# Towards a measurement theory in QFT: "Impossible" quantum measurements are possible but not ideal

From impossible measurements to
the Elegant Joint Measurement

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arXiv:2311.13644 Quantum 8, 1267 (2024) & arXiv:2307:06998 PR Research

# Impossible Measurements on Quantum Fields

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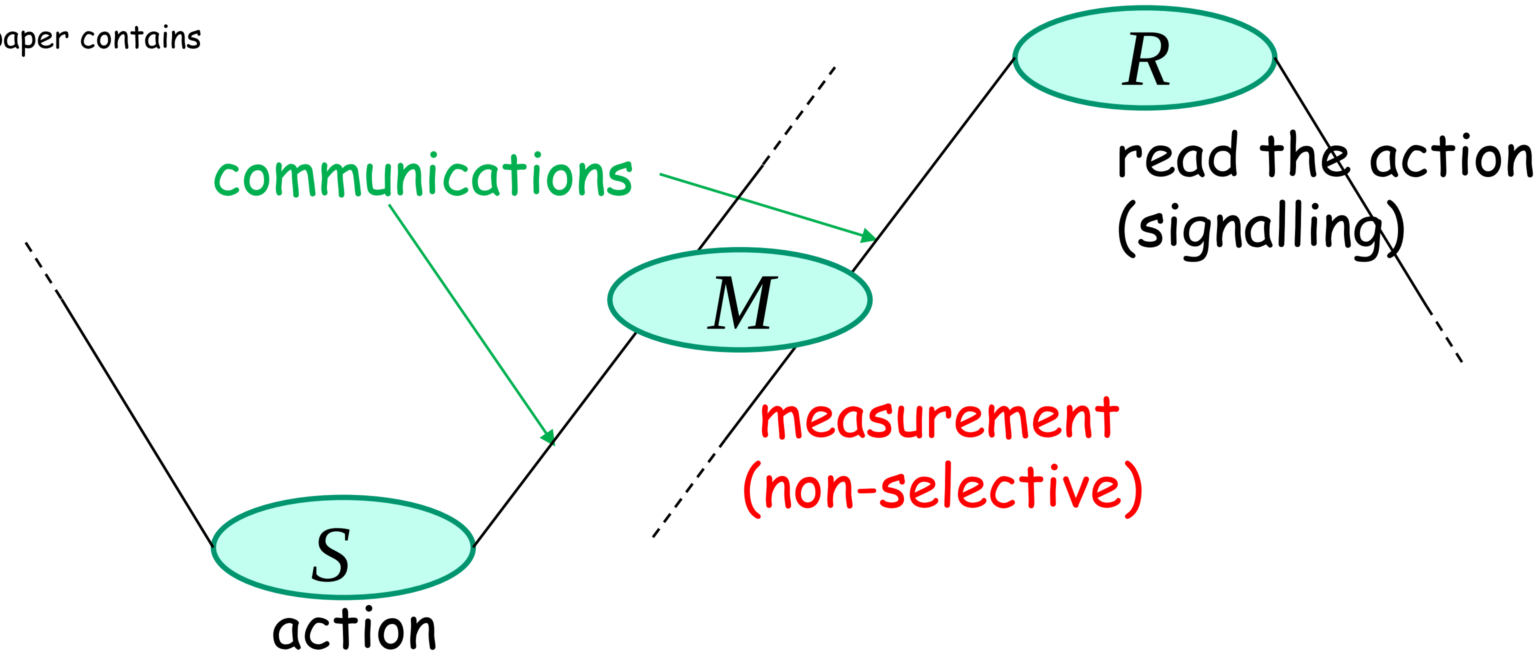
arXiv:gr-qc/9302018



note: this paper contains no figure.




- Clearly, it is not the communications that should be questioned, but the measurement.
- The measurement covers some space-like region, ifnot no impossible measurement. Hence, impossible measurements are measurements of NL (non-local) variables.

# Impossible Measurements on Quantum Fields

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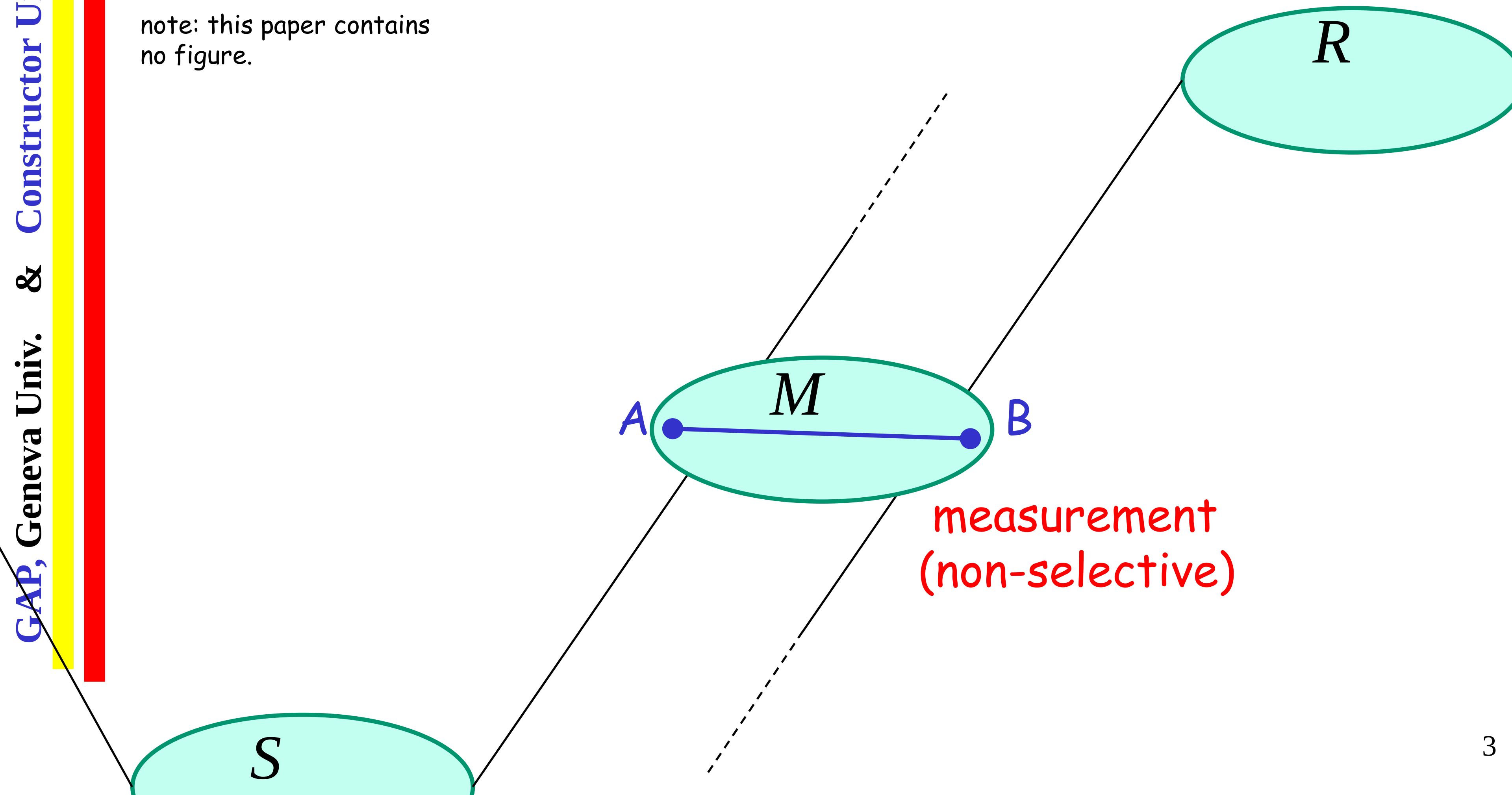


# Measurements of NL variables



Alice — NLM — Bob

NLM are defined by a basis

example: twisted basis: $|0,0\rangle, |0,1\rangle, |1,+\rangle, |1,-\rangle$

Assume initial state $|0,0\rangle$ and standard projective measurement
$\Rightarrow$ outcome $|0,0\rangle$ with probability 1.

Action: Alice flips her qubit before the measurement
$\Rightarrow$ outcome $|1,+\rangle$ & $|1,-\rangle$ each with prob. 1/2

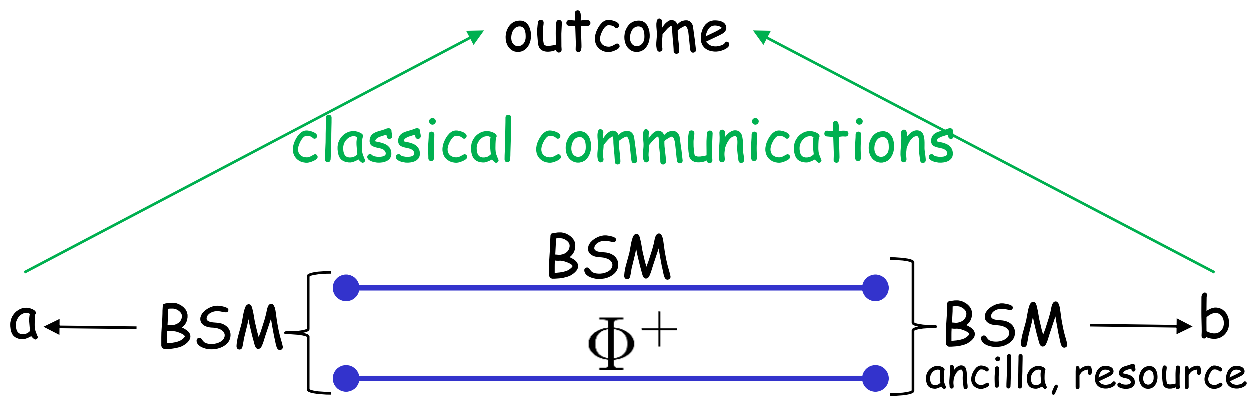
Bob's local state depends on Alice's action $\Rightarrow$ signalling !

**The essence of impossible measurements:**
**they are impossible (in relativity) because they signal (are acausal).**

also impossible in non-relativistic QM

Of course, one could bring the 2 qubits together. But the name of the game is to keep them at a distance and see whether the measurement is <u>localizable</u>, i.e. all the quantum parts can be done locally at Alice and Bob.

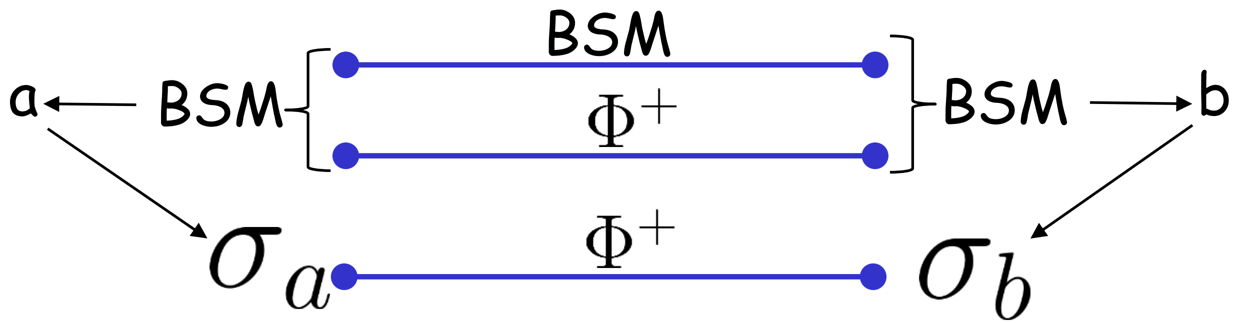
# BSM




The outcomes are s.t. the number of $\Phi$ is even
and the number of + is also even.

$\Rightarrow$ recovers the Born rule,
is not signalling,
is localizable (all quantum part is done locally),
but is not ideal (not immediately reproducible, not projective)

# Ideal BSM




Now, the BSM is localized and ideal (up to some swaps).

Theorem: (Popescu-Vaidman)
The BSM is the only joint NL measurement of 2 qubits that can be measured ideally without signalling.

Note: the BSM is not a typical measurement, but is exceptional.

Theorem: (Popescu-Vaidman)
All localizable ideal measurements must erase all local information.

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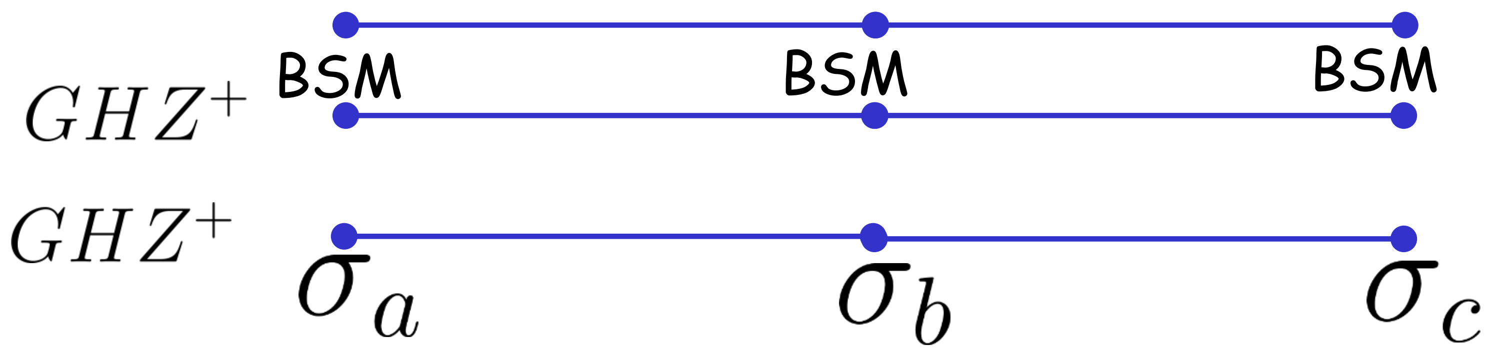
# Ideal GHZ Measurement

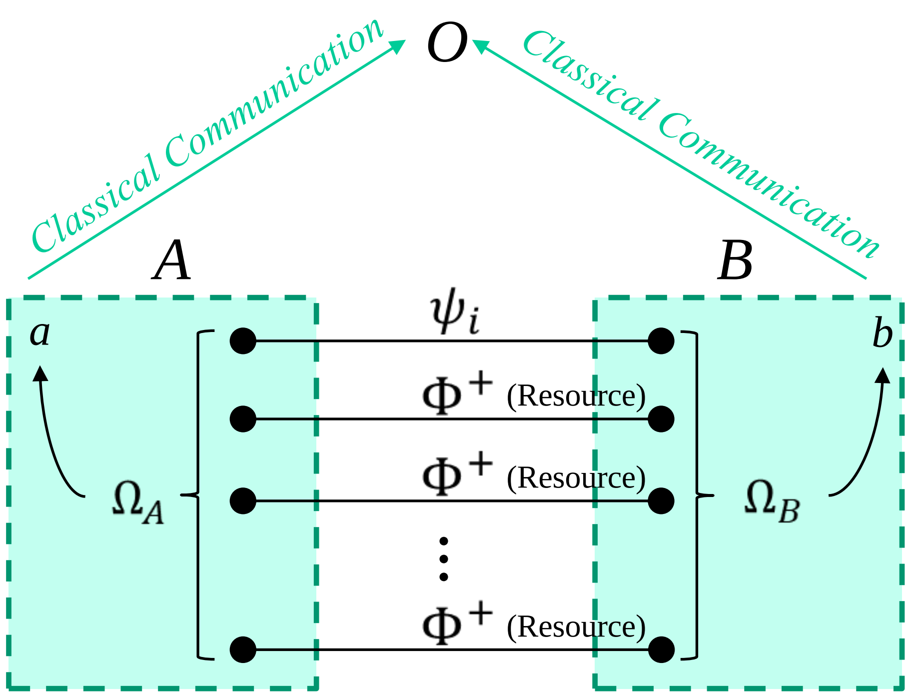



Theorem: All GHZM can be localized and measured ideally without signalling.

Theorem: all localizable ideal measurements must erase all local information.

What about $|0,0\rangle + |1,1\rangle + \epsilon|2,2\rangle$ ?

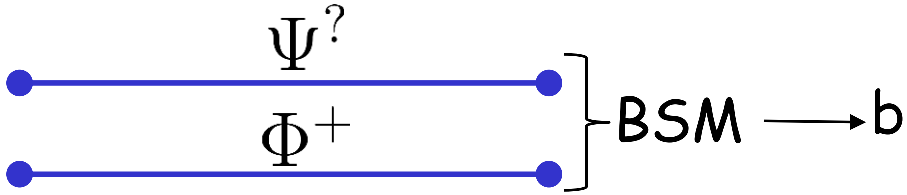
# Localizable Measurements



# Back to twisted basis: $|0,0\rangle, |0,1\rangle, |1,+\rangle, |1,-\rangle$



It can't be measured ideally. But is it localizable?



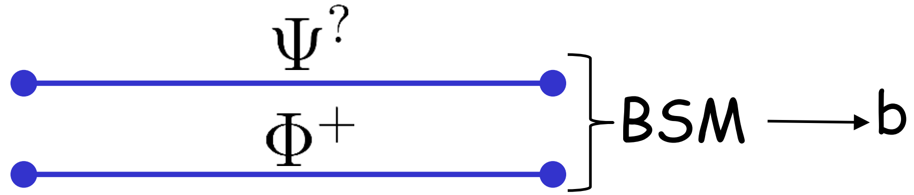

$$\mathbb{1} \otimes \sigma_b \Psi^?$$

Alice measures her 1st qubit in the Z-basis.
If a=0, then she measures her 2nd qubit in the Z-basis.
If a=1, then she measures her 2nd qubit in the X-basis.

The outcome is a function of b and Alice's 2 classical bits.

# π/4-twisted basis: $|0,0\rangle, |0,1\rangle, |1,\nearrow\rangle, |1,\swarrow\rangle$



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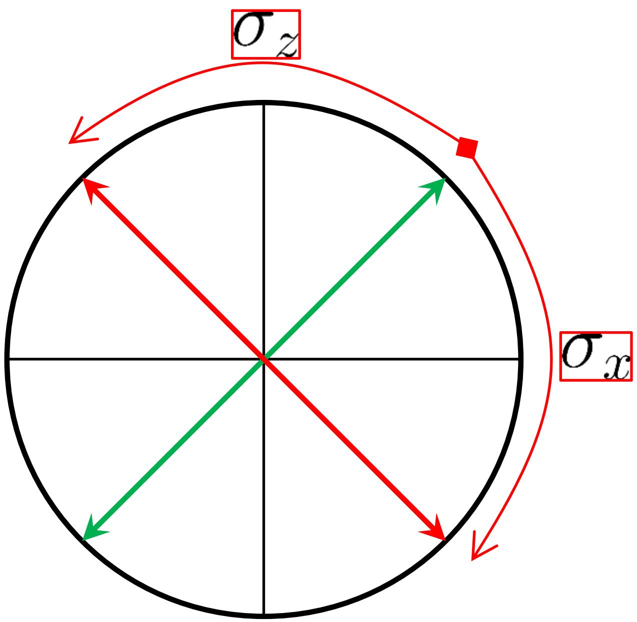
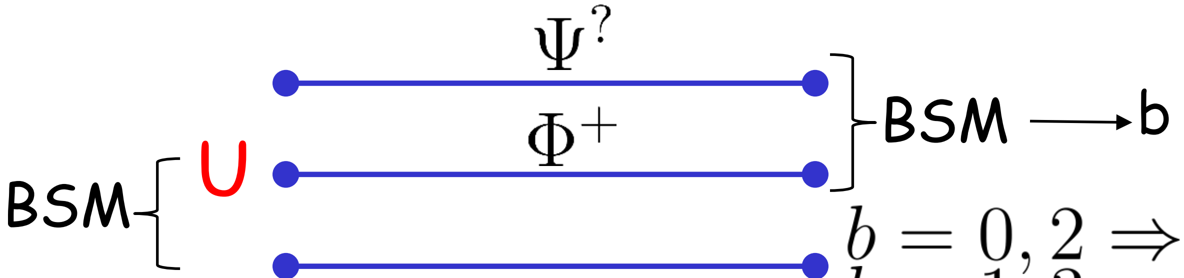
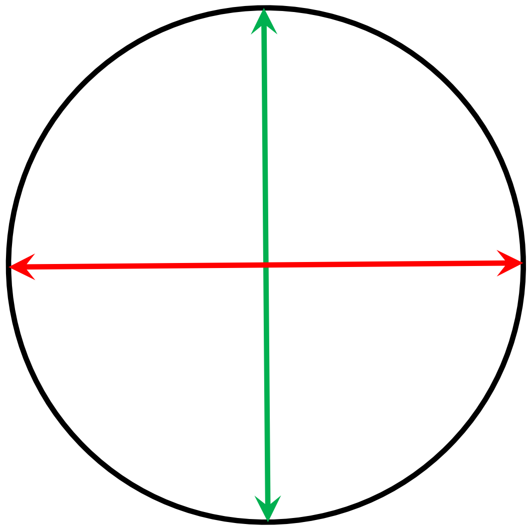


# π/4-twisted basis: $|0,0\rangle, |0,1\rangle, |1,\nearrow\rangle, |1,\swarrow\rangle$

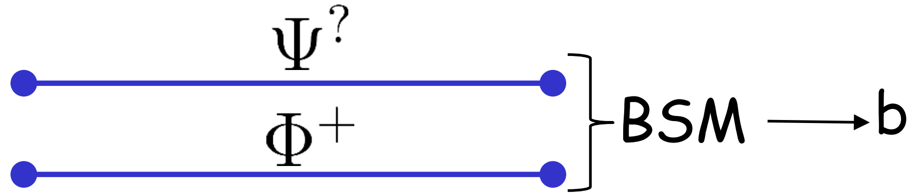



$$b = 0, 2 \Rightarrow \sigma_z$$
$$b = 1, 3 \Rightarrow \sigma_x$$

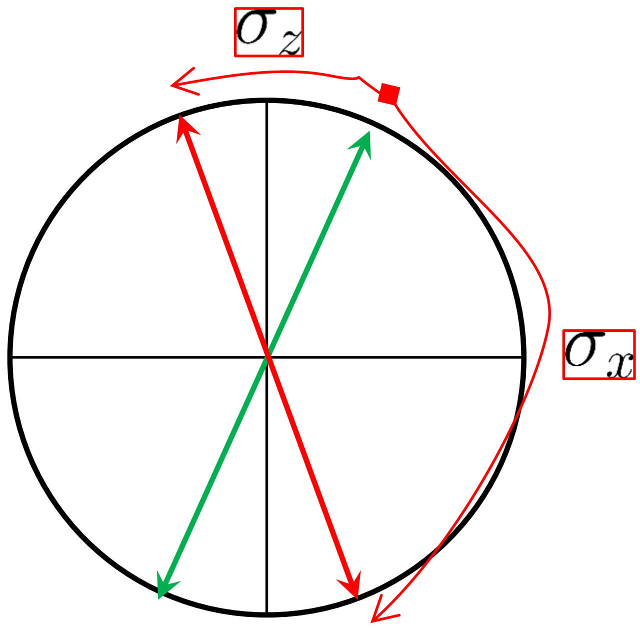


# $\pi$/8-twisted basis: $|0,0\rangle, |0,1\rangle, |1,\frac{\pi}{8}\rangle, |1,\frac{-7\pi}{8}\rangle$

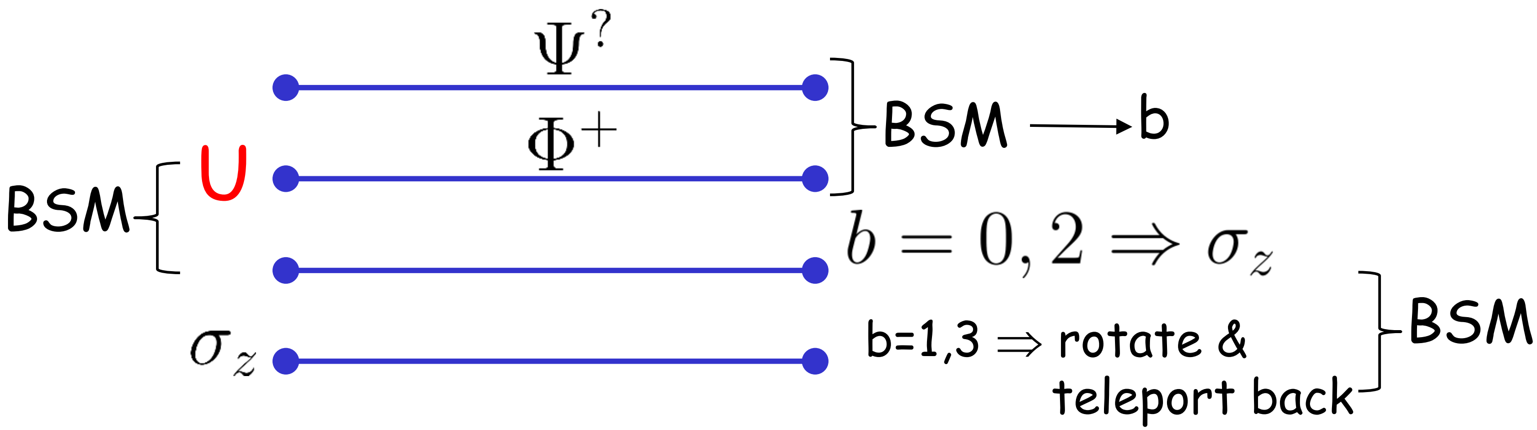




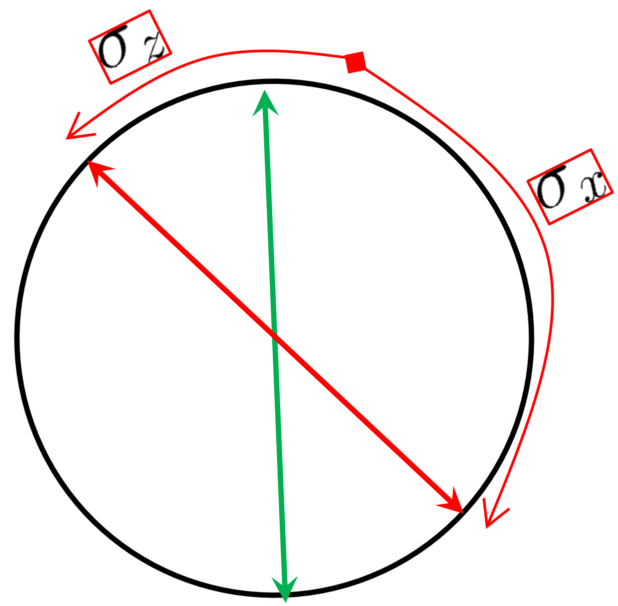

$\mathbb{1} \otimes \sigma_b \Psi^?$



# π/8-twisted basis: $|0,0\rangle, |0,1\rangle, |1,\frac{\pi}{8}\rangle, |1,\frac{-7\pi}{8}\rangle$




etc. one gets the idea that for all rational multiple of π-twisted bases it works with sufficiently many ebits. And for irrationals, one gets arbitrarily close.



# Main Theorem

<u>Theorem</u>: All measurements, any dimension, any nb of parties, are localizable. (Groisman-Reznik, Vaidman)

PHYSICAL REVIEW A **66**, 022110 (2002)

**Measurements of semilocal and nonmaximally entangled states**

Berry Groisman and Benni Reznik
*School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel*
(Received 15 November 2001; published 16 August 2002)

Consistency with relativistic causality narrows down dramatically the class of measurable observables. We argue that, by weakening the preparation role of ideal measurements, many of these observables become measurable. In particular, we show by applying entanglement assisted remote operations that all Hermitian observables of a $(2\times 2)$-dimensional bipartite system are measurable.

VOLUME 90, NUMBER 1 PHYSICAL REVIEW LETTERS week ending 10 JANUARY 2003

**Instantaneous Measurement of Nonlocal Variables**

Lev Vaidman
[1]*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel*
(Received 14 December 2001; revised manuscript received 15 April 2002; published 2 January 2003)

It is shown, under the assumption of the possibility to perform an arbitrary local operation, that all nonlocal variables related to two or more separate sites can be measured instantaneously, except for a finite time required for bringing to one location the classical records from these sites which yield the result of the measurement. It is a verification measurement: it yields reliably the eigenvalues of the nonlocal variables, but it does not prepare the eigenstates of the system.

In a nutshell: “impossible” measurements are localizable, hence possible, but can’t be ideal.

# Measurements as channels

PHYSICAL REVIEW A, VOLUME 64, 052309

**Causal and localizable quantum operations**

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[2]*Microsoft Corporation, One Microsoft Way, Redmond, Washington 98052*
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[4]*Center for Quantum Computer Technology, University of Queensland, Queensland 4072, Australia*
(Received 9 February 2001; published 12 October 2001)

We examine constraints on quantum operations imposed by relativistic causality. A bipartite superoperator is said to be *localizable* if it can be implemented by two parties (Alice and Bob) who share entanglement but do not communicate; it is *causal* if the superoperator does not convey information from Alice to Bob or from Bob to Alice. We characterize the general structure of causal complete-measurement superoperators, and exhibit examples that are causal but not localizable. We construct another class of causal bipartite superoperators that are not localizable by invoking bounds on the strength of correlations among the parts of a quantum system. A bipartite superoperator is said to be *semilocalizable* if it can be implemented with one-way quantum communication from Alice to Bob, and it is *semicausal* if it conveys no information from Bob to Alice. We show that all semicausal complete-measurement superoperators are semilocalizable, and we establish a general criterion for semicausality. In the multipartite case, we observe that a measurement superoperator that projects onto the eigenspaces of a stabilizer code is localizable.

Measurement channels implicitly assume post-measurements states. For example:

$$\rho \to \sum_k P_k \rho P_k$$

assumes that post-measurement states are given by the projection postulate.

# Terminologies

| | | |
|---|---|---|
| QFT | impossible | ??? |
| Aharonov gang | Contradict causality | Instantaneous verification |
| Gottesman-Preskil et al. | a-causal | localizable |
| Our suggestion | Signalling | localizable |

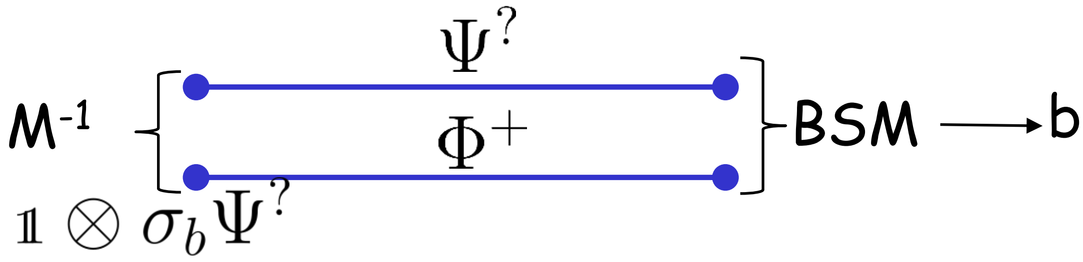
Seemingly, these different communities do not communicate.

QFT has no term for localizable / not ideal.
QFT has no measurement theory.
(though see C. J. Fewster, R. Verch, L. Borsten)

arXiv:2311.13644 Quantum 8, 1267 (2014)

# Localizable NLM with 1 ebit






$$M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M = \tilde{P}_b \Phi_b \equiv P_b$$

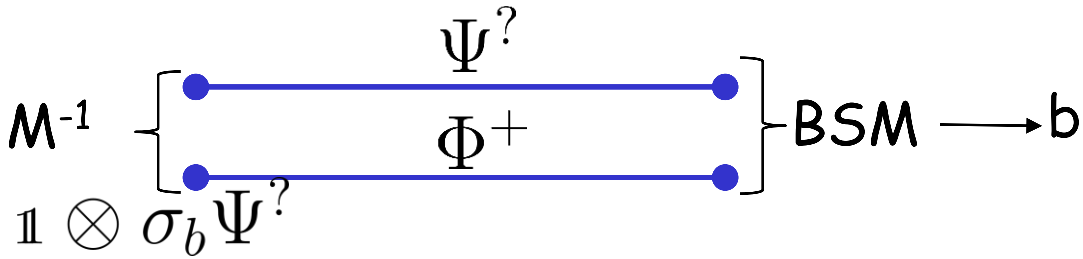
= permutation + phases of 00,01,10,11

<u>Theorem</u>

The only measurements localizable with 1 e-bit, but not with 0 e-bits are the

1. twisted basis measurement, and the
2. Bell State Measurement.

# Localizable NLM with 1 ebit




$$M^\dagger \cdot 1\!\!1 \otimes \sigma_b \cdot M = P_b$$

There are 24 permutations, but only 10 that are self-adjoint and have $\pm 1$ as eigenvalues.
So, we look for representations of SU(2) among these 10 permutations.

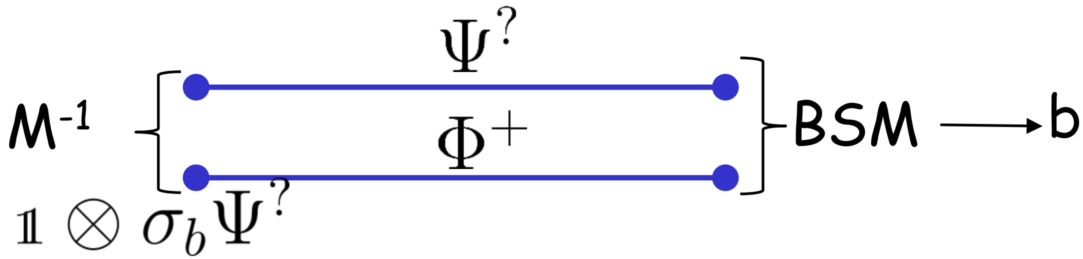
The equation has a "unique" solution:

$$M^\dagger \cdot 1\!\!1 \otimes \sigma_b \cdot M = N^\dagger \cdot 1\!\!1 \otimes \sigma_b \cdot N \qquad \text{for all b}$$

$$\Rightarrow (NM^\dagger) \cdot 1\!\!1 \otimes \sigma_b = 1\!\!1 \otimes \sigma_b \cdot (NM^\dagger)$$

$$\Rightarrow NM^\dagger = U \otimes 1\!\!1 \Rightarrow N = U \otimes 1\!\!1 \cdot M$$

# Localizable NLM with 1 ebit




$$\boxed{M^\dagger \cdot \mathbb{1}\otimes\sigma_b \cdot M = P_b} \Leftrightarrow \mathbb{1}\otimes\sigma_b \cdot M = M \cdot P_b$$

Linear equation, “easy” to solve.
Uniqueness of solution guaranties that the solution is unitary.

<u>Theorem</u>

The only measurements localizable with 1 e-bit, but not with 0 e-bits are the

1. twisted basis measurement, and the
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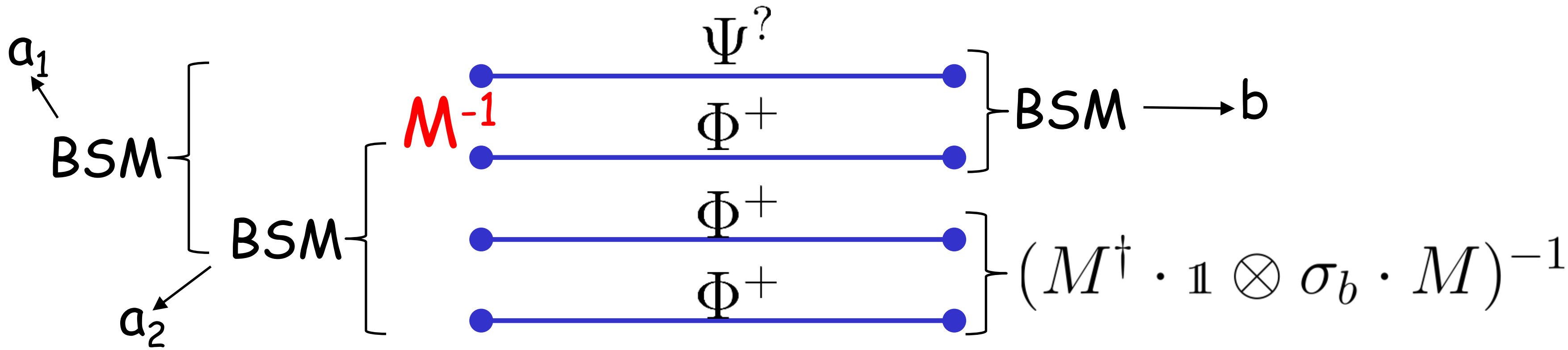
# Localizable NLM with 3 ebit



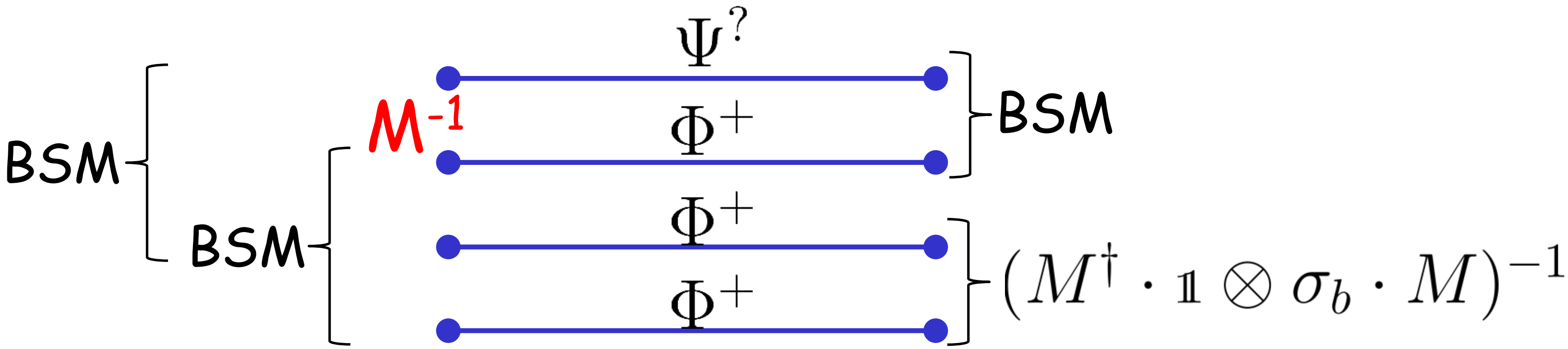



$$(M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M) \cdot \sigma_{a_1} \otimes \sigma_{a_2} \cdot (M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M) = P_b$$

= permutation+phases of 00,01,10,11

$$\Leftrightarrow M_b = M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M \quad \& \quad M_b^\dagger \cdot \sigma_{a_1} \otimes \sigma_{a_2} \cdot M_b = P_b$$

# Localizable NLM with 3 ebit




$$(M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M) \cdot \sigma_{a_1} \otimes \sigma_{a_2} \cdot (M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M)$$

= permutation+phases of 00,01,10,11

$\Rightarrow$ M = product measurement
M = twisted basis meas.
M = BSM
M = EJM (Elegant Joint Measurement)
and numerically "almost" nothing else. Can this be proven ?

# The Elegant Joint Measurement (EJM)

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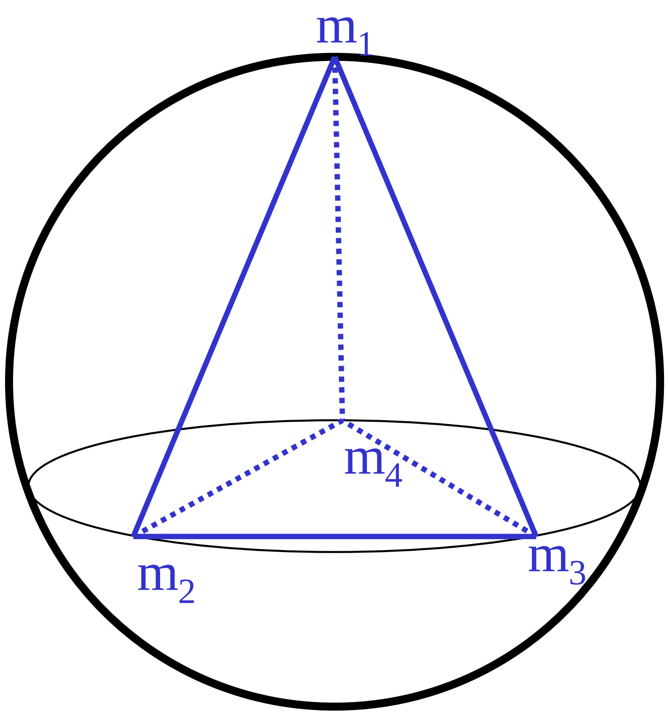



Look for 4 partially entangled and mutually orthogonal states with same degrees of entanglement and with partial states along the vertices of the tetrahedron.

$$\left|\Phi_j\right\rangle = c_0\left|\vec{m}_j, -\vec{m}_j\right\rangle + q_0\left|-\vec{m}_j, \vec{m}_j\right\rangle$$

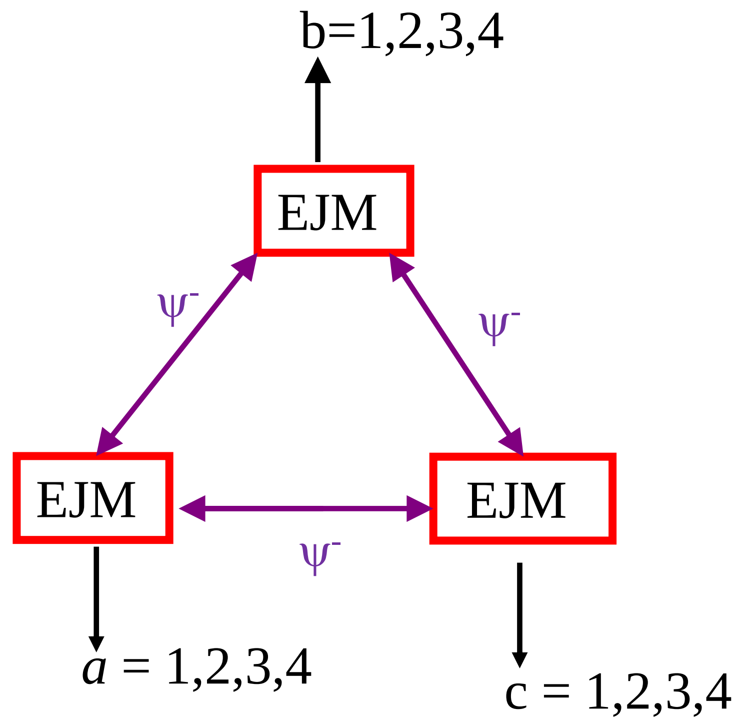
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}\left|\vec{m}_j, -\vec{m}_j\right\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}\left|-\vec{m}_j, \vec{m}_j\right\rangle$$

$\left\langle\Phi_j\middle|\Phi_i\right\rangle = \delta_{ji}$ & $c_0, c_1$ are real

$$Tr_B\left(\left|\Phi_j\right\rangle\left\langle\Phi_j\right|\right) = \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\vec{m}_j\right) \qquad Tr_A\left(\left|\Phi_j\right\rangle\left\langle\Phi_j\right|\right) = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\vec{m}_j\right)$$

NG, Entropy, «25 years of Q teleportation», 21,325 (2019)

# The Elegant Distribution




$$\Rightarrow p(a,b,c) = \begin{cases} \frac{25}{256} & \text{if} \quad a=b=c \\ \frac{1}{256} & \text{if} \quad a=b\neq c, \quad a=c \\ \frac{5}{256} & \text{if} \quad a\neq b\neq c\neq a \end{cases}$$

Challenge 4:
prove that the elegant distribution is local / non-local.

NG, Entropy, «25 years of Q teleportation», 21,325 (2019)

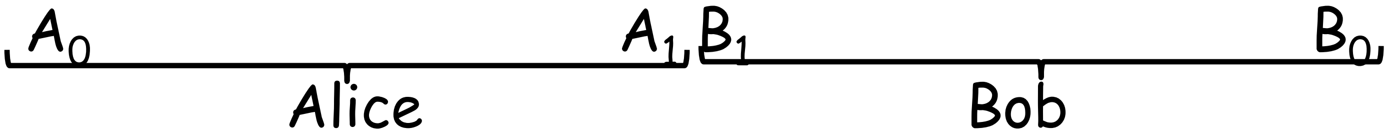
# Outlook



1. Higher dimensions? In particular EJM for qutrits.
2. More parties? In particular EJM for 3 qubits.
3. More e-bits: what classification of joint measurements does this approach provide?
4. Only perfectly overlapping systems can be jointly measured in an ideal (reproducible) way. What about partially overlapping systems? What is the best post-measured state (least disturbed) for given overlap?

# Model for partially overlapping qubits






Alice controls $A_0$ & $A_1$, Bob $B_0$ & $B_1$
Overlap at $A_1$-$B_1$

Logical qubits:

$$|\bar{0}\rangle_A = |0,0\rangle_{A_0,A_1}$$
$$|\bar{1}\rangle_A = c|1,0\rangle_{A_0,A_1} + s|0,1\rangle_{A_0,A_1}$$
$$|\bar{0}\rangle_B = |0,0\rangle_{B_1,B_0}$$
$$|\bar{1}\rangle_B = c|1,0\rangle_{B_1,B_0} + s|0,1\rangle_{B_1,B_0}$$

c=0 ⇒ perfect overlap        c=1 ⇒ fully distant

⇒ The twister basis can be measured ideally iff c=0, ie only if perfect overlap.

# Outlook

1. Higher dimensions? In particular EJM for qutrits.
2. More parties? In particular EJM for 3 qubits.
3. More e-bits: what classification of joint measurements does this approach provide?
4. Only perfectly overlapping systems can be jointly measured in an ideal (reproducible) way. What about partially overlapping systems? What is the best post-measured state (least disturbed) for given overlap?
5. Implications of the impossibility of ideal measurements for foundations of quantum theory.

# Implications for foundations ?

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Vol. 23 (1986) *REPORTS ON MATHEMATICAL PHYSICS* No. 3

**THE PROPERTY LATTICE OF SPATIALLY SEPARATED QUANTUM SYSTEMS**

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(*Received October 31, 1984 — Revised November 5, 1985*)

We consider a quantum system composed of two subsystems. Among the properties of this system we study the set of those that can be tested when the subsystems are spatially separated. We show that not all properties satisfy this criterion, but that there are enough such properties to characterize any pure state of the composed system.

# Implications for foundations ?



In the Jauch-Piron property-lattice approach to quantum physics (long before GPT) one considers lattices with certain natural axioms and then uses Piron's theorem to prove that the lattice is isomorphic to the closed subspaces of a linear space with superselection rules (that play the role of classical variables).

In my 1986 paper I proved that the lattice is atomic (has pure states). Today's results prove that it is also ortho-modular (each property has an orthogonal one). But it does not satisfy the covering law (projection of a pure state is not necessarily a pure state).

It is the covering law that bring in linearity:
Possibly, it is wrong to assume linearity for QFT ?!?!

# Conclusion

"Impossible" measurements are localizable,
hence possible but not ideal (not immediately reproducible).

The projection postulate does not always provide a good – not even a possible – description of quantum measurements.

The Bell state measurement is not a typical Q measurement.

Joint Q measurements could be ordered by the number of e-bits necessary to measure them non-locally (necessary to localize them).

The Elegant Joint Measurement is the simplest "typical" joint measurement with entangled eigenstates.

Coarse-grained measurements (eg partial BSM) can be obtained by merely adding the probabilities.

arXiv:2311.13644 Quantum 8, 1267 (2024) & arXiv:2307:06998 PR Research

# Importance of Measurements

- Physics is all about extracting information about

  **How Nature Does it**

- Extracting information = performing measurements.
- A physics theory must tell what is measurable and how – in principle – one should perform measurements.
- Hence the Quantum Measurement Problem is a serious physics problem:

  **Without a resolution, Q theory is not physics.**

- A resolution will lead to new physics.

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arXiv:1602.01497 & 1701.08300