

Birational geometry and dynamics



Sunday 30 June 2024 - Friday 5 July 2024

SRS

Scientific Programme

Mini-course

Topics on dynamical degrees by *Charles Favre* (CNRS, École Polytechnique)

Dynamical degrees are birational invariants associated to a dominant rational self-map of an algebraic variety. They play a crucial role in numerous problems in complex dynamics and in arithmetic dynamics. The goal of this course is to review their definitions and to cover some important recent results. In particular, we plan to describe the Banach spaces techniques developed by Dang-Favre to analyze the situation for polynomial maps of \mathbb{C}^3 ; and to explain the semi-continuity theorem of J. Xie.

Research talks

Potential density of rational points on the Hilbert cube of certain K3 surfaces by *Ekaterina Amerik* (HSE and Paris-Saclay)

Oguiso in 2009 has studied the following automorphism f of Hilb^2 of a K3 surface admitting two projective embeddings as a quartic: each of the two projective embeddings gives rise to an involution on Hilb^2 (the Beauville involution), and f is their product. At the time I made a remark that one could use f to prove the potential density of rational points on Hilb^2 . In this talk, we will consider the product of two Beauville involutions on Hilb^3 of a surface admitting two projective embeddings of degree 6, and use it to prove the potential density. This is a joint work in progress with my student Mikhail Lozhkin.

K3 surfaces of zero entropy by *Simon Brandhorst* (Universität des Saarlandes)

Automorphisms of K3 surfaces come in 3 flavours:

- 1) The orbit of every point is finite.
- 2) There exists a point with an infinite orbit, but no orbit is Zariski dense.
- 3) There is a Zariski dense orbit.

In the first and second case the automorphism has zero topological entropy while in the last case it is of positive entropy. We say that a surface has zero entropy if every of its automorphisms has zero entropy. In this talk we classify K3 surfaces of zero entropy yet with infinite automorphism group, equivalently, which have a unique elliptic fibration whose Jacobian has infinite Mordell-Weil group. Time permitting, I will report on work in progress with Matthias Zach to find equations for the automorphism of minimal entropy on a complex Enriques surface whose existence was proven by Oguiso and Yu.

(Joint work with Giacomo Mezzedimi)

Dynamics on Markov surfaces by *Serge Cantat* (CNRS, Université de Rennes)

Markov surfaces are certain complex cubic surfaces in the affine 3-space. The group of automorphisms of a Markov surface S is infinite countable: I shall discuss the dynamics of this group on S . (This is based on a joint work with Christophe Dupont and Florestan Martin-Baillon)

Example of rational mappings and spectrum of Laplacian on graphs by *Nguyen-Bac Dang* (Paris-Saclay)

In this talk, I will present some very specific rational maps related to some problems in analysis on groups and graphs. For a particular sequences of finite regular graphs, Grigorchuk, Bartholdi, Zuk, Sunic managed in the 80's considered the sequence of characteristic polynomials of their associated adjacency matrices. The question they raised is the distribution of zeros of these polynomials as the size of the graphs grow. For specific sequences, they showed that these characteristic polynomials satisfy a recursive formula involving a rational map, which can be $2/3/5$ dimensional. The computation of the zeros can then be reduced to a geometric question involving pulling back a specific hypersurface and intersecting with a special line. I will try to present a few instances, some for which these questions are solved and others which remain open.

About the dynamical number of base points of birational maps by *Julie Deserti* (Université d'Orléans)

A birational map of a projective smooth surface is regularisable if it is birationally conjugate to an automorphism. The dynamical number of base points is an invariant of conjugation associated to a birational map. This invariant allows to determine whether a birational map is regularisable but not only. I will give some of its properties and applications.

A transcendental birational dynamical degree by *Holly Krieger* (University of Cambridge)

In the study of rational self-maps of algebraic varieties, we wish to understand the integer sequence formed by the degrees of the iterates of the map. This sequence has growth rate measured by the dynamical degree, a birational invariant which controls the topological, arithmetic, and algebraic complexity of the system. I will discuss the surprising construction, joint with Bell, Diller, and Jonsson, of a transcendental dynamical degree for a birational map of projective 3-space, and how our work fits into a more general phenomenon of power series taking transcendental values at algebraic inputs.

Rational surfaces over perfect fields by *Stéphane Lamy* (Institut de Mathématique de Toulouse)

In this survey talk I will advertise the concept of "rank r fibration" that allows to put a natural partial order and a structure of simplicial complex on the set of Del Pezzo surfaces and conic bundles birational to a given surface X , defined over a perfect field. We will see how this relates to the Sarkisov program, which allows to describe (in principle) all birational maps between such surfaces, and so in particular also the Cremona group (taking $X = \mathbb{P}^2$, say over the field \mathbb{Q} of rational numbers, or over a finite field). I will also explain how this abstract simplicial complex can be realized via a polyhedral chamber decomposition in the space of divisors on a surface, which yields some strong connectedness properties for the complex.

A quaternionic Cremona transformation? by *John Lesieutre* (Penn State)

I will describe the construction of certain de Jonquieres-type maps on the quaternionic plane $\mathbb{H}\mathbb{P}^2$ using elementary projective geometry. Then I will discuss the geometry of the induced map on the corresponding Severi variety $\mathrm{Gr}(2,6)$ in more familiar algebro-geometric terms, and introduce some related maps on a quotient of this space.

Symmetries of Fano varieties by *Lena Ji* (University of Michigan)

Prokhorov and Shramov proved that the boundedness of Fano varieties (which Birkar later proved) implies the uniform Jordan property for automorphism groups of complex Fano varieties of fixed dimension. In particular, in each dimension n , there is an upper bound on the size of semisimple groups (i.e. those with no nontrivial normal abelian subgroups) acting on n -dimensional complex Fano varieties. In this talk, we investigate the action by a particular semisimple group: the symmetric group. This work is joint with Louis Esser and Joaquin Moraga.

Dynamics of Hurwitz correspondences on moduli space by *Rohini Ramadas* (University of Warwick)

The moduli space $M_{0,n}$ is a smooth affine variety parametrising point-configurations on \mathbb{P}^1 . I will introduce $M_{0,n}$ and some compactifications, and then give an overview of Hurwitz correspondences, which are a class of dynamical systems that act on $M_{0,n}$.

Finitely generated subgroups of algebraic elements of plane Cremona groups by *Christian Urech* (ETH)

To an algebraic surface S we associate its group of birational transformations $\text{Bir}(S)$. An element in $\text{Bir}(S)$ is called algebraic if it is contained in an algebraic subgroup of $\text{Bir}(S)$. In this talk, I will explain, why a finitely generated subgroup of $\text{Bir}(S)$ that consists only of algebraic elements is itself contained in an algebraic subgroup of $\text{Bir}(S)$. This answers a question of Charles Favre. I will explain why this technical result is interesting and use it to describe some dynamical properties of finitely generated subgroups of $\text{Bir}(S)$. This is joint work with Anne Lonjou and Piotr Przytycki.

Lightning talks

Marc Abboud: Rigidity of periodic points for loxodromic automorphisms of affine surfaces

Chase Bender: L^p -regularity of Bergman projection of monomial quotients

Pascal Fong: Strongly isotrivial elliptic surfaces

Giacomo Mezzedimi: Enriques surfaces of zero entropy

Qitong Jiang: Degeneration of a family of surface birational maps

Sokratis Zikas: On Gizatullin's problem

Richard Birkett: A universal property for algebraic stabilisation

Maxim Amirkhanov: Reducing finite subgroups of $\text{Bir}(\text{Severi-Brauer})$ to automorphisms

