Exercise 1
Equivariant localization in QFT
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For the conventions and explanations see the lectures.

1. Consider $\mathbb{C}$, $\mathbb{C}^n$ and S^2. Please write all torus actions explicitly and the corresponding vector fields. Write the examples of equivariantly closed forms for these spaces.

2. Remind yourself the definition of Grassmann integrations. Derive the following formula (modulo conventions)

$$\int d^{2n}\Psi \; e^{\Psi^i B_{ij} \Psi^j} \sim \sqrt{\det(B)} = \mathrm{Pf}(B)$$

3. Consider the supermanifold $T[1]M$ with the coordinates (X^μ, Ψ^μ). Please show that the measure

$$d^n X \; d^n \Psi = d^n \tilde{X} \; d^n \tilde{\Psi}$$

under diffeomorphisms of M.

4. Take the operation on the differential forms d_{dR}, d_v and $\mathcal{L}_v$ and rewrite them as operations on $T[1]M$

5. Using the supergeometry language please show that $\delta^2 W = 0$ implies $\mathcal{L}_v g = 0$ where W is defined

$$W = \Psi^\mu g_{\mu\nu} v^\nu$$

and δ corresponds to d_v (see the lectures).

6* Can you think how to derive the generalization of the Atyiah-Bott theorem for the case of non-isolated fixed points using the superlanguage [this may be a hard problem for some of you]