Birational geometry and dynamics

Les Diablerets, 30 June — 5 July 2024.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
9:30-10:30	Favre	Favre	Favre	Favre	Favre
10:40-11:15	Coffee	Coffee	Coffee	Coffee	Coffee
11:15-12:15	Amerik	Krieger	Ramadas	Deserti	Bac-Dang
12:30-14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:30-15:30	Urech	Ji	Brandhorst	**Hike**	
15:30-16:30	Coffee	PS/Coffee	PS/Coffee		
16:30-17:30	Lamy	Lesieutre	Cantat		
17:30-18:30	Free	Free	**Lightning talks**		
19:30-21:00	Dinner	Dinner	Dinner	Dinner	

Mini-course

Charles Favre (CNRS, École Polytechnique) — *Topics on dynamical degrees*

Dynamical degrees are birational invariants associated to a dominant rational self-map of an algebraic variety. They play a crucial role in numerous problems in complex dynamics and in arithmetic dynamics. The goal of this course is to review their definitions and to cover some important recent results. In particular, we plan to describe the Banach spaces techniques developed by Dang-Favre to analyze the situation for polynomial maps of $\mathbb{C}^3$; and to explain the semi-continuity theorem of J. Xie.

Talks

Katia Amerik (HSE, Paris-Saclay) — *Potential density of rational points on the Hilbert cube of certain K3 surfaces*

Oguiso in 2009 has studied the following automorphism f of Hilb^2 of a K3 surface admitting two projective embeddings as a quartic: each of the two projective embeddings gives rise to an involution on Hilb^2 (the Beauville involution), and f is their product. At the time I made a remark that one could use f to prove the potential density of rational points on Hilb^2. In this talk, we will consider the product of two Beauville involutions on Hilb^3 of a surface admitting two projective embeddings of degree 6, and use it to prove the potential density. This is a joint work in progress with my student Mikhail Lozhkin.

Simon Brandhorst (Universität des Saarlandes) — *K3 surfaces of zero entropy*

Automorphisms of K3 surfaces come in 3 flavours: 1) The orbit of every point is finite. 2) There exists a point with an infinite orbit, but no orbit is Zariski dense. 3) There is a Zariski dense orbit. In the first and second case the automorphism has zero topological entropy while in the last case it is of positive entropy. We say that a surface has zero entropy if every of its automorphisms has zero entropy. In this talk we classify K3 surfaces of zero entropy yet with infinite automorphism group, equivalently, which have a unique elliptic fibration whose Jacobian has infinite Mordell-Weil group. Time permitting, I will report on work in progress with Matthias Zach to find equations for the automorphism of minimal entropy on a complex Enriques surface whose existence was proven by Oguiso and Yu. Joint work with Giacomo Mezzedimi.

Serge Cantat (CNRS, Université de Rennes) — *Dynamics on Markov surfaces*

Markov surfaces are certain complex cubic surfaces in the affine 3-space. The group of automorphisms of a Markov surface S is infinite countable: I shall discuss the dynamics of this group on S. This is based on a joint work with Christophe Dupont and Florestan Martin-Baillon.

Nguyen-Bac Dang (Paris-Saclay) — *Example of rational mappings and spectrum of Laplacian on graphs*

In this talk, I will present some very specific rational maps related to some problems in analysis on groups and graphs. For a particular sequences of finite regular graphs, Grigorchuk, Bartholdi, Zuk, Sunic in the 80's considered the sequence of characteristic polynomials of their associated adjacency matrices. The question they raised is the distribution of zeros of these polynomials as the size of the graphs grow. For specific sequences, they showed that these characteristic polynomials satisfy a recursive formula involving a rational map, which can be 2/3/5 dimensional. The computation of the zeros can then be reduced to a geometric question involving pulling back a specific hypersurface and intersecting with a special line. I will try to present a few instances, some for which these questions are solved and others which remain open.

Julie Deserti (Université d'Orléans) — *About the dynamical number of base points of birational maps*
A birational map of a projective smooth surface is regularisable if it is birationally conjugate to an automorphism. The dynamical number of base points is an invariant of conjugation associated to a birational map. This invariant allows to determine whether a birational map is regularisable but not only. I will give some of its properties and applications.

Lena Ji (University of Michigan) — *Symmetries of Fano varieties*
Prokhorov and Shramov proved that the boundedness of Fano varieties (which Birkar later proved) implies the uniform Jordan property for automorphism groups of complex Fano varieties of fixed dimension. In particular, in each dimension n, there is an upper bound on the size of semisimple groups (i.e. those with no nontrivial normal abelian subgroups) acting on n-dimensional complex Fano varieties. In this talk, we investigate the action by a particular semisimple group: the symmetric group. This work is joint with Louis Esser and Joaquín Moraga.

Holly Krieger (University of Cambridge) — *A transcendental birational dynamical degree*
In the study of rational self-maps of algebraic varieties, we wish to understand the integer sequence formed by the degrees of the iterates of the map. This sequence has growth rate measured by the dynamical degree, a birational invariant which controls the topological, arithmetic, and algebraic complexity of the system. I will discuss the surprising construction, joint with Bell, Diller, and Jonsson, of a transcendental dynamical degree for a birational map of projective 3-space, and how our work fits into a more general phenomenon of power series taking transcendental values at algebraic inputs.

Stéphane Lamy (Institut de Mathématique de Toulouse) — *Rational surfaces over perfect fields*
In this survey talk I will advertise the concept of "rank r fibration" that allows to put a natural partial order and a structure of simplicial complex on the set of Del Pezzo surfaces and conic bundles birational to a given surface X, defined over a perfect field. We will see how this relates to the Sarkisov program, which allows to describe (in principle) all birational maps between such surfaces, and so in particular also the Cremona group (taking $X = \mathbb{P}^2$, say over the field of rational numbers, or over a finite field). I will also explain how this abstract simplicial complex can be realized via a polyhedral chamber decomposition in the space of divisors on a surface, which yields some strong connectedness properties for the complex.

John Lesieutre (Penn State) — *A quaternionic Cremona transformation?*
I will describe the construction of certain de Jonquieres-type maps on the quaternionic plane $\mathbb{HP}^2$ using elementary projective geometry. Then I will discuss the geometry of the induced map on the corresponding Severi variety $\mathrm{Gr}(2, 6)$ in more familiar algebro-geometric terms, and introduce some related maps on a quotient of this space.

Rohini Ramadas (Warwick Mathematics Institute) — *Dynamics of Hurwitz correspondences on moduli space*
The moduli space $M_{0,n}$ is a smooth affine variety parametrising point-configurations on $\mathbb{P}^1$. I will introduce $M_{0,n}$ and some compactifications, and then give an overview of Hurwitz correspondences, which are a class of dynamical systems that act on $M_{0,n}$.

Christian Urech (ETH) — *Finitely generated subgroups of algebraic elements of plane Cremona groups*
To an algebraic surface S we associate its group of birational transformations $\mathrm{Bir}(S)$. An element in $\mathrm{Bir}(S)$ is called algebraic if it is contained in an algebraic subgroup of $\mathrm{Bir}(S)$. In this talk, I will explain, why a finitely generated subgroup of $\mathrm{Bir}(S)$ that consists only of algebraic elements is itself contained in an algebraic subgroup of $\mathrm{Bir}(S)$. This answers a question of Charles Favre. I will explain why this technical result is interesting and use it to describe some dynamical properties of finitely generated subgroups of $\mathrm{Bir}(S)$. This is joint work with Anne Lonjou and Piotr Przytycki.

Lightning talks:

Marc Abboud (UniNE): Rigidity of periodic points for loxodromic automorphisms of affine surfaces
Chase Bender (University of Notre Dame): L^p-regularity of Bergman projection of monomial quotients
Pascal Fong (Paris-Saclay): Strongly isotrivial elliptic surfaces
Giacomo Mezzedimi (University of Bonn): Enriques surfaces of zero entropy
Qitong Jiang (Penn State): Degeneration of a family of surface birational maps
Sokratis Zikas (Université de Poitiers): On Gizatullin's problem
Richard Birkett (University of Illinois): A universal property for algebraic stabilisation
Maxim Amirkhanov (UniBAS): Reducing finite subgroups of Bir(Severi–Brauer) to automorphisms