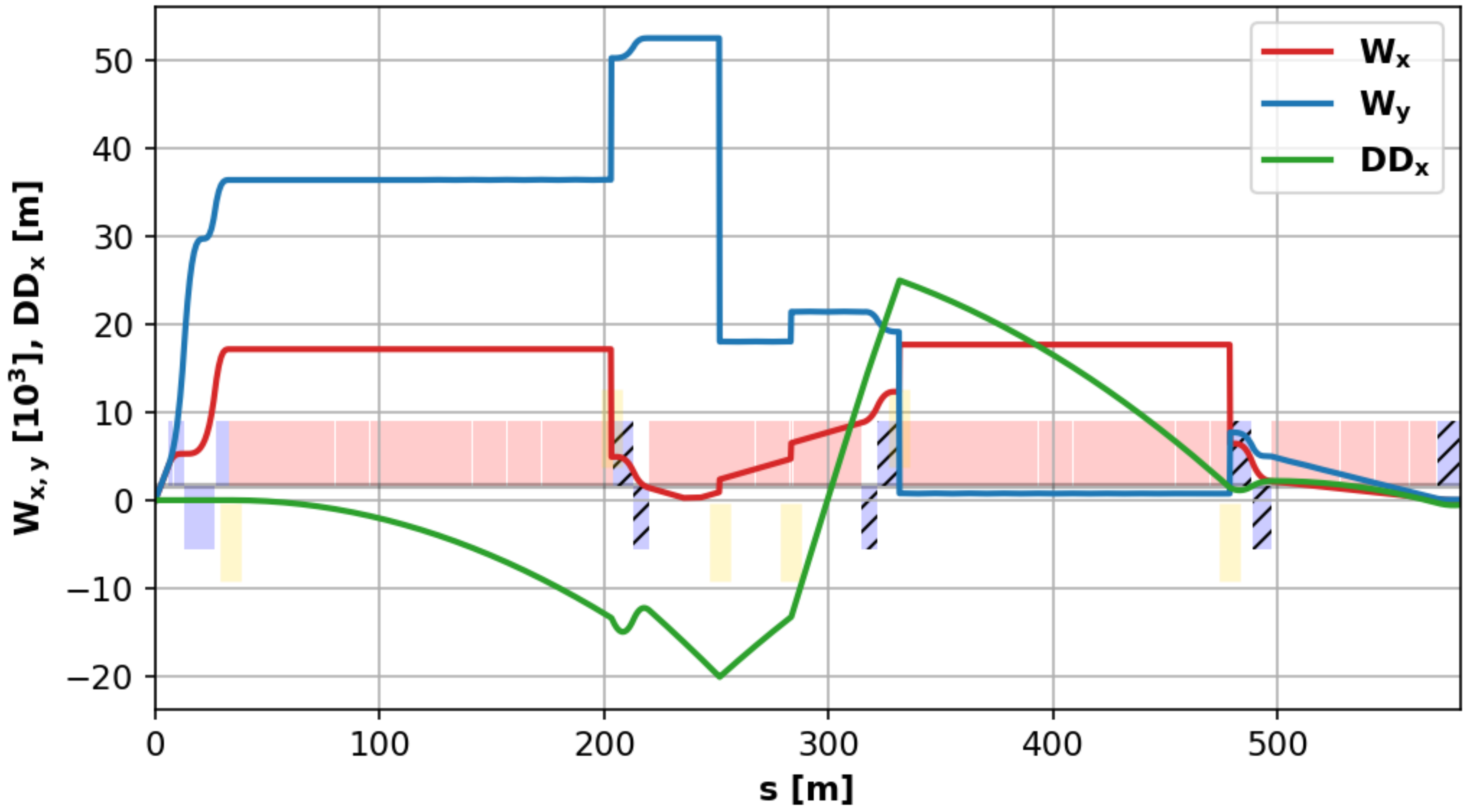




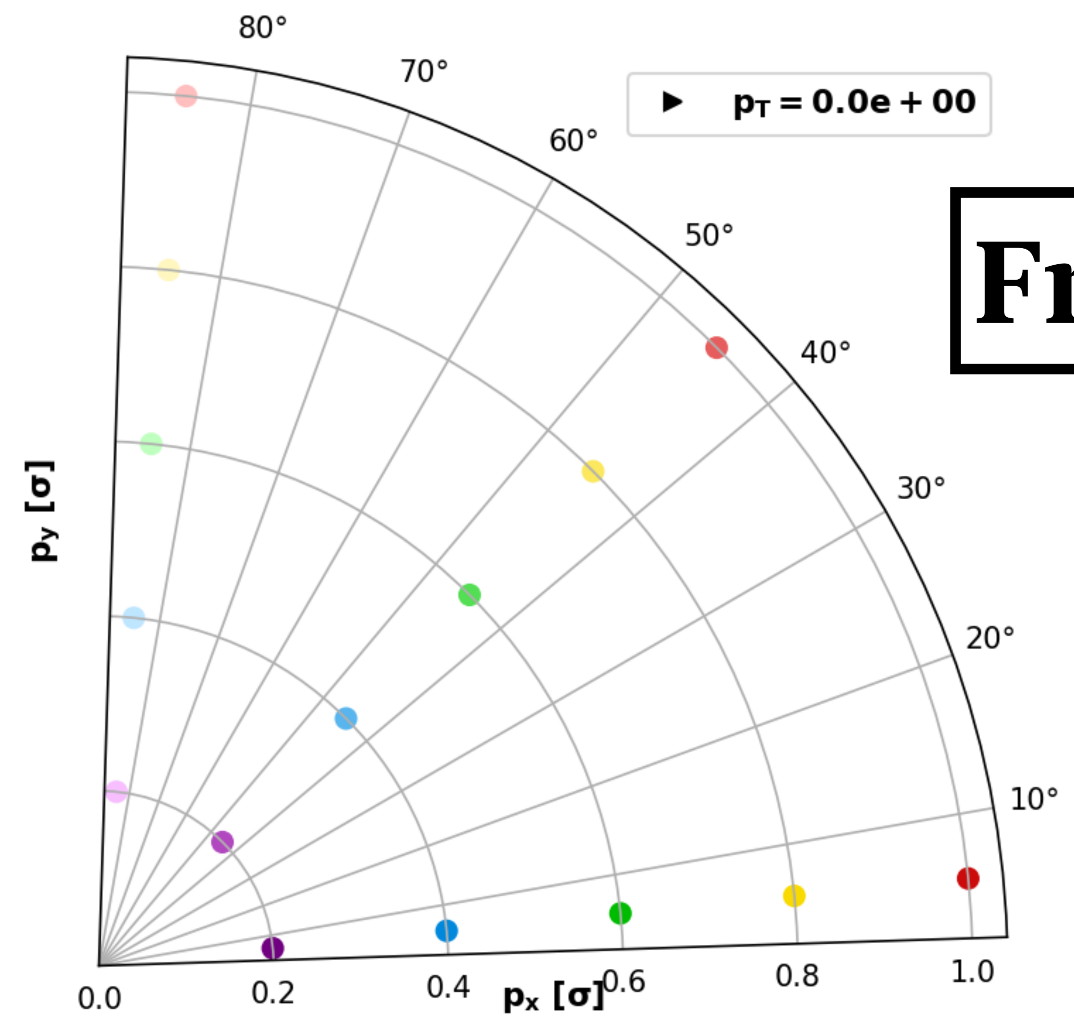
10TeV Muon Collider - Tracking Studies

- Alternative design of the CC with 1m long dipole-sextuple magnets with sextupolar components weaker than 0.2T (increase of dispersion with the addition of dipoles between sextuples of all sets).

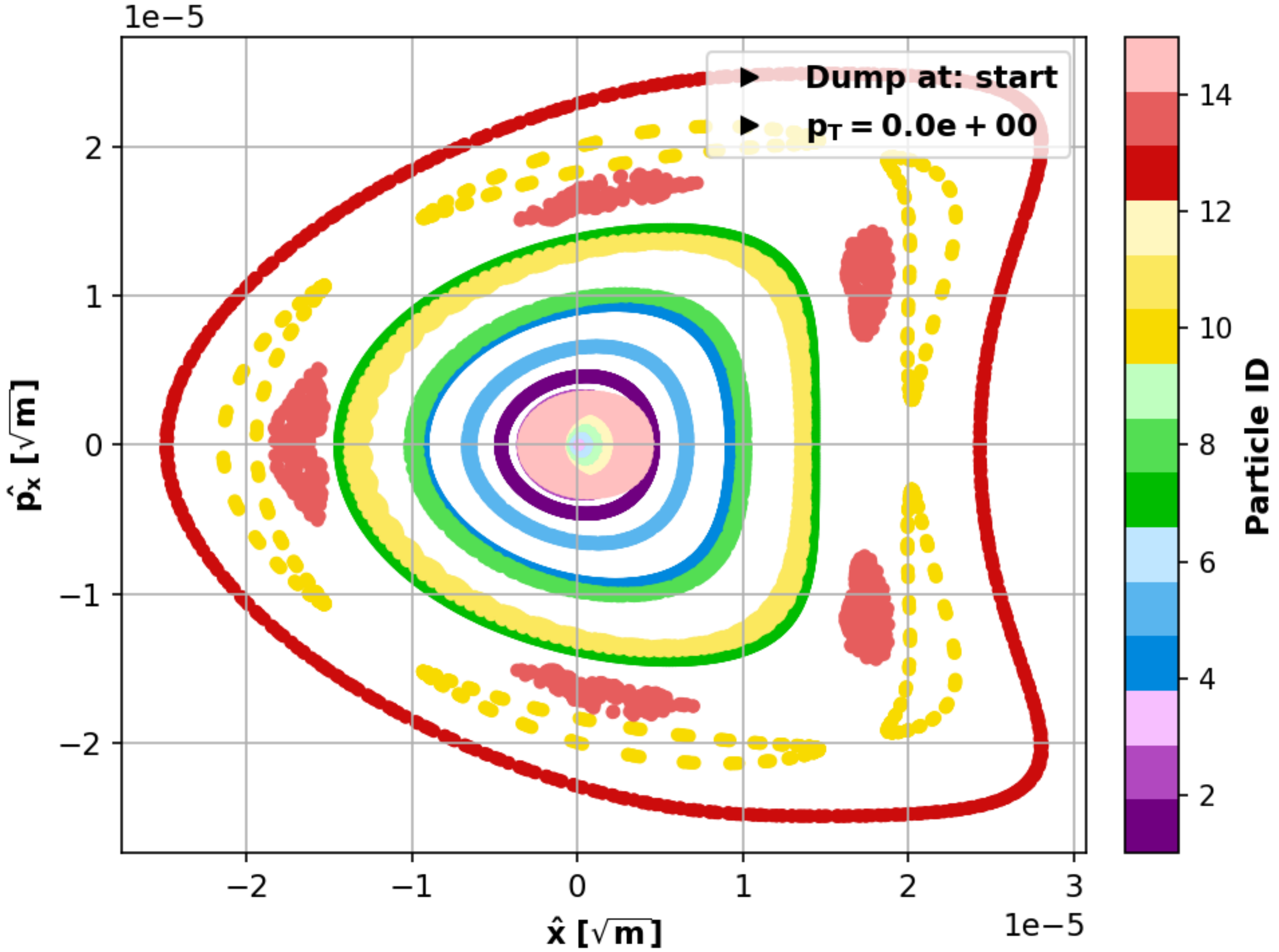


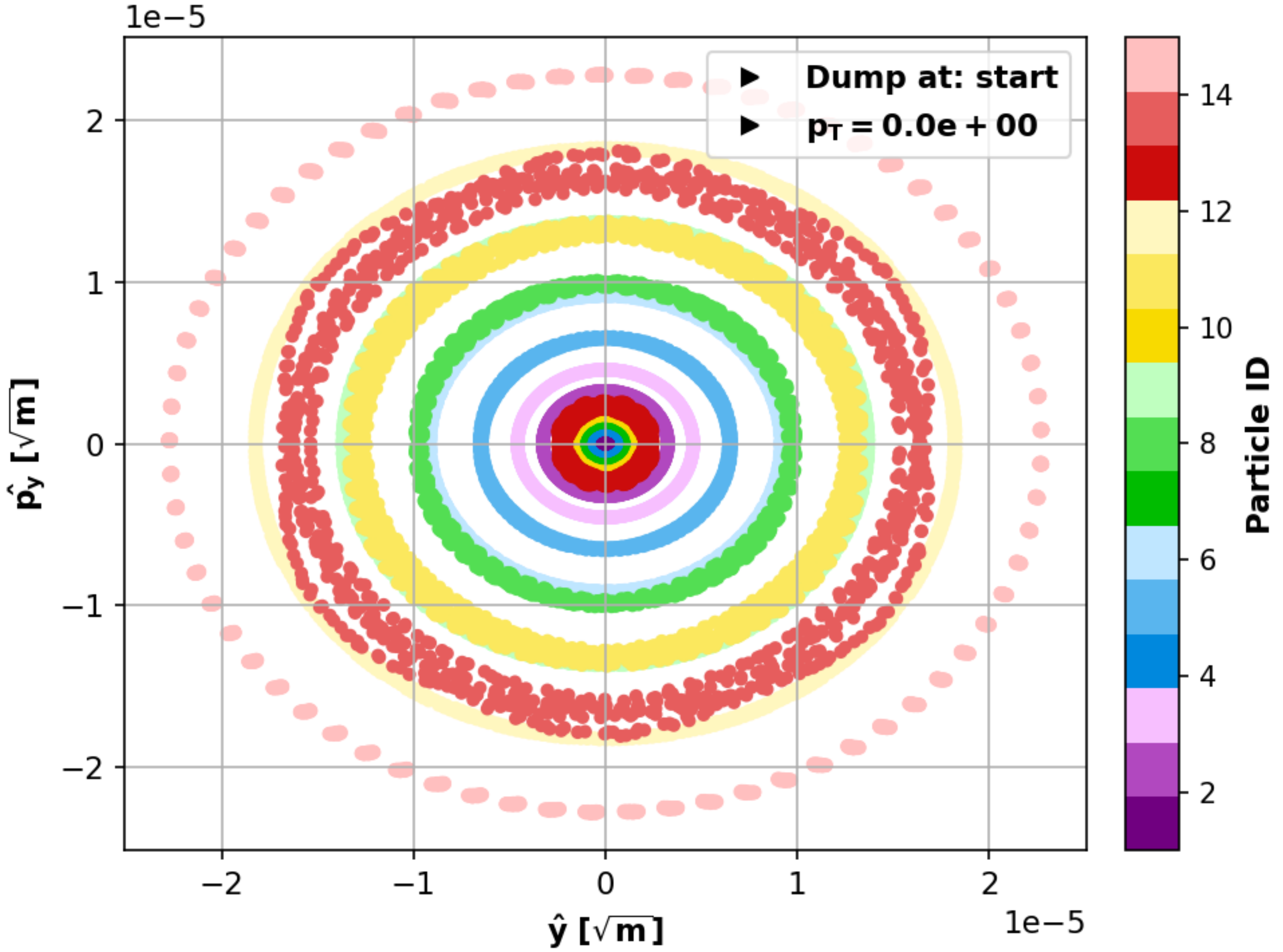


10TeV Muon Collider - Tracking Studies

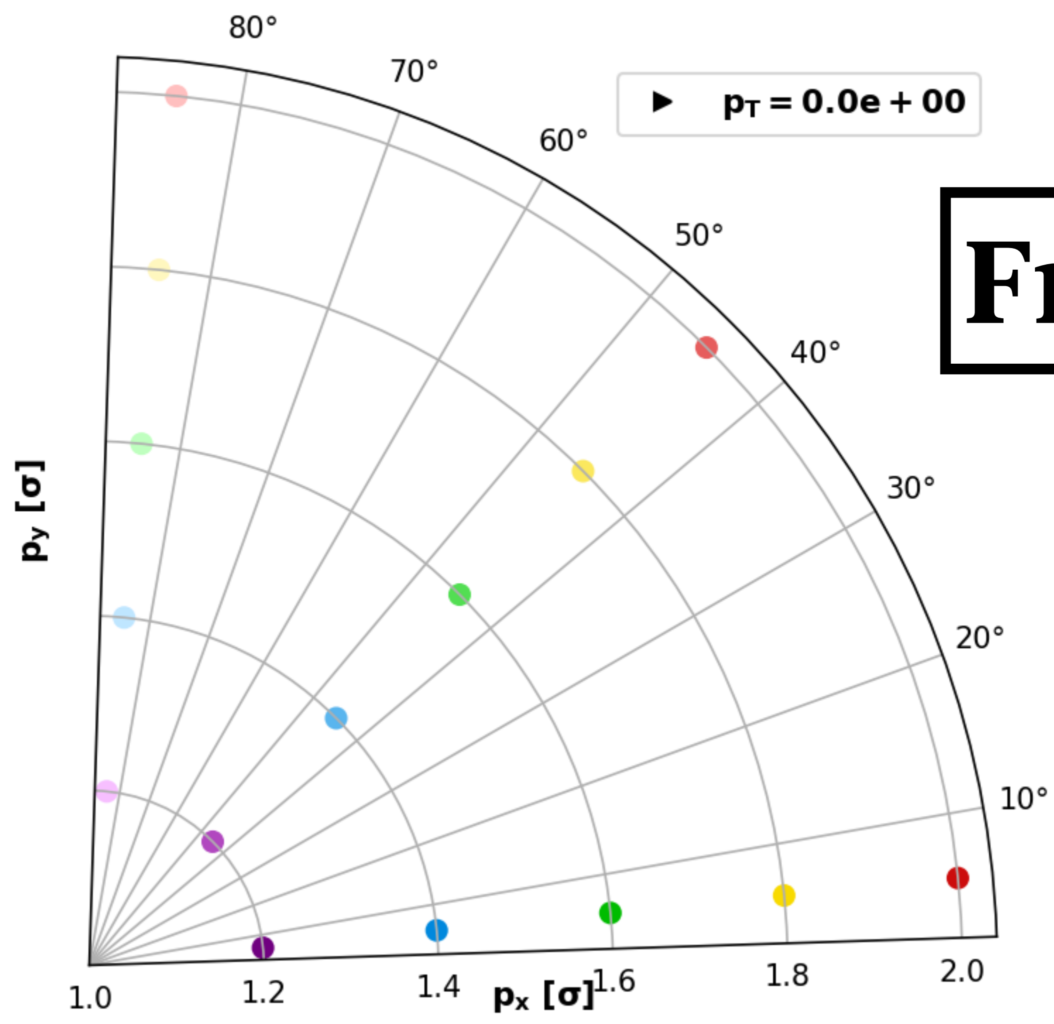


From 0 to 1σ

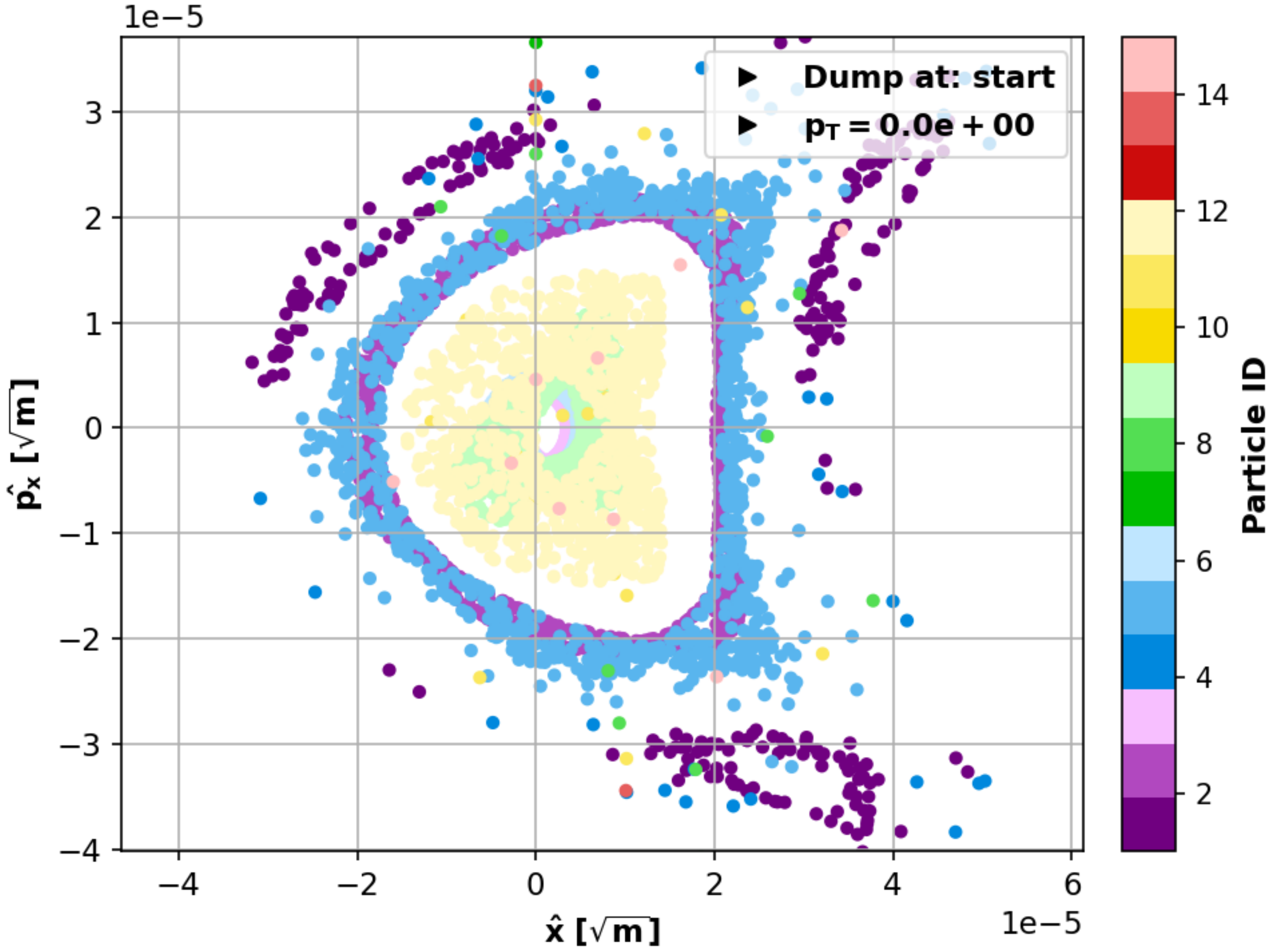


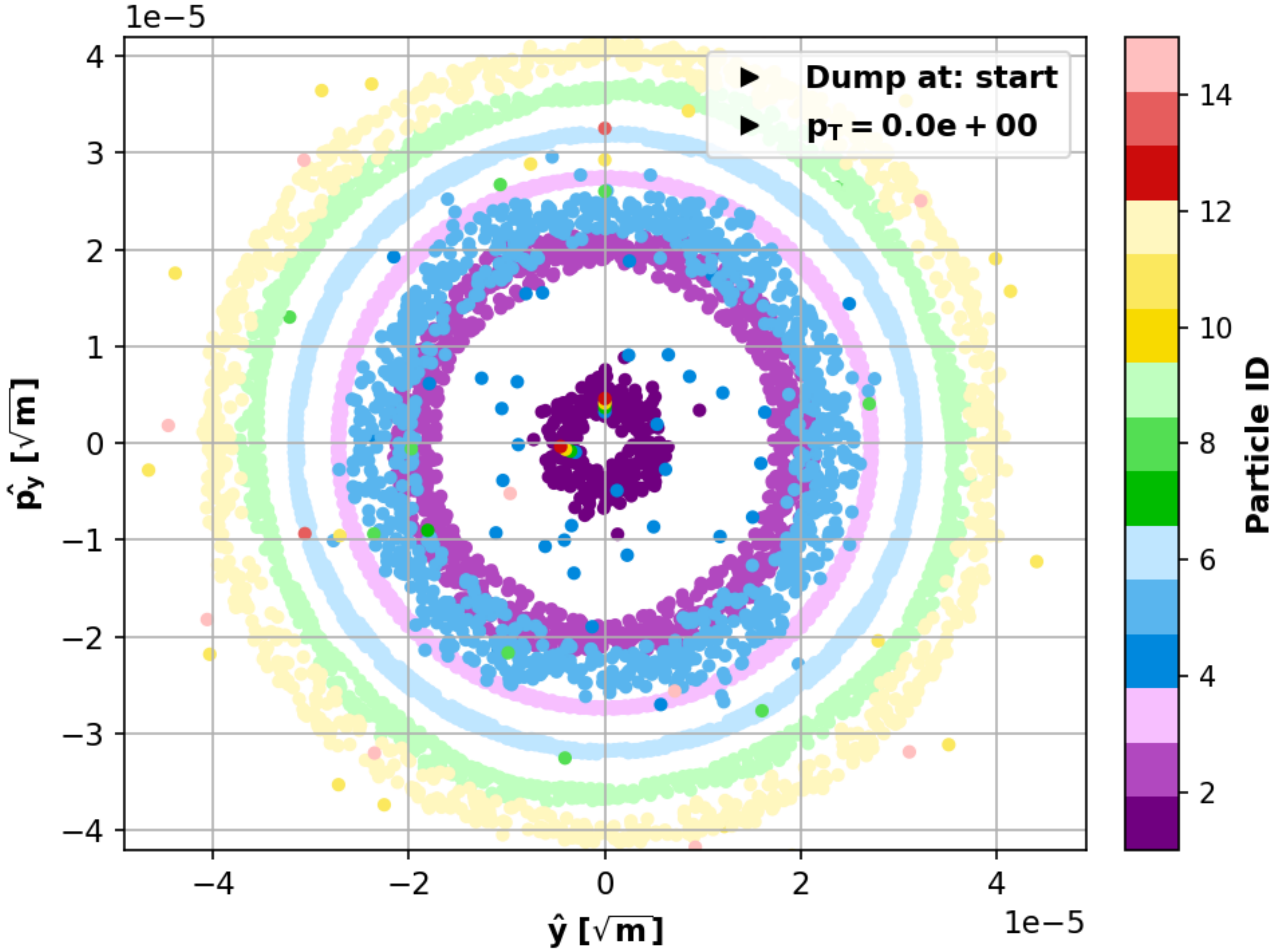


10TeV Muon Collider - Tracking Studies



From 1 to 2σ





10TeV Muon Collider - Tracking Studies

- Commuting the Lie transformations describing the lattice in order to find the impact of the non-zero Δμ based on the formulas:

$$\mathrm{Exp}[L_\nu]\,\mathrm{Exp}[L_\upsilon] = \mathrm{Exp}\left[:\mathrm{Exp}[L_\nu]\,\upsilon:\right]\mathrm{Exp}[L_\nu] = \mathrm{Exp}[L_\upsilon]\,\mathrm{Exp}\left[:\mathrm{Exp}[-L_\upsilon]\,\nu:\right]$$

$$e^Z = e^X e^Y \quad \texttt{with} \quad Z = X+Y+\frac{1}{2}\{X,Y\}+\frac{1}{12}(\{X,\{X,Y\}\}+\{Y,\{Y,X\}\})+\dots$$

Sextupole like contribution | Octupolar like contribution | Decapole like contribution

- Redesign the IR with smaller βs and more sextupole sets in the CC scheme.

To be addressed

- Longer CC section a good idea?
- Opinion of efforts for smaller βs outside IR (comprising inner triplet/quadruplet)
- Abandon new scheme with zero phase advance - Other comments on general approach?
- Other suggestions?

Thank you for your time!

All the **presented studies** are **work in progress** thus, any input is very welcome.

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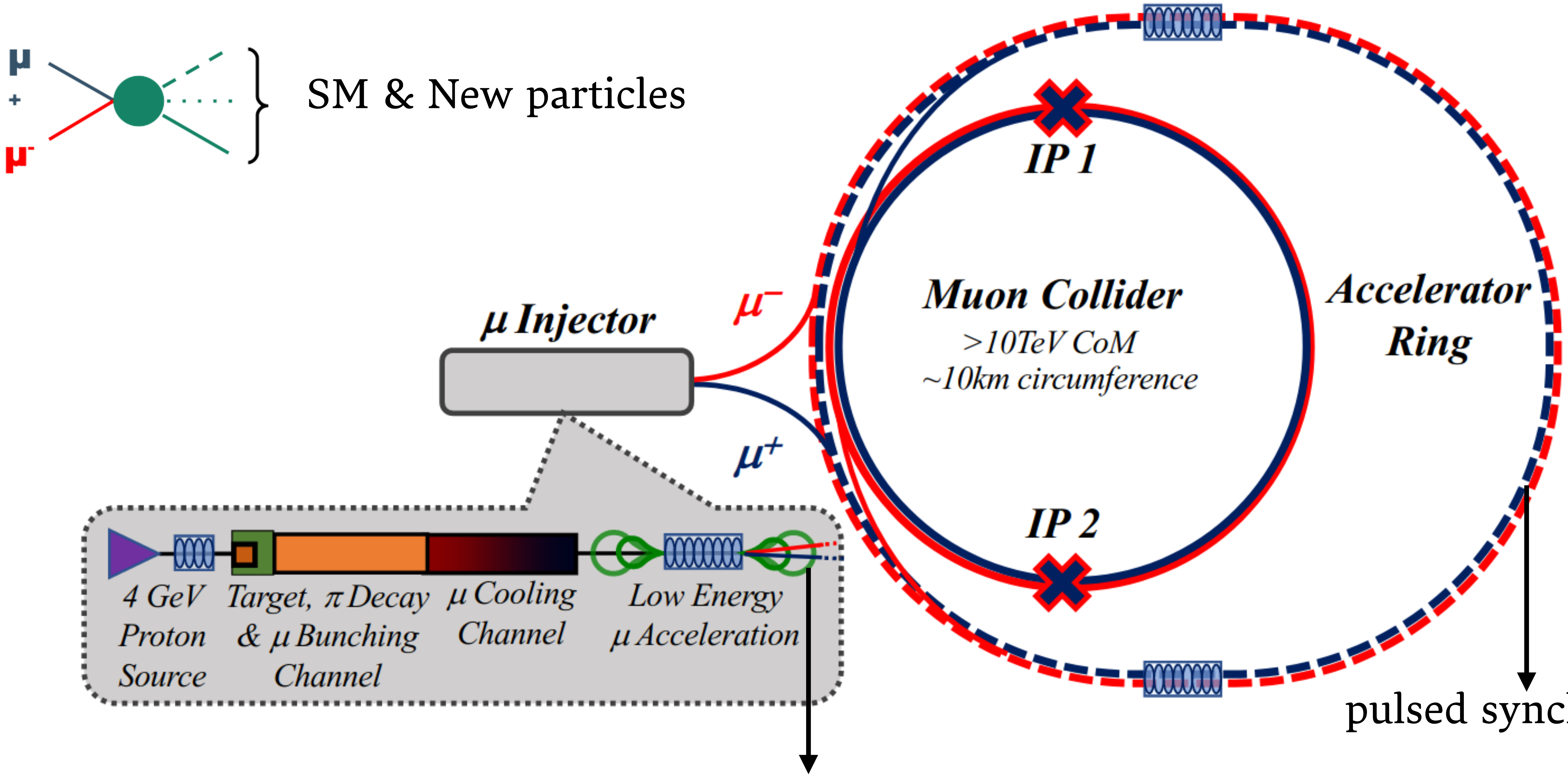
Backup

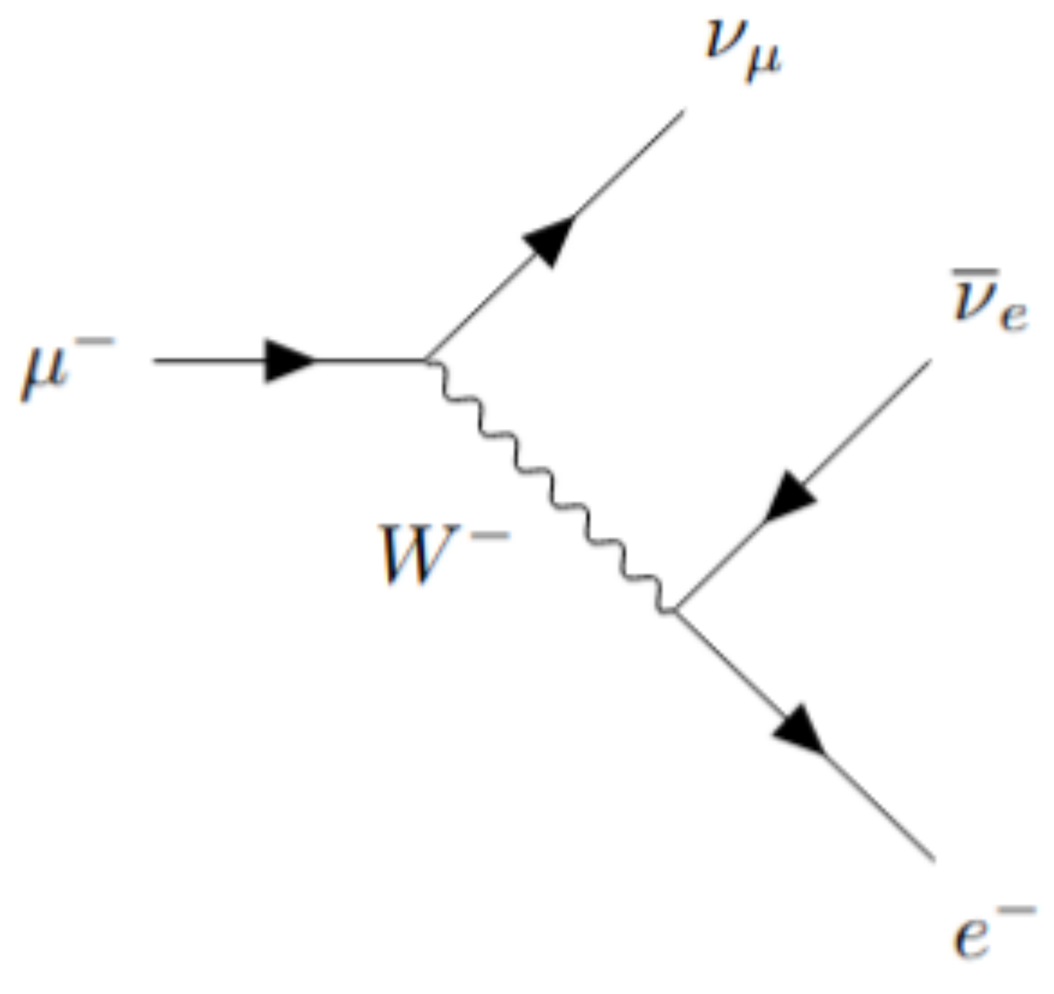
Overview of the Muon Collider Complex



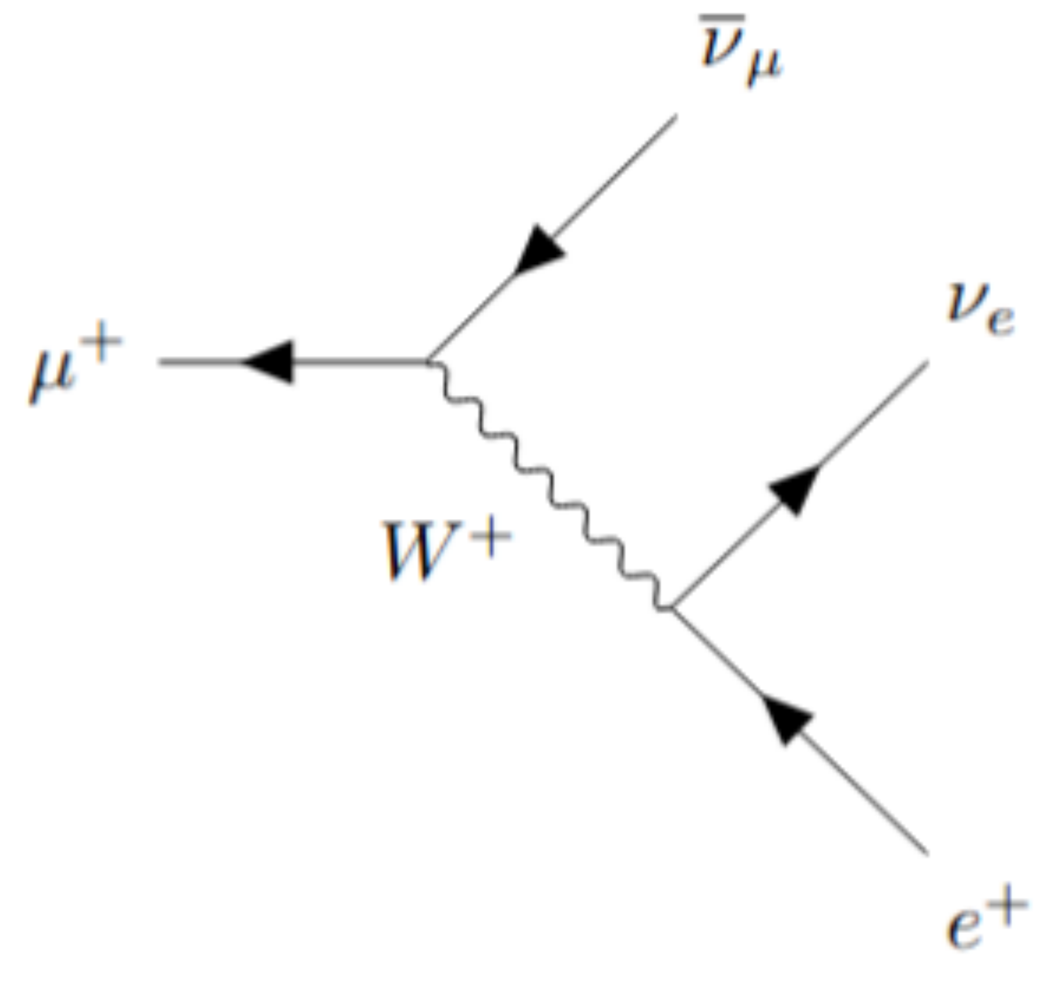
pulsed synchrotron

recirculating linacs

10TeV Muon Collider

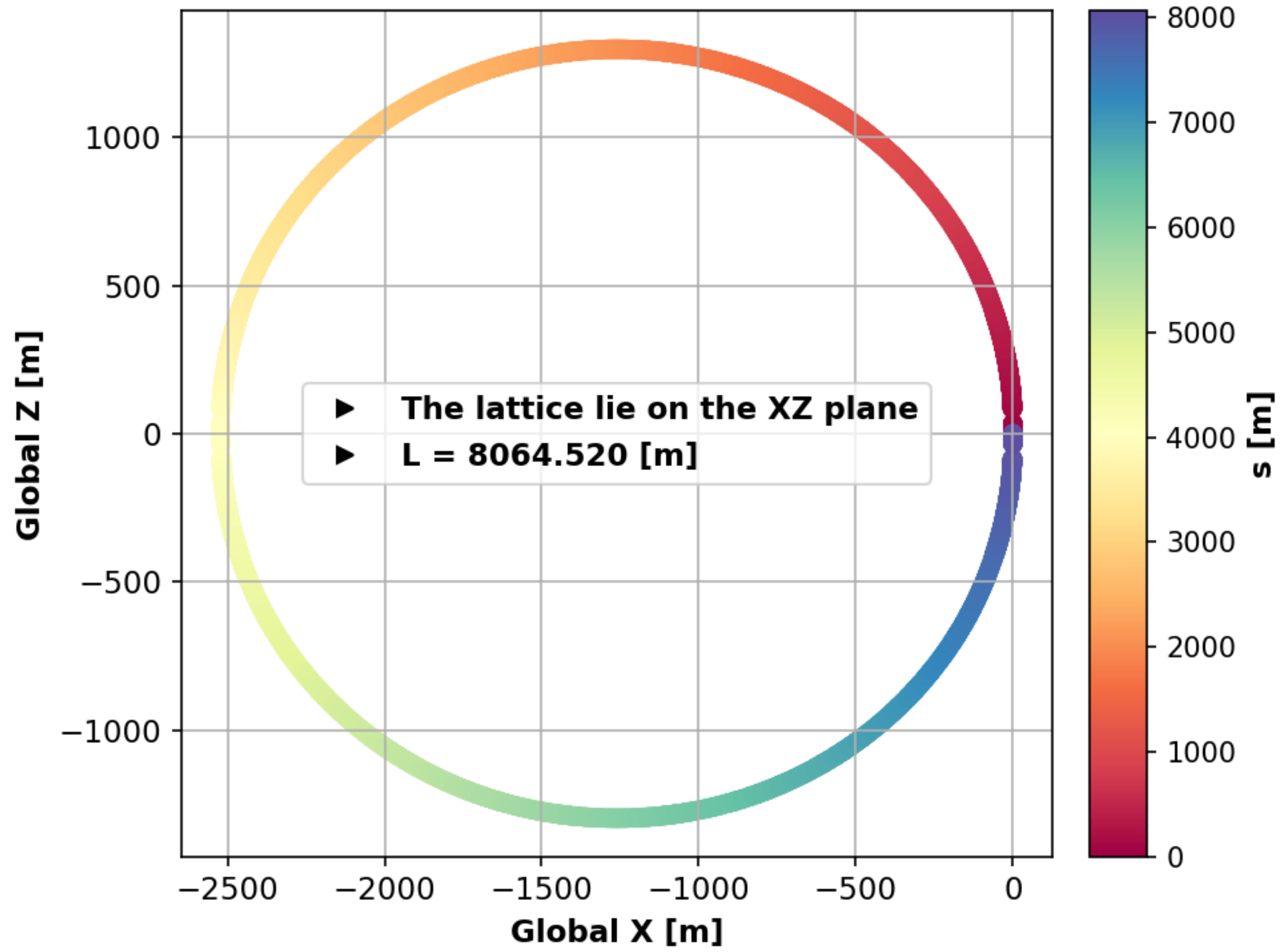


$\mu^- \to \nu_\mu + \overline{\nu}_e + e^-$



$\mu^+ \to \overline{\nu}_\mu + \nu_e + e^+$

- Due to muon decay (short lifetime τ_0~2.2μs or τ_{5TeV}~0.1s), the resulted neutrinos from a short piece of collider generate a narrow "radiation cone" that is an issue at the location, where it reach the earth surface (see talk by C. Carli) therefore, straight pieces (as in pure quads or X-poles) have to be avoided.
- Given that at least 2 straight sections are need (2 IPs), the planned shape of the collider is like a race track with extensive use of dipoles and combined function magnets.



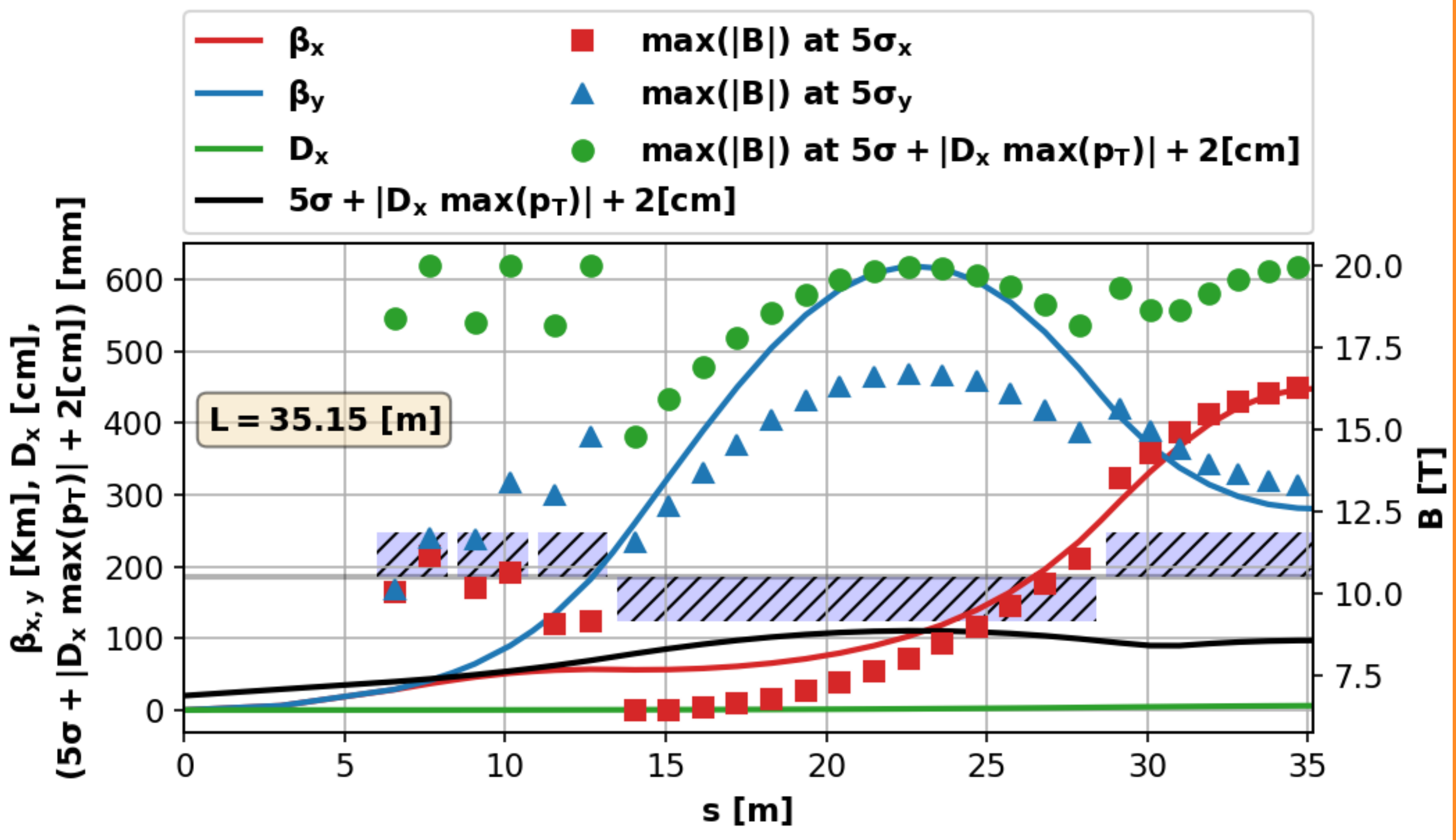
10TeV Muon Collider - Final Focusing Scheme

- If the 20T are not realistic and drop to 16T that is the FCC target, the same IR scheme (with similar quadrupole gradients) can be used with one or a combination of the following modifications:
  - reduction of the com energy to 8TeV, this configuration reaches the design luminosity for an 8TeV com muon collider (as β^* is inversely proportional to the energy)
  - reduction of the apertures (without significantly changing the gradients):
    - by reducing the beam envelope to 4σ+1.5cm, the luminosity degradation is negligible (less than 1%) but other consequences have to be understood
    - by increasing the β^* by a factor ~2.04 (a bit larger than (5/4)2)

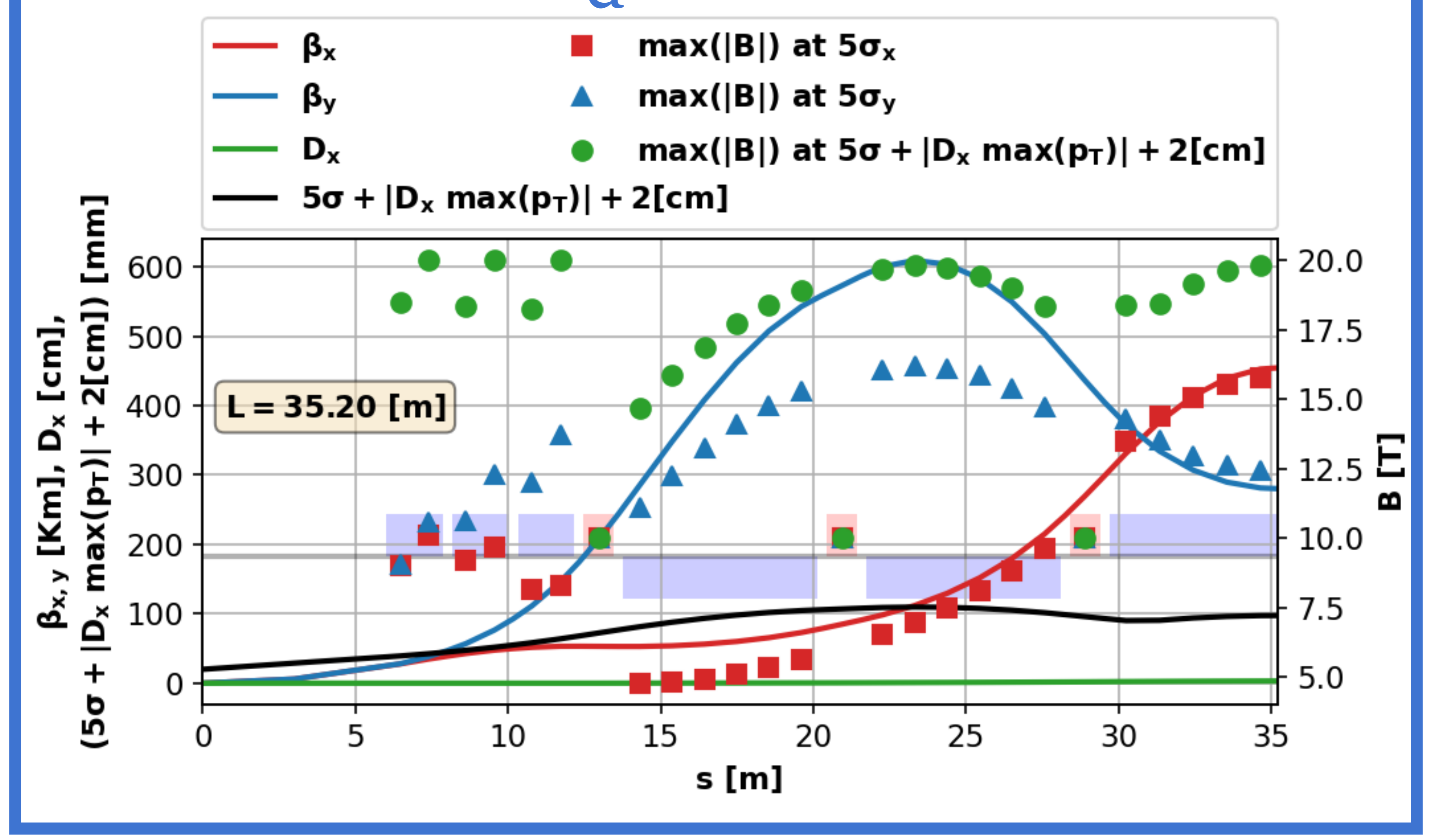
10TeV Muon Collider - Final Focusing Scheme

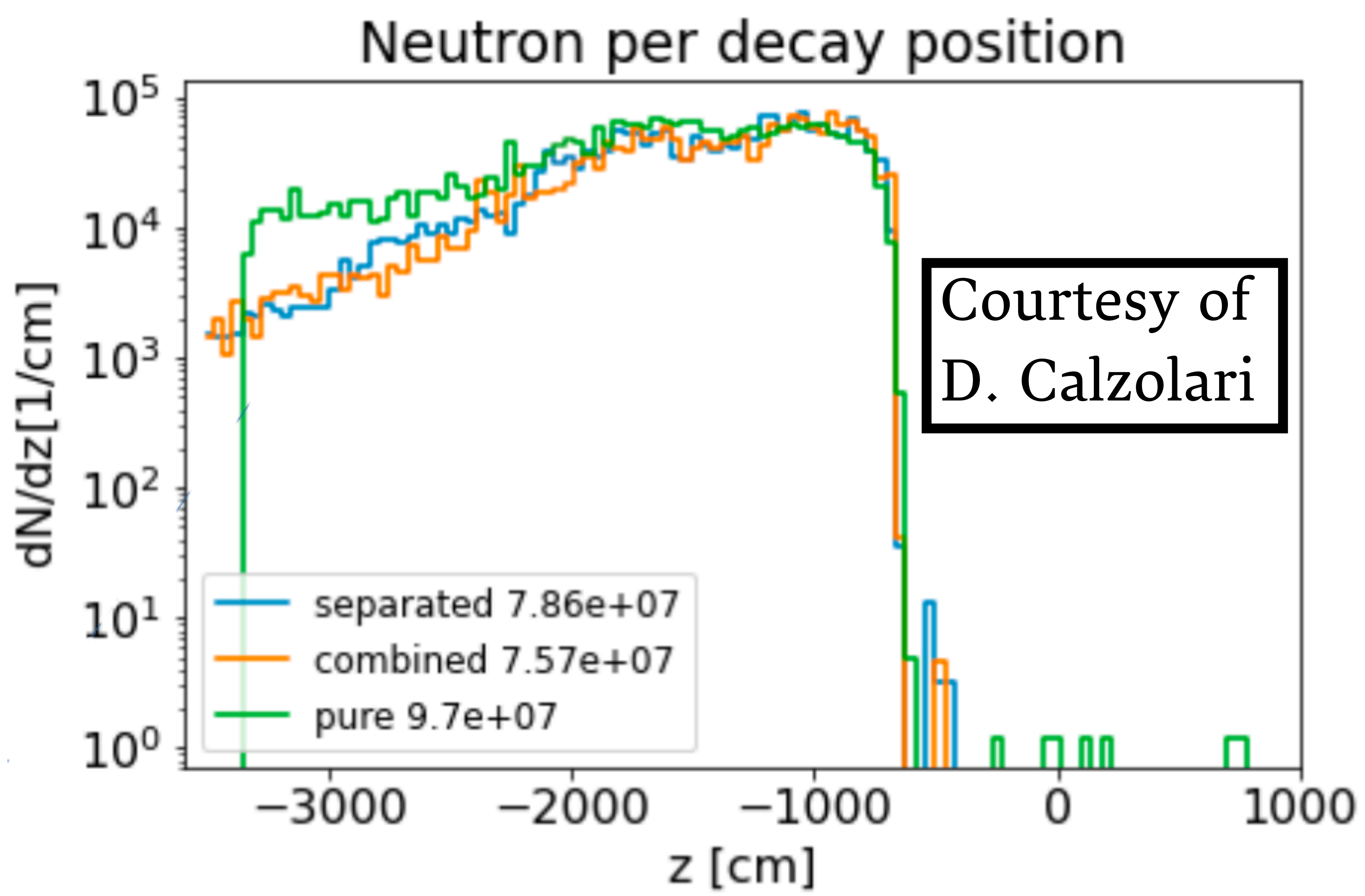
- Different FF schemes that include dipolar components or an elongated L* are designed and their effectiveness to mitigate the Beam Induced Background (BIB) is studied* by the FLUKA team.

B_d = 2T

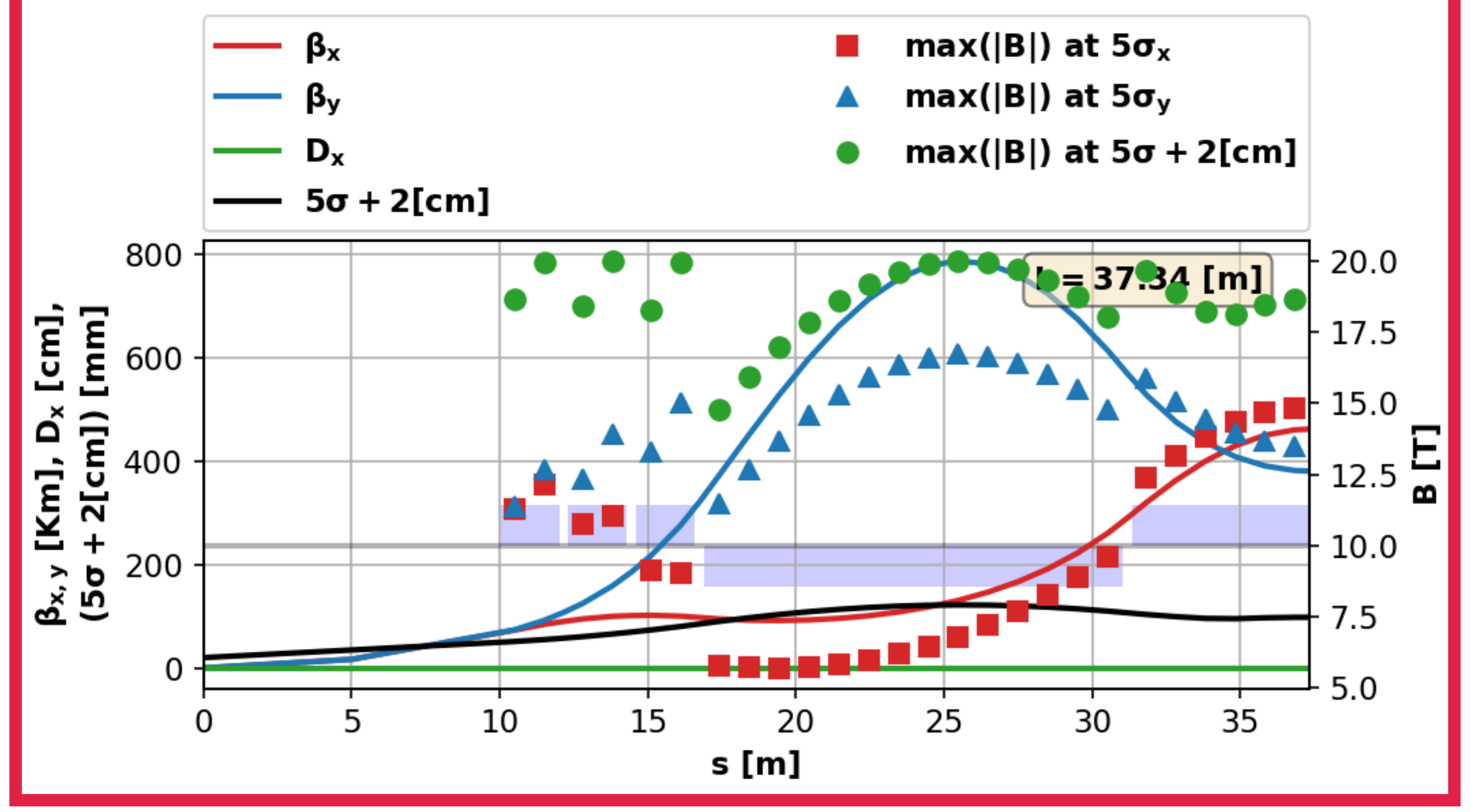


B_d = 10T





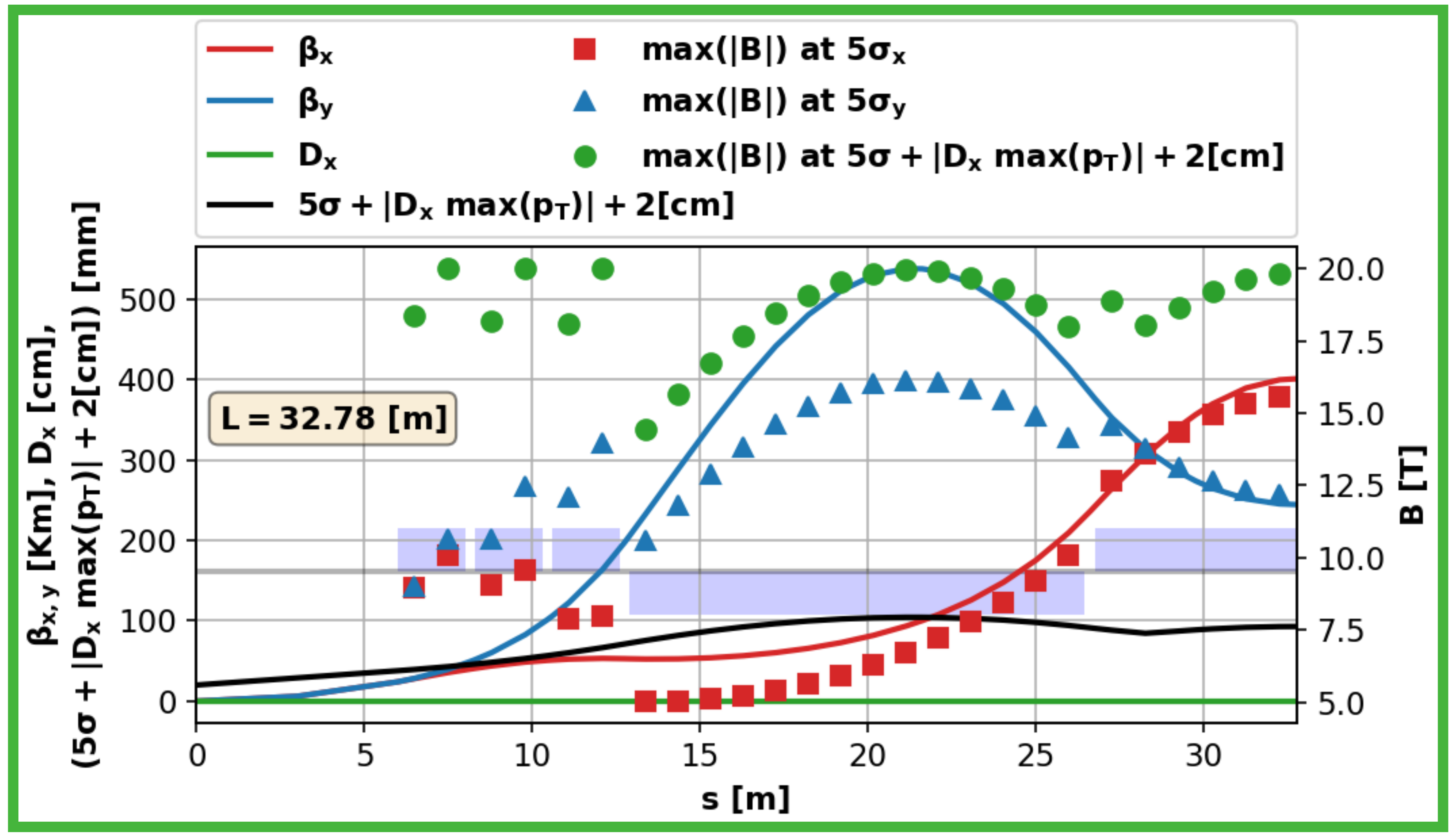
L* = 10m

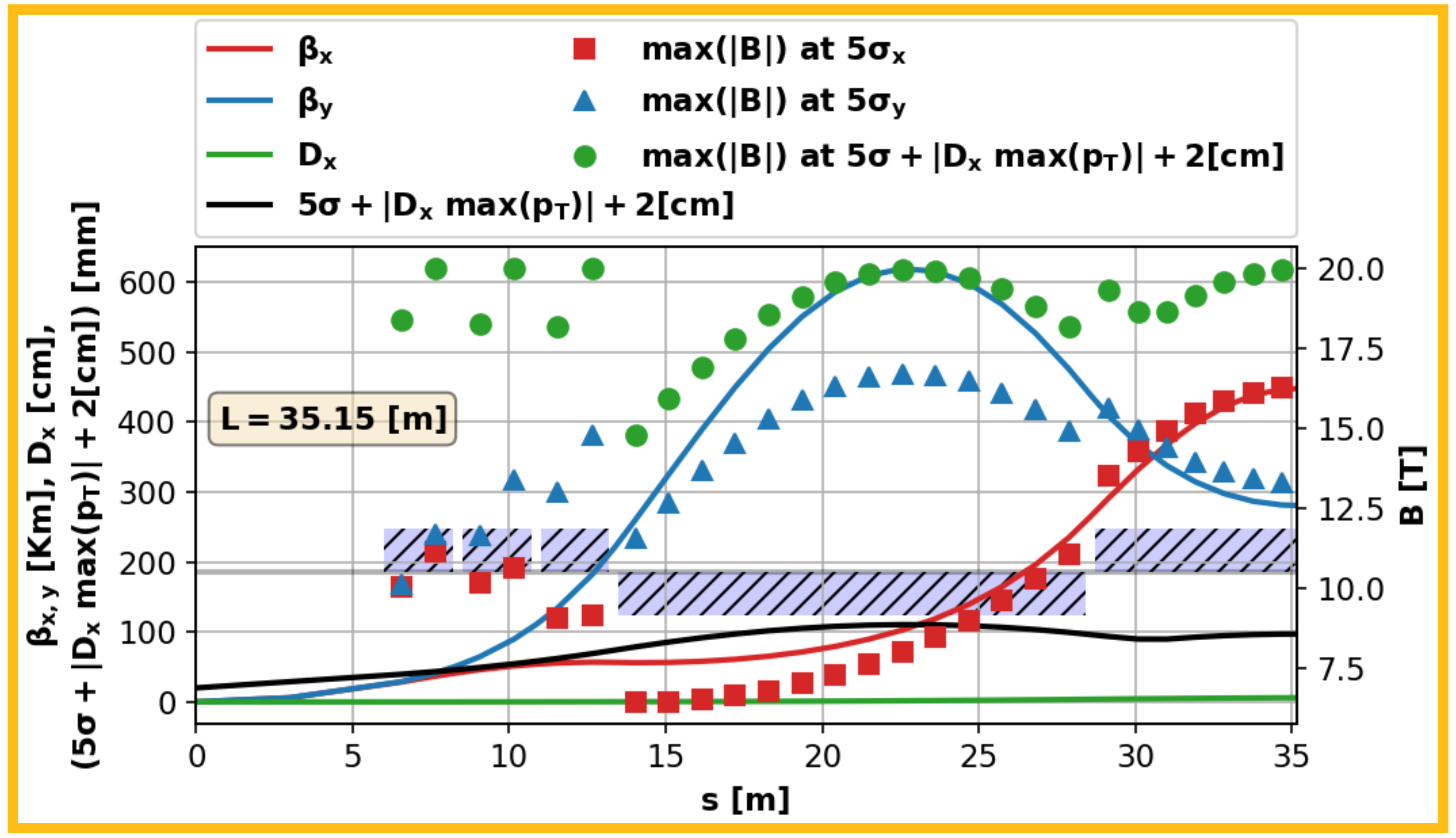


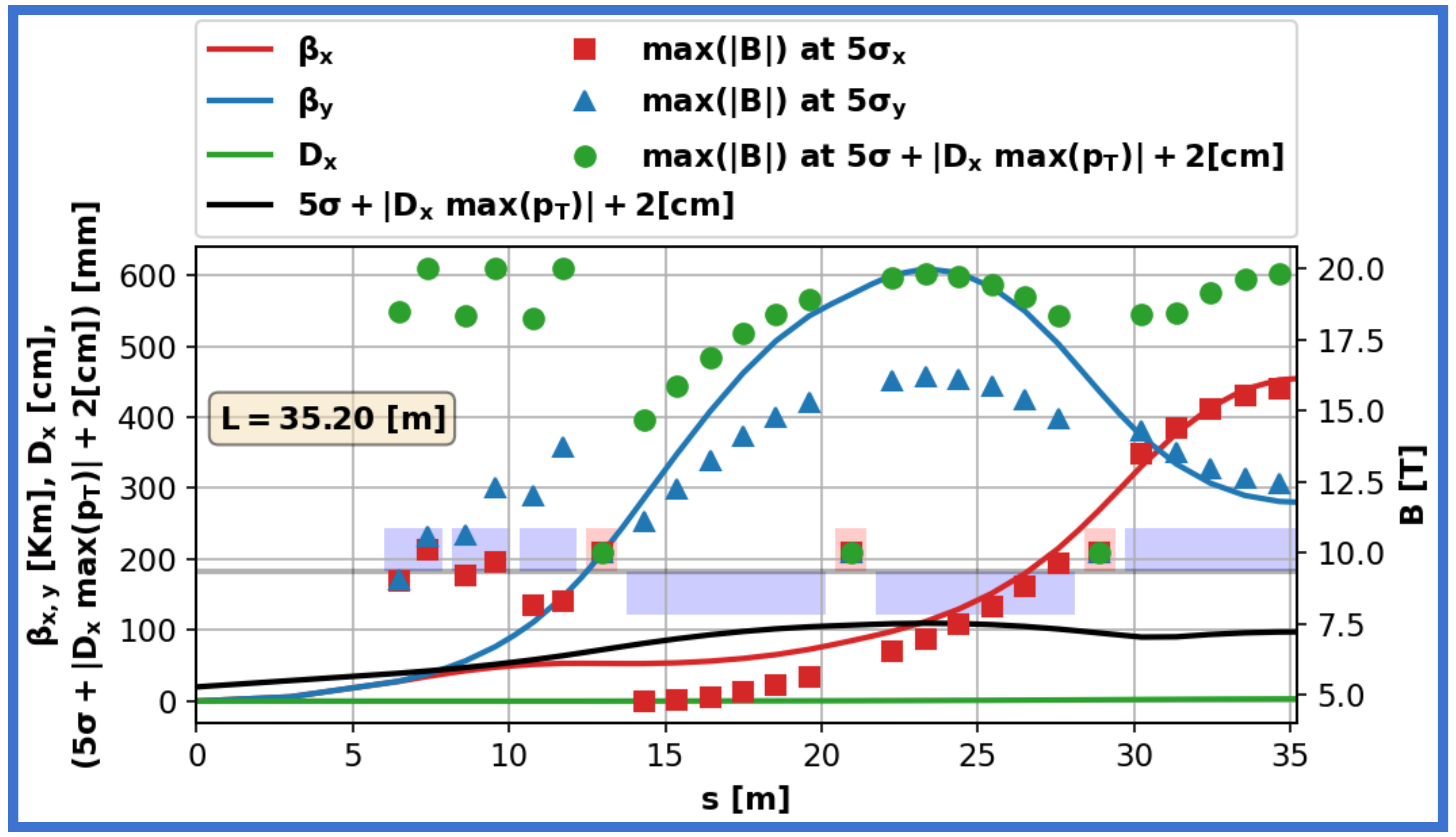
*see talk by D.Calzolari

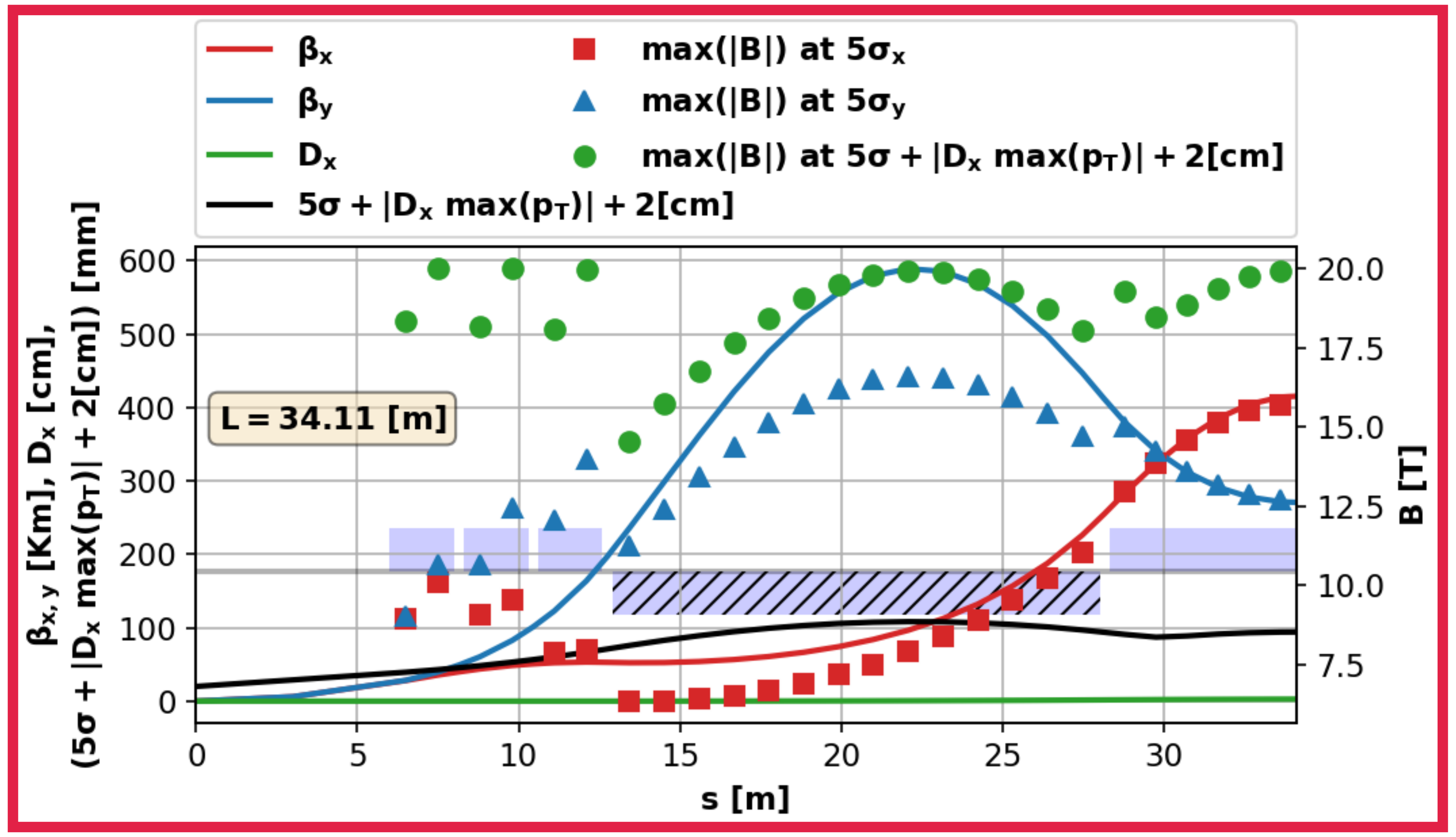
10TeV Muon Collider - Final Focusing Scheme

Due to muon decay along the interaction region, the Beam Induced Background (BIB) at the detectors area is significant thus in collaboration with the FLUKA team, the impact on BIB from the addition of dipolar components in the FF scheme is studied.









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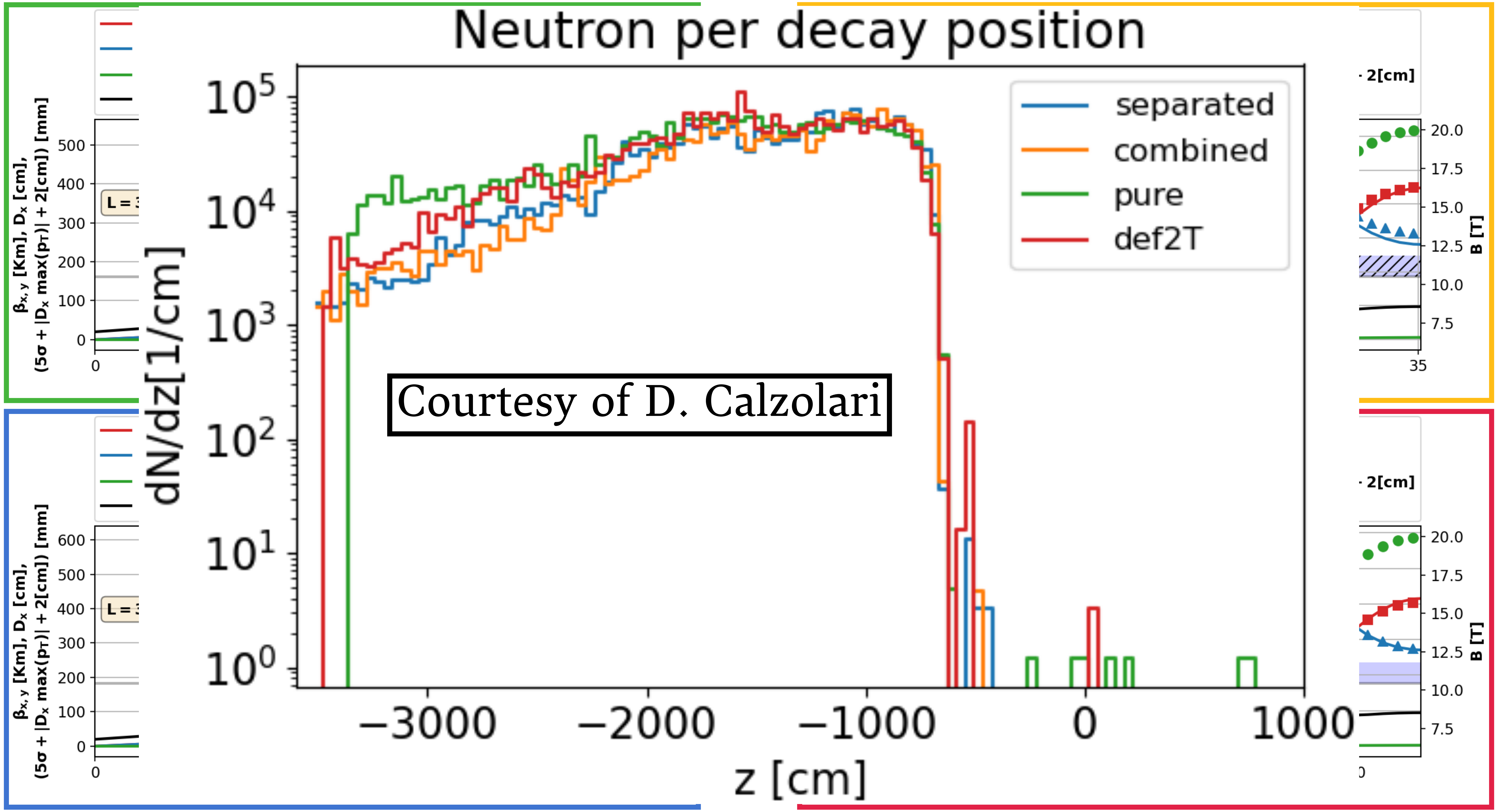
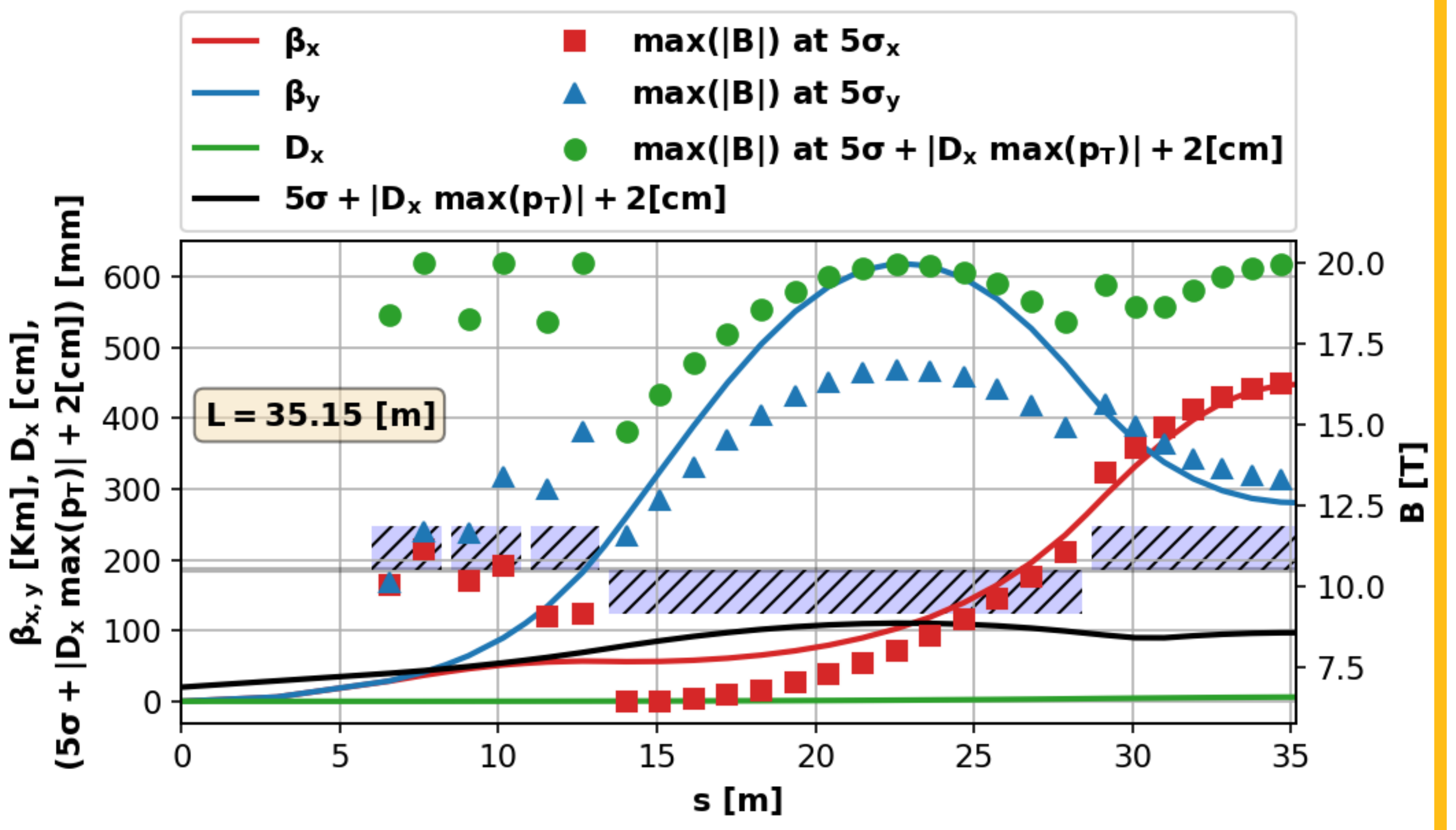
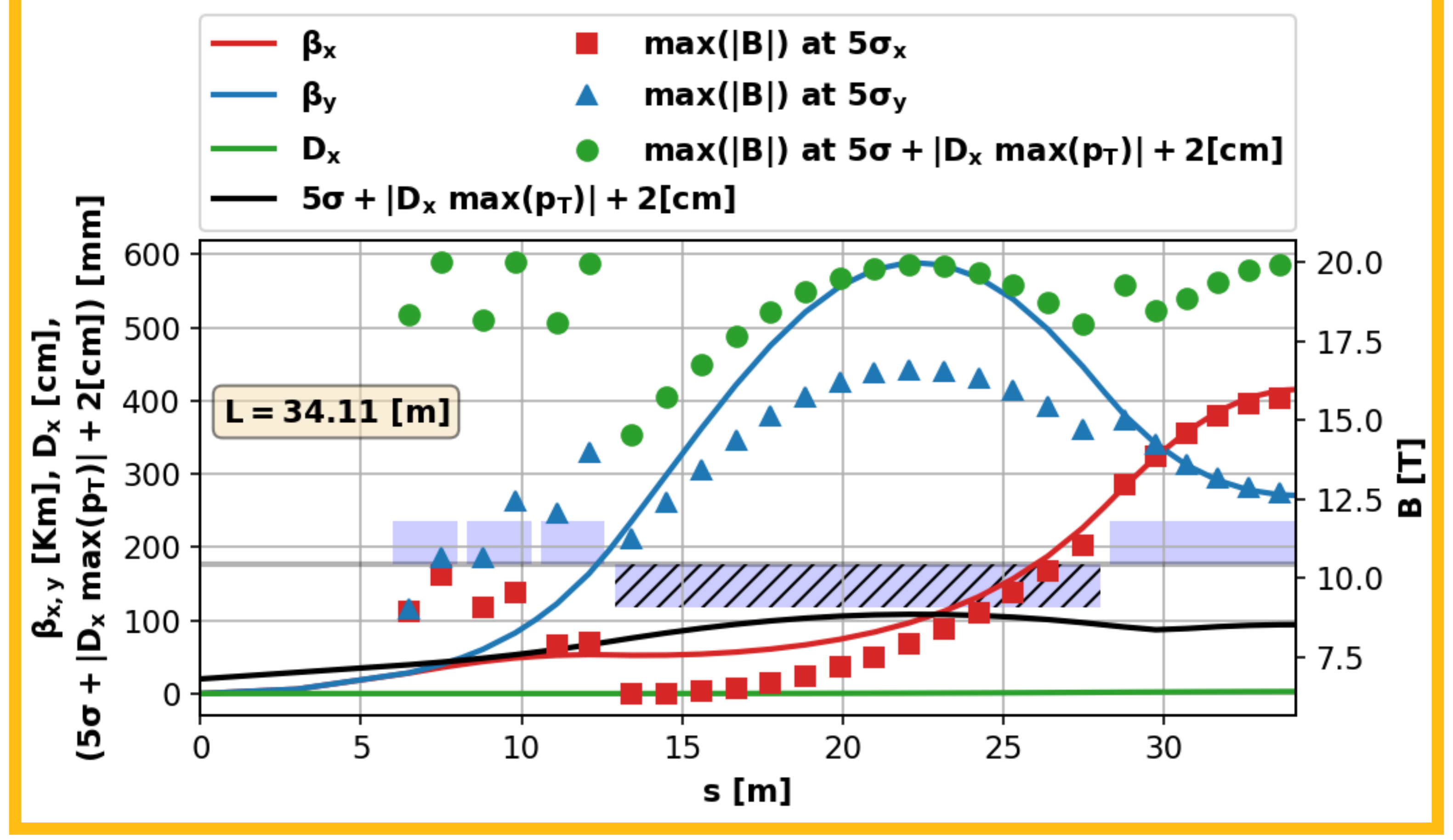


10TeV Muon Collider - Final Focusing Scheme

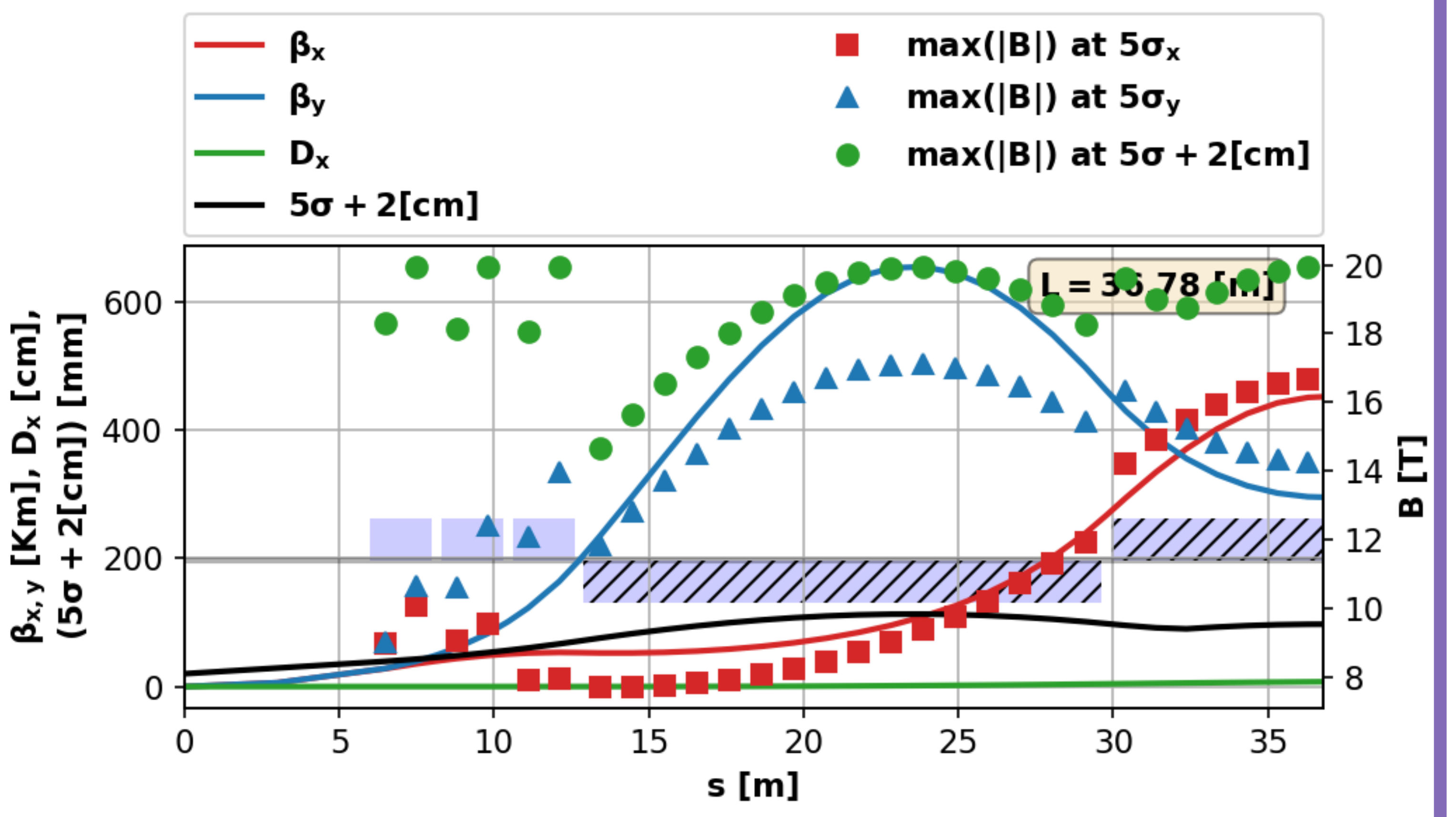
- Different FF schemes that include dipolar components or an elongated L* are designed for the mitigation of the Beam Induced Background (BIB).

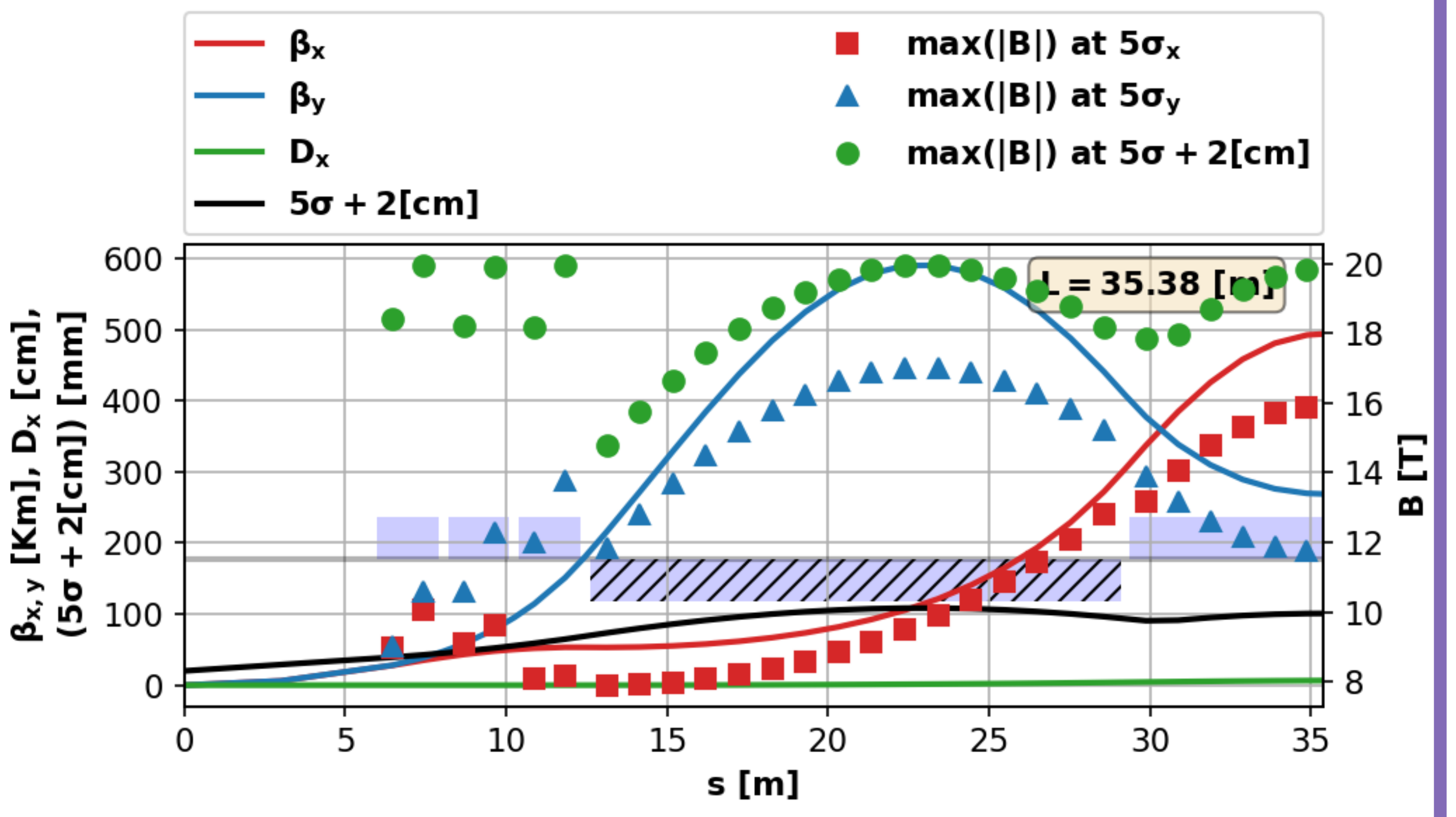
B_d = 2T



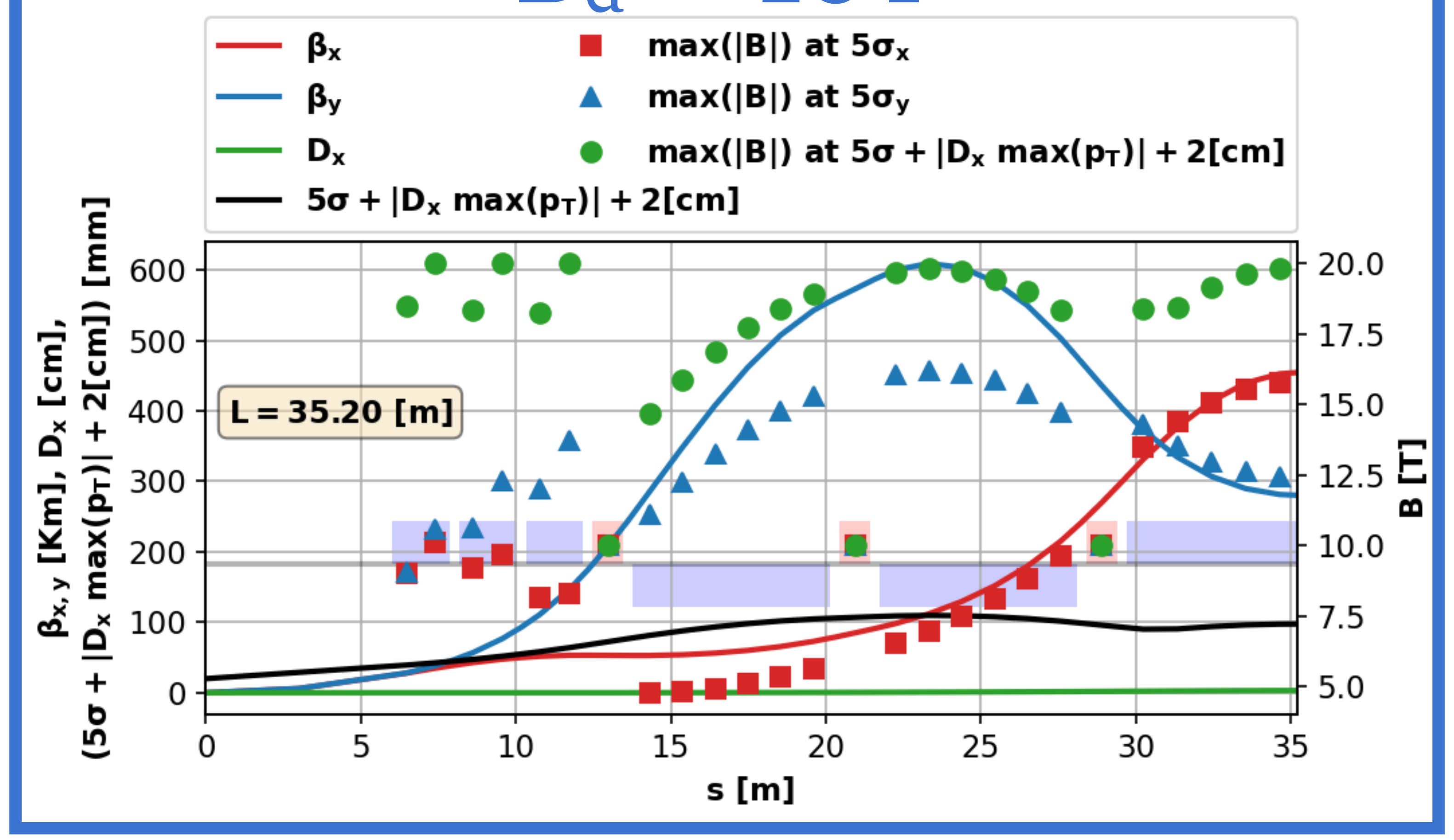


B_d = 4T

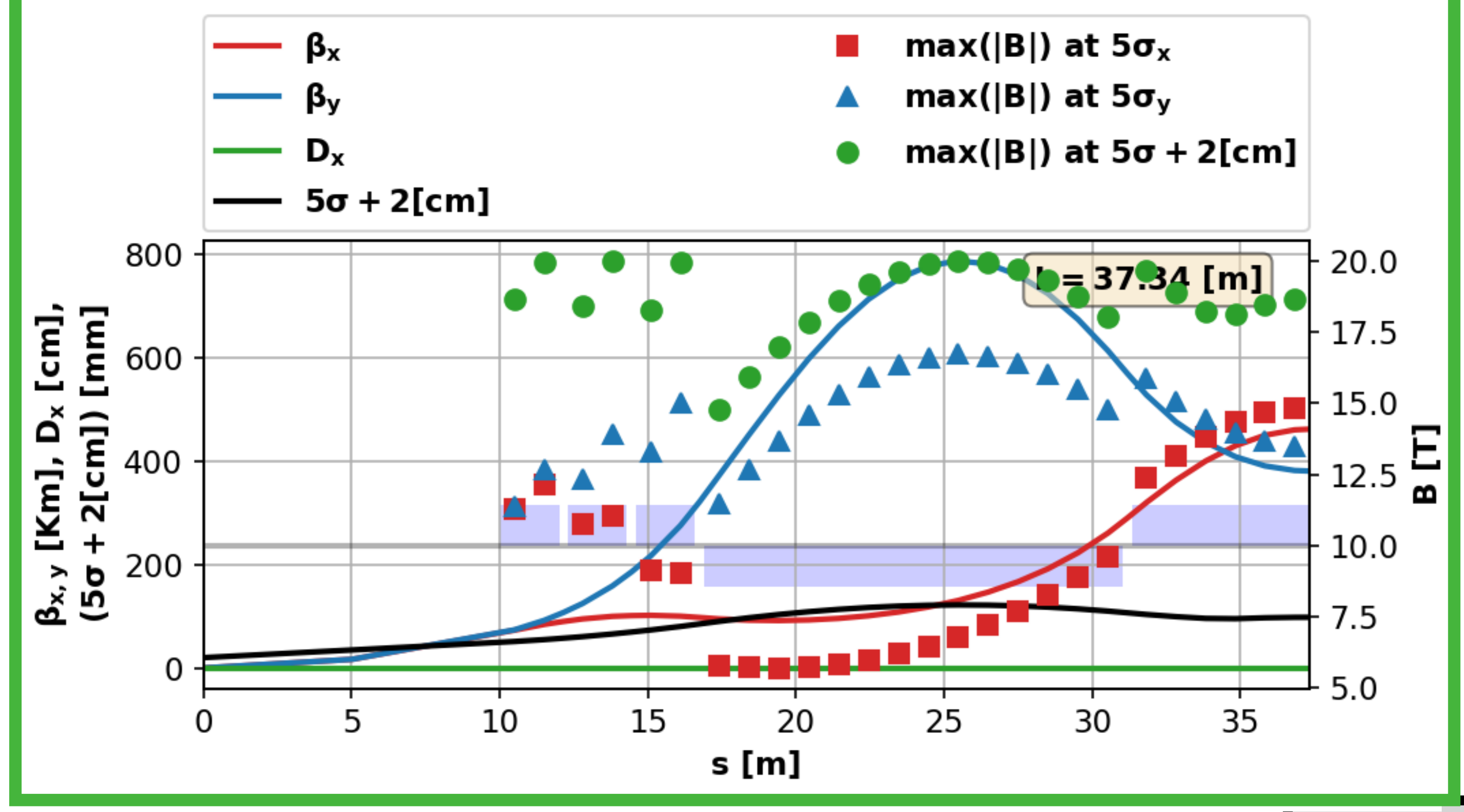




B_d = 10T

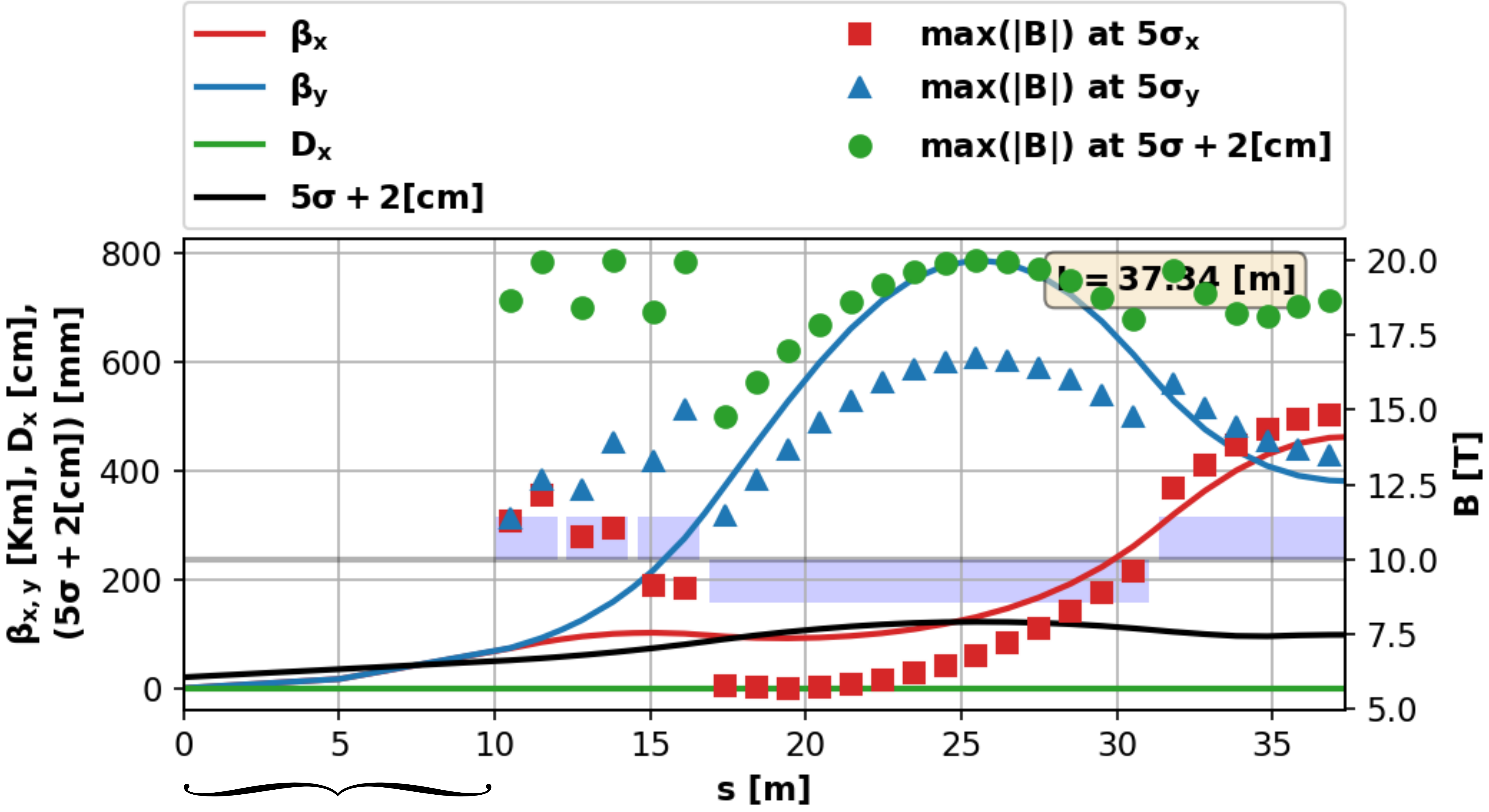


L* = 10m

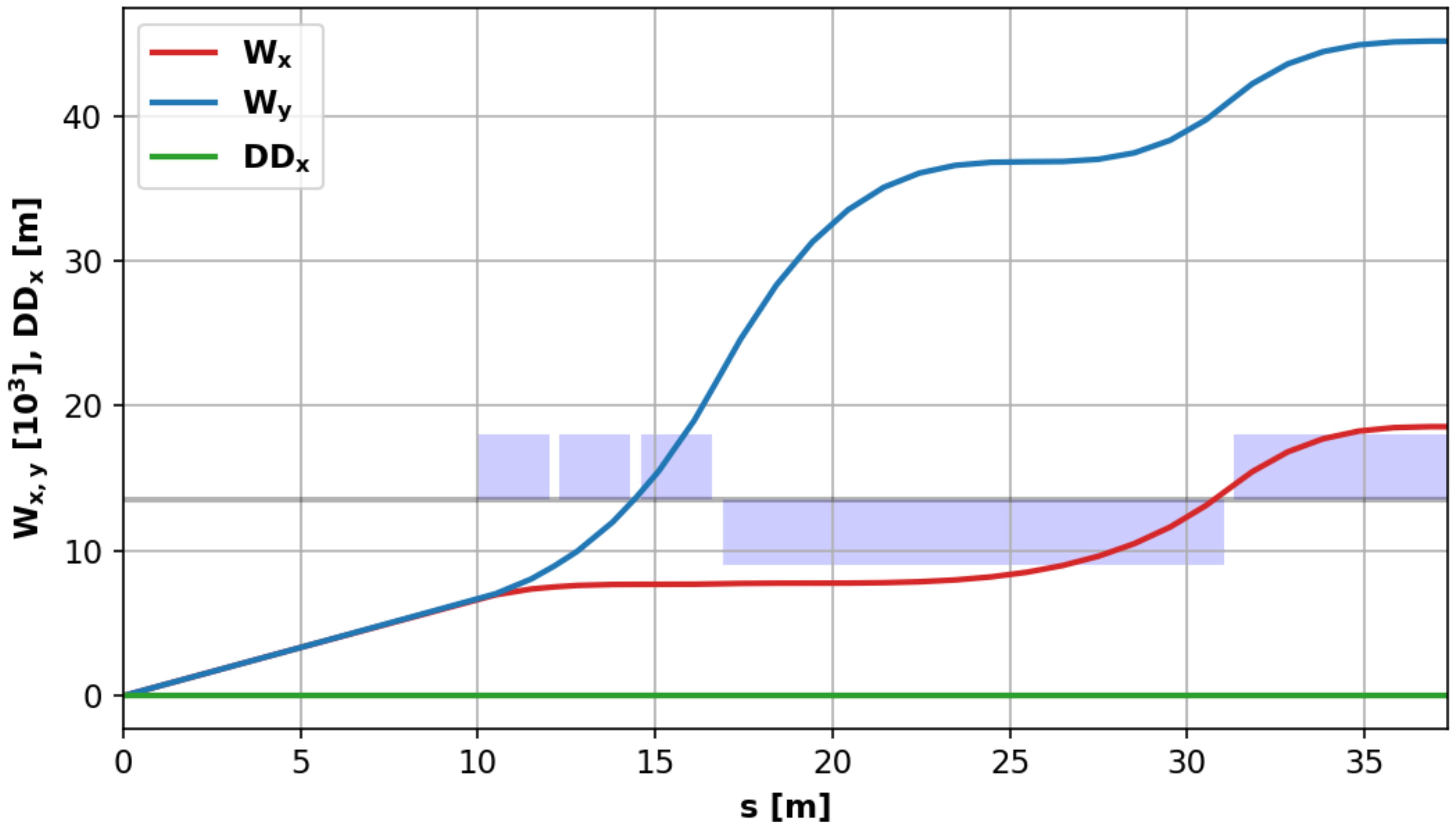


10TeV Muon Collider - Final Focusing Scheme

Due to muon decay along the interaction region, the Beam Induced Background (BIB) at the detectors area is significant thus in collaboration with the FLUKA team, the impact on BIB from the elongation of the L* is studied.



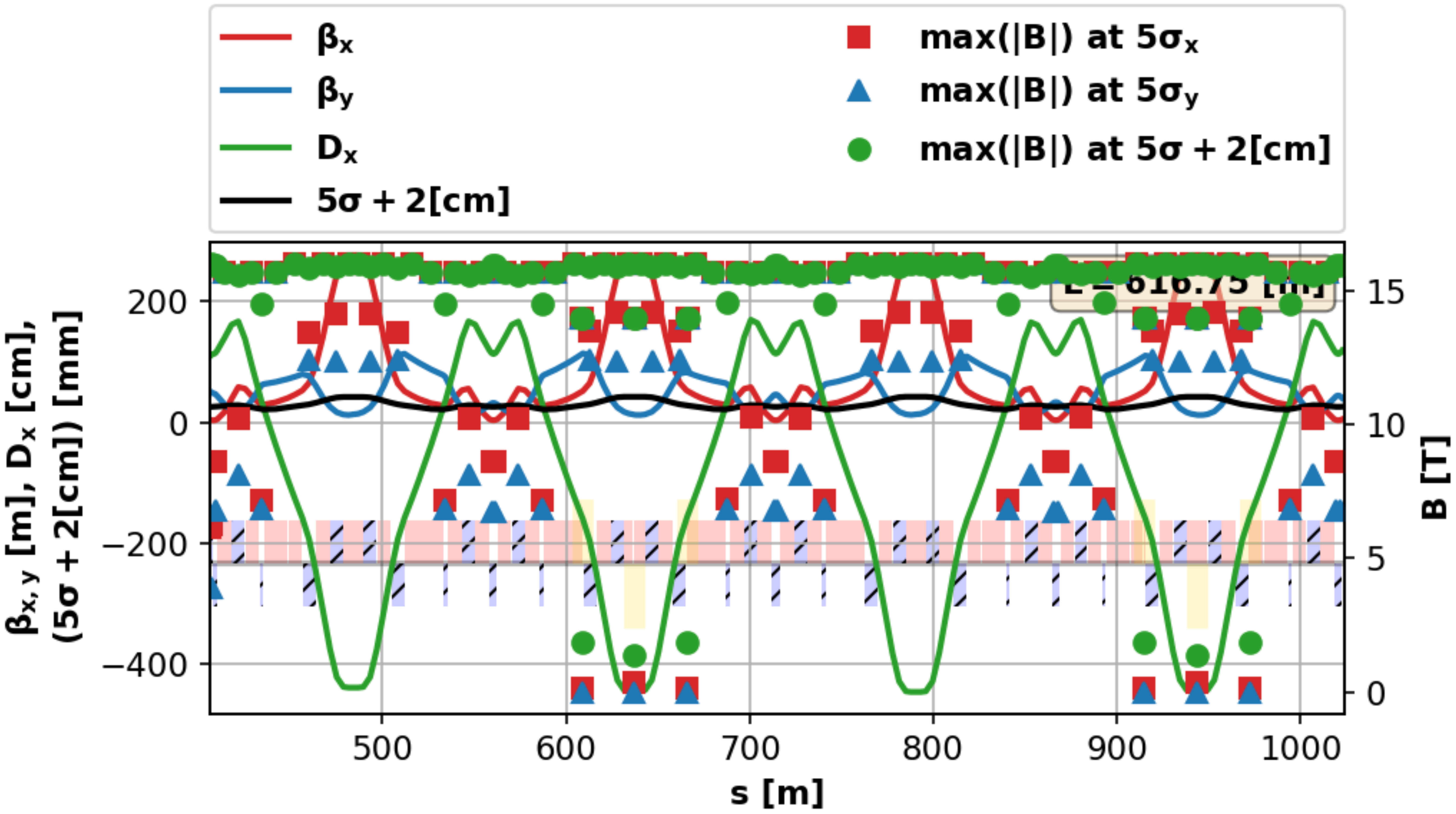
L*=10m

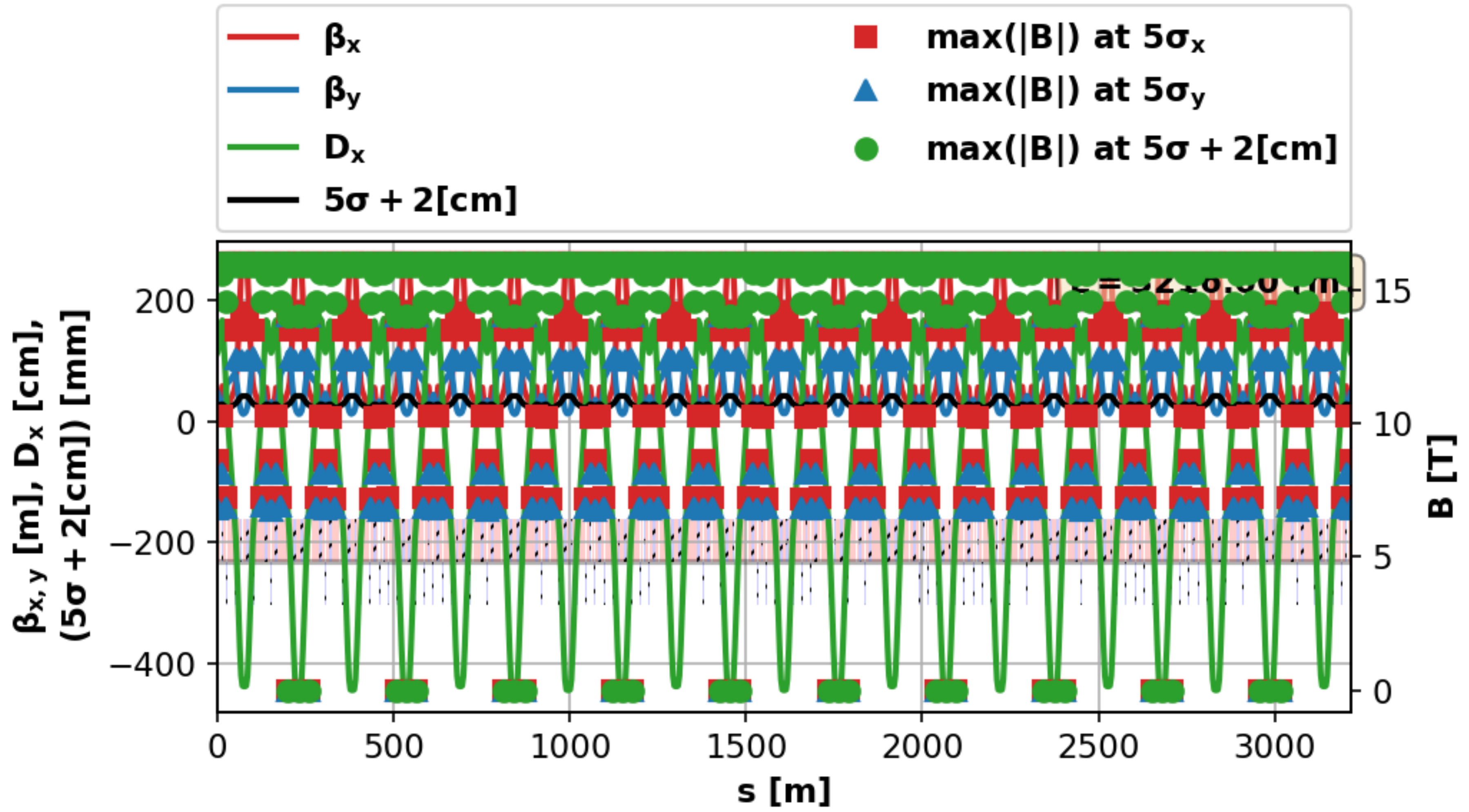


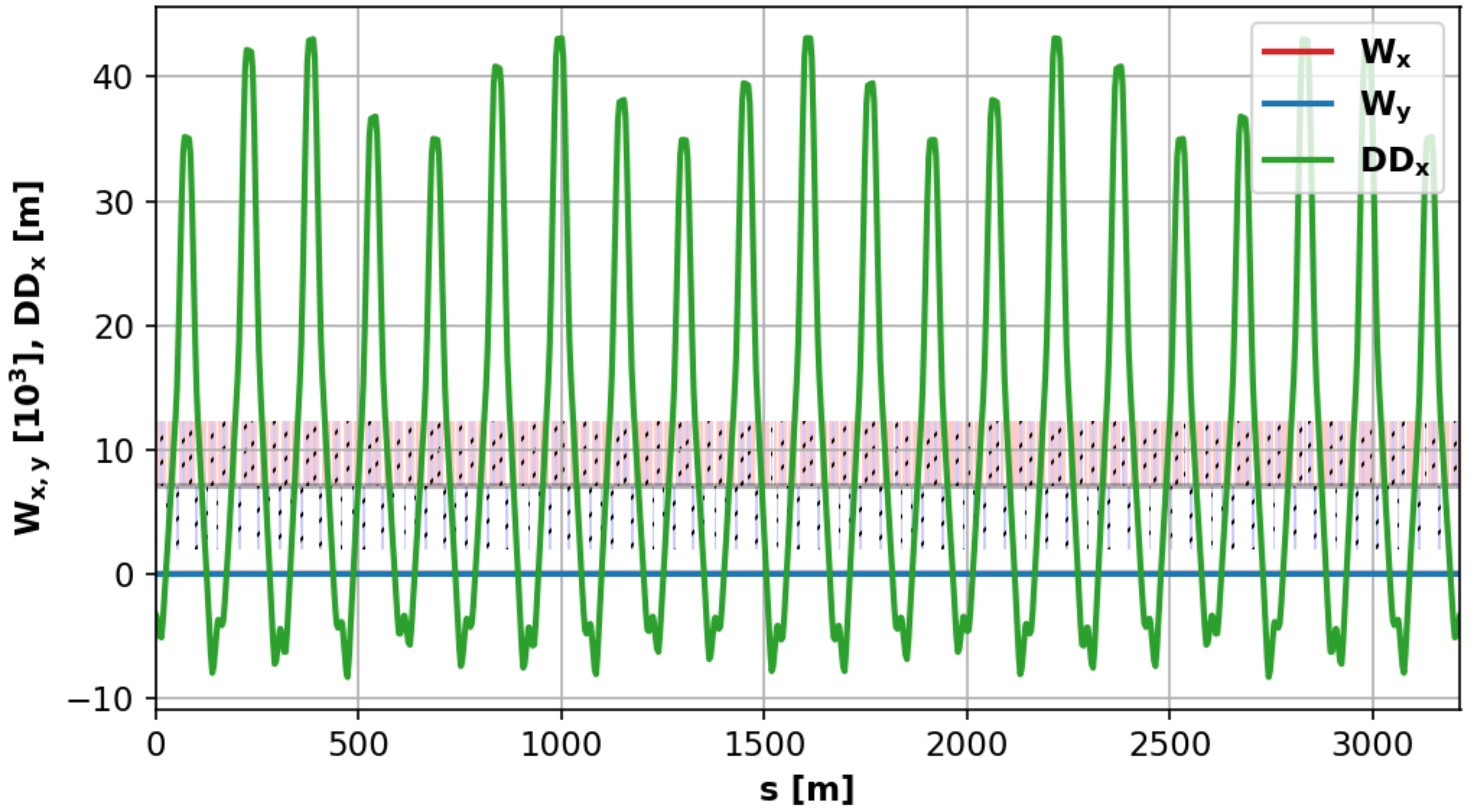
10TeV Muon Collider - Arc

- The CC scheme produces a large positive contribution to the momentum compaction factor (α_p) and phase slip ($\eta_p \sim \alpha_p - 4.5 \times 10^{-10}$) thus, a negative contribution from the arcs is generated in order to keep η_p small and stay below transition ($\eta_p, \alpha_p < 0$).
- The maximum allowed magnetic field is assumed to be the 16T.
- Each arc section consist of repeated Flexible Momentum Compaction (FMC) cells (each one is made out of 2 FODO cells).
- The integrated strength of a set of dipoles located at areas with negative dispersion controls the α_p while with another set of dipoles, the 2π closing of the trajectory is controlled.
- The linear chromaticity at x and y planes is controlled with a set of combined function dipole-sextupole magnets separated by a -I transform.
- The phase advance per FMC cell is $3\pi/2$ (-I transform every second cell).

10TeV Muon Collider - Arc



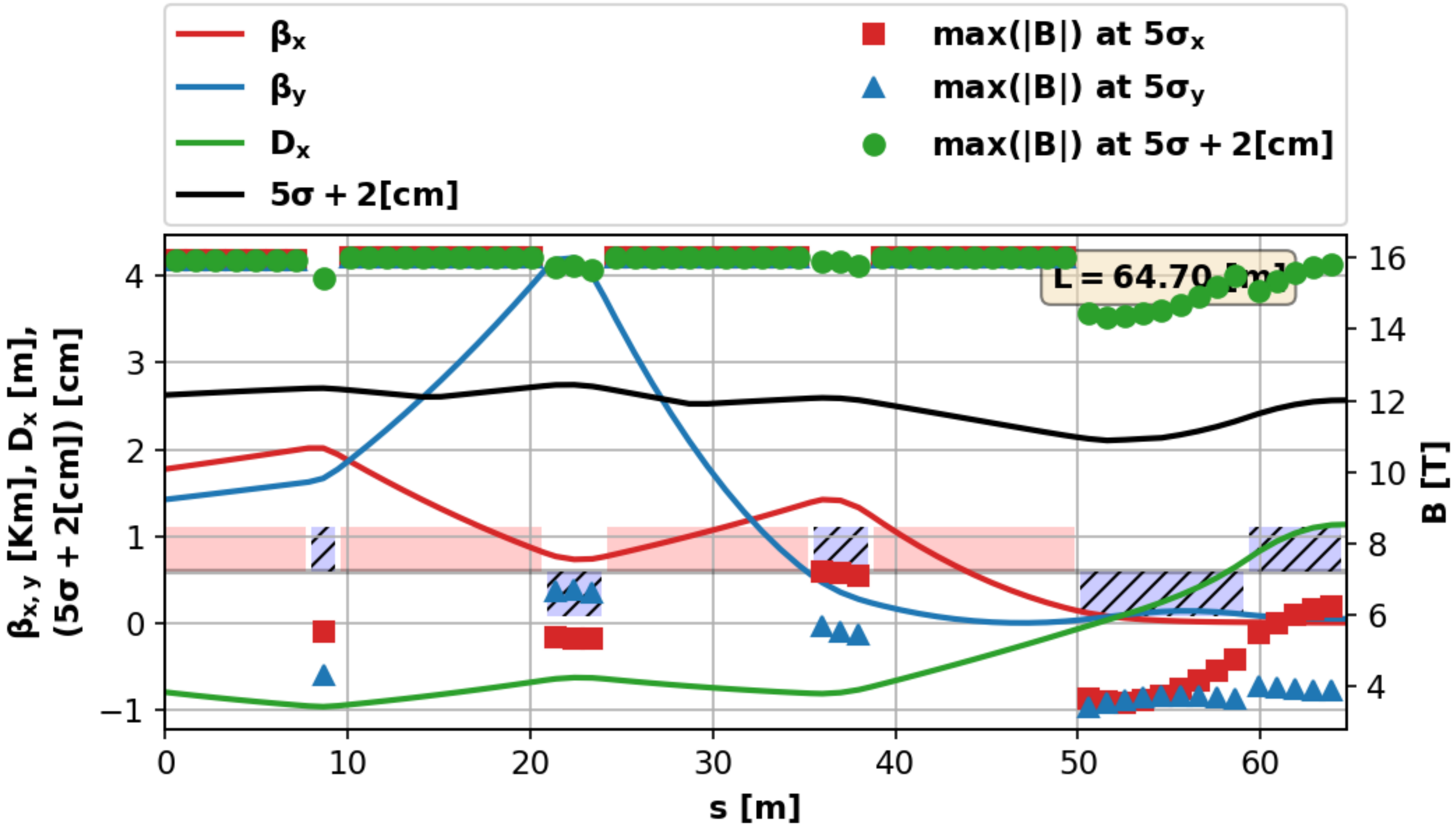


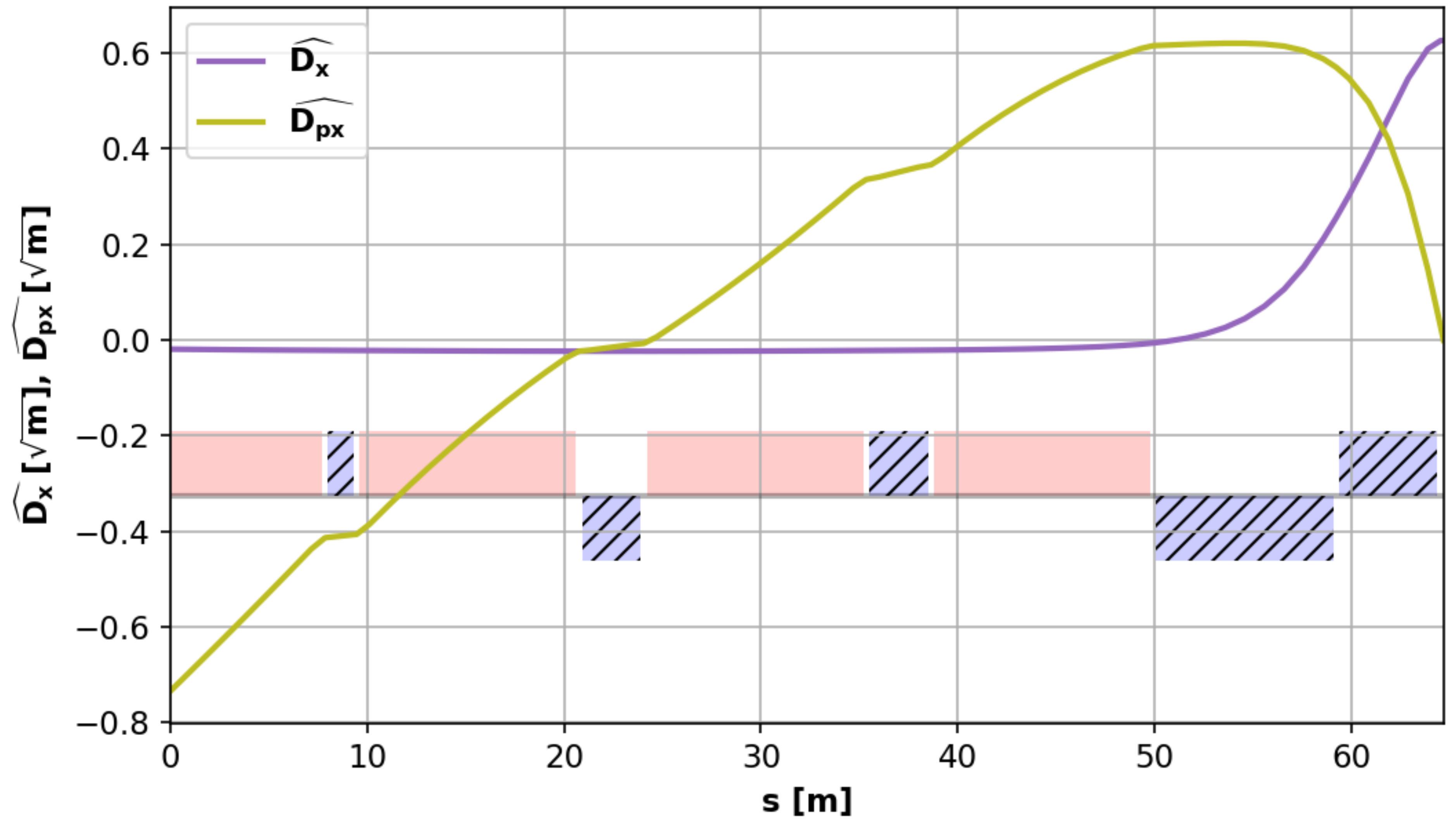


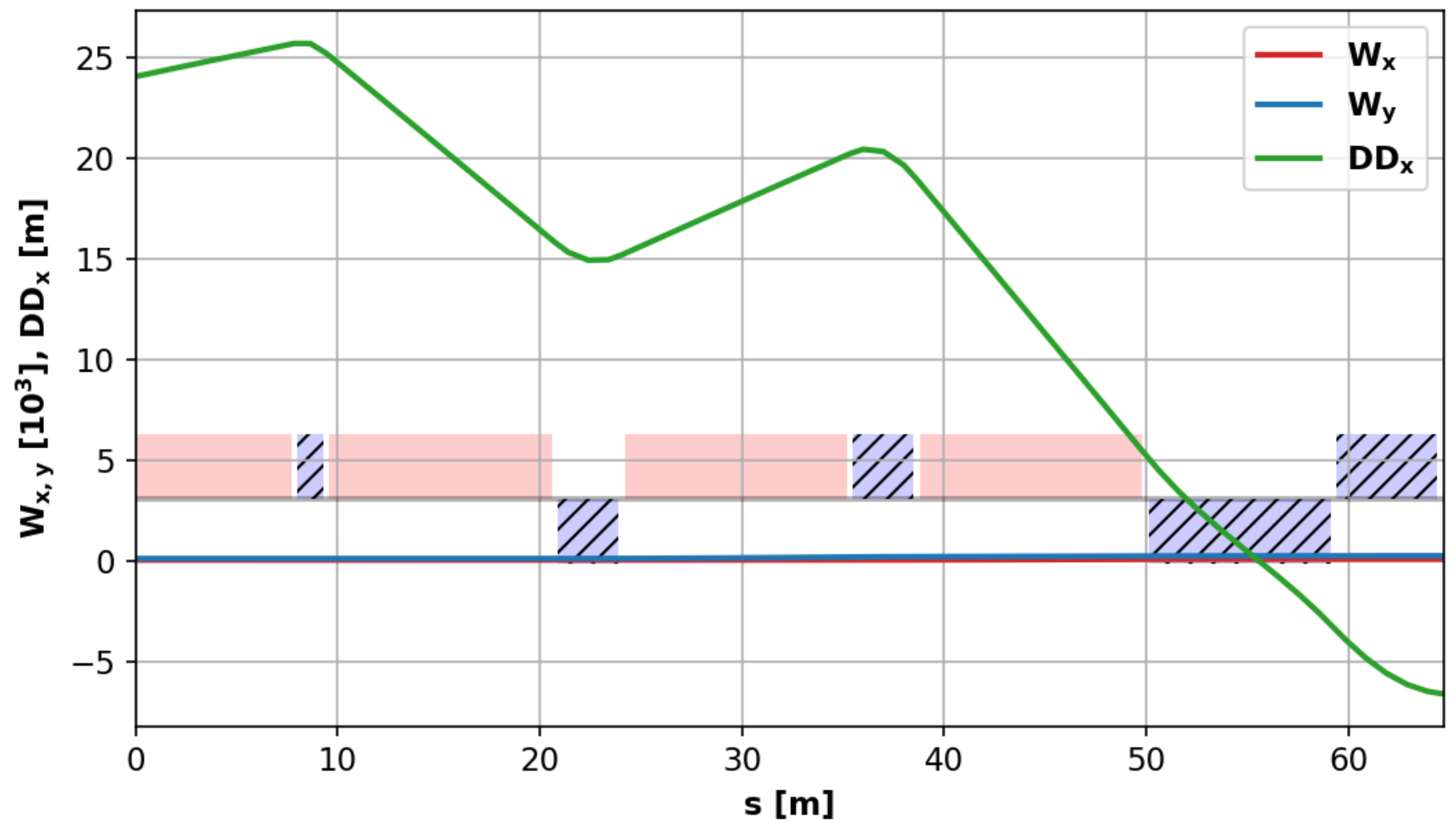
10TeV Muon Collider - Matching Section

- A matching section connecting the CC scheme and the arc is needed.
- The maximum allowed magnetic field is assumed to be the 16T.
- The $\beta_{x,y}$, $\alpha_{x,y}$, D_x and D_{px} are matched by controlling the integrated strength of five dipole-quadrupole and one dipole magnet.
- The matching of the D_{px} is facilitated by controlling its value at the end of the CC scheme (keeping it to small values).

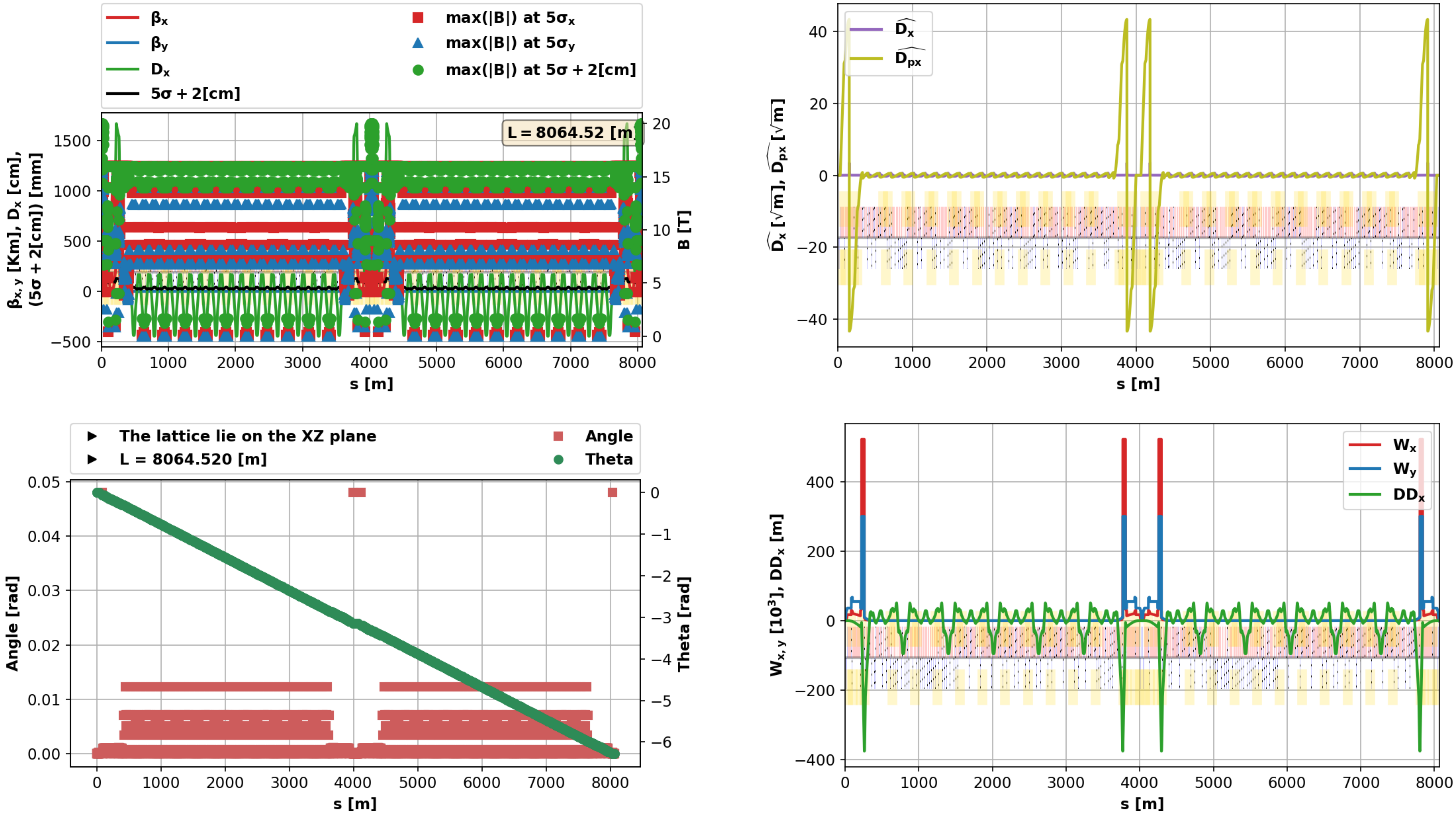
10TeV Muon Collider - Matching Section







10TeV Muon Collider - Full Lattice



Summary

- Minimization of the areas without dipolar components in order to evenly distribute the muon decay products (mostly the neutrino flux) and to minimize the collider length.
- Extensive use of combined function magnets (dipole-quadrupole, dipole- sextupole, etc) with independent control of their multipolar components.
- Different designs for the Final Focusing scheme that aim to mitigate the BIB. The BIB reduction due to dipolar components was so far found lower than expected, but the study is still ongoing.
- The Chromatic Correction scheme controls the Montague chromatic functions and the second order dispersion at the IPs with three dipole-sextupole doublets.
- Arc design with Flexible Momentum Compaction cells that control the momentum compaction factor, the linear chromaticity and the 2π closing of the trajectory with independent knobs.
- Each section of the collider relies on the information/restrictions constantly coming from the WP2/5/6/7 thus, the ring is constantly updated.

To be addressed

- Best location to include the straight sections.
- Potential improvement of the Final Focusing design for better control of the Beam Induced Background.
- Improve the phase advance sensitivity of the sextupole doublets in the Chromatic Correction scheme.
- Estimation of key parameters as well their tolerances for the:
  - minimum aperture (impedance - talk by D. Amorim, shielding - A. Lechner, cold bore, coil insulation, thermal insulation between shielding and cold bore, …)
  - maximum allowed magnetic fields - L. Bottura, the strength of each multipole component in combined function, fringe field, power supply stability
  - maximum beta values (outside the IR) -> collider length -> Luminosity
  - chromatisity values (for stability), use of octuples
  - …

10TeV Muon Collider - Arc

Arc dipole-quadrupole of v3 10TeV collider

Name	B_d [T]	G_1 [T/m]	Aperture - 2*(5σ+2cm) - [cm]
AQF1	12.3	87.153	8.289
AQD1	12.3	-120.325	5.967
AQF2	8	266.851	5.711
AQD2	6.5	-366.921	5.154

The above parameters may drastically change in the upcoming versions of the collider