Quantum Algorithms for Dynamical Systems

Why Dynamical Systems ?

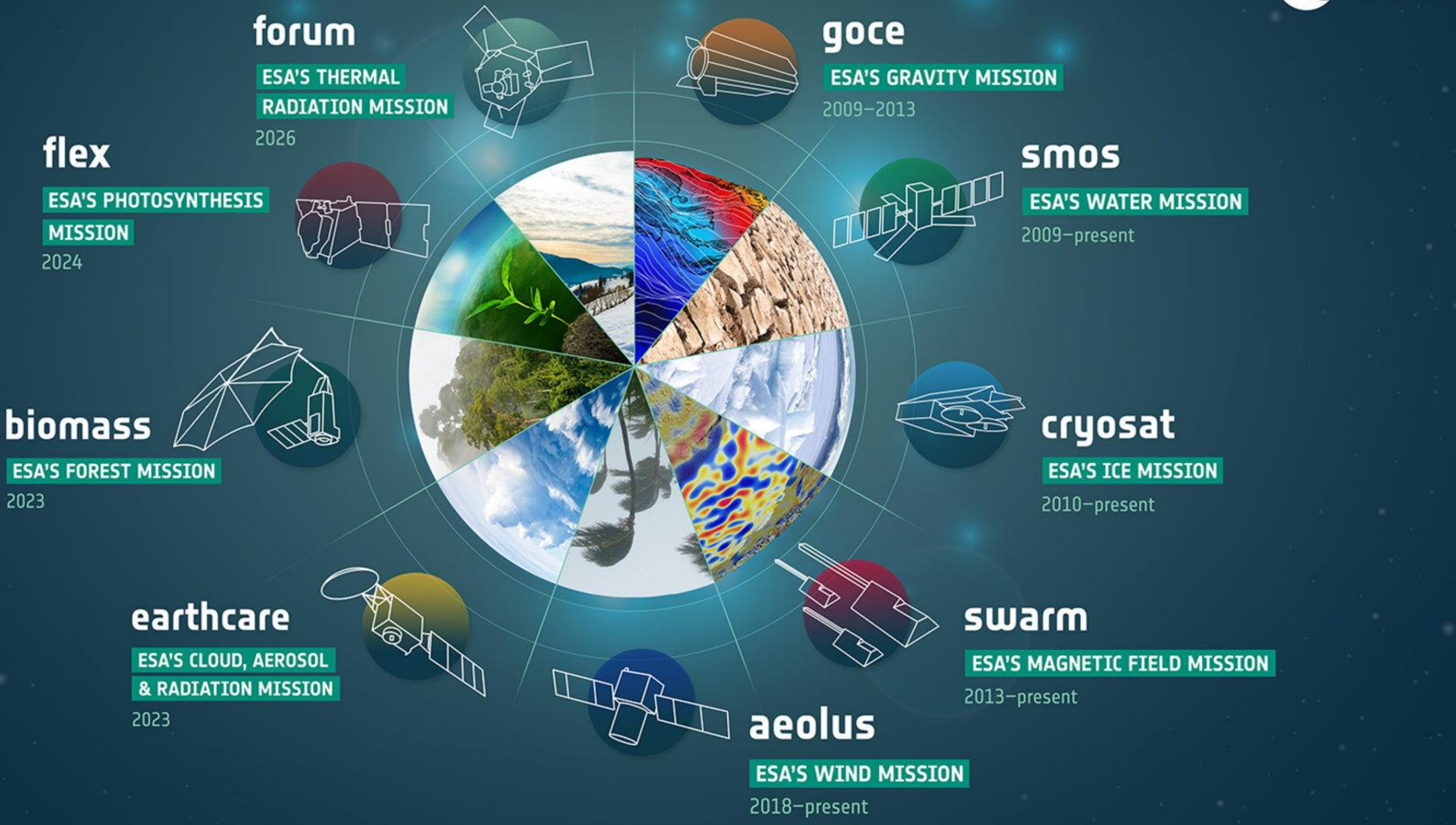
$$\rho\left[\frac{\partial V}{\partial t} + (V\cdot\nabla)V = -\nabla p + \rho g + \mu\nabla^2 V\right]$$

$$\frac{dV}{dt} = \frac{P}{P+1} - V$$
$$P = \max(P + \mu V, 0)$$

$$\frac{dS}{dt} = -\frac{\beta IS}{N}$$
$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

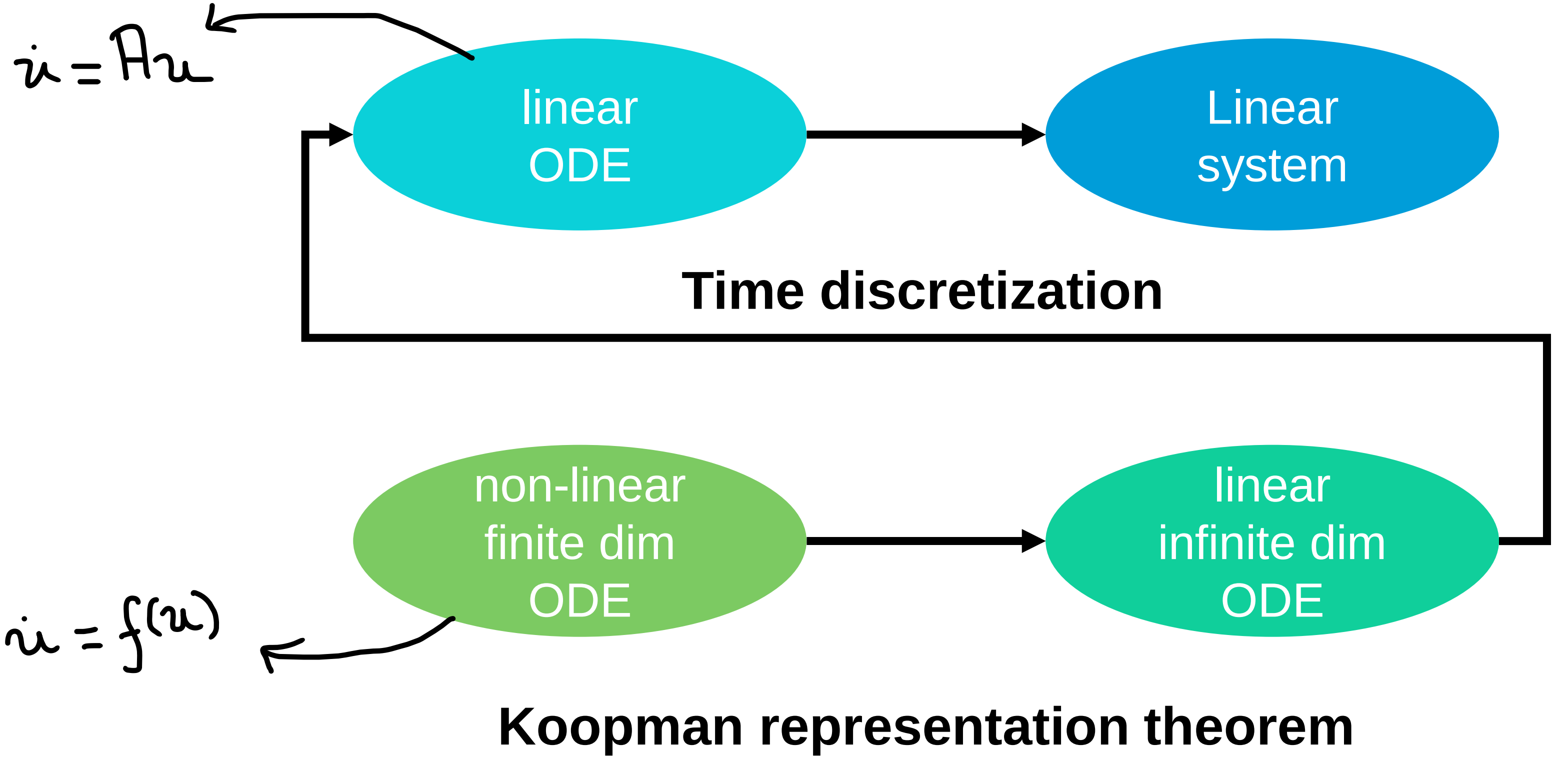
$$\frac{dx}{dt} = x(\alpha - \beta y)$$
$$\frac{dy}{dt} = y(\delta x - \gamma)$$

dataflows


forum
ESA'S THERMAL
RADIATION MISSION
2026
goce
ESA'S GRAVITY MISSION
2009–2013
flex
ESA'S PHOTOSYNTHESIS
MISSION
2024
smos
ESA'S WATER MISSION
2009–present
biomass
ESA'S FOREST MISSION
2023
cryosat
ESA'S ICE MISSION
2010–present
earthcare
ESA'S CLOUD, AEROSOL
& RADIATION MISSION
2023
swarm
ESA'S MAGNETIC FIELD MISSION
2013–present
aeolus
ESA'S WIND MISSION
2018–present

Why Quantum Computers ?

Quantum Computers solve linear systems in log(N) : Exponential Speedup



Quantum Algorithms to solve differential equations

Complexities :

optimal

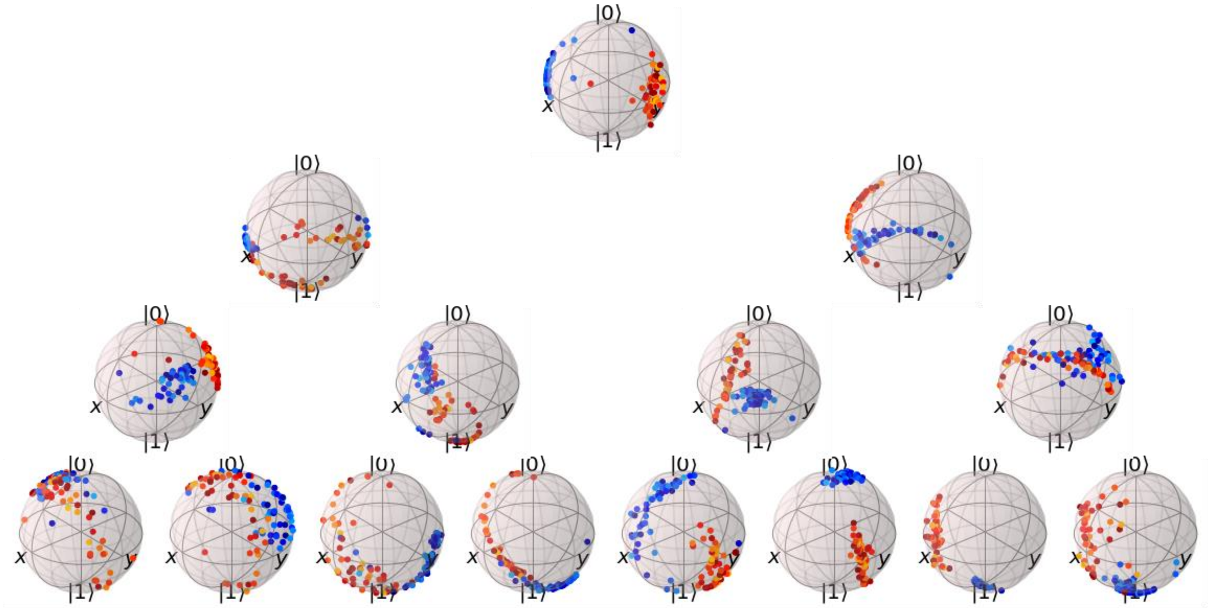
Hamiltonian	Qubitization [Low 19]	$\boldsymbol{O(\|A\|t + log(1/\epsilon))}$
Linear	non-quantumness overheads [An 22]	$\boldsymbol{\Omega(e^{\delta t} + \mu(A))}$
Quadratic	Carleman linearization [Childs 22]	$\boldsymbol{O\left(t^2 polylog(t, N)\|u_0\|/\|u(t)\|\right)}$
Generic nonlinear	Copies of states [Osborne 08]	$\boldsymbol{O(d^t \,\text{polylog}\, N)}$

Good scaling with dimensionality N but exponential with time t

qutree : Plot sets of multi-qubit pure states

AD

```
pip install qutree
from qutree import BBT
bbt = BBT(4)
bbt.add_data(psi)
bbt.plot_tree()
```



Visualization of a QML dataset on a qutree

https://github.com/CERN-IT-INNOVATION/qutree

Thank you

Questions ?